

T₁₃ 原问题等价于在一条线段上选两点, 两点位于
线段 $(\frac{1}{4}, \frac{1}{2}]$ 处和 $(\frac{1}{2}, \frac{3}{4})$ 处, 记为事件 A

$$P(A) = \frac{1}{4}$$

$$T_{14} \quad P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(B) - P(B-A)}{P(A \cup B) - P(B-A)} = \frac{P(B) - P(B-A)}{P(A \cup B) - P(B-A)} = \frac{4}{25}$$

$$T_{15} \quad P(\overline{A \cap B}) = P(\overline{A \cup B}) = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(AB))$$

$$P(B) = \frac{P(A)P(B|A)}{P(A|B)} = \frac{\frac{1}{4} \cdot \frac{1}{5}}{\frac{1}{2}} = \frac{1}{6} \quad P(AB) = P(A) \cdot P(B|A) = \frac{1}{12}$$

代入. $P(\overline{A \cap B}) = 1 - (\frac{1}{4} + \frac{1}{6} - \frac{1}{12}) = \frac{2}{3}$

T₁₆ 记至少一个点为事件 A, 两个点都为点为事件 B

$$P(B|A) = \frac{P(AB)}{P(A)} = \frac{P(B)}{P(A)} = \frac{\frac{C_2^2}{C_2^5}}{1 - \frac{C_2^3}{C_2^5}} = \frac{\frac{1}{10}}{\frac{3}{5}} = \frac{1}{6}$$

$$T_{17} \quad ① P(A|B) = \frac{P(AB)}{P(B)} = \frac{P(A) + P(B) - P(A \cup B)}{P(B)} \geq \frac{P(A) + P(B) - 1}{P(B)} = \frac{a+b-1}{b}$$

$$② P(A|B) + P(\overline{A}|B) = \frac{P(AB)}{P(B)} + \frac{P(\overline{A} \cap B)}{P(B)} = \frac{P(AB)}{P(B)} + \frac{1 + P(AB)}{P(B)} - \frac{P(A) + P(B)}{P(B)}$$

$$= \frac{P(B)(1 - P(B)) + P(AB) - P(A)P(B)}{P(B)(1 - P(B))} = 1 \quad \therefore P(AB) = P(A)P(B)$$

即 A 与 B 独立.

$$T_{18} \quad ① P_{1/2} = \frac{25}{100} \times \frac{5}{100} + \frac{31}{100} \times \frac{4}{100} + \frac{40}{100} \times \frac{2}{100} = 3.45\% = 0$$

$$② P_{1/2} = \frac{P(A) \cdot P(A|A)}{P(A)} = \frac{\frac{25}{100} \times \frac{5}{100}}{\frac{345}{1000}} = \frac{25}{69} = 0.362$$

