BIOS612_Project

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2024-11-20

We utilize the function `GLMMselect` from the R package `GLMMselect`, which is defined as follows:

`GLMMselect(Y, X, Sigma, Z, family, prior, offset, NumofModel, pip_fixed, pip_random)`

The arguments of this function are described below:

- `Y` : A numeric vector of binary or count data representing the response variable.
- `X` : A matrix of covariates.
- `Sigma` : A list of covariance matrices for the random effects.
- `Z` : A list of design matrices for the random effects.
- `family` : The distribution family for the response variable, either `"bernoulli"` or `"poisson"`.
- `prior` : The prior distribution for the variance components of the random effects, either `"AR"` or `"HM"`.
- `offset` : A known a priori component to be included in the linear predictor during model fitting.
- `NumofModel` : The number of models with the highest posterior probabilities to be reported.
- `pip_fixed` : A cutoff threshold; fixed effects with a posterior inclusion probability exceeding this threshold are included in the final model.
- `pip_random` : A cutoff threshold; random effects with a posterior inclusion probability exceeding this threshold are included in the final model.

Simulation Study Model

For the simulation study, we assume the following model:

$$y_{ij} = \beta \mathbf{x_{ij}} + u_i + s_i + \epsilon_{ij}$$

Where:

- y_{ij}: The response variable for the j-th observation in the i-th group.
- β: A vector of coefficients associated with the covariates.
- $\mathbf{x_{ij}}$: The covariate matrix for the j-th observation in the i-th group.
- $U = (u_1, \ldots, u_i)^T$: The overdispersion random effects for group i, where $U \sim N(0, \tau_2 \Sigma)$.
- $S = (s_1, \ldots, s_i)^T$: The spatial random effects for location i, modeled using a spatial covariance structure, where $S \sim N(0, \tau_1 \Sigma)$.
- ϵ_{ij}: The residual error term, where $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$.

Simulation Settings

For the simulation settings, we adopt the structure described in the referenced study. Specifically:

- The overdispersion random effects are modeled as $U \sim N(0, \tau_2 \Sigma)$.
- The spatial random effects are modeled as $S \sim N(0, \tau_1 \Sigma)$.
- The covariance matrix Σ is defined using a first-order neighborhood structure, consistent with the referenced study.
- The fixed effects coefficients are set to $\beta = (\beta_0, \beta_1, \beta_2, 0, 0)$.
- The covariates x_{ij} are sampled from a standard normal distribution.

Scenario One: Poisson Regression

In the first scenario, we set the parameters as follows:

- $\beta_0 = 1$, $\beta_1 = \beta_2 = 0.1$,
- $\tau_1 = 0.05$, $\tau_2 = 0.05$,
- Sample size: 100,
- Number of simulation iterations: 10.

This setup reflects a Poisson regression framework with specified parameter values and sample constraints.

```
### AR

library(MASS)
beta<-c(1,0.1,0.1,0,0)
result_best<-list()
result_postprob<-list()
tau=0.05
for (i in 1:10){
  set.seed(i)
  X1<-matrix(rnorm(400),nrow=100,ncol=4)
  X2<-cbind(1,X1)
  spatial_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[1]])
  overdispersion_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[2]])
  linear_predictor <- X2 %*% beta + spatial_random_effect + overdispersion_random_effect
  Y<-rpois(100,exp(linear_predictor))
  Model_selection_output <- GLMMselect(Y=Y, X=X1, Sigma=Sigma,
                          Z=Z, family="poisson", prior="AR", offset=NULL)
  result_best[[i]]<-Model_selection_output$BestModel
  result_postprob[[i]]<-Model_selection_output$PosteriorProb
}
```

```
print(result_best)
```

```
## [[1]]
## [[1]]$covariate_inclusion
## integer(0)
##
## [[1]]$random_effect_inclusion
## [1] 1
##
##
## [[2]]
## [[2]]$covariate_inclusion
## integer(0)
##
## [[2]]$random_effect_inclusion
## [1] 1
##
##
## [[3]]
## [[3]]$covariate_inclusion
## [1] 2
##
## [[3]]$random_effect_inclusion
## integer(0)
##
##
## [[4]]
## [[4]]$covariate_inclusion
## [1] 2
##
## [[4]]$random_effect_inclusion
## [1] 1
##
##
## [[5]]
## [[5]]$covariate_inclusion
## integer(0)
##
## [[5]]$random_effect_inclusion
## [1] 2
##
##
## [[6]]
## [[6]]$covariate_inclusion
## [1] 1 2
##
## [[6]]$random_effect_inclusion
## [1] 1
##
##
## [[7]]
## [[7]]$covariate_inclusion
## [1] 1
##
## [[7]]$random_effect_inclusion
```

```
## integer(0)
##
##
## [[8]]
## [[8]]$covariate_inclusion
## [1] 1
##
## [[8]]$random_effect_inclusion
## integer(0)
##
##
## [[9]]
## [[9]]$covariate_inclusion
## integer(0)
##
## [[9]]$random_effect_inclusion
## integer(0)
##
##
## [[10]]
## [[10]]$covariate_inclusion
## [1] 4
##
## [[10]]$random_effect_inclusion
## [1] 1
```

```
print(result_postprob)
```

```
## [[1]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  0 0.330
## 2    0  0  0  0  0  1 0.235
## 3    0  0  0  0  0  0 0.105
## 4    0  0  0  0  1  1 0.085
## 5    0  1  0  0  1  0 0.024
## 6    0  0  0  1  1  0 0.023
## 7    1  0  0  0  1  0 0.018
## 8    0  0  1  0  1  0 0.017
## 9    0  1  0  0  0  1 0.017
## 10   0  0  0  1  0  1 0.014
##
## [[2]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  0 0.409
## 2    1  0  0  0  1  0 0.148
## 3    0  0  0  0  1  1 0.066
## 4    1  1  0  0  1  0 0.040
## 5    0  0  0  1  1  0 0.036
## 6    1  0  0  1  1  0 0.035
## 7    0  1  0  0  1  0 0.035
## 8    0  0  1  0  1  0 0.022
## 9    1  0  1  0  1  0 0.020
## 10   1  1  0  1  1  0 0.020
##
## [[3]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  0 0.131
## 2    1  1  0  0  0  0 0.113
## 3    0  1  0  0  0  0 0.095
## 4    1  1  1  0  0  0 0.078
## 5    1  1  1  1  0  0 0.077
## 6    0  0  0  0  1  0 0.073
## 7    0  0  0  0  0  1 0.055
## 8    1  0  0  0  0  0 0.044
## 9    1  1  0  1  0  0 0.043
## 10   0  1  0  0  1  0 0.037
##
## [[4]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  0 0.280
## 2    0  1  0  0  1  0 0.132
## 3    1  1  0  0  1  0 0.052
## 4    0  0  0  0  1  1 0.047
## 5    0  0  0  0  0  1 0.046
## 6    0  1  0  0  0  1 0.036
## 7    0  1  0  1  1  0 0.031
## 8    0  0  0  1  1  0 0.029
## 9    1  0  0  0  1  0 0.028
## 10   0  1  0  0  0  0 0.024
##
## [[5]]
```

```
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  1 0.331
## 2    0  0  0  0  1  0 0.235
## 3    0  0  0  0  1  1 0.095
## 4    1  0  0  0  0  1 0.066
## 5    1  0  0  0  1  0 0.034
## 6    0  0  0  0  0  0 0.027
## 7    1  0  0  0  0  0 0.026
## 8    0  0  0  1  0  1 0.015
## 9    0  0  1  0  0  1 0.015
## 10   0  1  0  0  0  1 0.015
##
## [[6]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.253
## 2    1  1  0  0  0  0 0.106
## 3    1  1  0  1  1  0 0.090
## 4    1  1  1  0  1  0 0.083
## 5    1  1  1  1  1  0 0.075
## 6    0  1  0  0  1  0 0.072
## 7    1  1  1  1  0  0 0.070
## 8    1  1  0  1  0  0 0.053
## 9    1  1  1  0  0  0 0.050
## 10   1  0  0  0  1  0 0.022
##
## [[7]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  0  0  0  0  0 0.239
## 2    0  0  0  0  1  0 0.130
## 3    0  0  0  0  0  0 0.091
## 4    1  0  0  0  1  0 0.074
## 5    1  1  0  0  0  0 0.070
## 6    1  0  0  1  0  0 0.043
## 7    1  0  1  0  0  0 0.040
## 8    1  0  0  0  0  1 0.031
## 9    0  0  0  0  0  1 0.030
## 10   1  1  0  1  0  0 0.026
##
## [[8]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  0 0.131
## 2    1  0  0  0  0  0 0.122
## 3    1  1  1  1  0  0 0.089
## 4    1  0  1  0  0  0 0.074
## 5    0  0  0  0  0  1 0.058
## 6    1  1  1  0  0  0 0.051
## 7    0  0  0  0  1  0 0.050
## 8    1  0  1  1  0  0 0.047
## 9    1  1  0  0  0  0 0.042
## 10   1  0  0  1  0  0 0.039
##
## [[9]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  0 0.212
```

```
## 2    0  0  0  0  1  0 0.210
## 3    1  0  0  0  0  0 0.098
## 4    1  0  0  0  1  0 0.076
## 5    1  0  0  1  0  0 0.049
## 6    0  0  0  0  0  1 0.026
## 7    1  0  0  1  1  0 0.025
## 8    1  1  1  1  0  0 0.024
## 9    1  1  0  1  0  0 0.023
## 10   1  1  0  0  0  0 0.021
##
## [[10]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  0 0.283
## 2    0  0  0  1  1  0 0.121
## 3    0  0  0  0  1  1 0.051
## 4    0  1  0  1  1  0 0.049
## 5    0  1  0  0  1  0 0.038
## 6    0  0  0  1  0  1 0.035
## 7    0  0  0  1  0  0 0.031
## 8    0  0  0  0  0  1 0.031
## 9    0  1  1  1  1  0 0.030
## 10   0  0  1  1  1  0 0.028
```

```
### HC

beta<-c(1,0.1,0.1,0,0)
result_best<-list()
result_postprob<-list()
tau=0.05
for (i in 1:10){
  set.seed(i)
  X1<-matrix(rnorm(400),nrow=100,ncol=4)
  X2<-cbind(1,X1)
  spatial_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[1]])
  overdispersion_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[2]])
  linear_predictor <- X2 %*% beta + spatial_random_effect + overdispersion_random_effect
  Y<-rpois(100,exp(linear_predictor))
  Model_selection_output <- GLMMselect(Y=Y, X=X1, Sigma=Sigma,
                            Z=Z, family="poisson", prior="HC", offset=NULL)
  result_best[[i]]<-Model_selection_output$BestModel
  result_postprob[[i]]<-Model_selection_output$PosteriorProb
}
```

```
print(result_best)
```

```
## [[1]]
## [[1]]$covariate_inclusion
## integer(0)
##
## [[1]]$random_effect_inclusion
## [1] 1
##
##
## [[2]]
## [[2]]$covariate_inclusion
## integer(0)
##
## [[2]]$random_effect_inclusion
## [1] 1
##
##
## [[3]]
## [[3]]$covariate_inclusion
## [1] 2
##
## [[3]]$random_effect_inclusion
## integer(0)
##
##
## [[4]]
## [[4]]$covariate_inclusion
## integer(0)
##
## [[4]]$random_effect_inclusion
## [1] 1
##
##
## [[5]]
## [[5]]$covariate_inclusion
## integer(0)
##
## [[5]]$random_effect_inclusion
## [1] 2
##
##
## [[6]]
## [[6]]$covariate_inclusion
## [1] 1 2
##
## [[6]]$random_effect_inclusion
## [1] 1
##
##
## [[7]]
## [[7]]$covariate_inclusion
## [1] 1
##
## [[7]]$random_effect_inclusion
```

```
## integer(0)
##
##
## [[8]]
## [[8]]$covariate_inclusion
## [1] 1
##
## [[8]]$random_effect_inclusion
## integer(0)
##
##
## [[9]]
## [[9]]$covariate_inclusion
## integer(0)
##
## [[9]]$random_effect_inclusion
## integer(0)
##
##
## [[10]]
## [[10]]$covariate_inclusion
## integer(0)
##
## [[10]]$random_effect_inclusion
## [1] 1
```

```
print(result_postprob)
```

```
## [[1]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  0 0.327
## 2    0  0  0  0  0  1 0.244
## 3    0  0  0  0  0  0 0.119
## 4    0  0  0  0  1  1 0.074
## 5    0  1  0  0  1  0 0.023
## 6    0  0  0  1  1  0 0.022
## 7    1  0  0  0  1  0 0.017
## 8    0  1  0  0  0  1 0.016
## 9    0  0  1  0  1  0 0.016
## 10   0  0  0  1  0  1 0.014
##
## [[2]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  0 0.431
## 2    1  0  0  0  1  0 0.149
## 3    0  0  0  0  1  1 0.062
## 4    1  1  0  0  1  0 0.037
## 5    0  0  0  1  1  0 0.036
## 6    0  1  0  0  1  0 0.035
## 7    1  0  0  1  1  0 0.033
## 8    0  0  1  0  1  0 0.022
## 9    1  0  1  0  1  0 0.019
## 10   0  0  0  0  0  1 0.017
##
## [[3]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  0 0.138
## 2    1  1  0  0  0  0 0.119
## 3    0  1  0  0  0  0 0.100
## 4    1  1  1  0  0  0 0.082
## 5    1  1  1  1  0  0 0.081
## 6    0  0  0  0  1  0 0.068
## 7    0  0  0  0  0  1 0.054
## 8    1  0  0  0  0  0 0.046
## 9    1  1  0  1  0  0 0.045
## 10   0  1  0  0  1  0 0.033
##
## [[4]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  0 0.292
## 2    0  1  0  0  1  0 0.131
## 3    0  0  0  0  0  1 0.050
## 4    1  1  0  0  1  0 0.048
## 5    0  0  0  0  1  1 0.043
## 6    0  1  0  0  0  1 0.038
## 7    0  1  0  1  1  0 0.029
## 8    0  1  0  0  0  0 0.028
## 9    0  0  0  1  1  0 0.028
## 10   1  0  0  0  1  0 0.028
##
## [[5]]
```

```
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  1 0.347
## 2    0  0  0  0  1  0 0.235
## 3    0  0  0  0  1  1 0.084
## 4    1  0  0  0  0  1 0.066
## 5    1  0  0  0  1  0 0.032
## 6    0  0  0  0  0  0 0.030
## 7    1  0  0  0  0  0 0.030
## 8    0  0  0  1  0  1 0.015
## 9    0  0  1  0  0  1 0.015
## 10   0  1  0  0  0  1 0.015
##
## [[6]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.243
## 2    1  1  0  0  0  0 0.130
## 3    1  1  1  1  0  0 0.086
## 4    1  1  0  1  1  0 0.077
## 5    0  1  0  0  1  0 0.073
## 6    1  1  1  0  1  0 0.071
## 7    1  1  0  1  0  0 0.065
## 8    1  1  1  0  0  0 0.061
## 9    1  1  1  1  1  0 0.053
## 10   1  0  0  0  1  0 0.023
##
## [[7]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  0  0  0  0  0 0.254
## 2    0  0  0  0  1  0 0.123
## 3    0  0  0  0  0  0 0.097
## 4    1  1  0  0  0  0 0.074
## 5    1  0  0  0  1  0 0.067
## 6    1  0  0  1  0  0 0.045
## 7    1  0  1  0  0  0 0.043
## 8    0  0  0  0  0  1 0.030
## 9    1  0  0  0  0  1 0.029
## 10   1  1  0  1  0  0 0.028
##
## [[8]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  0 0.138
## 2    1  0  0  0  0  0 0.128
## 3    1  1  1  1  0  0 0.094
## 4    1  0  1  0  0  0 0.078
## 5    0  0  0  0  0  1 0.056
## 6    1  1  1  0  0  0 0.053
## 7    1  0  1  1  0  0 0.049
## 8    0  0  0  0  1  0 0.046
## 9    1  1  0  0  0  0 0.044
## 10   1  0  0  1  0  0 0.041
##
## [[9]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  0 0.227
```

```
## 2    0  0  0  0  1  0 0.201
## 3    1  0  0  0  0  0 0.105
## 4    1  0  0  0  1  0 0.071
## 5    1  0  0  1  0  0 0.053
## 6    1  1  1  1  0  0 0.026
## 7    0  0  0  0  0  1 0.026
## 8    1  1  0  1  0  0 0.025
## 9    1  1  0  0  0  0 0.022
## 10   1  0  1  1  0  0 0.022
##
## [[10]]
##      x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  0 0.296
## 2    0  0  0  1  1  0 0.121
## 3    0  0  0  0  1  1 0.046
## 4    0  1  0  1  1  0 0.045
## 5    0  1  0  0  1  0 0.037
## 6    0  0  0  1  0  0 0.037
## 7    0  0  0  1  0  1 0.037
## 8    0  0  0  0  0  1 0.034
## 9    1  1  1  1  0  0 0.028
## 10   0  0  1  1  1  0 0.026
```

Scenario Two: Poisson Regression

In this scenario, the values of τ_1 and τ_2 are fixed at 0.05. The vector of coefficients is updated to $\beta = (1, 0.5, 0.5, 0, 0)$.

```
### AR

library(MASS)
beta<-c(1,0.5,0.5,0,0)
result_best<-list()
result_postprob<-list()
tau=0.05
for (i in 1:10){
  set.seed(i)
  X1<-matrix(rnorm(400),nrow=100,ncol=4)
  X2<-cbind(1,X1)
  spatial_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[1]])
  overdispersion_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[2]])
  linear_predictor <- X2 %*% beta + spatial_random_effect + overdispersion_random_effect
  Y<-rpois(100,exp(linear_predictor))
  Model_selection_output <- GLMMselect(Y=Y, X=X1, Sigma=Sigma,
                              Z=Z, family="poisson", prior="AR", offset=NULL)
  result_best[[i]]<-Model_selection_output$BestModel
  result_postprob[[i]]<-Model_selection_output$PosteriorProb
}
```

```
print(result_best)
```

```
## [[1]]
## [[1]]$covariate_inclusion
## [1] 1 2 4
##
## [[1]]$random_effect_inclusion
## [1] 1
##
##
## [[2]]
## [[2]]$covariate_inclusion
## [1] 1 2
##
## [[2]]$random_effect_inclusion
## [1] 1
##
##
## [[3]]
## [[3]]$covariate_inclusion
## [1] 1 2 3
##
## [[3]]$random_effect_inclusion
## integer(0)
##
##
## [[4]]
## [[4]]$covariate_inclusion
## [1] 1 2 4
##
## [[4]]$random_effect_inclusion
## integer(0)
##
##
## [[5]]
## [[5]]$covariate_inclusion
## [1] 1 2 4
##
## [[5]]$random_effect_inclusion
## integer(0)
##
##
## [[6]]
## [[6]]$covariate_inclusion
## [1] 1 2 4
##
## [[6]]$random_effect_inclusion
## [1] 1
##
##
## [[7]]
## [[7]]$covariate_inclusion
## [1] 1 2 3 4
##
## [[7]]$random_effect_inclusion
```

```
## integer(0)
##
##
## [[8]]
## [[8]]$covariate_inclusion
## [1] 1 2
##
## [[8]]$random_effect_inclusion
## [1] 2
##
##
## [[9]]
## [[9]]$covariate_inclusion
## [1] 1 2 4
##
## [[9]]$random_effect_inclusion
## integer(0)
##
##
## [[10]]
## [[10]]$covariate_inclusion
## [1] 1 2 4
##
## [[10]]$random_effect_inclusion
## [1] 1
```

```
print(result_postprob)
```

```
## [[1]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  1  1  0 0.277
## 2    1  1  1  1  1  0 0.196
## 3    1  1  0  1  0  1 0.131
## 4    1  1  0  0  0  1 0.079
## 5    1  1  0  0  1  0 0.066
## 6    1  1  1  1  0  1 0.059
## 7    1  1  1  1  0  0 0.041
## 8    1  1  0  1  0  0 0.038
## 9    1  1  0  1  1  1 0.028
## 10   1  1  1  0  1  0 0.021
##
## [[2]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.503
## 2    1  1  0  1  1  0 0.190
## 3    1  1  1  0  1  0 0.128
## 4    1  1  1  1  1  0 0.116
## 5    1  1  0  0  1  1 0.030
## 6    1  1  0  0  0  1 0.011
## 7    1  1  0  1  1  1 0.007
## 8    1  1  1  0  1  1 0.005
## 9    1  1  1  1  1  1 0.002
## 10   1  1  0  1  0  1 0.002
##
## [[3]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  1  0  0 0.219
## 2    1  1  0  0  1  0 0.164
## 3    1  1  1  0  0  0 0.163
## 4    1  1  1  0  1  0 0.121
## 5    1  1  1  1  1  0 0.096
## 6    1  1  0  0  0  0 0.083
## 7    1  1  0  1  1  0 0.074
## 8    1  1  0  1  0  0 0.063
## 9    1  1  0  0  1  1 0.004
## 10   1  1  0  0  0  1 0.004
##
## [[4]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  0  0 0.300
## 2    1  1  1  1  0  0 0.261
## 3    1  1  0  1  0  0 0.227
## 4    1  1  1  0  0  0 0.114
## 5    1  1  0  0  1  0 0.035
## 6    1  1  0  0  0  1 0.015
## 7    1  1  0  1  1  0 0.015
## 8    1  1  1  1  1  0 0.011
## 9    1  1  1  0  1  0 0.009
## 10   1  1  0  1  0  1 0.005
##
## [[5]]
```

```
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  1  0  0 0.193
## 2    1  1  0  1  0  0 0.184
## 3    1  1  0  0  0  0 0.126
## 4    1  1  0  0  0  1 0.102
## 5    1  1  0  0  1  0 0.092
## 6    1  1  0  1  0  1 0.061
## 7    1  1  1  0  0  0 0.055
## 8    1  1  0  1  1  0 0.053
## 9    1  1  1  1  1  0 0.036
## 10   1  1  1  0  1  0 0.029
##
## [[6]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  1  1  0 0.291
## 2    1  1  0  0  1  0 0.212
## 3    1  1  1  1  1  0 0.191
## 4    1  1  1  1  0  0 0.078
## 5    1  1  0  1  0  0 0.078
## 6    1  1  1  0  1  0 0.064
## 7    1  1  0  0  0  0 0.035
## 8    1  1  1  0  0  0 0.015
## 9    1  1  0  0  0  1 0.009
## 10   1  1  0  0  1  1 0.008
##
## [[7]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  1  0  0 0.515
## 2    1  1  1  0  0  0 0.354
## 3    1  1  1  0  1  0 0.040
## 4    1  1  1  1  1  0 0.034
## 5    1  1  1  0  0  1 0.031
## 6    1  1  1  1  0  1 0.015
## 7    1  1  0  0  0  1 0.003
## 8    1  1  1  0  1  1 0.002
## 9    1  1  0  0  0  0 0.002
## 10   1  1  0  0  1  0 0.002
##
## [[8]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  0  1 0.306
## 2    1  1  0  0  0  0 0.096
## 3    1  1  0  1  0  1 0.095
## 4    1  1  0  0  1  0 0.091
## 5    1  1  1  1  0  0 0.075
## 6    1  1  0  1  0  0 0.072
## 7    1  1  1  0  0  1 0.064
## 8    1  1  1  1  0  1 0.045
## 9    1  1  1  0  0  0 0.036
## 10   1  1  0  1  1  0 0.033
##
## [[9]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  1  0  0 0.293
```

```
## 2    1  1  0  0  0  0 0.240
## 3    1  1  0  1  0  0 0.170
## 4    1  1  1  0  0  0 0.149
## 5    1  1  0  0  1  0 0.052
## 6    1  1  1  1  1  0 0.025
## 7    1  1  1  0  1  0 0.024
## 8    1  1  0  1  1  0 0.022
## 9    1  1  0  0  0  1 0.012
## 10   1  1  1  0  0  1 0.004
##
## [[10]]
##      x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.227
## 2    1  1  0  1  1  0 0.216
## 3    1  1  1  1  1  0 0.153
## 4    1  1  1  1  0  0 0.118
## 5    1  1  0  1  0  0 0.108
## 6    1  1  1  0  1  0 0.062
## 7    1  1  0  1  0  1 0.027
## 8    1  1  0  0  0  0 0.017
## 9    1  1  0  0  1  1 0.016
## 10   1  1  0  0  0  1 0.016
```

```
### HC

beta<-c(1,0.5,0.5,0,0)
result_best<-list()
result_postprob<-list()
tau=0.05
for (i in 1:10){
  set.seed(i)
  X1<-matrix(rnorm(400),nrow=100,ncol=4)
  X2<-cbind(1,X1)
  spatial_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[1]])
  overdispersion_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[2]])
  linear_predictor <- X2 %*% beta + spatial_random_effect + overdispersion_random_effect
  Y<-rpois(100,exp(linear_predictor))
  Model_selection_output <- GLMMselect(Y=Y, X=X1, Sigma=Sigma,
                            Z=Z, family="poisson", prior="HC", offset=NULL)
  result_best[[i]]<-Model_selection_output$BestModel
  result_postprob[[i]]<-Model_selection_output$PosteriorProb
}
```

```
print(result_best)
```

```
## [[1]]
## [[1]]$covariate_inclusion
## [1] 1 2 4
##
## [[1]]$random_effect_inclusion
## [1] 1
##
##
## [[2]]
## [[2]]$covariate_inclusion
## [1] 1 2
##
## [[2]]$random_effect_inclusion
## [1] 1
##
##
## [[3]]
## [[3]]$covariate_inclusion
## [1] 1 2 3
##
## [[3]]$random_effect_inclusion
## integer(0)
##
##
## [[4]]
## [[4]]$covariate_inclusion
## [1] 1 2 4
##
## [[4]]$random_effect_inclusion
## integer(0)
##
##
## [[5]]
## [[5]]$covariate_inclusion
## [1] 1 2 4
##
## [[5]]$random_effect_inclusion
## integer(0)
##
##
## [[6]]
## [[6]]$covariate_inclusion
## [1] 1 2 4
##
## [[6]]$random_effect_inclusion
## [1] 1
##
##
## [[7]]
## [[7]]$covariate_inclusion
## [1] 1 2 3 4
##
## [[7]]$random_effect_inclusion
```

```
## integer(0)
##
##
## [[8]]
## [[8]]$covariate_inclusion
## [1] 1 2
##
## [[8]]$random_effect_inclusion
## integer(0)
##
##
## [[9]]
## [[9]]$covariate_inclusion
## [1] 1 2 4
##
## [[9]]$random_effect_inclusion
## integer(0)
##
##
## [[10]]
## [[10]]$covariate_inclusion
## [1] 1 2 4
##
## [[10]]$random_effect_inclusion
## [1] 1
```

```
print(result_postprob)
```

```
## [[1]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  1  1  0 0.280
## 2    1  1  1  1  1  0 0.158
## 3    1  1  0  1  0  1 0.132
## 4    1  1  0  0  0  1 0.094
## 5    1  1  0  0  1  0 0.075
## 6    1  1  1  1  0  0 0.061
## 7    1  1  0  1  0  0 0.057
## 8    1  1  1  1  0  1 0.040
## 9    1  1  0  0  0  0 0.021
## 10   1  1  1  0  1  0 0.020
##
## [[2]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.555
## 2    1  1  0  1  1  0 0.183
## 3    1  1  1  0  1  0 0.123
## 4    1  1  1  1  1  0 0.087
## 5    1  1  0  0  1  1 0.024
## 6    1  1  0  0  0  1 0.012
## 7    1  1  0  1  1  1 0.004
## 8    1  1  1  0  1  1 0.003
## 9    1  1  0  1  0  1 0.002
## 10   1  1  1  0  0  1 0.002
##
## [[3]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  1  0  0 0.255
## 2    1  1  1  0  0  0 0.189
## 3    1  1  0  0  1  0 0.150
## 4    1  1  1  0  1  0 0.099
## 5    1  1  0  0  0  0 0.097
## 6    1  1  0  1  0  0 0.074
## 7    1  1  1  1  1  0 0.064
## 8    1  1  0  1  1  0 0.060
## 9    1  1  0  0  0  1 0.003
## 10   1  1  0  0  1  1 0.003
##
## [[4]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  0  0 0.309
## 2    1  1  1  1  0  0 0.268
## 3    1  1  0  1  0  0 0.233
## 4    1  1  1  0  0  0 0.118
## 5    1  1  0  0  1  0 0.028
## 6    1  1  0  0  0  1 0.013
## 7    1  1  0  1  1  0 0.011
## 8    1  1  1  0  1  0 0.006
## 9    1  1  1  1  1  0 0.006
## 10   1  1  0  1  0  1 0.004
##
## [[5]]
```

```
##     x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  1  1  0  0 0.222
## 2   1  1  0  1  0  0 0.212
## 3   1  1  0  0  0  0 0.145
## 4   1  1  0  0  0  1 0.096
## 5   1  1  0  0  1  0 0.083
## 6   1  1  1  0  0  0 0.063
## 7   1  1  0  1  0  1 0.049
## 8   1  1  0  1  1  0 0.042
## 9   1  1  1  1  1  0 0.023
## 10  1  1  1  0  1  0 0.023
##
## [[6]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  0  1  1  0 0.267
## 2   1  1  0  0  1  0 0.223
## 3   1  1  1  1  1  0 0.139
## 4   1  1  1  1  0  0 0.108
## 5   1  1  0  1  0  0 0.108
## 6   1  1  1  0  1  0 0.058
## 7   1  1  0  0  0  0 0.049
## 8   1  1  1  0  0  0 0.020
## 9   1  1  0  0  0  1 0.009
## 10  1  1  0  0  1  1 0.006
##
## [[7]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  1  1  0  0 0.541
## 2   1  1  1  0  0  0 0.371
## 3   1  1  1  0  1  0 0.029
## 4   1  1  1  0  0  1 0.023
## 5   1  1  1  1  1  0 0.020
## 6   1  1  1  1  0  1 0.008
## 7   1  1  0  0  0  1 0.002
## 8   1  1  0  0  0  0 0.002
## 9   1  1  0  0  1  0 0.001
## 10  1  1  0  1  0  0 0.001
##
## [[8]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  0  0  0  1 0.310
## 2   1  1  0  0  0  0 0.121
## 3   1  1  1  1  0  0 0.095
## 4   1  1  0  1  0  0 0.091
## 5   1  1  0  0  1  0 0.087
## 6   1  1  0  1  0  1 0.081
## 7   1  1  1  0  0  1 0.055
## 8   1  1  1  0  0  0 0.046
## 9   1  1  0  1  1  0 0.028
## 10  1  1  1  1  0  1 0.026
##
## [[9]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  1  1  0  0 0.305
```

```
## 2    1  1  0  0  0  0 0.251
## 3    1  1  0  1  0  0 0.178
## 4    1  1  1  0  0  0 0.155
## 5    1  1  0  0  1  0 0.043
## 6    1  1  1  0  1  0 0.018
## 7    1  1  0  1  1  0 0.016
## 8    1  1  1  1  1  0 0.015
## 9    1  1  0  0  0  1 0.010
## 10   1  1  1  0  0  1 0.003
##
## [[10]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.231
## 2    1  1  0  1  1  0 0.191
## 3    1  1  1  1  0  0 0.164
## 4    1  1  0  1  0  0 0.150
## 5    1  1  1  1  1  0 0.106
## 6    1  1  1  0  1  0 0.055
## 7    1  1  0  1  0  1 0.024
## 8    1  1  0  0  0  0 0.024
## 9    1  1  0  0  0  1 0.017
## 10   1  1  0  0  1  1 0.012
```

Scenario Three: Poisson Regression

In this scenario, the values of τ_1 and τ_2 are held constant to 0.05, and the coefficient vector is updated to $\beta = (4, 0.1, 0.1, 0, 0)$.

```
### AR

library(MASS)
beta<-c(4,0.1,0.1,0,0)
result_best<-list()
result_postprob<-list()
tau=0.05
for (i in 1:10){
  set.seed(i)
  X1<-matrix(rnorm(400),nrow=100,ncol=4)
  X2<-cbind(1,X1)
  spatial_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[1]])
  overdispersion_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau * Sigma[[2]])
  linear_predictor <- X2 %*% beta + spatial_random_effect + overdispersion_random_effect
  Y<-rpois(100,exp(linear_predictor))
  Model_selection_output <- GLMMselect(Y=Y, X=X1, Sigma=Sigma,
                           Z=Z, family="poisson", prior="AR", offset=NULL)
  result_best[[i]]<-Model_selection_output$BestModel
  result_postprob[[i]]<-Model_selection_output$PosteriorProb
}
```

```
print(result_best)
```

```
## [[1]]
## [[1]]$covariate_inclusion
## [1] 1 2
##
## [[1]]$random_effect_inclusion
## [1] 1
##
##
## [[2]]
## [[2]]$covariate_inclusion
## [1] 1
##
## [[2]]$random_effect_inclusion
## [1] 1 2
##
##
## [[3]]
## [[3]]$covariate_inclusion
## [1] 1 2
##
## [[3]]$random_effect_inclusion
## [1] 1
##
##
## [[4]]
## [[4]]$covariate_inclusion
## [1] 1 2
##
## [[4]]$random_effect_inclusion
## [1] 1 2
##
##
## [[5]]
## [[5]]$covariate_inclusion
## [1] 1
##
## [[5]]$random_effect_inclusion
## [1] 1 2
##
##
## [[6]]
## [[6]]$covariate_inclusion
## [1] 1 2
##
## [[6]]$random_effect_inclusion
## [1] 1 2
##
##
## [[7]]
## [[7]]$covariate_inclusion
## [1] 1
##
## [[7]]$random_effect_inclusion
```

```
## [1] 2
##
##
## [[8]]
## [[8]]$covariate_inclusion
## [1] 1 2 3
##
## [[8]]$random_effect_inclusion
## [1] 1 2
##
##
## [[9]]
## [[9]]$covariate_inclusion
## [1] 1 2
##
## [[9]]$random_effect_inclusion
## [1] 1
##
##
## [[10]]
## [[10]]$covariate_inclusion
## integer(0)
##
## [[10]]$random_effect_inclusion
## [1] 1 2
```

```
print(result_postprob)
```

```
## [[1]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.295
## 2    1  1  0  0  1  1 0.146
## 3    1  1  1  0  1  0 0.141
## 4    1  0  0  0  1  1 0.078
## 5    1  0  0  0  1  0 0.076
## 6    1  1  1  1  1  0 0.070
## 7    1  1  0  1  1  0 0.067
## 8    1  1  1  0  1  1 0.032
## 9    1  0  1  0  1  0 0.021
## 10   1  1  0  1  1  1 0.020
##
## [[2]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  0  0  0  1  1 0.416
## 2    0  0  0  0  1  1 0.138
## 3    1  0  0  0  1  0 0.137
## 4    1  1  0  0  1  1 0.045
## 5    1  0  1  0  1  1 0.040
## 6    0  0  0  0  1  0 0.039
## 7    1  0  1  0  1  0 0.032
## 8    1  1  0  0  1  0 0.031
## 9    1  0  0  1  1  1 0.026
## 10   1  0  0  1  1  0 0.015
##
## [[3]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.450
## 2    1  1  0  1  1  0 0.140
## 3    1  1  1  0  1  0 0.133
## 4    1  1  1  1  1  0 0.119
## 5    1  1  0  0  1  1 0.107
## 6    1  1  1  0  1  1 0.017
## 7    1  1  0  1  1  1 0.013
## 8    1  0  0  0  1  1 0.009
## 9    1  1  1  1  1  1 0.005
## 10   1  0  0  0  1  0 0.002
##
## [[4]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  1 0.350
## 2    0  1  0  0  1  1 0.104
## 3    1  1  0  0  0  1 0.093
## 4    1  1  0  0  1  0 0.073
## 5    1  1  0  1  1  1 0.069
## 6    1  1  0  1  0  1 0.058
## 7    1  1  1  0  1  1 0.040
## 8    1  1  0  1  1  0 0.035
## 9    0  1  0  0  0  1 0.029
## 10   1  1  1  1  0  1 0.019
##
## [[5]]
```

```
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  0  0  0  1  1 0.264
## 2    1  0  0  0  0  1 0.159
## 3    1  1  0  0  1  1 0.117
## 4    1  1  0  0  0  1 0.110
## 5    0  0  0  0  1  1 0.076
## 6    0  1  0  0  1  1 0.024
## 7    1  1  0  1  0  1 0.020
## 8    1  0  0  1  1  1 0.019
## 9    1  0  0  0  1  0 0.018
## 10   1  1  1  0  0  1 0.018
##
## [[6]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  1 0.268
## 2    1  1  0  0  1  0 0.218
## 3    1  1  0  0  0  1 0.104
## 4    1  1  1  0  1  0 0.091
## 5    1  1  0  1  1  0 0.087
## 6    1  1  1  1  1  0 0.084
## 7    1  1  1  0  1  1 0.046
## 8    1  1  0  1  1  1 0.036
## 9    1  1  1  0  0  1 0.022
## 10   1  1  0  1  0  1 0.018
##
## [[7]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  0  1 0.221
## 2    1  0  0  0  0  1 0.178
## 3    1  1  1  0  0  1 0.068
## 4    1  1  0  0  0  1 0.067
## 5    1  0  1  0  0  1 0.062
## 6    0  0  0  0  1  1 0.060
## 7    0  0  1  0  0  1 0.041
## 8    0  1  0  0  0  1 0.035
## 9    1  0  0  0  1  1 0.031
## 10   0  1  1  0  0  1 0.023
##
## [[8]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  1  1  0 0.210
## 2    1  1  1  0  1  1 0.205
## 3    1  1  1  0  1  0 0.161
## 4    1  1  0  0  1  1 0.149
## 5    1  1  1  1  1  1 0.076
## 6    1  1  0  0  1  0 0.055
## 7    1  1  0  1  1  0 0.043
## 8    1  1  0  1  1  1 0.034
## 9    0  1  0  0  1  1 0.021
## 10   0  1  1  0  1  1 0.011
##
## [[9]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  0 0.401
```

```
## 2    1  1  1  0  1  0 0.201
## 3    1  1  1  1  1  0 0.139
## 4    1  1  0  1  1  0 0.125
## 5    1  0  0  0  1  0 0.054
## 6    1  1  0  0  1  1 0.028
## 7    1  0  0  0  1  1 0.016
## 8    1  0  0  1  1  0 0.007
## 9    1  0  1  0  1  0 0.007
## 10   1  1  1  0  1  1 0.007
##
## [[10]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    0  0  0  0  1  1 0.328
## 2    1  0  0  0  1  1 0.137
## 3    0  0  0  0  1  0 0.112
## 4    1  1  0  0  1  0 0.056
## 5    1  1  0  0  1  1 0.056
## 6    0  1  0  0  1  0 0.053
## 7    0  1  0  0  1  1 0.051
## 8    1  0  0  0  1  0 0.050
## 9    1  1  0  1  1  0 0.015
## 10   1  1  1  0  1  0 0.013
```

Scenario Four: Poisson Regression

In this scenario, the values of the parameters are modified such that $\tau_1 = 0.01$ and $\tau_2 = 0$. Additionally, the coefficient vector is updated to $\beta = (1, 0.5, 0.5, 0, 0)$.

```
### AR

library(MASS)
beta<-c(1,0.5,0.5,0,0)
result_best<-list()
result_postprob<-list()
tau1=0.01
tau2=0
for (i in 1:10){
  set.seed(i)
  X1<-matrix(rnorm(400),nrow=100,ncol=4)
  X2<-cbind(1,X1)
  spatial_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau1 * Sigma[[1]])
  overdispersion_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau2 * Sigma[[2]])
  linear_predictor <- X2 %*% beta + spatial_random_effect + overdispersion_random_effect
  Y<-rpois(100,exp(linear_predictor))
  Model_selection_output <- GLMMselect(Y=Y, X=X1, Sigma=Sigma,
                           Z=Z, family="poisson", prior="AR", offset=NULL)
  result_best[[i]]<-Model_selection_output$BestModel
  result_postprob[[i]]<-Model_selection_output$PosteriorProb
}
```

```
print(result_best)
```

```
## [[1]]
## [[1]]$covariate_inclusion
## [1] 1 2 3
##
## [[1]]$random_effect_inclusion
## integer(0)
##
##
## [[2]]
## [[2]]$covariate_inclusion
## [1] 1 2 3 4
##
## [[2]]$random_effect_inclusion
## integer(0)
##
##
## [[3]]
## [[3]]$covariate_inclusion
## [1] 1 2 3
##
## [[3]]$random_effect_inclusion
## integer(0)
##
##
## [[4]]
## [[4]]$covariate_inclusion
## [1] 1 2
##
## [[4]]$random_effect_inclusion
## integer(0)
##
##
## [[5]]
## [[5]]$covariate_inclusion
## [1] 1 2
##
## [[5]]$random_effect_inclusion
## integer(0)
##
##
## [[6]]
## [[6]]$covariate_inclusion
## [1] 1 2
##
## [[6]]$random_effect_inclusion
## integer(0)
##
##
## [[7]]
## [[7]]$covariate_inclusion
## [1] 1 2 4
##
## [[7]]$random_effect_inclusion
```

```
## integer(0)
##
##
## [[8]]
## [[8]]$covariate_inclusion
## [1] 1 2
##
## [[8]]$random_effect_inclusion
## integer(0)
##
##
## [[9]]
## [[9]]$covariate_inclusion
## [1] 1 2
##
## [[9]]$random_effect_inclusion
## integer(0)
##
##
## [[10]]
## [[10]]$covariate_inclusion
## [1] 1 2 3 4
##
## [[10]]$random_effect_inclusion
## integer(0)
```

```
print(result_postprob)
```

```
## [[1]]
##    x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  1  1  0  0 0.348
## 2   1  1  1  0  0  0 0.345
## 3   1  1  0  0  0  0 0.187
## 4   1  1  0  1  0  0 0.069
## 5   1  1  1  0  1  0 0.013
## 6   1  1  0  0  1  0 0.010
## 7   1  1  1  1  1  0 0.008
## 8   1  1  0  0  0  1 0.007
## 9   1  1  1  0  0  1 0.006
## 10  1  1  1  1  0  1 0.003
##
## [[2]]
##    x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  1  1  0  0 0.331
## 2   1  1  0  0  0  0 0.245
## 3   1  1  1  0  0  0 0.187
## 4   1  1  0  1  0  0 0.150
## 5   1  1  0  0  1  0 0.023
## 6   1  1  0  0  0  1 0.014
## 7   1  1  1  1  1  0 0.014
## 8   1  1  1  0  1  0 0.012
## 9   1  1  0  1  1  0 0.010
## 10  1  1  1  0  0  1 0.005
##
## [[3]]
##    x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  0  0  0  0 0.301
## 2   1  1  1  1  0  0 0.257
## 3   1  1  1  0  0  0 0.254
## 4   1  1  0  1  0  0 0.131
## 5   1  1  0  0  1  0 0.019
## 6   1  1  1  0  1  0 0.011
## 7   1  1  0  0  0  1 0.008
## 8   1  1  1  1  1  0 0.006
## 9   1  1  0  1  1  0 0.005
## 10  1  1  1  0  0  1 0.003
##
## [[4]]
##    x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  0  0  0  0 0.365
## 2   1  1  1  1  0  0 0.183
## 3   1  1  0  1  0  0 0.154
## 4   1  1  1  0  0  0 0.150
## 5   1  1  0  0  1  0 0.072
## 6   1  1  1  0  1  0 0.020
## 7   1  1  0  1  1  0 0.019
## 8   1  1  0  0  0  1 0.014
## 9   1  1  1  1  1  0 0.013
## 10  1  1  0  1  0  1 0.003
##
## [[5]]
```

```
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  0  0 0.419
## 2    1  1  1  1  0  0 0.175
## 3    1  1  1  0  0  0 0.164
## 4    1  1  0  1  0  0 0.164
## 5    1  1  0  0  1  0 0.036
## 6    1  1  0  0  0  1 0.012
## 7    1  1  1  0  1  0 0.010
## 8    1  1  0  1  1  0 0.009
## 9    1  1  1  1  1  0 0.006
## 10   1  1  1  0  0  1 0.002
##
## [[6]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  0  0 0.322
## 2    1  1  1  1  0  0 0.229
## 3    1  1  0  1  0  0 0.183
## 4    1  1  1  0  0  0 0.165
## 5    1  1  0  0  1  0 0.039
## 6    1  1  1  0  1  0 0.015
## 7    1  1  0  1  1  0 0.015
## 8    1  1  1  1  1  0 0.013
## 9    1  1  0  0  0  1 0.009
## 10   1  1  0  1  0  1 0.002
##
## [[7]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  1  0  0 0.353
## 2    1  1  0  1  0  0 0.316
## 3    1  1  0  0  0  0 0.199
## 4    1  1  1  0  0  0 0.085
## 5    1  1  0  1  1  0 0.011
## 6    1  1  0  0  1  0 0.011
## 7    1  1  0  0  0  1 0.007
## 8    1  1  1  1  1  0 0.006
## 9    1  1  0  1  0  1 0.004
## 10   1  1  1  0  1  0 0.003
##
## [[8]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  0  0 0.320
## 2    1  1  1  1  0  0 0.232
## 3    1  1  1  0  0  0 0.195
## 4    1  1  0  1  0  0 0.137
## 5    1  1  0  0  0  1 0.038
## 6    1  1  0  0  1  0 0.027
## 7    1  1  1  0  1  0 0.010
## 8    1  1  1  0  0  1 0.010
## 9    1  1  0  1  0  1 0.009
## 10   1  1  0  1  1  0 0.007
##
## [[9]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  0  0 0.336
```

```
## 2    1  1  1  1  0  0 0.237
## 3    1  1  0  1  0  0 0.233
## 4    1  1  1  0  0  0 0.130
## 5    1  1  0  0  1  0 0.023
## 6    1  1  0  1  1  0 0.011
## 7    1  1  0  0  0  1 0.009
## 8    1  1  1  1  1  0 0.007
## 9    1  1  1  0  1  0 0.006
## 10   1  1  0  1  0  1 0.003
##
## [[10]]
##      x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  1  0  0 0.495
## 2    1  1  0  1  0  0 0.280
## 3    1  1  0  0  0  0 0.117
## 4    1  1  1  0  0  0 0.057
## 5    1  1  0  1  1  0 0.011
## 6    1  1  1  1  1  0 0.011
## 7    1  1  0  0  1  0 0.010
## 8    1  1  0  1  0  1 0.005
## 9    1  1  0  0  0  1 0.005
## 10   1  1  1  1  0  1 0.003
```

Scenario Five: Poisson Regression

In this scenario, the parameter values are set to $\tau_1 = \tau_2 = 1$, and the coefficient vector is specified as $\beta = (1, 1, 1, 0, 0)$.

```
library(MASS)
beta<-c(1,1,1,0,0)
result_best<-list()
result_postprob<-list()
tau1=1
tau2=1
for (i in 1:10){
  set.seed(i)
    X1<-matrix(rnorm(400),nrow=100,ncol=4)
  X2<-cbind(1,X1)
  spatial_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau1 * Sigma[[1]])
  overdispersion_random_effect <- mvrnorm(1, mu = rep(0, 100), Sigma = tau2 * Sigma[[2]])
  linear_predictor <- X2 %*% beta + spatial_random_effect + overdispersion_random_effect
  Y<-rpois(100,exp(linear_predictor))
  Model_selection_output <- GLMMselect(Y=Y, X=X1, Sigma=Sigma,
                            Z=Z, family="poisson", prior="AR", offset=NULL)
  result_best[[i]]<-Model_selection_output$BestModel
  result_postprob[[i]]<-Model_selection_output$PosteriorProb
}
```

```
print(result_best)
```

```
## [[1]]
## [[1]]$covariate_inclusion
## [1] 1 2
##
## [[1]]$random_effect_inclusion
## [1] 1
##
##
## [[2]]
## [[2]]$covariate_inclusion
## [1] 1 2
##
## [[2]]$random_effect_inclusion
## [1] 1
##
##
## [[3]]
## [[3]]$covariate_inclusion
## [1] 1 2
##
## [[3]]$random_effect_inclusion
## [1] 1
##
##
## [[4]]
## [[4]]$covariate_inclusion
## [1] 1 2 3
##
## [[4]]$random_effect_inclusion
## [1] 1 2
##
##
## [[5]]
## [[5]]$covariate_inclusion
## [1] 1 2
##
## [[5]]$random_effect_inclusion
## [1] 1 2
##
##
## [[6]]
## [[6]]$covariate_inclusion
## [1] 1 2
##
## [[6]]$random_effect_inclusion
## [1] 1 2
##
##
## [[7]]
## [[7]]$covariate_inclusion
## [1] 1 2 3
##
## [[7]]$random_effect_inclusion
```

```
## [1] 2
##
##
## [[8]]
## [[8]]$covariate_inclusion
## [1] 1 2
##
## [[8]]$random_effect_inclusion
## [1] 1 2
##
##
## [[9]]
## [[9]]$covariate_inclusion
## [1] 1 2 3
##
## [[9]]$random_effect_inclusion
## [1] 1 2
##
##
## [[10]]
## [[10]]$covariate_inclusion
## [1] 1 2
##
## [[10]]$random_effect_inclusion
## [1] 1
```

```
print(result_postprob)
```

```
## [[1]]
##    x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  0  0  1  0 0.356
## 2   1  1  0  0  1  1 0.216
## 3   1  1  0  1  1  0 0.117
## 4   1  1  1  0  1  0 0.107
## 5   1  1  1  1  1  0 0.083
## 6   1  1  0  1  1  1 0.050
## 7   1  1  1  0  1  1 0.047
## 8   1  1  1  1  1  1 0.025
## 9   1  1  0  0  0  1 0.000
## 10  1  1  1  0  0  1 0.000
##
## [[2]]
##    x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  0  0  1  1 0.236
## 2   1  1  0  0  1  0 0.199
## 3   1  1  1  1  1  0 0.119
## 4   1  1  0  1  1  0 0.116
## 5   1  1  1  0  1  0 0.108
## 6   1  1  0  1  1  1 0.090
## 7   1  1  1  0  1  1 0.075
## 8   1  1  1  1  1  1 0.057
## 9   1  0  0  0  1  1 0.000
## 10  1  0  0  0  1  0 0.000
##
## [[3]]
##    x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  0  0  1  0 0.377
## 2   1  1  1  0  1  0 0.154
## 3   1  1  0  0  1  1 0.151
## 4   1  1  0  1  1  0 0.108
## 5   1  1  1  1  1  0 0.104
## 6   1  1  1  0  1  1 0.051
## 7   1  1  0  1  1  1 0.032
## 8   1  1  1  1  1  1 0.024
## 9   1  1  0  0  0  1 0.000
## 10  1  1  1  0  0  1 0.000
##
## [[4]]
##    x1 x2 x3 x4 r1 r2   MPP
## 1   1  1  0  0  1  1 0.224
## 2   1  1  0  0  1  0 0.143
## 3   1  1  1  0  1  1 0.143
## 4   1  1  1  1  1  0 0.137
## 5   1  1  1  1  1  1 0.101
## 6   1  1  1  0  1  0 0.100
## 7   1  1  0  1  1  0 0.055
## 8   1  1  0  1  1  1 0.052
## 9   1  1  0  0  0  1 0.015
## 10  1  1  1  1  0  1 0.014
##
## [[5]]
```

```
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  1 0.280
## 2    1  1  0  1  1  1 0.128
## 3    1  1  0  0  1  0 0.123
## 4    1  1  0  1  1  0 0.098
## 5    1  1  1  1  1  0 0.072
## 6    1  1  1  1  1  1 0.069
## 7    1  1  0  0  0  1 0.066
## 8    1  1  1  0  1  1 0.065
## 9    1  1  1  0  1  0 0.037
## 10   1  1  0  1  0  1 0.029
##
## [[6]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  1 0.375
## 2    1  1  0  0  0  1 0.125
## 3    1  1  0  0  1  0 0.099
## 4    1  1  0  1  1  1 0.088
## 5    1  1  1  0  1  1 0.085
## 6    1  1  1  1  1  1 0.046
## 7    1  1  0  1  1  0 0.043
## 8    1  1  1  0  0  1 0.031
## 9    1  1  1  1  1  0 0.031
## 10   1  1  0  1  0  1 0.030
##
## [[7]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  1  0  0  1 0.365
## 2    1  1  1  1  0  1 0.296
## 3    1  1  1  0  1  1 0.111
## 4    1  1  1  1  1  1 0.086
## 5    1  1  0  0  0  1 0.081
## 6    1  1  0  1  0  1 0.026
## 7    1  1  0  0  1  1 0.021
## 8    1  1  0  1  1  1 0.006
## 9    1  1  1  1  1  0 0.005
## 10   1  1  1  0  1  0 0.003
##
## [[8]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  1 0.389
## 2    1  1  1  0  1  1 0.287
## 3    1  1  1  1  1  1 0.157
## 4    1  1  0  1  1  1 0.094
## 5    1  1  1  0  1  0 0.016
## 6    1  1  0  0  1  0 0.015
## 7    1  1  1  1  1  0 0.014
## 8    1  1  1  0  0  1 0.007
## 9    1  1  0  0  0  1 0.007
## 10   1  1  0  1  1  0 0.006
##
## [[9]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  1 0.225
```

```
## 2    1  1  1  0  1  1 0.189
## 3    1  1  1  0  1  0 0.129
## 4    1  1  1  1  1  0 0.110
## 5    1  1  1  1  1  1 0.106
## 6    1  1  0  0  1  0 0.076
## 7    1  1  0  1  1  1 0.054
## 8    1  1  0  0  0  1 0.038
## 9    1  1  0  1  1  0 0.027
## 10   1  1  1  0  0  1 0.022
##
## [[10]]
##     x1 x2 x3 x4 r1 r2   MPP
## 1    1  1  0  0  1  1 0.284
## 2    1  1  0  0  1  0 0.242
## 3    1  1  1  0  1  0 0.095
## 4    1  1  0  1  1  0 0.085
## 5    1  1  1  1  1  0 0.081
## 6    1  1  1  0  1  1 0.080
## 7    1  1  0  1  1  1 0.080
## 8    1  1  1  1  1  1 0.053
## 9    1  1  0  0  0  1 0.000
## 10   1  1  0  1  0  1 0.000
```

Case Study: Male Lip Cancer in Scotland

This dataset provides the number of male lip cancer cases in 56 counties of Scotland during the period 1975–1980. Additionally, it includes the percentage of the workforce employed in agriculture, fishing, or forestry (AFF) as a covariate.

The most general model considered is defined as follows:

$$y_i|\mu_i \sim_{\text{ind}} \text{Poisson}(\mu_i),$$

$$\log(\mu_i) = \log(n_i) + \mathbf{x_i}^T \boldsymbol{\beta} + \alpha_{1i} + \alpha_{2i}, \quad \boldsymbol{\alpha_1} \sim \text{Normal}(0, \tau_1 \Sigma), \quad \boldsymbol{\alpha_2} \sim \text{Normal}(0, \tau_2 \Sigma),$$

where:

- y_i: The observed number of lip cancer cases in the i-th county.
- n_i: The expected number of lip cancer cases in the i-th county, calculated based on age distributions across counties (treated as a constant).
- $\mathbf{x_i}$: The covariate vector, including AFF as a predictor.
- $\boldsymbol{\beta}$: The vector of fixed-effect coefficients.
- $\boldsymbol{\alpha_1}$: The overdispersion random effects, modeled as $\boldsymbol{\alpha_1} \sim \text{Normal}(0, \tau_2 \Sigma)$.
- $\boldsymbol{\alpha_2}$: The spatial random effects, modeled as $\boldsymbol{\alpha_2} \sim \text{Normal}(0, \tau_1 \Sigma)$.
- Σ: The covariance matrix associated with random effects.

```
lip_cancer_output <- GLMMselect(Y=lipcancer_Y, X=lipcancer_X, Sigma=lipcancer_Sigma, Z=lipcancer_Z,
                                family="poisson", prior="HC", offset=lipcancer_offset)
```

```
lip_cancer_output$BestModel
```

```
## $covariate_inclusion
## [1] 1
##
## $random_effect_inclusion
## [1] 1
```

```
lip_cancer_output$PosteriorProb
```

```
##   x1 r1 r2   MPP
## 1  1  1  0 0.905
## 2  0  1  0 0.084
## 3  1  1  1 0.009
## 4  0  1  1 0.002
## 5  1  0  1 0.000
## 6  0  0  1 0.000
## 7  1  0  0 0.000
## 8  0  0  0 0.000
```

```
lip_cancer_output <- GLMMselect(Y=lipcancer_Y, X=lipcancer_X, Sigma=lipcancer_Sigma, Z=lipcancer_Z,
                                family="poisson", prior="AR", offset=lipcancer_offset)
```

```
lip_cancer_output$BestModel
```

```
## $covariate_inclusion
## [1] 1
##
## $random_effect_inclusion
## [1] 1
```

The results indicate that incorporating spatial random effects is necessary, while overdispersion random effects do not contribute significantly to the model.