

T1

(1)

Jacobi

$$B = \begin{pmatrix} 1/5 & 0 & 0 \\ 0 & 1/4 & 0 \\ 0 & 0 & 1/10 \end{pmatrix} \cdot \begin{pmatrix} 0 & -2 & -1 \\ 1 & 0 & -2 \\ -2 & 3 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -0.4 & -0.2 \\ 0.25 & 0 & -0.5 \\ -0.2 & 0.3 & 0 \end{pmatrix}$$

$$\|B\|_{\infty} = 0.75 < 1$$

收斂

G-S

$$G = \begin{pmatrix} 5 & 0 & 0 \\ 1 & 4 & 0 \\ -2 & 3 & 10 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 0 & -2 & -1 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -0.4 & -0.2 \\ 0 & -0.1 & -0.55 \\ 0 & 0.05 & -0.125 \end{pmatrix}$$

$$\|G\|_{\infty} = 0.65 < 1$$

收斂

(2)

Jacobi

$$X^{(k+1)} = BX^{(k)} + f_1$$

$$f_1 = D^{-1}b = (-2.4 \quad 5 \quad 0.3)^T$$

$$X^{(15)} = (-3.9990 \quad 3.0002 \quad 1.9999)$$

$$X^{(16)} = (-4.0000 \quad 3.0001 \quad 2.0000)$$

$$\|X^{(16)} - X^{(15)}\|_{\infty} \leq 10^{-4}$$

$$X = (-4.0000 \quad 3.0001 \quad 2.0000)$$

G-S

$$X^{(k+1)} = BX^{(k)} + f_2$$

$$f_2 = (D - L)^{-1}b$$

$$X^{(7)} = \begin{pmatrix} -4.0000 & 3.0000 & 2.0000 \end{pmatrix}$$

$$X^{(8)} = \begin{pmatrix} -4.0000 & 3.0000 & 2.0000 \end{pmatrix}$$

$$\|X^{(8)} - X^{(7)}\|_{\infty} \leq 10^{-4}$$

$$X = \begin{pmatrix} -4.0000 & 3.0000 & 2.0000 \end{pmatrix}$$

T5

(a)

$$B = \begin{pmatrix} 0 & 0.4 & 0.4 \\ 0.4 & 0 & 0.8 \\ 0.4 & 0.8 & 0 \end{pmatrix}$$

$$-\lambda^3 + 4 * 0.4^3 + 6 * 0.4^2 * \lambda = 0$$

$$\rho(B) = \lambda_{MAX} = \frac{2}{5}(1 + \sqrt{3}) = 1.0928 > 1 \text{ 不收敛}$$

A 对称正定 收敛

(b)

$$B = \begin{pmatrix} 0 & 2 & -2 \\ 1 & 0 & 1 \\ 2 & 2 & 0 \end{pmatrix}$$

$$\lambda^3 = 0$$

$$\rho(B) = 0 < 1 \text{ 收敛}$$

$$G = \begin{pmatrix} 0 & 2 & -2 \\ 0 & 2 & -1 \\ 0 & 8 & -6 \end{pmatrix}$$

$$\lambda(\lambda+6)(2-\lambda)-8\lambda=0$$

$$\lambda^3+4\lambda^2-12\lambda=0$$

$$\rho(G)=2<1 \text{ 不收敛}$$

$$\mathbf{T9}$$

$$\begin{cases} X_1^{(k+1)} = & X_1^{(k)} + \frac{\omega}{4}(1-4X_1^{(k)}+X_2^{(k)}) \\ X_2^{(k+1)} = & X_2^{(k)} + \frac{\omega}{4}(4+X_1^{(k+1)}-4X_2^{(k)}+X_3^{(k)}) \\ X_3^{(k+1)} = & X_1^{(k)} + \frac{\omega}{4}(-3+X_2^{(k+1)}-4X_3^{(k)}) \end{cases} \tag{1}$$

$$\omega=1.03\rightarrow n=5$$

$$\omega=1\rightarrow n=6$$

$$\omega=1.1\rightarrow n=6$$

$$\mathbf{T14}$$

$$\text{(a)}$$

$$\det(\begin{vmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{vmatrix})=1-2a^3-3a^2=-(2a-1)(a+1)^2$$

$$-\frac{1}{2}< a < 1$$

$$\det |A|>0 \quad \mathbf{A正定}$$

$$\text{(b)}$$

$$B=\begin{pmatrix} 0 & a & a \\ a & 0 & a \\ a & a & 0 \end{pmatrix}$$

$$|\lambda I-B|=0$$

$$-\lambda^3+2a^3+3a^2\lambda=0$$

$$(\lambda+a)^2(\lambda-2a)=0$$

$$\rho(B) = 2|a| \quad -\frac{1}{2} < a < \frac{1}{2} \text{ 时收敛}$$

