T_1 $EX = -2\times 0.3 + 0\times 0.1 + 2\times 0.4 + 3\times 0.2 = 0.8$

$EX^2 = 4\times 0.7 + 9\times 0.2 = 4.6$

$DX = EX^2 - (EX)^2 = 4.6 - 0.64 = 3.96$

T_2 $EX = \int_{-1}^{2} x\cdot p_x(x)\,dx = 0$

$EY = \int_{-1}^{2} x^2 p_x(x)\,dx = \frac{5}{6}$

$Cov(X,Y) = \int_{-1}^{2} (x-0)(x^2-\frac{5}{6})\,p_x(x)\,dx$

$= \int_{-1}^{0} \frac{1}{2}(x^3-\frac{5}{6}x)\,dx + \int_{0}^{2} \frac{1}{4}(x^3-\frac{5}{6}x)\,dx$

$= \frac{1}{8} + \frac{5}{24} + 1 - \frac{5}{12} = \frac{11}{12}$

T_3 $\sqrt{D(X)} = \frac{1}{\lambda}$

$P(X > \sqrt{D(X)}) = \int_{\frac{1}{\lambda}}^{+\infty} \lambda e^{-\lambda x}\,dx$

$= -e^{-\lambda x}\Big|_{\frac{1}{\lambda}}^{+\infty} = \frac{1}{e}$

T_4 ①

U	1	2
P	$\frac{1}{9}$	$\frac{8}{9}$

V	1	2
P	$\frac{8}{9}$	$\frac{1}{9}$

② $E(U) = \frac{1}{9} + 2\times\frac{8}{9} = \frac{17}{9}$

$E(V) = \frac{8}{9} + 2\times\frac{1}{9} = \frac{10}{9}$

③ $cov(u,v) = E\{[u-E(u)][v-E(v)]\}$

$= E(uv) - E(u)E(v)$

$v \backslash u$	1	2
1	$\frac{1}{9}$	$\frac{4}{9}$
2	0	$\frac{4}{9}$

$= 1\times\frac{1}{9} + 2\times\frac{4}{9} + 4\times\frac{4}{9} - \frac{170}{81}$

$= \frac{5}{81}$

T5 ① $E(X) = 0.6$ $E(X^2) = 0.6$ $D(X) = E(X^2) - [E(X)]^2 = 0.24$

$E(Y) = 0.2$ $E(Y^2) = 0.5$ $D(Y) = E(Y^2) - [E(Y)]^2 = 0.46$

$E(XY) = -1\times 0.08 + 1\times 0.2 = 0.12$

$\rho = \frac{cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = 0$

② $E(X^2Y^2) = 1\times 0.08 + 1\times 0.2 = 0.28$

$cov(X^2, Y^2) = E(X^2Y^2) - E(X^2)E(Y^2)$

$= 0.28 - 0.3 = -0.02$

T6 $p(x,y) = \begin{cases} 2, & (0<x<1,\ 0<y<1,\ y>x) \\ 0 & \text{else} \end{cases}$

$u = x+y$ $\quad x = x$

$1<u<2$ 时 $P_u(u) = \int_{u-1}^{\frac{u}{2}} p(x, u-x)\,dx$ $\quad y = 3-y$

$\quad 1 \quad 0$

$= 2-u$

else $P_u(u) = 0$

T_7 $E(3X+Y)^2 = 9E(X^2) + E(Y^2) + 6E(XY)$

$= 20 + [\rho\sqrt{D(X)}\sqrt{D(Y)} + E(X)E(Y)] \times 6$

$= 20 + 6$

$= 26$

T_8 ①

$Y \backslash X$	0	1
0	$\frac{2}{3}$	$\frac{1}{6}$
1	$\frac{1}{12}$	$\frac{1}{12}$

② $\rho_{XY} = \dfrac{\mathrm{cov}(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \dfrac{E(XY)-E(X)E(Y)}{\sqrt{E(X^2)-[E(X)]^2}\sqrt{E(Y^2)-[E(Y)]^2}} = \dfrac{\frac{1}{12}-\frac{1}{4}\times\frac{1}{6}}{\sqrt{\frac{1}{4}-\frac{1}{16}}\sqrt{\frac{1}{6}-\frac{1}{36}}}$

$= \dfrac{\sqrt{15}}{15}$

T_9 ① $\rho_{XY} = \dfrac{\mathrm{cov}(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \dfrac{\frac{1}{4}E((X_1+X_2)(Y_1+Y_2)) - \frac{1}{4}E(X_1+X_2)P(Y_1+Y_2)}{\sqrt{\frac{1}{2}[D(X_1)+D(X_2)]}\sqrt{\frac{1}{2}[D(Y_1)+D(Y_2)]}} = 0$

② $P(X,Y) \neq P_X(X)P_Y(Y)$

T_{10} ① $\mathrm{cov}(X,Y) = E(XY) - E(X)E(Y)$

$= \frac{1}{4}\left[\int_{-2}^{-1} dx + \int_{-1}^{1}(-1)\cdot dx + \int_{1}^{2} dx\right]$

$= \frac{1}{4}$

$$X+Y = \begin{cases} -2 & -2\le u < -1 \\ 0 & -1 < u \le 1 \\ 2 & 1 < u \le 2 \end{cases}$$

② $D(X+Y) = E[(X+Y)^2] - [E(X+Y)]^2$

$= 2$

T11 ① $D(Y_i) = D(X_i - \bar{X}) = D\left(\frac{n-1}{n} X_i - \frac{1}{n}\sum_{j=1}^{n} X_j - \frac{1}{n} X_i\right)$

$= \frac{(n-1)^2}{n^2} + \frac{n-1}{n^2}$

$= \frac{n-1}{n}$

② $cov(Y_1, Y_n) = E(Y_1 Y_n) - \cancel{E(Y_1)E(Y_n)}$ 记 $X' = \frac{1}{n}\sum_{i=2}^{n-1} X_i$

$= E\left[\left(X' + \frac{n-1}{n}X_1 + \frac{1}{n}X_n\right)\left(X' + \frac{n-1}{n}X_n + \frac{1}{n}X_1\right)\right]$

$= E(X'^2) + \frac{n-1}{n^2}\left[E(X_1^2) + E(X_n^2)\right]$

$= \frac{n-2}{n^2} + \frac{2n-2}{n^2} = \frac{3n-4}{n^2}$

③ $P(Y_1 + Y_n \le 0) = \frac{1}{2}$

T12 $\rho_{XY} = \frac{cov(X,Y)}{\sqrt{D(X)}\sqrt{D(Y)}} = \frac{E(XY) - E(X)E(Y)}{D(X)} = \frac{E(XY) - (E(X))^2}{E(X^2) - (E(X))^2}$

$E(X^2) = \frac{1}{2^n}\sum_{i=1}^{n} i^2 C_n^i = \frac{1}{2^n} n(n+1) 2^{n-2} = \frac{1}{4} n(n-1)$

$E(XY) = \frac{1}{2^n}\sum_{i=1}^{n} i(n-i) C_n^i = \frac{n}{2^n}\cdot n \cdot 2^{n-1} - E(X^2) = \frac{1}{2}n^2 - \frac{1}{4}(n^2 - n)$

$= \frac{1}{4}(n^2 + n)$

$E(X) = \frac{1}{2^n}\sum_{i=1}^{n} i C_n^i = \frac{1}{2^n}\cdot n \cdot 2^{n-1} = \frac{n}{2}$

$\rho_{XY} = \frac{\frac{1}{4}n^2 + \frac{1}{4}n - \frac{n^2}{4}}{\frac{1}{4}n^2 - \frac{1}{4}n - \frac{n^2}{4}} = -1$

T_{13} ① $D(Y) = D(X_1) + 4D(X_2) + 9D(X_3)$

$= \frac{1}{12} \cdot 6^2 + 4 \times 4 + 9 \times 5$

$= 3 + 16 + 45$

$= 64$

② $\rho_{X_2 Y} = \frac{cov(X_2, Y)}{\sqrt{D(X_2)}\sqrt{D(Y)}} = \frac{E(X_2Y) - E(X_2)E(Y)}{2 \times 8} = \frac{-2 \cdot E(X_2^2) + E(X_1X_2) + 3E(X_2X_3)}{16}$

$= \frac{-8}{16} = -\frac{1}{2}$

T_{14} 令 $Y_i = X_i - \bar{X}$

$S^2 = \frac{n}{n-1} D(Y_i) \qquad E(S^2) = \frac{n}{n-1} E(D(Y_i))$

$D(Y_i) = D\left(\frac{n-1}{n} X_i + \frac{1}{n}\sum_{j=1}^{n} X_j - X_i\right)$

$= \frac{(n-1)^2}{n^2} D(X_i) + \frac{1}{n^2}\sum_{j=1}^{n} D(X_j)$

$D(X_i) = E(X_i^2) - [E(X_i)]^2$

$= \int_0^{+\infty} x^2 e^{-x} dx = -(x^2+2x+2)e^{-x}\Big|_0^{+\infty} = 2$

$D(Y_i) = \frac{(n-1)^2}{n^2} 2 + \frac{2(n-1)}{n}$

$E(S^2) = \frac{n}{n-1} \cdot \frac{2(n-1)}{n} = 2$

T_{15} 设进货量为 t

$$W = \begin{cases} 500X - 100(t-X) & t > X \\ 500t + 300(X-t) & t < X \end{cases}$$

$$W = \begin{cases} 600X - 100t & X < t \\ 300X + 200t & X \geq t \end{cases}$$

$t > 10$

$$E(W) = \frac{1}{21}\left[\sum_{i=10}^{t-1}(600i - 100t) + \sum_{j=t}^{30}(300j + 200t)\right]$$

$$= \frac{1}{21}\left[-100t(t-10) + 300\times(t+9)(t-10) + 150\cdot(t+30)(31-t) + 200t\cdot(31-t)\right]$$

$$= \frac{1}{21}\left(-100t^2 + 300t^2 - 150t^2 - 200t^2 + 1000t - 300t + 150t + 6200t - 27000 + 139500\right)$$

$$= \frac{1}{21}\left(-150t^2 + 7050t + 112500\right)$$

t 取 21

T16. $E(XY) = \frac{1}{\pi}\int_{-1}^{1}dx\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} xy\,dy$

$$= \frac{1}{\pi}\int_{-1}^{1}\left(\frac{1}{2}xy^2\right)\Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}dx$$

$$= 0$$

$E(X) = E(Y) = 0$

$\therefore \operatorname{cov}(X, Y) = E(XY) - E(X)E(Y) = 0$

$\therefore X, Y$ 不相关.

$p_X(x) = \int_{-\infty}^{+\infty} p(x,y)\,dy$　　同理. $p_Y(y) = \frac{2}{\pi}\sqrt{1-y^2}$

$= \frac{1}{\pi}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy$　　$p(x,y) \neq p_X(x)p_Y(y)$

$= \frac{2}{\pi}\sqrt{1-x^2}$

T17 交换对称 $E\left(\frac{X_i}{\sum_{j=1}^{n} X_j}\right) = E\left(\frac{X_k}{\sum_{j=1}^{n} X_j}\right)$　$i \neq k$

$\sum_{i=1}^{n} E\left(\frac{X_i}{\sum_{j=1}^{n} X_j}\right) = 1$　$\therefore E\left(\frac{X_i}{\sum_{j=1}^{n} X_j}\right) = \frac{1}{n}$　即可得 $E\left(\frac{X_1 + \cdots + X_k}{X_1 + X_2 + \cdots + X_n}\right) = \frac{k}{n}$

T18 ① $\sum_{k=1}^{+\infty} P(X \geq k) = \sum_{k=1}^{+\infty}\left[\sum_{j=k}^{+\infty} P_x(j)\right]$

$= \sum_{k=1}^{+\infty} P_x(k) \cdot \sum_{j=1}^{k} 1$

$= E(X)$

$\sum_{k=1}^{+\infty} (2k-1) \cdot P(X \geq k) = \sum_{k=1}^{+\infty} (2k-1) \sum_{j=k}^{+\infty} P_x(j)$

$= \sum_{k=1}^{+\infty} k^2 P_x(k)$

$= E(X^2)$

② $\int_0^{+\infty} P(X > x)\,dx$

$= \int_0^{+\infty} \int_x^{+\infty} p(y)\,dy\,dx$

$= \int_0^{+\infty} p(y)\,dy \int_0^{y} dx$

$= \int_0^{+\infty} y\,p(y)\,dy$

$= E(X)$

$2\int_0^{+\infty} x\,P(X > x)\,dx$

$= \int_0^{+\infty} 2x\,dx \int_x^{+\infty} p(y)\,dy$

$= \int_0^{+\infty} p(y)\,dy \int_0^{y} 2x\,dx$

$= \int_0^{+\infty} y^2\,p(y)\,dy$

$= E(x^2)$

T19 $E(X+Y)^2$

$= E(X^2) + E(Y^2) + 2E(XY)$

$E(XY) = E(X)E(Y)$

$= 2$

$E(X^2) = \sum_{k=1}^{+\infty} k^2 \cdot \frac{\lambda^k}{k!} e^{-\lambda}$

$= \sum_{k=1}^{+\infty} [k + k(k-1)] \frac{\lambda^k}{k!} e^{-\lambda}$

$= \lambda \sum_{k=1}^{+\infty} \frac{\lambda^{k-1}}{(k-1)!} e^{-\lambda} + \lambda^2 \sum_{k=1}^{+\infty} \frac{\lambda^{k-2}}{(k-2)!} e^{-\lambda}$

$= \lambda + \lambda^2$

$\therefore E(X+Y)^2 = 12$

$E(X) = \lambda$

$E(X^2) = 2$

$E(Y^2) = 6$

T20 $E(X) = \int_0^2 2\cdot 3\cdot e^{-3x}\,dx + \int_2^{+\infty} x\cdot 3\cdot e^{-3x}\,dx$

$= 6\int_0^2 e^{-3x}\,dx + \frac{1}{3}\int_6^{+\infty} 3x e^{-3x}\,d(3x)$

$= 2\cdot(1-e^{-3x})\Big|_0^2 + \left[-(y+1)e^{-y}\right]\Big|_6^{+\infty}$

$= 2(1-e^{-6}) + 7\cdot e^{-6}$

$= 2 + 5\cdot e^{-6}$

T21 $x=1$ 点有跳跃点.

$E(X) = \int_1^2 x(x-1)\,dx + \frac{1}{2} = \left(\frac{1}{3}x^3 - \frac{1}{2}x^2\right)\Big|_1^2 + \frac{1}{2} = \frac{7}{3} - 1 = \frac{4}{3}$

$E(X^2) = \int_1^2 x^2(x-1)\,dx + \frac{1}{2} = \left(\frac{1}{4}x^4 - \frac{1}{3}x^3\right)\Big|_1^2 + \frac{1}{2} = \frac{15}{4} - \frac{7}{3} + \frac{1}{2} = \frac{23}{12}$

$D(X) = E(X^2) - (E(X))^2 = \frac{23}{12} - \frac{16}{9} = \frac{5}{36}$

T22 $E = \int_1^{10} y\cdot\frac{2}{y^3}\,dy + 10\int_{10}^{+\infty}\frac{2}{y^3}\,dy$

$= 1 - \frac{2}{y})\Big|_1^{10} - 10\,\frac{1}{y^2}\Big|_{10}^{+\infty}$

$= 2 - \frac{1}{5} + \frac{1}{10}$

$= \frac{19}{10}$

T23 记 X 密度函数为 $p(x)$，不妨设 $x<0$ 时 $p(x)=0$

$P(|X| \geq x_0) = \int_{x_0}^{+\infty} p(x)\,dx$

$\frac{1}{f(x_0)}E[f(|X|)] = \int_0^{+\infty}\frac{f(x)}{f(x_0)}p(x)\,dx$

$= \int_0^{x_0}\frac{f(x)}{f(x_0)}p(x)\,dx + \int_{x_0}^{+\infty}\frac{f(x)}{f(x_0)}p(x)\,dx$

$\because f(x)$ 单调增且不为负.

$\therefore \frac{f(x)}{f(x_0)} > 0$

$\int_0^{x_0}\frac{f(x)}{f(x_0)}p(x)\,dx > 0$

$\therefore \frac{1}{f(x_0)}E[f(|X|)] \geq P(|X| \geq x_0)$