An Introduction to Abelian Categories

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Preliminary steps (i)

The intuition for abelian categories comes from the behaviour of a kind of mathematical object which is found everywhere in each science: **vector spaces**.

Vector spaces are considered to be well understood and have interesting categorical properties.

Preliminary steps (ii)

First of all, we will denote with Vect_K the category of vector spaces over the field K, whose objects are vector spaces and morphisms are linear maps.

We will denote with $\mathrm{FinDimVect}_K$ the subcategory of Vect_K which contains only finite dimensional K-vector spaces.

Preliminary steps (iii)

Vector spaces have a peculiar property: initial objects are isomorphic to final objects!

Moreover $\mathsf{FinDimVect}_K$ is self-dual (even though it's not a "natural duality"!), finite dimensional vector spaces share even more good properties.