Discrete Mathematics Homework 1

1. (a) In how many ways can eight people, denoted A, B, .. . , H be seated about the square table shown in Fig. 1, where Figs. 1(a) and 1(b) are considered the same but are distinct from Fig. 1(c)?

(b) If two of the eight people, say A and B, do not get along well, how many different seatings are possible with A and B not sitting on the same side?

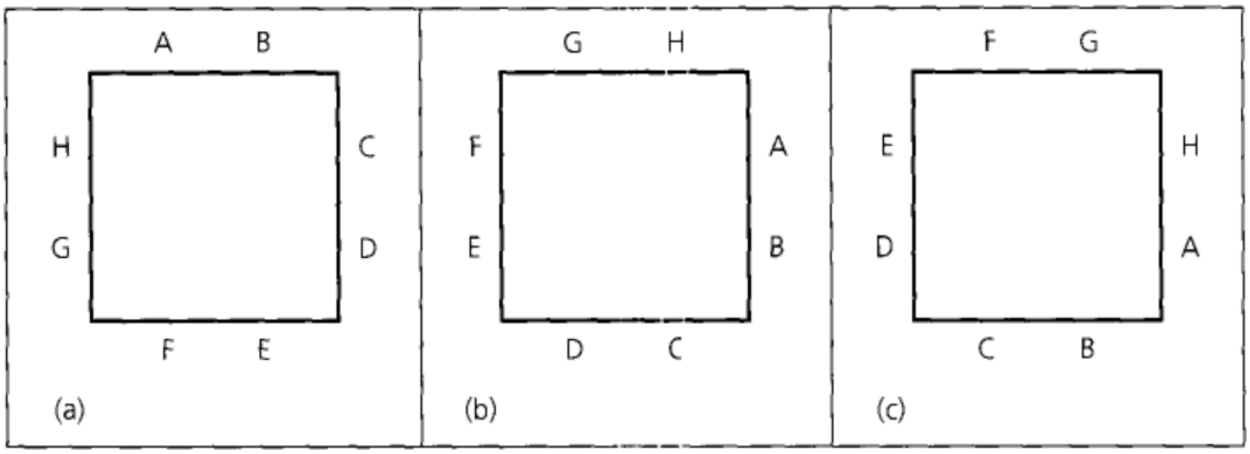

Figure 1

2 a) Find the number of ways to write 17 as a sum of 1's and 2's if order is relevant.

b) Answer part (a) for 18 in place of 17.

c) Generalize the results in parts (a) and (b) for *n* odd and for *n* even.

3. a) How many of the 9000 four-digit integers 1000, 1001, 1002, ..., 9998, 9999 have four distinct digits that are either increasing (as in 1347 and 6789) or decreasing (as in 6421 and 8653)?

b) How many of the 9000 four-digit integers 1000, 1001, 1002,... , 9998, 9999 have four digits that are either non-decreasing (as in 1347, 1226, and 7778) or nonincreasing (as in 6421, 6622, and 9888)?

4. How many distinct four-digit integers can one make from the digits 1, 3, 3, 7, 7, and 8?

5. a) In how many ways can a particle move in the *xy*-plane from the origin to the point (7, 4) if the moves that are allowed are of the form:

$$(R):(x,y)\to(x+1,y);\quad (U):(x,y)\to(x,y+1)?$$

b) Answer parts (a) if a third type of move

$$(D):(x,y)\to(x+1,y+1)$$

is also allowed.