Discrete Mathematics Homework 5

1. Find the recurrence relation of $D_n = \begin{vmatrix} a+b & ab & 0 & \cdots & 0 \\ 1 & a+b & ab & \ddots & \vdots \\ 0 & 1 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & a+b & ab \\ 0 & \cdots & 0 & 1 & a+b \end{vmatrix}_{n\times n}$. Hence calculate D_n.

2. Solve each of the following recurrence relations.

(a) $a_{n+2} + 3a_{n+1} + 2a_n = 3^n, \quad n \geq 0, \quad a_0 = 0, \quad a_1 = 1.$

(b) $a_{n+2} + 4a_{n+1} + 4a_n = 7, \quad n \geq 0, \quad a_0 = 1, \quad a_1 = 2.$

(c) $a_{n+2}^2 - 5a_{n+1}^2 + 6a_n^2 = 7n, \quad n \geq 0, \quad a_0 = 1, \quad a_1 = 1.$

(d) $a_{n+1} - 2a_n = 2^n, \quad n \geq 0, \quad a_0 = 1.$

(e) $a_n + 2a_{n-1} + 2a_{n-2} = 0, \quad n \geq 2, \quad a_0 = 1, \quad a_1 = 3.$

3. Let a_n be the number of expression (not answer) of $\underbrace{1-1-\cdots-1-1}_{n \text{ "}-\text{"}}$ together with ( ) for each subtraction. For instance,

$a_0 = 1$: 1
$a_1 = 1$: $1-1$
$a_2 = 2$: $(1-1)-1, \quad 1-(1-1)$
$a_3 = 5$: $\big((1-1)-1\big)-1, \quad \big(1-(1-1)\big)-1, \quad (1-1)-(1-1), \quad 1-\big((1-1)-1\big), \quad 1-\big(1-(1-1)\big)$

(a) List all possible cases for $n = 4$.

(b) Find the recurrence relation for a_n. Explain your relation in detail.

(c) Hence, solve the a_n.

4. Solve the systems of recurrence relation

$$a_{n+1} = -2a_n - 4b_n$$
$$b_{n+1} = 4a_n + 6b_n$$
$$n \geq 0,\ a_0 = 1,\ b_0 = 0$$

3. Find the recurrence relation of $D_n = \begin{vmatrix} 0 & 1 & 1 & 1 & \cdots & 1 \\ 1 & 0 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 1 & \vdots & \cdots & 1 & 0 & 1 \\ 1 & 1 & \cdots & 1 & 1 & 0 \end{vmatrix}_{n \times n}$. Hence calculate D_n.

Hint: $C_1 - C_n \to C_1$.