T_1 $F(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}p(u,v)\,du\,dv$

$x\le 0$ 或 $y\le 0$ 时 $F(x,y)=0$

$x\ge 1$ 且 $y\ge 1$ 时 $F(x,y)=1$

$0<x<1$ 且 $y>0$ 时 $F(x,y)=\begin{cases}\int_0^x\int_0^y 2\,du\,dv=2xy & x+y\le 1\\ \int_0^x\int_0^{1-\cancel{v}} 2\,du\,dv=2x-x^2 & x+y>1\end{cases}$

$0<y<1$ 且 $x>0$ 时 $F(x,y)=\begin{cases}2xy & x+y\le 1\\ 2y-y^2 & x+y>1\end{cases}$

T_2 ① $a+b+0.6=1$　　$P(X<0,Y\le 0)=a$　　(图上 X 值有误,应为-1,0,1)

$a+b=0.4$　　$a=0.2$　$b=0.2$

②

Z	-2	-1	0	1
P	0.2	0.1	0.4	0.3

③ $X=2$ 即 $Y>0$

$P_X(Y=0)=0.3$

T_3 $P(X+Y=2)=P(X=3,Y=-1)+P(X=2,Y=0)+P(X=1,Y=1)$

$=\frac{1}{8}\times\frac{1}{4}+\frac{1}{8}\times\frac{1}{2}+\frac{1}{4}\times\frac{1}{4}$

$=\frac{5}{32}$

T_4 ① $P(X>2Y)=\int_0^1\int_0^{\frac{u}{2}}(2-u-v)\,du\,dv$

$=\int_0^1\left[(2-u)\cdot\frac{u}{2}-\frac{1}{8}u^2\right]du$

$=\frac{1}{2}-\frac{5}{24}=\frac{7}{24}$

$0<z\le 1$ 时

② $$P(z) = \int_0^z \int_0^{z-v} (2-u-v)\,du\,dv$$
$$= \int_0^z \left[(2-v)(z-v) - \frac{1}{2}(z-v)^2\right] dv$$
$$= 2z^2 - \frac{1}{2}(2+z)z^2 + \frac{1}{3}z^3 - \frac{1}{6}z^3$$
$$= z^2\left(1-\frac{1}{3}z\right)$$

$1<z<2$ 时 $$P(z) = \int_0^{z-1}\int_0^1 (2-u-v)\,du\,dv + \int_{z-1}^{z}\int_0^{z-v} (2-u-v)\,du\,dv$$
$$= \int_0^{z-1} \left(\frac{3}{2}-v\right) dv + \int_{z-1}^{z} \left[(2-v)(z-v) - \frac{1}{2}(z-v)^2\right] dv$$
$$= \frac{3}{2}(z-1) - \frac{1}{2}(z-1)^2 + 2z - \frac{1}{2}(2+z)(2z-1) + \frac{1}{3}(3z^2-3z+1) + \frac{1}{6}$$
$$= -\frac{1}{2}z^2 + 2z - \frac{1}{2}$$

$$p_Z(z) = P'(z) = \begin{cases} 2z - z^2 & 0<z\le 1 \\ 2-z & 1<z<2 \\ 0 & \text{else.} \end{cases}$$

T5 ① $$p_X(x) = \int_{-\infty}^{+\infty} p(x,y)\,dy = \begin{cases} 2x & 0<x<1 \\ 0 & \text{else.} \end{cases}$$
$$p_Y(y) = \int_{-\infty}^{+\infty} p(x,y)\,dx = \begin{cases} 1-\frac{y}{2} & 0<y<2 \\ 0 & \text{else.} \end{cases}$$

② $$P(z) = \int_0^{2-z}\int_{\frac{1}{2}v}^{\frac{1}{2}z+\frac{1}{2}v} 1\cdot du\,dv + \int_{2-z}^{2}\int_{\frac{1}{2}v}^{1} 1\cdot du\,dv$$
$$= \int_0^{2-z} \frac{1}{2}z\,dv + \int_{2-z}^{2} \left(1-\frac{1}{2}v\right) dv$$
$$= \frac{1}{2}z(2-z) + z - \frac{1}{4}(4z-z^2)$$

$$p_Z(z) = P'(z) = 1 - \frac{1}{2}z$$

③ $$P\left(Y\le\frac{1}{2}, X\le\frac{1}{2}\right) = \int_0^{\frac{1}{2}}\int_{\frac{1}{2}v}^{\frac{1}{2}} 1\cdot du\,dv$$
$$= \int_0^{\frac{1}{2}} \left(\frac{1}{2}-\frac{1}{2}v\right) dv = \frac{3}{16}$$

$\frac{1}{2} - \frac{1}{16}$

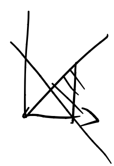
$$P(X\le \tfrac{1}{2}) = \int_0^{\frac{1}{2}} p_X(x)\,dx = \frac{1}{4}$$

$$P(Y\le \tfrac{1}{2} \mid X\le \tfrac{1}{2}) = \frac{P(Y\le \frac{1}{2}, X\le \frac{1}{2})}{P(X\le \frac{1}{2})} = \frac{3}{4}$$

T6 ① $p(x,y) = \begin{cases} \frac{1}{x}, & 0<y<x<1 \\ 0, & \text{else}. \end{cases}$

② $p_Y(y) = \begin{cases} \int_y^1 \frac{1}{x}\,dx = -\ln y, & 0<y<1 \\ 0, & \text{else}. \end{cases}$

③ 设 $Z = X+Y$

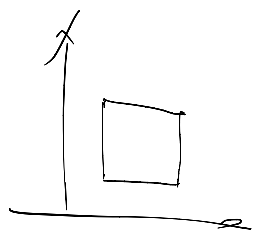


$$P(Z>1) = \int_{\frac{1}{2}}^{1}\int_{1-v}^{v} \frac{1}{v}\,du\,dv$$

$$= \int_{\frac{1}{2}}^{1} \left(2-\frac{1}{v}\right) dv$$

$$= 1-\ln 2$$

T7 设 $m = X-Y$ $\quad m\in[-2,2]$



$m>0$ 时

$$P(m) = \int_1^{3-m}\int_1^{v+m} \frac{1}{4}\,du\,dv + \int_{3-m}^{3}\int_1^{3} \frac{1}{4}\,du\,dv$$

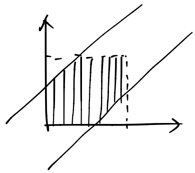
$$= 1-\frac{1}{4}(2-m)^2$$

$h(m) = \frac{1}{2}(2-m)$ $\qquad 1-\frac{1}{2}m$

T8 $P(X=2) = \frac{1}{2}\times\frac{1}{3}\times(1-\frac{1}{4}) + \frac{1}{2}\times(1-\frac{1}{3})\times\frac{1}{4} + (1-\frac{1}{2})\times\frac{1}{3}\times\frac{1}{4}$

$$= \frac{3}{24}+\frac{2}{24}+\frac{1}{24}$$

$$= \frac{1}{4}$$

T9 令 $Z = Y - X$ 

则Z服从的分布.

$$P(|Z| \le \tfrac{1}{2}) = \int_0^{\frac{1}{2}}\int_0^{u+\frac{1}{2}} 1 \cdot du\,dv + \int_{\frac{1}{2}}^{1}\int_{u-\frac{1}{2}}^{1} 1 \cdot du\,dv$$

$$= \int_0^{\frac{1}{2}} (v+\tfrac{1}{2})\,dv + \int_{\frac{1}{2}}^{1} (\tfrac{3}{2} - u)\,du$$

$$= \frac{1}{8} + \frac{1}{4} + \frac{3}{4} - \frac{1}{2}\cdot\frac{3}{4}$$

$$= \frac{3}{4}$$

T10 ① $$p(x,y) = p_{Y|X=x}(y) \cdot p_X(x) = \begin{cases} \frac{9}{x}y^2, & 0<y<x<1 \\ 0, & \text{else.} \end{cases}$$

② $$p_Y(y) = \int_y^1 \frac{9}{x} y^2\,dx = -9y^2 \ln y$$

T11 $x>0$ $y>0$ 时

$x = \sqrt{u}$ $y = \sqrt{v}$

$$p(u,v) = 4\sqrt{uv} \cdot \begin{vmatrix} \frac{1}{2\sqrt{u}} & 0 \\ 0 & \frac{1}{2\sqrt{v}} \end{vmatrix} = 1$$

$$\therefore p(u,v) = \begin{cases} 1, & 0<u<1, 0<v<1 \\ 0, & \text{else.} \end{cases}$$

T12 ① $1 = c\int_0^{\infty}\int_{-v}^{v}(v^2-u^2)e^{-v}\,du\,dv$

$$\frac{1}{c} = \int_0^{\infty}\left(+uv^2e^{-v} - \frac{1}{3}u^3e^{-v}\right)\Big|_{-v}^{v}\,dv$$

$$\frac{1}{c} = \int_0^{\infty} +\frac{4}{3}v^3e^{-v}\,dv$$

$$\frac{1}{c} = -\left(\frac{4}{3}v^3e^{-v} + 4v^2e^{-v} + 8ve^{-v} + 8e^{-v}\right)\Big|_0^{\infty}$$

$$\frac{1}{c} = 8 \qquad c = \frac{1}{8}$$

② $x\in(0,+\infty)$ 时

$$p_X(x) = \int_{-x}^{x} p(x,y)\,dy = \int_{-x}^{x}\frac{1}{8}(x^2-y^2)e^{-x}\,dy = \frac{1}{4}x^3e^{-x} - \frac{1}{12}x^3e^{-x} = \frac{1}{6}x^3e^{-x}$$

$$p_X(x) = \begin{cases}\frac{1}{6}x^3e^{-x}, & x>0\\ 0, & \text{else.}\end{cases}$$

$$p_Y(y) = \int_{|y|}^{\infty} p(x,y)\,dx = \int_{|y|}^{\infty}\frac{1}{8}(x^2-y^2)e^{-x}\,dx$$

$$= \frac{1}{8}y^2e^{-x}\Big|_{|y|}^{\infty} + \frac{1}{8}\left(-x^2e^{-x} - 2xe^{-x} - 2e^{-x}\right)\Big|_{|y|}^{\infty}$$

$$= \cancel{-\frac{1}{8}y^2e^{-|y|} + \frac{1}{8}y^2e^{-|y|}} + \frac{1}{4}|y|e^{-|y|} + \frac{1}{4}e^{-|y|}$$

$$= \frac{1}{4}(|y|+1)e^{-|y|}$$

不独立.

③ $$p_{X|Y=y}(x) = \frac{p(x,y)}{p_Y(y)} = \frac{x^2-y^2}{2(|y|+1)}e^{|y|-x} \qquad (0<|y|<x)$$

$$p_{Y|X=x}(y) = \frac{p(x,y)}{p_X(x)} = \frac{3(x^2-y^2)}{4x^3} \qquad (0<|y|<x)$$

T13 $f(x,y) = \begin{cases} e^{-y} & 0<x<1 \\ 0 & \text{else.} \end{cases}$

① $p_Z(z) = \int_0^{+\infty} f(z-y, y)\, dy$ $\qquad 0 < z-y < 1$

$0 < z \le 1$ 时 $\qquad y+1 > z > y$

$p_Z(z) = \int_0^{z} e^{-y}\, dy$

$= 1 - e^{-z}$

$z > 1$ 时

$p_Z(z) = \int_{z-1}^{z} e^{-y}\, dy$

$= (e-1) e^{-z}$

$$p_Z(z) = \begin{cases} 1-e^{-z} & 0<z\le 1 \\ (e-1)e^{-z} & z>1 \end{cases}$$

② $p_R(r) = \int_0^{+\infty} |y| f(yr, y)\, dy$ $\qquad 0 < yr < 1$

$= \int_0^{\frac{1}{r}} y e^{-y}\, dy$

$= \left(-ye^{-y} - e^{-y}\right)\Big|_0^{\frac{1}{r}}$

$= 1 - \left(1+\frac{1}{r}\right) e^{-\frac{1}{r}}$

③ $p_{Z|X=x}(z) = \dfrac{p(x,z)}{p_X(x)}$

$0 < x < 1$ 时

原式 $= e^{-(z-x)}$ $\qquad z > 0$

T14 $X+bY$ 与 $X-bY$ 独立.

$\because \operatorname{cov}(X+bY, X-bY)=0$

$$\operatorname{cov}(X,X)+b\operatorname{cov}(X,Y)-b\operatorname{cov}(X,Y)-b^2\operatorname{cov}(Y,Y)=0$$

$$\operatorname{cov}(X,X)=b^2\operatorname{cov}(Y,Y)$$

$$\sigma_1^2=b^2\sigma_2^2$$

$$b=\pm\frac{\sigma_1}{\sigma_2}$$

T15 ① $p_1(x,y)>0 \quad p_2(x,y)>0$

$\therefore p(x,y)>0$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p(x,y)\,dx\,dy = 0.4\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p_1(x,y)\,dx\,dy + 0.6\times\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} p_2(x,y)\,dx\,dy$$

$$=0.4+0.6=1$$

得证.

② $$p_X(x)=0.4\int_{-\infty}^{+\infty} p_1(x,y)\,dy+0.6\int_{-\infty}^{+\infty} p_2(x,y)\,dy$$

$$=(0.4+0.6)\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}$$

$$=\frac{1}{\sqrt{2\pi}\sigma_1}e^{-\frac{(x-\mu_1)^2}{2\sigma_1^2}}$$

T16 $P(X=k)=C_{10}^{k}(0.3)^k(0.7)^{10-k}$

$P(Y=k)=C_5^{k}(0.3)^k(0.7)^{10-k}$

$$P(X+Y=k)=\sum_{i=0}^{t} C_{10}^{i}C_5^{k-i}\cdot(0.3)^k(0.7)^{10-k} \qquad t=\begin{cases}k & k\le 5\\ 5 & k>5\end{cases}$$

$$=C_{15}^{k}(0.3)^k(0.7)^{10-k}$$

易得.

T17 $p(x,y) = \lambda^2 e^{-\lambda(x+y)}$

① $p_Z(z) = \int_0^{+\infty} p(x, z-x)\, dx$

$= \int_0^{z} \lambda^2 e^{-\lambda z}\, dx$

$= \lambda^2 z e^{-\lambda z}$

$P(Z \geq \frac{2}{\lambda}) = 1 - \int_0^{\frac{2}{\lambda}} \lambda^2 z e^{-\lambda z}\, dz$

$= 1 + (\lambda z e^{-\lambda z} + e^{-\lambda z})\Big|_0^{\frac{2}{\lambda}}$

$= 1 + 2e^{-2} + e^{-2} - 1$

$= 3e^{-2}$

② $P(Y - X \geq \frac{1}{X}) = \iint\limits_{y-x \geq \frac{1}{x}} \lambda^2 e^{-\lambda(x+y)}\, dx dy$

$= \int_0^{+\infty} \int_{x+\frac{1}{x}}^{+\infty} \lambda^2 e^{-\lambda(x+y)}\, dy\, dx$

$= \int_0^{+\infty} -\lambda e^{-\lambda(x+y)} \Big|_{x+\frac{1}{x}}^{+\infty}\, dx$

$= \int_0^{+\infty} +\lambda e^{-\lambda(2x+\frac{1}{x})}\, dx$

$= -\frac{1}{2} e^{-\lambda(2x+\frac{1}{x})} \Big|_0^{+\infty}$

$= \frac{1}{2e}$

T18 ① $y = \frac{z}{x}$

$p(x,z) = p(x, \frac{z}{x}) \cdot |J(x,z)|$

$= \begin{cases} \frac{1}{x}, & 0 < z < x < 1 \\ 0, & \text{else.} \end{cases}$

$0 < \frac{z}{x} < 1$

$0 < z < x$

② $p_Z(z) = \int_{-\infty}^{+\infty} p(x,z)\, dx$

$= \begin{cases} -\ln z, & 0 < z < 1 \\ 0, & \text{else.} \end{cases}$

T19 $p_M(z) = p_X(z)F_Y(z) + p_Y(z)F_X(z) = 2z$

$p_N(z) = p_X(z)(1-F_Y(z)) + p_Y(z)(1-F_X(z)) = 2(1-z)$

$0 < z < 1$

T20 $\begin{cases} u = a_1 x_1 + a_2 x_2 \\ v = x_2 \end{cases}$ $\begin{cases} x_1 = \frac{1}{a_1}(u - a_2 v) \\ x_2 = v \end{cases}$

$$p_u(u) = \int_{-\infty}^{+\infty} |J| p(u, v) \, dv$$

$\begin{matrix} \frac{1}{a_1} & -\frac{a_2}{a_1} \\ 0 & 1 \end{matrix}$

$$= \int_{-\infty}^{+\infty} \frac{1}{a_1} \frac{1}{2\pi} e^{-\frac{1}{2}\left(\frac{1}{a_1^2}(u-a_2 v)^2 + v^2\right)} \, dv$$

$=$

T21 $P(X=k) = \frac{e^{-30} \, 30^k}{k!}$

$$p(y) = \sum_{x=y}^{+\infty} C_x^y p^y (1-p)^{x-y} \frac{e^{-30} \cdot 30^x}{x!}$$