

Jacobi Method: First Two Iterations

We are given the system:

$$\begin{cases} 3x_1 - x_2 + x_3 = 1 \\ 3x_1 + 6x_2 + 2x_3 = 0 \\ 3x_1 + 3x_2 + 7x_3 = 4 \end{cases} \quad \text{with } X^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Rewriting the equations:

$$x_1 = \frac{1 + x_2 - x_3}{3}, \quad x_2 = \frac{-3x_1 - 2x_3}{6}, \quad x_3 = \frac{4 - 3x_1 - 3x_2}{7}$$

Iteration 0:

$$X^{(0)} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Iteration 1:

$$\begin{aligned} x_1^{(1)} &= \frac{1 + 0 - 0}{3} = \frac{1}{3} \\ x_2^{(1)} &= \frac{-3(0) - 2(0)}{6} = 0 \\ x_3^{(1)} &= \frac{4 - 3(0) - 3(0)}{7} = \frac{4}{7} \end{aligned}$$

$$X^{(1)} = \begin{pmatrix} \frac{1}{3} \\ 0 \\ \frac{4}{7} \end{pmatrix}$$

Iteration 2:

$$\begin{aligned} x_1^{(2)} &= \frac{1 + 0 - \frac{4}{7}}{3} = \frac{1 - \frac{4}{7}}{3} = \frac{\frac{3}{7}}{3} = \frac{1}{7} \\ x_2^{(2)} &= \frac{-3 \cdot \frac{1}{3} - 2 \cdot \frac{4}{7}}{6} = \frac{-1 - \frac{8}{7}}{6} = \frac{-\frac{15}{7}}{6} = -\frac{15}{42} = -\frac{5}{14} \\ x_3^{(2)} &= \frac{4 - 3 \cdot \frac{1}{3} - 3 \cdot 0}{7} = \frac{4 - 1}{7} = \frac{3}{7} \end{aligned}$$

$$X^{(2)} = \begin{pmatrix} \frac{1}{7} \\ -\frac{5}{14} \\ \frac{3}{7} \end{pmatrix}$$

Final Answer:

$$X^{(1)} = \left(\frac{1}{3}, 0, \frac{4}{7} \right), \quad X^{(2)} = \left(\frac{1}{7}, -\frac{5}{14}, \frac{3}{7} \right)$$