

## COMPLEX NUMBERS.

1. If  $z + z^{-1} = 2 \cos \theta$ ,  $\theta \in \mathbb{R}$ , show that

$$z^m + z^{-m} = 2 \cos(m\theta), \quad m \in \mathbb{N}.$$

2. Given that  $13 = 2^2 + 3^2$  and  $74 = 5^2 + 7^2$ , express the product  $13 \times 74$  as a sum of two squares.

3. If

$$\left| \frac{z-9}{z-1} \right| = 3,$$

show that  $|z| = 3$ .

4. Find  $z \in \mathbb{C}$ , if

$$\left| \frac{z}{z+1} \right| = 1 \quad \text{and} \quad \frac{z}{\bar{z}} = i.$$

5. Identify geometrically each of the following sets in  $\mathbb{C}$ . Sketch the graph in each case:

(a)  $A := \left\{ z \in \mathbb{C} : \left| \frac{z-a}{z-b} \right| = 1, a \neq b \right\};$

(b)  $A := \{ z \in \mathbb{C} : 0 \leq \operatorname{Re}(iz) < 1 \};$

(c)  $A := \{ z \in \mathbb{C} : |z| + \operatorname{Re}(z) \leq 1 \}.$