

T₂₀ ① 记得取两颗球都为事件 A

$$\begin{aligned} P(A) &= 0.8 + 0.1 \times \left(1 - \frac{C_1^1 C_{19}^3}{C_{20}^4}\right) + 0.1 \times \left(1 - \frac{C_2^1 C_8^3 + C_2^2 C_{18}^2}{C_{20}^4}\right) \\ &= 0.8 + 0.08 + 0.1 \times \frac{12}{19} \\ &= 0.943 \end{aligned}$$

$$\frac{\cancel{2} \times \cancel{18} \times \cancel{19} \times 16}{\cancel{2} \times \cancel{2}} + \frac{\cancel{18} \times \cancel{19}}{\cancel{2}}$$

$$\cancel{20 \times 19 \times 18 \times 17} \quad \cancel{4 \times 3 \times 2}$$

② 记得取一颗球为事件 B

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(B)}{P(A)} = 0.848$$

T₂₁ 记得取一颗球为红色的事件 A，取另外一颗球为黄色的事件 B

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{1}{3}$$

T₂₃ 记得取到两颗球事件 A

$$\begin{aligned} P(A) &= \frac{C_1^1}{C_3^3} \left(\frac{C_2^2}{C_{10}^2} \cdot \frac{C_5^1}{C_{10}^1} \right) + \frac{C_1^1}{C_3^3} \left(\frac{C_4^2}{C_{10}^2} \cdot \frac{C_6^1}{C_{10}^1} + \frac{C_4^1 C_6^1}{C_{10}^2} \cdot \frac{C_5^1}{C_{10}^1} \right) + \frac{C_1^1}{C_3^3} \left(\frac{C_6^1 C_6^1}{C_{10}^2} \cdot \frac{C_5^1}{C_{10}^1} + \frac{C_2^2}{C_{10}^2} \cdot \frac{C_5^1}{C_{10}^1} \right) \\ &= \frac{1}{2} \left[\frac{2}{15} + \frac{2}{3} \left(\frac{2}{15} + \frac{8}{15} \right) + \frac{1}{3} \left(\frac{8}{15} + \frac{1}{3} \right) \right] \\ &= \frac{13}{30} \end{aligned}$$

T₂₅ $P(A-B) = P(B-A)$ $\therefore P(A \cup B) = P(A) + P(B)$

$$\Rightarrow P(A) - P(AB) = P(B) - P(AB) \quad P(A) = P(B) = \frac{1}{3}$$

$$P(A) = P(B)$$

$$P(A) = \frac{1}{3}$$

T₂₆ ① 记得取两颗球事件 A $P(A) = 1 - (0.4)^3 = 0.936$

$$\textcircled{2} \quad 1 - (0.4)^x > 0.99$$

$$\text{记 } (0.4)^x < 0.01 \quad x = 6$$

T₂₈ 三點的機率為何？ $\rightarrow A$

$$P(A) = \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^5 \\ = \frac{15}{32}$$

T₂₉ $P(A) = P(B) = \frac{1}{2}$ $P(AB) = \frac{3 \times 3}{6 \times 6} = \frac{1}{4} = P(A)P(B)$ $\therefore A, B$ 互斥事件。

$$P(AC) = P(A\bar{B}) = P(A)P(\bar{B}) = \frac{1}{4} \quad (A, B \text{ 互斥})$$

$$P(C) = \frac{6 \times 3}{6 \times 6} = \frac{1}{2} \quad \therefore P(AC) = P(A)P(C) \quad \therefore A, C \text{ 互斥}$$

同理可證 B, C 互斥。 $\therefore A, B, C$ 互斥事件。

$$P(ABC) = 0 \quad (\frac{1}{2} + \frac{1}{2} = 1) \quad P(ABC) = 0$$

$$\neq P(A)P(B)P(C) \quad \therefore A, B, C \text{ 互斥事件}$$

T₃₁ $P = [1 - (1-p)^2]^n$

$$= p^n (2-p)^n$$

T₃₅ 三點的機率為何？ $\rightarrow A$

$$P(A) = 1 - [C_{10}^0 \times (0.8)^0 + C_{10}^1 \times (0.2) \times (0.8)^9 + C_{10}^2 \times (0.2)^2 \times (0.8)^8 + C_{10}^3 \times (0.2)^3 \times (0.8)^7] \\ = 0.121$$

T₃₇ 三點的機率為何？ $\rightarrow A$, $\rightarrow B$ 互斥事件

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(B)}{P(A)} = \frac{1 - [(\frac{1}{6})^0 + C_{10}^1 \times (\frac{1}{6}) \times (\frac{5}{6})^9]}{1 - (\frac{1}{6})^0} \\ = 3 - 2 \times 1.1926 \\ = 0.6148$$

Ch 02

$$T_1 \quad P(X=1) = \frac{4 \times 3 \times 2}{4^3} = \frac{3}{8}$$

$$P(X=2) = \frac{5 \times 4 \times 3}{4^3} = \frac{9}{16}$$

$$P(X=3) = \frac{4}{4^3} = \frac{1}{16}$$

X	1	2	3
$P(X)$	$\frac{3}{8}$	$\frac{9}{16}$	$\frac{1}{16}$

$$T_2 \quad 3 \leq T \leq 7$$

$$\min\{a_1, a_2, a_3\} = 1$$

$$P(T=3) = \frac{9 \times 6 \times 3}{9 \times 8 \times 7} = \frac{9}{28}$$

$$P(T=4) = \frac{9 \times 6 \times 3 (4+2)}{9 \times 8 \times 7 \times 6} = \frac{9}{28}$$

$$P(T=5) = \frac{9 \times 6 \times 3 (4+3+2+3+2)}{9 \times 8 \times 7 \times 6 \times 5} = \frac{9}{14}$$

$$P(T=6) = \frac{9 \times 6 \times 3 (4+3+2+2+3+2+2+2)}{9 \times 8 \times 7 \times 6 \times 5 \times 4} = \frac{3}{28}$$

$$P(T=7) = \frac{9 \times 6 \times 3 (4+3+2+2+3+2+2+2+2)}{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3} = \frac{1}{28}$$

X	3	4	5	6	7
P	$\frac{9}{28}$	$\frac{9}{28}$	$\frac{9}{14}$	$\frac{3}{28}$	$\frac{1}{28}$

$$T_3 \quad \textcircled{1} \quad P_1 + P_2 + P_3 = 1$$

$$C \left(\frac{2}{3} + \frac{4}{9} + \frac{8}{27} \right) = 1$$

$$C = \frac{27}{38} \quad |$$

$$\textcircled{2} \quad P_1 + P_2 + P_3 = 1$$

$$C \left(1 + \frac{\lambda^2}{2} + \dots + \frac{\lambda^k}{k!} + \dots \right) = 1$$

$$C = \frac{1}{e^\lambda - 1}$$

$$T_4 \quad c=0 \quad d=\frac{1}{4} \quad a+\frac{1}{4} = \frac{3}{4} \quad a=\frac{1}{2} \quad b=1-\frac{1}{4}-a=\frac{1}{4}$$

$$\left\{ \begin{array}{l} a=\frac{1}{2} \\ b=\frac{1}{4} \\ c=0 \\ d=\frac{1}{4} \end{array} \right.$$

$$\begin{array}{r} 49 \\ 7 \\ \hline 343 \end{array}$$

$$\begin{array}{r} 0.79 \\ 0.7 \\ \hline 0.49 \\ 0.49 \\ \hline 0.49 \end{array}$$

T₅ 令 X 为命中次数

$$P(X=0) = (1-p)^3$$

$$P(X=1) = C_3^1 p(1-p)^2$$

$$P(X=2) = C_3^2 p^2 (1-p)$$

$$P(X=3) = p^3$$

$$P = 0.6$$

X	0	1	2	3
P	0.064	0.288	0.432	0.216

$$P = 0.7$$

X	0	1	2	3
P	0.027	0.189	0.441	0.343

① 计算人射概率的和为事件 A

$$P(A) = 0.064 \times 0.027 + 0.288 \times 0.189 + 0.432 \times 0.441 + 0.216 \times 0.343$$

$$= 0.3208$$

② 计算乙射中次数为事件 B

$$P(B) = 0.288 \times 0.027 + 0.432 \times (0.027 + 0.189) + 0.216 \times (0.027 + 0.189 + 0.441)$$

$$= 0.243$$

T₇ ① 计算随机事件并计算 $P(A \cap B) = P(A)P(B)$ 为事件 A

$$P(A) = \frac{C_4^4}{C_8^4} = \frac{1 \times 3 \times 2}{28 \times 7 \times 6 \times 5} = \frac{1}{280}$$

② 计算随机事件 10 次并取 2 和 3 次为事件 B

$$P(B) = C_10^3 [P(A)]^3 \times [1 - P(A)]^7 = \frac{10 \times 9 \times 8}{3 \times 2} \times \left(\frac{1}{280}\right)^3 \times \left(\frac{27}{280}\right)^7 < 0.01$$

中等概率事件

T8 ① 求 $\{X \leq 3\}$ 的概率

$$\begin{aligned} P(A) &= 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) \\ &= 1 - e^{-2.5} \left(1 + 2.5 + \frac{(2.5)^2}{2} + \frac{(2.5)^3}{6} \right) \\ &= 0.2425 \end{aligned}$$

$$② P(X) = \begin{cases} 0.082 & x \leq 0 \\ 0.287 & x \leq 1 \\ 0.544 & x \leq 2 \\ 1 & x \leq 3 \end{cases}$$

T ₁₀	① $1 = a \int_{-\infty}^{+\infty} e^{-\lambda x} dx$	$\int_0^{\infty} e^{-x} dx = \frac{1}{2a}$	$a = \frac{1}{2}$
②	$1 = a \int_0^{2\pi} \sin \frac{x}{2} dx$	$\int_0^{2\pi} \sin x dx = \frac{1}{2a}$	$a = \frac{1}{4}$
③	$1 = \int_0^a \cos x dx$	$\sin a = 1$	$a = 2k\pi + \frac{\pi}{2}, k \in \mathbb{N}$
④	$1 = a \int_{-1}^1 \frac{1}{1+x^2} dx$	$\arctan 1 = \frac{1}{2a}$	$a = \frac{\pi}{4}$

T₁₁ $F(0) = 0 \quad A + B = 0$

$$F(+\infty) = 1 \quad A = 1 \quad B = -1$$

$$P(-1 < X \leq 1) = F(1) - F(-1) = 1 - e^{-\frac{1}{2}}$$