HW0 Solutions

1.

$$\begin{aligned} M(n) &= \sum_{1\leqslant i\leqslant n}\sum_{0\leqslant j\leqslant i-1}\sum_{4\leqslant k\leqslant 9}(2) \\ &= 2\sum_{1\leqslant i\leqslant n}\sum_{0\leqslant j\leqslant i-1}\sum_{4\leqslant k\leqslant 9}(1) \\ &= 2\sum_{1\leqslant i\leqslant n}\sum_{0\leqslant j\leqslant i-1}(9-4+1) \\ &= 12\sum_{1\leqslant i\leqslant n}\sum_{0\leqslant j\leqslant i-1}(1) \\ &= 12\sum_{1\leqslant i\leqslant n}(i-1-0+1) \\ &= 12\sum_{1\leqslant i\leqslant n}(i) \\ &= 6n(n+1). \end{aligned}$$

2. $n \in o(n \lg n)$ because

$$\begin{aligned} \lim_{n\to\infty}\frac{n}{n\lg n} &= \lim_{n\to\infty}\frac{1}{\lg n} \\ &= 0. \end{aligned}$$

$n \lg n \in o(n^2)$ because

$$\begin{aligned} \lim_{n\to\infty}\frac{n\lg n}{n^2} &= \lim_{n\to\infty}\frac{\lg n}{n} \\ &= \lim_{n\to\infty}\frac{\frac{1}{(\ln 2)\, n}}{1} \\ &= \lim_{n\to\infty}\frac{1}{(\ln 2)n} \\ &= 0. \end{aligned}$$

3. $\lg(\sqrt{n})$ grows faster because

$$\begin{aligned} \lim_{n\to\infty}\frac{\lg(\sqrt{n})}{\sqrt{\lg n}} &= \lim_{n\to\infty}\frac{\frac{1}{2}\lg n}{(\lg n)^{\frac{1}{2}}} \\ &= \frac{1}{2}\lim_{n\to\infty}(\lg n)^{\frac{1}{2}} \\ &= \infty. \end{aligned}$$