Discrete Mathematics Homework 7

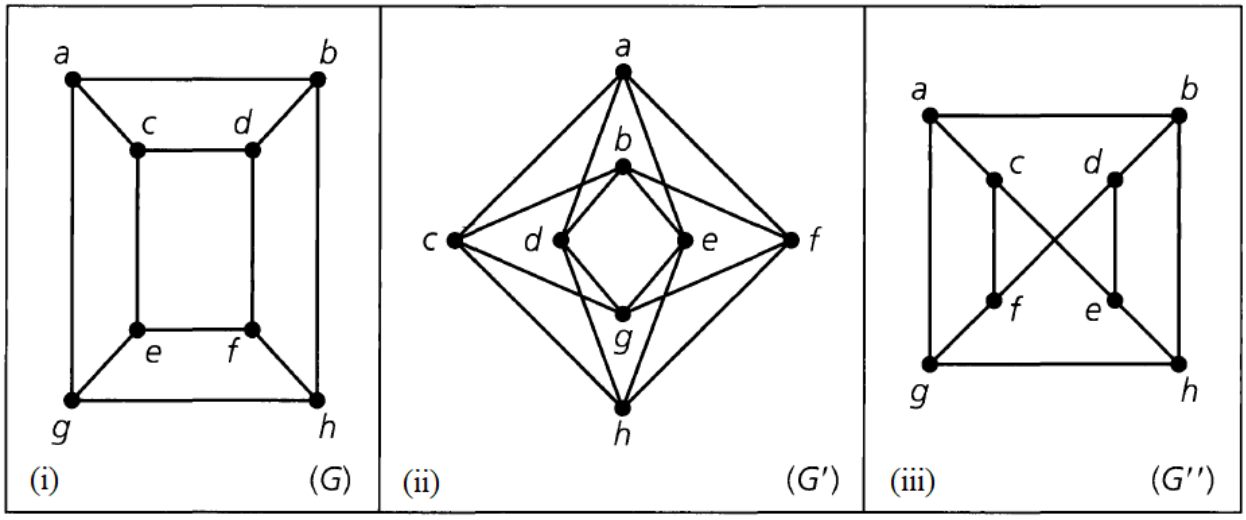
1. Let $G = (V, E)$ be a loop-free connected graph with $|V| = v$. If $|E| > (v/2)^2$, prove that G cannot be bipartite.

2. a) Let $G = (V, E)$ be a loop-free connected graph with $|V| \geq 11$. Prove that either G or its complement $\bar{G}$ must be nonplanar.

b) Find a counterexample to part (a) for $|V| = 8$.

3 (a) Prove that a graph is bipartite if and only if it contains no odd cycles.

(b) Which is of the following graph is bipartite? If bipartite, give 2 subsets V_1 and V_2 such that $V = V_1 \cup V_2$ and $V_1 \cap V_2 = \emptyset$.



4. Let G be a connected planar graph such that degree of every vertex is 3. The length of boundary of every region in G is 4, 6 or 8. Every vertex lies on one region of length 4, one region of length 6, and one region of length 8.

Determine the number of regions of each length and the number of regions of G (including infinite region).

5. Let $G = (V, E)$ be an undirected connected loop-free graph. Suppose further that G is planar and determines 53 regions. If, for some planar embedding of G, each region has at least five edges in its boundary, prove that $|V| \geq 82$.