矩估计：

T1 $\mu_1 = EX = np$

$p = \frac{\mu_1}{n}$ $\mu_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$

$p = \frac{\overline{X}}{n}$

极大似然估计： $(x_1, x_2 \cdots x_n)$ 为 $(X_1, X_2 \cdots X_n)$ 一组观测值.

$x_i = 1, 0$

$P(X=x) = p^x (1-p)^{1-x}$

$L(p) = \prod_{i=1}^{n} p^{x_i} (1-p)^{1-x_i}$

$= p^{\sum_{i=1}^{n} x_i} (1-p)^{n-\sum_{i=1}^{n} x_i}$

$\ln L(p) = (\sum_{i=1}^{n} x_i)\ln p + (n-\sum_{i=1}^{n} x_i)\ln(1-p)$

$\frac{d \ln L p}{dp} = \frac{\sum_{i=1}^{n} x_i}{p} + \frac{n-\sum_{i=1}^{n} x_i}{1-p} = 0$

$p = \frac{1}{n^2}\sum_{i=1}^{n} x_i$

$p = \frac{\overline{X}}{n}$

T2 矩估计：

$\mu_1 = EX = \frac{\alpha+1}{\alpha+2}$

$1 - \frac{1}{\alpha+2} = \frac{1}{n}\sum_{i=1}^{n} X_i$

$\alpha = \frac{1}{1-\frac{1}{n}\sum_{i=1}^{n} X_i} - 2$

$= \frac{2\overline{X}-1}{1-\overline{X}}$

极大似然估计：

$(x_1, x_2, \cdots x_n)$ 为 $(X_1, X_2 \cdots X_n)$ 一组观测值。

$$L(\alpha) = (\alpha+1)^n \left(\prod_{i=1}^{n} x_i\right)^\alpha$$

$$\ln L(\alpha) = n\ln(\alpha+1) + \alpha\ln\left(\prod_{i=1}^{n} x_i\right)$$

$$\frac{d\ln L(\alpha)}{d\alpha} = 0$$

$$\frac{n}{\alpha+1} = -\frac{1}{\ln\left(\prod_{i=1}^{n} x_i\right)}$$

$$\alpha = -n\ln\left(\prod_{i=1}^{n} x_i\right) - 1$$

$$\therefore \alpha = -n\sum_{i=1}^{n}\ln X_i - 1$$

T_3

$$\int_0^{+\infty} \exp\left\{-\frac{1}{2\sigma^2}(\ln x-\mu)^2\right\} dx$$

$$= \int_0^{+\infty} x\exp\left\{-\frac{1}{2\sigma^2}(\ln x-\mu)^2\right\} d\ln x$$

$$= \int_{-\infty}^{+\infty} e^t \exp\left\{-\frac{1}{2\sigma^2}(t-\mu)^2\right\} dt$$

$$= \int_{-\infty}^{+\infty} e^{-\frac{1}{2\sigma^2}t^2 + (\frac{\mu}{\sigma^2}+1)t - \frac{\mu^2}{2\sigma^2}} dt$$

$$= e^{\mu+\frac{\sigma^2}{2}} \int_{-\infty}^{+\infty} e^{-\frac{1}{2\sigma^2}t^2} dt$$

$$= \sqrt{2\pi}\,\sigma\, e^{\mu+\frac{\sigma^2}{2}}$$

$$\mu_1 = e^{\mu+\frac{\sigma^2}{2}}$$

$$\mu_1 = \sum_{i=1}^{n} X_i$$

$$\mu_2 = \sum_{i=1}^{n} X_i^2$$

$$\int_0^{+\infty} x^2 \exp\left\{-\frac{1}{2\sigma^2}(\ln x-\mu)^2\right\} d\ln x$$

$$= \int_{-\infty}^{+\infty} e^{2t} \cdot e^{-\frac{1}{2\sigma^2}(t-\mu)^2} dt$$

$$= \int_{-\infty}^{+\infty} e^{-\frac{1}{2\sigma^2}(t^2 - 2(\mu+2\sigma^2)t + \mu^2)} dt$$

$$= \int_{-\infty}^{+\infty} e^{-\frac{1}{2\sigma^2}(t-\mu-2\sigma^2)^2} \cdot e^{2\sigma^2+2\mu} dt$$

$$= \sqrt{2\pi}\sigma \cdot e^{2\sigma^2+2\mu}$$

$$\mu_2 = e^{2\sigma^2+2\mu}$$

$$\sigma^2 = \ln\mu_2 - 2\ln\mu_1$$

$$\mu = 2\ln\mu_1 - \frac{1}{2}\ln\mu_2$$

$$L(\mu,\sigma^2) = (2\pi\sigma^2)^{-\frac{n}{2}} \cdot \frac{1}{\prod_{i=1}^n x_i} \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^n (\ln x-\mu)^2\right\}$$

$$\ln L = -\frac{n}{2}\ln(2\pi\sigma^2) - \sum_{i=1}^n \ln x_i - \frac{1}{2\sigma^2}\sum_{i=1}^n (\ln x_i - \mu)^2$$

$$\frac{\partial L}{\partial \mu} = 0 \qquad \frac{1}{\sigma^2}\sum_{i=1}^n (\mu - \ln x_i) = 0 \qquad \mu = \frac{1}{n}\sum_{i=1}^n \ln x_i$$

$$\mu = \frac{1}{n}\sum_{i=1}^n \ln X_i$$

$$\frac{\partial L}{\partial \sigma^2} = 0 \qquad -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^n (\ln x_i - \mu)^2 = 0$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^n \left(\ln X_i - \frac{1}{n}\sum_{i=1}^n \ln X_i\right)$$

T4 矩估计:

$$\frac{1}{\theta}\int_{\mu}^{+\infty} x e^{-\frac{x-\mu}{\theta}}\,dx$$

$$= \theta\int_{0}^{+\infty} \left(m+\frac{\mu}{\theta}\right) e^{-m}\,dm$$

$$= \theta\left[-me^{-m}\Big|_{0}^{+\infty} + (1-\mu/\theta)e^{-m}\Big|_{0}^{+\infty}\right]$$

$$= \mu+\theta$$

$$\frac{1}{\theta}\int_{\mu}^{+\infty} x^2 e^{-\frac{x-\mu}{\theta}}\,dx$$

$$= \theta^2\cdot e^{\frac{\mu}{\theta}}\int_{\mu/\theta}^{+\infty} m^2 e^{-m}\,dm$$

$$= \theta^2\cdot e^{\frac{\mu}{\theta}}\left(-m^2e^{-m}-2me^{-m}-2e^{-m}\right)\Big|_{\mu}^{+\infty}$$

$$= \theta^2\left(\mu^2/\theta^2+2\mu/\theta+2\right)$$

$$= \mu^2+2\mu\theta+2\theta^2$$

$$S^* = \sqrt{\mu^2+2\mu\theta+2\theta^2-(\mu+\theta)^2} = \theta$$

$$\theta = S^*$$

$$\mu = \overline{X} - S^*$$

极大似然估计： $(x_1, x_2 \cdots x_n)$ 为 $(X_1, X_2 \cdots X_n)$ 一组观测值.

$$L(\mu, \theta) = \frac{1}{\theta^n} \exp\left\{-\left(\sum_{i=1}^{n} x_i - n\mu\right)/\theta\right\}$$

$$\frac{\partial L}{\partial \mu} = \frac{n}{\theta^{n+1}} \exp\left\{-\left(\sum_{i=1}^{n} x_i - n\mu\right)/\theta\right\} \quad \text{单调增}.$$

又 $\because \mu \le \min\{x_1, x_2 \cdots x_n\}$

$\therefore \mu = \min\{x_1, x_2 \cdots x_n\}$ $\qquad \mu = \min\{X_1, X_2 \cdots X_n\}$

$$\frac{\partial L}{\partial \theta} = -\frac{n}{\theta^{n+1}} \exp\left\{-\left(\sum_{i=1}^{n} x_i - n\mu\right)/\theta\right\} + \frac{\sum_{i=1}^{n} x_i - n\mu}{\theta^{n+2}} \exp\left\{-\left(\sum_{i=1}^{n} x_i - n\mu\right)/\theta\right\}$$

令 $\frac{\partial L}{\partial \theta} = 0$ $\qquad n(\theta + \mu) = \sum_{i=1}^{n} x_i$

$$\theta = \bar{X} - \min\{X_1, X_2 \cdots X_n\}$$

T5. 矩估计：

$\mu_1 = \theta$

$\theta = \bar{X}$

极大似然：$(x_1, x_2 \cdots x_n)$ 为 $(X_1, X_2 \cdots X_n)$ 一组观测

θ 可取 $\left[\max\{X_1, X_2 \cdots X_n\} - \frac{1}{2}, \min\{X_1, X_2 \cdots X_n\}\right]$

上任意值.

T6 矩估计：

$$\mu_1 = \frac{3}{2}\theta$$

$$\frac{3}{2}\theta = \overline{X}$$

$$\theta = \frac{2}{3}\overline{X}$$

最大似然估计：

$$\theta = [\min\{X_1, X_2 \cdots X_n\}, \ \frac{1}{2}\max\{X_1, X_2, \cdots X_n\}]$$

T7 矩估计

$$3-4\theta = 2$$

$$\theta = \frac{1}{4}$$

极大似然.

$$L(\theta) = \theta^2 \cdot [2\theta(1-\theta)]^2 \cdot \theta^2 (1-2\theta)^4$$

$$\ln L(\theta) = \ln 4 + 6\ln\theta + 2\ln(1-\theta) + 8\ln(1-2\theta)$$

$$\frac{3}{\theta} - \frac{1}{1-\theta} - \frac{4}{1-2\theta} = 0$$

$$6\theta^2 - 9\theta + 3 + 2\theta^2 - \theta + 4\theta^2 - 4\theta = 0$$

$$12\theta^2 - 14\theta + 3 = 0$$

$$\theta = \frac{14-2\sqrt{13}}{24}$$

$$\theta = \frac{7-\sqrt{13}}{12}$$

T8 $P(X=0)=e^{-\lambda}$

$(x_1, x_2, \cdots x_n)$ 为 $(X_1, X_2 \cdots X_n)$ 一组观测值.

$$L(\lambda)=\prod_{i=1}^{n}\frac{\lambda^{x_i}}{x_i!}e^{-\lambda}$$

$$\ln L(\lambda) = -n\lambda + (\sum_{i=1}^{n}x_i)\cdot\ln\lambda - \sum_{i=1}^{n}\ln(x_i!)$$

$$\frac{d\ln L(\lambda)}{d\lambda}=0 \qquad -n+\frac{1}{\lambda}\sum_{i=1}^{n}x_i=0$$

$$\lambda=\frac{1}{n}\sum_{i=1}^{n}x_i$$

$$\lambda=\overline{X}$$

$$P(X=0)=e^{-\overline{X}}$$

T9. ① $E(\hat{\mu}_1)=\frac{1}{4}EX_1+\frac{3}{4}EX_2$

$=\frac{1}{4}\mu+\frac{3}{4}\mu$

$=\mu$

$M=D(\hat{\theta}(X_1, X_2))$

$=\sigma_1^2+\sigma_2^2$

$=\frac{1}{16}+\frac{9}{16}$

$=\frac{5}{8}$

② $E(\hat{\mu}_2)=\frac{1}{2}EX_1+\frac{1}{2}EX_2$

$=\frac{1}{2}\mu+\frac{1}{2}\mu$

$=\mu$.

$M=D(\hat{\theta}(X_1, X_2))$

$=\sigma_1^2+\sigma_2^2$

$=\frac{1}{4}+\frac{1}{4}$

$=\frac{1}{2}$.

① > ②

T10 $\sigma^2 = c^2\sigma_1^2 + (1-c)^2\sigma_2^2$

$$= (\sigma_1^2+\sigma_2^2)c^2 - 2\sigma_2^2 c + \sigma_2^2$$

$$c = \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2} \text{ 时}$$

T11 $E\left(\sum_{i=1}^{n-1}(X_{i+1}-X_i)^2\right)$

$$= E\left[\sum_{i=1}^{n}(X_i-\bar{X})^2 + \sum_{i=2}^{n-1}(X_i-\bar{X})^2 + 2\sum_{i=1}^{n-1}(X_i-\bar{X})(X_{i+1}-\bar{X})\right]$$

$$= 2(n-1)\sigma^2$$

$$\therefore c = \frac{1}{2(n-1)}$$

T12 $(x_1, x_2, \ldots x_n)$ 为 $(X_1, X_2 \cdots X_n)$ 一组观测值.

$$L(\sigma^2) = (2\pi\sigma^2)^{-\frac{n}{2}} \cdot e^{-\frac{1}{2\sigma^2}\cdot\sum_{i=1}^{n}(x_i-\mu)^2}$$

$$\ln L(\sigma^2) = -\frac{n}{2}(\ln 2\pi + \ln\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\frac{dL}{d\sigma^2} = 0 \qquad -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 = 0$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i-\mu)^2$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-\mu)^2$$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\mu)^2$$

$$S^2 > \sigma^2$$

T13 $u=\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

$$P\left(-u_{\alpha/2} < \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} < u_{\alpha/2}\right)=1-\alpha$$

区间为 $\left(\bar{X}-u_{\alpha/2}\dfrac{\sigma}{\sqrt{n}},\ \bar{X}+u_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}\right)$

$$\frac{2\sigma u_{\alpha/2}}{\sqrt{n}} < L$$

$$n > \frac{4\sigma^2 u_{\alpha/2}^2}{L^2}$$

T14 $\dfrac{\sqrt{n}(\bar{X}-\mu)}{S} \sim t(n-1)$

区间为 $\left(\bar{X}-t_{\alpha/2}(n-1)\dfrac{S}{\sqrt{n}},\ \bar{X}+t_{\alpha/2}(n-1)\dfrac{S}{\sqrt{n}}\right)$

$\bar{X}=6 \quad S^2=0.33$

区间为 $(5.558,\ 6.442)$

T15 $\dfrac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$

σ^2置信区间为 $\left(\dfrac{(n-1)S^2}{\chi^2_{\alpha/2}(n-1)},\ \dfrac{(n-1)S^2}{\chi^2_{1-\alpha/2}(n-1)}\right)$

即 $(6.99\times10^{-4},\ 6.63\times10^{-3})$

T16 区间为 $\left(\bar{X}-\bar{Y}-t_{\alpha/2}(n_1+n_2-2)S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}},\right.$

$\left.\bar{X}-\bar{Y}+t_{\alpha/2}(n_1+n_2-2)S_w\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right)$

$$S_w=\sqrt{\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}}$$

即为 $(-0.002, 0.006)$

T17 $\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t(n-1)$ $\quad p=\mu$

$P\left(-t_{\alpha/2}(n-1)<\frac{\bar{X}-\mu}{S/\sqrt{n}}<t_{\alpha/2}(n-1)\right)$

区间为 $\left(\bar{X}-t_{\alpha/2}(n-1)\cdot\frac{S}{\sqrt{n}}, \bar{X}+t_{\alpha/2}(n-1)\frac{S}{\sqrt{n}}\right)$

即为 $(0.203, 0.235)$

T18 $\frac{\bar{X}-\lambda}{S/\sqrt{n}} \sim N(0,1)$

$\left(-u_{\alpha/2}<\frac{\bar{X}-\lambda}{S/\sqrt{n}}<u_{\alpha/2}\right)$ $\quad S=\bar{X}$

区间为 $\left(\bar{X}-u_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}, \bar{X}+u_{\alpha/2}\sqrt{\frac{\bar{X}}{n}}\right)$