# CS1010S Programming Methodology

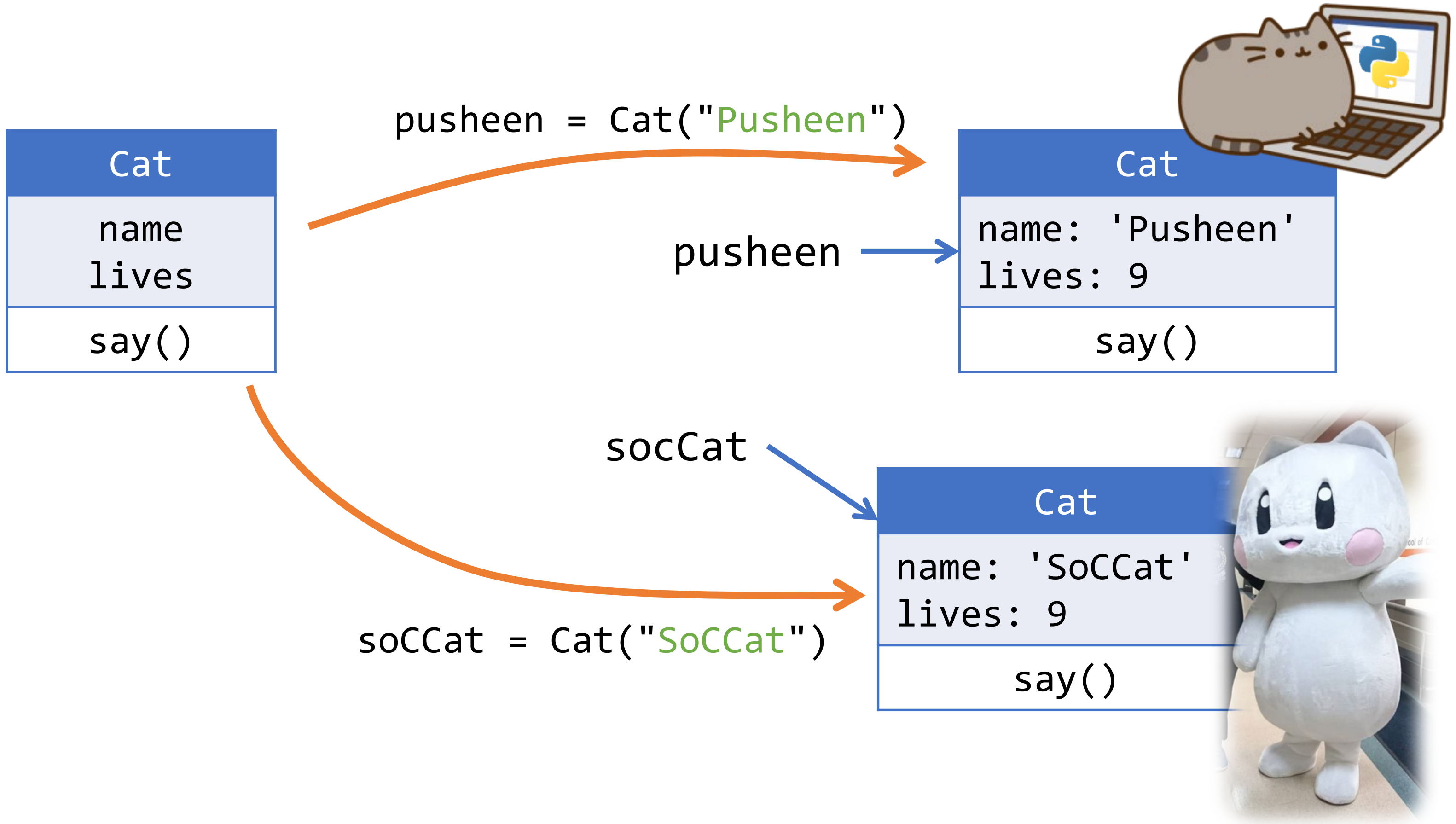
## Lecture 10

## Memoization, Dynamic Programming & Exception Handling

28 Oct 2020

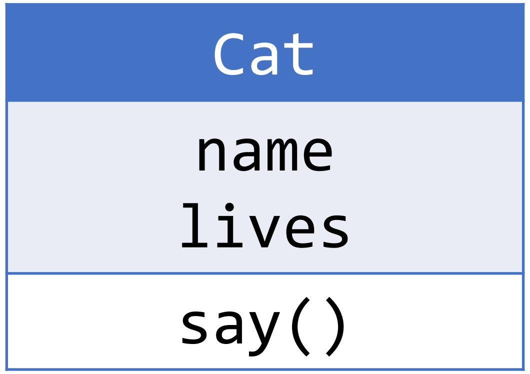
# Recap: OOP Concepts

- Class vs Instance
  - Class is a template, which is used to create instances



# Recap: Class

- Contains
  1. Fields/Properties
  2. Methods/Functions

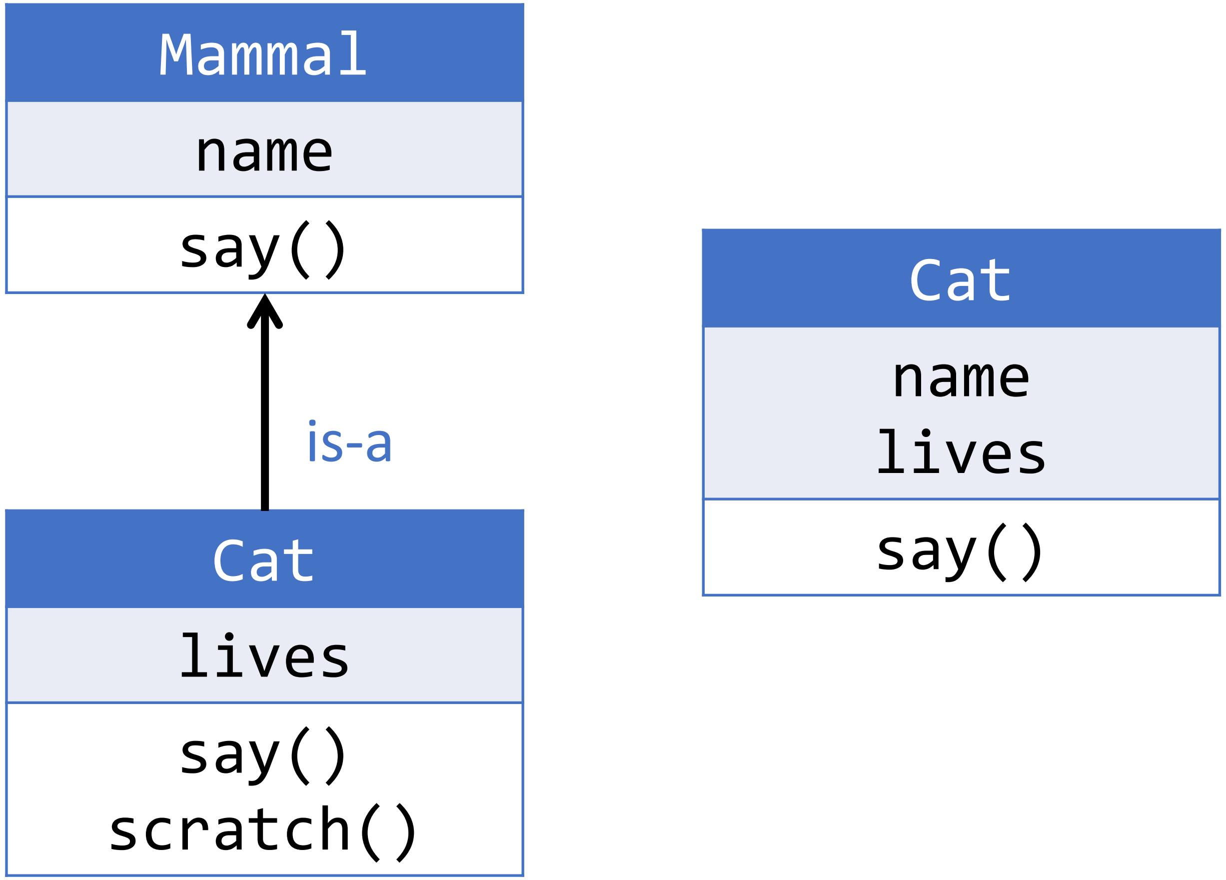



```python
class Cat:
    def __init__(self, name):
        self.name = name
        self.lives = 9

    def say(self):
        print("meow")

pusheen = Cat("Pusheen")
pusheen.say()
```

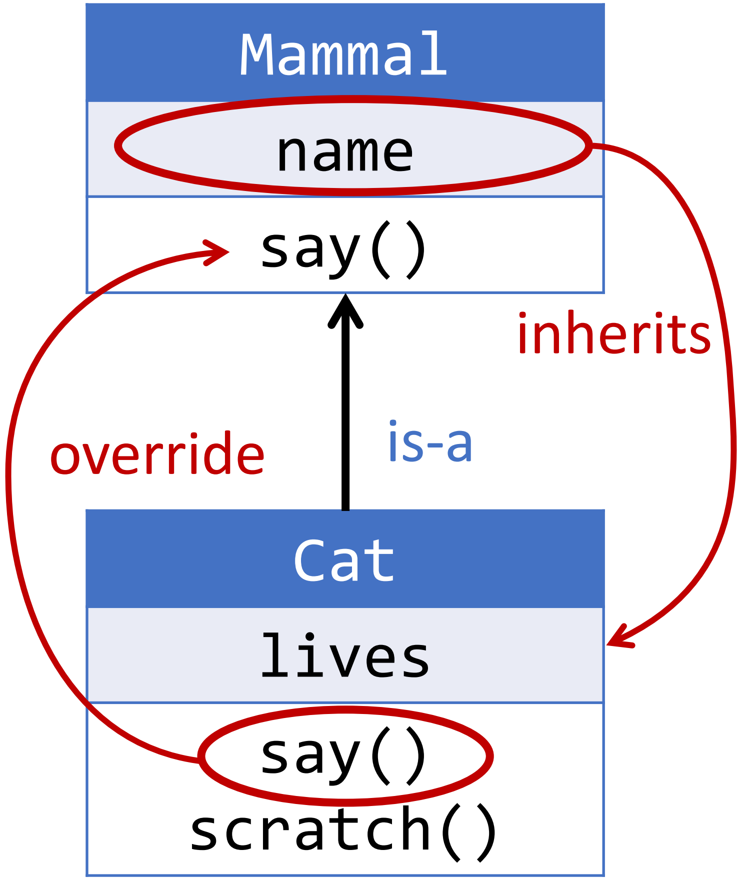
__init__ method
is called on creation

```
Meow
```

# Recap: Inheritance

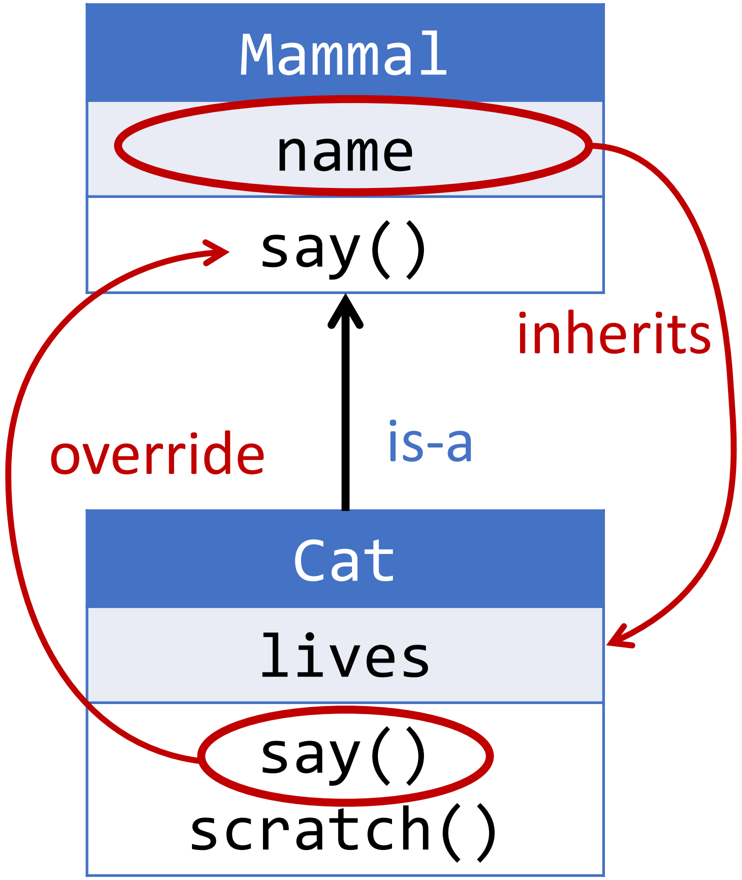


# Recap: Inheritance



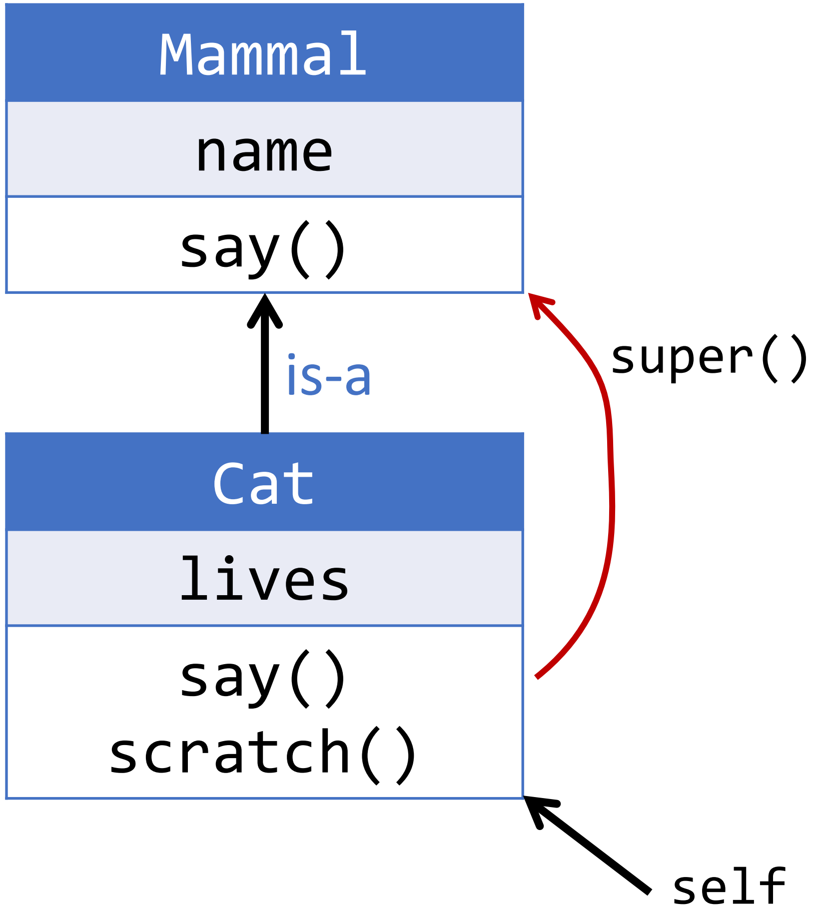

```python
class Mammal:
    def __init__(self, name):
        self.name = name
    def say(self):
        print(self.name + " say")

class Cat(Mammal):
    def __init__(self, name):
        super().__init__(name)
        self.lives = 9
    def say(self):
        print("meow")
    def scratch(self):
        print(self.name,
              "scratch!")
```

# Recap: Inheritance




```
>>> pusheen = Cat("Pusheen")
>>> pusheen.say()
meow

>>> pusheen.scratch()
Pusheen scratch!

>>> pusheen.lives
9
```

# Recap: super()




`super()` refers to superclass

`self` refers to instance

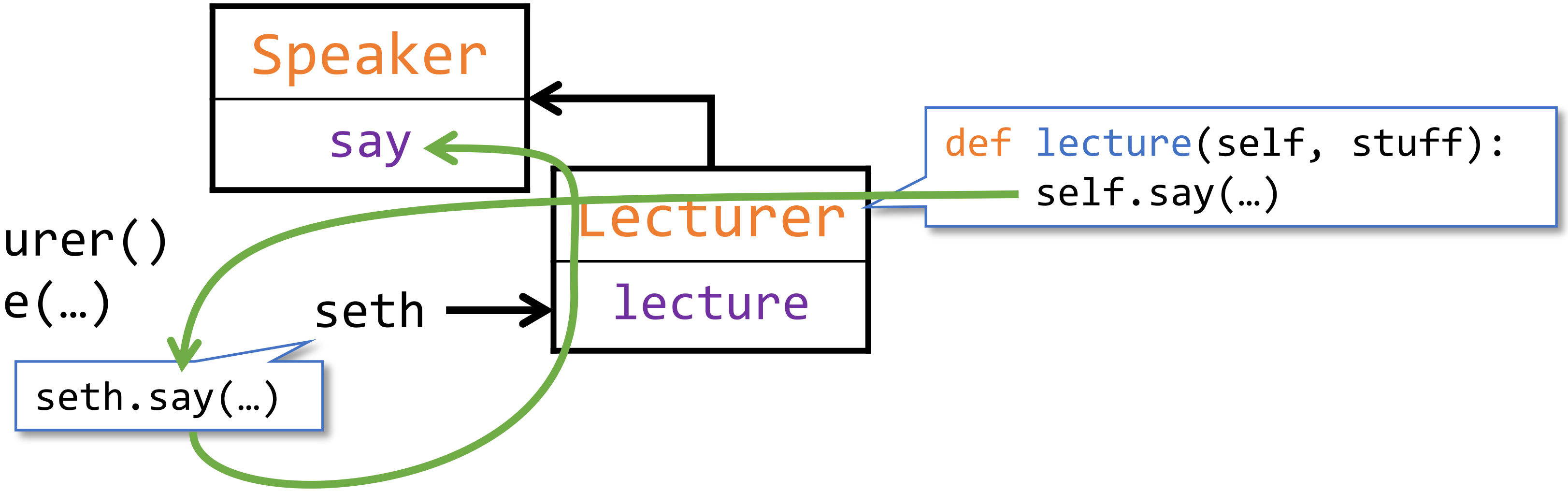
```
class Cat(Mammal):
    …
    def say(self):
        super().say()
        print("meow")
```

```
>>> pusheen.say()
Pusheen say
meow
```

# Polymorphism

- Which superclass does `super()` resolve to?
  - Depends on the type of the object/instance

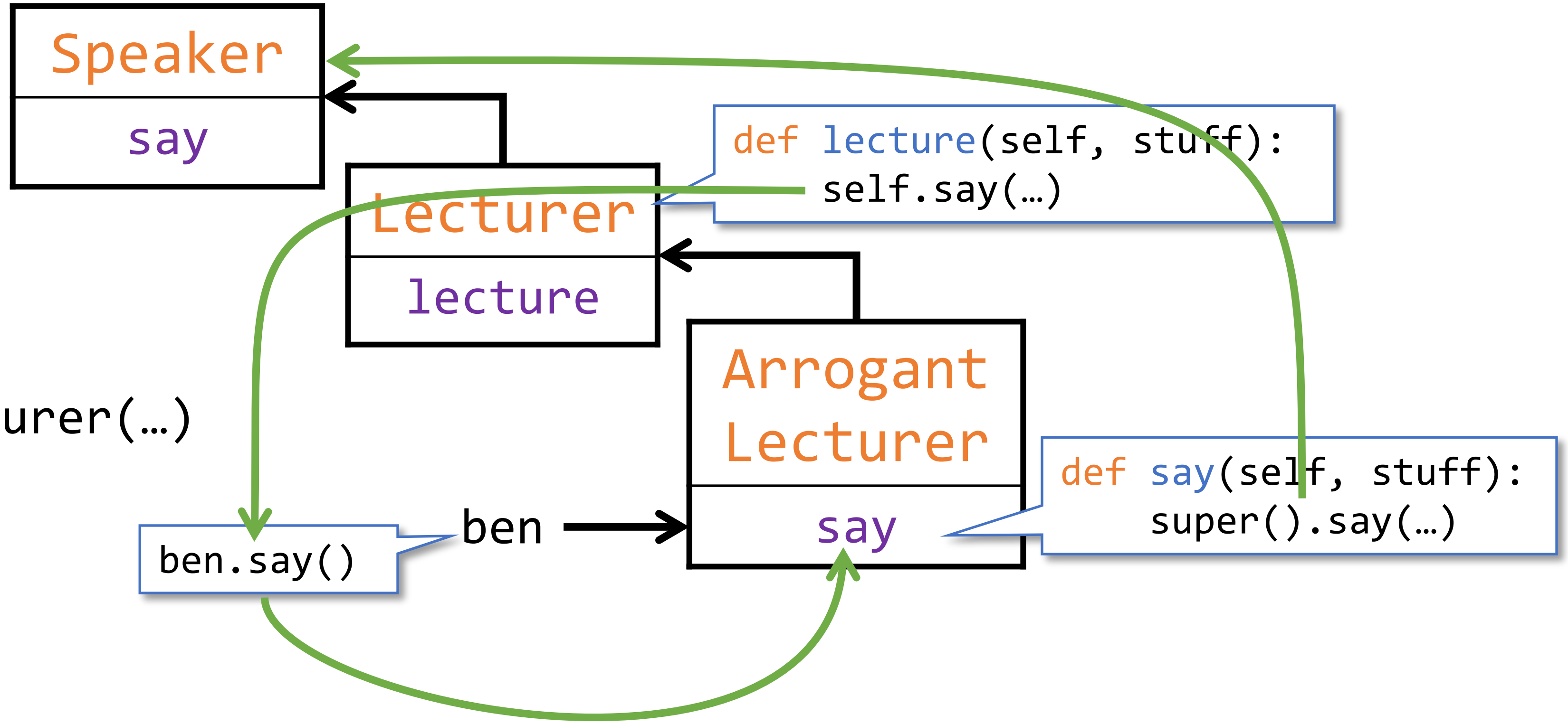
```
>>> seth = Lecturer()
>>> seth.lecture(…)
```



# Polymorphism

- Which superclass does `super()` resolve to?
  - Depends on the type of the object/instance

```
>>> seth = Lecturer()
>>> seth.lecture(…)

>>> ben = ArrogantLecturer(…)
>>> ben.lecture(…)
```

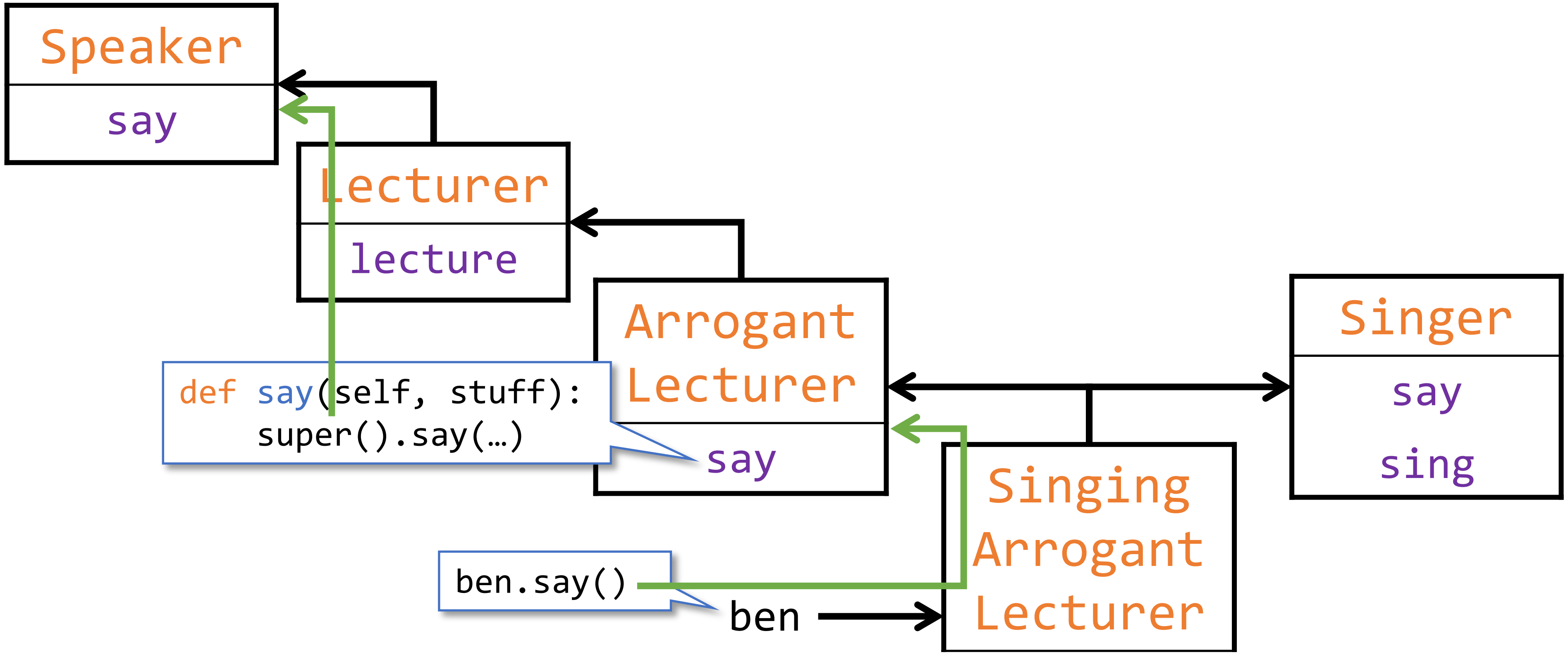



`Lecturer.lecture` has different behaviour

# Multiple Inheritance

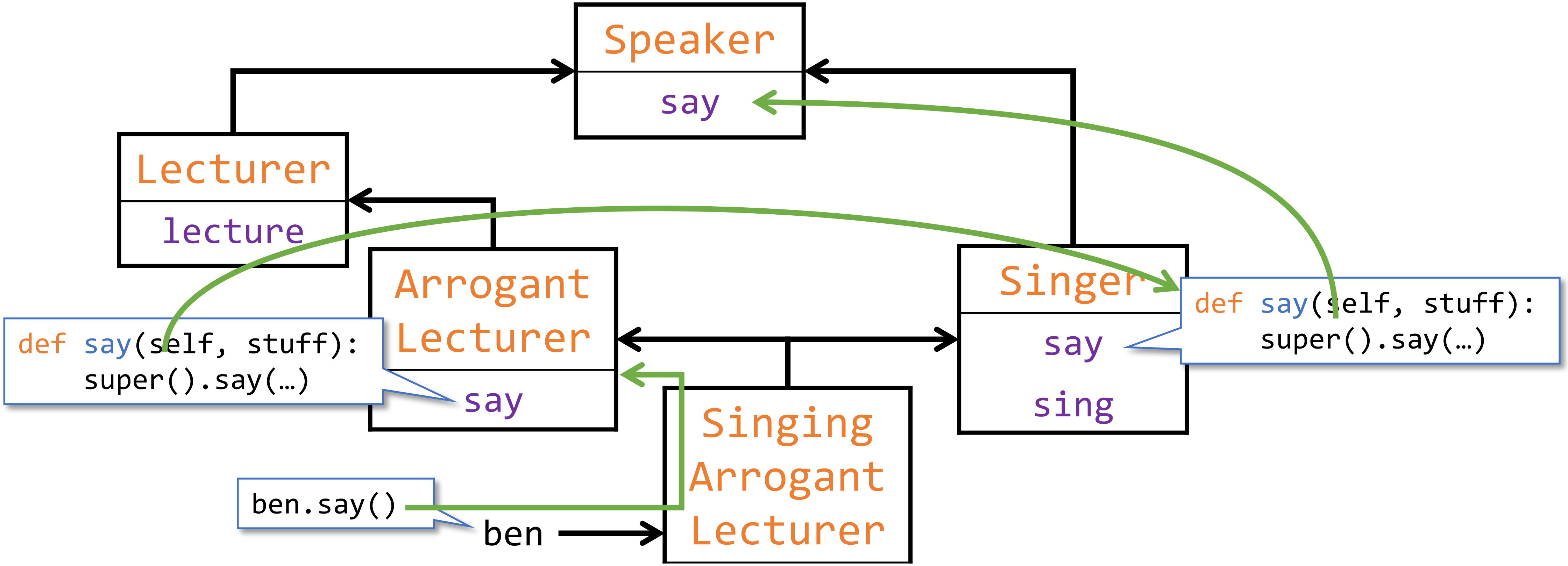
- Which superclass does `super()` resolve to?
  - Order of declaration

```
class SingingArrogantLecturer(ArrogantLecturer, Singer)
```



# Diamond Hierachy

- Intuition: All subclasses' method must be called before superclass'
  - Allows polymorphism to occur naturally



# Today’s Agenda

- Optional Arguments
- Memoization
- Dynamic Programming
- Exception Handling

# Optional Arguments

- Consider the function zip
  - takes as inputs a sequence of n-sequences
  - “zips” them up into a sequence of n-tuples
- Example:

```
zip([( 0,  1,  2,  3,  4),
     ( 5,  6,  7,  8,  9),
     (10, 11, 12, 13, 14)])
```

⇨

```
[(0, 5, 10),
 (1, 6, 11),
 (2, 7, 12),
 (3, 8, 13),
 (4, 9, 14)]
```

# How to implement zip

```
def zip(seqs):
  result = []
  for i in range(min(map(lambda x: len(x), seqs))):
    result.append(tuple(map(lambda x: x[i], seqs)))
  return result
```

# Suppose we want to map

- The function nmap
  - takes as inputs a function and a sequence of sequences
  - applies the function on the n-tuple
- Example

```
>>> nmap(lambda x, y, z: x+y+z,
         [( 0,  1,  2,  3,  4),
          ( 5,  6,  7,  8,  9),
          (10, 11, 12, 13, 14)])
[15, 18, 21, 24, 27]
```

# How to implement nmap

```
def nmap(fn, seqs):
  result = []
  for seq in zip(seqs):
    result.append(fn(seq))
  return result
```

Error: fn does not take in a tuple

In fact, what does fn take?

It depends on what is passed in. i.e. beyond our control

# The * notation

- Call a function for which you don't know in advance how many arguments there will be using.

```
def funky(op, args):
    return op(*args)
print(funky(lambda x: x*x, (2,))) → 4
print(funky(lambda x,y: x+y, (2, 1))) → 3
```

# The * notation

- Can also specify that a function takes in optionally many arguments

```
def f(x, y, *z):
    <body>
```

- function `f` can be called with 2, or more arguments.

- Calling `f(1, 2, 3, 4, 5, 6)`: in the body,
  - `x` → `1`,
  - `y` → `2`,
  - `z` → `(3, 4, 5, 6)`

# Improved nmap

```
def nmap(fn, *seqs):
  result = []
  for seq in zip(seqs):
    result.append(fn(*seq))
  return result

>>> nmap(lambda x, y, z: x+y+z,
         ( 0,  1,  2,  3,  4),
         ( 5,  6,  7,  8,  9),
         (10, 11, 12, 13, 14))
[15, 18, 21, 24, 27]
```

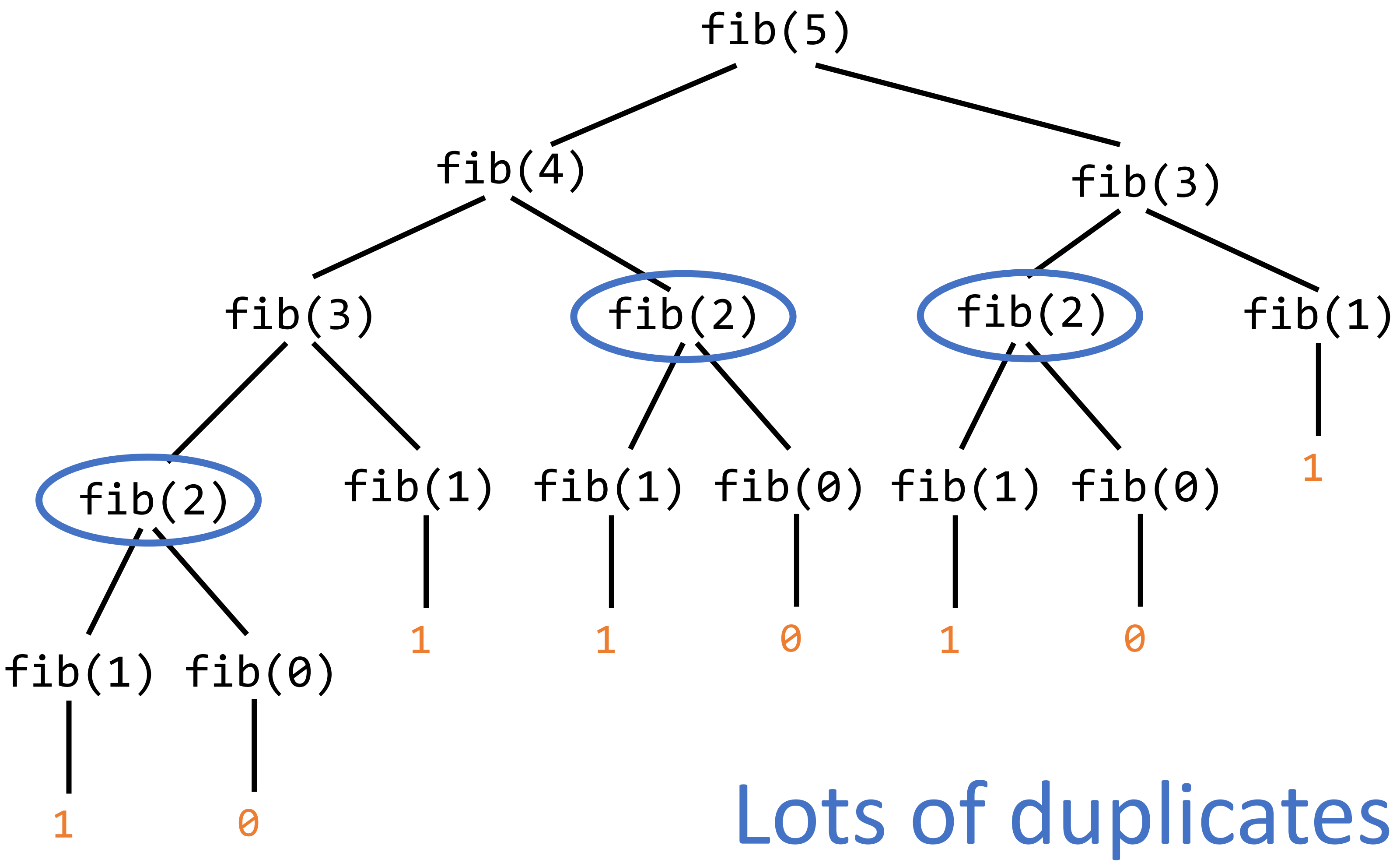
# Recall: Fibonacci

```
def fib(n):
    if n == 0:
        return 0
    elif n == 1:
        return 1
    else:
        return fib(n-1) + fib(n-2)
```

Time complexity = $O(\Phi^n)$ (exponential!)

How can we do better?

# Computing Fibonacci




Lots of duplicates

What’s the obvious way to do better?

Remember what you had earlier computed!

# Memoization

Notice the spelling,

NOT memorization

# Simple Idea!!

Function records, in a table, values that have previously been computed.

- A memoized function:
  - Maintains a table in which values of previous calls are stored
  - Use the arguments that produced the values as keys

- When the memoized function is called, check table to see if the value exists:
  - If so, return value.
  - Otherwise, compute new value in the ordinary way and store this in the table.

# Implementing Memoization

```
memoize_table = {}
def memoize(f, name):
    if name not in memoize_table:
        memoize_table[name] = {}
    table = memoize_table[name]
    def helper(*args):
        if args in table:
            return table[args]
        else:
            result = f(*args)
            table[args] = result
            return result
    return helper
```

Name to store in reference table

# Fibonacci with Memoization

- Now that we have memoize, the obvious thing to do is:

```
memo_fib = memoize(fib, "fib")
```

- What's the time complexity now?

Still exponential!

HUH??

# Fibonacci with Memoization

- Now that we have memoize, the obvious thing to do is:

```
memo_fib = memoize(fib, "fib")
```

- Problem: recursive step in fib will call `fib` instead of `memo_fib`.

# Implementing Memoization

```
memoize_table = {}
def memoize(f, name):
    if name not in memoize_table:
        memoize_table[name] = {}
    table = memoize_table[name]
    def helper(*args):
        if args in table:
            return table[args]
        else:
            result = f(*args)
            table[args] = result
            return result
    return helper
```

recursive step in fib will call fib instead of memo_fib.

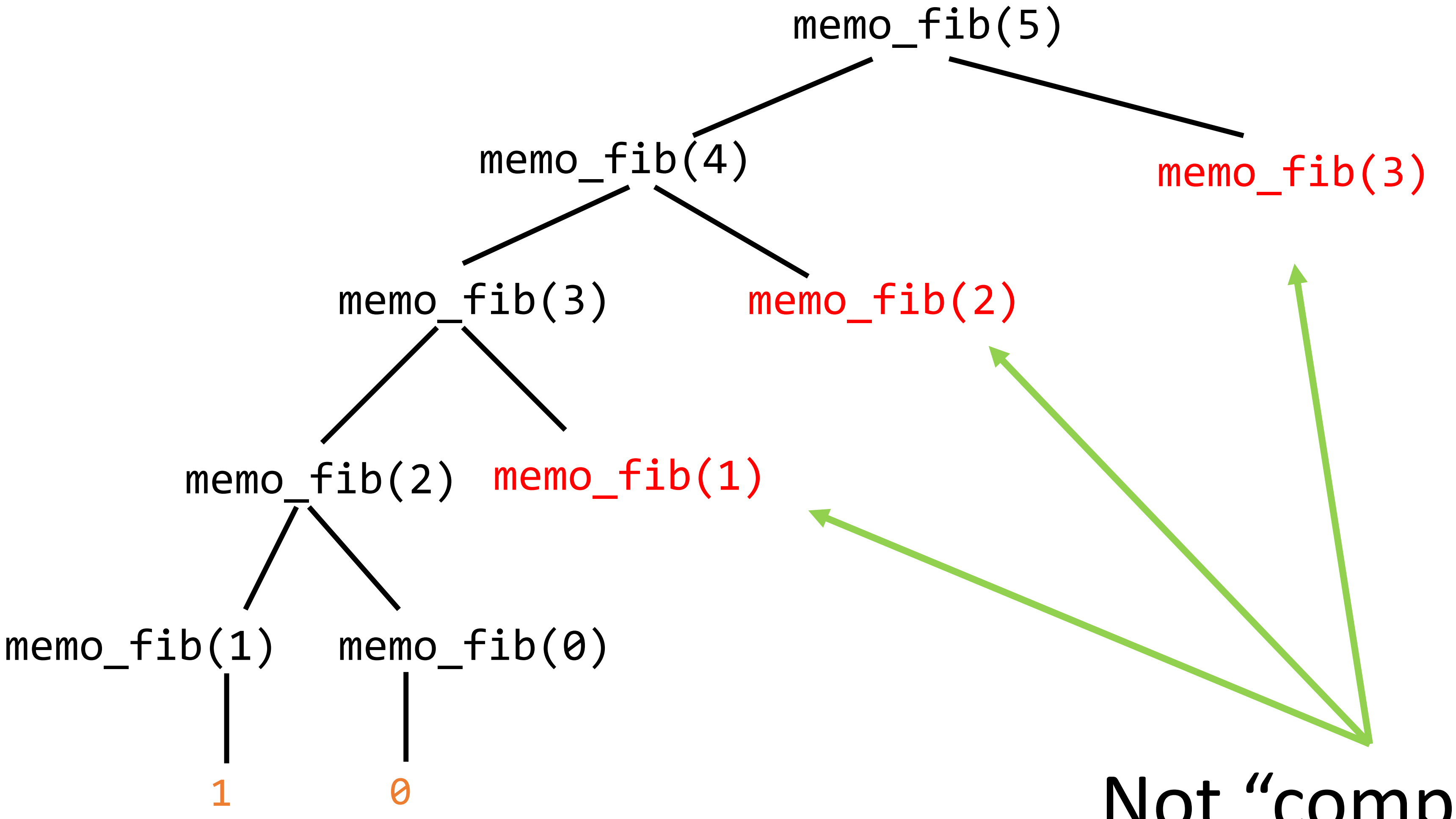
# Doing it Right!

```
def memo_fib(n):
    def helper(n):
        if n == 0:
            return 0
        elif n == 1:
            return 1
        else:
            return memo_fib(n-1) +
                   memo_fib(n-2)
    return memoize(helper, "memo_fib")(n)
```

Why??

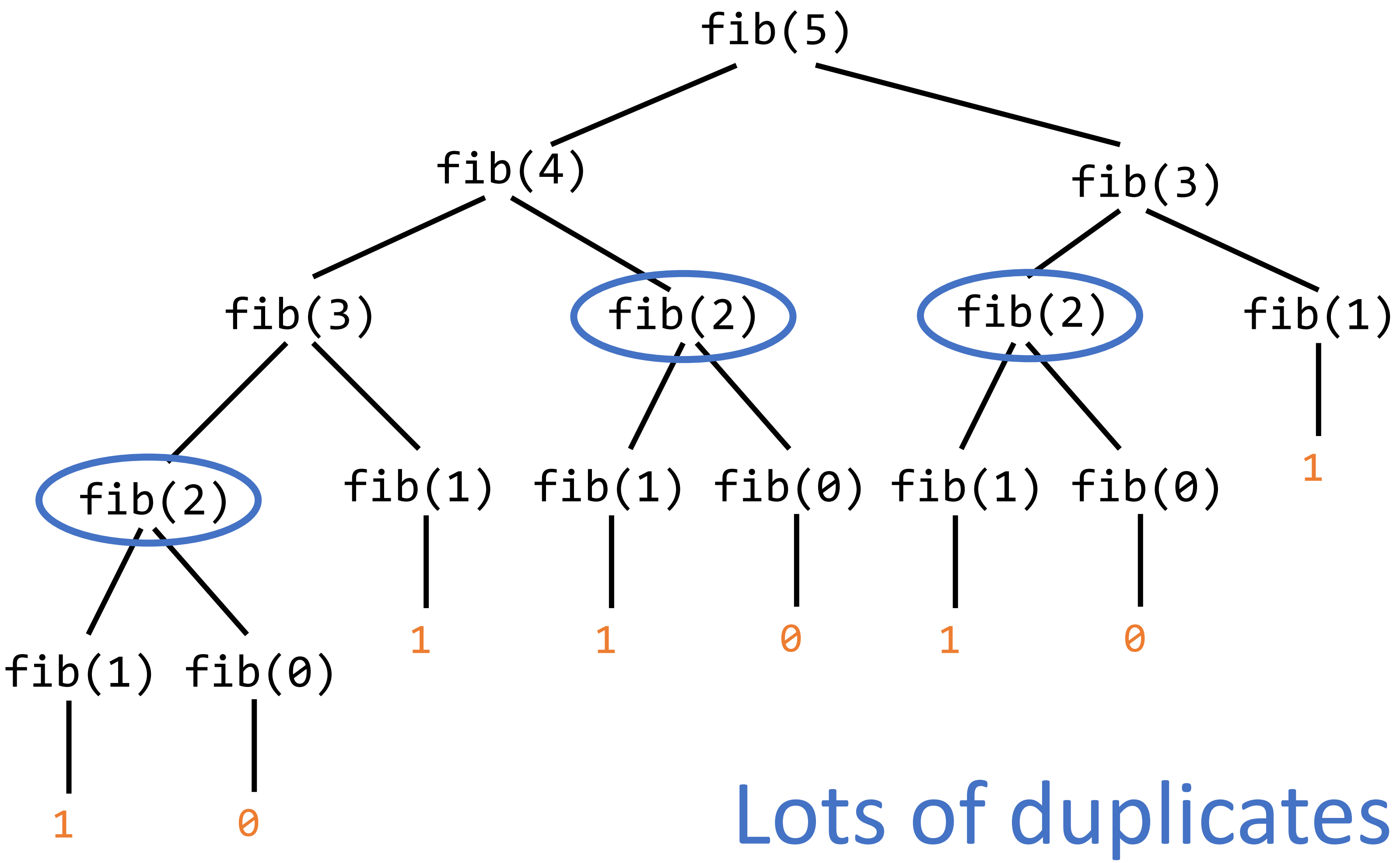
What's the time complexity now? $O(n)$ (linear)!

# Computing Fibonacci




Not "computed" but by table lookup!

# Compare to the Original Version




Lots of duplicates

# Doing it Right!

```
def memo_fib(n):
    def helper(n):
        if n == 0:
            return 0
        elif n == 1:
            return 1
        else:
            return memo_fib(n-1) +
                   memo_fib(n-2)
    return memoize(helper, "memo_fib")(n)
```

Each `fib(n)` is computed only once!

# Doing it Right!

```
def memo_fib(n):
    def helper(n):
        if n == 0:
            return 0
        elif n == 1:
            return 1
        else:
            return memo_fib(n-1) +
                   memo_fib(n-2)
    return memoize(helper, "memo_fib")(n)
```

Efficiency of table lookup is important: Table lookup should be $O(1)$, i.e. hash table.

What happens to time complexity if table lookup is not constant, say $O(n)$?

# Another Example: $C_k^n$

```
def choose(n, k):
    if k > n:
        return 0
    elif k==0 or k==n:
        return 1
    else:
        return choose(n-1, k) +
               choose(n-1, k-1)
```

Why is the recursion true?

# Remember Count-Change?

- Consider one of the elements $x$. $x$ is either chosen or it is not.
- Then number of ways is sum of:
  - Not chosen. Ways to choose $k$ elements out of remaining $n - 1$ elements; and
  - Chosen. Ways to choose $k - 1$ elements out of remaining $n - 1$ elements

# Another Example: $C_k^n$

```
def choose(n, k):
    if k > n:
        return 0
    elif k==0 or k==n:
        return 1
    else:
        return choose(n-1, k) +
               choose(n-1, k-1)
```

What is the order of growth?

How can we speed up the computation?

# Memoization!

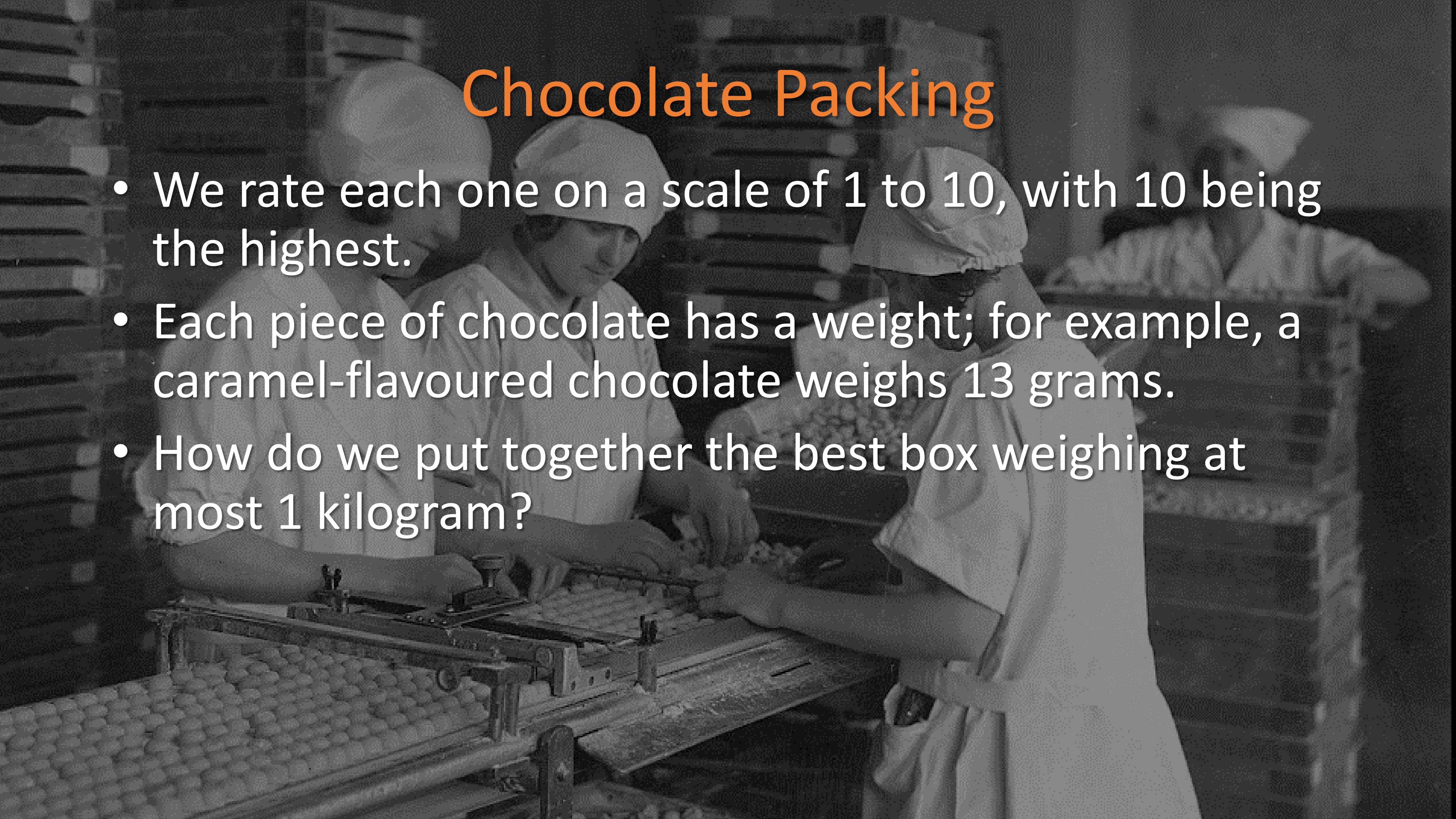
# Memoized Choose

```
def memo_choose(n, k):
    def helper(n, k):
        if k > n:
            return 0
        elif k==0 or k==n:
            return 1
        else:
            return memo_choose(n-1, k) +
                   memo_choose(n-1, k-1)
    return memoize(helper, "choose")(n, k)
```

Don't need to use memoize function. Can just use a dictionary!

# Chocolate Packing

- Suppose we are at a chocolate candy store and want to assemble a kilogram box of chocolates.
- Some of the chocolates (such as the caramels) at this store are absolutely the best, and others are only so-so.



# Chocolate Packing

- We rate each one on a scale of 1 to 10, with 10 being the highest.
- Each piece of chocolate has a weight; for example, a caramel-flavoured chocolate weighs 13 grams.
- How do we put together the best box weighing at most 1 kilogram?

# Abstract Data Type for Chocolates

```
def make_chocolate(desc,weight,value):
    return (desc,weight,value)

def get_description(choc):
    return choc[0]
def get_weight(choc):
    return choc[1]
def get_value(choc):
    return choc[2]
```

# Here are the Chocolates

```
shirks_chocolates =
  (make_chocolate('caramel dark', 13, 10),
   make_chocolate('caramel milk', 13, 3),
   make_chocolate('cherry dark', 21, 3),
   make_chocolate('cherry milk', 21, 1),
   make_chocolate('mint dark', 7, 3),
   make_chocolate('mint milk', 7, 2),
   make_chocolate('cashew-cluster dark', 8, 6),
   make_chocolate('cashew-cluster milk', 8, 4),
   make_chocolate('maple-cream dark', 14, 1),
   make_chocolate('maple-cream milk', 14, 1))
```

# Implementing a Box

```
def make_box(list_of_choc, weight, value):
    return (list_of_choc, weight, value)

def make_empty_box():
    return make_box((), 0, 0)

def box_chocolates(box):
    return box[0]
def box_weight(box):
    return box[1]
def box_value(box):
    return box[2]
```

# Implementing a Box

```
def add_to_box(choc, box):
  return make_box(
          box_chocolates(box) + (choc,),
          box_weight(box) + get_weight(choc),
          box_value(box) + get_value(choc))

def better_box(box1, box2):
    if box_value(box1) > box_value(box2):
        return box1
    else:
        return box2
```

# How to Solve this Problem?

- Enumerate all the possible boxes (constrained by weight limit)
- Compute value of each packing
- Pick box with highest value

# Simple Solution

```
def pick(chocs, weight_limit):
    if chocs==() or weight_limit==0:
        return make_empty_box()
    elif get_weight(chocs[0]) > weight_limit: # 1st too heavy
        return pick(chocs[1:], weight_limit)
    else:
        # none of 1st kind
        box1 = pick(chocs[1:], weight_limit)
        # at least one of 1st kind
        box2 = add_to_box(chocs[0],
                          pick(chocs,
                               weight_limit -
                               get_weight(chocs[0])))
        return better_box(box1, box2)
```

# Simple Solution

- What is the order of growth?
  - Exponential! $O(2^n)$

- Again, a lot of repeat computations.

- Think memoization!

# Original Simple Solution

```
def pick(chocs, weight_limit):

        if chocs==() or weight_limit==0:
            return make_empty_box()
        elif get_weight(chocs[0]) > weight_limit:
            return pick(chocs[1:], weight_limit)
        else:
            box1 = pick(chocs[1:], weight_limit)
            box2 = add_to_box(chocs[0],
                              pick(chocs,
                                   weight_limit -
                                   get_weight(chocs[0])))
            return better_box(box1, box2)
```

# Memoized Version

```
def memo_pick(chocs, weight_limit):
    def helper(chocs, weight_limit):
        if chocs==() or weight_limit==0:
            return make_empty_box()
        elif get_weight(chocs[0]) > weight_limit:
            return memo_pick(chocs[1:], weight_limit)
        else:
            box1 = memo_pick(chocs[1:], weight_limit)
            box2 = add_to_box(chocs[0],
                              memo_pick(chocs,
                                        weight_limit -
                                       get_weight(chocs[0])))
            return better_box(box1, box2)
    return memoize(helper, "pick")(chocs, weight_limit)
```

# Recap: Memoization

- Two Steps:
  1. Write function to perform required computation
  2. Add wrapper that stores the result of the computation ($O(1)$ lookup table)

- When you wrap your function, just make sure that he recursive calls go to the wrapped version and not the raw form.

# Homework

Re-factor the chocolate packing code into OOP format.

# Design Pattern: Wrapper (also called Decorator)

- Memoization as an idea is simply to remember the stuff you have done before so that you don't do the same thing twice
- However, the method that we used to implement memoization is also an important concept

# Design Pattern: Wrapper (also called Decorator)

- Design Pattern: Wrapper (also known as Decorator)
- Key idea is that you add an extra layer to introduce additional functionality and use the original function to do "old work"

# Revisting `memo_choose`

- Consider `memo_choose(8, 4)`
- The following is the table of values at the end of the computation:

| | | | | |
|---|---|---|---|---|
| 1 | #f | #f | #f | #f |
| 1 | 1 | #f | #f | #f |
| 1 | 2 | 1 | #f | #f |
| 1 | 3 | 3 | 1 | #f |
| 1 | 4 | 6 | 4 | 1 |
| 1 | 5 | 10 | 10 | 5 |
| #f | 6 | 15 | 20 | 15 |
| #f | #f | 21 | 35 | 35 |
| #f | #f | #f | 56 | 70 |

# Recall Pascal's Triangle

- If we were to fill up the table, we expect

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 2 | 1 | 0 | 0 |
| 1 | 3 | 3 | 1 | 0 |
| 1 | 4 | 6 | 4 | 1 |
| 1 | 5 | 10 | 10 | 5 |
| 1 | 6 | 15 | 20 | 15 |
| 1 | 7 | 21 | 35 | 35 |
| 1 | 8 | 28 | 56 | 70 |

# Dynamic Programming

- Idea: why don't we compute choose by filling up this table from the bottom?
- Fancy name for this simple idea — Dynamic Programming :-)
- What is the order of growth then?

Why??

$O(n)$ or more accurately $O(nk)$

# Dynamic Programming: choose

```
def dp_choose(n, k):
    row = [1] * (k+1)
    table = []
    for i in range(n+1):
        table.append(row.copy())

    for j in range(1, k+1):
        table[0][j] = 0

    for i in range(1, n+1):
        for j in range(1, k+1):
            table[i][j] = table[i-1][j-1]
                        + table[i-1][j]
    return table[n][k]
```

allocate space for table (all 1's)

Fill first row with 0's

fill the rest

return answer

# Let’s check out the table

```
row = [1] * (k+1)
table = []
for i in range(n+1):
  table.append(
            row.copy())
```

$$\xrightarrow{k+1}$$

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |

# Let’s check out the table

```
row = [1] * (k+1)
table = []
for i in range(n+1):
  table.append(
              row.copy())
```

$$C_0^0 \xrightarrow{\;k+1\;} C_k^0$$

n+1

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$C_0^n$ $C_k^n$

# Let’s check out the table

```
for j in range(1,k+1):
  table[0][j] = 0
```

j

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Let’s check out the table

```
for j in range(1,k+1):
  table[0][j] = 0
```

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Let’s check out the table

```
for i in range(1,n+1):
  for j in
    range(1,k+1):
        table[i][j] =
    table[i-1][j-1] +
    table[i-1][j]
```

j

i

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Let’s check out the table

```
for i in range(1,n+1):
  for j in
    range(1,k+1):
        table[i][j] =
    table[i-1][j-1] +
    table[i-1][j]
```

j

i

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Let’s check out the table

```
for i in range(1,n+1):
  for j in
    range(1,k+1):
        table[i][j] =
    table[i-1][j-1] +
    table[i-1][j]
```

j

i

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Let’s check out the table

```
for i in range(1,n+1):
    for j in
        range(1,k+1):
                table[i][j] =
        table[i-1][j-1] +
        table[i-1][j]
```

j

i

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Let’s check out the table

```
for i in range(1,n+1):
  for j in
    range(1,k+1):
            table[i][j] =
 table[i-1][j-1]
  + table[i-1][j]
```

j

i

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |

# Let’s check out the table

```
for i in range(1,n+1):
  for j in
    range(1,k+1):
        table[i][j] =
    table[i-1][j-1] +
    table[i-1][j]
```

$$
\begin{array}{c|ccccc}
 & & & j \rightarrow & & \\
\hline
i \downarrow & 1 & 0 & 0 & 0 & 0 \\
 & 1 & 1 & 0 & 0 & 0 \\
 & 1 & 2 & 1 & 1 & 1 \\
 & 1 & 1 & 1 & 1 & 1 \\
 & 1 & 1 & 1 & 1 & 1 \\
 & 1 & 1 & 1 & 1 & 1 \\
 & 1 & 1 & 1 & 1 & 1 \\
 & 1 & 1 & 1 & 1 & 1
\end{array}
$$

# Fast Forward

| | | | | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 2 | 1 | 0 | 0 |
| 1 | 3 | 3 | 1 | 0 |
| 1 | 4 | 6 | 4 | 1 |
| 1 | 5 | 10 | 10 | 5 |
| 1 | 6 | 15 | 20 | 15 |
| 1 | 7 | 21 | 35 | 35 |

Can we adopt a dynamic programming approach for solving the chocolate packing problem?

YES (of course)!

# How?

# Let's take another look at the chocolate packing problem…

Remember the table .... dynamic programming is about filling up some table efficiently so that the total time complexity is $O(size\ of\ table)$

# What does the chocolate packing table look like?

- Table of list of chocolates vs weight limit
- Each table entry is the optimal choice for a given list and weight limit.

# What does the chocolate packing table look like?

- Observation: If we have no chocolates, answer is, easy: Empty set ( ).
- Otherwise, the answer for a list of $x$ types of chocolates is the better between:
  1. Additional chocolate + Optimal choice with remaining $x$ types of chocolates and reduced weight limit
  2. Optimal choice with remaining $x - 1$ types of chocolates and current weight limit.

# Building the chocolate packing table

Idea: build a table from the smaller cases to larger cases

| Weight Limit | () | | | |
|---|---|---|---|---|
| 0 | () | | | |
| 1 | () | | | |
| 2 | () | | | |
| 3 | () | | | |
| 4 | () | | | |
| 5 | () | | | |
| 6 | () | | | |
| 7 | () | | | |
| 8 | () | | | |
| … | () | | | |

# Building the chocolate packing table

| Weight Limit | () | ((A 4 2)) | | |
|---|---|---|---|---|
| 0 | () | () | | |
| 1 | () | () | | |
| 2 | () | () +A | | |
| 3 | () | () | | |
| 4 | () | (A) | | |
| 5 | () | (A) | | |
| 6 | () | (A)+A | | |
| 7 | () | (A) | | |
| 8 | () | (A,A) | | |
| ⋮ | () | ⋮ | | |

# Building the chocolate packing table

| Weight Limit | () | ((A 4 2)) | ((A 4 2)<br>(B 5 3 )) | |
|---|---|---|---|---|
| 0 | () | () | () | |
| 1 | () | () | () | |
| 2 | () | () | () Cannot | |
| 3 | () | () | () add B | |
| 4 | () | (A) | (A) | |
| 5 | () | (A) | (B) | |
| 6 | () | (A) | | |
| 7 | () | (A) | | |
| 8 | () | (A,A) | | |
| ⋮ | () | ⋮ | | |

# Building the chocolate packing table

| Weight Limit | () | ((A 4 2)) | ((A 4 2)<br>(B 5 3 )) | |
|---|---|---|---|---|
| 0 | () | () | () | |
| 1 | () | () | () | |
| 2 | () | () | () | |
| 3 | () | () | () | |
| 4 | () | (A) | (A) +B | |
| 5 | () | (A) | B | |
| 6 | () | (A) | B +B | |
| 7 | () | (A) | B | |
| 8 | () | (A,A) | | |
| ⋮ | () | ⋮ | | |

# Building the chocolate packing table

| Weight Limit | () | ((A 4 2)) | ((A 4 2)<br>(B 5 3 )) | |
|---|---|---|---|---|
| 0 | () | () | () | |
| 1 | () | () | () | |
| 2 | () | () | () | |
| 3 | () | () | () | |
| 4 | () | (A) | (A) | |
| 5 | () | (A) | B | |
| 6 | () | (A) | B | |
| 7 | () | (A) | B +B | |
| 8 | () | (A,A) | (A,A) | |
| ⋮ | () | ⋮ | ⋮ | |

Challenge of the Day: Write `dp_pick_chocolates` over the weekend. ☺

# Prime Numbers

- In recitation, we defined a function `is_prime` to check whether a number is prime
- But what if we wanted to list ALL of the numbers that are prime, in the interval $[0, \ldots, n]$?

# Prime Numbers: Naïve Solution

```
def is_prime(n): # O(n^0.5)
    if n==0 or n==1:
        return False
    elif n == 2:
        return True
    for i in range(2, int(sqrt(n))+1):
        if n % i == 0:
            return False
    return True

def naive_prime(n): # O(n^1.5)
    return [is_prime(i) for i in range(n+1)]
```

# Prime Numbers

Idea: why don’t we compute the primes by filling up a table from the bottom?

# Prime Numbers: DP

```
def dp_prime(n):
    bitmap = [True]*(n+1)
    bitmap[0] = False # 0 is not prime
    bitmap[1] = False # 1 is not prime
    for i in range(2,n):
        if bitmap[i] == True:
            for j in range(2*i,n+1,i):
                bitmap[j] = False
    return bitmap
```

# Prime Numbers: DP

How does it work?

~~0~~, ~~1~~, 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, 9, ~~10~~, 11, ~~12~~, 13

~~0~~, ~~1~~, 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13

~~0~~, ~~1~~, 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13

~~0~~, ~~1~~, 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13

~~0~~, ~~1~~, 2, 3, ~~4~~, 5, ~~6~~, 7, ~~8~~, ~~9~~, ~~10~~, 11, ~~12~~, 13

# Errors and Exceptions

# Errors and Exceptions

- Until now error messages haven’t been more than mentioned, but you have probably seen some
- Two kinds of errors (in Python):
  1. syntax errors
  2. exceptions

# Syntax Errors

```
>>> while True print('Hello world')
SyntaxError: invalid syntax
```

# Exceptions

- Errors detected during execution are called exceptions
- Examples:
  - ZeroDivisonError,
  - NameError,
  - TypeError

# ZeroDivisionError

```
>>> 10 * (1/0)
Traceback (most recent call last):
  File "<pyshell#3>", line 1, in <module>
    10 * (1/0)
ZeroDivisionError: division by zero
```

# NameError

```
>>> 4 + spam*3
Traceback (most recent call last):
  File "<pyshell#4>", line 1, in <module>
    4 + spam*3
NameError: name 'spam' is not defined
```

# TypeError

```
>>> '2' + 2
Traceback (most recent call last):
  File "<pyshell#5>", line 1, in <module>
    '2' + 2
TypeError: Can't convert 'int' object to str
implicitly
```

# ValueError

```
>>> int('one')
Traceback (most recent call last):
  File "<pyshell#2>", line 1, in <module>
    int('one')
ValueError: invalid literal for int() with base 10: 'one'
```

# Handling Exceptions

The simplest way to catch and handle exceptions is with a try-except block:

```python
x, y = 5, 0
try:
    z = x/y
except ZeroDivisionError:
    print("divide by zero")
```

# Try-Except (How it works I)

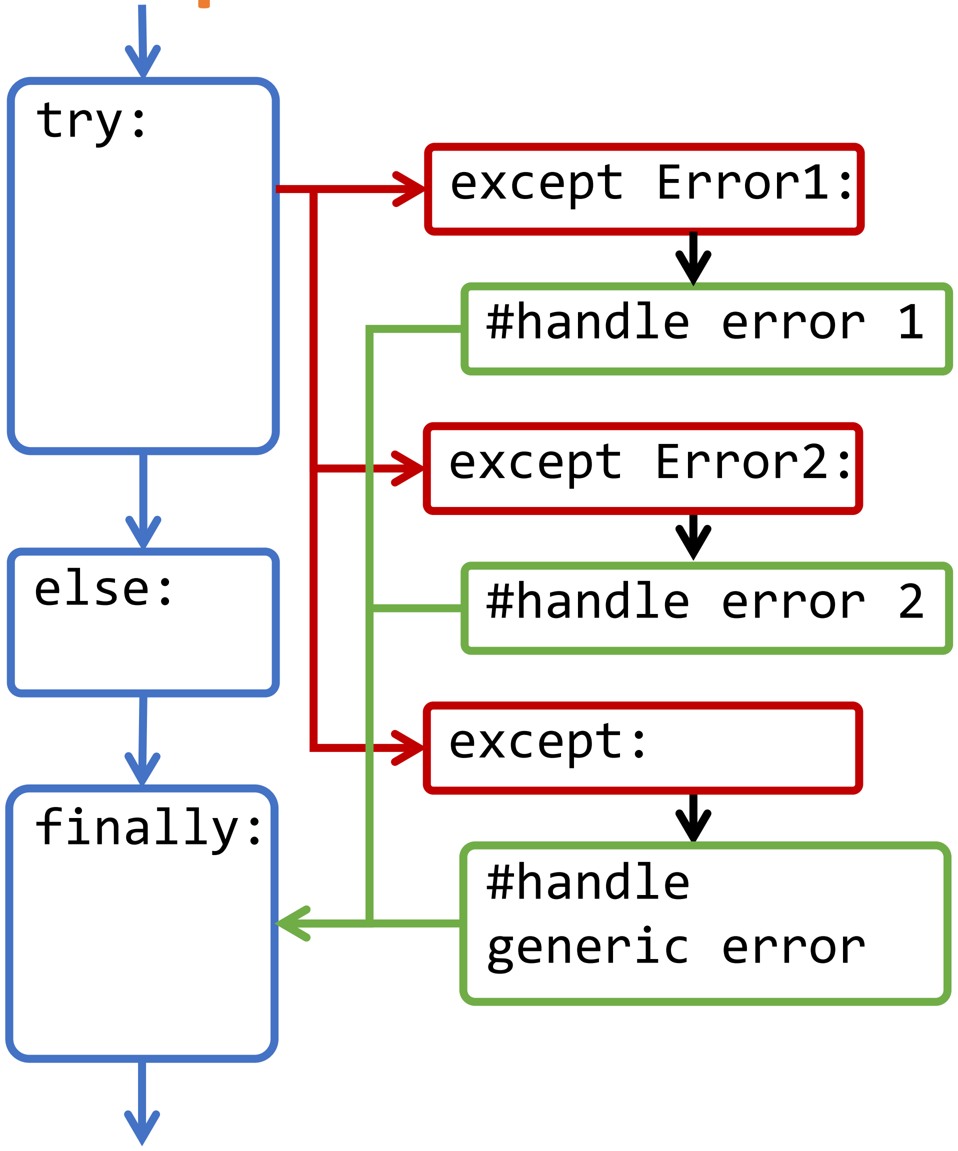
- The `try` clause is executed
- If an exception occurred, skip the rest of the `try` clause, to a matching `except` clause
- If no exception occurs, the `except` clause is skipped (go to the else clause, if it exists)
- The `finally` clause is always executed before leaving the `try` statement, whether an exception has occurred or not.

# Try-Except

- A `try` clause may have more than 1 `except` clause, to specify handlers for different exception.
- At most one handler will be executed.
- Similar with `if-elif-else`
- `finally` will always be executed

# Try-Except

```
try:
    # statements
except Error1:
    # handle error 1
except Error2:
    # handle error 2
except: # wildcard
    # handle generic error
else:
    # no error raised
finally:
    # always executed
```



# Try-Except Example

```python
def divide_test(x, y):
    try:
      result = x / y
    except ZeroDivisionError:
      print("division by zero!")
    else:
      print("result is", result)
    finally:
      print("executing finally clause")
```

# Try-Except Blocks

```
>>> divide_test(2, 1)
result is 2.0
executing finally clause

>>> divide_test(2, 0)
division by zero!
executing finally clause

>>> divide_test("2", "1")
executing finally clause
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
  File "<stdin>", line 3, in divide
TypeError: unsupported operand type(s) for /: 'str' and 'str'
```

```
def divide_test(x, y):
    try:
      result = x / y
    except ZeroDivisionError:
      print("division by zero!")
    else:
      print("result is", result)
    finally:
      print("executing finally
            clause")
```

# Raising Exceptions

The `raise` statement allows the programmer to force a specific exception to occur:

```
>>> raise NameError('HiThere')
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
NameError: HiThere
```

# Exception Types

- Built-in Exceptions: https://docs.python.org/3/library/exceptions.html
- User-defined Exceptions

# User-defined Exceptions I

```
class MyError(Exception):
    def __init__(self, value):
        self.value = value
    def __str__(self):
        return repr(self.value)
```

# User-defined Exceptions II

```
try:
    raise MyError(2*2)
except MyError as e:
    print('Exception value:', e.value)
Exception value: 4

raise MyError('oops!')
Traceback (most recent call last):
  File "<stdin>", line 1, in ?
__main__.MyError: 'oops!'
```

# Why use Exceptions?

In the good old days of C, many procedures returned special ints for special conditions, i.e. -1

# Why use Exceptions?

- But Exceptions are better because:
  - More natural
  - More easily extensible
  - Nested Exceptions for flexibility

# Summary

- Memoization dramatically reduces computation.
  - Once a value is computed, it is remembered in a table (along with the argument that produced it).
  - The next time the procedure is called with the same argument, the value is simply retrieved from the table.
- `memo_fib` takes time = $O(n)$
- `memo_choose` takes ?? time?

# Memoization vs Dynamic Programming

- Sometimes DP requires more computations
- DP requires the programmer to know exactly which entries need to be computed
- For smart programmer, DP can however be made more space efficient for some problems, i.e. limited history recurrences like Fibonacci