T_1 ①③ 不是; ②④ 是,

T_2 $E(\bar{X}) = \frac{1}{n}\sum_{i=1}^{n} E(X_i) = \frac{1}{2}$

$D(\bar{X}) = \frac{\sigma^2}{n} = \frac{1}{12n}$

T_3 ① $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$

$= \frac{1}{n-1}\left(\sum_{i=1}^{n} X_i^2 + n\bar{X}^2 - 2\bar{X}\sum_{i=1}^{n} X_i\right)$

$= \frac{1}{n-1}\sum_{i=1}^{n} X_i^2 - n\bar{X}^2$

② $E(S^2) = E\left(\frac{1}{n-1}\sum_{i=1}^{n} X_i^2 - n\bar{X}^2\right)$

$= \frac{1}{n-1}\left[\sum_{i=1}^{n} E(X_i^2) - nE(\bar{X}^2)\right]$

$= \frac{1}{n-1}\left[n(\sigma^2+\mu^2) - n\left(\frac{\sigma^2}{n} + \mu^2\right)\right]$

$= \sigma^2$

T_4 $E(\chi^2) = \sum_{i=1}^{n} E(X_i^2) = n(\sigma^2+\mu^2) = n$

$D(\chi^2) = \sum_{i=1}^{n} D(X_i^2) = \sum_{i=1}^{n}\left[E(X_i^4) - E^2(X_i^2)\right] = 2n$

T_5 令 $\chi^2 = \sum_{i=1}^{10} X_i^2/\sigma^2$ $\quad P(\chi^2(10) > 16) = 0.1$

T6 $p(x) = \frac{\Gamma(\frac{n+1}{2})}{\sqrt{n\pi}\,\Gamma(\frac{n}{2})}\left(1+\frac{x^2}{n}\right)^{-\frac{n+1}{2}}$

$$\lim_{n\to\infty} p(x) = \frac{1}{\sqrt{n\pi}} \cdot e^{-\frac{x^2}{2}} \frac{\Gamma(\frac{n}{2}+\frac{1}{2})}{\Gamma(\frac{n}{2})}$$
$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

T7 $Z_i \sim \frac{1}{2}X_i$

$$Y = \frac{\sum_{i=1}^{10} Z_i^2}{2\sum_{i=11}^{15} Z_i^2} = \frac{\chi_1^2(10)}{2\chi_2^2(5)} = \frac{\chi_1^2(10)/10}{\chi_2^2(5)/5} = F(10,5)$$

T8 $\bar{X} = \frac{1}{2n}\sum_{i=1}^{2n} X_i$, $Y = \sum_{i=1}^{n}(X_i + X_{n+i} - 2\bar{X})^2$

$$= \sum_{i=1}^{2n} X_i^2 + 4n\bar{X}^2 - 4\sum_{i=1}^{n}(X_i + X_{n+i})\cdot\bar{X}$$
$$= \sum_{i=1}^{2n} X_i^2 - 4n\bar{X}^2$$

$$E(Y) = \sum_{i=1}^{2n} E(X_i^2) = 2n(\sigma^2+\mu^2)$$

T9 $X_{n+1} - \bar{X} \sim N(\mu-\mu, \sigma^2 + \frac{\sigma^2}{n}) \sim N(0, \frac{n-1}{n}\sigma^2)$

$$\sim \sqrt{\frac{n+1}{n}}\,\sigma \cdot N(0,1)$$

$$S \sim \frac{\sigma}{\sqrt{n-1}}\chi^{\cancel{2}}(n-1)$$

$$\therefore \frac{X_{n+1}-\bar{X}}{S}\sqrt{\frac{n}{n+1}} = \frac{\cancel{\sqrt{\frac{n+1}{n}}}\,\cancel{\sigma}\,N(0,1)}{\cancel{\sigma}\sqrt{S^2/(n-1)}} \cdot \cancel{\sqrt{\frac{n}{n+1}}} = t(n-1)$$

T10 ① $P(\bar{X} > 13)$ $\quad \bar{X} \sim N(12, \frac{4}{5})$

$= P(\frac{\bar{X}-\mu}{\sigma} > \frac{\sqrt{5}}{2})$

② $P = 1 - [P(X_i > 10)]^5$

③ $P = 1 - [P(X_i < 15)]^5$

T11 $\bar{x}' = \frac{n_1 \bar{x} + n_2 \bar{y}}{n_1 + n_2}$

$S' = \frac{(n-1)S_1^2 + (n-1)S_2^2}{n_1 + n_2 - 1}$