

Implementation of a Carry Lookahead Fast Adder

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Abstract—Carry lookahead adders are one of the most common fast-adder architectures used in high performance computing to date. They allow for arbitrarily large summations with only $O(\log(n))$ complexity, rather than the $O(n)$ complexity of a traditional ripple-carry adder.

The purpose of this project is to implement a simulation of the execution of a carry lookahead adder using the Intel 8051 architecture. Although this simulation cannot achieve the performance of a hardware implementation, it serves as a proof of concept for a carry lookahead adder.

I. Introduction

The carry lookahead adder was originally patented by IBM[1] in 1972. As time passed and technology improved, this method of addition was shown to be even more useful than originally thought, as it allows for the quick computation of numbers that would take a non-insignificant amount of time using the traditional ripple-carry method.

Our goal in this project is to implement a simulation of a carry lookahead adder on the 8051 microcontroller as a proof of concept. Since execution is serial, this simulation will not be faster than the built-in adding operations, as we are unable to calculate multiple things at once, as we could were we building a hardware model of a carry lookahead adder.

II. Approach

The carry value of a certain bit addition can be determined by looking at the previous pair of bits, finding their generate and propagate values, and comparing the propagate value to the carry of the previous bit operation. After a string of carries is created then all that must be done is two XOR operations because XORing the first input with the second input and XORing the result of that operation with the carry string gives the logical equivalent of adding the two numbers directly.

III. Results and Discussion

By first generating our Propagate(P) and Generate(G) strings from the input, and then stepping through our carry string and deriving each bit from the P and G strings and the previous carry bit, we emulate the stages of operation of a carry lookahead adder.

The P string is created by XORing each pair of respective input bits, and the G by ANDing together each pair of input bits. This would be done in hardware with blocks of two logic gates all operating in parallel, and would take approximately $2\Delta t$ to complete.

The next stage, wherein the P and carry bit are XORed together to create the sum, is emulated by a loop that XORs bytes of the carry string with bytes of the P string. While this adds another n time in the software solution, it would implement only another $2\Delta t$ in a hardware solution, since it would be only two more logic gates for each bit, all operating in parallel.

Algorithm 1 shows the basic implementation of this simulation:

Algorithm 1 Pseudocode for CLA Adder

```

R3 → carry string
R6 → Input A
R7 → Input B
for all bytes  $P_i \in P, G_i \in G, RX_i \in RX$  do
   $P \leftarrow R6 \oplus R7$ 
   $G \leftarrow R6 \wedge R7$ 
end for
for all bits  $C_i \in C, P_i \in P, G_i \in G$  do
   $C \leftarrow G \vee (P \wedge C)$ 
end for
for all bytes  $SUM_i \in SUM, C_i \in C, P_i \in P$  do
   $SUM_i \leftarrow C_i \oplus P_i$ 
end for

```

TABLE I
Inputs, Sum, and Computation time

Operand 1	Operand 2	Sum	Time(clock pulses)
0x8A	0x25	0xAF	0xA0
0x1FFF	0xA222	0xC111	0x12E
0x3EFF	0x8AAA	0xC9A9	0x12E
0x254621	0xAAAAAA	0xCFF0CB	0x01D4
0x8532AA	0x243699	0xA96943	0x01D5
0x23456789	0x98765432	0xBBBBBBBB	0x026A
0xABABABAB	0x11111111	0xBCBCBCBC	0x04C2
0xC1C1C1C1C1	0xC1C1C1C1C1	0xDDDDDDDDDD	0x0300
0x123456789ABC	0x123456789ABC	0x2468ACF13578	0x0396
0x1234567C1C1C1C	0x11AAAAA8532AA	0x23DF0126A14EC6	0x042E

Operand Length vs Computation Time

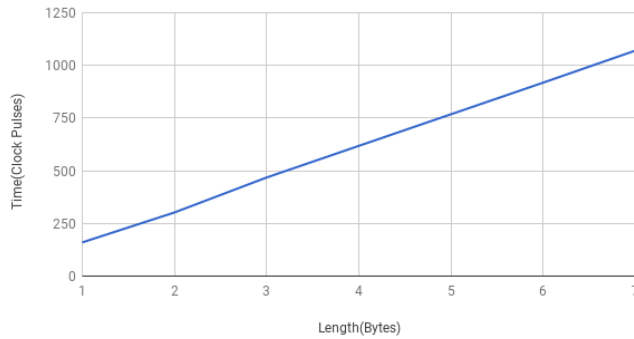


Fig. 1. Inputs vs Computation Time

As can be seen in Fig. 1, this software simulation falls far short of the speed of its hardware implementation. However, it does match very closely the speed of a ripple-carry add on the 8051 microcontroller, changing only the constant in the time-complexity.

IV. Summary and Conclusion

Upon testing we found that because the carry lookahead adder must be executed in serial, the simulation doesn't end up being any faster than simply adding the two values together. With that said, we were still successful in creating and implementing the carry lookahead adder, and, if given access to different hardware, we would be able to lower the execution time for the addition.

V. Source Code

```

MOV 40H, #00H ; Input A
MOV 41H, #00H
MOV 42H, #00H
MOV 43H, #00H
MOV 44H, #00H
MOV 45H, #00H
MOV 46H, #00H
MOV 47H, #00H

MOV 48H, #00H ; Input B
MOV 49H, #00H
MOV 4AH, #00H
MOV 4BH, #00H
MOV 4CH, #00H
MOV 4DH, #00H
MOV 4EH, #00H
MOV 4FH, #00H

MOV R2, #8H ; Length of longest operand in bytes

MOV R0, #40H
MOV R1, #48H
MOV A, R2
MOV R5, A ; length of operands stored in R2

SETUP: MOV SCON, #10000010B
        MOV TMOD, #00010000B
        MOV TL1, #00H ; start timer at 0
        MOV TH1, #00H ; start timer at 0

INPUTA: JNB TI, $ ; wait until ready to transmit
        CLR TI
        MOV A, @R0
        MOV C, P
        MOV TB8, C ; set odd parity bit
        MOV SBUF, A ; output byte of A
        INC R0 ; move to next byte
        DJNZ R5, INPUTA
        MOV R0, #40H ; reset to beginning of A

        MOV A, R2 ; reset counter
        MOV R5, A

INPUTB: JNB TI, $ ; wait until ready to transmit
        CLR TI
        MOV A, @R1
        MOV C, P
        MOV TB8, C ; set odd parity bit
        MOV SBUF, A ; output byte of B
        INC R1
        DJNZ R5, INPUTB

```

```

MOV R1, #48H ; reset to beginning of B

SETB TR1 ; start timer
MOV A, R2 ; init counter
MOV R5, A
LOAD: MOV A, @R0 ; temp hold for byte of R6 data
      MOV R4, A
      MOV A, @R0
      XRL A, @R1 ; propagate
      MOV @R0, A

      MOV A, @R1
      ANL A, R4 ; generate
      MOV @R1, A
      INC R0 ; move to next bit of P
      INC R1 ; move to next bit of G
      DJNZ R5, LOAD

      MOV A, R2 ; reset R5
      MOV R5, A
      CLR C ; C will be used as Ci in boolean equation
      MOV A, R2
      DEC A
      ADD A, #40H
      MOV R0, A ; point at least significant byte
      MOV A, R2
      DEC A
      ADD A, #48H
      MOV R1, A ; point at least significant byte

CARRY: MOV B, @R1
        MOV A, @R0
        MOV R4, #8H ; set counter for 8 rotations
        MOV @D6H, C ; store carry-in
        BITE: ANL C, @D0H ; intermediate = Ci AND P(i+1)
              ORL C, @F0H
              MOV @F0H, C ; save Ci into C as well as R3.0
              RR A ; rotate P byte
              MOV @R1, A ; store P byte in mem
              MOV A, B ; move C/C to A for rotation
              RR A ; rotate C/C byte
              MOV B, A ; replace byte in R3
              MOV A, @R1 ; reload P from mem
              DJNZ R4, BITE ; repeat for whole byte
              MOV A, B
              MOV @D6H, C ; store carry-out
              CLR C
              RLC A ; rotate C/G string to align carries over correct bits
              MOV C, @D6H ; restore carry-out
              JNB @D6H, NOINC
              INC A ; increment if there was a carry-in
              NOINC: MOV @R1, A ; replace C/G in memory

              DEC R0
              DEC R1
              DJNZ R5, CARRY

              MOV R0, #40H ; return to beginning of P
              MOV R1, #48H ; return to beginning of C/G
              MOV A, R2
              MOV R5, A
              SUM: MOV A, @R0 ; compute final sum
                   XRL A, @R1 ; move to next bit of P
                   INC R0 ; move to next bit of Carry string
                   INC R1
                   DJNZ R5, SUM
                   CLR TR1 ; stop timer

TIME: JNB TI, $ ; wait until ready to transmit
      CLR TI
      MOV A, TH1
      MOV C, P
      MOV TB8, C ; set odd parity bit
      MOV SBUF, A ; output high byte of time
      JNB TI, $ ; wait until ready to transmit
      CLR TI
      MOV A, TL1
      MOV C, P
      MOV TB8, C ; set odd parity bit
      MOV SBUF, A ; output low byte of time

      MOV A, R2
      MOV R5, A
      OUT: MOV R0, #40H ; reset R0 to beginning of result string
          JNB TI, $ ; wait until ready to transmit
          CLR TI
          MOV A, @R0
          MOV C, P
          MOV TB8, C ; set odd parity bit
          MOV SBUF, A ; output byte of result
          INC R0
          DJNZ R5, OUT

      FLUSH: MOV A, #0FFH
             MOV C, P
             MOV TB8, C ; set odd parity bit
             MOV SBUF, A ; output dummy byte to flush output
             END

```

References

- [1] Franz, S. and Dieter, S. Parallel binary carry look-ahead adder system. [US Patent 3,700,875]. <https://www.google.com/patents/US3700875> 1972.