Assignment 2 **MA-4093**

COMPLEX NUMBERS.

1. Write the following complex numbers in polar form:

$$z_1 = 3 + 3i, \quad z_2 = 2 - i3\sqrt{2}, \quad z_3 = -3 - \sqrt{3}i.$$

2. Find each of the following numbers and give your answer in the form $z = x + iy$:

$$z_1 = \sqrt{-8i}, \quad z_2 = \sqrt{-11 + 60i}, \quad z_3 = \sqrt[4]{-1}.$$

3. Find all solutions of the equations

$$iz^2 + 2iz + 2 + i = 0, \tag{1}$$
$$z^4 + z^2 + 1 = 0. \tag{2}$$

4. Describe the image $f(D)$ of the set $D \subset \mathbb{C}$ under the mapping f, if:

   (a) $f(z) = \overline{z}$ and $D := \{z \in \mathbb{C} : |z - (2 + i)| \leq 1\}$;
   (b) $f(z) = (\mathrm{Re}\,(z))^2 + 2i$ and $D := \{z \in \mathbb{C} : |z| \leq 1\}$;
   (c) $f(z) = z + 6$ and $D := \{z \in \mathbb{C} : \mathrm{Re}\, z > 0\}$;
   (d) $f(z) = 1/z$ and $D := \{z \in \mathbb{C} : 0 < |z| < 1\}$;
   (e) $f(z) = z^2$ and $D := \{z \in \mathbb{C} : 1 \leq \mathrm{Re} \ \leq 2\}$;

5. Determine the following complex numbers:

$$z_1 = 3\log(1 + i\sqrt{3}), \quad z_2 = \log(1 + i\sqrt{3})^3, \quad z_3 = \log(1 + i)^{\pi i}.$$

6. Solve each of the equations:

$$e^z = 1 + i, \tag{3}$$
$$\mathrm{Log}\,(1 + z) = i\pi, \tag{4}$$
$$e^{z+1} + 2 = 0. \tag{5}$$