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Instructions:

1. You should finish and hand-in the in-class exercise **in class**.
2. If you have any questions about the questions, you can ask the instructor during the class time.
3. Late submission of in-class exercise will NOT be entertained and 0 mark will be given for the in-class exercises.
4. Please provide a difficulty level (DL) score between 0 and 10 to indicate how difficult you found this exercise after completing it. 0: extremely easy. 10: extremely difficult.

Questions:

1. Select a number X randomly from the set 1,2,3,4. Then, select a number Y randomly from the set $1, 2, \dots X$. Find the joint probability distribution of (X, Y) and $P(X = Y)$.

$$P(X=x) = \frac{1}{4} \qquad (x = 1, 2, 3, 4).$$

$$P(Y=y \mid X=x) = \frac{1}{x} \qquad (y = 1, 2, \cdots x).$$

$$P(Y=y, X=x) = P(Y=y \mid X=x) \cdot P(X=x) = \frac{1}{4x} \quad \text{for } y \leq x.$$

$$P(Y=1, X=1) = \frac{1}{4}$$

$$P(Y=1, X=2) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$P(Y=2, X=2) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

$$P(Y=1, X=3) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

$$P(Y=2, X=3) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

$$P(Y=3, X=3) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$$

$$P(Y=1, X=4) = \frac{1}{16}$$

$$P(Y=2, X=4) = \frac{1}{16}$$

$$P(Y=3, X=4) = \frac{1}{16}$$

$$P(Y=4, X=4) = \frac{1}{16}$$

$$\text{and } P(X=y) = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{16}$$

$$= \frac{12+6+4+3}{48}$$

$$= \frac{25}{48}$$

2. There are n balls in a bag, numbered from 1 to n. A ball is drawn randomly from the bag. If the ball with number 1 is drawn, you get 1 point and stop drawing. If the ball with number i (where $i \geq 2$) is drawn, you get i points, and the ball is put back into the bag, and you draw again. This process continues until you get the ball with number 1. Find the expected total score.

Let X be the expected score, Y be the first ball to be drawn

$$E(X) = E(X | Y=1) \cdot \frac{1}{n} + \sum_{i=2}^{n} E(X | Y=i) \cdot \frac{1}{n}$$

You get i grades and process repeat

$$= \frac{1}{n} + \sum_{i=2}^{n} (i + E) \cdot \frac{1}{n}$$

$$= \frac{1}{n} + \left(\frac{(2+n)(n-1)}{2} + (n-1)E \right) \cdot \frac{1}{n}$$

$$\frac{\#}{n} E = \frac{1}{n} \cdot \left(\frac{(2+n)(n-1)+2}{2} \right)$$

$$E = \frac{n^2 + n}{2}$$

3. Why did you choose this course? What are your expectations for this course?

To enhance my knowledge of probability and statistics because I'm poor in these topics. To learn more statistics knowledge

Marks: