FINAL EXAM
CSCI 163/COEN 179: THEORY OF ALGORITHMS
SPRING 2009

1. (5 points) Given a sorted array `A[1..n]` of distinct integers, write an $O(\log n)$ algorithm to determine whether `A[i] == i` for some `i`.

2. (5 points) Write an $O(|V| + |E|)$ algorithm `bool is_single_cycle(V, E)` to determine whether the input graph $G = (V, E)$ consists of a single simple cycle.

3. (5 points) Write an algorithm `int Min(int H[1..n])` to return the smallest element in a max-heap `H` of size `n`. What is the asymptotic running time of your algorithm ?

4. (5 points) Write an $O(\log n)$ algorithm `unsigned powerof3(unsigned n)` to compute 3^n.

5. (5 points) Write an $O(\log a + \log b)$ algorithm `unsigned lcm(unsigned a, unsigned b)` to compute the least common multiple of two positive integers `a` and `b`.

6. (5 points) Give an $O(n^2)$ algorithm to find the transitive closure of an undirected graph.

7. (5 points) Write an algorithm to find the *maximum-weight* spanning tree of an input graph $G = (V, E, W)$.

8. (5 points) Write an $O(mn)$ algorithm to determine whether it is possible to make change for a value n using denominations $d[1..m]$ *so that each denomination is used at most once.*

9. (5 points) Find a Huffman code for the alphabet {`A, C, G, T`} whose probabilities are 0.35, 0.2, 0.05 and 0.4 respectively.

10. (5 points) Show that the LONGEST PATH problem is in NP:

    - INPUT: an undirected graph G and a positive integer L;
    - OUTPUT: yes if and only if G has a simple path of length L.