SCHOOL OF COMPUTING

ASSESSMENT FOR
Special Term I, 2015/2106

Solutions for CS1010X — PROGRAMMING METHODOLOGY

June 2016 Time Allowed: 2 Hours

INSTRUCTIONS TO STUDENTS

1. Please write your Student Number only. Do not write your name.
2. The assessment paper contains **FIVE (5) questions** and comprises **TWENTY-TWO (22) pages**.
3. Weightage of questions is given in square brackets. The maximum attainable score is 100.
4. This is a **CLOSED** book assessment, but you are allowed to bring **TWO** double-sided A4 sheets of notes for this exam.
5. Write all your answers in the space provided in this booklet.
6. **Please write your student number below.**

STUDENT NO: ____________________

(this portion is for the examiner's use only)

Question	Marks	Remark
Q1		
Q2		
Q3		
Q4		
Q5		
Total		

Question 1: Parallel Computations [25 marks]

Ben Bitdiddle was sitting around one day and he decided that it would be a great idea to be able to create some functions that will be flexible enough to perform a computation on an arbitrary number of arguments. In particular, he came up with 3 functions `triple`, `add_one` and `filter_odd` that operates on an arbitrary number of arguments and returns the answers as a tuple, as shown:

```
>>> triple(1,2) # multiplies arguments by 3
(3,6)

>>> triple(1,2,3)
(3,6,9)

>>> add_one(2,3) # adds one to the arguments
(3,4)

>>> add_one(1,2,3)
(2,3,4)

>>> filter_odd(1,2,3)  # keep only the odd numbers
(1,3)

>>> filter_odd(2,4,6)
()
```

A. Give a possible implementation for the function `triple`. [3 marks]

```
def triple(*args):
    return tuple(map(lambda x: 3*x,args))
```

B. Give a possible implementation for the function `add_one`. [3 marks]

```
def add_one(*args):
    return tuple(map(lambda x: x+1,args))
```

C. Give a possible implementation for the function `filter_odd`. [4 marks]

```
def filter_odd(*args):
    return tuple(filter(lambda x: x%2==1,args))
```

Ben then decided that it would good to be able to string all the operations together and tried:

```
>>> filter_odd(add_one(triple(1,2,3)))
```

Unfortunately, this resulted in a `TypeError`. So, Ben decided to come up with a new function that he called `composen` to solve the problem. In particular, with `composen`, Ben managed to do:

```
>>> alpha = composen(triple,add_one)
>>> alpha(1,2,3)
(6, 9, 12)

>>> beta = composen(add_one,triple)
>>> beta(1,2,3)
(4, 7, 10)

>>> composen(alpha,beta)
>>> gamma(1,2,3)
(15, 24, 33)
```

D. Give a possible implementation for the function `composen`. [5 marks]

```
def composen(f,g):
    def helper(*args):
        ans = g(*args)
        return f(*ans)
    return helper
```

E. Now suppose:

```
>>> delta = composen(alpha,composen(filter_odd,beta))
```

What is the value of `delta(1,2,3,4,5)`? Explain. [5 marks]

```
(24,42)

(1,2,3,4,5) => (3,6,9,12,15) => (4,7,10,13,16) [beta]
=> (7,13) [filter_odd]
=> (8,14) => (24,42) [alpha]
```

Ben has a friend, Alyssa P Hacker, who heard about Ben's problem with the `TypeError`. Alyssa told Ben, "Actually, we can use just Python's Exception handling mechanism to deal with the `TypeError` using a function called `wrapper`. In particular, we can then do the following:"

```
>>> new_triple = wrapper(triple)
>>> new_add_one = wrapper(add_one)
>>> new_add_one(new_triple(1,2,3))
(4, 7, 10)
```

F. Give a possible implementation for the function `wrapper`. [5 marks]

```
def wrapper(f):
    def helper(*args):
        try:
            return f(*args)
        except:
            args = args[0]
            return f(*args)
    return helper
```

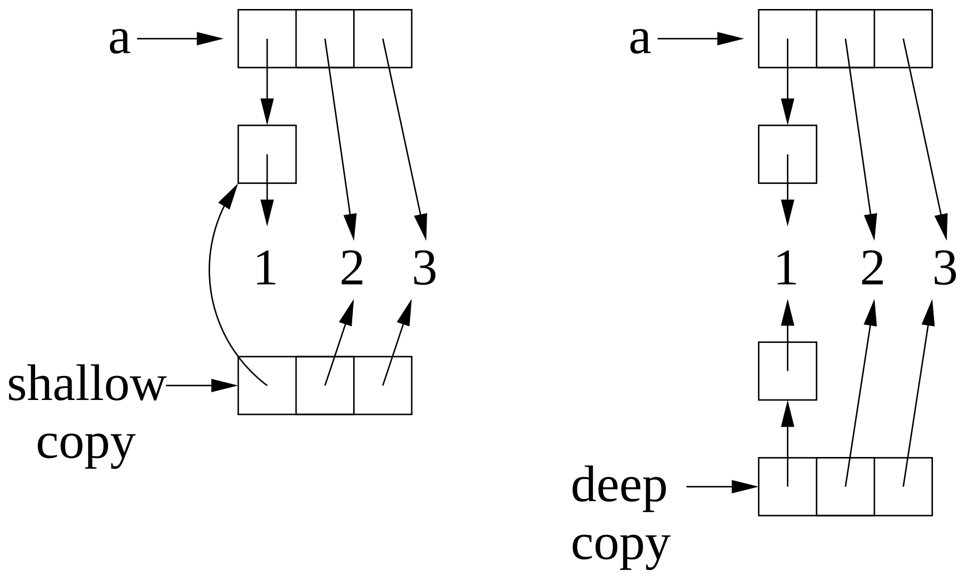
-3 marks if the `except` clause is not correct. Some students have the `except` clause wrap outside `def` which is completely wrong, aka zero.

It is actually also possible to do this without Exception Handling:

```
def wrapper(f):
    def helper(*args):
        if len(args) == 1 and type(args[0]) == tuple:
            args = args[0]
        return f(*args)
    return helper
```

Question 2: How Deep Is Your Copy? [17 marks]

We discussed shallow and deep copy in class. In particular, a shallow copy only copies the first layer, while a deep copy will copy every list or tuple element. This is illustrated in the following figure:



A. [**Warm Up**] Give a possible implementation for `deep_copy` for a list. [3 marks]

```
def deep_copy(lst):
    if lst == []:
        return []
    elif type(lst[0]) == list:
        return [deep_copy(lst[0])] + deep_copy(lst[1:])
    else:
        return [lst[0]] + deep_copy(lst[1:])
```

zero marks for those who tried to do `import copy`. This question is already a giveaway, please don't try to push your luck.

B. One question after you do a deep copy is whether you actually did the copy correctly. So, what we want is to have a function `deep_copy_check` that checks whether one list is a deep copy of the other. For example:

```
>>> a = [[[1],2],3,4]
>>> b = deep_copy(a)
>>> deep_copy_check(a,b) # assuming deep_copy is correct
True

>>> b[0][0] = a[0][0]
>>> deep_copy_check(a,b)
False

>>> b[0] = a[0]
>>> deep_copy_check(a,b)
False

>>> c = deep_copy(b)
>>> deep_copy_check(a,c)
True

>>> d = c.copy()
>>> deep_copy_check(c,d)
False
```

Give a possible implementation for deep_copy_check. [6 marks]

```
def deep_copy_check(lst1,lst2):
    if lst1 != lst2 or lst1 is lst2:
        return False
    else:
        result = True
        for i in range(len(lst1)):
            if type(lst1[i]) == list:
                result = result and deep_copy_check(lst1[i],lst2[i])
            else:
                result = result and lst1[i] is lst2[i]
        return result
```

This solution looks simple, but this is probably the hardest question of this exam and the one that is designed to identify the A+ students.

Alternative:

```
def deep_copy_check(lst1,lst2):
    if lst1 != lst2 or lst1 is lst2:
        return False
    else:
        for i in range(len(lst1)):
            if type(lst1[i]) == list and \
               not deep_copy_check(lst1[i],lst2[i]):
                return False
            elif lst1[i] is not lst2[i]:
                return False
        return True
```

C. [**Replacements**] Next, what we want to do is to replace elements within a tree consisting of trees with a function called `deep_replace`, which takes in a list and two arguments, a reference and a target and replaces all instances of reference with the target in the tree, i.e.

```
>>> a = [[[1],2],3,4]
>>> deep_replace(a,2,3)
>>> a
[[[1], 3], 3, 4]

>>> deep_replace(a,3,5)
>>> a
[[[1], 5], 5, 4]

>>> deep_replace(a,5,1)
>>> a
[[[1], 1], 1, 4]

>>> deep_replace(a,1,9)
>>> a
[[[9], 9], 9, 4]
```

Give a possible implementation for `deep_replace`. [4 marks]

```
def deep_replace(lst,a,b):
    for i in range(len(lst)):
        if type(lst[i]) == list:
            deep_replace(lst[i],a,b)
        elif lst[i] == a:
            lst[i] = b
```

Zero for students who do string slicing, which will create new lists instead of modifying the input list `lst`.

D. [**Counting Replacements**] Next, we don't just want to replace elements, we actually want to count the number of elements that got replaced in the process, i.e.

```
>>> a = [[[1],2],3,4]
>>> counting_deep_replace(a,2,3)
1
>>> a
[[[1], 3], 3, 4]

>>> counting_deep_replace(a,2,3)
0

>>> counting_deep_replace(a,3,5)
2
>>> a
[[[1], 5], 5, 4]

>>> counting_deep_replace(a,5,1)
2
>>> a
[[[1], 1], 1, 4]

>>> counting_deep_replace(a,1,9)
3
>>> a
[[[9], 9], 9, 4]
```

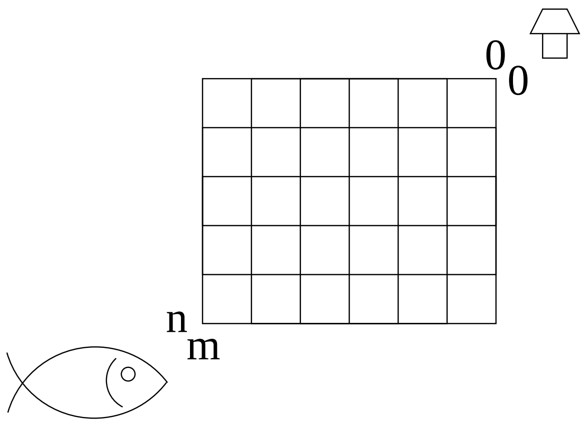
Give a possible implementation for `counting_deep_replace`. [4 marks]

```
def counting_deep_replace(lst,a,b):
    count = 0
    for i in range(len(lst)):
        if type(lst[i]) == list:
            count += counting_deep_replace(lst[i],a,b)
        elif lst[i] == a:
            lst[i] = b
            count += 1
    return count
```

No error carry forward, i.e. full credit, if the student got tthe answer for Part (C) wrong but modified Part (C) correctly to count the number of replacements.

Question 3: Finding Dory [24 marks]

In this question, you will help Dory (a fish) find her way home. For simplicity, we model the paths between Dory and home as an $m \times n$ grid as shown:



A. [**Warm Up**] We assume that Dory can only swim up or right and the number possible paths home is given by the function `paths`, which takes in 2 positive integers m and n as follows:

```
>>> paths(1,1)
2

>>> paths(2,2)
6

>>> paths(3,1)
4

>>> paths(3,2)
10

>>> paths(3,3)
20
```

Give a possible implementation for the function `paths`. [4 marks]

```
def paths(m,n):
    if m==0 or n==0:
        return 1
    else:
        return paths(m,n-1)+paths(m-1,n)
```

Some students would probably come up with the formula for $\binom{m+n}{m}$ and we will give to them for this question if they get their formula right, but it would be very very difficult to find a mathematical solution to the subsequent questions and so going down this path will be very dangerous.

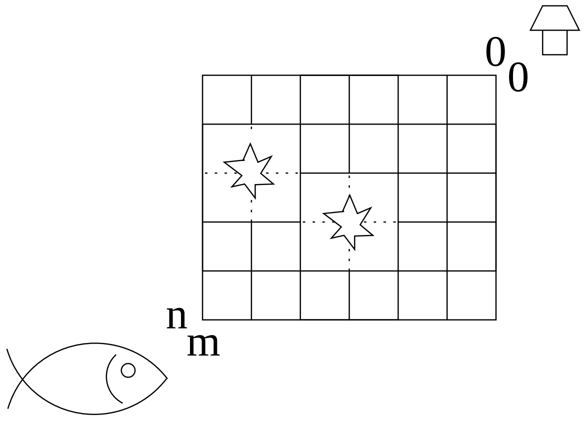
B. What is the order of growth in time and space for the function `paths` that you implemented in Part(A)? Explain. [4 marks]

Time: $O(2^{m+n})$, since it's some sort of tree recursion with 2 branches and where the maximum depth is determined by the larger of m and n.

Space: $O(m+n)$ since maximum depth is $m+n$.

We were particularly generous this year and gave full credit to answers $O(2^n)$ and $O(n)$.

C. [**Blocked Paths**] But alas, life is never simple and Dory sometimes runs into some obstacles that blocks off some junctions. The function `blocked_paths` takes in m and n and a tuple of blocked junctions which are coordinate pairs (tuples) and returns the number of possible paths.



```
>>> blocked_paths(1,1,((0,0),))
0
>>> blocked_paths(1,1,((1,0),))
1
>>> blocked_paths(1,1,((0,1),))
1
>>> blocked_paths(1,1,((1,1),))
0
>>> blocked_paths(2,2,((1,1),))
2
>>> blocked_paths(2,2,((1,1),(0,1)))
1
```

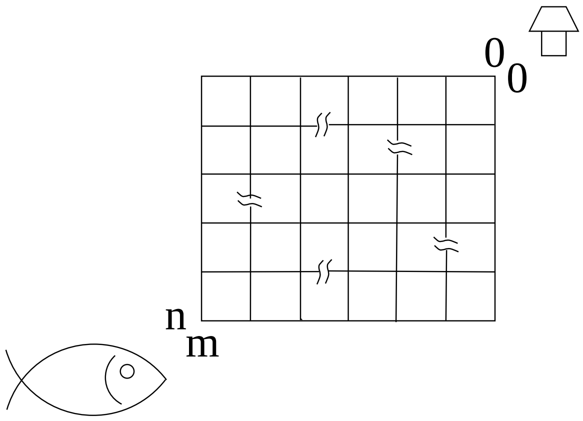
Give a possible implementation for the function `blocked_paths`. [6 marks]

```
def blocked_paths(m,n,blocks):
    if (m,n) in blocks:
        return 0
    elif m==0 and n==0:
        return 1
    elif m==0:
        return blocked_paths(0,n-1,blocks)
    elif n==0:
        return blocked_paths(m-1,0,blocks)
    else:
        return blocked_paths(m,n-1,blocks)+blocked_paths(m-1,n,blocks)
```

-1 mark for students who got the cases for `m==0` and `n==0` wrong, i.e. they just returned `1`.

Quite a few students got this Part and Part (D) wrong probably because they thought that this question was similar to the Red Riding Hood question from a past year. Well, no. The whole point of this question is to test that students have internalized recursion well enough to solve such problems from first principle.

D. [**Broken Paths**] Other times, obstacles only block off the path between junctions instead of the whole junction. The function `broken_paths` takes in *m* and *n* and a tuple of blocked connections which are sets of 4 coordinates $(from_x, from_y, to_x, to_y)$ which specifies which connections are broken.



```
>>> broken_paths(1,1,((1,1,1,0),))
1

>>> broken_paths(1,1,((1,1,0,1),))
1

>>> broken_paths(1,1,((1,0,0,0),))
1

>>> broken_paths(1,1,((0,1,0,0),))
1

>>> broken_paths(2,2,((0,1,0,0),))
3

>>> broken_paths(2,2,((1,1,0,1),))
4

>>> broken_paths(2,2,((1,1,0,1),(1,1,1,0)))
2
```

Give a possible implementation for the function `broken_paths`. [5 marks]

```
def broken_paths(m,n,blocks):
    if m==0 and n==0:
        return 1
    else:
        ans = 0
        if (m,n,m,n-1) not in blocks and n>0:
            ans += broken_paths(m,n-1,blocks)
        if (m,n,m-1,n) not in blocks and m>0:
            ans += broken_paths(m-1,n,blocks)
        return ans
```

Actually, the answer is incredibly simple. Just check at each juncture which path still works. Students need to learn to think simply and work on first principles.

E. [**Dynamic Programming**] Give a Dynamic programming implementation for the function `broken_paths`. If your implementation in Part(D) is already DP and correct, you will get the marks to this question for free (power to you!) [5 marks]

```
def dp_broken_paths(m,n,blocks):
    t = []
    row = [0]*(m+1)
    for i in range(n+1):
        t.append(row.copy())

    for i in range(m+1):
        for j in range(n+1):
            if i==0 and j==0:
                t[i][j] = 1
            else:
                ans = 0
                if (i,j,i,j-1) not in blocks and j>0:
                    ans += t[i][j-1]
                if (i,j,i-1,j) not in blocks and i>0:
                    ans += t[i-1][j]
                t[i][j] = ans
    return t[m][n]
```

This question is testing whether a student knows how to translate a recursive solution into DP. Student will still get credit if the logic in Part(D) is wrong, but this part is the correct translation of the code in Part(D) into a DP formulation.

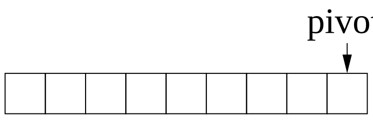
Up to 2 points is given to students who can set up the matrix correctly. Some students got 1 point because they forgot to copy each row.

Question 4: In-Place Quicksort [30 marks]

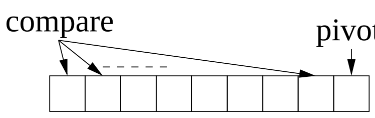
Quicksort is one of the classic sorts that we did not discuss in class. No problem. In this question, you will learn about it. How cool is that?

```
>>> a = [4,2,56,23,12,1,32,5,7]
>>> quicksort(a)
>>> a
[1, 2, 4, 5, 7, 12, 23, 32, 56]
```

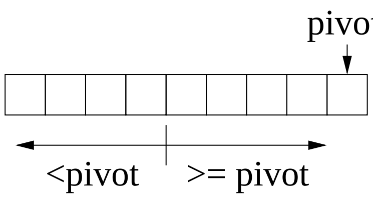
We call the last element of the list that we want to sort the *pivot*.



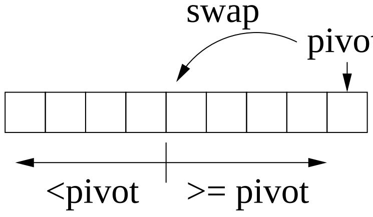
Next we compare first $n-1$ elements of the list with the pivot one at a time.



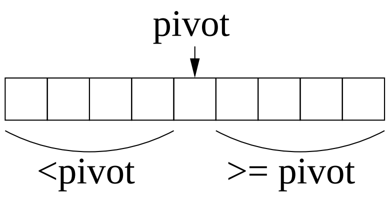
As we do so, we do some swaps so that by the time we are done, the elements are divided into 2 segments: the first segments are elments that are smaller than the pivot and the second are elements that are bigger or equal to the pivot.



We swap the pivot with the element at the boundary, so that the pivot is in between the two seqments.



We recursively apply the above process to the two segments on the left and right of the pivot. At some point, the list will get sorted.



Got it?

A. [**Sanity Check**] Explain what you understand by an *in-place* sort. [3 marks]

An in-place algorithm is an algorithm which transforms input using no auxiliary data structure. However a small amount of extra storage space is allowed for auxiliary variables. Basically, doesn't use storage or at most a small constant amount.

B. Implement the function `quicksort` that will do an in-place sort on an input list. **Hint:** If you really cannot figure out how to do this in-place, you can do it not-in-place and take a small penalty. [7 marks]

```
def quicksort(lst):
    def helper(start,end):
        if start >= end: # Base case
            return

        pivot = lst[end]
        current = start
        for i in range(start,end):
            if lst[i] < pivot:
                lst[i],lst[current] = lst[current],lst[i]
                current += 1
        lst[current],lst[end] = lst[end],lst[current]

        helper(start,current-1)
        helper(current+1,end)
    helper(0,len(lst)-1)
```

Students who gave up on in-place Quicksort to implement non-in-place Quicksort, i.e. by creating 2 new lists and calling recursion on them, get a 2 point penalty. However, non-in-place Quicksort is extremely cumbersome to implement in C, so these students will typically get Part (E) massively wrong.

C. What is the order of growth in time and space for the sort you implemented in Part(B) in terms of the number of elements n in the average case? Explain. [4 marks]

Time: $O(n \log n)$ (average case), this is because with high likelihood we get something similar to Merge sort.

Space: $O(1)$ since this is an in-place sort!

D. What is the worst case order of growth in time? When does it happen? Explain. [3 marks]

Worst case is $O(n^2)$ when the pivot is so bad that it never divides the list into 2 sets at each step. One example is either a sorted or a reverse sorted list.

E. Is the sort you implemented in Part(B) a *stable* sort? Explain. [3 marks]

Not stable. It is possible that in the swapping 2 equivalent elements might get swapped. Example is [3,7,9,7,5] $\rightarrow$ [3,5,9,7,7]. 7 in position 2 get swapped to rear.

The answer in this Part needs to follow the answer in Part (A). Those who implemented non-in-place Quicksort would typically have implemented a stable sort. Zero if the answer here is non-compliant with Part (A).

F. Explain how you would modify the code in Part(B) to get a reverse-sorted list. Explaining in words is good enough. Don't need to write code here. [3 marks]

Simply, flip the sign in the line `if lst[i] < pivot`. Other accepted solutions include:

1. Do big + pivot + small instead of small + pivot + big on combination.
2. reverse the list before returning at the end.

G. [*Quicksort in C*] Consider the following snippet of code in C:

```
#include <stdio.h>

void quicksort(int lst[],int start,int end);

void print_array(int a[], int length){
    printf("[");
    for (int i=0; i<length; i++){
        if (i!=length-1) {
            printf(" %d,", a[i]);
        } else{
            printf(" %d", a[i]);
        }
    }
    printf("]\n");
}

int main()
{
    int a[] = {4,2,56,23,12,1,32,5,7};
    quicksort(a,0,8);
    print_array(a,9);
    return 0;
}
```

Give a possible implementation for quicksort in C so that the above code works. Basically, translate your code in Part(B) to C! [7 marks]

```
void quicksort(int lst[],int start,int end)
{
    if (start >= end){
        return;
    }

    int pivot = lst[end];
    int current = start;
    int temp;
    for (int i=start; i<end; i++) {
        if (lst[i] < pivot){
            temp = lst[i];
            lst[i] = lst[current];
            lst[current++] = temp;
        }
    }
    temp = lst[end];
    lst[end] = lst[current];
    lst[current] = temp;
    quicksort(lst,start,current-1);
    quicksort(lst,current+1,end);
}
```

This is pretty straightforward. Only difference between C and Python is the need to create a temp variable. Credit will be given if code in Part(B) has a minor bug but the translation is correct.

This question was a disaster for most students. Essentially, most got some credit depending on how much of a disaster it was. Those who did non-in-place Quicksort in Part (B) had a lot of trouble with this part. Actually, this question would have hinted at the need to create a helper q function in Part (B).

Question 5: How Do We Do The Right Thing? [4 marks]

In his article *"How Will You Measure Your Life?,"* Professor Clayton Christensen recounted the following story:

> *I'd like to share a story about how I came to understand the potential damage of "just this once" in my own life. I played on the Oxford University varsity basketball team. We worked our tails off and finished the season undefeated. The guys on the team were the best friends I've ever had in my life. We got to the British equivalent of the NCAA tournament – and made it to the final four. It turned out the championship game was scheduled to be played on a Sunday. I had made a personal commitment to God at age 16 that I would never play ball on Sunday. So I went to the coach and explained my problem. He was incredulous. My teammates were, too, because I was the starting center. Every one of the guys on the team came to me and said, "You've got to play. Can't you break the rule just this one time?"*
>
> *I'm a deeply religious man, so I went away and prayed about what I should do. I got a very clear feeling that I shouldn't break my commitment–so I didn't play in the championship game.*
>
> *In many ways that was a small decision–involving one of several thousand Sundays in my life. In theory, surely I could have crossed over the line just that one time and then not done it again. But looking back on it, resisting the temptation whose logic was "In this extenuating circumstance, just this once, it's OK" has proven to be one of the most important decisions of my life. Why? My life has been one unending stream of extenuating circumstances. Had I crossed the line that one time, I would have done it over and over in the years that followed.*
>
> *The lesson I learned from this is that it's easier to hold to your principles 100% of the time than it is to hold to them 98% of the time. If you give in to "just this once," based on a marginal cost analysis, as some of my former classmates have done, you'll regret where you end up. You've got to define for yourself what you stand for and draw the line in a safe place."*

If you have come across a situation like this in your own life, describe it and tell us how you found the inner strength to do the right thing. Or if you didn't, repent now and tell us how you think you would have convinced yourself to do better if you had the chance to relive that moment. If not, think of a situation where you will be tempted to "take the slippery slope," and explain how you would convince yourself to **do the right thing** and not the convenient thing.

The student will be awarded points as long as he/she is coherent and doesn't say something obviously wrong. Exactly how many points depends on the effort and thoughtfulness put into writing this mini-essay. Amuse the prof and you get full credit.

The point of this question is to encourage students to reflect critically on what they read.

— E N D O F P A P E R —

Scratch Paper

– H A P P Y   H O L I D A Y S ! –