Assignment 2

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Problem 2

Part 1:

Step 1:
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\lambda\ \_\ .\ x\ \textsf{val}}\ (\text{D-Lam})}{\{x \to D\} \vdash (\lambda\ x\ .\ \lambda\ \_\ .\ x)\ L \mapsto \lambda\ \_\ .\ x}\ (\text{D-App-Done})}{\{x \to D\} \vdash (\lambda\ x\ .\ \lambda\ \_\ .\ x)\ L\ * \mapsto (\lambda\ \_\ .\ x)\ *}\ (\text{D-App-Lam})}{\varnothing \vdash (\lambda\ x\ .\ (\lambda\ x\ .\ \lambda\ \_\ .\ x)\ L\ *)\ D \mapsto (\lambda\ x\ .\ (\lambda\ \_\ .\ x)\ *)\ D}\ (\text{D-App-Body})$$

Step 2:
$$\dfrac{\dfrac{\dfrac{x \to D \in \{x \to D,\ \_ \to *\}}{\{x \to D,\ \_ \to *\} \vdash x \mapsto D}\ (\text{D-Var})}{\{x \to D\} \vdash (\lambda\ \_\ .\ x)\ * \mapsto (\lambda\ \_\ .\ D)\ *}\ (\text{D-App-Body})}{\varnothing \vdash (\lambda\ x\ .\ (\lambda\ \_\ .\ x)\ *)\ D \mapsto (\lambda\ x\ .\ (\lambda\ \_\ .\ D)\ *)\ D}\ (\text{D-App-Body})$$

Step 3:
$$\dfrac{\dfrac{D\ \textsf{val}}{\{x \to D\} \vdash (\lambda\ \_\ .\ D)\ * \mapsto D}\ (\text{D-App-Done})}{\varnothing \vdash (\lambda\ x\ .\ (\lambda\ \_\ .\ D)\ *)\ D \mapsto (\lambda\ x\ .\ D)\ D}\ (\text{D-App-Body})$$

Step 4:
$$\dfrac{D\ \textsf{val}}{\varnothing \vdash (\lambda\ x\ .\ D)\ D \mapsto D}\ (\text{D-App-Done})$$

Part 2:

$$\dfrac{\Gamma, x \to e_{\textsf{var}} \vdash e_{\textsf{body}} \mapsto e'_{\textsf{body}}}{\Gamma \vdash \textsf{let}\ x = e_{\textsf{var}}\ \textsf{in}\ e_{\textsf{body}} \mapsto \textsf{let}\ x = e_{\textsf{var}}\ \textsf{in}\ e'_{\textsf{body}}}\ (\text{D-Let-Body})$$

$$\dfrac{e_{\textsf{body}}\ \textsf{val}}{\Gamma \vdash \textsf{let}\ x = e_{\textsf{var}}\ \textsf{in}\ e_{\textsf{body}} \mapsto e_{\textsf{body}}}\ (\text{D-Let-Done})$$

Problem 3

We claim that the let statement is type safe.

For all Γ, e, τ, assume $\Gamma, x : \tau_{\mathsf{var}} \vdash e_{\mathsf{body}} : \tau_{\mathsf{body}}, e = (\mathsf{let}\ x : \tau_{\mathsf{var}} = e_{\mathsf{var}}\ \mathsf{in}\ e_{\mathsf{body}}), \tau = \tau_{\mathsf{body}}$, then:

Progress: if $e : \tau$, then either e val or there exists an e' such that $e \mapsto e'$.

Proof. By D-Let, $e \mapsto [x \to e_{\mathsf{var}}]\ e_{\mathsf{body}}$, therefore e will always step. □

Preservation: if $e : \tau$ and $e \mapsto e'$ then $e' : \tau$.

Proof. There is only one way to step: by D-Let. By the substitution typing lemma, $x : \tau_{\mathsf{var}} \vdash e_{\mathsf{body}} : \tau_{\mathsf{body}} \implies [x \to e_{\mathsf{var}}]e_{\mathsf{body}} : \tau_{\mathsf{body}}$. □

rec statement is not type safe because there is no operational semantics that can be applied to $\mathsf{rec}(e_{\mathsf{base}};\ x_{\mathsf{num}}.x_{\mathsf{acc}}.e_{\mathsf{acc}})(n)$ when $n < 0$, so $\mathsf{rec}(e_{\mathsf{base}};\ x_{\mathsf{num}}.x_{\mathsf{acc}}.e_{\mathsf{acc}})(-1)$ is not a value, and it can not step, so it violates progress.