Discrete Mathematics Homework 2

1. Determine how many integer solutions there are to $x_1 + x_2 + x_3 + x_4 = 19,$ if

(a) $0 \le x_i$ for all $1 \le i \le 4$

(b) $0 \le x_i < 8$ for all $1 \le i \le 4$

(c) $0 \le x_1 \le 5, 0 \le x_2 \le 6, 3 \le x_3 \le 7, 3 \le x_4 \le 8.$

2. Find the rook polynomials for the shaded chessboards in Fig. 1. Explain your answer.

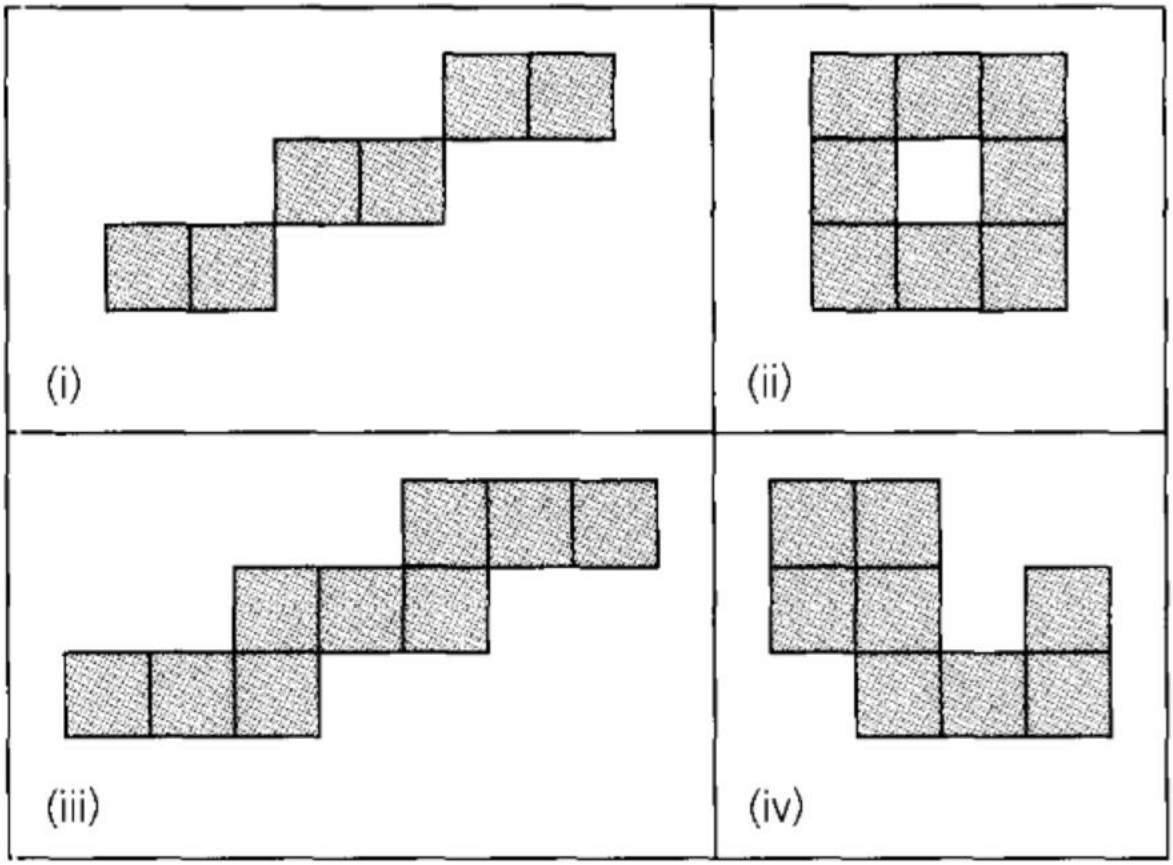

Fig. 1

3. In how many ways can one arrange the letters in CORRESPONDENTS so that

(a) there is no pair of consecutive identical letters?

(b) there are exactly two pairs of consecutive identical letters?

(c) there are at least three pairs of consecutive identical letters?

4. (a) In how many ways can the integers 1, 2, 3, …, n be arranged in a line so that none of the patterns 12, 23, 34, . . . , $(n-1)n$ occurs?

(b) Show that the result in part (a) equals $d_{n-1} + d_n$?

$$\left[d_n = n! - \binom{n}{1}(n-1)! + \binom{n}{2}(n-2)! - \cdots + (-1)^n \binom{n}{n}(n-n)! \right]$$
$$= \text{the number of darangements of } 1, 2, 3, \cdots, n \Big]$$

5. Find the number of ways to arrange the letters in LAPTOP so that none of the letters L, A, T, O is in its original position and the letter P is not in the third or sixth position.