Homework 3 Solutions

1.

```
template <class Item>
Item dc-max(vector<Item> v, int lo, int hi)
{
    if (lo == hi)
        return v[lo];

    int mid = lo + (hi - lo) / 2;
    ml = dc-max(v, lo, mid);
    mr = dc-max(v, mid+1, hi);
    return (ml > mr ? : ml : mr);
}
```

Let $C(n)$ be the worst-case number of array element comparisons.

$$
\begin{aligned}
C(1) &= 0 \\
C(n) &= 2C\left(\frac{n}{2}\right) + 1, \ n = 2, 4, 8, ...
\end{aligned}
$$

Applying the master theorem to $a = 2$, $b = 2$, $d = 0$, we have $C(n) \in \Theta(n^{\log_2 2}) = \Theta(n)$.

2.

```
template <class Item>
Item dc-majority(vector<Item> v, int lo, int hi)
{
    if (lo == hi)
        return v[lo];

    int mid = lo + (hi - lo)/2;
    ml = dc-majority(v, lo, mid);
    mr = dc-majority(v, mid+1, hi);

    int cl(0), cr(0);
    if (ml != NULL_ITEM)
        for (auto e: v)
            if (e == ml)
                ++cl;
    if (mr != NULL_ITEM)
        for (auto e: v)
            if (e == mr)
                ++cr;
    if (cl > (hi - lo + 1)/2)
        return ml;
    if (cr > (hi - lo + 1)/2)
        return mr;
    return NULL_ITEM;
}
```

Let $C(n)$ be the worst-case number of array element comparisons.

$$\begin{aligned}
C(1) &= 0 \\
C(n) &= 2C\left(\frac{n}{2}\right) + \sum_{\text{lo} \leqslant i \leqslant \text{hi}} \quad (2) \\
&= 2C\left(\frac{n}{2}\right) + 2n, \quad n = 2, 4, 8, \ldots
\end{aligned}$$

Applying the master theorem to $a = 2, b = 2, d = 1$, we have $C(n) \in \Theta(n^1 \lg n) = \Theta(n \lg n)$.