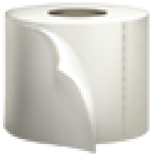

Pilgrim Paper

Pilgrim: An Atomic Valuation Framework for Low Liquidity Assets

Version v0.2

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Abstract. Pilgrim is an **atomic valuations framework** for assets with low liquidity, including but not limited to NFTs, credit, real estate, and unlisted corporate/DAO shares. We present a novel mechanism for creating liquidity for illiquid asset types by assuming virtual liquidity values set by the initial pool deployer, and generating liquidity on-the-fly as demand for liquid borrow positions (LBPs) increase (and vice versa). Bonding curves based on Uniswap and mechanisms taken from money market and synthetic asset logic are used to recursively determine pricing of illiquid assets locked as collateral, allowing anyone holding collateral position derivatives to access asset-specific features, and gain partial price exposure without liquidating the asset through money market borrows against locked LBPs. This also allows for anyone to either bet for (long) or against (short) a given illiquid asset by purchasing LBP derivatives, called **rounds**, that are only liquidated either when a target liquidation threshold is given and activated, or when the base LBP is liquidated. The authors believe that Pilgrim will not only enable dynamic composability of illiquid assets with new and existing on-chain financial primitives, but

also allow for novel valuation, funding, market construction, trading and liquidity control mechanisms not possible with any other financial instrument.

I. Introduction

While illiquid and/or non-fungible assets are an essential component for modern financial infrastructure, atomically determining valuation of such assets remains a challenge. Auction and orderbook-based models are primary means of agreeing upon pricing for illiquid assets, such as real estate and art. Although this approach work well for use cases that do not require composability and automation, auctions are not desirable for novel applications that require a live, atomic price feed like those of stock markets and cryptocurrencies — as auctions often require days (if not weeks or months) before valuation could be finalized.

One potential solution to this problem is fractionalizing illiquid assets or bags of such assets into **shares**, of which then could be freely traded on the market. However, such an approach poses the following issues:

- **Market liquidity must be manually maintained.** Even if illiquid assets are fractionalized into shares or shards, they are still derivatives of the underlying asset and therefore require liquidity in order for them to be traded. As liquidity requirements inherently force some shards to be locked indefinitely with corresponding liquid assets even though there are a limited number of shards available, this is very capital inefficient and difficult to manage in the long term.
- **Reconstitution of underlying assets is difficult.** Shard-based models often require either a certain percentage of shard owners to coordinate or participate in governance proposals to retrieve the underlying asset. Not only is this participation requirement rigorous and time-consuming, such an approach comes with optimistic coordination assumptions that may result in problems as described below. Reconstitution difficulties while shards are being traded also prohibit assets from being actively used with DApps that allocate certain features, such as with Axie Infinity or other NFT-based games.
- **Coordination assumptions are often optimistic.** Performing actions collectively under the underlying NFT under a shards model require coordination that, inevitably, result in multiple parties with conflicting interests. For instance, people that bought shards below a given reconstitution value will most likely agree to redeem their shards, but others that bought shards above this value will likely be against redemption. While approaches such as involving gradual inflation of shards given to active participants may be able to solve this particular issue, fundamentally it still stands true that fractionalization-based models result in other inefficiencies listed here.
- **This still does not fix nonatomic liquidation properties of illiquid assets.** Unlike fungible and liquid assets, illiquid assets require a time-consuming auction to liquidate and gain access to liquid assets in return. Fractionalization based

approaches do not resolve this problem, as reconstitution must be triggered in order to liquidate assets (of which has coordination issues in itself).

Therefore, the following features are desirable when constructing an atomic valuation system for illiquid or non-fungible assets:

- **Liquidity should be generated and withdrawn programmatically in proportion to market demand.** As there could be an infinite number of assets in existence, requiring every single one of them to manually manage market liquidity is extremely inefficient. Therefore, an automated liquidity control mechanism is required, whereby liquid assets provided with buy (or short) orders act as both collateral and liquidity for buy bids, implemented on top of a prediction market-based price curve as those of Uniswap.
- **All features of the underlying asset should be accessible even when it is held by a custody contract for valuation.** Fractionalization-based mechanisms effectively block access to the underlying asset, especially in cases when an asset is allocated certain features by an external application. Therefore a mechanism should exist to relay the concept of ownership to a staking derivative-like asset that could act as a functional pointer to its underlying asset while it is actively being traded.
- **No coordination assumptions should be made outside of financial incentives.** Coordination is difficult to resolve through non-incentivized governance processes that must be executed manually. Ideally, outside of financial incentives and free market principals, effective coordination should not be one of the assumptions for designing such a valuation system.
- **Liquidity should be readily available when requested by the underlying asset's original owner.** There should exist a mechanism where the owner of an asset may borrow a portion of locked liquidity generated from market demand. This would allow for partial liquidity exposure for owners of illiquid assets, even when an auction or a sale on the asset is not finalized.

II. The Pilgrim Trading Protocol

The Pilgrim dynamic AMM (dynAMM) defines a base valuation mechanism where:

- market liquidity is generated in proportion to demand
- valuation of illiquid assets locked as collateral is recursively determined based on an AMM-like prediction market system
- locked market liquidity may be partially borrowed against illiquid collateral
- a staking derivative-like asset used for borrows, pool liquidations, and feature access, is minted on pool deployment

Uniswap has introduced a novel mechanism to construct markets on-chain based on prediction market models, except that liquid assets are used as collateral against a prediction market curve within a pooled token pair contract. **Pilgrim builds on top of this**

automated-market-maker mechanism to construct atomic price curves for illiquid assets as well.

Once an illiquid asset is deposited with Pilgrim, the Pilgrim Money Market internally defines a **Liquid Borrow Position** (LBP), which is equivalent to reverse mortgages. Traditional mortgage loans have a set loan-to-value ratio (LTV) used to borrow a **fixed amount** of liquid assets with an illiquid asset specified as collateral with fluctuating value, either set by supply-and-demand bids or public auctions. **LBPs are the exact opposite: we allow implicit trading of a liquid mortgage borrow position with a fixed LTV, thus allowing the underlying collateral value to be determined based on the current valuation of its corresponding borrow position.**

We first take the standard Uniswap bonding curve

$$x \cdot y = k$$

with two major modifications:

- **Pool liquidity is dynamically adjusted per trade.** Under the Uniswap model, the liquidity parameter k only changes when liquidity is explicitly provided or withdrawn from the pool. However, this assumes that assets provided on both sides are liquid and fungible assets; such a model results in liquidity fragmentation for asset types that are not fungible. To compensate for this, we define the liquidity parameter k to
    - **increase** when there is more demand for the corresponding asset
    - **decrease** when there is less demand for the corresponding asset

    Defining L as the base round liquidity parameter, and p as the current exchange rate per round, this could be expressed as

    $$L^2 \cdot p = z$$

    with $L \cdot (p - p_{genesis})$ as the total amount of liquidity locked.

    Simplifying this as

    $$x \cdot y = z$$

    with z defined as a dynamic product and not a constant, buying r units of LBP rounds therefore should then result in:

    $$\frac{1}{x} \cdot (x + r)^2 \cdot y = z$$

    therefore, required liquid assets to buy r units of LBP rounds equate to

$$\Delta y = y' - \frac{y \cdot (x+r)}{x} = \frac{y \cdot r \cdot (x+r)}{x^2}$$

and vice versa. Buying LBPs with n units of liquid assets equate to

$$n = \Delta y = \frac{y \cdot r \cdot (x+r)}{x^2}$$

therefore the amount of LBP rounds returned from this pool is

$$r = \frac{x \cdot (\sqrt{1 + \frac{4n}{y}} - 1)}{2}$$

and vice versa.

The following is a simple example of the logic illustrated above:

	x	y	z
`initial condition`	100	200	20,000
`liquidity added`	110	220	24,200
`swapped`	**100**	**242**	24,200

Note that x stays constant, being the base round liquidity parameter L; while y increases, reflecting added liquidity within this pool. Therefore the **spot price per round** has increased from 2 to 2.42, while the **execution price** stays at 2.

- **Slippage should be counter-uniform across the entire pool.** This to protect against slippage delta-based attacks; i.e. trading nK units of liquid assets (assuming $n \in N$ & $K \in N$) should be equal to executing K unit worth of trading operations n times. This operation could be generalized as
  - **minting** new rounds: let *required liquidity in* equal to $f_{nM}(y)$

$$f_{nM}(y) = \frac{(x+k)^{2n+1} - x^{2n}(x+k)}{(2x+k)x^{2n}} y$$

  - **burning** new rounds: let *required liquidity out* equal to $f_{nB}(y)$

$$f_{nB}(y) = \frac{(x+k)^{2n} - x^{2n}}{(2x+k)(x+k)^{2n-1}}y$$

The above is proven with a separate Arbitrary Rounds Trading paper.

As trading is executed at point of liquidity provision, maximum allowed slippage per unit decreases exponentially as liquidity is adjusted against the value of liquid assets already locked with a pool contract. Therefore, price movement is **exponential against liquidity deposited —** unlike the Uniswap curve that defines a standard supply-to-demand relationship.

This bonding curve is also unique in the sense that minted rounds **effectively equate with liquidity tokens** under the Uniswap model; current shares of round tokens represent relative ownership of liquidity within a particular Pilgrim pool. As Uniswap defines the initial number of liquidity tokens minted as $\sqrt{x \cdot y}$ and mints liquidity proportional to existing liquidity tokens afterwards, the **effective collateral value** of Pilgrim round tokens may also be calculated as such.

III. metaNFTs and Pool Ownership

The Pilgrim protocol defines a staking derivative-like asset minted to the creator of a particular Pilgrim pool contract, called **metaNFTs**. metaNFTs represent illiquid collateral within a Pilgrim pool, while rounds represent ownership over liquid collateral, each backing market valuation of the other.

Owners of metaNFTs may:

- relay features of its underlying NFT to a third-party application that supports such,
- accept a **full pool buyout bid** to liquidate the entire pool and its corresponding LBP for reclaiming its underlying NFT,
- collect trading fees paid by round traders,
- and potentially compose them with third-party mechanisms based on its dynamic price feed enabled by the Pilgrim trading protocol, such as borrowing liquid assets within a particular liquidity pool with metaNFTs as collateral, allowing for dynamic liquidation implementations on illiquid-to-liquid money markets.

metaNFTs also **share the same metadata** as its underlying NFT, such that one may consider metaNFTs as “upgraded” versions of the original NFT with a dynamic price feed that earns yield, and enhanced composability with third-party decentralized applications.

IV. Pool Buyouts and Liquidations

Unlike auction-based valuation protocols, Pilgrim enables atomic price feeds for illiquid assets, which inherently affect how full buyouts work.

Buying the entire underlying NFT is equivalent to liquidating an existing LBP position, which liquidates all liquid asset positions (rounds) and illiquid asset positions (metaNFTs) back to the base token paid by the liquidator.

Multiple liquidation bids may exist at a time on a bidder queue. The metaNFT owner has the right to approve **one and only one bid** that exists on the bidder queue for this Pilgrim pool that: (i) has a higher valuation than the current spot price set by the Pilgrim trading protocol, and (ii) has the lowest bid timestamp value. metaNFT owners do **not** have the right to choose from multiple bids.

V. The Pilgrim Token & Financial Coordination

The Pilgrim Token (PIL) exists to capture value from the Pilgrim protocol, amplify protocol rewards for those that have aligned incentives with the protocol, and govern the protocol in a decentralized manner.

- Protocol fees generated from Pilgrim pools are used to buy PIL from Uniswap, subsidized with additional PIL from the Protocol Treasury at a predefined ratio **based on relative pool TVL against the protocol's total liquidity**, and are distributed to both metaNFT holders and round holders.
    - Total protocol rewards for a particular Pilgrim pool is defined as `REWARD_CONSTANT * TOTAL_VALUE_LOCKED * TRADING_VOLUME` .
- Protocol fees generated from Pilgrim pools with PIL as its base token has a higher PIL fee subsidization ratio than (i).
- Protocol fees generated from Pilgrim pools with PIL as its base token and paired with a PIL - ETH Uniswap v3 liquidity position has a higher PIL fee subsidization ratio than (ii).
- Protocol fees generated from Pilgrim pools with PIL as its base token and paired with a PIL - xPIL Uniswap v3 liquidity position has a higher PIL fee subsidization ratio than (iii).
- A portion of all Pilgrim protocol fees are distributed to xPIL holders, which are locked PIL positions committed for a predefined period. xPIL holders also receive additional PIL fee subsidizations from the Protocol Treasury, of which cannot be claimed until lockup expiry.

The above characteristics create financial coordination opportunities, whereby:

- to receive higher protocol fees and therefore higher PIL subsidizations,
    - metaNFT holders must lock as much liquidity as possible from round traders. They also should set their base token to PIL for receiving additional protocol incentives.
    - round holders must either hold round positions for significant periods of time to gain further exposure to this particular Pilgrim pool's TVL growth, or actively

rebalance their positions to be aligned with Pilgrim pools that they consider undervalued.

- this creates financial gain opportunities for active traders on both sides;
    - pool buyout bids are inherently affected by current round prices, and vice versa; round prices are a direct outcome of TVL competition coordinated by multiple groups of metaNFT holders and round position holders.
    - however, for buyout bidders, pool TVL should be lower for potential gains. as metaNFT holders are the only ones that may approve their bids, and only the bid that are closest to the current NFT valuation may be approved, buyout bidders should actively rebalance their bid positions. This also affects metaNFT and round token valuation, and therefore the amount of PIL rewards they receive.
    - not only this encourages community-level coordination and competition to gain more exposure for subsidized protocol rewards, position rebalancing is also encouraged due to constantly changing buyout bids. This increases overall protocol fees, increasing PIL buybacks and fees being distributed to both protocol users and xPIL holders.

This coordination mechanism may be forked and be used alongside existing PIL token incentives by third-party protocols to incentivize coordination within their own communities.

VI. Potential Applications & Conclusion

Pilgrim enables atomic valuation of any primitive that may be expressed on-chain. This allows for additional composability and programmability not yet available with any other financial instrument, including but not limited to:

- Atomic valuation of illiquid financial positions, including Uniswap v3 liquidity positions, Maker vaults and Compound borrow positions.
- Leverage and hedging through combined atomic valuation of multiple illiquid positions:

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8/ same for money markets, @compoundfinance = long <COLLATERAL> plus borrow interest, @anchor_protocol = long <BASSET> minus future staking income. of course borrowing in itself creates leverage but does not guarantee anything in itself

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9/ if one's effectively longing you could match it with a short on Uniswapv3, and vice versa. Process:

- Mint an NFT for a position on DeFi protocol X
- Exchange p% of the NFT for q% of a counterposition NFT on DeFi protocol Y
- p/q is effectively counteruniform

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- decentralized, community-coordinated valuation of corporate and DAO shares for funding.
- credit valuation and historical logs for both individuals and DAOs for under-collateralized borrows.

Future work includes:

- shorting mechanisms for traders to allow NFT valuation to drop below its listed price, while still incurring positive trading fees and TVL
- native collateralized borrows against protocol liquidity utilizing metaNFTs.

Revision History

v0.1: Initial release.

v0.2: Added visualization examples, unified terms with other Pilgrim public resources.