VIDEO 5.7

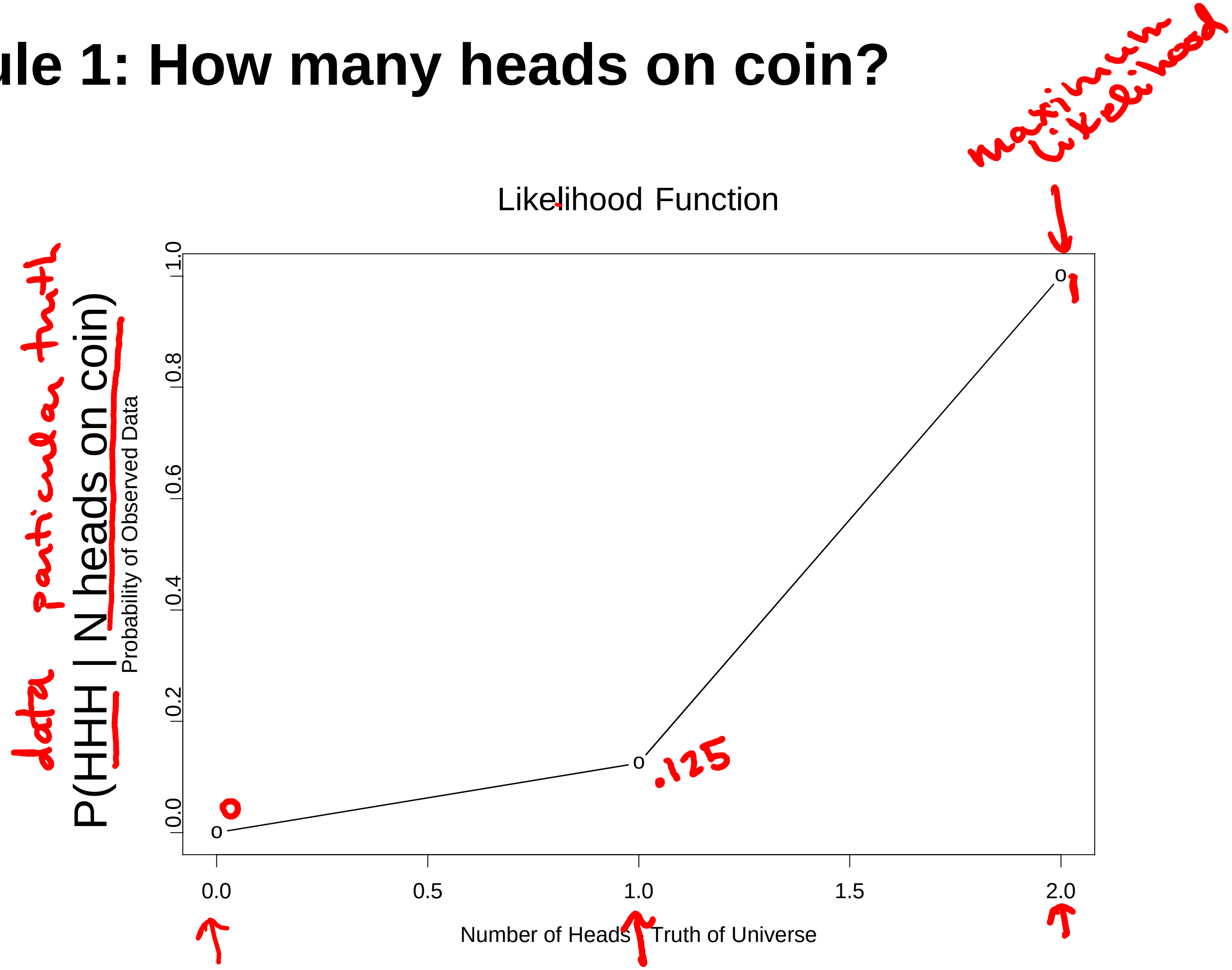
Maximum likelihood estimation

- Fitting logistic regression using maximum likelihood

Estimating coefficients in logistic regression

Recall from Module 1: How many heads on coin?

data: HHH



Maximum likelihood to estimate coefficients

Likelihood for coin flips:

p = unknown probability of an H on a single flip

Collect data: 10 flips ⇒ 8 H's

$$L(p) = \underbrace{p\cdot p\cdot p\cdot p\cdot p\cdot p\cdot p\cdot p}_{8\text{ H's}}\cdot\underbrace{(1-p)\cdot(1-p)}_{2\text{ T's}} = \underline{p^8(1-p)^2}$$

assume flips are independent

Likelihood for logistic regression:

β_0 and β_1 are unknown "true" coefficient values in the population

Collect data: NNIPS-II

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> table(death180, gestage)
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gestage

	Death	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46
A	0	84	111	140	151	214	319	419	475	585	830	1193	1330	1064	743	531	329	267	194	107
D	1	18	32	37	31	36	35	30	44	37	41	52	60	45	27	20	17	21	6	8

$$L(\beta_0,\beta_1) = P(\text{data} \mid \beta_0,\beta_1)$$

assume infants are independent

Estimation of regression coefficients by method of maximum likelihood

$\rightarrow L(\beta_0, \beta_1) = P(\text{data} \mid \beta_0, \beta_1)$

$= P(D \mid GA=28)^{18} \cdot P(A \mid GA=28)^{84}$

$\cdot P(D \mid GA=29)^{32} \cdot P(A \mid GA=29)^{111}$

• continue for all GA in our data

$$= \left[\frac{e^{\beta_0+\beta_1(28)}}{1+e^{\beta_0+\beta_1(28)}}\right]^{18} \left[\frac{}{1+e^{\beta_0+\beta_1(28)}}\right]^{84} \left[\quad\right]^{32} \left[\quad\right]^{111} \cdots$$

for all GA

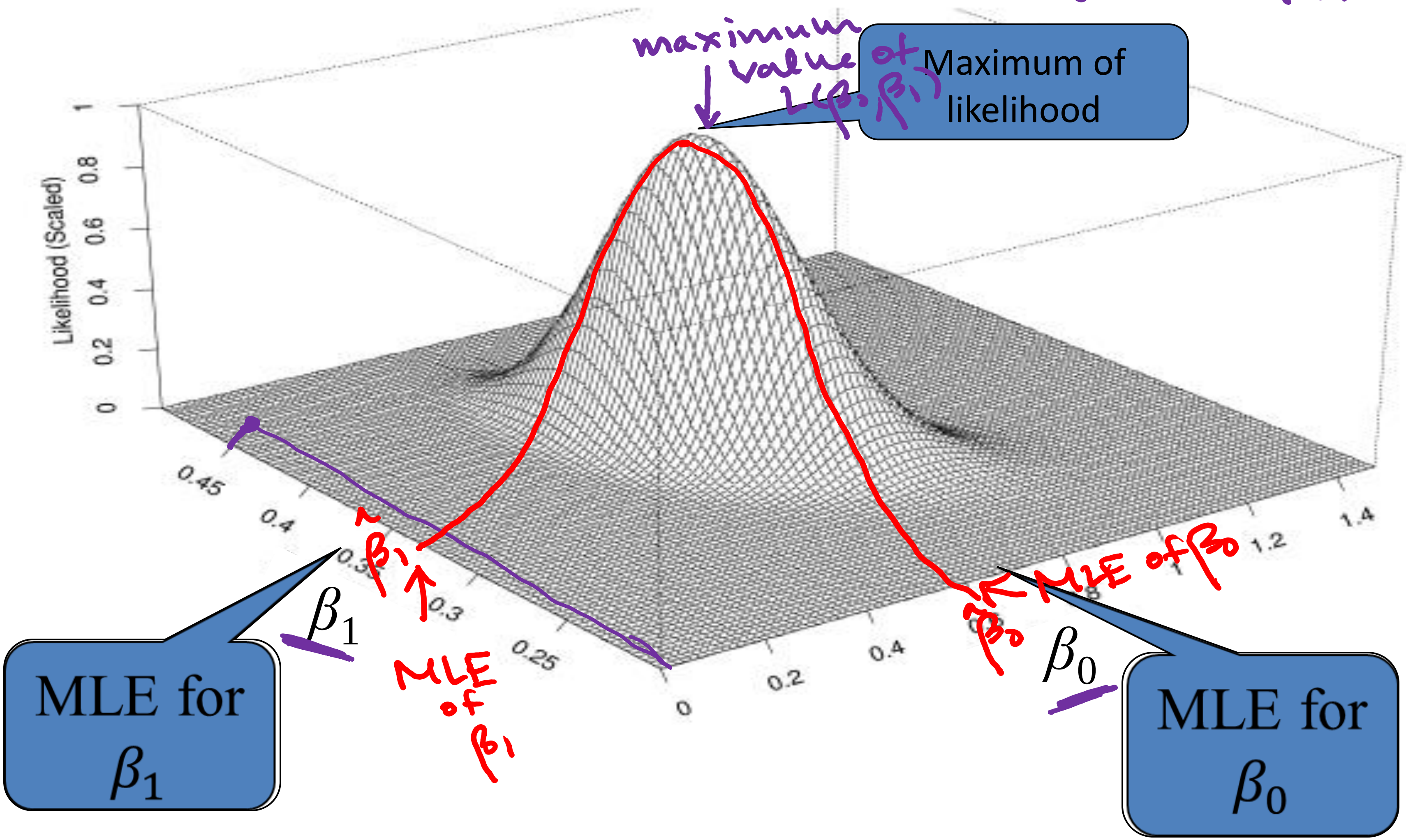
• we use calculus to maximize $L(\beta_0, \beta_1)$ with respect to β_0, β_1

• this gives us the values $\hat{\beta}_0, \hat{\beta}_1$ that give the maximum likelihood

• so $\hat{\beta}_0, \hat{\beta}_1$ are the maximum likelihood estimates (MLEs)

Example likelihood function

• try different pairs of (β_0, β_1)
• graph $L(\beta_0, \beta_1)$



⇒ Note: this is an example, not the actual likelihood function for our data