Discrete Mathematics Homework 4

1. Show that the number of partitions of a positive integer n where no summand appears more than twice equals the number of partitions of n where no summand is divisible by 3.

2. (a) Find the exponential generating function for the number of ways to arrange n letters, $n \geq 0$, selected from each of the following words.

(i) HAWAII

(ii) MISSISSIPPI

(iii) ISOMORPHISM

(b) For (ii) of part (a), what is the exponential generating function if the arrangement must contain at least two I's?

3. In how many ways can we select seven nonconsecutive integers from $\{1, 2, 3, \ldots, 50\}$?

4. How many 20-digit quaternary (0, 1, 2, 3) sequences are there where:

(a) There is at least one 2 and an odd number of 0's?

(b) No symbol occurs exactly twice?

(c) No symbol occurs exactly three times?

(d) There are exactly two 3's or none at all?

5. For a positive integer n, we partition n into summands of 1, 2 and 3 with **ordering**. For example, 3 can be partitioned into

$$3 = 1 + 1 + 1 = 1 + 2 = 2 + 1$$

First two of these partitions have an odd number of parts, and the last two have an even number of parts.

(a) Let $p(n)$ be the number of partitions (with ordering) of n. Find a recurrence relation of $p(n)$, $n \geq 4$. Explain your relation in detail. Calculate $p(7)$.

(b) Find the generating function of partitions (with ordering) of n that have an odd number of parts. Find the generating function of partitions (with ordering) of n that have an even number of parts.

(c) Show that the number of partitions (with ordering) of $4n - 1$ that have an odd number of parts is equal to the number of partitions (with ordering) of $4n - 1$ that have an even number of parts.

(d) List all partitions (with ordering) of the integer 7 into odd number and even number of parts into summands of 1, 2 and 3.