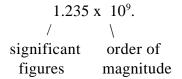# Scientific Notation and the Size of Things

1 Scientific Notation

Physics is a quantitative science. Every observation, every experiment, every theory, in the end must produce numbers. Every theory must produce numbers that actually occur in observations or experiments. New observations demand new theories to explain them. A new theory drives new experiments to test the new theory - how close do the numbers produced by the theory match those of the experiment or observation? There is a constant drive for ever increasingly refined measurement tools and methods - increased accuracy and increased precision. Since numbers are so important we must have the most effective and efficient ways to represent numbers. That's called scientific notation.

Scientific notation separates a numerical quantity into two parts - significant figures and order of magnitude. The number 1,234,872,492,091, or to four figures 1.235 billion is represented as

$$1.235 \times 10^9.$$

$$\underset{\substack{\text{significant} \\ \text{figures}}}{\diagup} \qquad \underset{\substack{\text{order of} \\ \text{magnitude}}}{\diagdown}$$

The order of magnitude is powers of ten. It tells us how big the units. The significant figures is a digital part usually with one figure to the left of the decimal point. It tells us how many of those units. Sometimes it is written in the exponential form as

$$1.235 \ E+09 \ ,$$

where the "E+09" says that the order of magnitude is +9. Notice that our language carries this notation when we say 1.235 billion. In the long way of "1 billion, 2hundred and 35 million" , the 1 billion leads.

Let's look at an example. Suppose you found out that a body from outer space was coming into the earth at 90,000 mph and was to crash where you are sitting or standing right now. Compare your concerns and reactions when you are told that its mass is $9.3 \times 10^{+9}$ kg; its mass is 9.3×10^{-9} kg. Which number catches your attention? At first do you really care about the 9.3 here?

The scientific notation makes computations easier because the significant figures and the orders of magnitude are handled separately. The handling of the significant figures depends on the nature of the numbers. There are exact numbers and measured numbers. An exact number has arbitrary accuracy. On the other hand, the accuracy of measured numbers is limited by the precision of the measuring instruments and the accuracy of the measuring techniques. So, the

number of significant figured retained in calculations is different for the two, in fact opposite. For exact numbers the maximum number of significant figures are retained. But, for measured numbers, the smallest number of significant figures are retained. We will illustrate this in the examples below.

For addition and subtraction we deal only with the significant figure after all terms have been converted to same order of magnitude. Then readjust order of magnitude to put the significant figure in standard form.

$$1.235\text{x}10^9 + 2.831\text{x}10^7 = 1.235\text{x}10^9 + (2.831/100)\text{x}(10^7 \text{ x}100)$$
<div align="center">divide by   multiply by
100 to get order of magnitude the same</div>

for exact numbers
$$= 1.235\text{x}10^9 + 0.02831\text{x}10^9 \ = 1.26331\text{x}10^9 ,$$
<div align="center">keep all.</div>

But, for measured numbers
$$= 1.235\text{x}10^9 + 0.02831\text{x}10^9 \ = 1.263\text{x}10^9 ,$$
<div align="right">keep only the smallest number
of significant figures.</div>

1.235
0.028<u>31 lost, dropped</u>

1.263   ~~31 gone!~~

Or, alternatively,

$$1.235\text{x}10^9 + 2.831\text{x}10^7 = 123.5\text{x}10^7 + 2.831\text{x}10^7 = 126.3\text{x}10^7 = 1.263\text{x}10^9$$

For multiplication and division we operate on the two parts separately, then make order of magnitude adjustments after all is done. Operating with the order of magnitude is particularly simple - it follows the laws of exponents ,

<div align="center">
m- times n-times   m- times
$a^m * a^n = (a...a)*(a...a) = a^{m+n}$ and $a^m/a^n = (a.....a)/(a...a) = a^{m-n}$
<div align="right">n-times</div>
</div>

For examples,

1. $1.235\text{x}10^9 * 8.32\text{x}10^4 = 10.275\text{x}(10^{9+4}) = 1.0275\text{x}10\text{x}(10^{13}) = 1.0275\text{x}10^{14}$
<div align="center">for exact numbers.</div>

<div align="center">$1.03\text{x}10^{14},$ for measured numbers.</div>

2. $1.235\text{x}10^9/8.32\text{x}10^4 = (1.235/8.32) \text{ x } (10^{9-4}) = 0.148375\text{x}(10^5) = 1.48375\text{x}10^{-1}\text{x}10^5$

<div align="center">$= 1.48375\text{x}10^4$ for exact numbers.</div>

$$= 1.48 \times 10^4 \qquad \text{for measured numbers.}$$

Examples of exact numbers

Number like ½ , 1/3 , 100, e, π are examples of exact numbers. Even though the values of e, π and other such numbers are usually listed with a limited number of digits, their values can be calculated to as many digits as you want. Both e and π have been calculated millions of digits.

Examples of measured numbers

All physical constants are measured numbers. In the United States, it is the mission of the National Bureau of Standard to maintain and improve the accuracy of all of the numbers. They are always fighting to get values by one more significant figure. When the 10 significant figure is secured, they go for the eleventh, and so on.

Rounding

Rounding is best explained by example. Given the number

3.4652754	
3.465275	rounded to 7 digits
3.46528	rounded to 6 digits
3.4653	rounded to 5 digits
3.465	rounded to 4 digits
3.47	rounded to 3 digits
3.5	rounded to 2 digits
4.	rounded to 1 digit.

5 or larger go up by1. Less that 5 drop down by 1.

3.2 Power of 10
The BIG, the LARGE

unit name (prefix)	unit value	power of 10	Exponential
units	$1 = 10/10$	10^0	1.0E+00
tens (deca-)	$10 = 1*10$	10^1	1.0E+01
hundreds (hecto-)	$100 = 1*10*10$	10^2	1.0E+02
thousands(kilo-) ten	$1,000 = 1*10*10*10$	10^3	1.0E+03
thousands	$10,000 = 1*10*10*10*10$	10^4	1.0E+04
hundred thousands	$100,000 = 1*10*10*10*10*10$	10^5	1.0E+05
		5, 0's	|<--10, 5 times >|		
millions (mega-)	$1,000,000 = 1*10*10*10*10*10*10$	10^6	1.0E+06
		6, 0's	|<---10, 6 times ---->|		
ten millions	$10,000,000 = 1*10*10*10*10*10*10*10$	10^7	1.0E+07
		7, 0's	|<-----10, 7 times ---->|		
hundred millions	$100,000,000 = 1*10*10*.....*10*10*10$	10^8	1.0E+08
		8, 0's	|<------ 10, 8 times---- --->|		
billions (giga-)	$1,000,000,000 = 1*10*10*.............*10*10$	10^9	1.0E+09
		nine 0's	|<----- 10, 9 times ----->|		
ten billions	$10,000,000,000 = 1*10*.............*10$	10^{10}	1.0E+10
		ten 0's	|<--- 10 times -->|		
hundred billions	$100,000,000,000 = 1*10*.............*10$	10^{11}	1.0E+11
		eleven 0's	|<--- 11 times -->|		
trillions (tera-)	$1,000,000,000,000 = 1*10*.............*10$	10^{12}	1.0E+12
		twelve 0's	|<--- 12 times -->|		
ten trillions	$10,000,000,000,000 = 1*10*.............*10$	10^{13}	1.0E+13
		thirteen 0's	|<--- 13 times -->|		
hundred trillion	$100,000,000,000,000=10*10*10*.......10*10*10$	10^{14}	1.0E+14
		fourteen 0's	|<---10, 14 times -->|		
quadrillions (peta-)	$1,000,000,000,000,000 = 10*10*............*10*10$	10^{15}	1.0E+15
		fifteen 0's	|<---10, 15 times -->|		

The SMALL, the TINY of IT ALL - the LAND of "THS"

unit name (prefix)	unit value	power of 10	Exponential
tenths (deci-)	$0.1=1/10$	10^{-1}	1.0E-01
hundredths (centi-)	$0.01=1/100$	10^{-2}	1.0E-02
thousandths(milli-) ten	$0.001=1/1,000$	10^{-3}	1.0E-03
thousandths	$0.0001=1/10,000$	10^{-4}	1.0E-04
hundred thousandths	$0.00001= 1/100,000$	10^{-5}	1.0E-05
		4, 0's			
millionths (micro-)	$0.000001= 1/1,000,000$	10^{-6}	1.0E-06
		5, 0's			
ten millionths	$0.0000001= 1/10,000,000$	10^{-7}	1.0E-07
		6, 0's			
hundred millionths	$0.00000001= 1/100,000,000$	10^{-8}	1.0E-08
		7, 0's			
billionths (nano-)	$0.000000001=1/1,000,000,000$	10^{-9}	1.0E-09
		8, 0's			
ten billionths	$0.0000000001=1/10,000,000,000$	10^{-10}	1.0E-10
		nine 0's			
hundred billionths	$0.00000000001=1/100,000,000,000$	10^{-11}	1.0E-11
		ten 0's			
trillionths (pico-)	$0.000000000001=1/1,000,000,000,000$	10^{-12}	1.0E-12
		eleven 0's			
ten trillionths	$0.0000000000001= 1/10,000,000,000,000$	10^{-13}	1.0E-13
		twelve 0's			
hundred trillionths	$0.00000000000001=1/100,000,000,000$	10^{-14}	1.0E-14
		thirteen 0's			
quadrillionths (femto-)	$0.000000000000001=1/1,000,000,000,000,000$	10^{-15}	1.0E-15
		fourteen 0's			