Assignment 3 **MA-4093**

Complex Numbers.

1. Show that for any complex number $z \in \mathbb{C}$ the following relations hold:

$$\cos(2z) = \cos^2 z - \sin^2 z, \tag{1}$$
$$\sin z = \sin(\pi - z). \tag{2}$$

2. Show that the set

$$S := \{z \in \mathbb{C} : |z| < 1 \quad \text{and} \quad \operatorname{Re} z > 0\},$$

is open.

3. Express the function $f = f(z)$, $z = x + iy$ in the form $f(z) = u(x, y) + iv(x, y)$ where u, v are real-valued functions, if:

(a) $f(z) = z^2 + 3z^3$;

(b) $f(z) = \dfrac{z}{1+z}, \quad z \neq -1$;

(c) $f(z) = i\overline{z} + \operatorname{Im}\left(\dfrac{i}{z}\right), \quad z \neq 0$.

4. Show by definition that if $\lim_{z \to z_0} f(z) = L$, then $\lim_{z \to z_0} \operatorname{Re} f(z) = \operatorname{Re} L$.

5. Show that function $f(z)$ does not have any limit as $z \to 0$, if

$$(a) \quad f(z) = \frac{\operatorname{Re} z}{z}; \quad (b) \quad f(z) = \frac{\overline{z}}{|z|^2}.$$

6. Find the limit

$$\lim_{z \to i} \frac{iz^3 - 1}{z + i}.$$