Assignment 4 **MA-4093**

DIFFERENTIABILITY. CAUCHY-RIEMANN EQUATIONS

1. Show by definition that the functions $f_1(z) = \operatorname{Re} z$, $f_2(z) = \operatorname{Im} z$, $z = x + iy$ are not complex differentiable at any point $z \in \mathbb{C}$.

2. Show that each of the following functions $f = f(z), z = x + iy$ is nowhere differentiable:

   (a) $f(z) = \cos y - i \sin y$;

   (b) $f(z) = 2y - ix$;

   (c) $f(z) = \operatorname{Im}(z) + 2i\operatorname{Re}(z)$.

3. Find the derivatives of the following functions:

   (a) $(z^2-1)^n$; (b) $1/(z^2+3)$; (c) $z/(z^3-5)$; (e) $(az+b)/(cz+d)$.

4. Show that

$$1 + 2z + \cdots + nz^{n-1} = \frac{1 - z^n}{(1 - z)^2} - \frac{nz^n}{1 - z}.$$

5. Determine the points $z \in \mathbb{C}$, where the function $f(z)$, $z = x + iy$ is differentiable and find its derivative at each such point.

$$(a)\ f(z) = z|z|^2; \quad (b)\ f(z) = \cos x + i \sin y.$$

6. Show that the function $f(z) = \cos z$ is differentiable at any point $z \in \mathbb{C}$ and find its derivative.