ADVANCED TOPICS IN COMPUTER SCIENCE SERIES

MODERN HEURISTIC TECHNIQUES FOR COMBINATORIAL PROBLEMS

Edited by
COLIN R REEVES BSc, MPhil
Department of Statistics and Operational Research
School of Mathematical and Information Sciences
Coventry University

McGRAW-HILL BOOK COMPANY

London · New York · St Louis · San Francisco · Auckland
Bogotá · Caracas · Lisbon · Madrid · Mexico · Milan
Montreal · New Delhi · Panama · Paris · San Juan · São Paulo
Singapore · Sydney · Tokyo · Toronto

Contents

1 Introduction 1

1.1 Combinatorial Problems 1

1.1.1 Links with linear programming 3

1.2 Local and Global Optima 4

1.3 Heuristics 5

1.3.1 The case for heuristics 7

1.3.2 Modern methods 11

1.3.3 Evaluation of heuristics 14

2 Simulated Annealing 20

2.1 Introduction 20

2.2 The Basic Method 24

2.2.1 Local optimization 24

2.2.2 The annealing algorithm 26

2.2.3 A brief overview of the theory 27

2.2.4 Generic decisions 29

2.2.5 Problem-specific decisions 32

2.2.6 Examples 34

2.2.7 Aids to fine-tuning 40

2.3 Enhancements and Modifications 42

2.3.1 Acceptance probability 43

2.3.2 Cooling 44

2.3.3 The neighbourhoods 46

2.3.4 Sampling 48

2.3.5 The cost function 49

2.3.6 In combination with other methods 51

2.3.7 Parallel implementations 53

2.4 Applications 55

2.4.1 Classical problems 56
2.4.2 VLSI and computer design 57
2.4.3 Sequencing and scheduling 59
2.4.4 Other problems 62
2.5 Conclusions 63

3 Tabu Search 70
3.1 Introduction 70
3.2 The Tabu Search Framework 71
3.2.1 An illustrative example 71
3.2.2 Notation and problem description 82
3.2.3 Neighbourhood search 83
3.2.4 Tabu search characteristics 85
3.2.5 Tabu search memory 88
3.2.6 Recency-based tabu memory functions 94
3.2.7 Aspiration criteria 98
3.2.8 Frequency-based memory 104
3.2.9 Frequency-based memory in simple intensification and diversification processes 109
3.3 Broader Aspects of Intensification and Diversification . 111
3.3.1 Diversification versus randomization 112
3.3.2 Reinforcement by restriction 114
3.3.3 Extrapolated relinking 117
3.3.4 Solutions evaluated but not visited 118
3.3.5 Interval-specific penalties and incentives 119
3.3.6 Candidate list procedures 119
3.3.7 Compound neighbourhoods 121
3.3.8 Creating new attributes—vocabulary building and concept formation 122
3.3.9 Strategic oscillation 125
3.4 Tabu Search Applications 127
3.5 Connections and conclusions 136
3.5.1 Simulated annealing 136
3.5.2 Genetic algorithms 138
3.5.3 Neural networks 141

4 Genetic Algorithms 151
4.1 Introduction 151
4.2 Basic Concepts 152

4.2.1 Intrinsic parallelism and the schema theorem . 154
4.2.2 Recent developments 159
4.3 A Simple Example 162
4.4 Extensions and Modifications 164
4.4.1 Population-related factors 165
4.4.2 Modified operators 170
4.4.3 Chromosome coding and representation 175
4.4.4 Hybridization 179
4.4.5 Parallel implementations 180
4.4.6 Computer software 181
4.5 Applications 181
4.5.1 Travelling salesman problem 181
4.5.2 Sequencing and scheduling 182
4.5.3 Graph colouring 184
4.5.4 Steiner trees 184
4.5.5 Knapsack problems 185
4.5.6 Set covering problems 185
4.5.7 Bin packing 186
4.5.8 Neural networks 186
4.5.9 Other problems 187
4.6 Conclusions 188

5 Artificial Neural Networks **197**
5.1 Introduction 197
5.2 Neural Networks 198
5.2.1 Biological neural networks 198
5.2.2 Artificial neural networks 201
5.3 Combinatorial Optimization Problems 204
5.4 The Graph Bisection Problem 205
5.4.1 Neural mapping 205
5.4.2 The mean field equations 207
5.4.3 Mean field dynamics 210
5.5 The Graph Partition Problem 211
5.5.1 Neuron multiplexing—Ising representation . . . 211
5.5.2 K-state neurons—Potts representation 213
5.5.3 Mean field Potts equations 214
5.5.4 Mean field dynamics 215
5.5.5 A generic algorithm 220
5.5.6 Numerical results 221

5.6 The Travelling Salesman Problem 221
5.7 Scheduling Problems 223
5.7.1 A synthetic example 223
5.7.2 A realistic example 226
5.8 Deformable Templates 229
5.9 The Knapsack Problem 233
5.10 Summary 239

6 Lagrangean Relaxation 243
6.1 Introduction 243
6.2 Overview 245
6.2.1 Techniques 245
6.2.2 Review of the literature 246
6.3 Basic Methodology 246
6.3.1 Introduction 246
6.3.2 Lagrangean relaxation 247
6.3.3 Set covering problem 249
6.3.4 Example Lagrange multiplier values 253
6.3.5 Advanced Lagrangean relaxation 253
6.4 Lagrangean Heuristics and Problem Reduction 260
6.4.1 Lagrangean heuristic 260
6.4.2 Problem reduction 261
6.4.3 Remarks 266
6.5 Determination of Lagrange Multipliers 266
6.5.1 Subgradient optimization 267
6.5.2 Advanced subgradient optimization 270
6.5.3 Multiplier adjustment 273
6.6 Dual Ascent 276
6.6.1 Basic concepts 276
6.6.2 Example dual ascent algorithm 278
6.6.3 Connections 279
6.6.4 Comparisons 280
6.6.5 Conclusions 281
6.7 Tree Search 282
6.7.1 Tree structure 283
6.7.2 Branching node 283
6.7.3 Backtracking 283
6.7.4 Forward branching rule 283
6.7.5 Lower bound computation 285

6.7.6 Computer programs 286
6.8 Applications 290
6.8.1 Journals 290
6.8.2 Summary 297
6.9 Conclusions 298

7 Evaluation of Heuristic Performance 304
7.1 Introduction 304
7.2 Analytical Methods 305
7.2.1 Worst-case and average performance analysis . 305
7.2.2 Bounds 307
7.3 Empirical Testing 308
7.4 Statistical Inference 311
7.5 Conclusion 313