

Remedial

Higher Order Functions

```
def foo(x, y):  
    return lambda y: x(y, y)  
def bar(t):  
    return foo(lambda x, y: x+y+t, t)  
print(bar(bar(3)(2))(3))
```

$\text{bar}(\text{bar}(3)(2))(3) \rightarrow \text{bar}(f_{g3}(2))(3)$

$\text{bar}(f_{g3}(2))(3) \rightarrow \text{bar}(7)(3)$

$\text{bar}(7)(3) \rightarrow f_{g7}(3)$

$f_{g7}(3) \rightarrow 13$

$\text{foo}(x, y) \rightarrow f_x, \quad f_x(a) = x(a, a)$

$\text{bar}(t) \rightarrow \text{foo}(g_t, t), \quad g_t(x, y) = x + y + t$
 $\rightarrow f_{g_t}, \quad f_{g_t}(a) = g_t(a, a) = 2a + t$

Higher Order Functions

```
once = lambda f: lambda x: f(x)
twice = lambda f: lambda x: f(f(x))
thrice = lambda f: lambda x: f(f(f(x)))
print(thrice(twice)(once)(lambda x: x + 2)(9))
```

`thrice(twice)(once)(lambda x: x + 2)(9)` \rightarrow `htwice(once)(plustwo)(9)`

`htwice(once)(plustwo)(9)` \rightarrow `twice(twice(twice(once)))(plustwo)(9)`

`twice(twice(twice(once)))(plustwo)(9)` \rightarrow `twice(twice(gonce))(plustwo)(9)`

`twice(twice(gonce))(plustwo)(9)` \rightarrow `twice(?) (plustwo)(9)`

`once(f) \rightarrow f,` `f(x) = f(x)`

`twice(f) \rightarrow gf,` `gf(x) = f(f(x))`

`thrice(f) \rightarrow hf,` `hf(x) = f(f(f(x)))`

Higher Order Functions

```
once = lambda f: lambda x: f(x)
twice = lambda f: lambda x: f(f(x))
thrice = lambda f: lambda x: f(f(f(x)))
print(thrice(twice)(once)(lambda x: x + 2)(9))
```

```
twice(once)(f)
= once(once(f))
= once(f)
= f
```

```
twice(twice(once))(f)
= once(once(once(once(f))))
= once(once(once(f)))
= once(once(f))
= once(f)
= f
```

```
twice(twice(twice(once)))(plustwo)(9)
= plustwo(9)
= 11
```