

## Mid-Term Test

28 March 2015

**Time allowed:** 1 hour 45 minutes

**Matriculation No:**

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### Instructions (please read carefully):

1. Write down your matriculation number on the **question paper**. **DO NOT WRITE YOUR NAME ON THE QUESTION SET!**
2. This is **an open-sheet test**. You are allowed to bring one A4 sheet of notes (written on both sides).
3. This paper comprises **FIVE (5) questions** and **SIXTEEN (16) pages**. The time allowed for solving this test is **1 hour 45 minutes**.
4. The maximum score of this test is **100 marks**. The weight of each question is given in square brackets beside the question number.
5. All questions must be answered correctly for the maximum score to be attained.
6. All questions must be answered in the space provided in the answer sheet; no extra sheets will be accepted as answers.
7. The back-sides of the sheets and the pages marked “scratch paper” in the question set may be used as scratch paper.
8. You are allowed to un-staple the sheets while you solve the questions. Please make sure you staple them back in the right order at the end of the test.
9. You are allowed to use pencils, ball-pens or fountain pens, as you like (no red color, please).

## GOOD LUCK!

Question	Marks	Remark
Q1		
Q2		
Q3		
Q4		
Q5		
Total		

**Question 1: Python Expressions [30 marks]**

There are several parts to this problem. Answer each part **independently and separately**. In each part, one or more Python expressions are entered into the interpreter (Python shell). Determine the response printed by the interpreter for the final expression entered. If the interpreter produces an error message, or enters an infinite loop, explain why.

**A.**

```
y = 97%4
if y%3:
    print("Hello")
else:
    print("There!")
```

[5 marks]

**B.**

```
x = 123
y = 1000
def foo(x,y):
    y = 100
    print(x//y)
foo(y,x)
```

[5 marks]

**C.**

```
def f1(x,y,z):
    return x(y,z)
def f2(x,y):
    return y(x)
print(f1(lambda x,y: x+1, 2, lambda x: x**2))
```

[5 marks]

**D.**

```
def count():
    for i in range(10):
        total = i
        for j in range(10):
            total += j
            if j%2==0:
                continue
            elif j+i>15:
                return total
print(count())
```

[5 marks]

**E.** `x = ("a", "b", "c")`  
`y = ("a")`  
`z = (x, ("a", "c"), ("a", "b"), ("b", "c"))`  
`if x in z:`  
    `if y in x:`  
        `print("Good")`  
    `elif y in z:`  
        `print("Bad")`  
`print("Done")`

[5 marks]

**F.** `def p():`  
    `return lambda: print("Confused?")`  
`if not p:`  
    `p()`  
`elif p:`  
    `p()()`  
`else:`  
    `print(not p()())`

[5 marks]

**Question 2: Game of Mystery Boxes [20 marks]**

In the Game of Mystery Boxes, contestants are given a sequence of boxes. Each box contains either a positive or negative cash amount. For every box the contestant opens, the amount of cash in the box will be added (or subtracted if the amount is negative) to the contestant's winnings. The contestant can decide at any time whether to continue to open the next box, or walk away with the current winnings. The contestant starts with \$0 and the game ends if the current winnings goes below 0 and the contestant will leave the game with \$0 in winnings.

**A.** Write a function `winnings` that takes as input a tuple of integers which represent the cash amount of the sequence of boxes that a contestant can open and returns the final winnings if all the boxes were to be opened (until the game ends). You may assume the contestant chooses to open boxes until there are no more boxes left, or he loses with \$0. [6 marks]

```
def winnings(seq):
```

**B.** What is the order of growth in terms of time and space for the function you wrote in Part (A) in terms of  $n$  where  $n$  is the length of `seq`. Explain your answer. [4 marks]

Time:

Space:

**C.** Hindsight is 20/20. Write a function `max_winnings(seq)` that takes as input a tuple of integers `seq` which represent the cash amount of the boxes in sequence, and return the maximum possible winnings, i.e. we assume that the contestant will decide whether to open the boxes based on the optimal strategy. [6 marks]

```
def max_winnings(seq):
```

**D.** Now suppose instead there are an infinite number of boxes, and that there is a function `next_box()` that returns the cash value of the next box that is opened. Assume that the contestant will keep opening boxes until his winnings fall below zero and the game ends. Write a function `max_num` that returns the number of boxes that a contestant can open before the game ends. [4 marks]

```
def max_num():
```

**Question 3: Higher-Order Functions [24 marks]**

Consider the following higher-order function that we call `choosy`:

```
def choosy(select, a, b, op, n, base):
    if n == 1:
        return base
    elif select(n):
        return op(a(n), choosy(select, a, b, op, n-1, base))
    else:
        return op(b(n), choosy(select, a, b, op, n-1, base))
```

**A. [Warm-up]** Suppose the function `sum_integers(n)` computes the sum of integers from 1 to  $n$  (inclusive) and `sum_integers(n)` is defined as follows:

```
def sum_integers(n):
    return choosy(<T1>,
                  <T2>,
                  <T3>,
                  <T4>,
                  <T5>,
                  <T6>)
```

Please provide possible implementations for T1, T2, T3, T4, T5 and T6.

[8 marks]

<T1>:  
[2 marks]

--

<T2>:  
[1 mark]

--

<T3>:  
[1 mark]

--

<T4>:  
[2 marks]

--

<T5>:  
[1 mark]

--

<T6>:  
[1 mark]

--

**B.** Suppose the function `sum_even_integers(n)` computes the sum of even integers from 1 to  $n$  (inclusive) and `sum_even_integers(n)` is defined as follows:

```
def sum_even_integers(n):
    return choosy(<T7>,
                  <T8>,
                  <T9>,
                  <T10>,
                  <T11>,
                  <T12>)
```

Please provide possible implementations for T7, T8, T9, T10, T11 and T12.

[8 marks]

<T7>:  
[2 marks]

--

<T8>:  
[1 mark]

--

<T9>:  
[1 mark]

--

<T10>:  
[2 marks]

--

<T11>:  
[1 mark]

--

<T12>:  
[1 mark]

--



**C.** Suppose the function `is_prime(n)` returns `True` if the number  $n$  is prime, for  $n \geq 2$ , and `False` otherwise, and `is_prime` is defined as follows:

```
def is_prime(n):
    def helper(x):
        return choosy(<T13>,
                      <T14>,
                      <T15>,
                      <T16>,
                      <T17>,
                      <T18>)
    return helper(n-1)
```

Please provide possible implementations for T13, T14, T15, T16, T17 and T18. [8 marks]

Hint: The boolean operator `and` will return `True` if and only if both inputs are `True`, and `False` otherwise.

<T13>:  
[2 marks]

--

<T14>:  
[1 mark]

--

<T15>:  
[1 mark]

--

<T16>:  
[2 marks]

--

<T17>:  
[1 mark]

--

<T18>:  
[1 mark]

--

**Question 4: Tic-Tac-Toe [23 marks]**

We are all familiar with this childhood game called Tic-Tac-Toe, which is played on a  $3 \times 3$  grid and two players take turns to put in either a symbol “X” or “O” in one of the squares. The first player to place 3 of his symbols in a row either vertically, horizontally, or diagonally, wins. In this problem, you will model this game.

The following is a brief description of the problem:

- `make_state()` will create an initially empty state object for the game.
- `get_square(s, x, y)` will return the symbol in the game state object `s` at the grid location  $(x, y)$ , where  $0 \leq x, y \leq 2$ . If the grid location is empty, `get_square(s, x, y)` will return `None`.
- `move(s, x, y, player)` will return a new state object after a player (where `player` is either “X” or “O”) makes a move at the grid location  $(x, y)$  for a game state object `s`. If the move is invalid, `False` is returned instead.

**A.** Decide on an implementation for the Tic-Tac-Toe game state object and implement `make_state`. Describe how the state is stored in your implementation and the conditions that a valid game state object in your implementation must satisfy. [4 marks]

**Note:** You are limited to using tuples for this question, i.e. you cannot use lists and other Python data structures.

```
def make_state():
```

**B.** Implement a function `check_rep(s)`, that will return `True` if `s` is a valid game state object for the representation you described in Part(A) above, or `False` otherwise. You can assume that `s` is a tuple. [4 marks]

```
def check_rep(s):
```

**C.** Implement a function `get_square(s, x, y)`, that will return the symbol in the square at grid position  $(x, y)$ ,  $0 \leq x, y \leq 2$  for the game state object `s`. If the square is empty, return `None`. [3 marks]

```
def get_square(s, x, y):
```

**D.** Implement a function `move(s, x, y, player)`, that will return a new state object after a player (where player is either "X" or "O") makes a move at the grid location  $(x, y)$  for a game state object `s`. If the move is invalid, `False` is returned instead. [6 marks]

```
def move(s, x, y, player):
```

**E.** Implement a function `check_win(s)`, that will return the symbol for the winning player (where player is either "X" or "O" for a game state object `s`, or `False` if neither player has won. [6 marks]

```
def check_win(s):
```

**Question 5: What is the purpose of education to you? [3 marks]**

*Finally, the end product [of education] is the good citizen, the man or woman who has had the maximum of nurturing of his or her natural talent to fit him or her to earn his or her livelihood in our society and who can bring up a family and care for them.*  
– Mr Lee Kuan Yew

Do you agree? Explain. (Basically, tell us: why are you coming to university?)

## Appendix

The following are some functions that were introduced in class. For your reference, they are reproduced here.

```
def sum(term, a, next, b):
    if (a > b):
        return 0
    else:
        return term(a) + sum(term, next(a), next, b)

def product(term, a, next, b):
    if a > b:
        return 1
    else:
        return term(a) * product(term, next(a), next, b)

def fold(op, f, n):
    if n==0:
        return f(0)
    else:
        return op(f(n), fold(op, f, n-1))

def enumerate_interval(low, high):
    return tuple(range(low,high+1))

def filter(pred, seq):
    if seq == ():
        return ()
    elif pred(seq[0]):
        return (seq[0],) + filter(pred, seq[1:])
    else:
        return filter(pred, seq[1:])

def accumulate(fn, initial, seq):
    if seq == ():
        return initial
    else:
        return fn(seq[0], accumulate(fn, initial, seq[1:]))
```

Scratch Paper

— END OF PAPER —