Volatility and Under-Insurance in Economies with Limited Pledgeability: Evidence from a Frost Shock*

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Abstract: We use transaction-level data on payments, credit, and insurance to examine how Brazilian farmers responded to the severe frost of July 2021, a shock that affected coffee, a perennial crop whose plants are a major component of farm value. The frost shock reduced both output and the pledgeable value of farmers' collateral. We find that insured farmers increased investment in the years following the shock, while uninsured farmers reduced investment and borrowing. We show how this pattern is consistent with models of imperfect pledgeability of a firm's collateral, where constrained firms neither insure (ex ante) nor fully recover from a shock (ex post). Limited commitment endogenously generates under-insurance through the combination of upfront payment of the insurance premium with the tightening of borrowing constraints post-shock due to the decrease in total collateral. We discuss two equilibrium implications of this mechanism: the inefficacy of emergency credit lines in targeting liquidity constrained firms and the amplification of output volatility from the rising risk of extreme weather shocks.

The views expressed in this working paper are those of the authors and do not necessarily reflect those of the Central Bank of Brazil.

1 Introduction

Under-insurance is widespread among financially constrained households and small firms, even when insurance premiums are subsidized below actuarially fair rates. This pattern is observed in settings ranging from health and home insurance to crop and climate risk insurance. Much of the existing literature seeks behavioral explanations or estimates insurance demand in reduced form. In contrast, we show that under-insurance can arise as a constrained-optimal outcome in a general equilibrium setting with collateral constraints. We also show that insurance coverage meaningfully alters the propagation

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of liquidity shocks. Using rich microdata from Brazil's agricultural sector in response to an extreme weather shock, we estimate how the response of credit, investment, and insurance demand differs depending on insurance status, and we show how these results are consistent with our model.

This paper provides the first empirical evidence with both financial and output microdata that shows how collateral constraints reduce hedging in response to a large negative one-time shock, amplifying and extending the time horizon of the shock. We study the aftermath of the July 2021 frost in Brazil, which severely damaged perennial crops, most notably coffee, substantially reducing the productive capital stock of affected farmers. Coffee trees, which require years to mature, are a core part of both production and collateral value for these farmers. We document that uninsured farmers experiencing more severe shocks reduced debt and investment significantly more than their insured counterparts, and that take-up of insurance after the shock declined among those most affected. This occurred despite community-rated premiums and unchanged interest rates. Our findings are inconsistent with models that treat credit and insurance as substitutes. Instead, our findings are consistent with a constrained environment where borrowing capacity and insurance interact dynamically through collateral constraints.

To interpret these empirical patterns, we develop a general equilibrium model of a production economy with limited commitment and collateral constraints, building on the framework of Rampini and Viswanathan (2022). Farmers face shocks to their capital stock, motivated by natural disasters, that simultaneously reduce both output and pledgeable collateral. In contrast to their setting with income shocks, our model emphasizes capital stock shocks, which are empirically relevant in climate-exposed sectors and affect both contemporaneous production and future borrowing capacity. The core friction is limited commitment: farmers can walk away with current output and a fraction of future collateral, generating endogenously incomplete markets. Insurance enables the reallocation of resources across states but also tightens collateral constraints ex ante, reducing borrowing for non-insurance motives. We show that the optimal state-contingent contract with limited commitment is equivalent to a problem with endogenous borrowing limits, similarly to Alvarez and Jermann (2000), but where default does not trigger exclusion from future borrowing, consistent with our institutional setting. In line with Rampini and Viswanathan (2022), our model predicts that low-net-worth agents use all available financial slack and do not buy insurance to preserve borrowing capacity. We extend these insights by predicting that (1) uninsured farmers reduce borrowing more than insured farmers in response to a capital shock, and (2) uninsured farmers invest less in the short term. We incorporate endogenous prices: aggregate supply determines the relative price of output, feeding back into individual farmers' investment and consumption choices. These price movements generate additional amplification when exposure to the aggregate shock is heterogeneous. We solve the model numerically by iterating on the marginal utility of wealth, using a dynamic programming formulation with capital and debt as state variables and collateral constraints indexed by realized productivity. Similarly to Sastry et al. (2024), we show empirical evidence of the Rampini and Viswanathan (2022) result that borrowing constrains risk management, but we differ from Sastry et al. (2024) in methodology and mechanism. Sastry et al. (2024) focus on households' insurance decisions and estimate demand elasticities across constrained and unconstrained groups. In contrast, we study firms' dynamic responses to realized liquidity and collateral shocks, quantifying how borrowing constraints distort not just insurance takeup, but also credit and investment behavior following a shock in a general equilibrium setting.

Our model contributes to the theoretical literature on the causes and consequences of under-insurance. Caballero and Krishnamurthy (2003) show that financially constrained firms undervalue insurance when the marginal value of liquidity in production exceeds the market price. Dávila and Korinek (2018) highlight how financial frictions generate both distributive and collateral externalities, resulting in under-insurance and over-borrowing in decentralized equilibria. Martins-Da-Rocha et al. (2022) emphasize that limited pledgeability can lead to inefficient equilibria when strategic default affects equilibrium pricing, and demonstrate that tighter borrowing constraints can improve welfare. While the amplification in those papers arises through endogenous interest rates, our mechanism operates even with fixed rates through equilibrium price effects in the output market when farmers have heterogeneous exposure to aggregate shocks.

More broadly, our paper adds to a literature documenting that financial frictions amplify the real effects of shocks through lower investment, distorted insurance decisions, and endogenous price dynamics. Alfaro et al. (2024) show that financial frictions more than double the impact of aggregate uncertainty shocks on investment and financial variables. Unlike their framework focusing on volatility shocks and cross-sectional firm sensitivity, our model focuses on the interaction of capital destruction, collateral constraints, and price feedback in general equilibrium. Other papers, such as Arellano et al. (2019), Ottonello and Winberry (2020), and Giroud and Mueller (2017) show how financial frictions propagate shocks through investment, credit, and labor channels, but do not study the role of hedging. Our findings show that constrained under-insurance is not only a byproduct of frictions but can amplify real responses through both extensive and intensive margins of credit and investment.

The model rationalizes four key empirical findings: (i) uninsured farmers experience larger declines in borrowing after the shock than insured farmers; (ii) uninsured farmers reduce investment more; (iii) insurance take-up declines most for the most affected uninsured farmers; and (iv) financial amplification arises through endogenous prices and borrowing constraints, with feedback from heterogeneity in exposure to aggregate shocks. We run event study regressions on three sets of outcomes: credit, investment, and insurance. We find that among uninsured coffee farmers, experiencing a 1 percentage point more severe shock resulted in 1.4 percent less debt in the two years after the shock. These farmers also renegotiated debt; the mean maturity of outstanding debt of the most severely affected uninsured farmers was 20 months longer relative to baseline than the debt of unaffected uninsured farmers. In turn, each percentage point of shock severity coincided with 0.6 percent less investment in the form of purchases from upstream sectors to rebuild their capital stock. Among uninsured coffee farmers, those who experienced a more severe shock are less likely than less affected farmers nearby to subsequently purchase insurance, even though insurance pricing is community-rated, meaning that a farmer's history of claims does not affect her insurance premium. By comparison, there is no relationship between insurance takeup and frost shock severity for farmers of annual crops. We replicate our empirical findings through model simulation. Unlike models with endogenous exclusion or interest rate spreads, here under-insurance arises from collateral-based limits, consistent with the Brazilian setting where borrowers are not excluded after default, and loan rates are effectively fixed by policy.

In addition to the contribution of the model linked to the empirics, this paper contributes to two empirical strands of the literature. The first is an empirical literature on the interaction between credit and insurance. One strand focuses on the role of ex ante financial constraints that can be alleviated by financing the insurance premiums (Cole and Xiong, 2017; Casaburi and Willis, 2018). In our setting, subsidized lending is linked to subsidized insurance premiums, which McIntosh et al. (2013) show increases willingness to pay for insurance but Annan (2022) show can exacerbate moral hazard. Yet insurance takeup is still low, so we believe that financing premiums is not a sufficient explanation. Lane (2024) follow a long literature that studies whether credit can substitute for insurance. We show that credit and insurance are complements after a shock despite appearing substitutable ex ante, and we show how this arises from collateral constraints that bind ex post rather than ex ante. We are also the first to construct a comprehensive dataset linking credit and insurance from both formal (firm) and informal (individual) contracts with banks and insurers at the transaction level.

The second is a long empirical literature in corporate finance, with a burgeoning literature in climate finance, on the impact of weather shocks on farmers through financial channels. Bergman et al. (2020) estimate the elasticities of land prices and revenue to yields using a weather shock instrument, in the setting of the farm debt crisis in the 1980s in Iowa before the introduction of crop insurance. Brown et al. (2021) estimate how firms respond to severe weather shocks using credit lines, and find that heavier snow at a firm's headquarters decreases cash flow and increases short-term lending with no impact on investment nor capital stock. Cortés and Strahan (2017) show that there is reallocation of lending across locations in response to a natural disaster shock as banks' balance sheets deteriorate. In our setting, there is a dominant nationwide lender in the agricultural sector backstopped by the government, so lending rates were unchanged, allowing us a clean test of the insurance mechanism. In general, our setting and mechanism differ from each of these papers: the physical shock, causing a reduction in the capital stock, interacted with insurance is what generates our financial and hedging results.

The first paper to formally describe the coffee cycle using a theoretical model and historical data was Netto (1959). Netto (1959) discussed how coffee production experiences periods of expansion followed by crises, driven by backward-looking farmers and the time required to grow coffee plants. During periods of high demand or supply shocks, prices rise sharply, triggering investments. However, as production exceeds demand, stocks accumulate, leading to price declines and a crisis. As stocks deplete and prices recover, the expansion resumes, continuing the boom and bust cycle. Netto (1959) argued that policies aimed at maintaining prices during high production amplify cycle volatility, with negative macroeconomic effects, particularly due to exchange rate volatility during a time when coffee represented 50% of Brazil's exports and 80% of global production. Our model shows that boom and bust cycles can also arise with forward-looking behavior, as unaffected farmers experience relaxed borrowing constraints during shocks, while affected farmers face reduced collateral and borrow less, leading to a slow recovery and high prices before the eventual decline. Moreover, in our model, price stabilization policies may disproportionately benefit unaffected (or insured) farmers, further exacerbating the cycle and increasing volatility.

Our setting is well-suited for our study for three reasons. The first reason is the comprehensive availability of insurance, combined with the common puzzle of low takeup despite subsidized premiums and insurance-linked financing for the premiums. In most settings, revenue insurance is unavailable. In agriculture in Brazil, every farmer can access public insurance, and most can choose to buy additional insurance from private providers. Many other countries offer crop insurance subsidies, with global subsidies exceeding $20 billion, yet under-insurance is pervasive (Hazell and Varangis, 2020). The second reason is the detailed data available on farm production and assets that we merge to payments and credit microdata, allowing us to trace the domestic impacts of the shock at a granular level. Finally, agriculture experiences shocks large enough to impact aggregate output, but also unpredictably heterogeneous enough over nearby locations to provide quasi-exogenous variation for estimation.

The remainder of the paper is organized as follows. Section 2 describes the institutional setting in Brazil, the data, and the timeline of the frost shock. Section 3 introduces the empirical methodology and presents the regression results. Section 4 contains the baseline model, calibration, and simulation results. Section 5 displays the general equilibrium model, still a work-in-progress. Section 6 considers policy implications. Section 7 concludes.

2 Setting and Data

Firms demand insurance due to revenue volatility and the desire to hedge against risk, which can directly arise from preferences or indirectly arise from the cost of default. While price insurance exists for commodities through derivatives markets, there are no available revenue insurance contracts in most sectors. This could be due to a combination of unraveling induced by adverse selection or moral hazard, credit constraints to pay the insurance premium upfront, and limited commitment by insurers to truthfully fulfill claims. In the many countries, including the US and our setting in Brazil, the government offers insurance not just for farmers' revenues, but also yields (quantities) and costs (input expenditures). Despite the widespread provision of subsidies, with insurance premiums an average of 30% below actuarially fair in Brazil, insurance takeup is as low as 20% for crops like coffee. While salience and search costs may be large enough frictions to result in low takeup among small "household" farmers, the phenomenon of low takeup is true for large "industrial" farmers as well in Brazil.

In addition, agriculture is a setting where liquidity shocks are particularly salient because farmers receive most of their income around harvest time. A shock that occurs between harvests can leave farmers in a difficult situation: current and future income decreases, but current expenditure increases to respond to the shock. Even if the farmers have sufficient assets, they can face a cash flow shortage. This suggests that farmers' insurance takeup decision is inherently linked to their credit access and borrowing constraints.

2.1 Data

There are three components to our main dataset: the rural credit registry, crop insurance, and other financial data. The Rural Credit Registry (SICOR) provides contract-level data for agricultural credit and the linked public crop insurance program. The contracts list the crop, the area, the expected production, the actual production, the insurance premium, the insured value, the coverage level, and the precise coordinates of the farm. We observe insurance claims that quantify the amount of the loss, the amount paid to farmers, and a description of the reason for the insurance payout. We supplement the credit and insurance data from SICOR with data from the Ministry of Agriculture (MAPA) on insurance contracts from the main private crop insurers. In MAPA, we observe similar variables to SICOR for premiums, claims, and farm characteristics. To measure hedging beyond insurance, we use data on foreign exchange futures and derivatives contracts, which are cleared and registered through the Brazilian exchange B3, and reported to the Central Bank of Brazil (BCB). Coffee-specific contracts exist because coffee is an internationally traded commodity.

For firms' supply chain linkages and borrowing from banks, we merge transaction-level datasets of interfirm payments, invoices, and credit operations from the BCB. The BCB credit registry (SCR) contains all firms and individuals whose total debt since June 2016 exceeds 200 BRL, equal to around 40 USD at current exchange rates. Using the crosswalk between the BCB datasets and the administrative registry from the federal revenue service, we observe each counterparty's municipality and 7-digit CNAE sector, which is similar in specificity to a 6-digit Harmonized System (HS) code. We classify specific CNAE sectors as upstream of coffee farming, based both on the CNAE descriptions and payment shares, and split the sectors into investment purchases (e.g. seedlings) and material purchases (e.g. fertilizer). See Appendix B for more details.

A key challenge in constructing the combined dataset of contracts at the farmer level was the creation of a crosswalk across insurance and credit contracts that small-scale farmers took under their personal identifier rather than their firm identifier. We believe that we are among the first to construct such a crosswalk for all farmers in any country.

For municipality-level statistics on weather and farm output that we only feature in Appendix A, we merge weather station data from Brazil's National Institute of Meterology to agricultural survey data from the Municipal Agricultural Production (PAM) and Systematic Survey of Agricultural Production (LSPA) from the Brazilian Institute of Geography and Statistics (IBGE).

Table 1 shows summary statistics from our main dataset. Insured and uninsured farmers differ along most dimensions: uninsured farmers pay higher interest rates, default at higher rates, have smaller farms, and receive less revenue.[1] However, insured and uninsured farmers were similarly affected by the frost shock; there is little correlation between the frost shock magnitude and any systematic differences in insurance takeup across the country or within small geographical regions.

[1]We define default the standard way in Brazil, as debt that has not been paid at least 90 days after the due date.

Table 1. Summary Statistics on the Frost Shock, Coffee Production, and Hedging

	Mean	25th Percentile	Median	75th Percentile	Number of Coffee Farmers
Farm Area (Ha)					
Insured	42.61	6.56	14.22	34.9	20,472
Uninsured	23.88	2.62	5.45	14.08	80,376
Revenue Share from Coffee (%)					
Insured	92%	97%	100%	100%	20,472
Uninsured	91%	100%	100%	100%	80,376
Frost Shock Magnitude (Damage Share)					
Insured	26%	-	-	-	20,472
Uninsured	27%	-	-	-	80,376
Insured Value (Thousand USD)					
Insured	122	25	49	125	20,472
Uninsured	-	-	-	-	80,376
Claim Payout (Share of Total Insured Value)					
Insured	30%	5%	20%	46%	20,472
Uninsured	-	-	-	-	80,376
FX Hedge (Share of Farmers)					
Insured	1.4%	-	-	-	20,472
Uninsured	0.06%	-	-	-	80,376
Outstanding Debt (Thousand USD)					
Insured	80	0	45	112	20,472
Uninsured	31	0	5.6	32	80,376
Default Rate (Share of Debt Over 90 Days Overdue)					
Insured	2.5%	-	-	-	20,472
Uninsured	6.2%	-	-	-	80,376
Interest Rate (per Annum)					
Insured	5.6%	1.1%	4.4%	6.5%	15,747
Uninsured	9.5%	2%	4.2%	5.5%	60,760
Outstanding Debt Maturity (Years)					
Insured	7.5	5.9	7.3	8.8	15,747
Uninsured	8.1	5.9	8.8	9.9	60,760
Annual Payment Outflows (Thousand USD)					
Insured	743	11	43	191	20,472
Uninsured	275	1.6	5.58	18.6	80,376
Agricultural Inputs Payments (Share of Total Payments)					
Insured	48%	20%	48%	75%	20,472
Uninsured	45%	11%	45%	78%	80,376

Notes: These summary statistics use data from the BCB and SICOR. The damage share of coffee plants is the insurer's assessment of the share of future coffee bean production that is lost due to damage from the frost. Insured value and claims are only available for insured farmers; the values are zero and undefined for uninsured farmers. The interest rates are low because the government heavily subsidizes agricultural lending in Brazil.

2.2 Frost Shock

We study the July 2021 frost in Brazil, which led to an unexpected decrease in the capital stock for affected coffee farmers. Three severe waves of frost affected the states of São Paulo, Minas Gerais, and Paraná between the end of June 2021 and the beginning of August 2021. The frost was multiple standard deviations outside the range of outcomes that farmers typically consider during planting season. Due to timing and geography, the frost primarily affected perennial crops like oranges and coffee, rather than annual crops like soybeans and maize. As a result, estimating the aggregate economic impact of the frost shock would be difficult from harvest data alone[2]. Measuring frost with weather data can be imprecise because small differences in geography and soil, in combination with small differences in temperature, can correspond to large differences in losses.[3]

We interpret the frost as a capital stock shock, rather than solely a productivity or cash flow shock, because coffee is a perennial crop. The frost killed coffee plants, whose value is included in the collateral that farmers post for loans. The timing of the frost, at the end of the harvest season, meant that the 2021 coffee harvest was not heavily disrupted. We define the shock at the farmer-crop level using insurance claims as a proportion of insured value. We believe that insurance claims are the best available granular metric of the shock, and better than the typical weather-based or region-level output based metrics in the literature. Insurance claims are only paid after an agronomist visits the lot and validates the extent of the damages. We observe the cause of each claim, and we observe the precise coordinates of each insured plot.[4]

We define the frost shock metric FS at the farmer level by matching each farmer i who grew coffee to nearby insured coffee farmers $i' \in \mathcal{N}_j$ who received an insurance claim only for frost in 2021. We observe farm coordinates x_i and the ratio fs_i of frost insurance claim to insured value. We define the frost shock for farmer j as

$$\mathrm{FS}_i = \frac{\sum_{i' \in \mathcal{N}_i} w_{ii'}(x_i) I_{i'}}{\sum_{i' \in \mathcal{N}_i} w_{ii'}(x_i)}, \quad w_{ii'}(x_i) = \frac{1}{d(x_i, x_{i'})^{\beta}}, \tag{1}$$

where I_i is the insurance claim normalized as a share of underlying value for insured farmer i, $d(x_i, x_{i'})$ is the distance between farms, and $\mathcal{N}_i$ is the set of k-nearest neighbors of insured farmers to farmer i. Note that (1) is well-defined for all farmers i, regardless of whether farmer i purchased insurance. Both β and k are chosen by cross-validation.

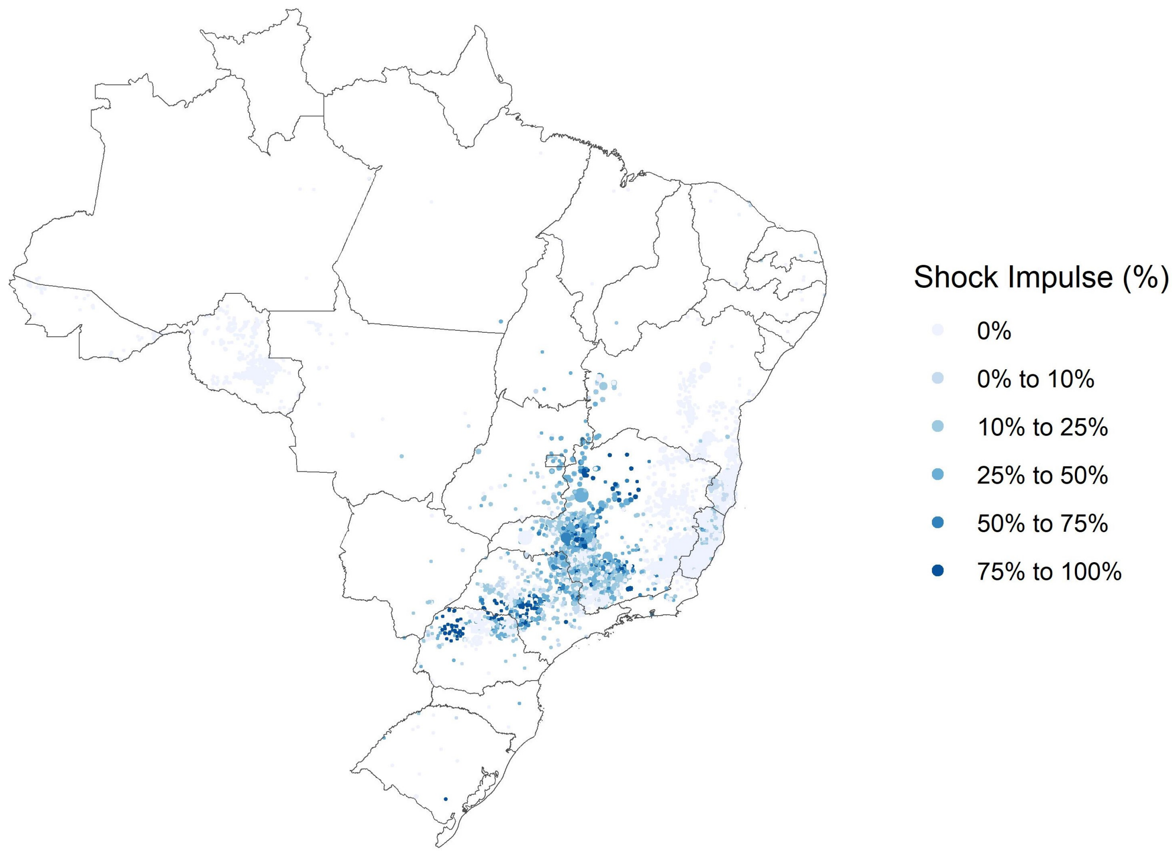
[2]Indeed, the challenge of estimating the effect of frost without detailed microdata at the coffee farm level has been recognized for a long time. See, for example, Stevens (1955) for a historical discussion.

[3]See Appendix A for more details about the measurement of the frost shock and the geographical distribution of crops in Brazil.

[4]The fact that insurance can be contingent on a frost shock is unsurprising, as frost events, though rare, tend to have significant impacts on coffee production in Brazil. There were at least 17 frost shock events in the 20th century.

Figure 1 shows the geographical distribution of the frost shock. Although some coffee-growing regions (e.g. northern Paraná state through northwestern Minas Gerais state) were affected more than others (e.g. eastern Minas Gerais state through Bahia state),[5] there was variation in the severity of the frost shock even within each municipality, an administrative division that is on average half the size of a county in the contiguous US.

Figure 1. Frost Shock Impulse Based on Coffee Insurance Claims



Notes: The insurance data are from SICOR. Each dot represents a farm that grew coffee in 2021. The size of the dot scales in proportion to the size of the farm. The color of the dot represents the shock impulse, equal to the magnitude of the shock, as measured by the proportion of coffee plants that were damaged, where darker colors represent higher magnitudes. The coffee-growing regions that were unaffected by the frost shock, with shock impulse equal to 0%, were Rondônia, eastern Minas Gerais, Espírito Santo, and eastern Bahía; these regions can be seen clearly in Figure A1 in Appendix A.

3 Empirical Methodology and Results

Our identifying assumption is that the granular impulse of the frost was as good as random, conditional on farmer and crop by municipality by year fixed effects. Our justification is that the frost was not anticipated at the time of planting, the most severe

[5] See Figure A1 in Appendix A for a map of coffee production in Brazil.

since the advent of modern weather stations, and the weather forecast did not foresee it until a week beforehand, after all planting and insurance decisions were sunk. Even then, forecasts did not capture the granular spatial variation in the realized shock.

Our regressions encompass three sets of outcome variables. The credit regressions examine affected farmers' outcomes y_{ijt}, outstanding debt and maturity, with separate event study coefficients by whether the farmers had existing insurance contracts in place. Our fixed effects control for farm characteristics as well as other shocks that affected any combination of lender, location, and time. Let i be farmer, j be municipality, let g denote insured versus uninsured, and t be quarter. Let $s_{i\tau}$ be the interaction of our preferred shock metric FS_i, defined in equation (1), with an indicator for whether the shock occured in the given period: $s_{i\tau} = \text{FS}_i \mathbb{1}\{t = \tau\}$. Let Ins_i be an indicator of whether a firm purchased insurance at the beginning of the growing season of the shock.

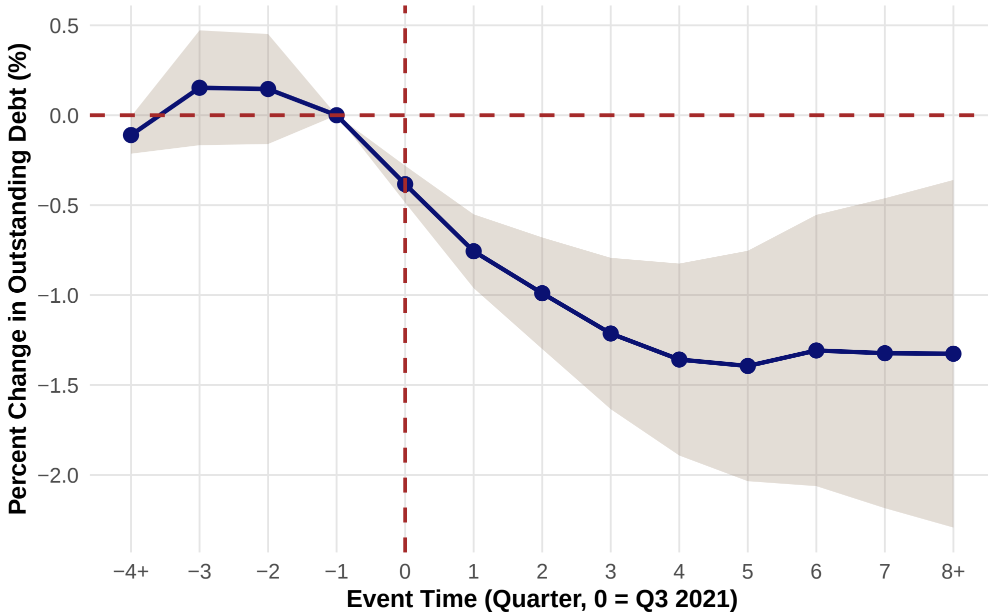
$$y_{ijt} = \sum_{\tau=2019Q1}^{2023Q4} \beta_\tau^C s_{ij\tau} \text{Ins}_i + \sum_{\tau=2019Q1}^{2023Q4} \gamma_\tau^C s_{ij\tau} \text{No\_Ins}_i + \alpha_i + \alpha_{jgt} + \epsilon_{ijt}. \tag{2}$$

In other words, the β_τ^C coefficients in equation (2) compare shocked insured farmers to non-shocked insured farmers, and the γ_τ^C coefficients compare shocked uninsured farmers to non-shocked uninsured farmers. Our rationale is that conditional on the granular fixed effects, the magnitude of the frost shock was as good as random, and differences in outcomes are reflective of the differences in damages through farmer's reduced net worth. We do not compare insured to uninsured farmers in this regression because insurance status is not randomly assigned. Figure 2 shows the event study coefficients for the $\{\gamma_\tau^C\}$ coefficients, across event time τ on the horizontal axis, for the outcome y_{ijt} outstanding debt balance in Figure 2a and the outcome y_{ijt} debt maturity in Figure 2b.[6]

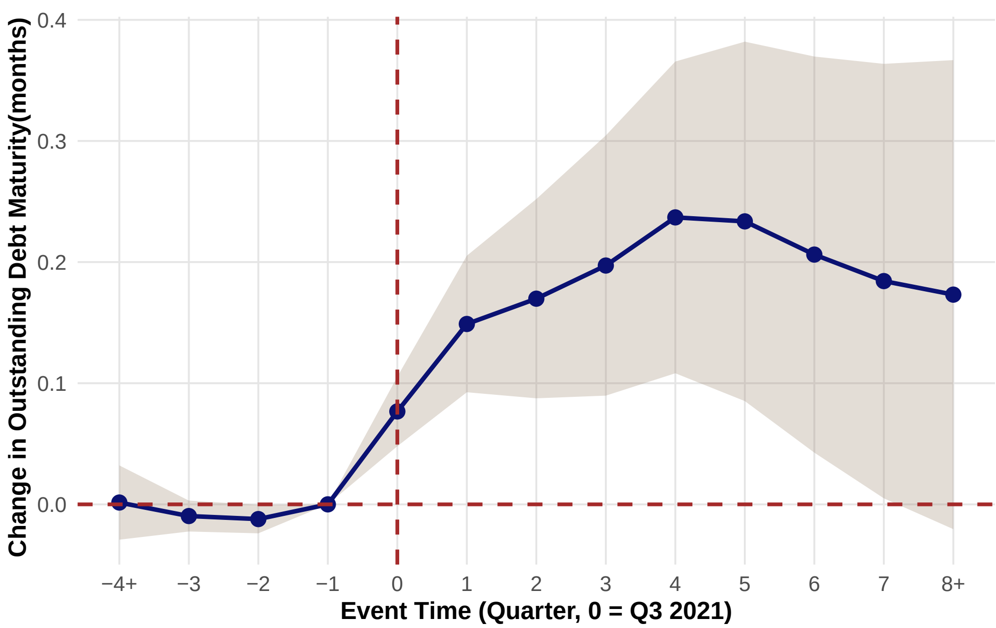
The takeaway from Figure 2 is that each additional percentage point of coffee plant damage for uninsured farmers, comparing against uninsured farmers with zero damage, corresponds to a 0.4% immediate reduction and 1.4% long-run in outstanding debt balance. Our interpretation is that farmers' net worth decreased, so their borrowing constraints tightened and reduced their debt load despite having higher marginal returns to capital. The takeaway from Figure 2b is that each additional percentage point of coffee plant damage for uninsured farmers, comparing against uninsured farmers with zero damage, corresponds to a 0.08 month immediate increase and 0.2 month long-run increase in outstanding debt maturity. Our interpretation is that farmers renegotiated debt and received forbearance on their loans.

[6]See Figure A4 in Appendix A for the results for the $\{\beta_\tau^C\}$ coefficients.

Figure 2. Credit Regression Results of $\{\gamma_\tau^C\}$ for Uninsured Farmers



(a) Outstanding debt decreased



(b) Debt maturity increased

Notes: These regressions use credit registry data from the BCB. The event study coefficients correspond to the cumulative effects across $\{\gamma_\tau^C\}$ in equation (2), comparing credit outcomes for shocked uninsured farmers to non-shocked uninsured farmers. The interpretation of the shock magnitude $s_{ij\tau}$ is one percentage point increase in coffee plant damage. The outcome variables y_{ijt} are outstanding debt balance in panel (a) and outstanding debt maturity in panel (b).

By comparison, we show in Figure A4 in Appendix A that the results for $\{\beta_\tau^C\}$ in regression (2) are zero across all event time, as we expect; insurance cushions farmers from the financial impacts of the shock. In addition, we find no significant effects on other credit outcomes: the debt interest rate, the default rate, and the composition of farmers' debt.

The payment regressions examine how farmer's purchases changed, specifically focusing on purchases from suppliers in sectors that are consistent with investment in re-growing crops or in the quality of the land.[7]

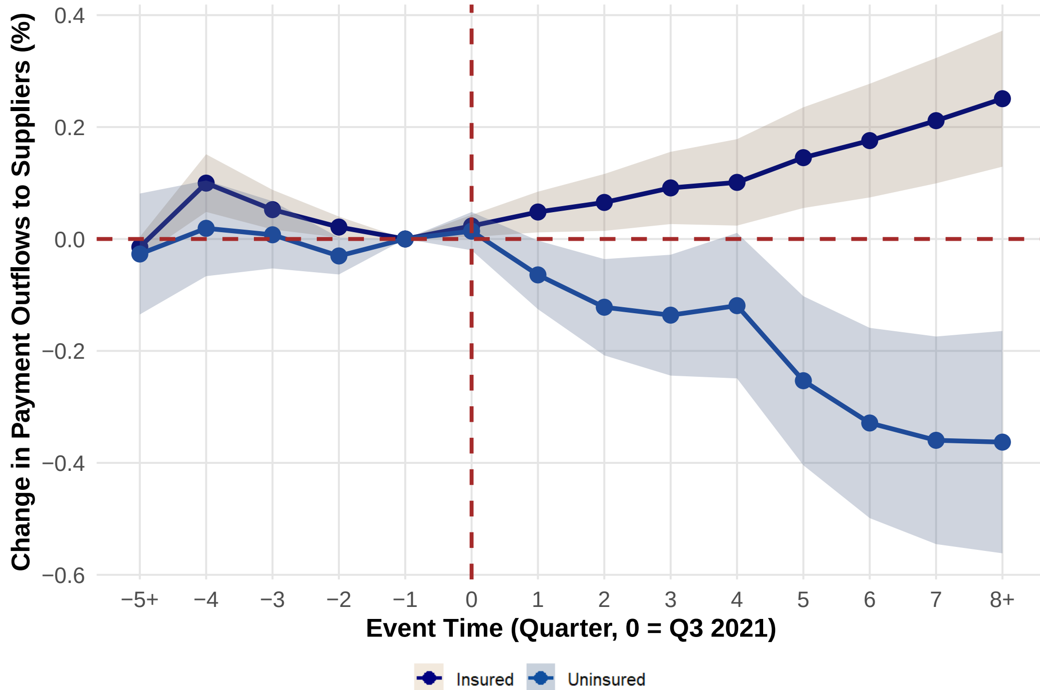
$$
\begin{aligned}
y_{ijst} = &\sum_{\tau=2019Q1}^{2023Q4} \beta_\tau^P s_{ij\tau} \text{Ins}_i \text{Upstream}_s + \sum_{\tau=2019Q1}^{2023Q4} \gamma_\tau^P s_{ij\tau} \text{No\_Ins}_i \text{Upstream}_s \\
&+ \alpha_{is} + \alpha_{jmt} + \epsilon_{ijt},
\end{aligned} \tag{3}
$$

where subscript i is farmer, j is municipality, s is the supplier, t and τ are at the quarterly level, and m is the supplier's 7-digit CNAE code. The fixed effects control for farmer-supplier linkages as well as other shocks that affect any combination of municipality, time, and supplier sector. The outcome y_{ijst} is log payment *outflows* to suppliers of agricultural inputs (e.g. seeds and fertilizers) and similar agricultural services that are consistent with investments to increase future farm output. Similarly to the credit regressions, we do not compare insured to uninsured farmers because insurance status is not randomly assigned.

Figure 3 shows the event study coefficients for the $\{\gamma_\tau^P\}$ and $\{\beta_\tau^P\}$ coefficients from equation (3), across event time τ on the horizontal axis. For each percentage point of additional damage, affected non-insured farmers decrease investment over time compared to unaffected non-insured farmers, with a long-run decrease of 0.38 percentage points. Our interpretation is that marginal investments are likely to be financed, and firms' borrowing constraints tighten as net worth decreases due to the shock. By comparison, affected insured farmers increase investment over time compared to unaffected insured farmers, with a long-run increase of 0.24 percentage points per 1 percentage point of damage. We believe that this is due to the liquidity injection from the timing of the insurance payout, which is typically as soon as an insurance adjuster can reach the farm, versus typical farm revenues that occur months later.

[7]We do not use payments to proxy for farmers' sales because many of the farmers' sales are made through advance purchase (CPR) contracts that are not directly affected by the shock.

Figure 3. Payment Outflow (Investment) Regression Results of $\{\gamma^P_\tau\}$ and $\{\beta^P_\tau\}$



Notes: These regressions use payment data from the BCB. The event study coefficients correspond to the cumulative effects across $\{\gamma^P_\tau\}$ and $\{\beta^P_\tau\}$ in equation (3). Coefficients $\{\gamma^P_\tau\}$ compare shocked uninsured farmers to non-shocked uninsured farmers, where shocked uninsured farmers decreased investment more, particularly in the next growing season 4-8 quarters later. Coefficients $\{\beta^P_\tau\}$ compare shocked insured farmers to non-shocked insured farmers, where shocked insured farmers invested more. The interpretation of the shock magnitude $s_{ij\tau}$ is one percentage point increase in coffee plant damage.

The insurance regression in equation (4) examines how farmers' subsequent uptake of insurance and other hedging changed in response to the shock:

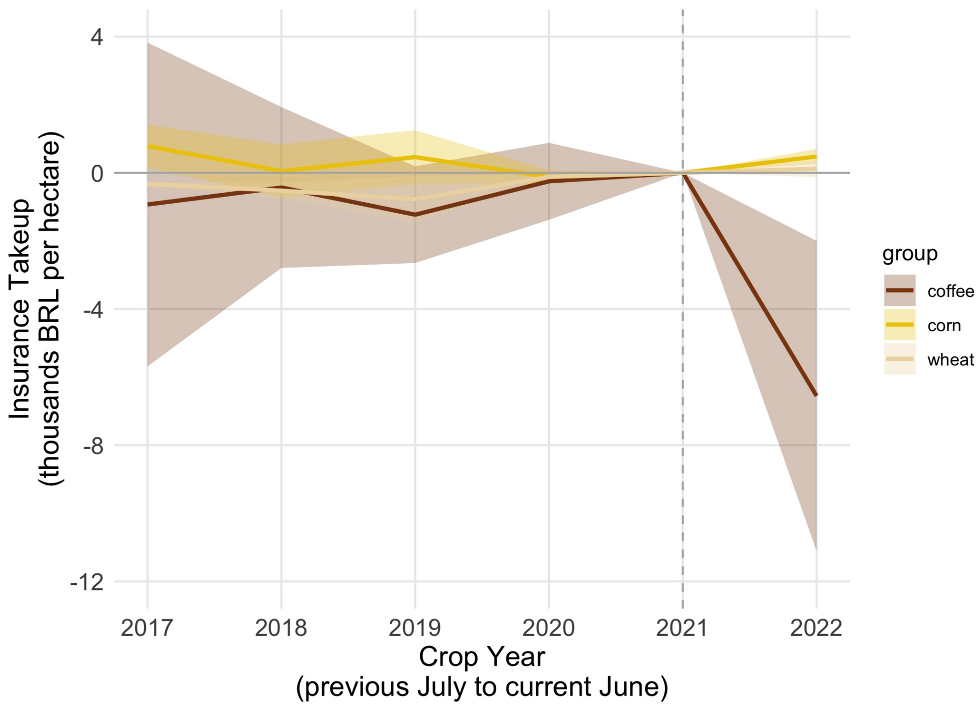
$$y_{ijct} = \sum_{\tau=2017}^{2022} \beta^I_{c\tau} s_{ijc\tau} \mathrm{Ins}_{ic} + \alpha_i + \alpha_{jct} + \epsilon_{ijct}, \tag{4}$$

where i is farmer, c is crop, j is municipality, t and τ are at the annual (growing season) level, and Ins_{ic} is an indicator for whether farmer i's plot of crop c was insured at the time of the frost shock. The fixed effects control for farm characteristics as well as any other shocks that affect any combination of municipality, crop, and time. Figure 4 shows that affected farmers subsequently took up less insurance than unaffected farmers for coffee, which is a perennial crop where the shock had lasting effects on net worth. However, subsequent insurance takeup for the annual crops corn and wheat were unchanged, since farmers plant new seeds every growing season, even though insured losses for corn and wheat were substantial.[8] In the model, we rationalize the subsequent decrease in coffee

[8]Note that soybeans, the primary crop in Brazil, were unaffected by the frost shock because of the timing

insurance takeup among farmers who experienced the shock through the lens of under-insurance as an endogenous consequence of net worth shocks in the presence of financial constraints.

Figure 4. Insurance takeup comparing frost-affected farmers to unaffected



Notes: The regressions are run on insurance data from SICOR and correspond to equation (4) for each of the three main crops c affected by the frost shock: corn, coffee, and wheat. Only coffee has a significant insurance response because coffee is a perennial crop, for which the frost shock reduced affected farmers' net worth, while corn and wheat are annual crops.

In Appendix A, we show additional results that complement the aforementioned results: we find no significant differences in default rates in Figure A6, interest rates in Figure A5, nor insurance premiums in Figure A7 between affected and unaffected farmers, in both the insured and uninsured groups. We interpret these findings as consistent with a setting where the financial market anticipates farmers' ability to repay debt in the event of shocks and allows for some form of state-contingent repayment. This may occur through mechanisms such as mandatory insurance or automatic debt renegotiation, as observed in certain rural credit contracts. However, this does not imply complete risk-sharing. As we will discuss in the model, when farmers face limited commitment constraints, both insurance and credit uptake will be constrained.

In conclusion, the frost shock led to a reduction in outstanding debt among uninsured farmers, with an increase in debt maturity through renegotiation and forbearance. After

of the soybean growing season.

the shock, uninsured affected farmers borrowed from subsidized credit lines to smooth consumption, but did not increase expenditure on investment goods, unlike insured affected farmers. We also find that uninsured affected farmers reduced labor demand, both on the intensive and extensive margin, unlike insured affected farmers. Total value insured decreased for affected farmers but increased for nearby unaffected ones. The credit channel alone was insufficient to compensate for the lack of insurance, even though emergency credit lines were extended in the period. See appendix for more details.

In Section 4, we present a model that formalizes these mechanisms by introducing limited pledgeability in borrowing contracts. We show how this friction leads to market incompleteness, where the ability to insure is closely linked to the ability to borrow. The frost shock further tightens borrowing constraints, particularly for farmers with lower net worth, reducing their capacity to hedge against future negative shocks through insurance or savings and to borrow in response to the shock, constraining their ability to reinvest after the adverse event and amplifying the real effects of the shock.

4 Baseline Model

We consider an economy with risk-averse farmers, subject to shocks to the capital stock. The key friction is limited commitment: farmers cannot commit to paying back any debt, and can run away with production and a fraction of collateral (capital). Farmers can only borrow up to the remainder of the collateral value, and a negative shock to collateral value tightens their debt limit. The main takeaway is that limited pledgeability of collateral results in coffee farmers with low net worth neither saving to use internal funds nor hedging against negative shocks

We build upon the models from Rampini and Viswanathan (2013, 2022), but we differ by considering a production economy that experiences capital stock shocks similar to Caballero and Krishnamurthy (2003) and Lorenzoni (2008). Firms, who cannot commit to repay, write contracts with risk-neutral financial intermediaries, who are unconstrained and can commit. These contracts condition on state (shock) realization under limited pledgeability. We show the equivalence between the constrained optimal contract maximizing farmers' utility and a market featuring one-period state-contingent savings (insurance) and non-state-contingent debt contracts, all subject to collateral constraints. Markets are endogenously incomplete because it is not constrained optimal to have state-contingent debt contracts.

4.1 Setup

Preferences: Farmer i has risk-averse preferences over consumption

$$U^E_{it}(\{c_{it}\}) := \mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k u(c_{it})\right],$$

where $\beta \in (0,1)$. Period utility $u(\cdot)$ is strictly increasing, strictly concave, continuously differentiable, and satisfies the Inada conditions.

Production Technology: Each farmer i produces a common good with production function

$$y_{it} = A_{it}k^{\alpha}_{it}, \tag{5}$$

where k_{it} is capital. We normalize the price of the investment good, and the unit of valuation of the capital stock, to be 1. Capital evolves according to the law of motion

$$k_i(s^t_i) = \theta(s_{it})\left(i_{i,t-1} + (1-\delta)k_{i,t-1}\right),$$

where i_t is investment, δ is depreciation, $\theta(s_t) \in \Theta$ is the realization of the capital shock, Θ is a finite set, and $s^t = (s_1, \dots, s_t)$ is the history of states. The price of the common good is potentially state dependent $p(s_t)$.

Contract: We follow the assumption from Rampini and Viswanathan (2022) by assuming that farmers can write contracts with risk-neutral financial intermediaries conditioning on state (shock) realization under limited pledgeability. Farmers can abscond with their income as well as a fraction $1-\lambda$ of total capital stock in the event of default. Farmers cannot be excluded from financial markets. Lenders are risk neutral and discount the future at a rate $\frac{1}{R}$ and have deep pockets, so lenders are willing to offer any state-contingent claims with an expected return of at least R.

In the next subsection we discuss the contract that maximizes farmers' welfare subject to the repayment incentive constraint. We show that the optimal contract can be implemented in a decentralized market and characterize the set of debt and insurance contracts that achieve this. In other words, we show that the same results from Rampini and Viswanathan (2022) hold in a setting with capital shocks and risk-averse farmers.

4.2 Optimal Contract and Endogenously Incomplete Markets

For clarity of exposition, henceforth we exclude the farmer subscripts i. We can write the optimal contract problem as a problem of choosing a sequence $\{c(s^t), f(s^t), k^*(s^t)\}_{t \geqslant \tau}$ of consumption c, state-contingent payment flow f, and capital k to maximize expected discounted utility

$$E_\tau \left[\sum_{t=\tau}^{\infty} \beta^{(t-\tau)} u\left(c_t\right) \right] \tag{6}$$

subject to the current and future period budget constraints:

$$W\left(s^\tau\right) \geqslant c\left(s^\tau\right) + f\left(s^\tau\right) + k^*(s^\tau), \tag{7}$$

$$\tilde{p}(s^t) k^* \left(s^{t-1}\right)^\alpha + (1-\delta)\theta(s^t) k^* \left(s^{t-1}\right) \geqslant c\left(s^t\right) + f\left(s^t\right) + k^*(s^t), \quad \forall t > \tau, \tag{8}$$

the lender participation constraint:

$$E_\tau \left[\sum_{t=\tau}^{T} R^{-(t-\tau)} f_t \right] \geqslant 0 \tag{9}$$

and the limited commitment constraint:

$$E_{\tau'} \left[\sum_{t=\tau'}^{\infty} \beta^{\left(t-\tau'\right)} u\left(c_t\right) \right] \geqslant E_{\tau'} \left[\sum_{t=\tau'}^{\infty} \beta^{\left(t-\tau'\right)} u\left(\hat{c}_t\right) \right], \quad \forall \tau' \geqslant \tau, \forall \{\hat{c}\left(s^t\right)\}, \tag{10}$$

where $\left\{\hat{c}\left(s^t\right)\right\}_{t=\tau'}^{\infty}$ is a solution to the optimal contract when the net worth is given by

$$\hat{W}\left(s^{\tau'}\right) = \tilde{p}(s^t) k^* \left(s^{\tau'-1}\right)^\alpha + (1-\lambda)(1-\delta)\theta(s^{\tau'}) k^* \left(s^{\tau'-1}\right),$$

and $\tilde{p}(s^t)$ is the state-contingent equilibrium price. Constraints (7)-(8) represent the budget constraints. Constraint (9) is the lender's participation constraint, while (10) is the repayment incentive constraint. In case of default, the farmer loses a fraction λ of the capital stock, leaving the net worth as $\hat{W}(\cdot)$. Since farmers cannot be excluded from the credit market, they would engage in a new contract with an intermediary after default, making the contract recursive. The next proposition shows that this contract can be implemented in a decentralized market.

Proposition 1: A consumption allocation is the outcome of the optimal contract if and only if the allocation is the outcome of an economy where farmers only have access to a sequence of one-period state contingent savings contracts $\{h(s^t)\}_{t \geqslant \tau}$ and one period (not state contingent) debt contracts $\{d_t\}_{t \geqslant \tau}$, satisfying:

$$\underbrace{d_t}_{\text{Debt}} \leqslant \underbrace{\lambda(1-\delta)\theta(s_t)k(s^t)}_{\text{Collateral constraint}} + \underbrace{h(s^t)}_{\text{Insurance}}, \quad \text{for all } s^t \tag{11}$$

$$d_t \geqslant 0, \quad h(s^t) \geqslant 0, \quad \text{for all } s^t.$$

Proof. See Appendix C. □

The result shows that a farmer's borrowing is constrained by collateral and upfront insurance due to limited commitment, consistent with our empirical setting where insurance is often mandatory for rural credit. The next section presents a numerical solution to the model, which helps interpret our empirical findings in the context of endogenous market incompleteness.

4.3 Numerical Solution

For the numerical simulation of the model, we consider an economy with a continuum of mass 1 of farmers. At each state farmers can either have capital shock $\theta = 0.3$, or no shock $\theta = 1$. We consider four states. In states s_1 and s_2 no one is subject to capital shock. In states s_3, s_4 half of the farmers are *randomly* receive the capital shock. Table 2 summarizes the probability of shocks and prices. We interpret the state s_4 as the frost shock state, where prices are high due to the shock affecting a significant fraction of farmers.

State (s)	Price $(p(s))$	$\mathbb{P}(\theta(s) = 0.3)$
s_1	1.0	0
s_2	1.5	0
s_3	1.0	0.5
s_4	1.5	0.5

Table 2. Price and probability of shock in each state

Table 3 shows the calibrated parameters. We set the transition probabilities in a way to achieve the following unconditional distribution of states:

$$P_S = [0.1; 0.1; 0.6; 0.2].$$

Following Proposition 1, defining $b(s) := h(s) - d$, we can write the farmer's recursive

Parameter	Value
α	0.3
λ	0.4
δ	0.05
β	0.85
R	1.1

Table 3. Parameter values

problem as:

$$V(W, s) = \max_{c,k,l,\{W(s')\},\{b(s')\}} u(c) + \beta\mathbb{E}[V(W(s'), s')|s]$$

$$\text{s.t} \quad W + \sum_{s'} \pi(s'|s)\frac{b(s')}{R} \geqslant c + k,$$

$$p(s')(\theta(s')k)^{\alpha} + (1-\delta)\theta(s')k - b(s') \geqslant W(s'), \quad \text{for all } s',$$

$$\lambda(1-\delta)\theta(s')k \geqslant b(s'), \quad \text{for all } s'.$$

Then we can use the standard dynamic programming techniques to find the policy and value functions. Numerically, we consider a sequence of aggregate states over time:

$$S = [1, \underbrace{4}_{\text{Frost Shock}}, \underbrace{2, 2}_{\text{High Price}}] \tag{12}$$

Based on the model's policy functions, there exists a critical net worth level, $\underline{W}$, such that if a farmer enters period 3 with a net worth $W \leqslant \underline{W}$, they will be uninsured in period 2, defined as covering less than 20% of the damage. The initial net worth distribution is assumed to be uniform, with uninsured farmers having net worth distributed in the range $[0, \underline{W}]$ and insured farmers in the range $[\underline{W}, \overline{W}]$. We simulate the model by drawing 1000 farmers in each group (insured vs non-insured) according to the uniform distribution of net worth just described.

To reproduce the empirical specification with model generated data we consider the following specification:

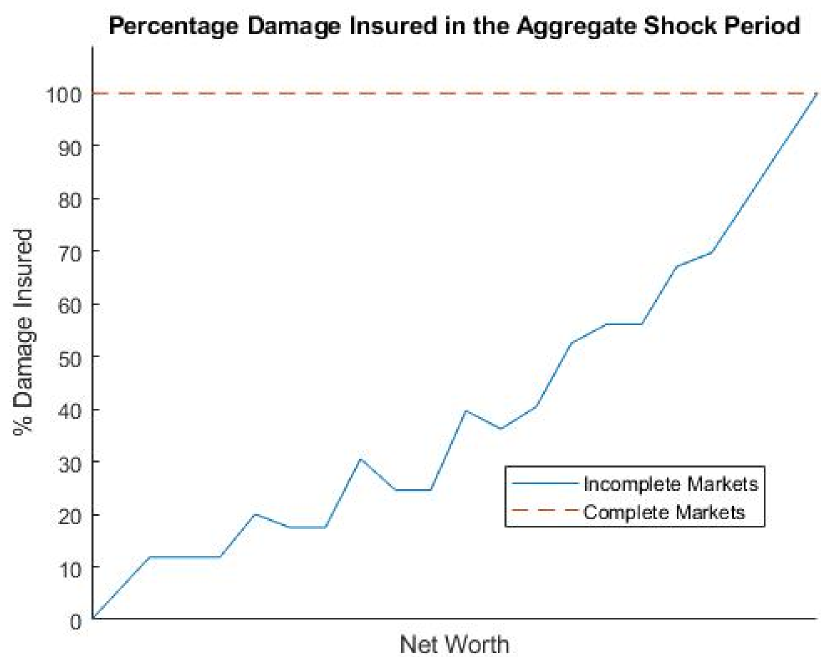
$$y_{it} = \sum_{\tau=-1}^{2} \beta_{\tau}\text{Shock}_{i\tau}\text{Ins}_i + \sum_{\tau=-1}^{2} \beta_{\tau}\text{Shock}_{i\tau}(1 - \text{Ins}_i) + \alpha_i + \alpha_{gt} + \epsilon_{it}. \tag{13}$$

Where i is farmer, g is insurance status, t is period, Ins_i is an indicator for insured at the time of the shock, y_{it} is the output of interest (debt or investment). The next section summarizes the numerical results from the model. First, we present the relationship between net worth and insurance, followed by the event studies in response to the capital shock. This includes a counterfactual exercise exploring how relative responses would change if markets became complete after the shock (i.e., no limited commitment).

4.4 Baseline Model Results

Figure 5 shows the model-predicted relationship between a farmer's net worth and insurance coverage. Under complete markets, the farmer always seeks full insurance with any risk aversion in utility or subsidy to the insurance premium. Under incomplete markets, there is incomplete insurance with a positive relationship.

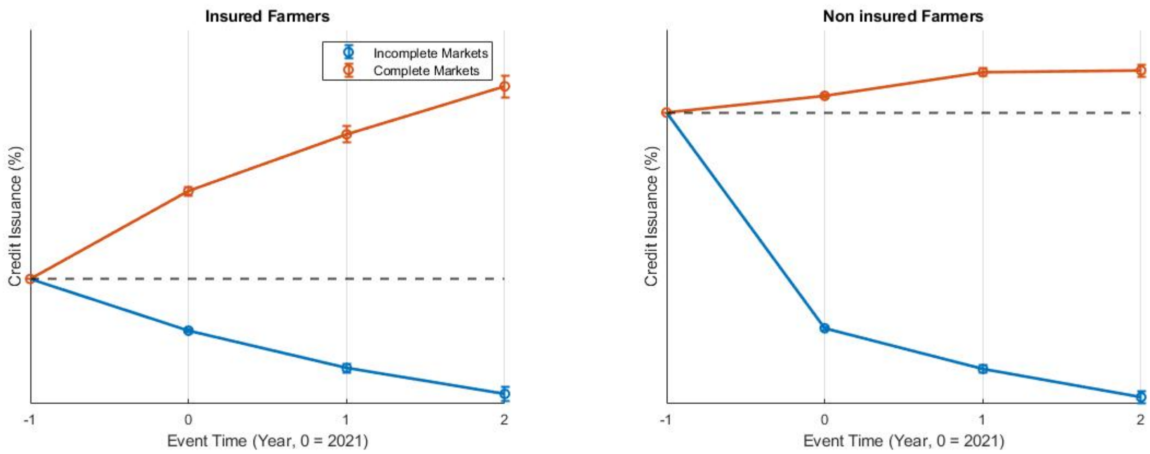
Figure 5. Model-Implied Relationship Between Insurance and Net Worth



Notes: This figure shows the model-implied optimal insurance coverage for farmers under two scenarios: complete markets, where it is always optimal to purchase actuarially fair insurance, and incomplete markets, where the binding collateral constraint results in low insurance demand when net worth is low, and a positive relationship between insurance demand and net worth.

Figure 6 shows the model-implied event studies cumulative effects for borrowing by insured farmers on the left and uninsured farmers on the right. Insured farmers' borrowing is increasing in time after the shock under complete markets, but slightly decreasing in time under incomplete markets. Uninsured farmers' borrowing sharply decreases under incomplete markets.

Figure 6. Model-Implied Impulse Response in Borrowing by Insured vs Uninsured Farmers



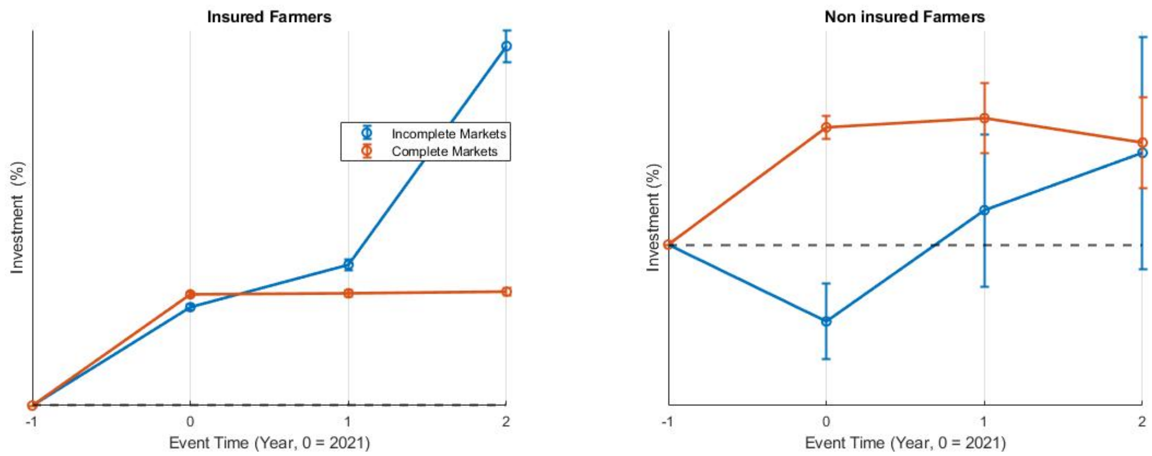
(a) Debt of Insured Farmers

(b) Debt of Uninsured Farmers

Notes: The event study plots are generated based on model-simulated data. We consider a sequence of aggregate shocks over time, including a frost shock and a high-price period, see equation (12). Farmers are categorized as insured or uninsured, depending on whether their net worth exceeds a critical threshold $\underline{W}$. We simulate the net worth distribution of 1000 farmers in each group, drawing from a uniform distribution. The empirical specification in (13) relates the shock impacts on debt and investment to insurance status, using fixed effects to control for farmer and group-time variation. The plots show the dynamic responses before and after the shock. On the left, we plot the cumulative effects on total outstanding debt for the insured group, and on the right, for the uninsured group.

Figure 7 shows the model-predicted change in investment. On the left are insured farmers, whose investment increases similarly under complete and incomplete markets. On the right are uninsured farmers, whose investment decreases at the time of the shock despite having the highest marginal returns.

Figure 7. Model Predicted Change in Investment by Insured vs Uninsured Farmers



(a) Investment by Insured Farmers (b) Investment by Uninsured Farmers

Notes: The event study plots are generated based on model-simulated data. We consider a sequence of aggregate shocks over time, including a frost shock and a high-price period, see equation (12). Farmers are categorized as insured or uninsured, depending on whether their net worth exceeds a critical threshold $\underline{W}$. We simulate the net worth distribution of 1000 farmers in each group, drawing from a uniform distribution. The empirical specification in (13) relates the shock impacts on debt and investment to insurance status, using fixed effects to control for farmer and group-time variation. The plots show the dynamic responses before and after the shock. On the left, we plot the cumulative effects on total investment for the insured group, and on the right, for the uninsured group.

Overall, the results show that market incompleteness amplifies the real effects of the frost shock. Due to collateral constraints, uninsured firms—those that should invest more after the shock to replenish lost capital—are unable to do so. The shock reduces both their borrowing capacity (as collateral diminishes) and their ability to insure against future shocks, as lower net worth slows the recovery. In the next subsection, we explore the policy implications of this channel.

5 General Equilibrium Model

To interpret the empirical patterns and evaluate aggregate implications, we now develop a general equilibrium model with capital shocks, borrowing constraints, and endogenous price determination. We build on the partial equilibrium framework in section 4 and in Rampini and Viswanathan (2022) by allowing output prices to be determined endogenously as a function of aggregate supply, capturing the feedback between individual investment decisions and relative prices.

The model features a continuum of risk-averse farmers who produce a common good using capital. Farmers face aggregate and idiosyncratic shocks to their capital stock and make investment and insurance decisions subject to collateral constraints. Financial intermediaries are risk-neutral and can offer state-contingent contracts, but only those enforceable under limited commitment. The collateral constraint limits the farmer's ability to shift resources across states, leading to under-insurance and under-investment when net worth is low. Endogenous price feedback magnifies this effect: farmers most exposed to the shock also face tighter borrowing limits and higher prices for inputs, exacerbating inequality in recovery trajectories.

The remainder of this section formalizes the setup, characterizes the optimal contract under limited commitment, and presents the decentralized equilibrium. We then describe our solution method, calibrate the model, and simulate a sequence of shocks to compare the response of insured and uninsured farmers. Finally, we quantify how market incompleteness distorts farmers' recovery and amplifies the aggregate real effects of capital destruction.

5.1 Setup

Preferences: Farmers' preferences are given by

$$U_t^E(\{c_t\}) = \mathbb{E}\left[\sum_{k=0}^{\infty} \beta^k u(c_t)\right],$$

where $\beta \in (0,1)$, $u(\cdot)$ is strictly increasing, strictly concave, continuously differentiable, and satisfies the Inada conditions.

Production Technology: Each farmer produces a common good with production function

$$y_{it} = k_{it}^{\alpha}, \tag{14}$$

where k_t is farmers' capital stock. We normalize the price of the investment good, and the unit of valuation of the capital stock, to be 1. Capital evolves according to the following law of motion

$$k_i(s_i^t) = \theta(s_{it})\left(i_{i,t-1} + k_{i,t-1}\right),$$

where i_t is investment, δ is depreciation, $\theta(s_t) \in \Theta$ is the realization of the capital shock, Θ is a finite set, and $s^t = (s_1, \ldots, s_t)$ is the history of states.

Frictions: We assume farmers have access to one-period state-contingent bonds b and are subject to collateral constraints.[9] The collateral constraints are of the form:

$$\lambda p(s^t) y(s_i^t) \geqslant b\left(s_i^t\right), \tag{15}$$

where $\frac{1}{\beta} > R > 0$, is an exogenous constant.

Price determination: The price of coffee is given by

$$p(s^t) = c_p Y(s^t)^{-\frac{1}{\gamma}},$$

where $c_p > 0, \gamma > 0$ are constants and $Y(s^t)$ is the aggregate supply, given by:

$$Y(s^t) = \int_0^1 y_i(s_i^t) di.$$

5.2 Solution Method

With states ordered from best to worst as $s_1, s_2, ..., s_n$, we can write the problem as:

$$\begin{aligned} V(W, s) = & \max_{k, \{b(s')\}} u(c) + \beta \sum_{s'} \pi(s'|s) V(W(s'), s') \\ \text{s.t} \quad & \lambda \tilde{\theta}(s')^{\alpha} k^{\alpha} \geqslant b(s'), \quad \text{for all } s' \\ & W(s') = \tilde{\theta}(s')^{\alpha} k^{\alpha} + \theta(s') k - b(s'), \quad \text{for all } s', \\ & c = W + \sum_{s'} \pi(s'|s) \frac{b(s')}{R} - k, \end{aligned}$$

where $\tilde{\theta}(s)$ can include price effects. Note that while W appears to have the interpretation of net worth, it is static and does not account for differences in future flows of production or utility.[10] We can write the first order condition with respect to k, $b(s')$ and the envelope

[9] We follow the assumption from Rampini and Viswanathan (2022) by assuming that households can abscond with their income as well as a fraction $1 - \lambda$ of their pledged revenues. Farmers cannot be excluded from financial markets. Lenders are risk neutral and discount the future at a rate $\frac{1}{R}$ and have deep pockets, so lenders are willing to offer any state-contingent claims with an expected return of at least R. We show in the appendix that the optimal contract is equivalent to this collateral constraint.

[10] Preview of our findings: if there are two idiosyncratic states, low $\underline{s}$ and high $\overline{s}$ capital stock after the shock, then holding fixed the aggregate state we find that more productive farmers endogenously insure, have lower $W(\overline{s})$ and higher $W(\underline{s})$, but higher consumption in both states. This is driven by the insurance decision. The balance sheet analog of net worth is a probability-weighted average of $W(\overline{s})$ and $W(\underline{s})$, and the dynamic analog is the value function V.

condition as:

$$
\begin{aligned}
&u'(c) = \sum_{s'} \pi(s'|s)\beta V_w(W(s'), s') \left[\tilde{\theta}(s')^\alpha \alpha k^{\alpha-1} + \theta(s')\right] + \sum_{s'} \mu(s')\lambda\theta(s')^\alpha \alpha k^{\alpha-1}, \\
&\beta V_w(W(s'), s') + \frac{\mu(s')}{\pi(s'|s)} = \frac{u'(c)}{R}, \quad \text{for all } s', \\
&\mu(s') \geqslant 0, \quad \mu(s')\left[\lambda\tilde{\theta}(s')^\alpha k^\alpha - b(s')\right] = 0, \quad \text{for all } s', \\
&V_w(W, s) = u'(c).
\end{aligned}
$$

The solution method is based on iterating in the marginal utility of wealth, by creating a grid for next period state variable W in the best state $\bar{s}$, solving for the current wealth (and marginal utility of wealth) using the first order conditions and the current and updating the guess. See Appendix D for more details.

5.3 General Equilibrium Simulation

We consider a continuum of farmers with unit mass. Each farmer receives an idiosyncratic shock to productivity, which takes values $\theta = 0.3$ (bad) or $\theta = 0.95$ (good). The economy features two aggregate states. In the good state, 20% of farmers receive the bad shock. In the bad state, 50% of farmers receive the bad shock.

We calibrate the model with the following parameter values: $\alpha = 0.3$, $\lambda = 0.7$, $\beta = 0.9$, $R = 1.05$, and CRRA utility $u(c_t) = c_t^{1-\sigma}/(1-\sigma)$ with $\sigma = 2$. The transition matrix is set such that the unconditional distribution over states is:

$$P_S = [0.8,\ 0.2]. \tag{16}$$

To solve the model, we use a global method that combines the Krusell–Smith algorithm with an endogenous grid method. We approximate the law of motion for aggregate output using a set of moments m summarizing the distribution of $A_i k_{it}^\alpha$. Farmers form expectations over next period's output using:

$$Y_{t+1}(S_{t+1}) = \int_0^1 A_i \theta(s_{it+1})^\alpha k_{it+1}^\alpha \, di = f(m, S_t, a), \tag{17}$$

where $f(\cdot)$ is a known functional form and a is a finite-dimensional parameter vector summarizing beliefs. We start with an initial guess a_0 for these belief parameters, solve the model given a_0, simulate the economy to generate data, and use this data to update the belief parameters. This process is repeated until convergence. We simulate the model economy for 11,000 periods, with 10,000 farmers. The first 1,000 periods are discarded to eliminate transients. We define the shock indicator $\text{Shock}_{it} = 1$ if the aggregate state is bad in period t and farmer i receives the idiosyncratic shock. We estimate the following event study specification using simulated data:

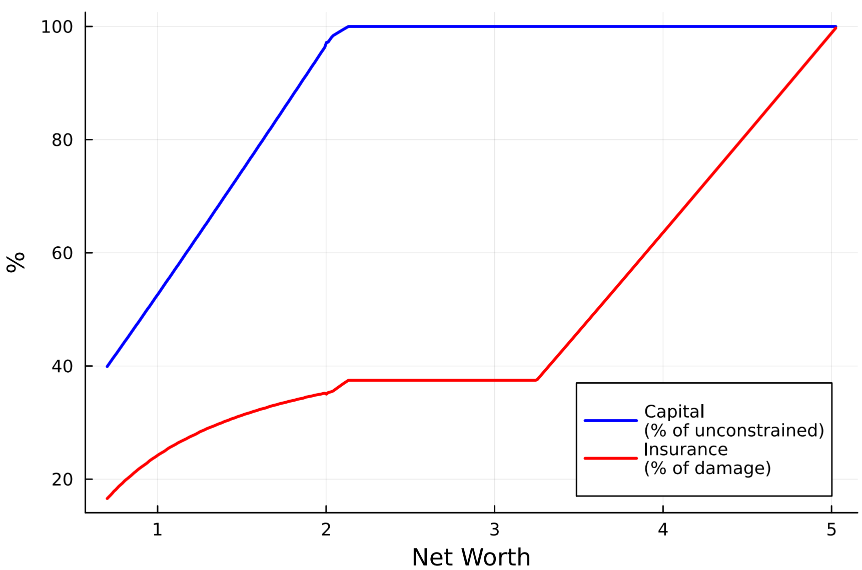
$$y_{it} = \sum_{\tau=-G}^{M} \beta_\tau^{NI} \text{Shock}_{i\tau} + \alpha_i + \alpha_t + \epsilon_{it}, \tag{18}$$

where $\text{Shock}_{i\tau}$ is an indicator for whether the farmer received the shock at time τ, i indexes farmers, t indexes quarters, and y_{it} is the outcome of interest. In this setup, all farmers are uninsured.

5.4 General Equilibrium Model Results

A key implication of the model is that insurance coverage increases with farmers' net worth when markets are incomplete due to limited pledgeability. This arises because wealthier farmers are better able to self-insure and access credit, reducing their vulnerability to negative shocks. Figure 8 shows that the model reproduces a positive relationship between initial net worth and insurance uptake, consistent with empirical patterns.

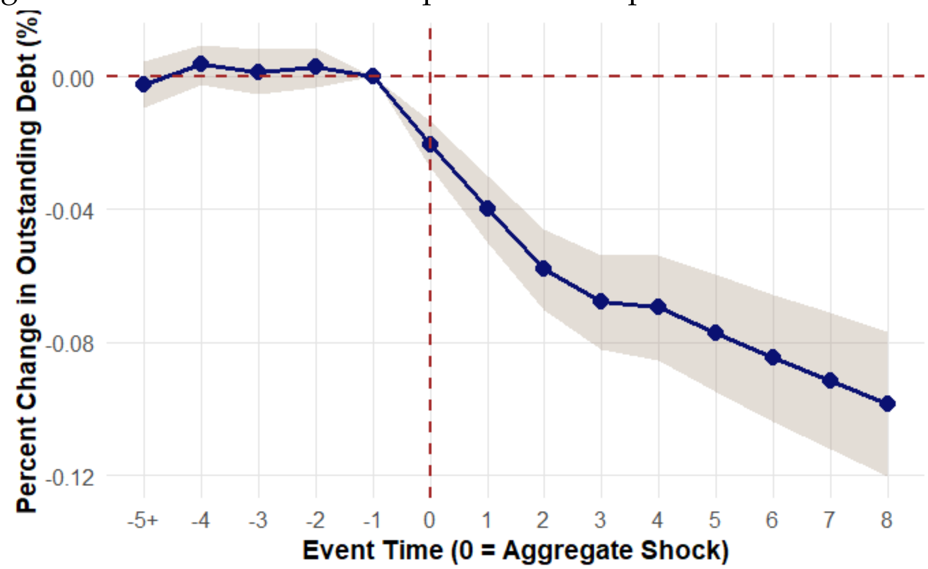
Figure 8. Insurance Coverage Increases with Farmer Net Worth



Notes: The plot shows the relationship between farmer net worth and insurance uptake under incomplete markets. Farmers with higher wealth can borrow and insure more easily, while poorer farmers face tighter borrowing constraints and lower insurance coverage.

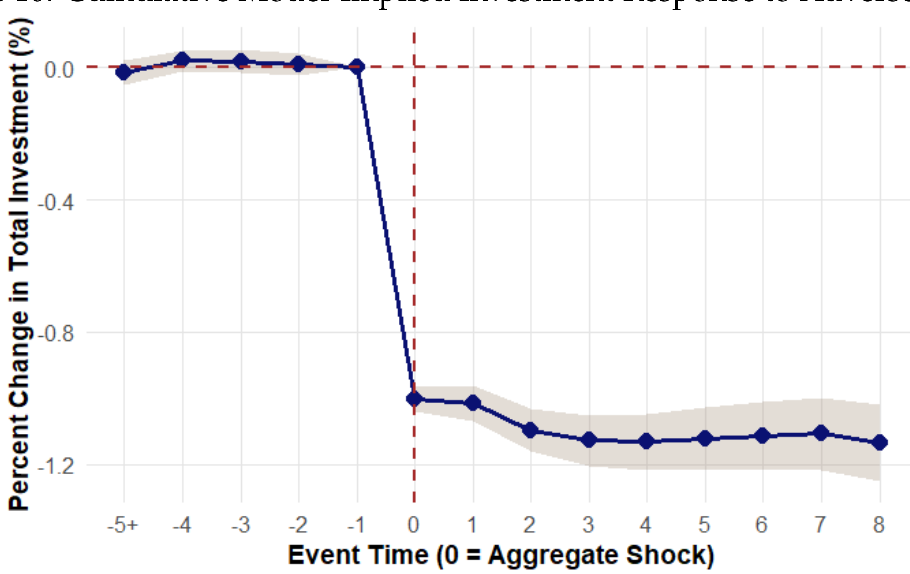
We compute the cumulative responses of farmers' debt load to the shock using the event study coefficients β_τ^{NI} in (18). Figure 9 shows that the simulated results have a persistent decline in borrowing following the shock, with persistence through general equilibrium price feedback shown in Figure A8.

Figure 9. Cumulative Model-Implied Debt Response to Adverse Shock



Notes: Based on model-simulated data, we estimate the cumulative effect of an adverse shock on debt load using a fixed-effects event study. The estimated coefficients β_{τ}^{NI} capture the dynamic adjustment in debt relative to the quarter of the shock. This is for the set of farmers who are assumed to be uninsured.

Figure 10. Cumulative Model-Implied Investment Response to Adverse Shock



Notes: The plot shows the dynamic response of investment following a negative shock, based on the same simulated economy used for the debt results. The investment path exhibits a persistent decline as farmers cut back due to tighter credit conditions and increased uncertainty. The specification mirrors that of the debt response, with farmer and time fixed effects.

Analogously, we examine the cumulative impact of the shock on investment. Figure 10 shows that the model predicts a prolonged drop in investment following adverse shocks, reflecting both constrained borrowing and heightened uncertainty. This delayed recovery in capital accumulation is a key channel through which liquidity constraints generate persistent real effects.

Relative to the baseline model in Section 4, the general equilibrium model introduces an additional amplification channel through endogenous price responses. In the baseline model, uninsured farmers face binding borrowing constraints and underinsure due to low net worth, leading to sharp drops in both debt and investment after the shock, in Figures 6 and 7. In the general equilibrium model, these same mechanisms are reinforced by price feedback: when many farmers experience a capital shock, aggregate output falls, which raises the price of coffee. While this price increase benefits unconstrained farmers, it further distorts recovery by widening the gap between insured and uninsured farmers, as shown in Figures 8, 9, and 10. The response of credit and investment is not only driven by individual constraints but also shaped by aggregate dynamics, which induce persistent differences in outcomes across farmers. Compared to the baseline, the general equilibrium results highlight the systemic nature of liquidity constraints and underscore the macroeconomic importance of financial market incompleteness given a capital destruction shock.

6 Policy Implication for Emergency Credit Lines

We examine the effectiveness of emergency credit lines within the model's framework. Following the frost shock, the Central Bank of Brazil allocated 1.3 billion BRL (approximately 250 million USD) to be disbursed as credit lines to coffee farmers impacted by the event. Financial intermediaries, such as banks and credit cooperatives, were responsible for disbursing the funds. However, only 49% of the total amount was actually lent, and, according to credit registry microdata, a disproportionate share went to farmers already holding some form of quantity insurance: 40% of all lending went to the 20% of farmers who had purchased insurance.[11] Given the extent of the frost shock, why did the emergency credit not reach those who needed it most?

The model highlights a key channel: emergency credit lines, intermediated by the financial sector, are ineffective because the constraints lie with the farmers, not the financial intermediaries. While credit and insurance might appear to be substitutes, emergency credit lines cannot replace insurance when the same underlying friction limits both credit access and insurance take-up. These financial constraints amplify the impact of weather shocks on output volatility, leading to a slow recovery for affected farmers, even

[11]For the total amount allocated, see Resolution of the National Monetary Council No. 4,954 of October 21, 2021. For the total amount disbursed, see a report from the Ministry of Agriculture.

when efforts are made to extend credit lines.

These insights align with other international experiences regarding emergency credit lines. For instance, Joaquim and Wang (2022) examine the Paycheck Protection Program (PPP) during the COVID crisis in the U.S., where emergency credit lines similarly failed to effectively reach the most constrained firms. As in our context, they found that firms most in need of financial support often had limited access to emergency credit due to pre-existing financial constraints. In both cases, whether through the PPP or credit lines following the frost shock, the core issue was not the availability of credit but rather the inability of financially constrained firms, whether farmers or businesses, to utilize it. This reinforces the idea that financial constraints, when coupled with exogenous shocks, exacerbate economic volatility and hinder recovery. Credit extensions, especially when indirectly targeted through financial intermediaries, cannot replace the liquidity provided by insurance or more flexible financial mechanisms.

Recently, these findings were echoed in the corporate emergency credit lines following the flood in Rio Grande do Sul, where only a fraction of the available funds were disbursed due to collateral requirements imposed by financial intermediaries. Both cases highlight a potential flaw in relying primarily on credit-based responses intermediated by financial institutions. Without addressing the underlying constraints that prevent firms from securing credit or insurance, emergency credit may still lack effective targeting.

7 Conclusion

This paper provides new evidence on how agents respond to net worth shocks based on insurance status in an incomplete market, and builds a model that rationalizes the results. The model shows that collateral constraints lead to under-insurance, which in turn can exacerbate misallocation because the shocked uninsured agents have high marginal return to capital yet do not invest. The empirical results show that these shocked uninsured farmers reduce both borrowing and investment, and do not subsequently take up insurance as much as nearby unshocked farmers.

We also show how the under-insurance can increase the propagation of shocks up the supply chain, amplifying the negative impact on aggregate output. In the general equilibrium extension of our model, with endogenous feedback between insurance, investment, and prices, we quantify the aggregate effect of net worth shocks under limited collateral pledgeability and demonstrate how emergency credit lines and transfers could mitigate these effects. The aggregate impact is lower investment and greater output volatility.

Our findings have important policy implications for the efficacy of subsidies for insurance, linked credit, and emergency credit lines in environments with limited pledgeable collateral. Future research should use quasi-exogenous variation in the pricing of each component to estimate the elasticities of firms' production decisions to net worth shocks, as part of a broader exploration of the interactions between credit, insurance, and investment.

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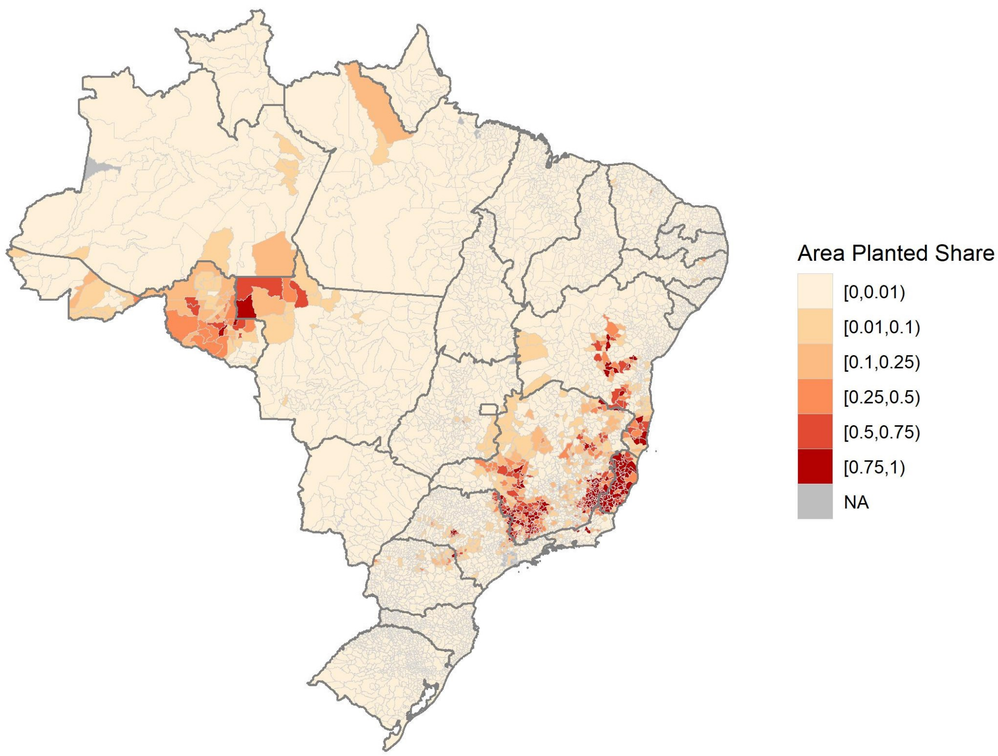
A Empirical Appendix

The purpose of this section is to provide additional context on coffee farming in Brazil and on the frost shock. Coffee is a perennial crop whose coffee beans are harvested from coffee cherries that typically grow on the plant each year. Coffee plants can live for many decades, and typically do not begin producing cherries until the third growing season. The growing season of coffee in south central Brazil begins in September to November, with harvest in June through August. Coffee plants are sensitive to cold temperatures, with low frost tolerance. The primary coffee growing regions in Brazil, depicted in red in Figure A1, are in regions where nighttime winter temperatures rarely drop below 10 Celsius.

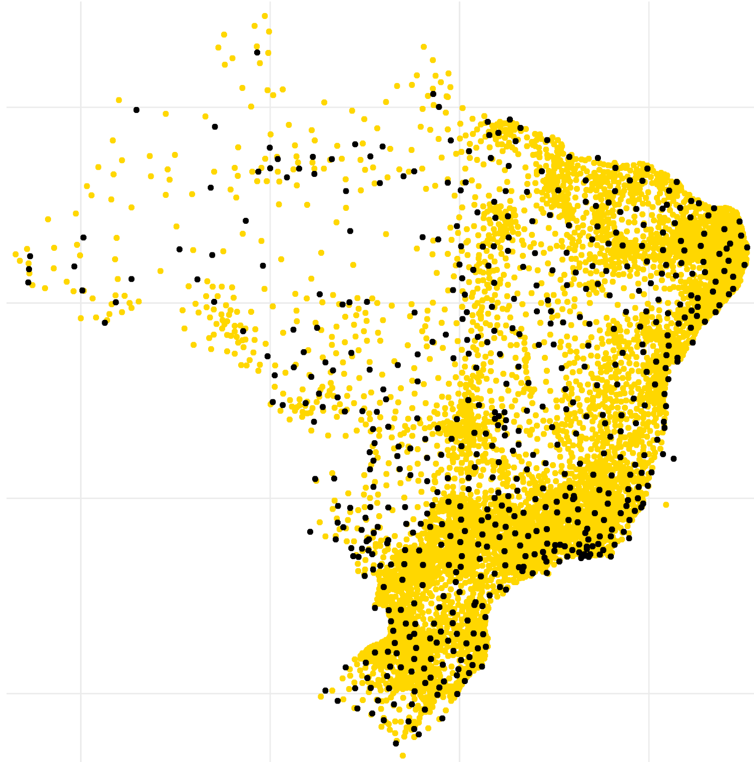
Figure A1. Geographical Distribution of Coffee Production in Brazil in 2021



Notes: This figure uses data from the Municipal Agricultural Production (PAM) survey to plot the municipality-level share of agricultural land in 2021 where coffee was planted. In Brazil, the municipality is the administrative division that is roughly analogous to the county in the US.

However, during the frost shock, nighttime temperatures were lower than climatic averages by more than 10 degrees Celsius for multiple periods of consecutive days. The first frost did not appear in weather forecasts until 3 days before the frost on June 28, 2021, and anecdotes from news articles suggest that farmers had limited options to protect their trees, given the scale of coffee farming and the fragility of coffee plants. To assign the distribution of weather variables to each municipality, we follow the methodology of Deschênes and Greenstone (2007) and subsequent papers in environmental economics.

Figure A2. Locations of Weather Stations in Brazil in July 2021



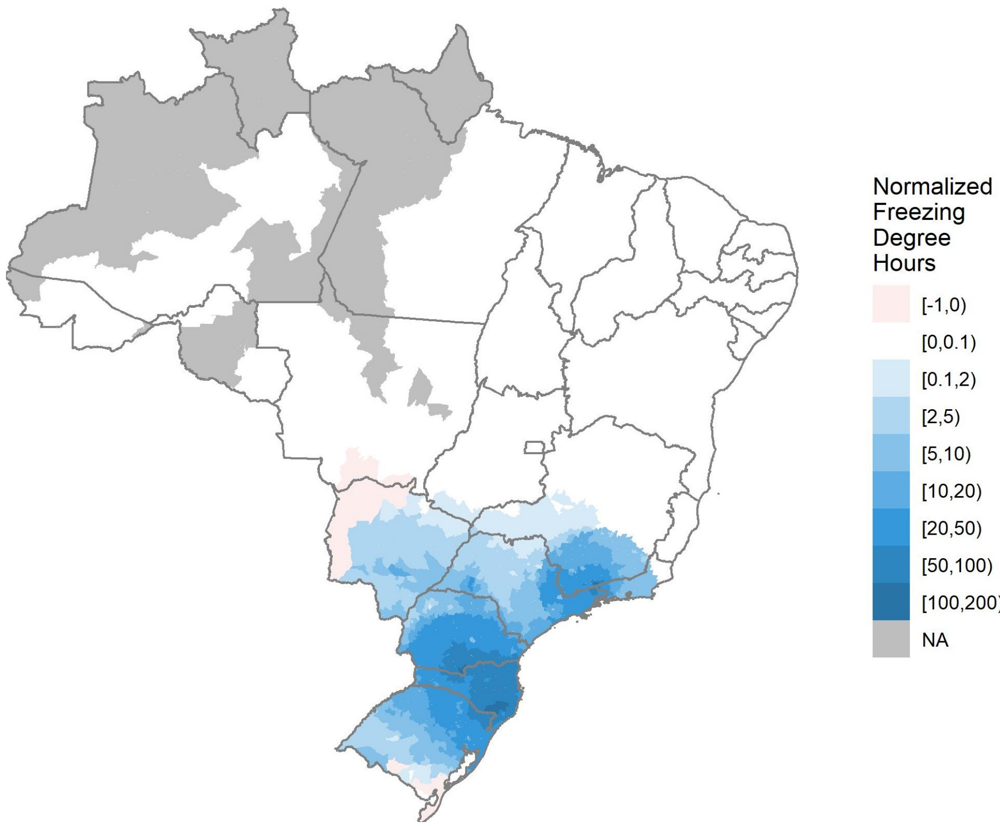
Notes: This figure shows the locations of weather stations from Brazil's National Institute of Meterology in yellow, as well as municipality centroids in black.

Using station-by-hour data from weather stations monitored by Brazil's National Institute of Meterology, whose distribution is shown in Figure A2, we count the number of hours over the frost shock period of June 28 through August 2, 2021 in each temperature bin of width one degree Celsius. Then, we use inverse distance weights $\omega_{s,i}$ between municipality i centroids and weather stations s with a 100 kilometer radius to define the municipality-level count of hours in each temperature bin. We define freezing-degree-hours (FDH) similar to growing-degree-days:

$$\text{FDH}_i = \sum_{s \text{ near } i} \omega_{s,i} \sum_{\text{hour } h} \max\{0, -T_{s,h}\}. \tag{19}$$

where $T_{s,h}$ is mean hourly temperature at weather station s. Figure A3 shows the distribution of FDH_i across municipalities i with sufficient weather data. However, the sensitivity of coffee plants to cold weather is highly non-linear, which a quasi-linear variable like FDH cannot accurately capture. Coffee plants begin to experience defoliation and frost burn at temperatures as high as 5 Celsius, which a much wider swath of central Brazil experienced during the frost shock. The extent of damage varies with wind, and even the nutrient composition of the soil, in addition to temperature. As a result, we believe that Figure 1 captures the impulse of the frost shock more accurately than any weather-based metric, like Figure A3, that follows the standard methods from the environmental economics literature or the climate finance literature.

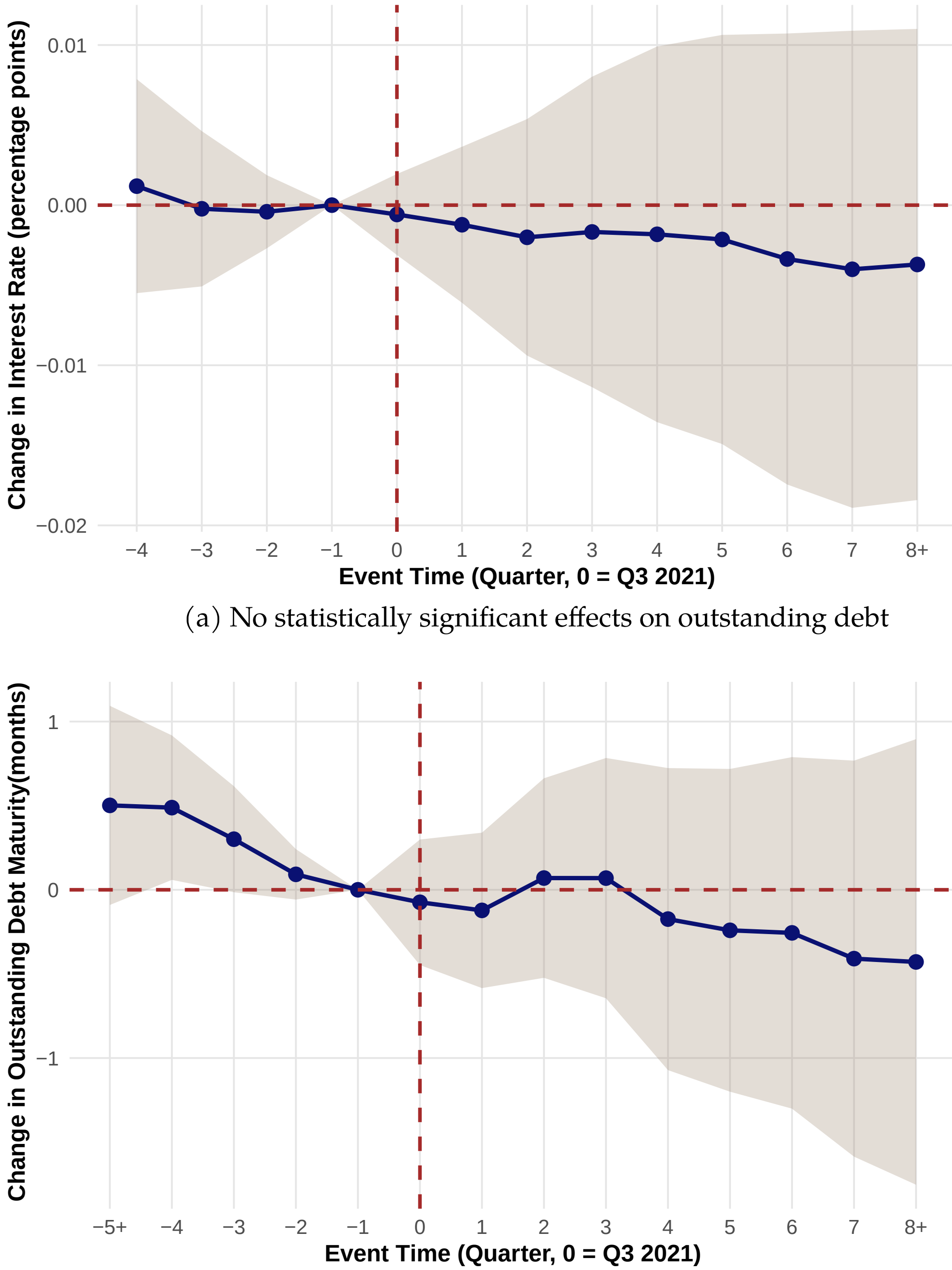
Figure A3. Municipality-Level Freezing-Degree-Hours During the Frost Shock Period



Notes: This figure shows the geographical distribution of freezing-degree-hours (FDH), normalized by the 2011-2020 average, in each municipality. We compute FDH following equation (19) using weather station data from Brazil's National Institute of Meterology.

A.1 Additional Event Studies Results

Figure A4. Credit Regression Results of $\{\beta^C_\tau\}$ for Insured Farmers

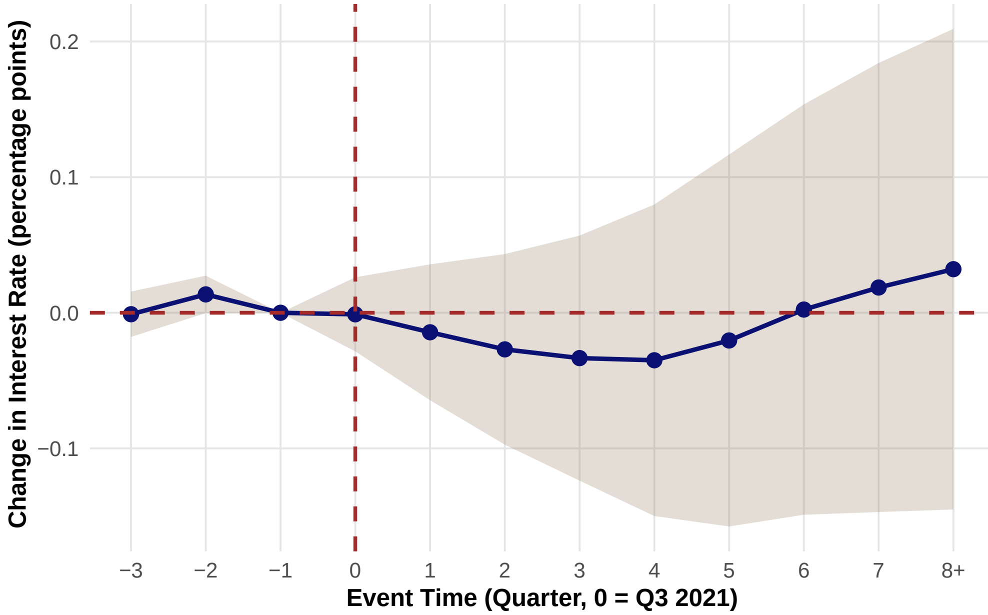


(a) No statistically significant effects on outstanding debt

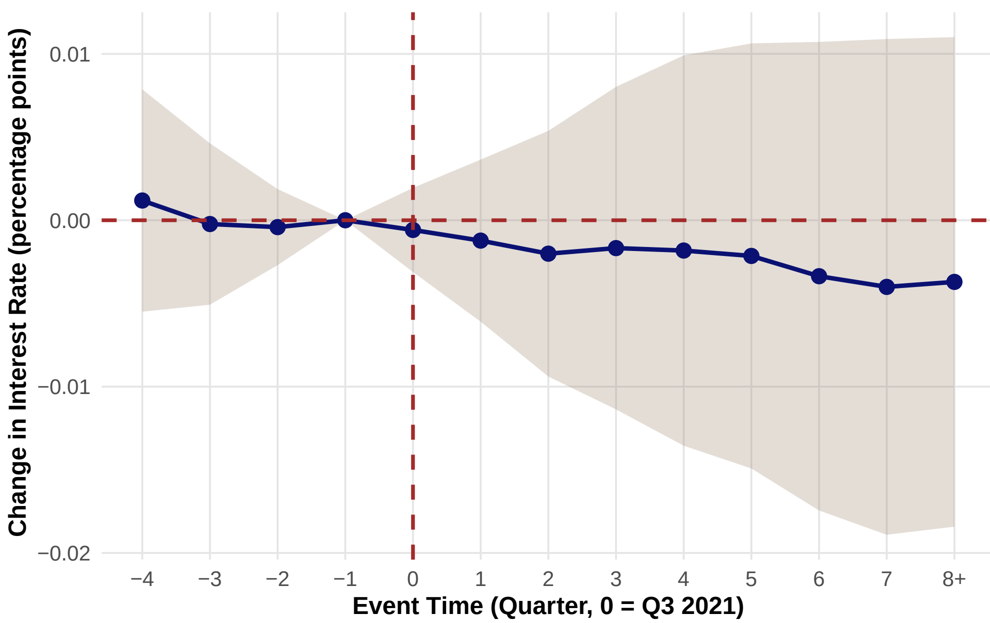
(b) No statistically significant effects on debt maturity

Notes: These regressions use credit registry data from the BCB. The event study coefficients correspond to the cumulative effects across $\{\beta^C_\tau\}$ in equation (2), comparing credit outcomes for shocked insured farmers to non-shocked uninsured farmers. The interpretation of the shock magnitude $s_{ij\tau}$ is one percentage point increase in coffee plant damage. The outcome variables y_{ijt} are outstanding debt balance in panel (a) and outstanding debt maturity in panel (b).

Figure A5. Interest Rate Regression Results for Uninsured Farmers and Insured Farmers



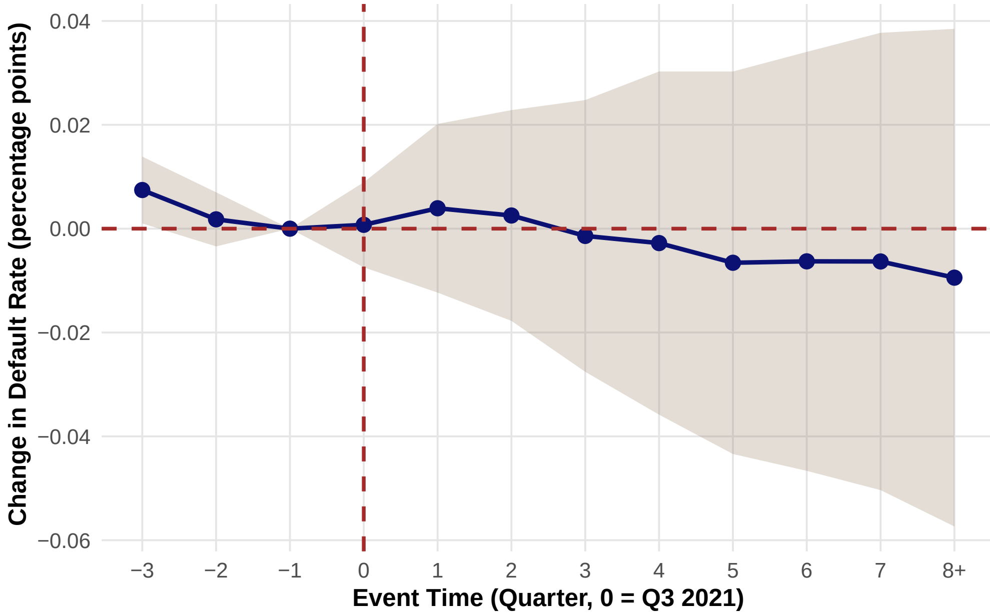
(a) No statistically significant effects on uninsured farmers' debt interest rate.



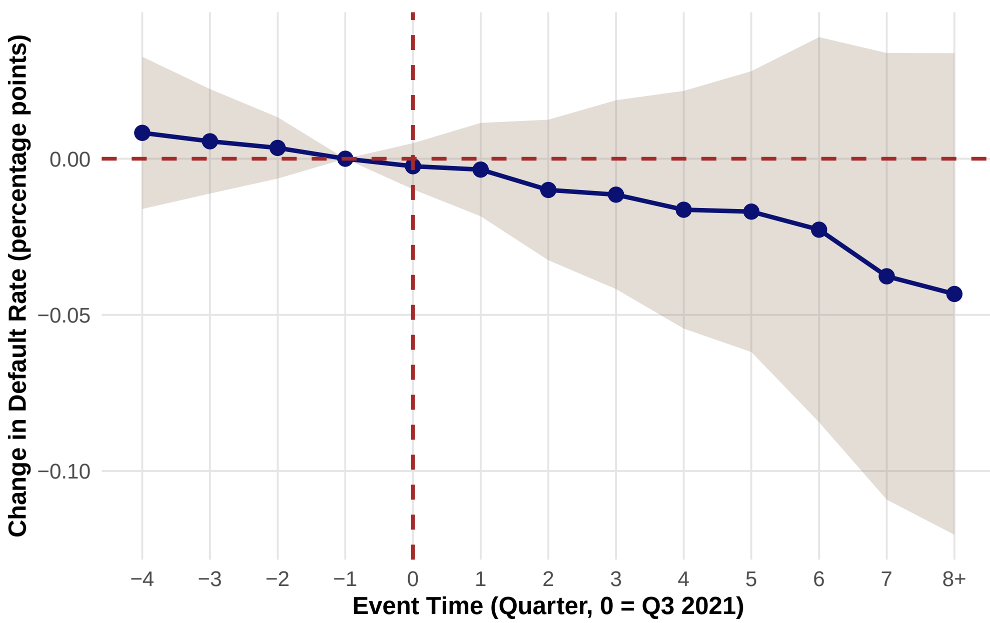
(b) No statistically significant effects on insured farmers' debt interest rate.

Notes: These regressions use credit registry data from the BCB. The event study coefficients represent the cumulative effects comparing the interest rates (on outstanding debt) for shocked farmers to non-shocked farmers. The shock magnitude corresponds to a one percentage point increase in coffee plant damage. The outcome variable, y_{ijt}, is the outstanding volume-weighted average interest rate on all farmers' credit operations. Plot (a) shows the results for uninsured farmers, while plot (b) shows the results for insured farmers.

Figure A6. Default Rate Results for Uninsured Farmers and Insured Farmers



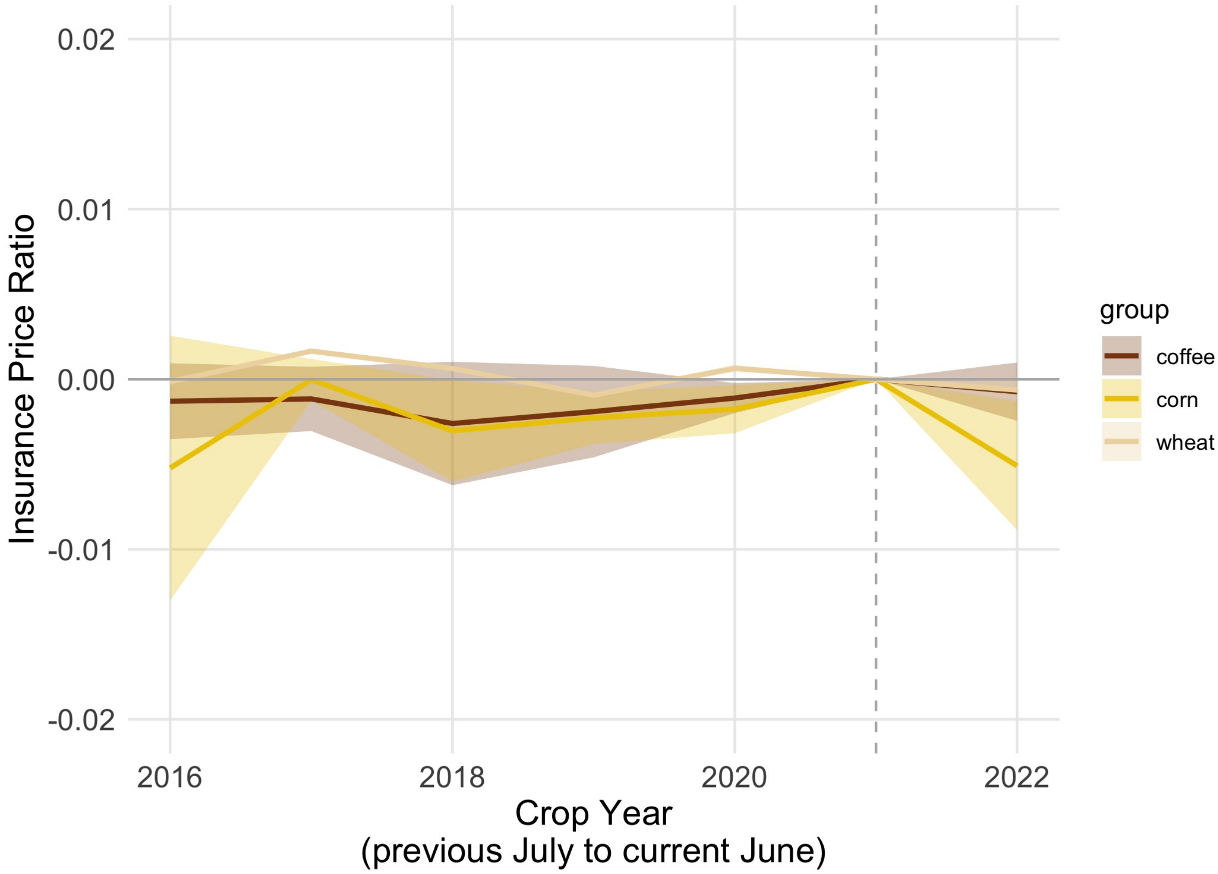
(a) No statistically significant effects on uninsured farmers' default rate.



(b) No statistically significant effects on insured farmers' default rate.

Notes: These regressions use credit registry data from the BCB. The event study coefficients represent the cumulative effects comparing the default rates (on outstanding debt) of shocked farmers to non-shocked farmers. The shock magnitude corresponds to a one percentage point increase in coffee plant damage. The outcome variable, y_{ijt}, is the default rate, defined as the total volume in default (more than 90 days late payments) divided by the total outstanding debt. Plot (a) shows the results for uninsured farmers, while plot (b) shows the results for insured farmers.

Figure A7. Insurance Premium of Frost-Affected Farmers to Unaffected



Notes: The regressions are run on insurance data from SICOR and correspond to equation (4) for each of the three main crops c affected by the frost shock: corn, coffee, and wheat. Outcome s given by $\left(\frac{\text{Premium}}{\text{Insured Value}}\right)$, which is our measure of insurance premium. There is no significant difference on any crops insurance premium between affected and non affected farmers.

B Data Appendix

We combine all electronic payments at the Central Bank of Brazil: boletos, bank transfers, and instant payments. Because boletos can be settled via cash, and it is standard to pay by boleto when issuing a nota fiscal for tax purposes, we believe that we cover almost all of the farmers' input payments. We classify the following CNAE codes as upstream of coffee farming, whose code is 0134-2/00, for the sake of the payment regressions that we interpret as input purchases proxying for investment:

- **0141-5/01**: Production of certified seeds, except forage for pasture.
- **0142-3/00**: Production of certified coffee seedlings, including genetically modified ones.

- **0161-0/02**: Pruning services in coffee plantations.
- **0161-0/03**: Contracted land preparation, cultivation, and harvesting services.
- **2831-3/00**: Manufacture of machinery and equipment for agriculture and livestock, parts and accessories.
- **2832-1/00**: Manufacture of machine tools, parts, and accessories.
- **3314-7/11**: Maintenance and repair of machinery and equipment for agriculture and livestock.
- **3314-7/12**: Maintenance and repair of machine tools.
- **4530-7/05**: Retail trade of tires and inner tubes.
- **4661-3/00**: Wholesale trade of machinery, devices, and equipment for agricultural use; parts and pieces.
- **7731-4/00**: Rental of agricultural machinery and equipment without operator.
- **7739-0/99**: Rental of other machinery and equipment not previously specified, without operator.
- **7490-1/03**: Intermediation and agency activities of services and businesses in general, except real estate.
- **0159-8/99**: Production of other plant-based products not previously specified.
- **0161-0/99**: Support activities for agriculture not previously specified.
- **4611-7/00**: Commercial representatives and trade agents of agricultural raw materials and live animals.

C Baseline Model Appendix

Optimal Contract: We can write the optimal contract problem as a problem of choosing a sequence $\{c(s^t), f(s^t), k^*(s^t)\}_{t\geqslant\tau}$ to maximize

$$E_\tau\left[\sum_{t=\tau}^{\infty}\beta^{(t-\tau)}u\left(c_t\right)\right] \tag{20}$$

subject to the current and future period budget constraints:

$$W\left(s^\tau\right) \geqslant c\left(s^\tau\right) + f\left(s^\tau\right) + k^*(s^\tau), \tag{21}$$

$$\tilde{p}(s^t)k^*\left(s^{t-1}\right)^\alpha + (1-\delta)\theta(s^t)k^*\left(s^{t-1}\right) \geqslant c\left(s^t\right) + f\left(s^t\right) + k^*(s^t), \quad \forall t > \tau, \tag{22}$$

the lender participation constraint:

$$E_\tau\left[\sum_{t=\tau}^{T}R^{-(t-\tau)}f_t\right] \geqslant 0 \tag{23}$$

and the limited commitment constraint:

$$E_{\tau'}\left[\sum_{t=\tau'}^{\infty}\beta^{\left(t-\tau'\right)}u\left(c_t\right)\right] \geqslant E_{\tau'}\left[\sum_{t=\tau'}^{\infty}\beta^{\left(t-\tau'\right)}u\left(\hat{c}_t\right)\right], \quad \forall\tau' \geqslant \tau, \forall\{\hat{c}\left(s^t\right)\}, \tag{24}$$

where $\left\{\hat{c}\left(s^t\right)\right\}_{t=\tau'}^{\infty}$ is a solution to the optimal contract when the net worth is given by

$$\hat{W}\left(s^{\tau'}\right) = \tilde{p}(s^t)k^*\left(s^{\tau'-1}\right)^\alpha + (1-\lambda)(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right).$$

Endogenous debt limit framework:

We can write the farmer problem as choosing a sequence $\{c(s^t), b(s^t), k^*(s^t)\}_{t\geqslant\tau}$ to maximize

$$U\left(\{c(s^t)\} \mid s^\tau\right) := E_\tau\left[\sum_{t=\tau}^{\infty}\beta^{(t-\tau)}u\left(c_t\right)\right] \tag{25}$$

subject to the sequential budget constraint

$$\begin{aligned}&\tilde{p}(s^t)k^*\left(s^{t-1}\right)^\alpha + (1-\delta)\theta(s^t)k^*\left(s^{t-1}\right) + \sum_{s^{t+1}}\frac{1}{R}\pi(s^{t+1}|s^t)b(s^{t+1}) \geqslant \\ &c\left(s^t\right) + k^*(s^t) + b(s^t), \quad \forall t \geqslant \tau,\end{aligned} \tag{26}$$

and the debt limit is given by a process $\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}$:

$$\lambda(1-\delta)\theta(s^t)k^*\left(s^{t-1}\right) - b\left(s^t\right) \geqslant -\widetilde{D}^i\left(s^t\right), \forall t > \tau. \tag{27}$$

On the following, we define the continuation utility, conditional on repaying debt equal to x at state s^τ and having debt limit given by $\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}$ as $\widetilde{V}^i\left(\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}, x \mid s^\tau\right)$. We say that a sequence of debt limit is *not too-tight* if for all s^t,

$$\widetilde{V}^i\left(\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}, x \mid s^\tau\right) = \widetilde{V}^i\left(\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}, 0 \mid s^\tau\right)$$

Following the arguments from Section 3.2 of Martins-Da-Rocha et al. (2022), we can show that a sequence of debt limits is not too tight if and only if $\widetilde{D}^i\left(s^t\right) = 0$ for all s^t.

Lemma 1. (*Endogenously incomplete markets*) *A consumption allocation is the outcome of the optimal contract if and only if, it is the outcome of the endogenous debt limit framework.*

Proof. ($\Rightarrow$) Fix a sequence $\{c(s^t), b(s^t), k^*(s^t)\}_{t\geqslant\tau}$ that solves the farmer's optimal contract problem. We first show that if a sequence $\{b(s^t)\}_{t\geqslant\tau}$ satisfies (33) and (34) then

$$\lambda(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right) \geqslant \mathbb{E}_{\tau'}\left[\sum_{t=\tau'}^{\infty}\frac{b_t}{R^{t-\tau'}}\right], \quad \text{for all } \tau' \geqslant \tau. \tag{28}$$

Assume by way of contradiction that there exist $\tau', s^{\tau'}$ such that

$$\lambda\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right) < \mathbb{E}_{\tau'}\left[\sum_{t=\tau'}^{\infty}\frac{b_t}{R^{t-\tau'}}\right].$$

Consider the following deviation: a farmer defaults on τ' and enters a contract with the following allocation schedule

$$\hat{b}(s^t) = \begin{cases} b(s^t) & \text{if } t > \tau' \\ -\mathbb{E}_{\tau'}\left[\sum_{t=\tau'+1}^{\infty}\frac{b_t}{R^{t-\tau'}}\right] & \text{if } t = \tau' \end{cases}$$

$$\hat{k}^*(s^t) = k^*(s^t), \quad \text{for all } t \geqslant \tau'$$

First note that by construction $\hat{c}(s^t) = c(s^t)$ for all $t > \tau'$. We choose $\hat{c}_\tau$ such that the budget constraint holds at τ. Moreover, the definition of $\hat{p}$ ensures that (33) holds. Finally, note that in the optimal contract, the budget constraint has to hold:

$$\begin{aligned}\hat{c}_t - c_t &= b(s^{\tau'}) - \hat{t}(s^{\tau'}) + \lambda(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right) \\ &= \left[\sum_{t=\tau'}^{\infty} \frac{b_t}{R^{t-\tau'}}\right] - \lambda(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right) > 0,\end{aligned}$$

which is a contradiction as the farmer is better off with this deviation. Now, the proof is analogous to Rampini and Viswanathan (2013). Define

$$b\left(s^{\tau'}\right) := E_{\tau'}\left[\sum_{t=\tau'}^{\infty} R^{-\left(t-\tau'\right)} b_t\right] \leqslant \lambda(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right), \quad \forall \tau' > \tau.$$

Then $b_{\tau'}$ equals $b(s^{\tau'}) - \sum_{s^{\tau'+1}} \frac{1}{R}\pi(s^{\tau'+1}|s^{\tau'})b(s^{\tau'+1})$ for all $\tau' \geqslant \tau$, with $b(\tau) = 0$. The budget constraints (36) become equivalent to the budget constraints (31). Moreover, (37) holds, so the sequence of consumption is feasible in the economy with endogenous debt limit.

($\Leftarrow$) Fix a sequence $\{c(s^t), b(s^t), k^*(s^t)\}_{t \geqslant \tau}$ that solves farmer's maximization problem under the endogenous debt limit framework. Define the repayment values, for all $t > \tau$:

$$t(s^t) := b(s^t) - \sum_{s^{t+1}} \frac{1}{R}\pi(s^{t+1}|s^t)b(s^{t+1}),$$

And consider the sequence $\{t(s^t)\}_{t \geqslant \tau}$. This sequence satisfies the budget constraint (31) by definition. Moreover, it is also true that (33) is satisfied.[12] Because:

$$\mathbb{E}_\tau\left[R^{-(t-\tau)}t_t\right] = 0.$$

Now assume by way of contraction that $\{t(s^t)\}_{t \geqslant \tau}$ do not satisfy (34). Then there exists $\tau' \geqslant \tau$ such that $U\left(\{c(s^t)\} \mid s^{\tau'}\right) < U\left(\{\hat{c}(s^t)\} \mid s^{\tau'}\right)$, where $\{\hat{c}(s^t)\}$ is a solution to the optimal contract problem where the initial net worth is given by $\tilde{p}(s^{\tau'})k^*\left(s^{\tau'-1}\right)^\alpha + (1-\delta)(1-\lambda)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right)$. Let the associated transfers be $\hat{t}(s^t)$, by the previous part of the

[12] Too see why, note that we can write $t(s^t) = b(s^t) - \frac{1}{R}\mathbb{E}\left[b(s^{t+1})\right]$, and decompose, by the law of iterated expectations:

$$\mathbb{E}_\tau\left[R^{-(t-\tau)}t_t\right] = t(s^\tau) + \sum_{k=1}^{\infty} \frac{1}{R^k}\mathbb{E}_\tau\left[\mathbb{E}_{\tau+k}\left[t_{\tau+k+1}|s^{\tau+k}\right]\right],$$

and the fact that $b(s^t)$ is bounded from above, given that the production function satisfies the inada conditions.

proof it's true that

$$\hat{b}(s^t) := E_t\left[\sum_{k=t}^{\infty} R^{-(k-\tau')}\hat{t}_k\right], \forall t > \tau'$$

is feasible in the endogenous debt limit, when the $b(s^{\tau'}) = 0$. Now define the alternative consumption plan,

$$\tilde{c}(s^t) = \begin{cases} c(s^t) & \text{if } t < \tau', \\ \hat{c}(s^t) & \text{if } t \geqslant \tau'. \end{cases}$$

This consumption plan is feasible under the endogenous debt limit problem, since at τ' we have that $b(\tau') \leqslant (1-\delta)(1-\lambda)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right)$, which implies that $U\left(\{c(s^t)\} \mid s^\tau\right) < U\left(\{\tilde{c}(s^t)\} \mid s^\tau\right)$, contradicting optimality. □

Corollary 1. (*Alternative characterization of endogenously incomplete markets*) *A consumption allocation is the outcome of the optimal contract if and only if the allocation is the outcome of an economy where farmers only have access to a sequence of one-period state contingent savings contracts* $\{h(s^t)\}_{t\geqslant\tau}$ *and one period* (*not state contingent*) *debt contracts* $\{d(s^t)\}_{t\geqslant\tau}$, *satisfying :*

$$\lambda\theta(s^t)(1-\delta)k^*\left(s^{t-1}\right) + h(s^t) \geqslant d\left(s^{t-1}\right), \quad \forall t > \tau \tag{29}$$

$$d(s^t) \geqslant 0, \quad h(s^t) \geqslant 0, \quad \textit{for all } t > \tau$$

Proof. We can get the result by defining, on the endogenous debt limit framework, for all $\tau' > t$, $s^{\tau'}$

$$d(s^{\tau'-1}) := \left[\max_{s^{\tau'}} b(s^{\tau'})\right]^+$$

$$h(s^{\tau'}) := d(s^{\tau'-1}) - b(s^{\tau'}),$$

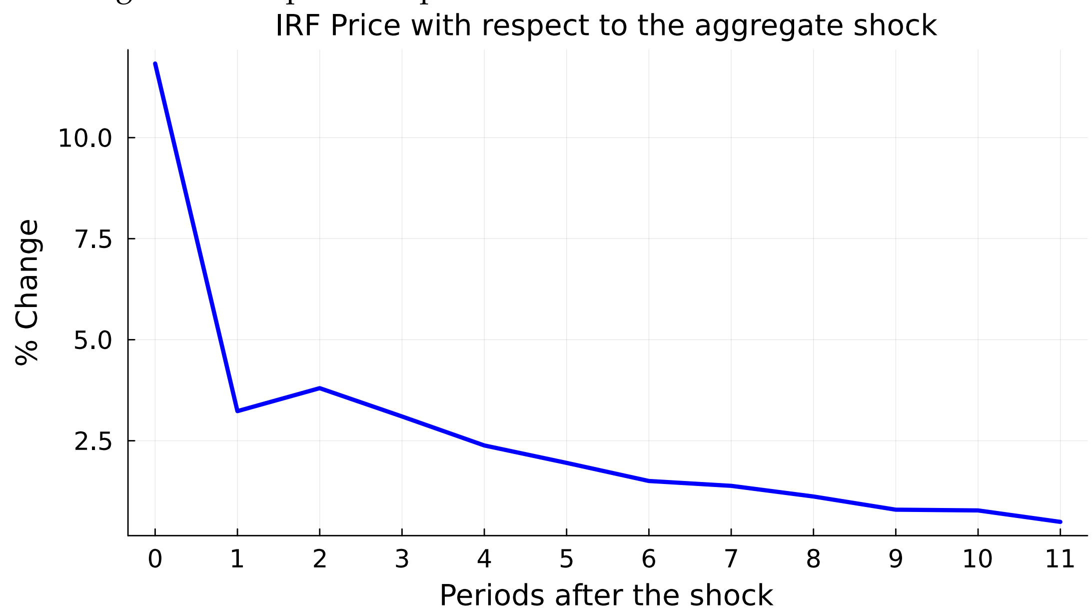
and appeal to Lemma 2. □

D General Equilibrium Model Appendix

D.1 Price Response

The model features a general equilibrium channel where adverse shocks trigger an endogenous response in the relative price of coffee. When farmers reduce investment and output falls, the resulting contraction in supply leads to an increase in prices. This price response partially mitigates the output loss but also amplifies cross-sectional heterogeneity.

Figure A8. Impulse Response of Coffee Prices to Adverse Shocks



Notes: The plot displays the model-implied impulse response of the coffee price following a negative shock to farmer productivity. As output declines, the aggregate supply of coffee contracts, generating upward pressure on prices. These dynamics are endogenous to the model and contribute to the persistence of real effects.

D.2 Optimal Contract

We can write the optimal contract problem as a problem of choosing a sequence $\{c(s^t), t(s^t), l(s^t), k^*(s^t)\}_{t \geqslant \tau}$ to maximize,

$$E_\tau \left[\sum_{t=\tau}^{\infty} \beta^{(t-\tau)} u\left(c_t\right) \right] \tag{30}$$

subject to the current and future period budget constraints:

$$W\left(s^\tau\right) \geqslant c\left(s^\tau\right) + t\left(s^\tau\right) + wl(s^\tau) + k^*(s^\tau), \tag{31}$$

$$\tilde{p}(s^t)k^*\left(s^{t-1}\right)^\alpha l\left(s^t\right)^{1-\alpha} + (1-\delta)\theta(s^t)k^*\left(s^{t-1}\right) \geqslant c\left(s^t\right) + t\left(s^t\right) + wl(s^t) + k^*(s^t), \quad \forall t > \tau, \tag{32}$$

the lender participation constraint:

$$E_\tau \left[\sum_{t=\tau}^{T} R^{-(t-\tau)} t_t \right] \geqslant 0 \tag{33}$$

and the limited commitment constraint:

$$E_{\tau'}\left[\sum_{t=\tau'}^{\infty}\beta^{\left(t-\tau'\right)}u\left(c_t\right)\right] \geqslant E_{\tau'}\left[\sum_{t=\tau'}^{\infty}\beta^{\left(t-\tau'\right)}u\left(\hat{c}_t\right)\right], \quad \forall\tau' \geqslant \tau, \forall\{\hat{c}\left(s^t\right)\}, \tag{34}$$

where $\left\{\hat{c}\left(s^t\right)\right\}_{t=\tau'}^{\infty}$ is a solution to the optimal contract when the state variable W is given by

$$\hat{W}\left(s^{\tau'}\right) = \tilde{p}(s^t)k^*\left(s^{\tau'-1}\right)^{\alpha} l(s^{\tau'})^{1-\alpha} + (1-\lambda)(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right).$$

Endogenous debt limit framework We can write the farmer problem as:

of choosing a sequence $\{c(s^t), b(s^t), l(s^t), k^*(s^t)\}_{t\geqslant\tau}$ to maximize,

$$U\left(\{c(s^t)\} \mid s^{\tau}\right) := E_{\tau}\left[\sum_{t=\tau}^{\infty}\beta^{(t-\tau)}u\left(c_t\right)\right] \tag{35}$$

subject to the sequential budget constraint

$$\tilde{p}(s^t)k^*\left(s^{t-1}\right)^{\alpha} l\left(s^t\right)^{1-\alpha} + (1-\delta)\theta(s^t)k^*\left(s^{t-1}\right) + \sum_{s^{t+1}}\frac{1}{R}\pi(s^{t+1}|s^t)b(s^{t+1}) \geqslant$$
$$c\left(s^t\right) + wl(s^t) + k^*(s^t) + b(s^t), \quad \forall t \geqslant \tau, \tag{36}$$

and the debt limit is given by a process $\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}$:

$$\lambda(1-\delta)\theta(s^t)k^*\left(s^{t-1}\right) - b\left(s^t\right) \geqslant -\widetilde{D}^i\left(s^t\right), \forall t > \tau. \tag{37}$$

On the following, we define the continuation utility, conditional on repaying debt equal to x at state s^{τ} and having debt limit given by $\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}$ as $\widetilde{V}^i\left(\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}, x \mid s^{\tau}\right)$. We say that a sequence of debt limit is *not too-tight* if for all s^t,

$$\widetilde{V}^i\left(\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}, x \mid s^{\tau}\right) = \widetilde{V}^i\left(\{\widetilde{D}^i\left(s^t\right)\}_{t>\tau}, 0 \mid s^{\tau}\right)$$

Following the arguments from Section 3.2 of Martins-Da-Rocha et al. (2022) , we can show that a sequence of debt limits is not too tight if and only if $\widetilde{D}^i\left(s^t\right) = 0$ for all s^t.

Lemma 2. (*Endogenously incomplete markets*) *A consumption allocation is the outcome of the optimal contract if and only if, it is the outcome of the endogenous debt limit framework.*

Proof. ($\Rightarrow$) Fix a sequence $\{c(s^t), t(s^t), l(s^t), k^*(s^t)\}_{t\geqslant\tau}$ that solves farmer's optimal contract problem. We first show that if a sequence $\{t(s^t)\}_{t\geqslant\tau}$ satisfies (33) and (34) then

$$\lambda(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right) \geqslant \mathbb{E}_{\tau'}\left[\sum_{t=\tau'}^{\infty}\frac{t_t}{R^{t-\tau'}}\right], \quad \text{for all } \tau' \geqslant \tau. \tag{38}$$

Assume by way of contradiction that there exists $\tau', s^{\tau'}$ such that

$$\lambda\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right) < \mathbb{E}_{\tau'}\left[\sum_{t=\tau'}^{\infty}\frac{t_t}{R^{t-\tau'}}\right].$$

Consider the following deviation. Farmer default on τ', enters a contract with the following allocation schedule

$$\hat{t}(s^t) = \begin{cases} t(s^t) & \text{if } t > \tau' \\ -\mathbb{E}_{\tau'}\left[\sum_{t=\tau'+1}^{\infty}\frac{t_t}{R^{t-\tau'}}\right] & \text{if } t = \tau' \end{cases}$$

$$\hat{k}^*(s^t) = k^*(s^t),\ \hat{l}(s^t) = l(s^t), \quad \text{for all } t \geqslant \tau'$$

First note that by construction $\hat{c}(s^t) = c(s^t)$ for all $t > \tau'$. We choose $\hat{c}_\tau$ such that the budget constraint holds at τ. Moreover, the definition of $\hat{p}$ ensures that (33) holds. Finally, note that in the optimal contract, the budget constraint has to hold, then:

$$\begin{aligned}\hat{c}_t - c_t &= t(s^{\tau'}) - \hat{t}(s^{\tau'}) + \lambda(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right) \\ &= \left[\sum_{t=\tau'}^{\infty}\frac{t_t}{R^{t-\tau'}}\right] - \lambda(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right) > 0,\end{aligned}$$

which is a contradiction as the farmer is better off with this deviation. Now, the proof is analogous to Rampini and Viswanathan (2013) . Define

$$b\left(s^{\tau'}\right) := E_{\tau'}\left[\sum_{t=\tau'}^{\infty} R^{-\left(t-\tau'\right)}t_t\right] \leqslant \lambda(1-\delta)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right), \quad \forall\tau' > \tau.$$

Then $t_{\tau'} = b(s^{\tau'}) - \sum_{s^{\tau'+1}}\frac{1}{R}\pi(s^{\tau'+1}|s^{\tau'})b(s^{\tau'+1})$ for all $\tau' \geqslant \tau$, with $b(\tau) = 0$. Then the budget constraints (36) become equivalent to the budget constraints (31). Moreover, (37) holds, so the sequence of consumption is feasible in the economy with endogenous debt limit.

$(\Leftarrow)$ Fix a sequence $\{c(s^t), b(s^t), l(s^t), k^*(s^t)\}_{t\geqslant\tau}$ that solves farmer's maximization problem under the endogenous debt limit framework. Define the repayment values, for all $t > \tau$:

$$t(s^t) := b(s^t) - \sum_{s^{t+1}} \frac{1}{R}\pi(s^{t+1}|s^t)b(s^{t+1}),$$

And consider the sequence $\{t(s^t)\}_{t\geqslant\tau}$. This sequence satisfies the budget constraint (31) by definition. Moreover, it is also true that (33) is satisfied.[13] Because:

$$\mathbb{E}_\tau\left[R^{-(t-\tau)}t_t\right] = 0.$$

Now assume by way of contraction that $\{t(s^t)\}_{t\geqslant\tau}$ do not satisfy (34). Then there exists $\tau' \geqslant \tau$ such that $U\left(\{c(s^t)\} \mid s^{\tau'}\right) < U\left(\{\hat{c}(s^t)\} \mid s^{\tau'}\right)$, where $\{\hat{c}(s^t)\}$ is a solution to the optimal contract problem where the initial state variable is given by $\tilde{p}(s^{\tau'})k^*\left(s^{\tau'-1}\right)^\alpha l\left(s^{\tau'}\right)^{1-\alpha} + (1-\delta)(1-\lambda)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right)$. Let the associated transfers be $\hat{t}(s^t)$, by the previous part of the proof it's true that

$$\hat{b}(s^t) := E_t\left[\sum_{k=t}^{\infty} R^{-\left(k-\tau'\right)}\hat{t}_k\right], \forall t > \tau'$$

is feasible in the endogenous debt limit, when the $b(s^{\tau'}) = 0$. Now define the alternative consumption plan,

$$\tilde{c}(s^t) = \begin{cases} c(s^t) & \text{if } t < \tau', \\ \hat{c}(s^t) & \text{if } t \geqslant \tau'. \end{cases}$$

This consumption plan is feasible under the endogenous debt limit problem, since at τ' we have that $b(\tau') \leqslant (1-\delta)(1-\lambda)\theta(s^{\tau'})k^*\left(s^{\tau'-1}\right)$, which implies that $U\left(\{c(s^t)\} \mid s^\tau\right) < U\left(\{\tilde{c}(s^t)\} \mid s^\tau\right)$, contradicting optimality. □

Corollary 2. (*Alternative characterization of endogenously incomplete markets*) *A consumption allocation is the outcome of the optimal contract if and only if the allocation is the outcome of an economy where farmers only have access to a sequence of one-period state contingent savings*

[13] Too see why, note that we can write $t(s^t) = b(s^t) - \frac{1}{R}\mathbb{E}\left[b(s^{t+1})\right]$, and decompose, by the law of iterated expectations:

$$\mathbb{E}_\tau\left[R^{-(t-\tau)}t_t\right] = t(s^\tau) + \sum_{k=1}^{\infty}\frac{1}{R^k}\mathbb{E}_\tau\left[\mathbb{E}_{\tau+k}\left[t_{\tau+k+1}|s^{\tau+k}\right]\right],$$

and the fact that $b(s^t)$ is bounded from above, given that the production function satisfies the inada conditions.

contracts $\{h(s^t)\}_{t\geqslant\tau}$ *and one period* (*not state contingent*) *debt contracts* $\{d(s^t)\}_{t\geqslant\tau}$, *satisfying :*

$$\lambda\theta(s^t)(1-\delta)k^*\left(s^{t-1}\right)+h(s^t)\geqslant d\left(s^{t-1}\right),\quad \forall t>\tau \tag{39}$$

$$d(s^t)\geqslant 0,\quad h(s^t)\geqslant 0,\quad \textit{for all } t>\tau$$

Proof. We can get the result by defining, on the endogenous debt limit framework, for all $\tau'>t$, $s^{\tau'}$

$$d(s^{\tau'-1}):=\left[\max_{s^{\tau'}} b(s^{\tau'})\right]^+$$
$$h(s^{\tau'}):=d(s^{\tau'-1})-b(s^{\tau'}),$$

and appeal to Lemma 2. □

D.3 General Solution Technique

Below is an algorithm to solve the problem

$$\begin{aligned}
V(W,s)=&\max_{c,k,l,\{W(s')\},\{b(s')\}} u(c)+\beta\mathbb{E}[V(W(s'),s')|s]\\
\text{s.t}\quad & W+\sum_{s'}\pi(s'|s)\frac{b(s')}{R}\geqslant c+k+wl,\\
&\tilde{p}(s')k^\alpha l^{1-\alpha}+(1-\delta)\theta(s')k\geqslant W(s')+b(s'),\quad \text{for all } s',\\
&\lambda(1-\delta)\theta(s')k\geqslant b(s'),\quad \text{for all } s'.
\end{aligned}$$

Note that we can rewrite this equation as

$$\begin{aligned}
V(W,s)=&\max_{k,l,\{b(s')\}} u\left(W+\sum_{s'}\pi(s'|s)\frac{b(s')}{R}-k-wl\right)\\
&+\beta\sum_{s'}\pi(s'|s)V(\tilde{p}(s')k^\alpha l^{1-\alpha}+(1-\delta)\theta(s')k-b(s'),s')\\
\text{s.t}\quad &\lambda(1-\delta)\theta(s')k\geqslant b(s'),\quad \text{for all } s'.
\end{aligned}$$

The algorithm is as follows:

- **Input:** Parameters. Grid for $W(s_1)$, $\mathcal{W}:=(W_l,\dots,W_h)$. Compute the capital, such that for each $w_g\in\mathcal{W}$:

$$k(w_g,s)=\min\left\{\tilde{k}(w_g,s),k^*(s)\right\},$$

where

$$k^*(s) = \left(\alpha \frac{\sum_{s'} \pi(s'|s)\theta(s')^\alpha}{R - \sum_{s'} \pi(s'|s)\theta(s')}\right)^{\frac{1}{1-\alpha}},$$

$$w_g - (1-\lambda)\theta(s)^\alpha \tilde{k}(w_g, s)^\alpha - \theta(s)\tilde{k}(w_g, s) = 0$$

- **Input:** Initial guess for $V_w(W(s), s)$ for all s and $w_g \in \mathcal{W}$. Interpolation method for all points outside the grid.
- **Loop 1** for $s \in s_1, ..., s_n$
- Set $i = 0$.
- **Loop 2** For $w_g \in W_h \to W_l$ (in that order):
  1. For $j \leqslant i$: $W(s_j) = (1-\lambda)\theta(s_j)^\alpha k(w_g, s)^\alpha + \theta(s_j)k(w_g, s)$
  2. Compute

$$u'(c) = \frac{\sum_{j\leqslant i} \pi(s_j|s)\beta R V_w(W(s_j), s_j)[(1-\lambda)\theta(s_j)^\alpha \alpha k(w_g, s)^{\alpha-1} + \theta(s_j)]}{R - \sum_{j\leqslant i} \pi(s_j|s)\lambda\theta(s_j)^\alpha \alpha k(w_g, s)^{\alpha-1} - \sum_{j>i} \pi(s_j|s)[\theta(s_j)^\alpha \alpha k(w_g, s)^{\alpha-1} + \theta(s_j)]}$$

$$c = u'^{-1}(u'(c))$$

  3. For $j > i$: find $W(s_j)$, such that

$$\beta R V_w(W(s_j), s_j) = u'(c)$$

  4. For all j: $b(s_j) = \theta(s_j)^\alpha k(w_g, s)^\alpha + \theta(s_j)k(w_g, s) - W(s_j)$
  5. If $i < n$ and $b(i+1) > \lambda\theta(s_{i+1})^\alpha k(wg, s)^\alpha$, set $i = i+1$ go back to step 1
  6. Compute

$$w = c + k(w_g, s) - \sum_j \frac{\pi(s_j|s)b(s_j)}{R}$$

  7. Update $V_w^{\text{next}}(w, s) = u'(c)$
- **End Loop 1**
- $\varepsilon = \|V_w - V_w^{\text{next}}\|_{\infty,\mathcal{W}}$, update $V_w = V_w^{\text{next}}$. If $\varepsilon >$ tol go back to the start of **Loop 1**.
- **End Loop 2**

D.4 Model Calibration

1. Cobb-Douglas capital parameter α : Farmer's expenditure on capital (interest payments on debt + outbound payments in the specified CNAE codes that we interpret as investment) as a fraction of total expenditure (interest payment + RAIS wages if we have that + all outbound payments)

2. R : Average (real) borrowing interest rate.

3. Demand elasticity γ : Coffee price elasticity with respect to Brazil's coffee production

$$\gamma = \frac{\gamma_w}{s_B},$$

where γ_w is the world price elasticity with respect to total quantity, and s_B is the average share of Brazilian coffee exports.

4. Shock magnitude and transition probabilities θ, π : Optimal transport based quantization.

5. Borrowing constraint parameter λ : Iterate over model to match the debt event studies.

6. Distribution of productivity $A_{i,t}$: Two types A_{un} and A_{ins}. Set $A_{un} = 1$ and A_{ins} to match the share of insured and uninsured in the data.

7. Discount rate β and CRRA risk aversion σ: See below.

Let $a_i(s_i^t)$ denote the sum of the value of the capital stock $k_i(s_i^t)$ and income net of the insurance claim $y(s_i^t)$ We jointly calibrate σ and β using these two moments: 1) unconstrained (insured) farmers' consumption response to the net worth shock inclusive of insurance payout

$$\theta_a = \frac{\partial \Delta \log c}{\partial \Delta \log a}$$

and 2) their steady-state ratio of consumption to income (1− savings rate). If the event study is not identified or there are other objections, use the ratio $\rho := \frac{\text{Var}\,(\Delta \log c)}{\text{Var}\,(\Delta \log y)} = (1-\theta_a)^2$. We proxy consumption using outbound payments to CNAE codes that are purchases of final goods, defined as the purchase share to CPFs (vs CNPJs) being above a threshold. We proxy income using inbound boletos, scaled by the municipality-level data on coffee production multiplied by the coffee price.

In addition to the previous setup, assume that income shocks are i.i.d. log-normal with variance σ_ε^2:

$$y_t = (1+\varepsilon_t)\,\bar{y}, \quad \varepsilon_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}\big(-\tfrac{1}{2}\sigma_\varepsilon^2,\, \sigma_\varepsilon^2\big).$$

For unconstrained households, the Euler equation binds. Guess and check linear consumption out of wealth

$$c_t = \kappa m_t, \qquad m_t := k_t + y_t, \qquad 0 < \kappa < 1. \tag{40}$$

Substitute (40) into the intertemporal budget constraint

$$m_{t+1} = R(1-\kappa)\, m_t + (1+\varepsilon_{t+1})\,\bar{y}.$$

The Euler equation under (40) is

$$1 = \beta R\, \mathbb{E}_t\left[\left(\frac{c_{t+1}}{c_t}\right)^{-\sigma}\right] = \beta R\, \mathbb{E}_t\Big[\big(R(1-\kappa) + \tfrac{y_{t+1}}{m_t}\big)^{-\sigma}\Big].$$

Define $Z_{t+1} := \ln\big[R(1-\kappa) + y_{t+1}/m_t\big]$. For small income variance relative to next period net worth, Z_{t+1} is approximately normal, so $\mathbb{E}[e^{-\sigma Z}] = \exp\big(-\sigma\mathbb{E}[Z] + \tfrac{1}{2}\sigma(\sigma{+}1)\,\mathrm{Var}(Z)\big)$. Up to $\mathcal{O}(\sigma_\varepsilon^2)$,

$$1 \approx \beta R^{1-\sigma}\,\big(R(1-\kappa)\big)^{-\sigma}\, \exp\Big\{\tfrac{\sigma+1}{2}\,\sigma_\varepsilon^2\Big\}. \tag{41}$$

Assume (41) holds with equality and solve for the consumption parameter κ:

$$\kappa(\sigma,\beta,R,\sigma_\varepsilon^2) = 1 - \beta^{1/\sigma}\, R^{(1-\sigma)/\sigma}\, \exp\Big\{\tfrac{\sigma+1}{2\sigma}\,\sigma_\varepsilon^2\Big\}.$$