Discrete Mathematics Homework 6

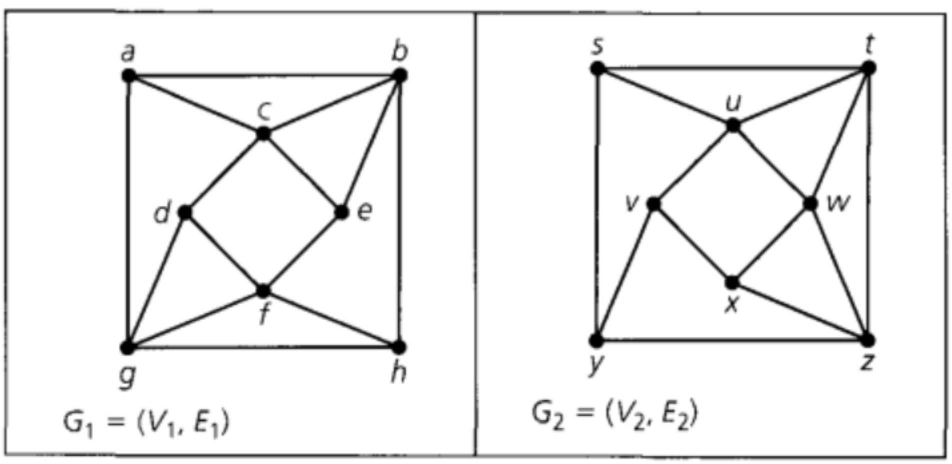
1. (a) Let G be an undirected graph with n vertices. If G is isomorphic to its own complement $\overline{G}$, how many edges must G have? (Such a graph is called self-complementary.)

(b) Find an example of a self-complementary graph on four vertices and two examples on five vertices.

(c) If G is a self-complementary graph on n vertices, where $n > 1$, prove that $n = 4k$ or $n = 4k + 1$, for some $k \in \boldsymbol{N}$.

(d) Let G be a cycle on n vertices. Prove that G is self-complementary if and only if $n = 5$.

2. Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be the loop-free undirected connected graphs in below figure.



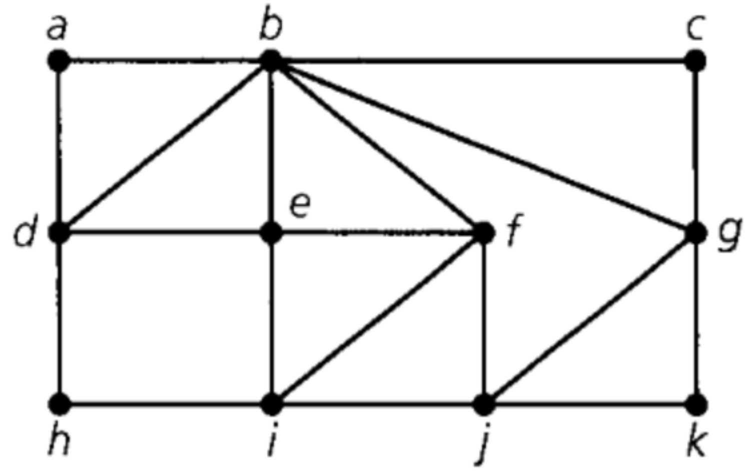
(a) Determine $|V_1|, |E_1|, |V_2|,$ and $|E_2|$.

(b) Find the degree of each vertex in V_1. Do likewise for each vertex in V_2.

(c) Are the graphs G_1 and G_2 isomorphic?

3 (a) Find an Euler circuit for the graph in below figure.

(b) If the edge $\{d, e\}$ is removed from this graph, find an Euler trail for the resulting subgraph.



4 (a) Let $G = (V, E)$ be a directed graph or multigraph with no isolated vertices. Prove that G has a directed Euler circuit if and only if G is connected and $od(v) = id(v)$ for all $v \in V$.

(b) If $G = (V, E)$ is a directed graph or multigraph with no isolated vertices, prove that G has a directed Euler trail if and only if

(i) G is connected;

(ii) $od(v) = id(v)$ for all but two vertices x, y in V; and

(iii) $od(x) = id(x) + 1$, $id(y) = od(y) + 1$.

5. Let G be a directed graph on n vertices. If the associated undirected graph for G is K_n, prove that $\sum_{v\in V}\left(od(v)^2\right) = \sum_{v\in V}\left(id(v)^2\right)$.