CS3243 INTRODUCTION TO ARTIFICIAL INTELLIGENCE

ADVERSARIAL SEARCH

Adapted from Nicholas Teh

CS3243 INTRODUCTION TO ARTIFICIAL INTELLIGENCE

CONTENT SUMMARY

nicholas.teh@u.nus.edu

KEY CONCEPTS

- **Adversarial Search**
  - Tracing tree with Alpha-Beta pruning algorithm
    - Detect nodes that are pruned/not evaluated
    - Knowing values/range of values for which the nodes will/will not be pruned
    - Knowing what happens when some values change (could solve by brute force) **Key is to be fast!**
  - The Minimax algorithm

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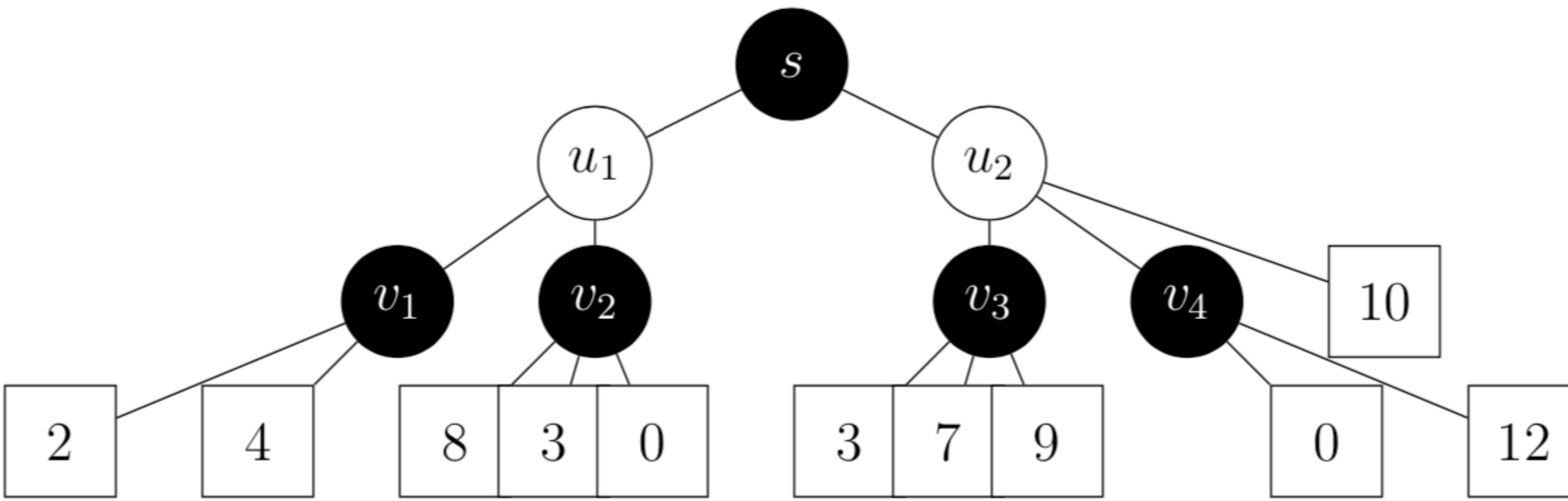
ADVERSARIAL SEARCH

MASTERING ADVERSARIAL SEARCH GRAPHS

- α: worst case (lower bound) for MAX $\alpha \geq \#$
  β: worst case (upper bound) for MIN $\beta \leq \#$
- Have α for every MAX node, start with $-\infty$
  Have β for every MIN node, start with ∞
- When propagating upwards (or deep-compare),
  PRUNING: If [child] $\alpha \geq \beta$ [parent], then prune child's remaining
  PRUNING: If [child] $\beta \leq \alpha$ [parent], then prune child's remaining
  If prune, value don't copy upwards
- Compare all the way up for conflict if deep tree

TUTORIAL 5 QUESTION 1

- Assume that we iterate over nodes from **right to left**; mark with an 'X' all ARCS that are pruned by α-β pruning, if any.



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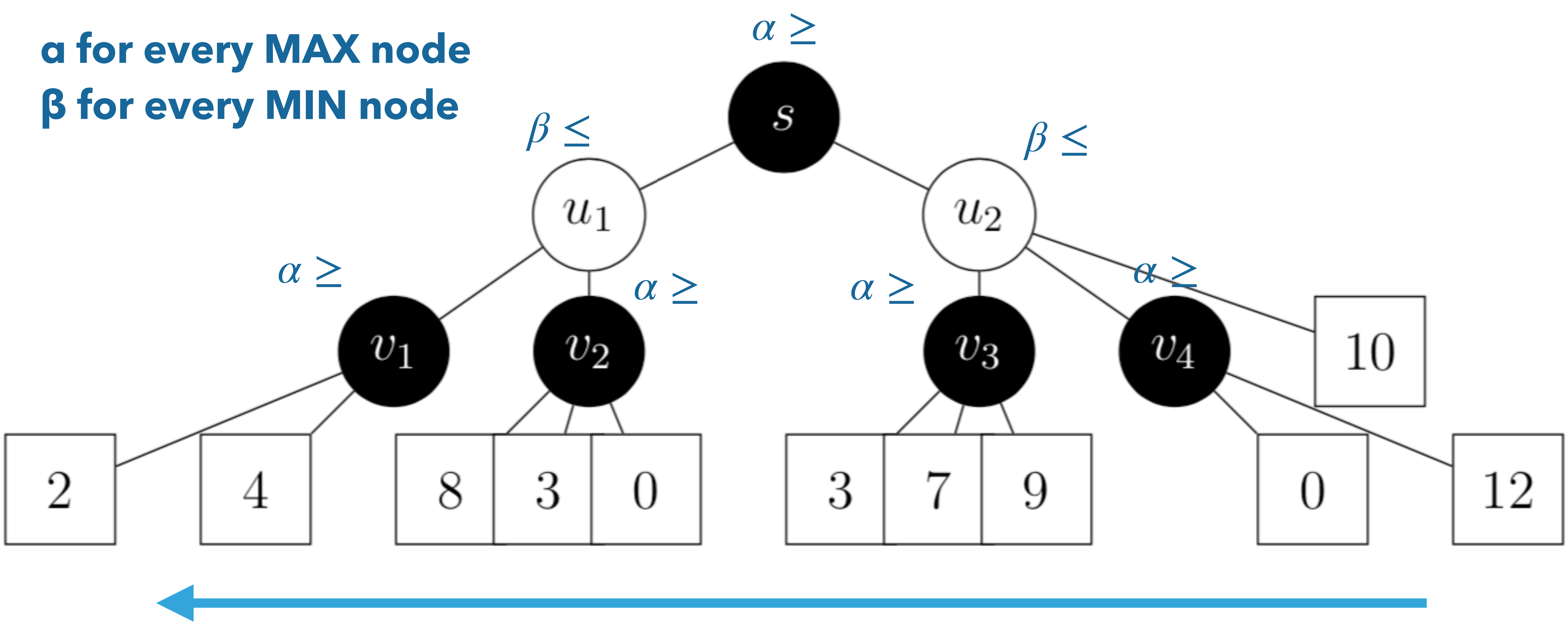
α for every MAX node
β for every MIN node



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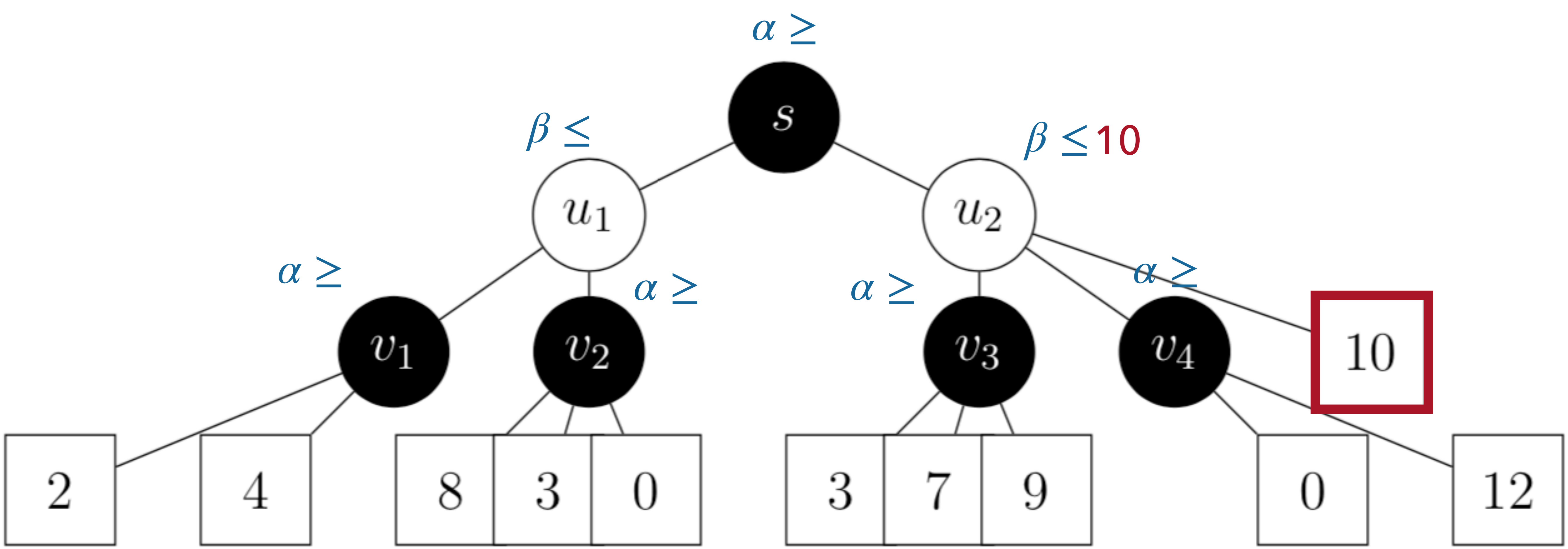

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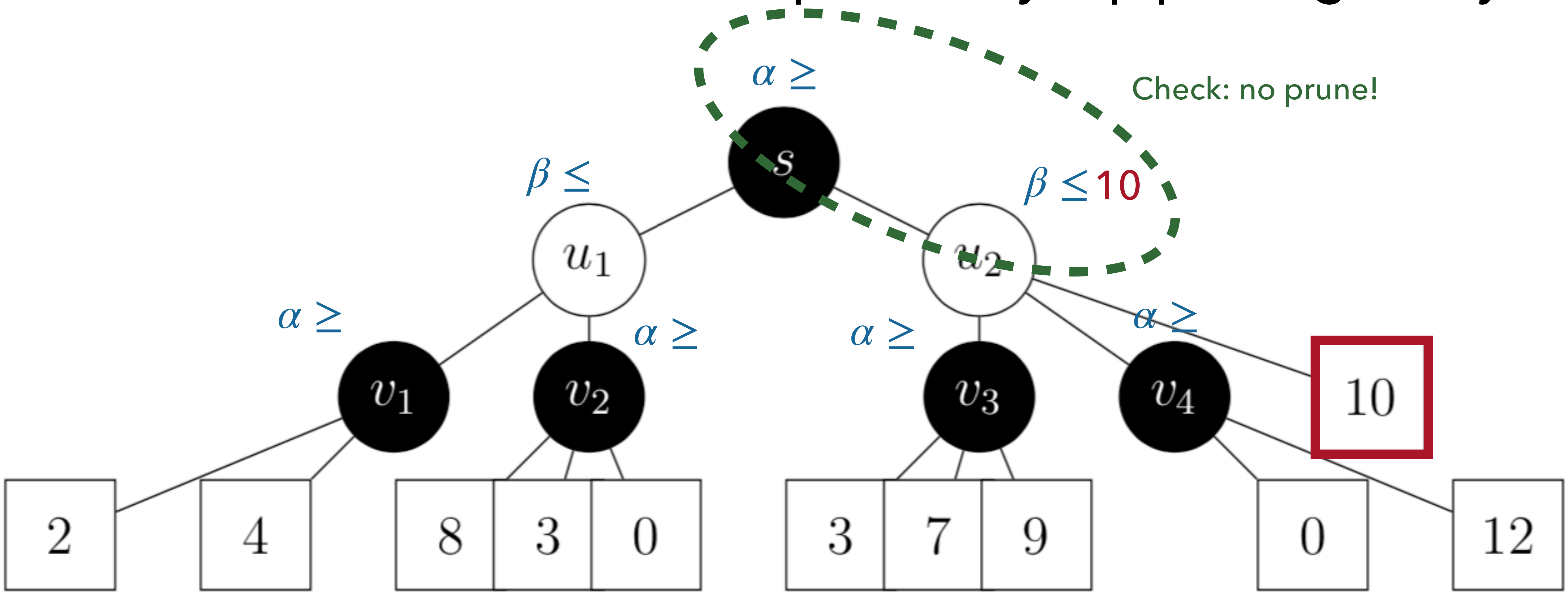

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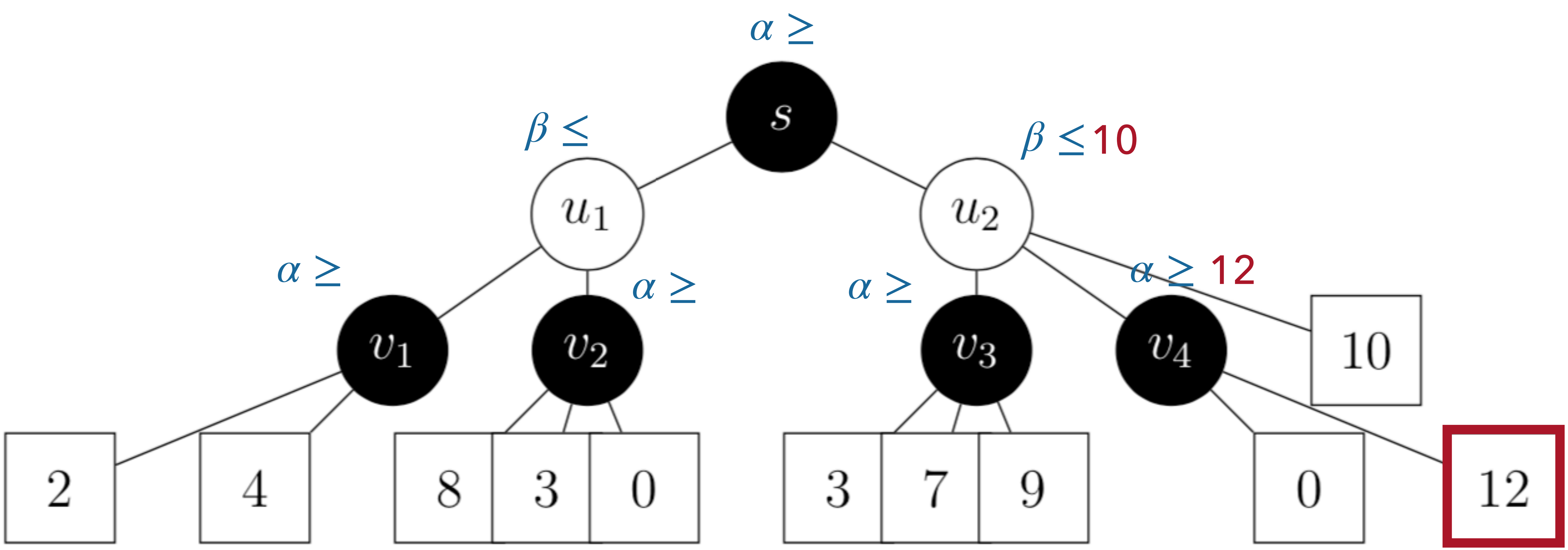

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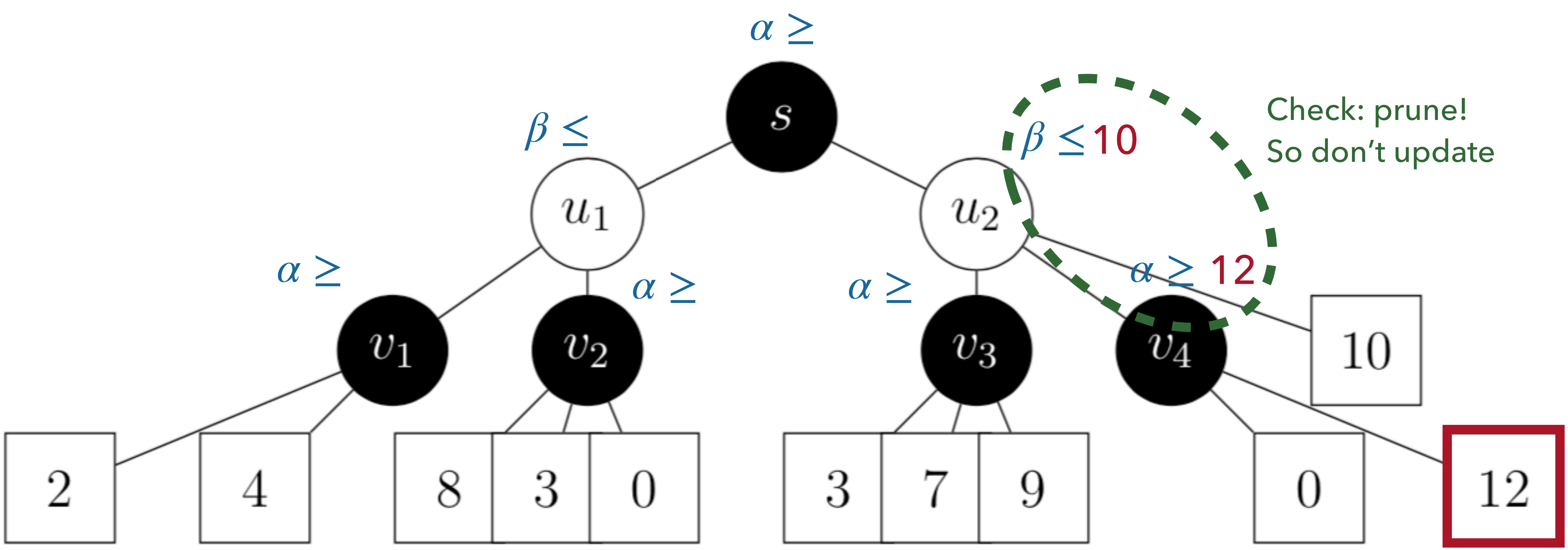

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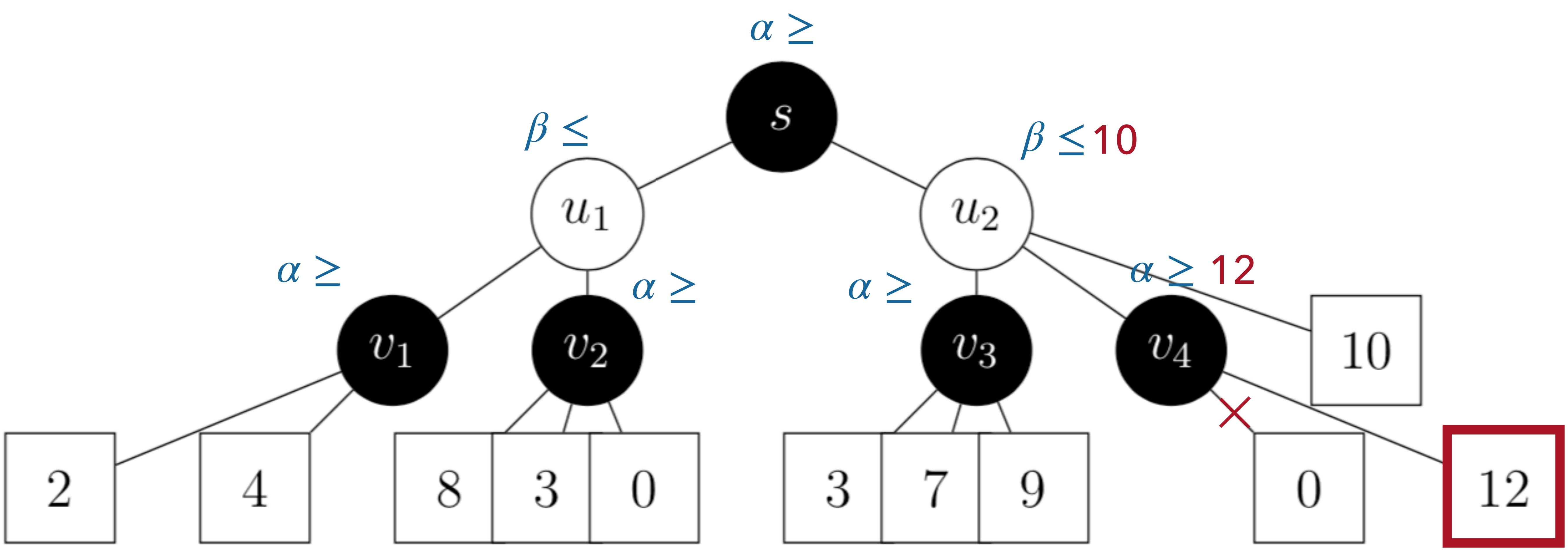

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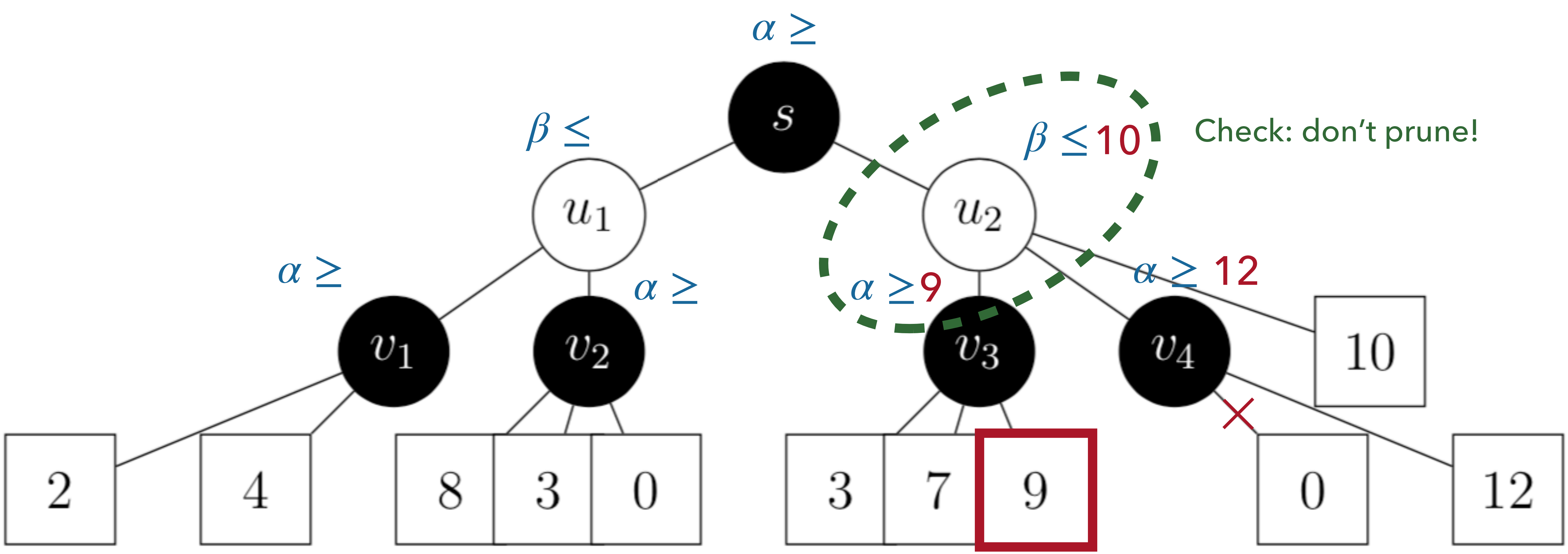

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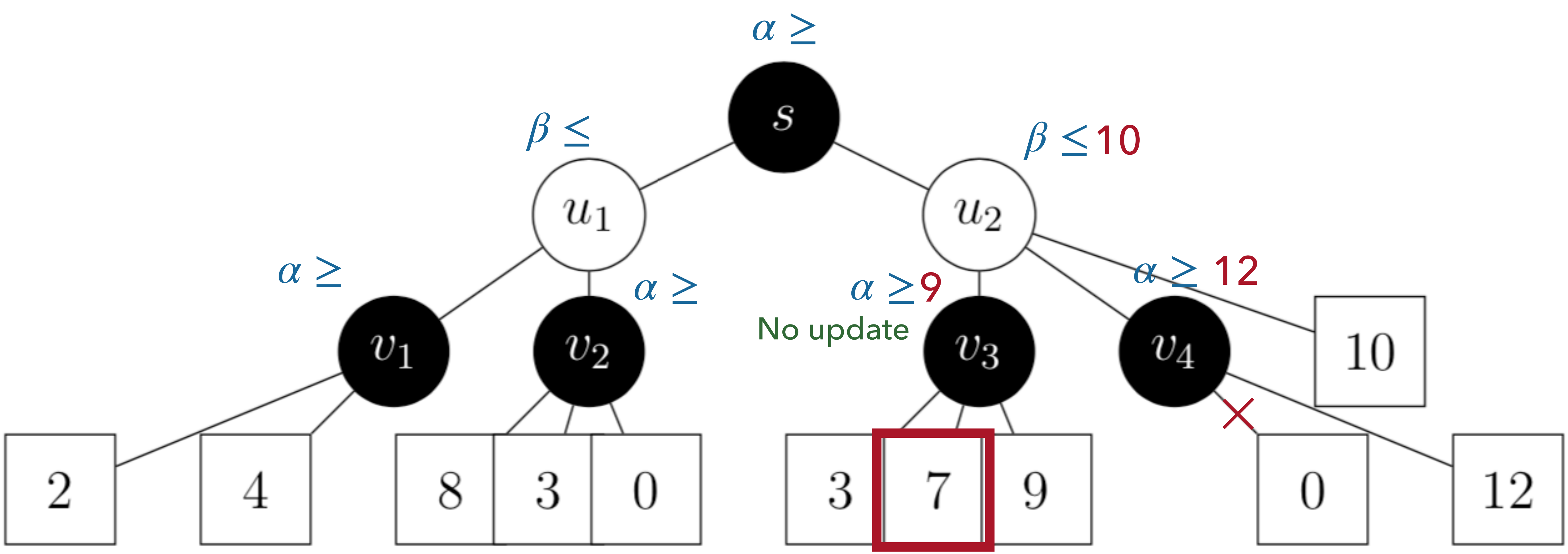

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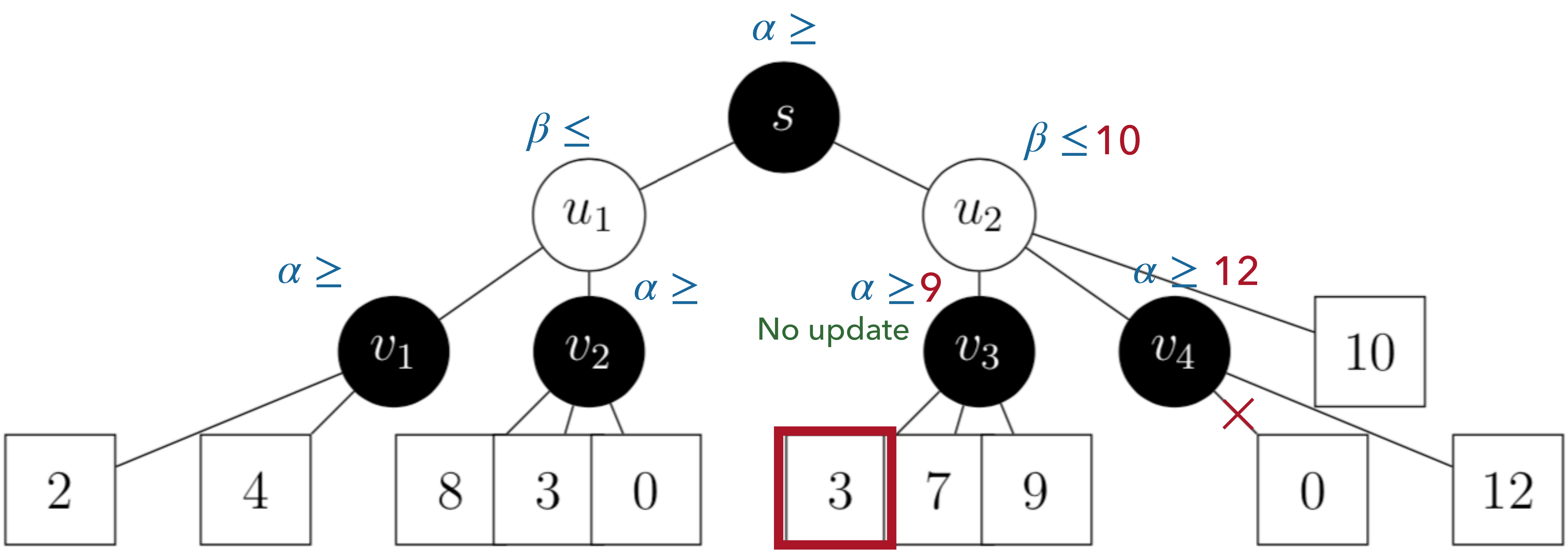

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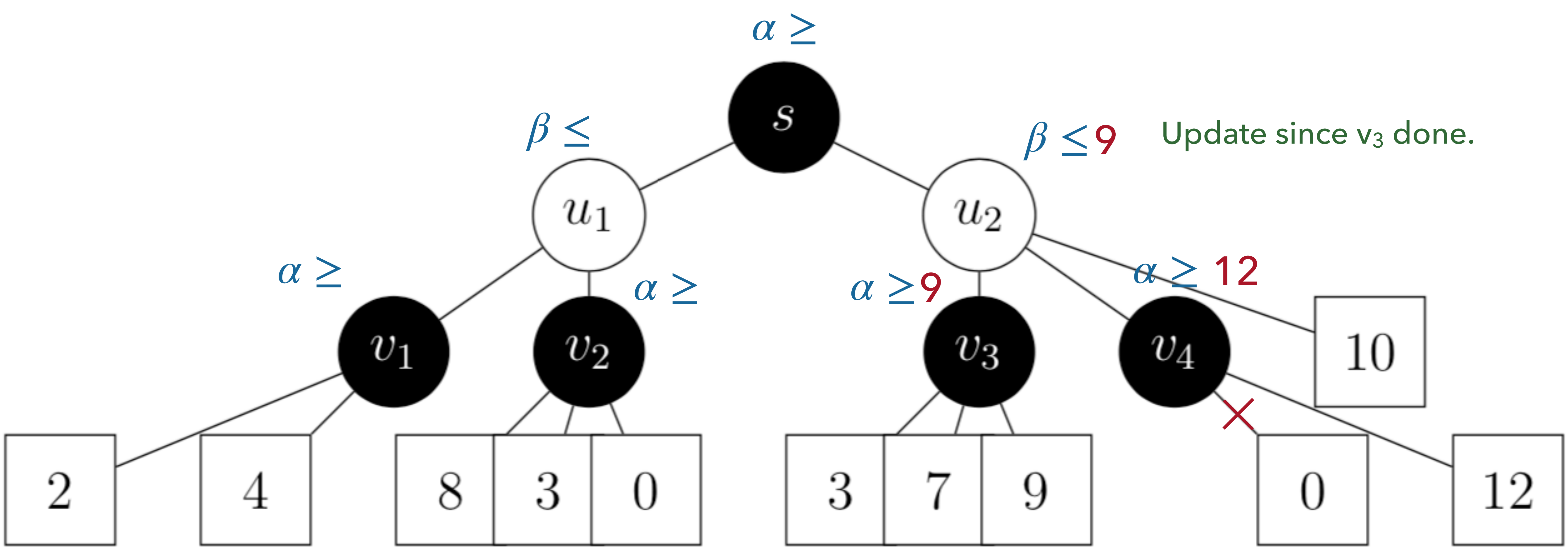

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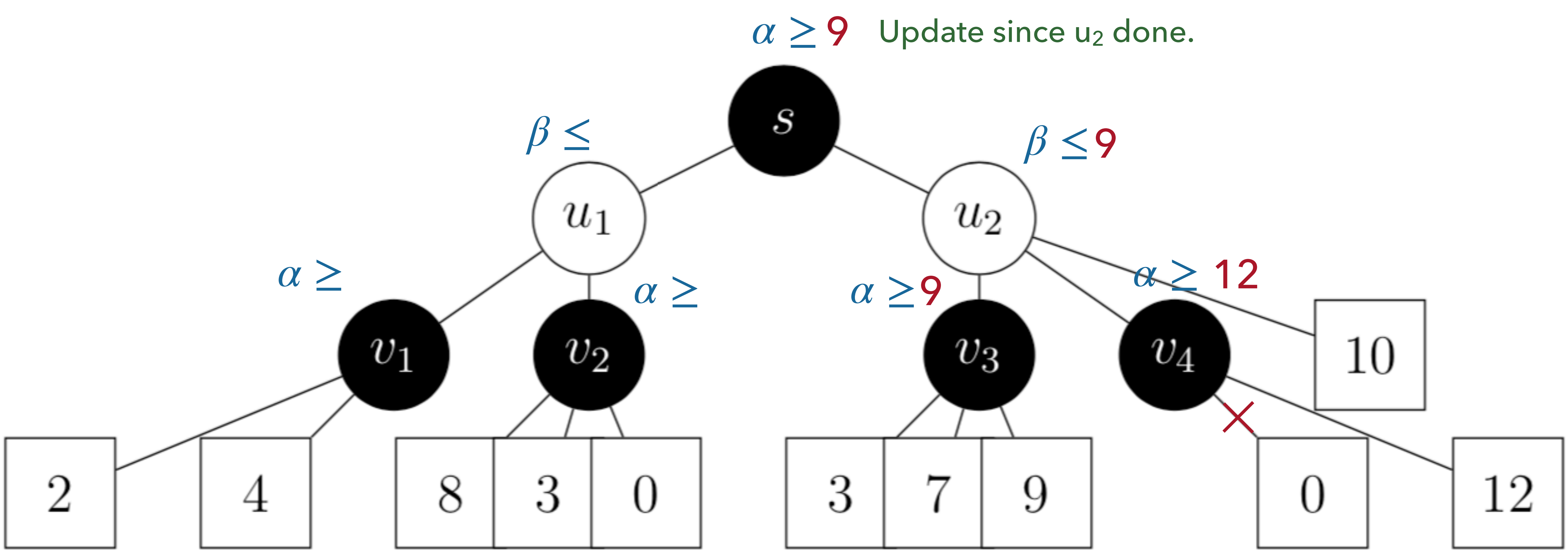

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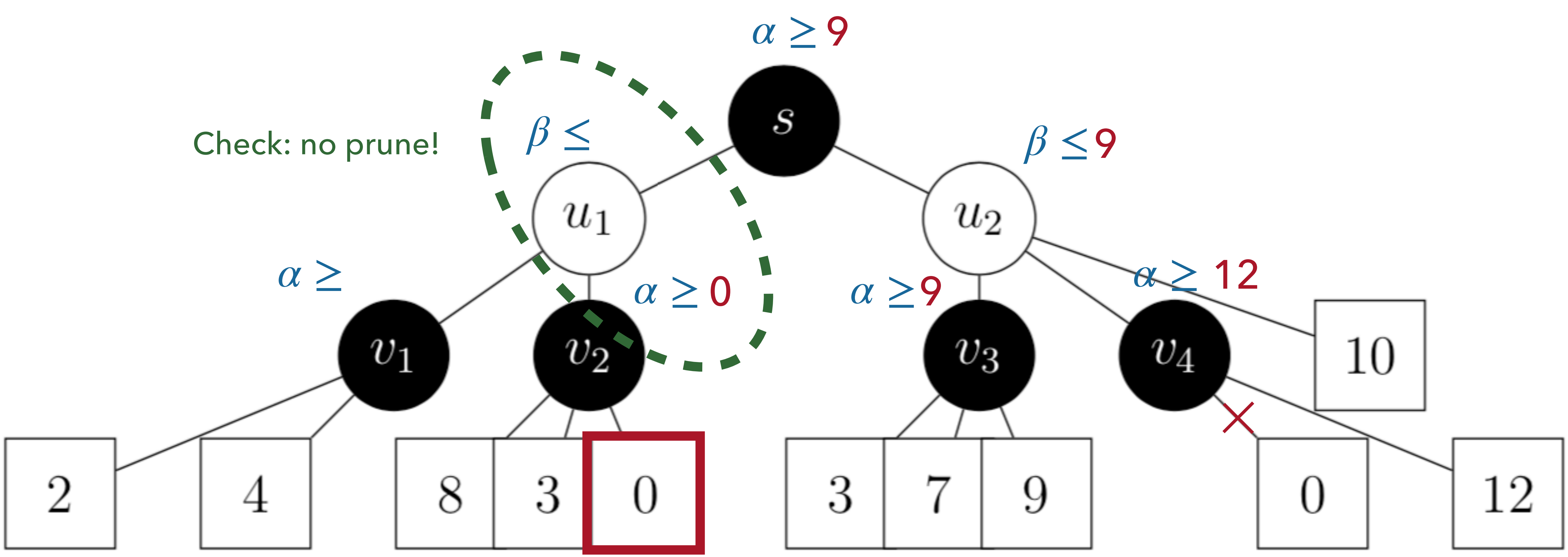

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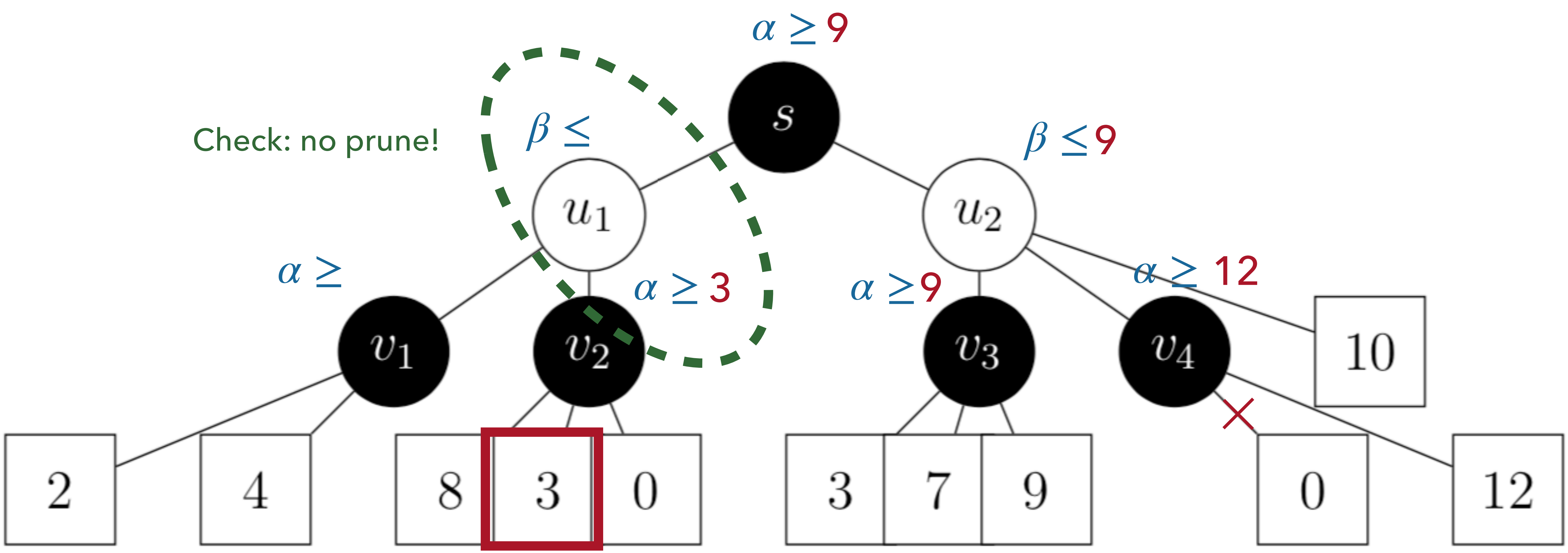

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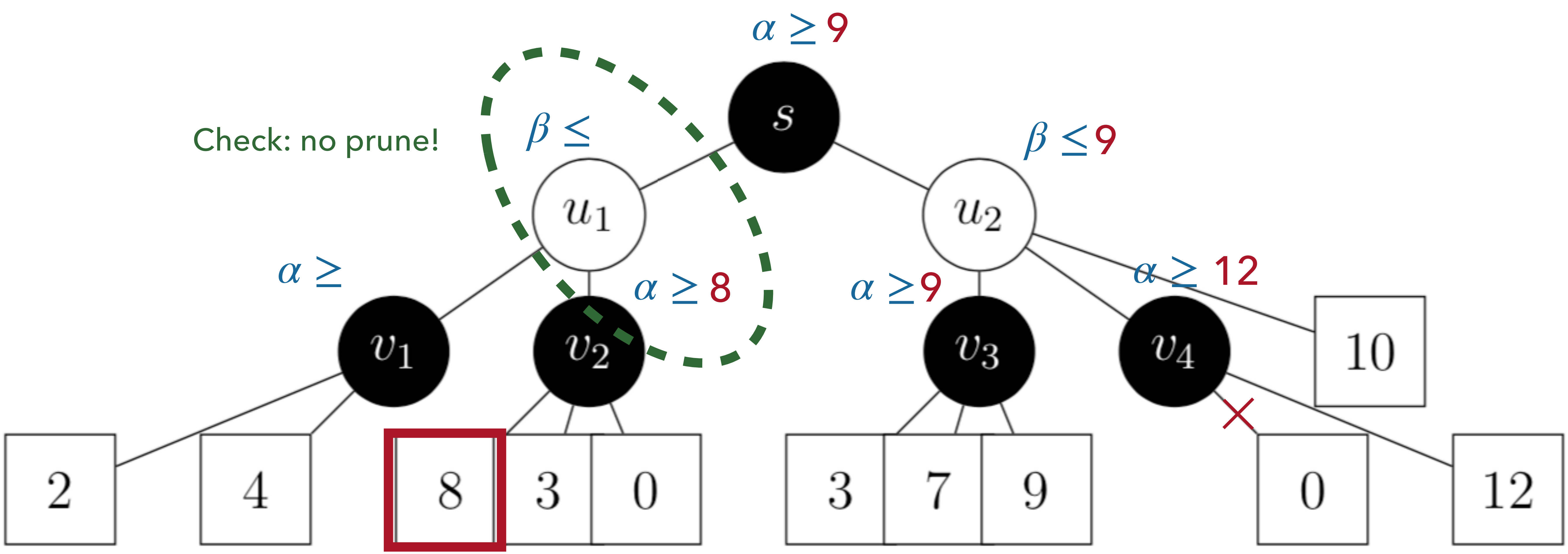

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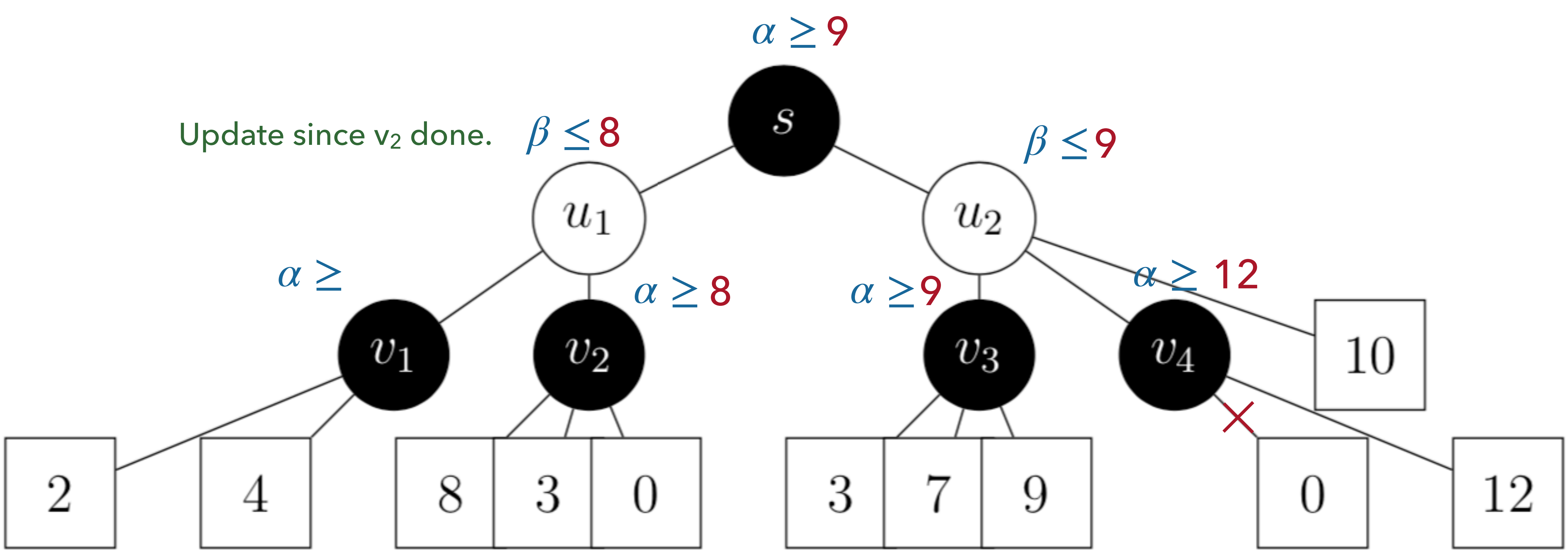

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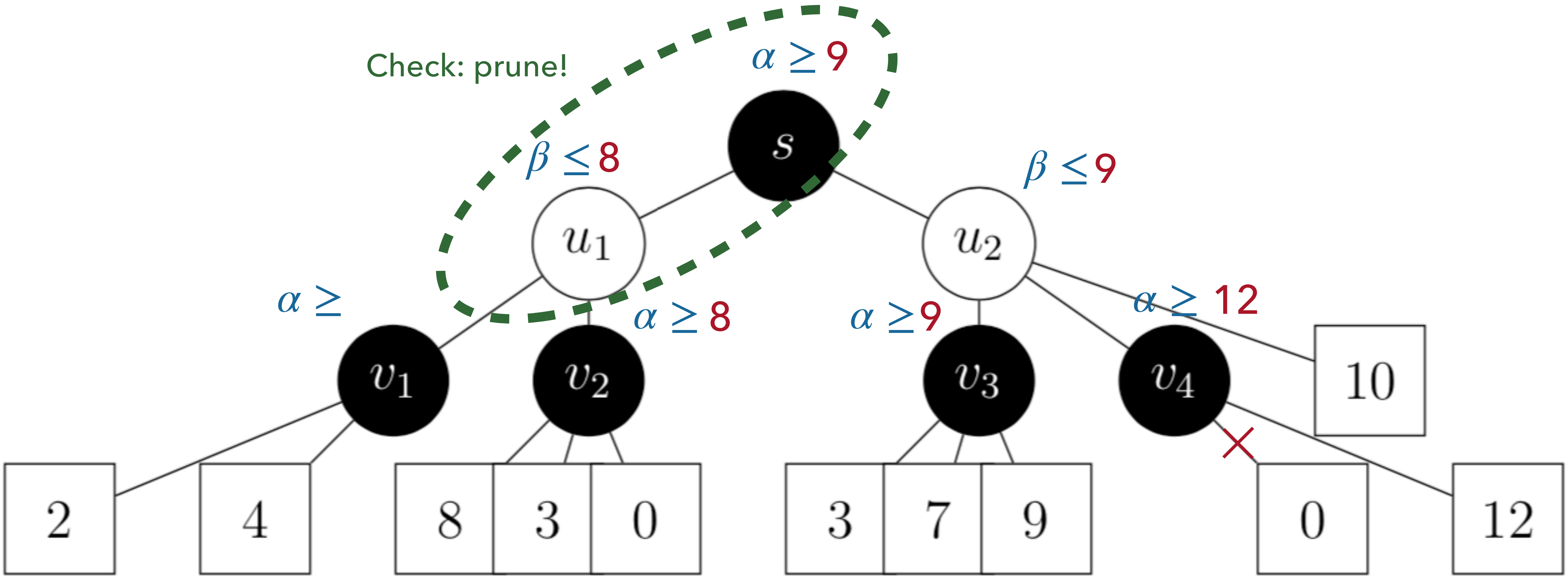

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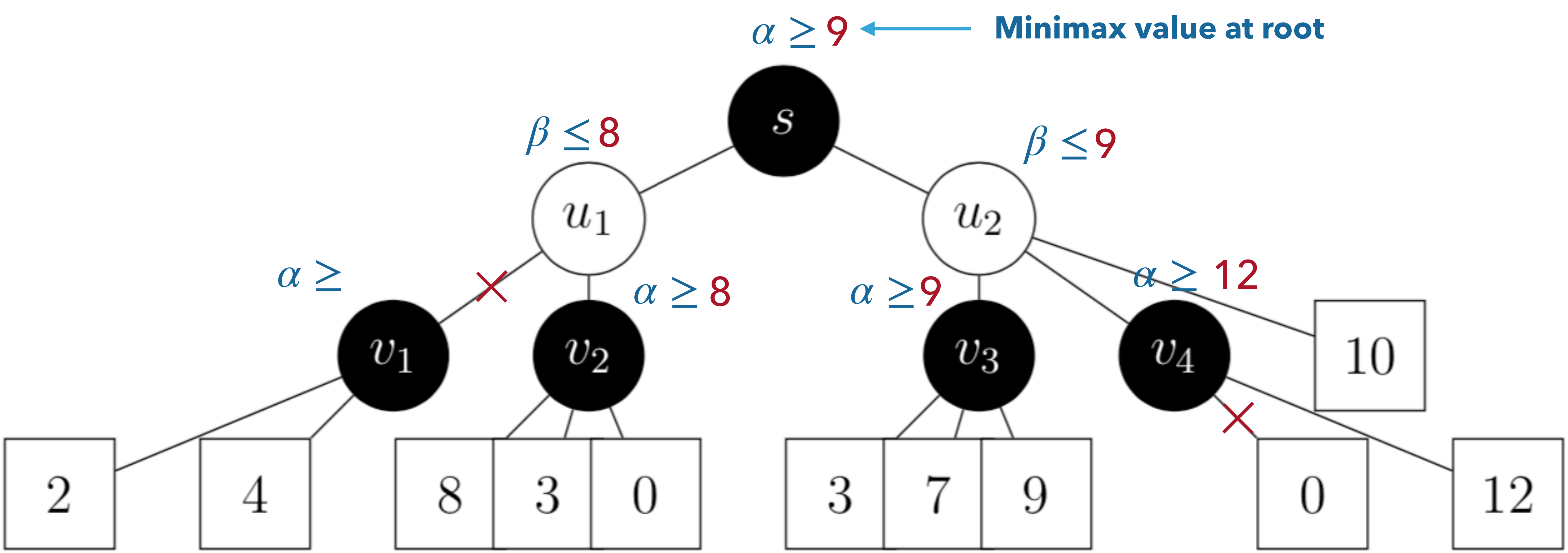

For justification, tell us the alpha/beta values of the immediate node will do.

MOCK MID-TERM

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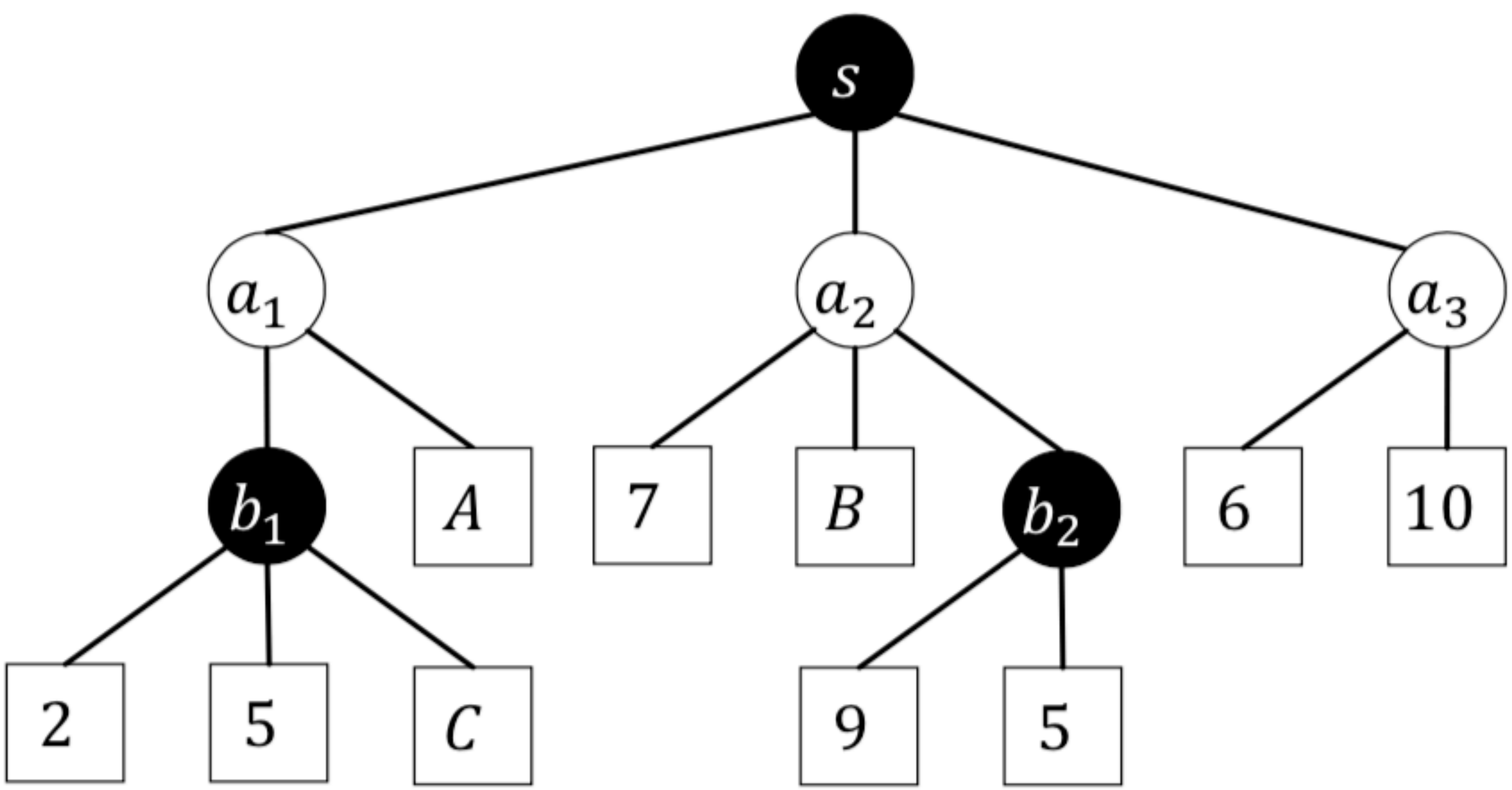

A > ______

B > ______

C < ______

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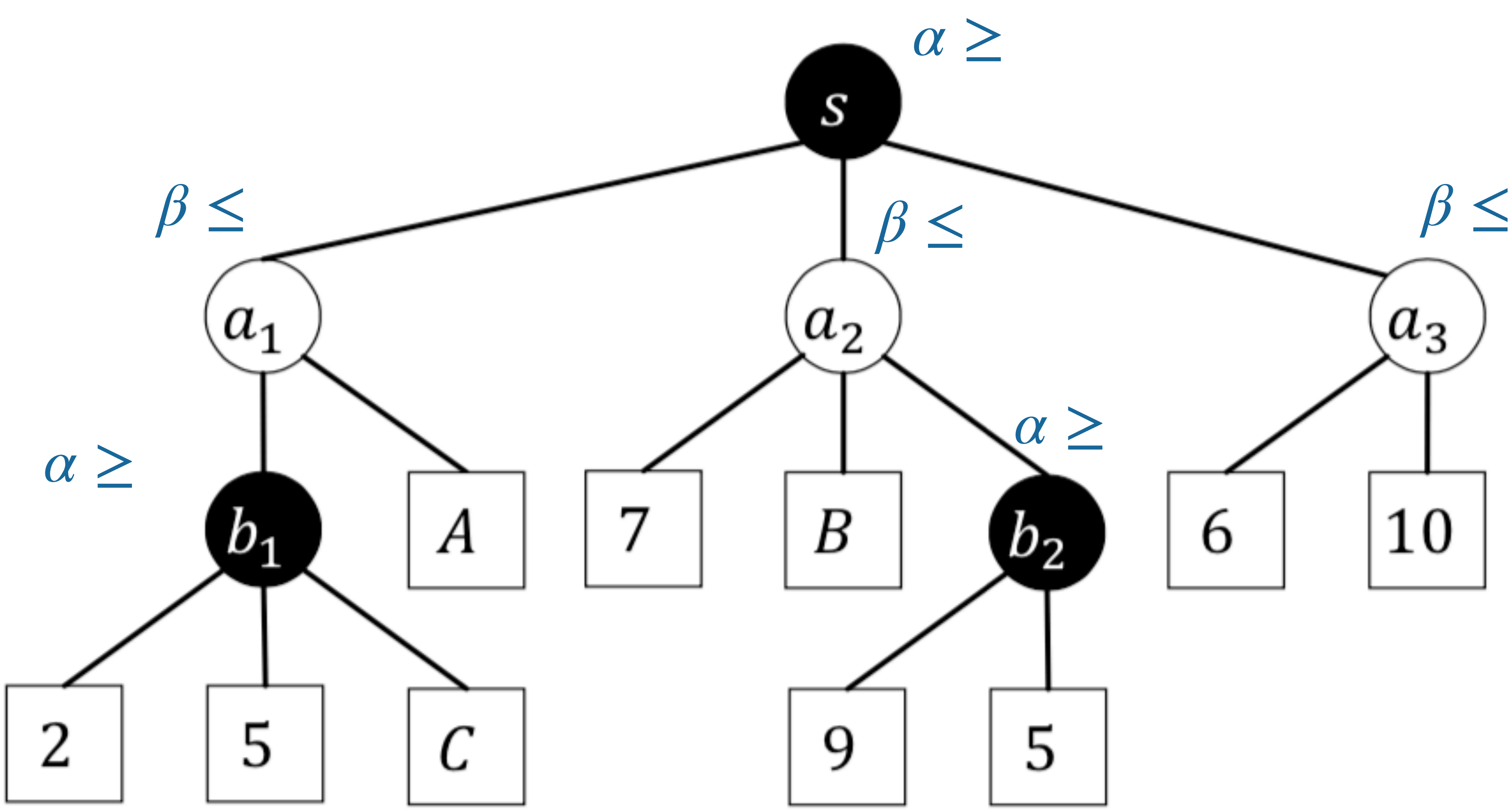

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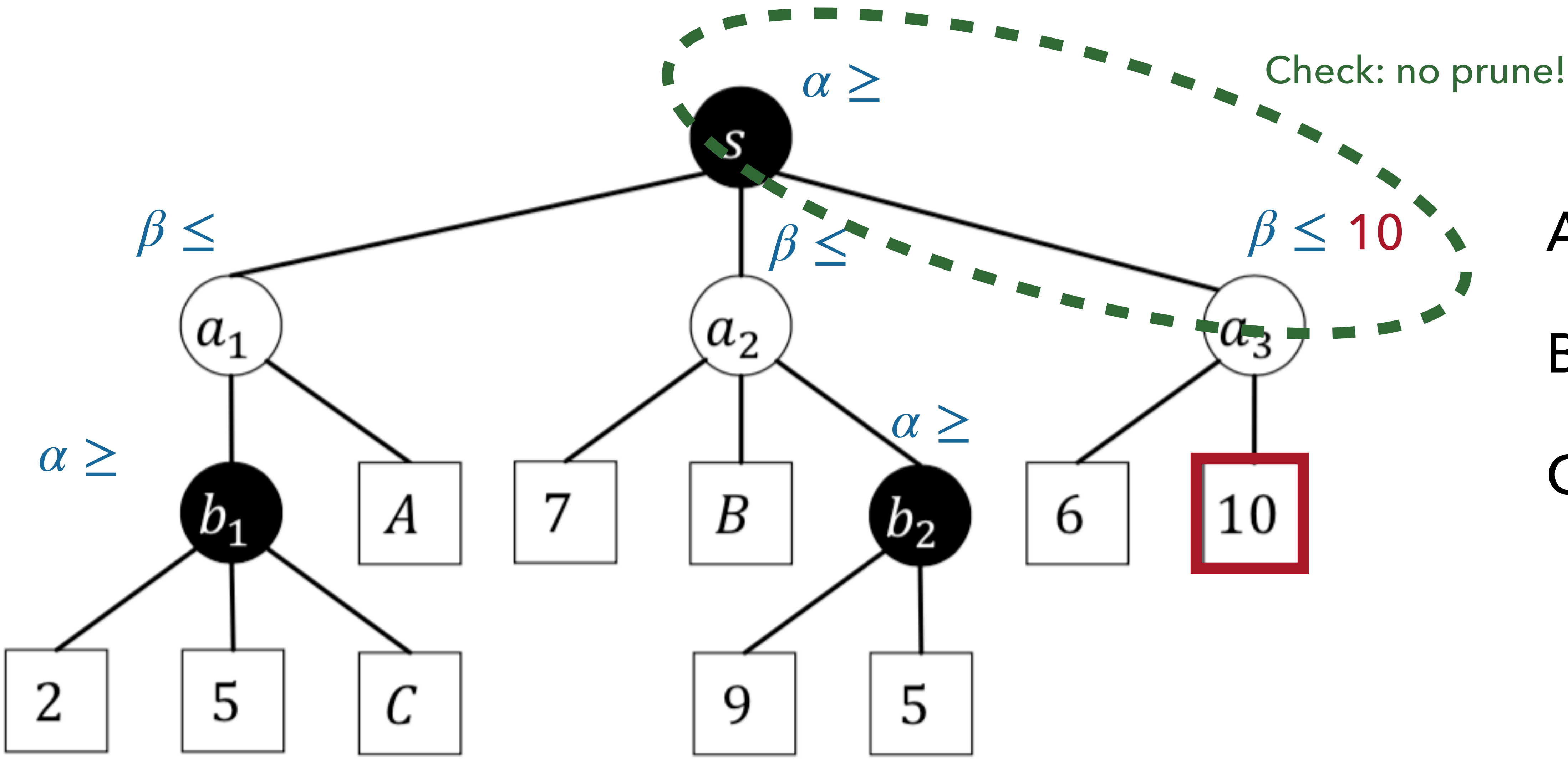

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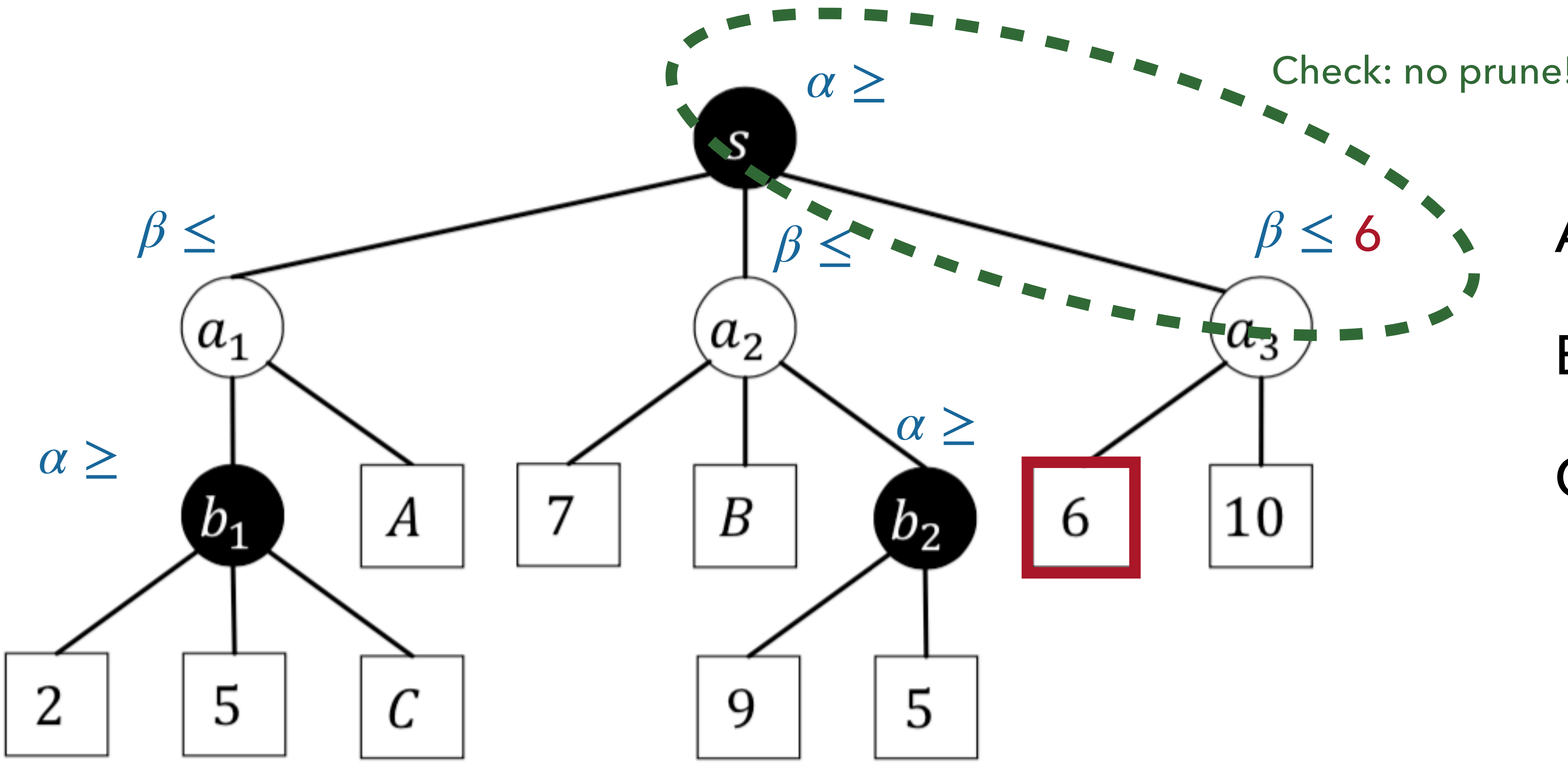

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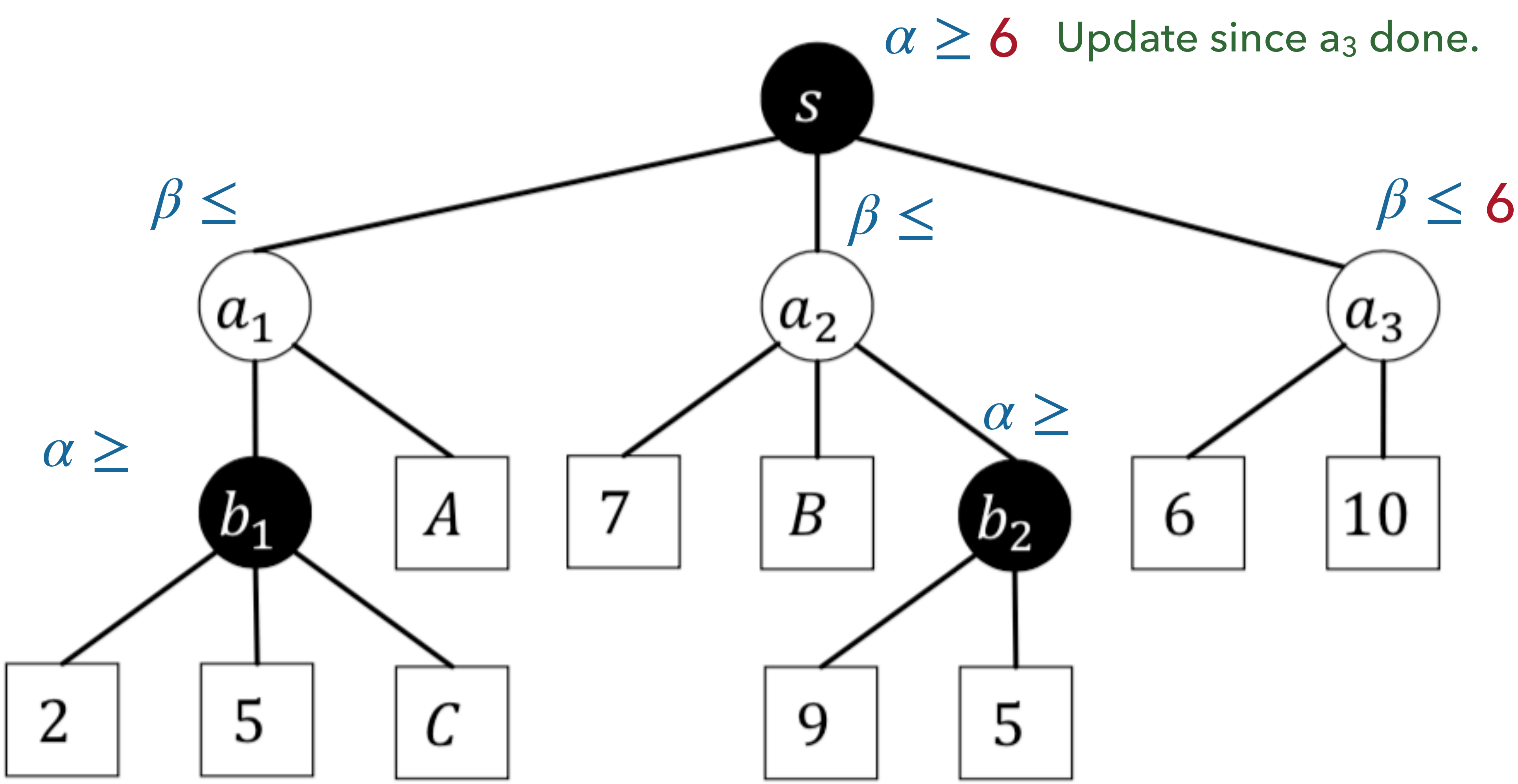

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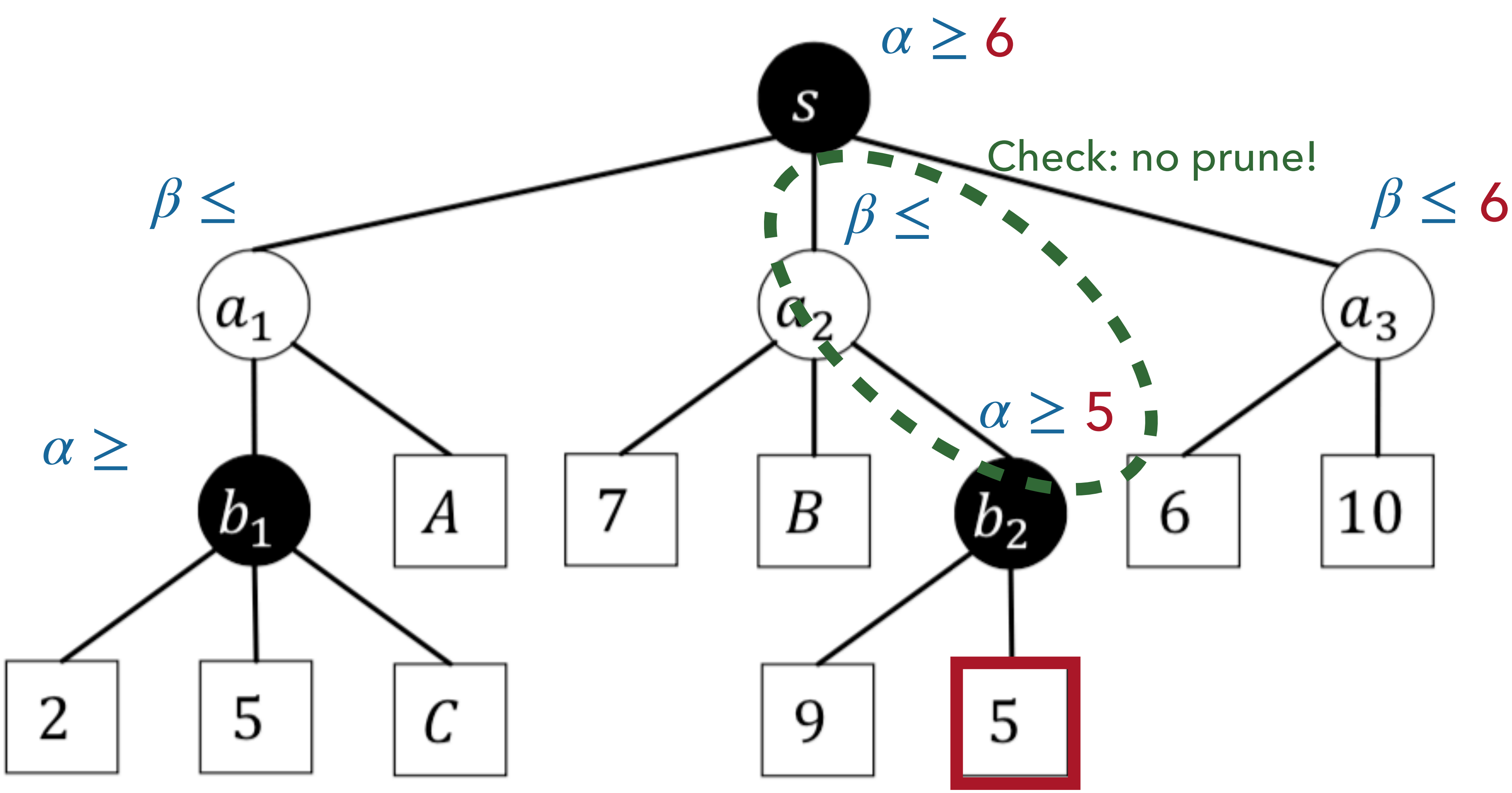

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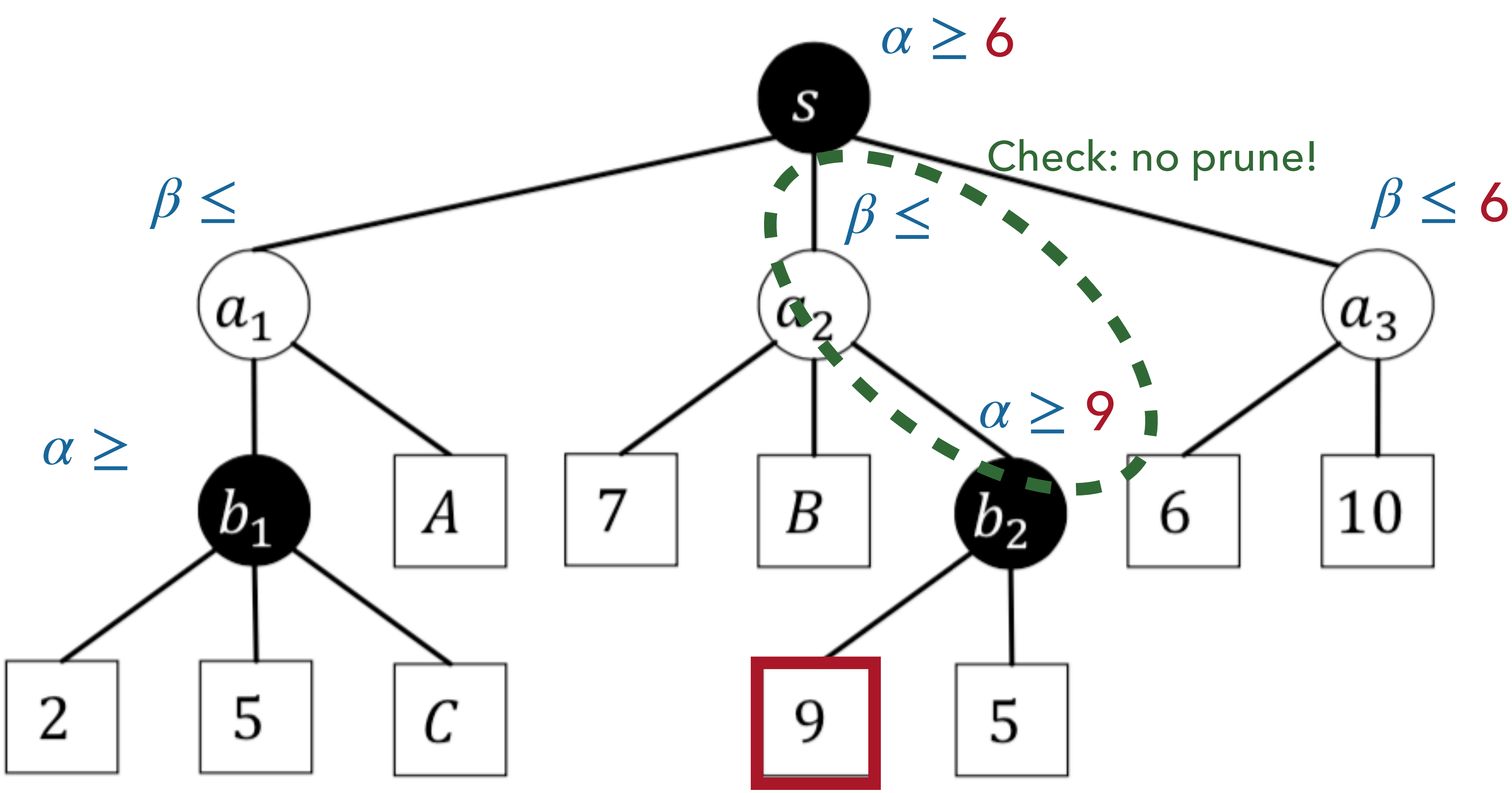

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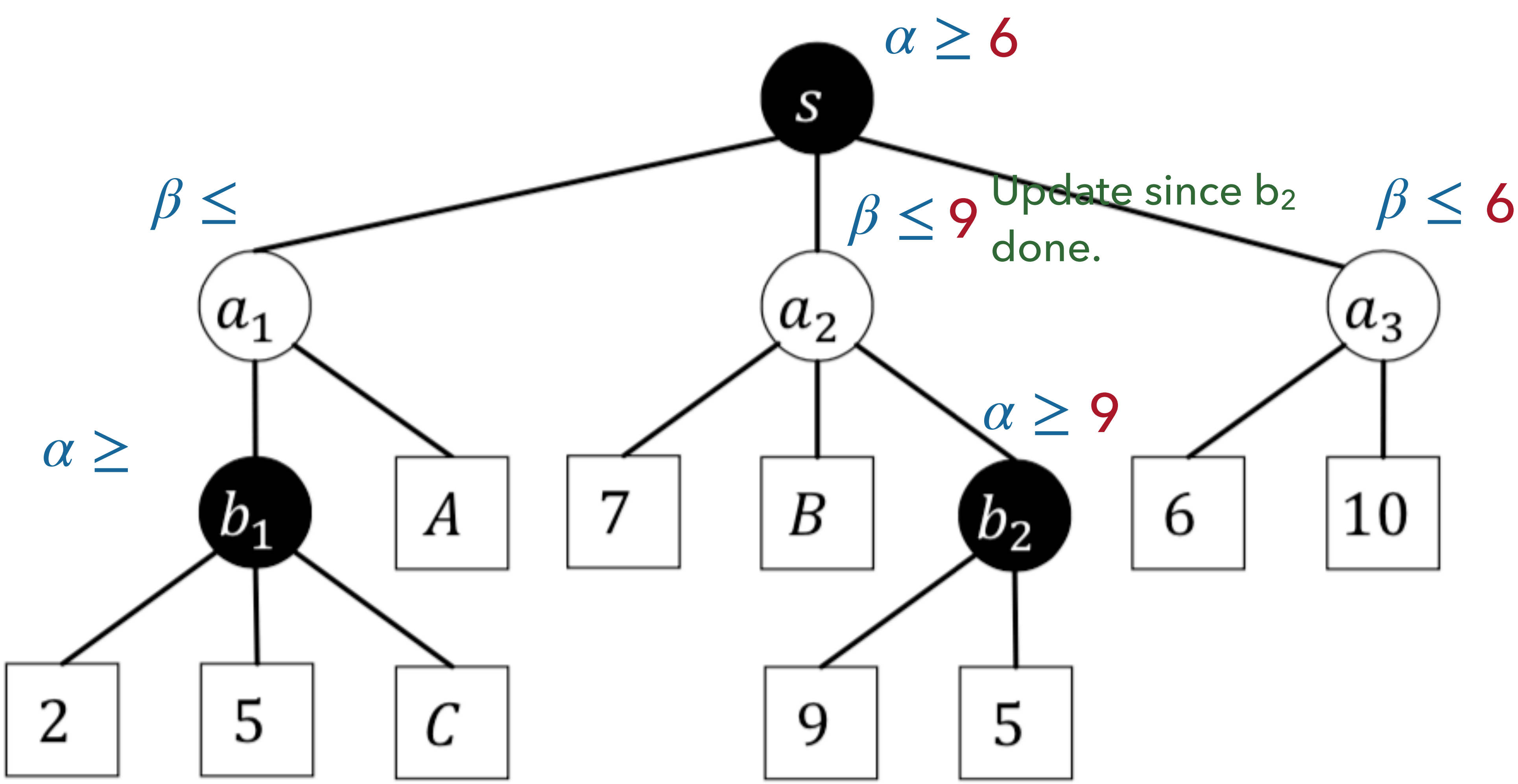

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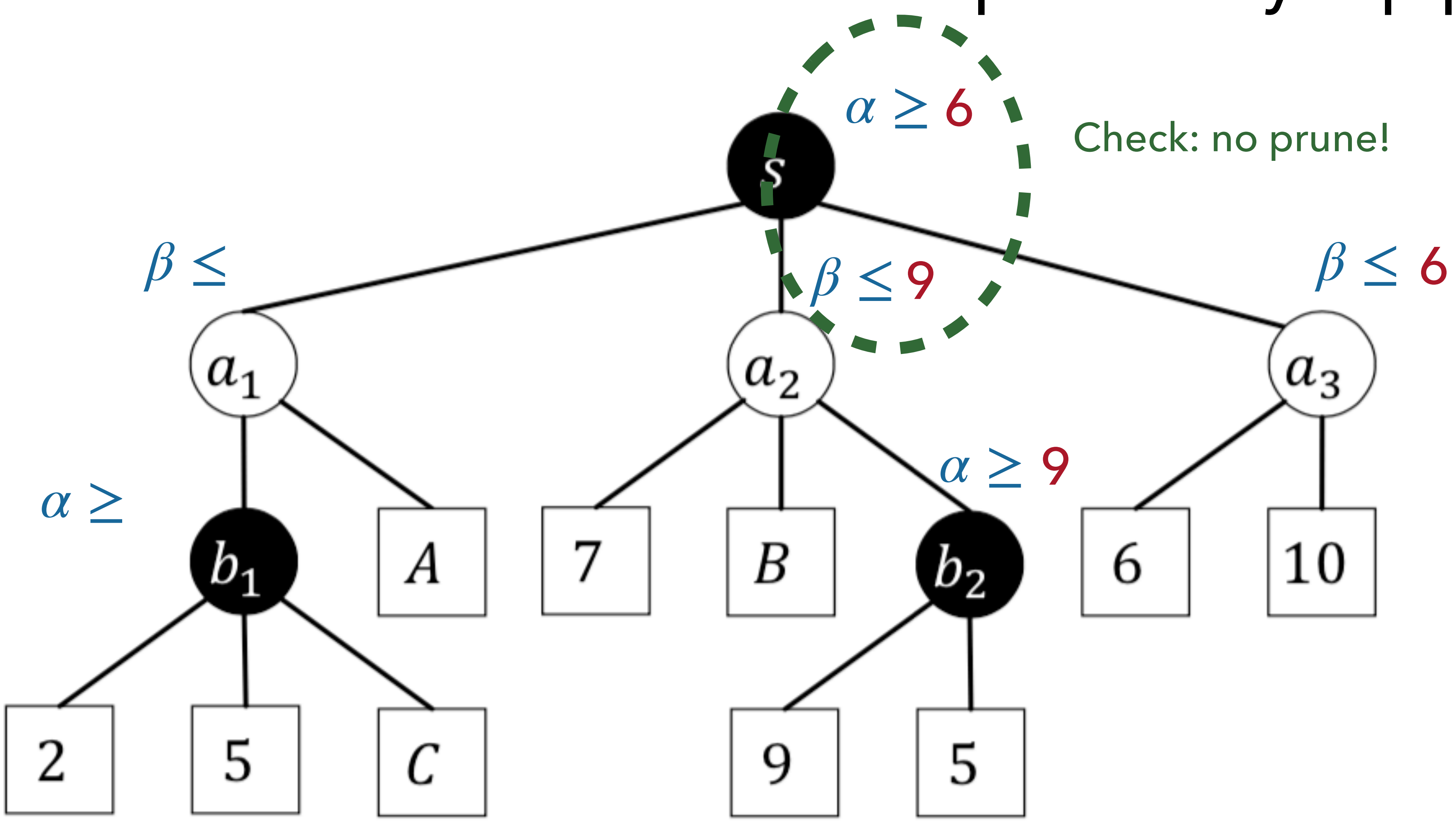

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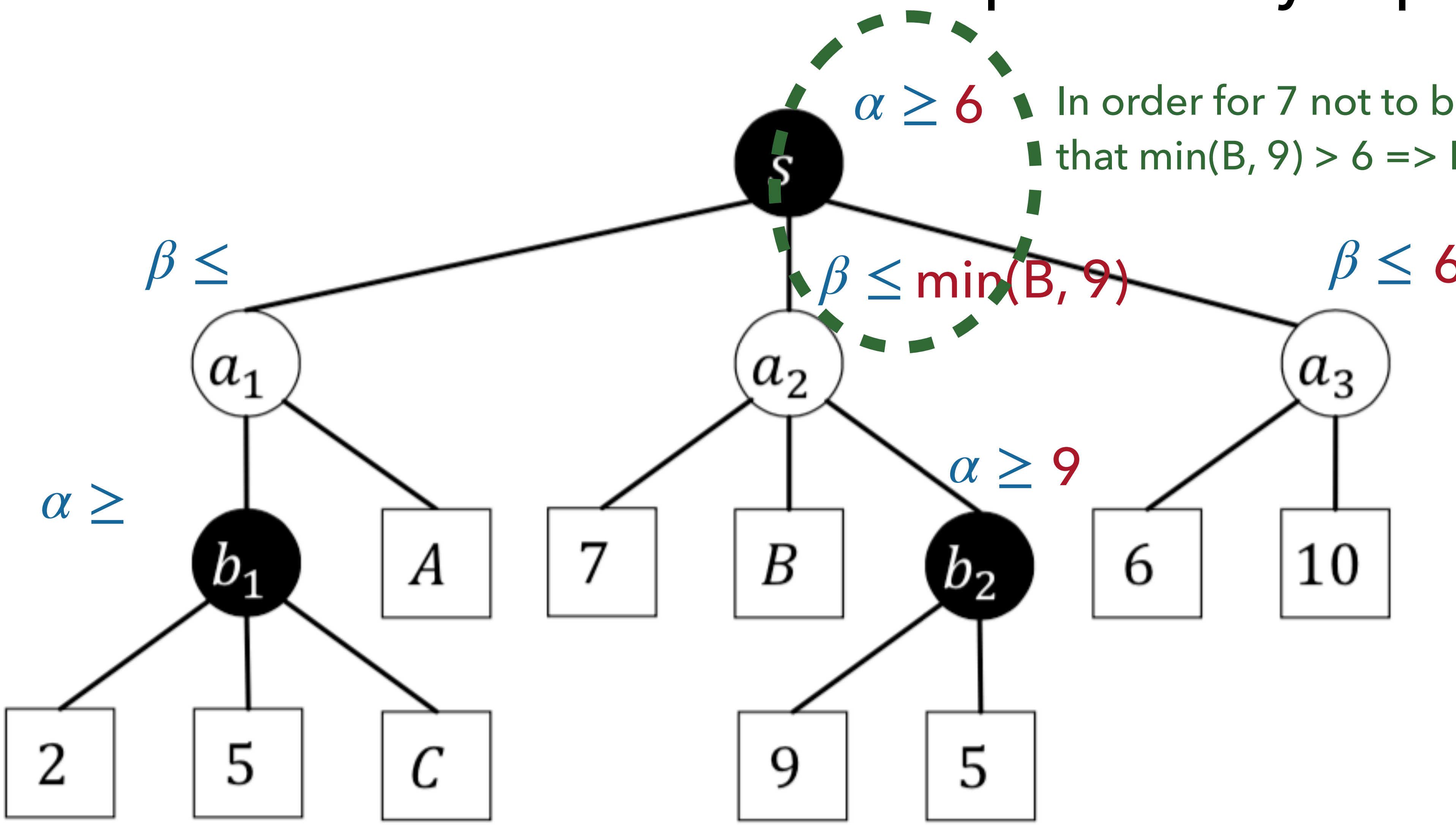

In order for 7 not to be pruned, it must be that min(B, 9) > 6 => B > 6

A > ______

B > 6

C < ______

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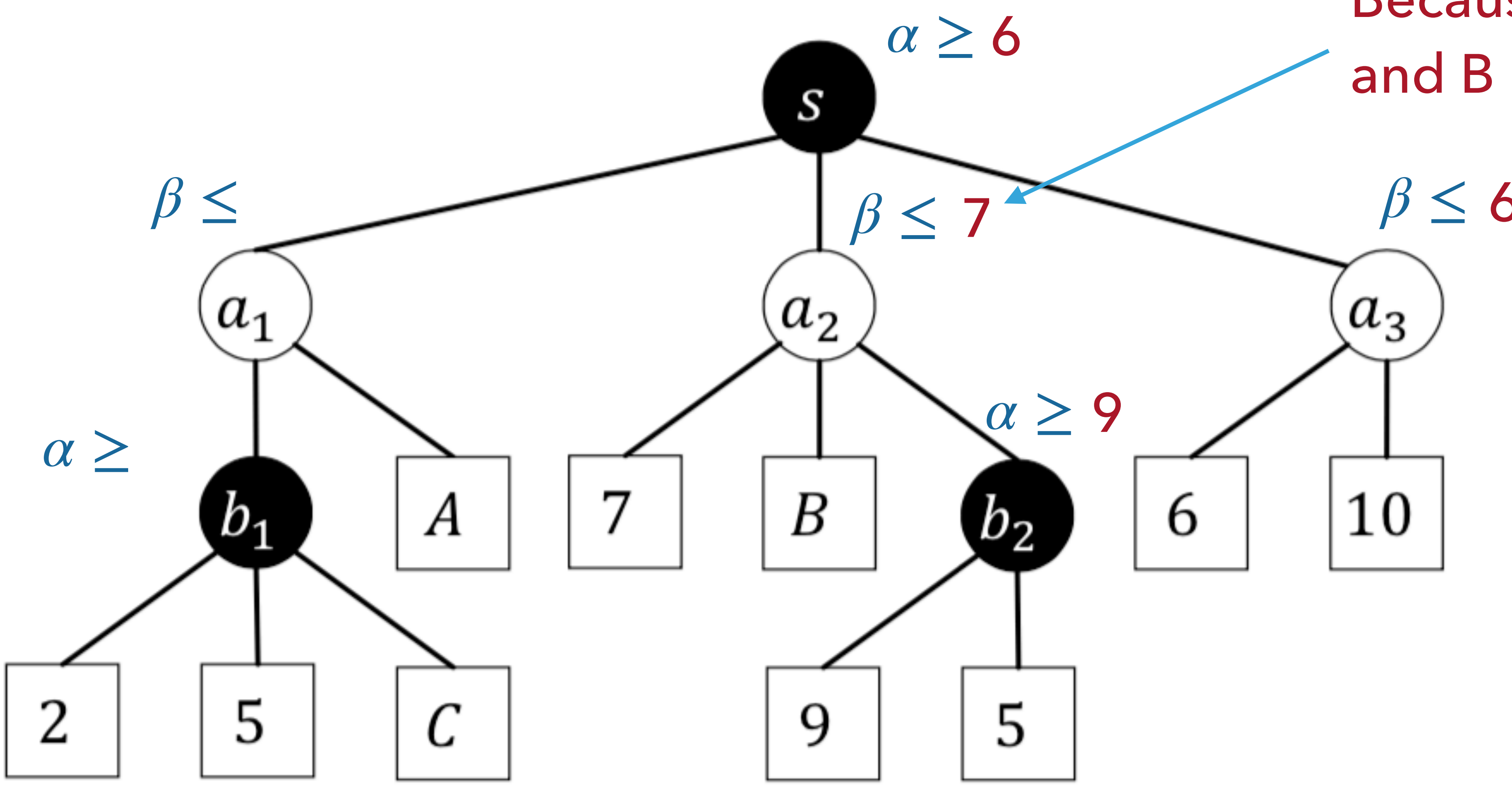

Because min(B, 7, 9) = 7
and B > 6 => B ≥ 7

A > ______

B > 6

C < ______

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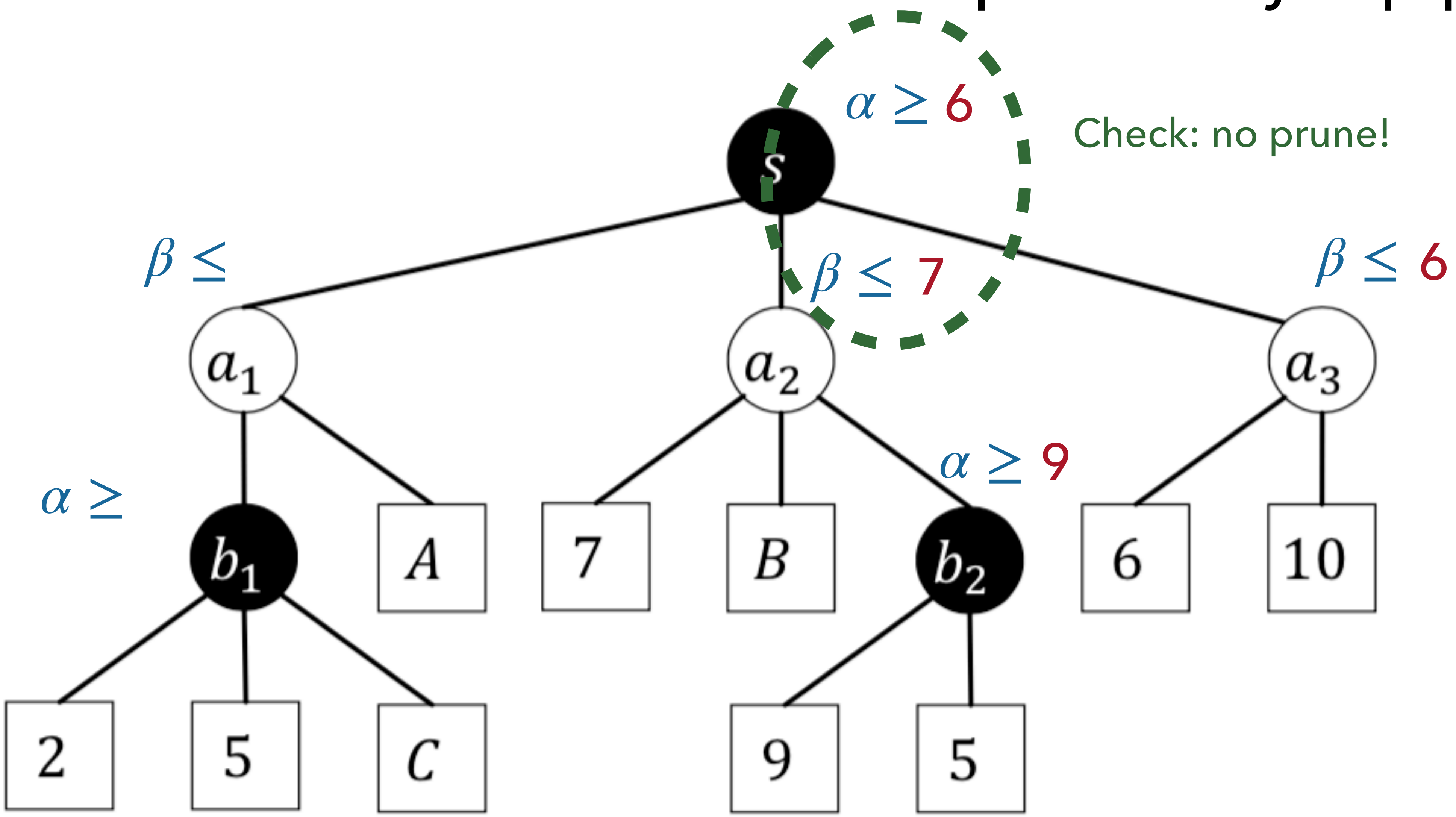

A > ______

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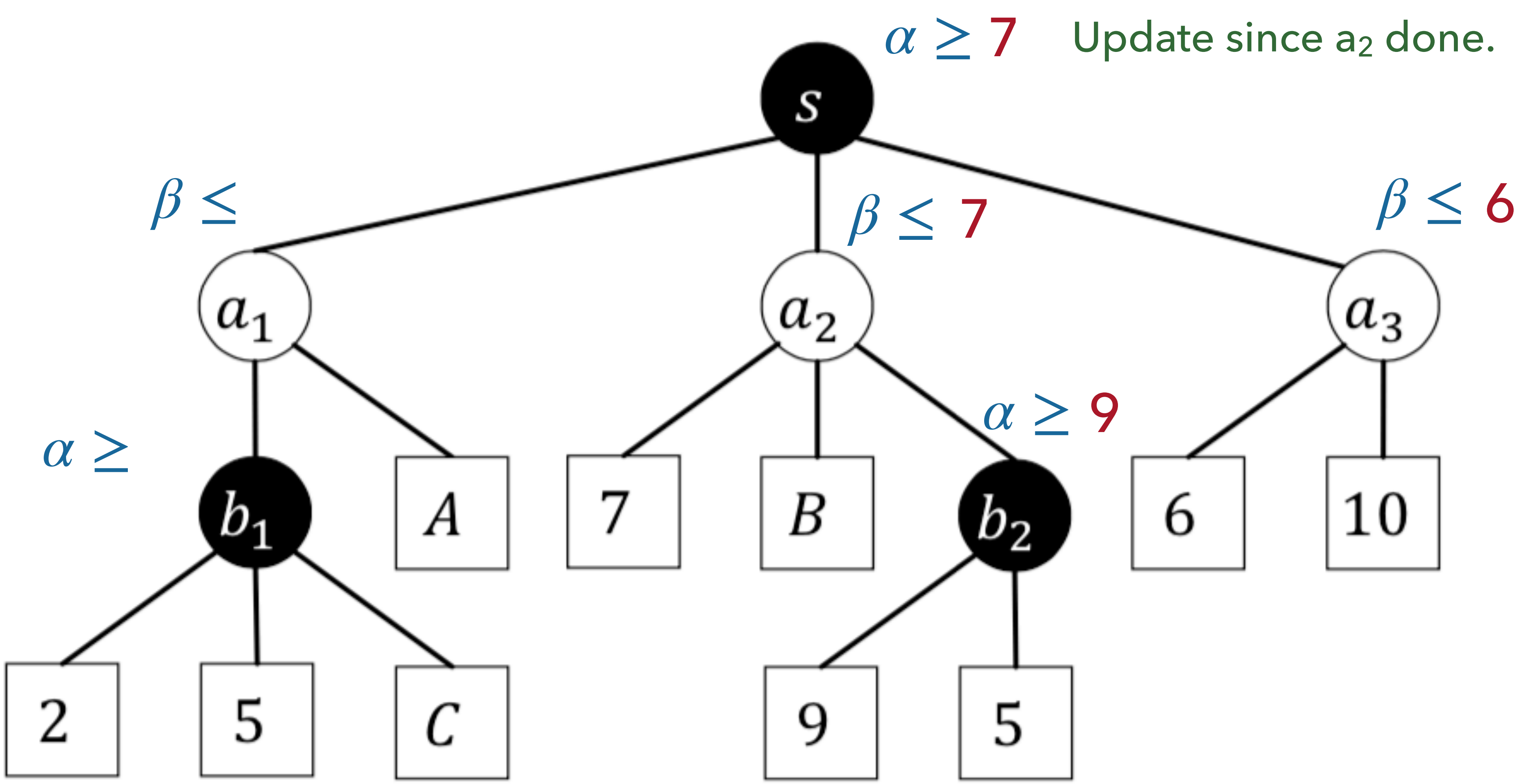

A > ______

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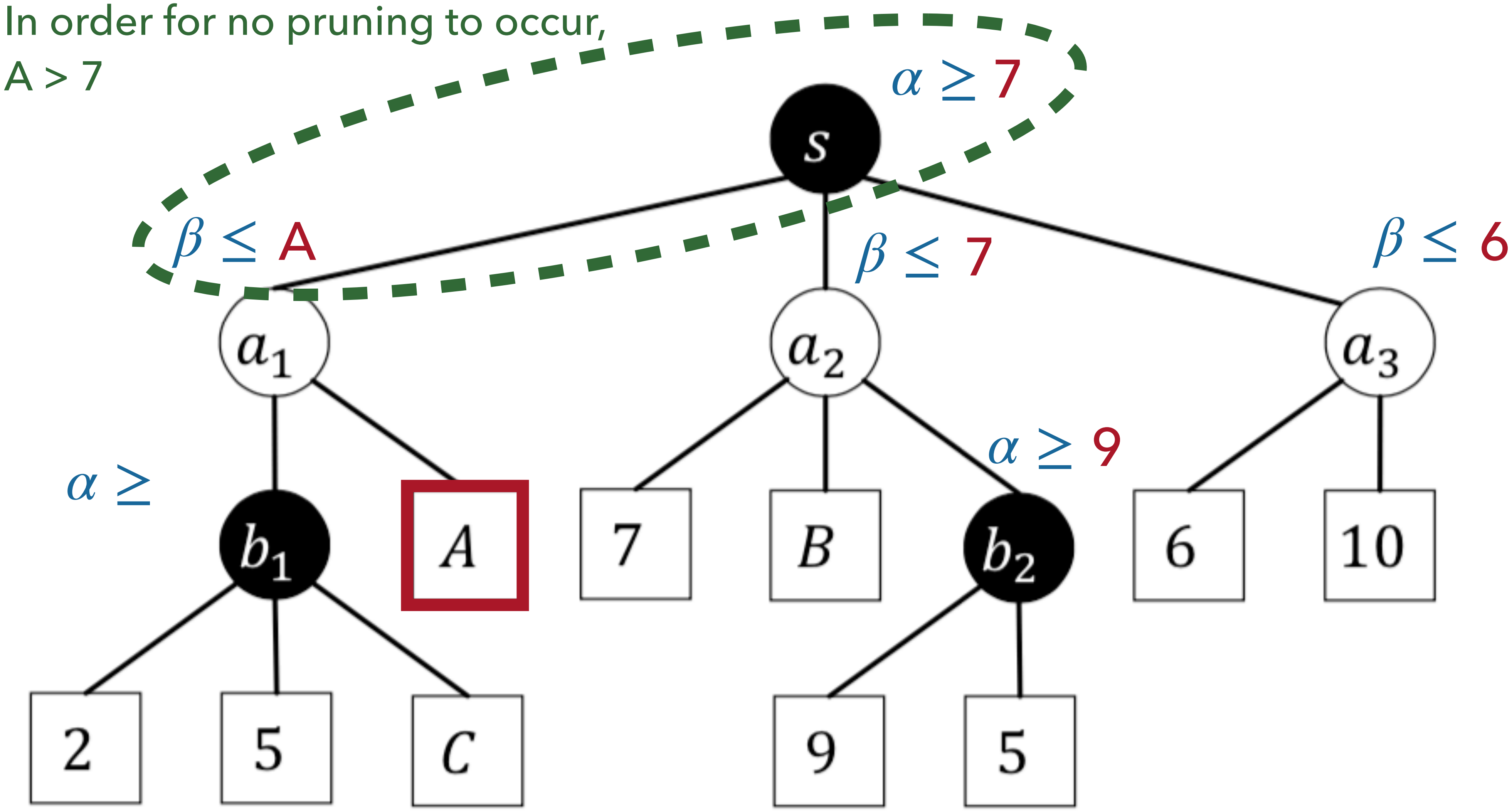
In order for no pruning to occur,
A > 7



A > 7

B > 6

C < ______

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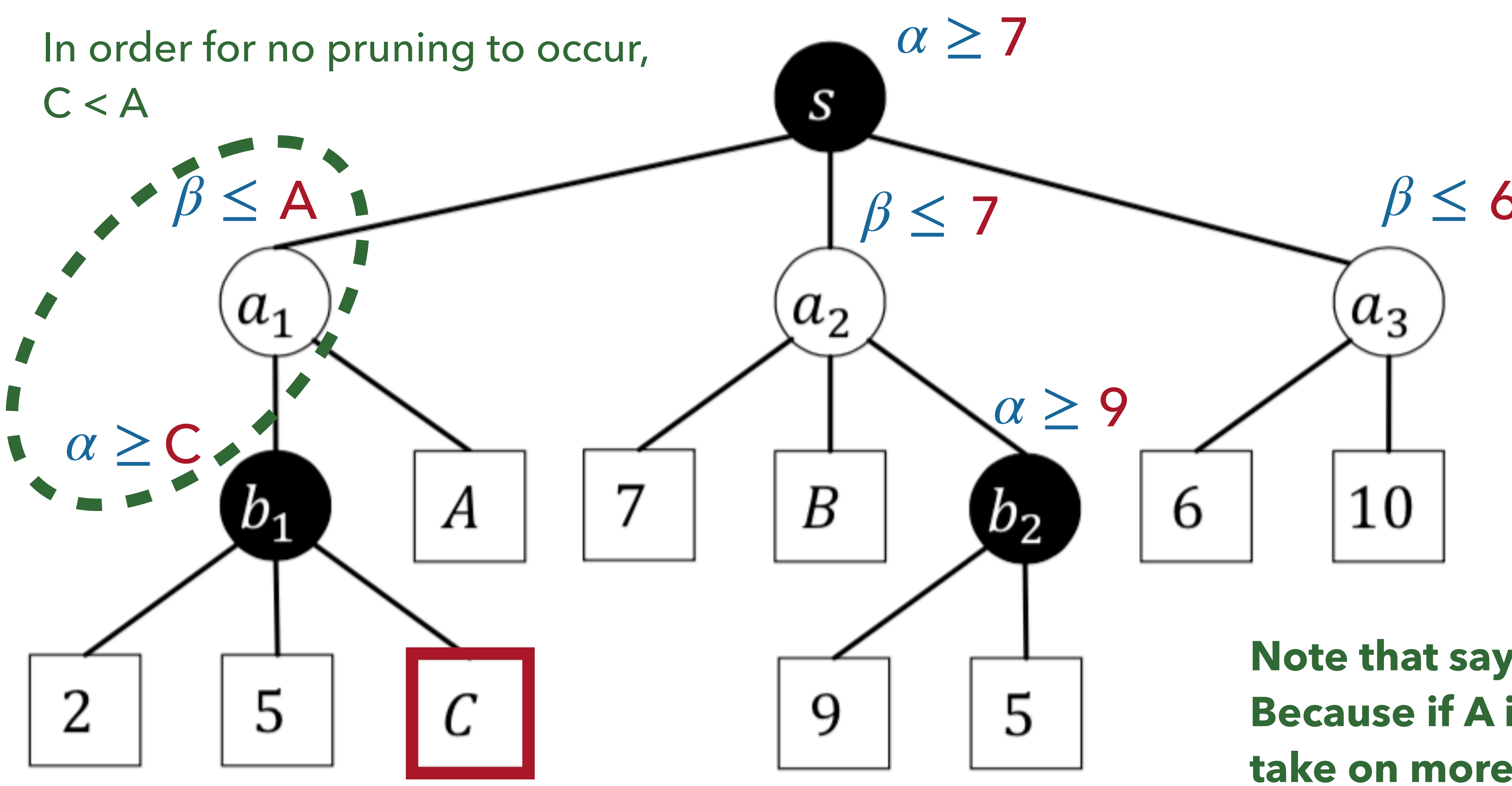

A > 7

B > 6

C < A

Note that saying C < 8 is insufficient. Because if A is very large, then C can take on more values than merely C < 8.

EXTRA QUESTION

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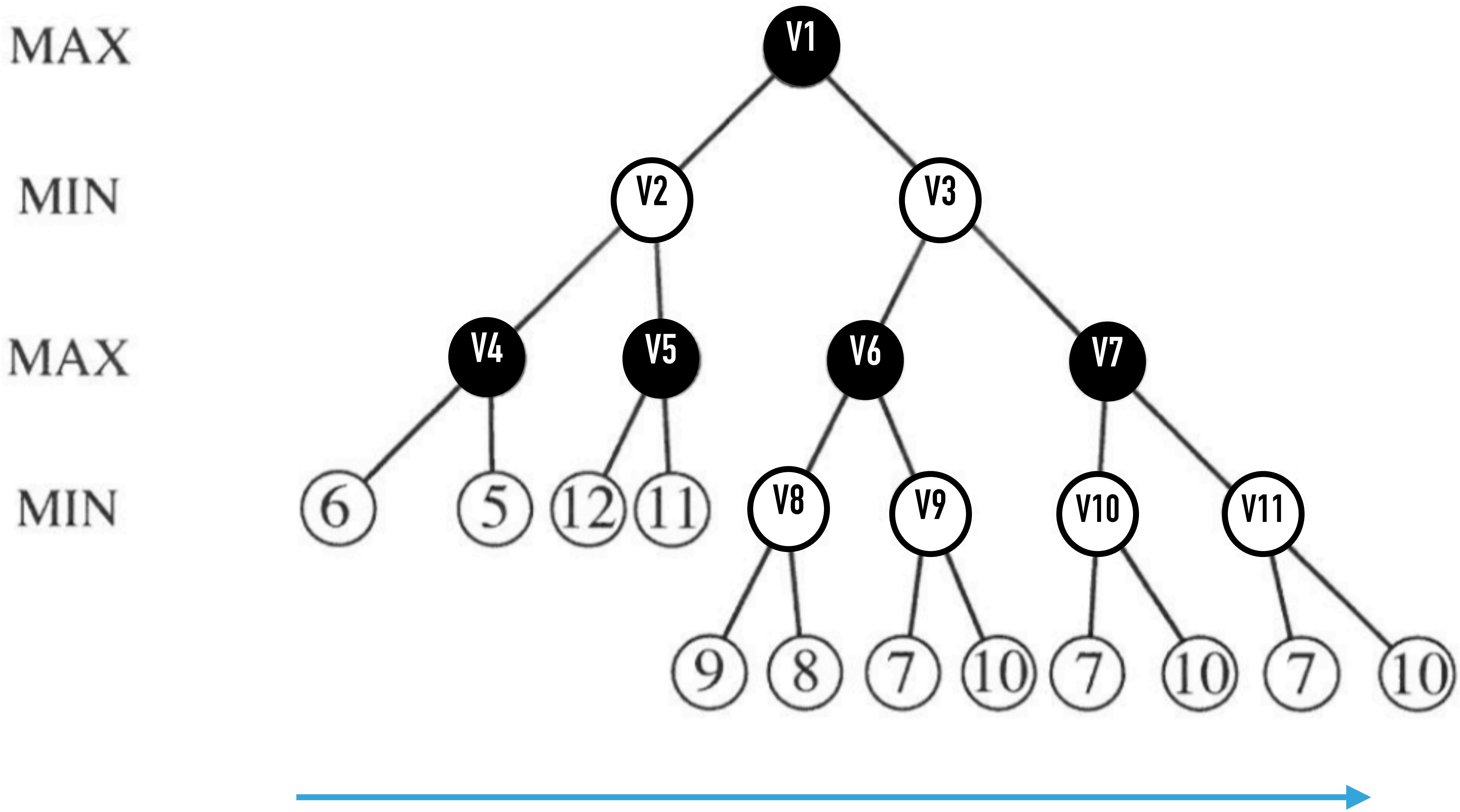

Show which arcs are pruned

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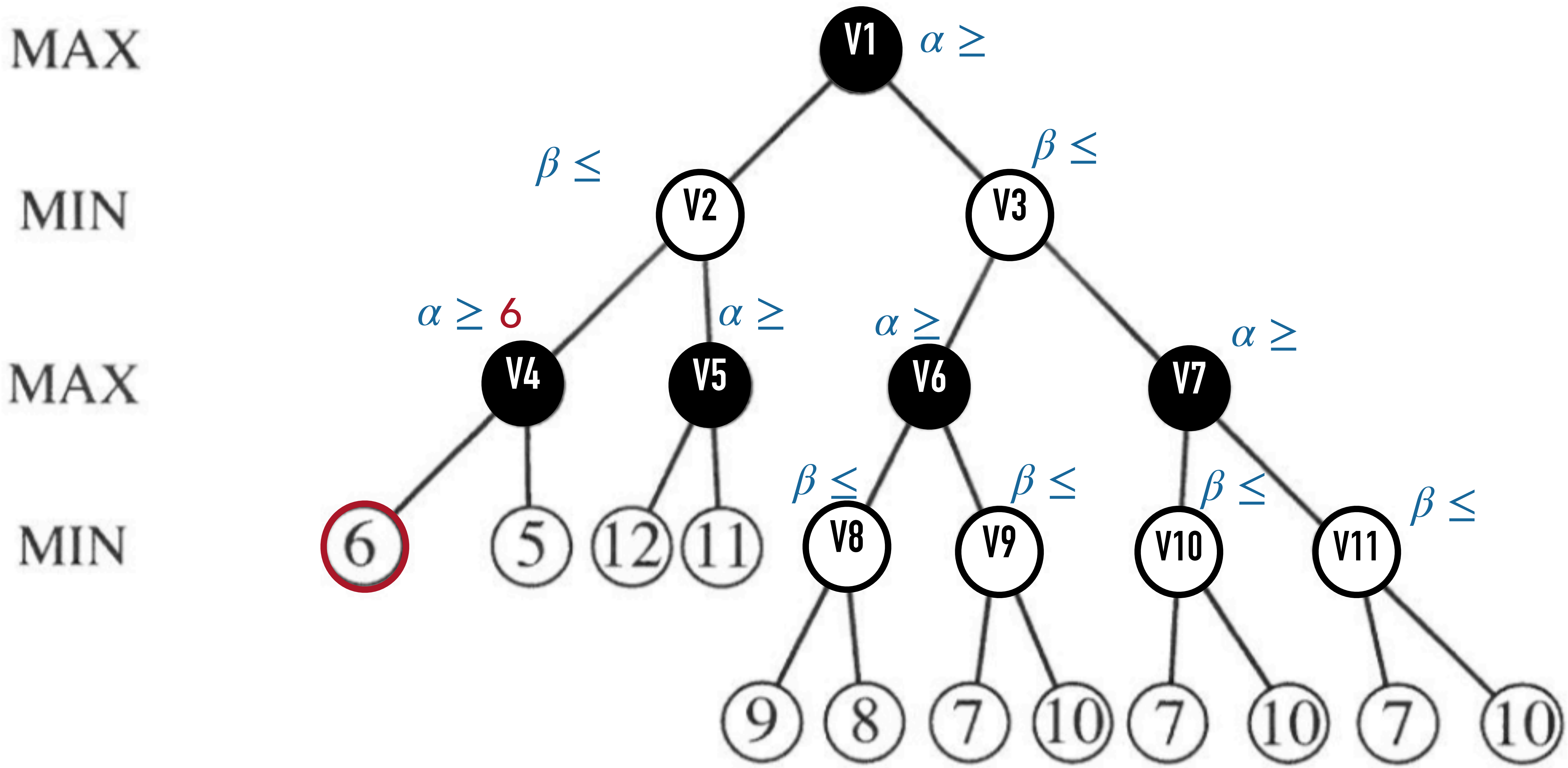

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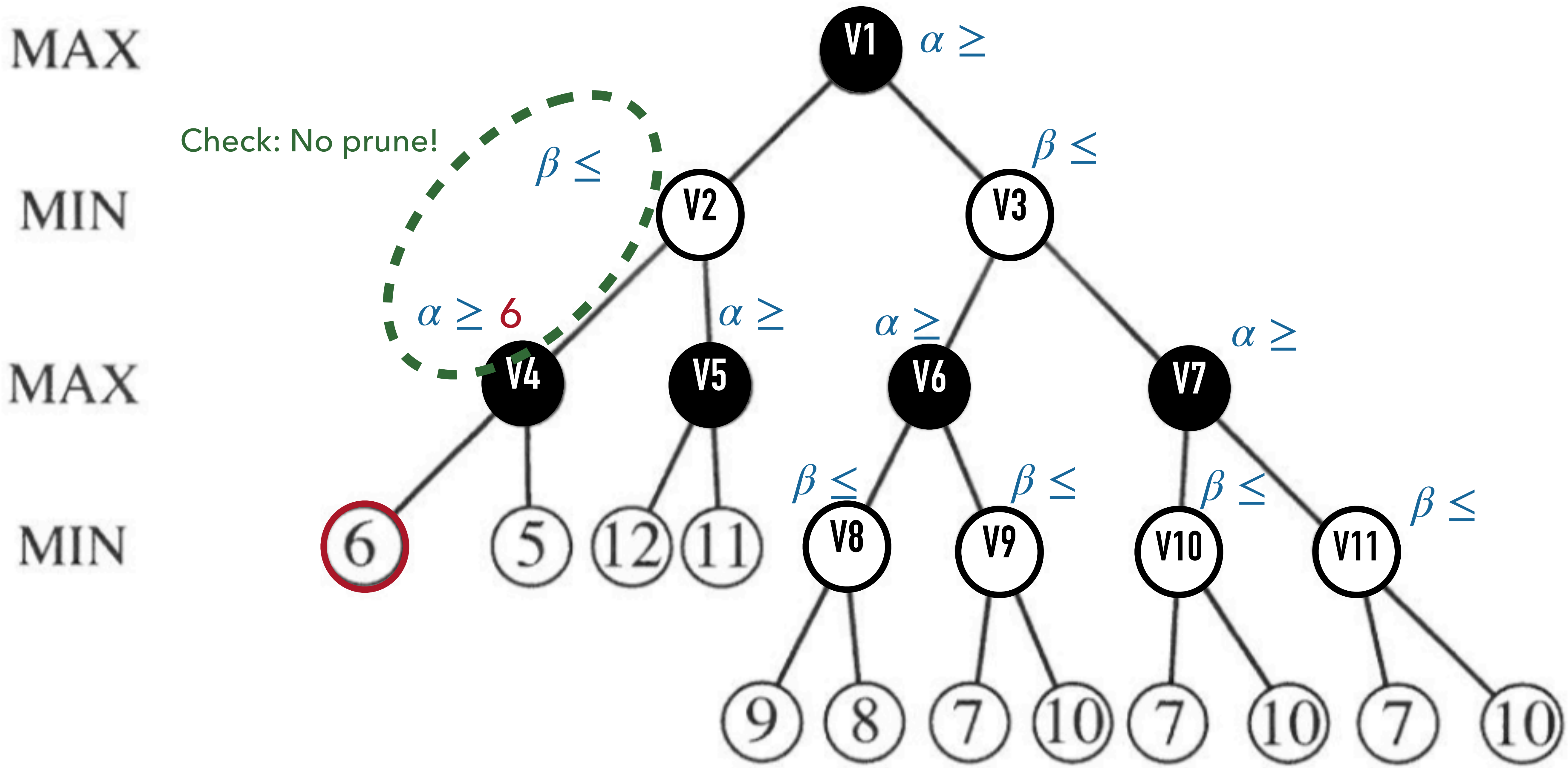

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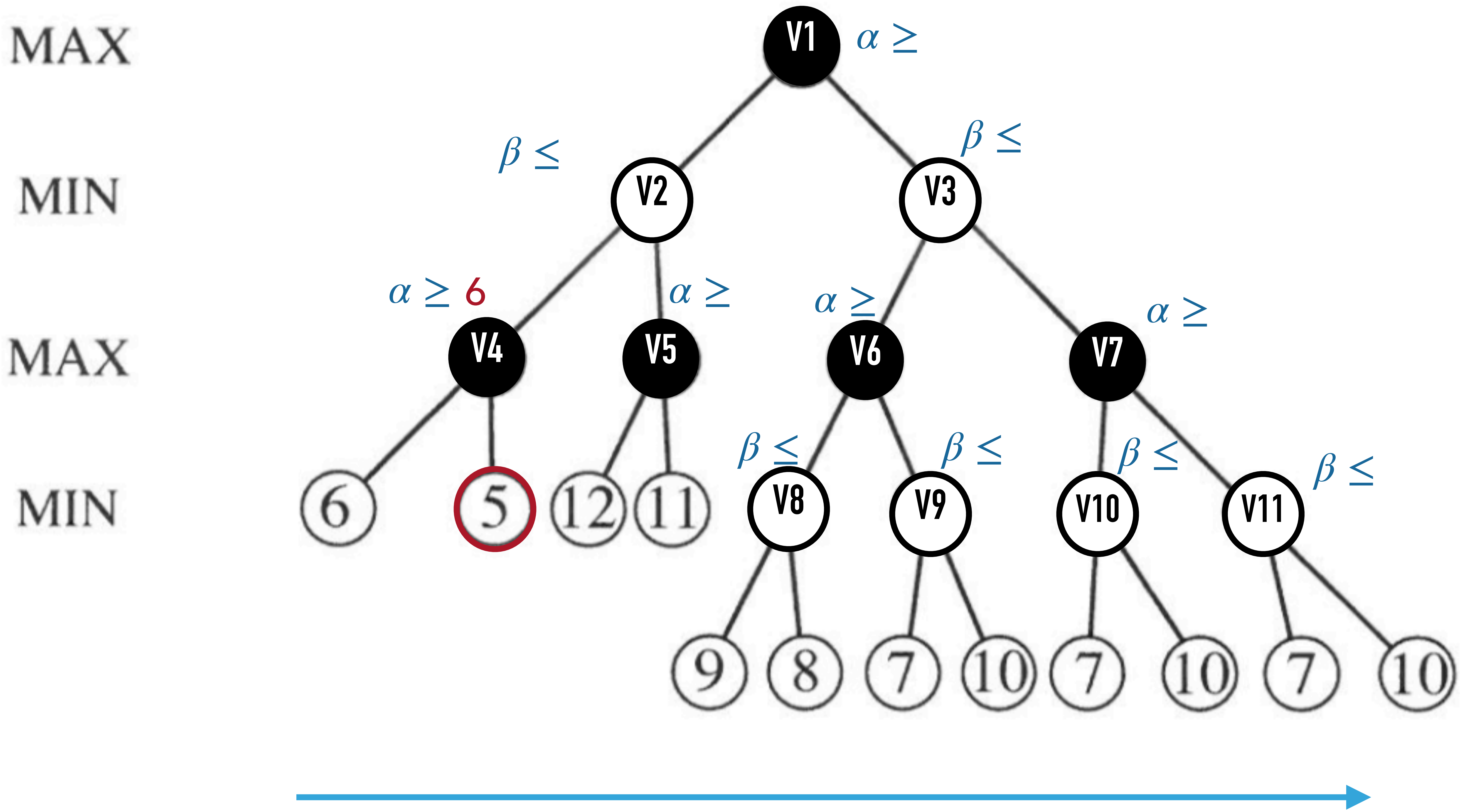

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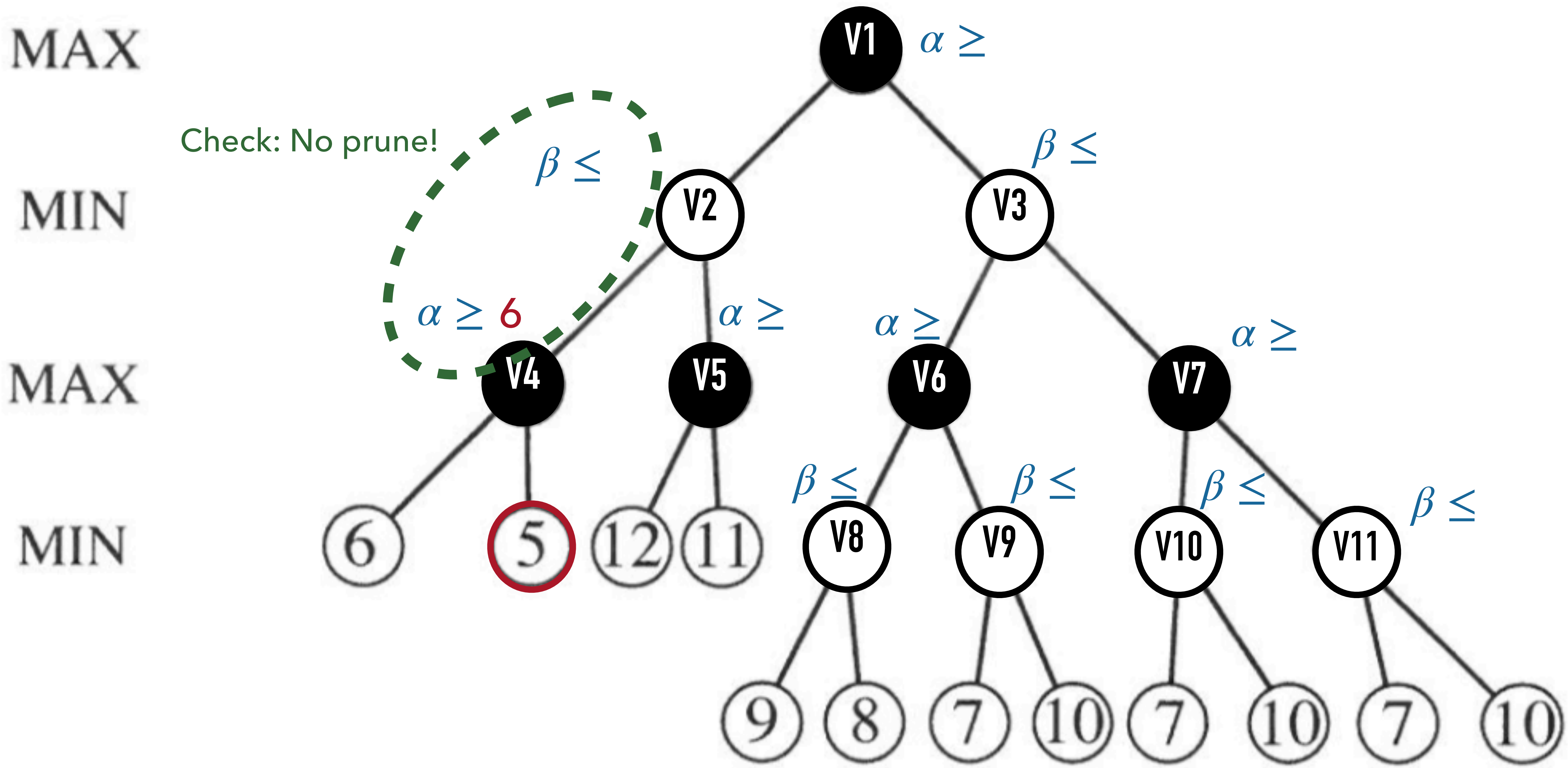

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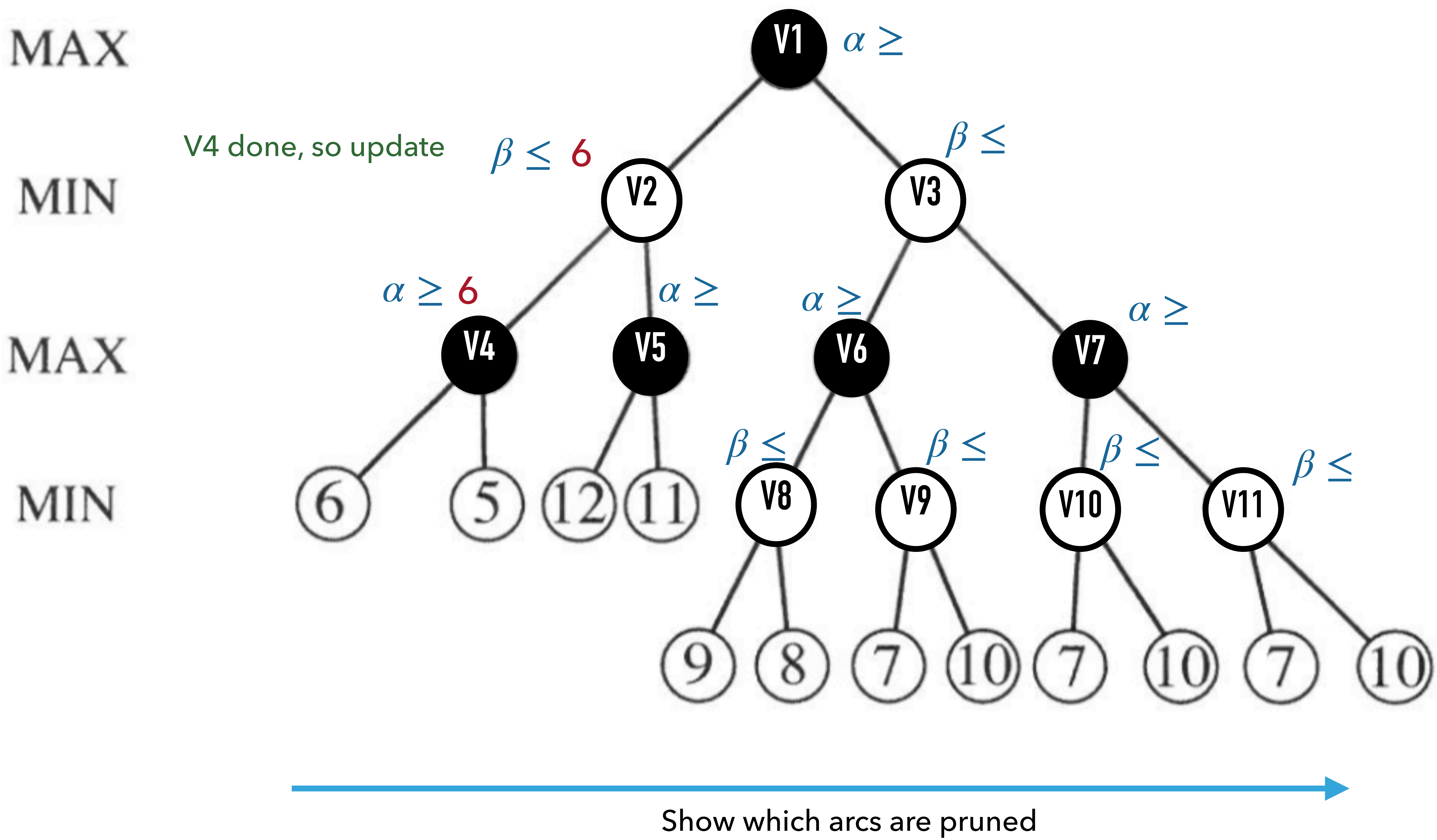

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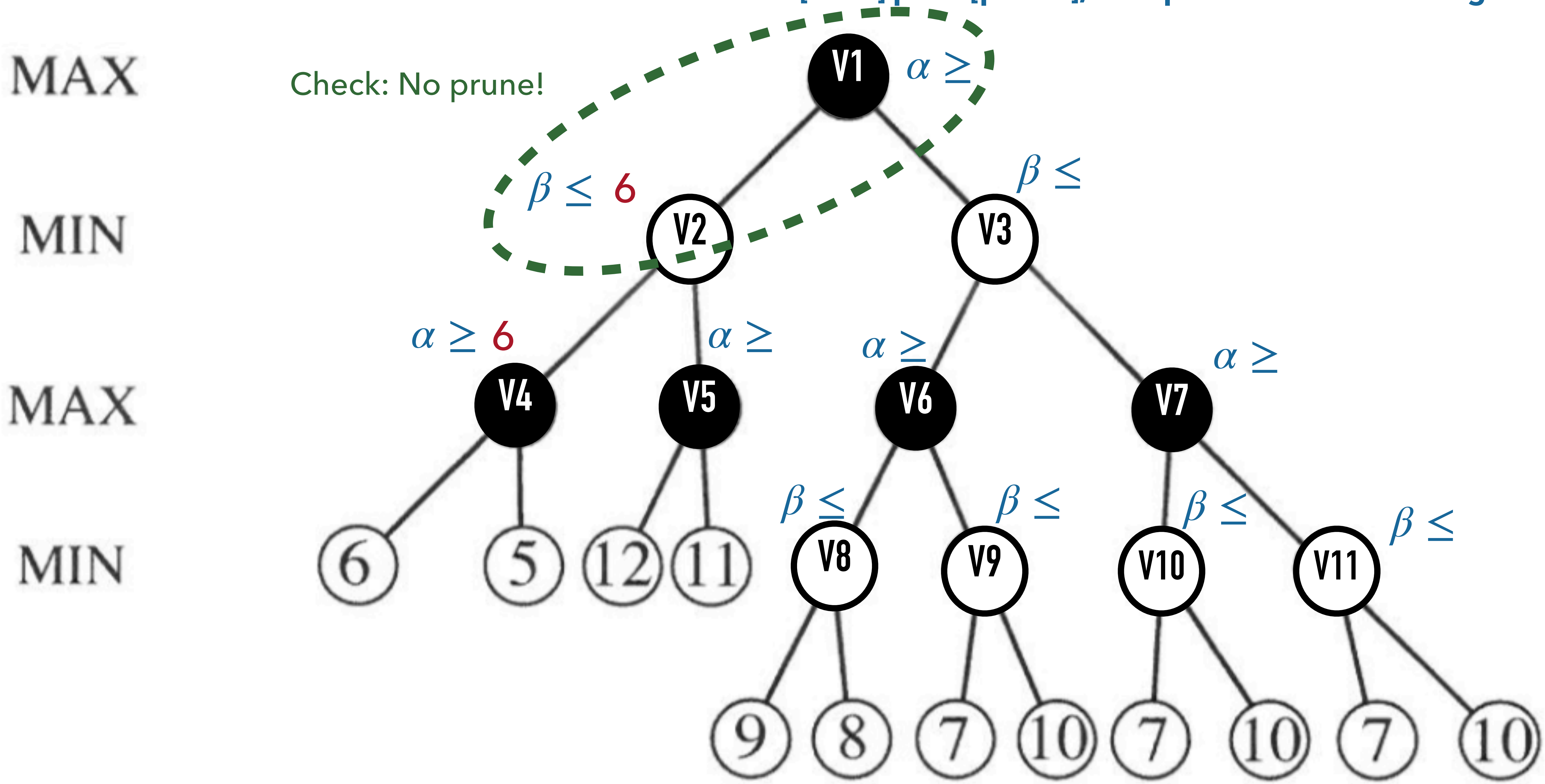

Show which arcs are pruned

EXTRA QUESTION

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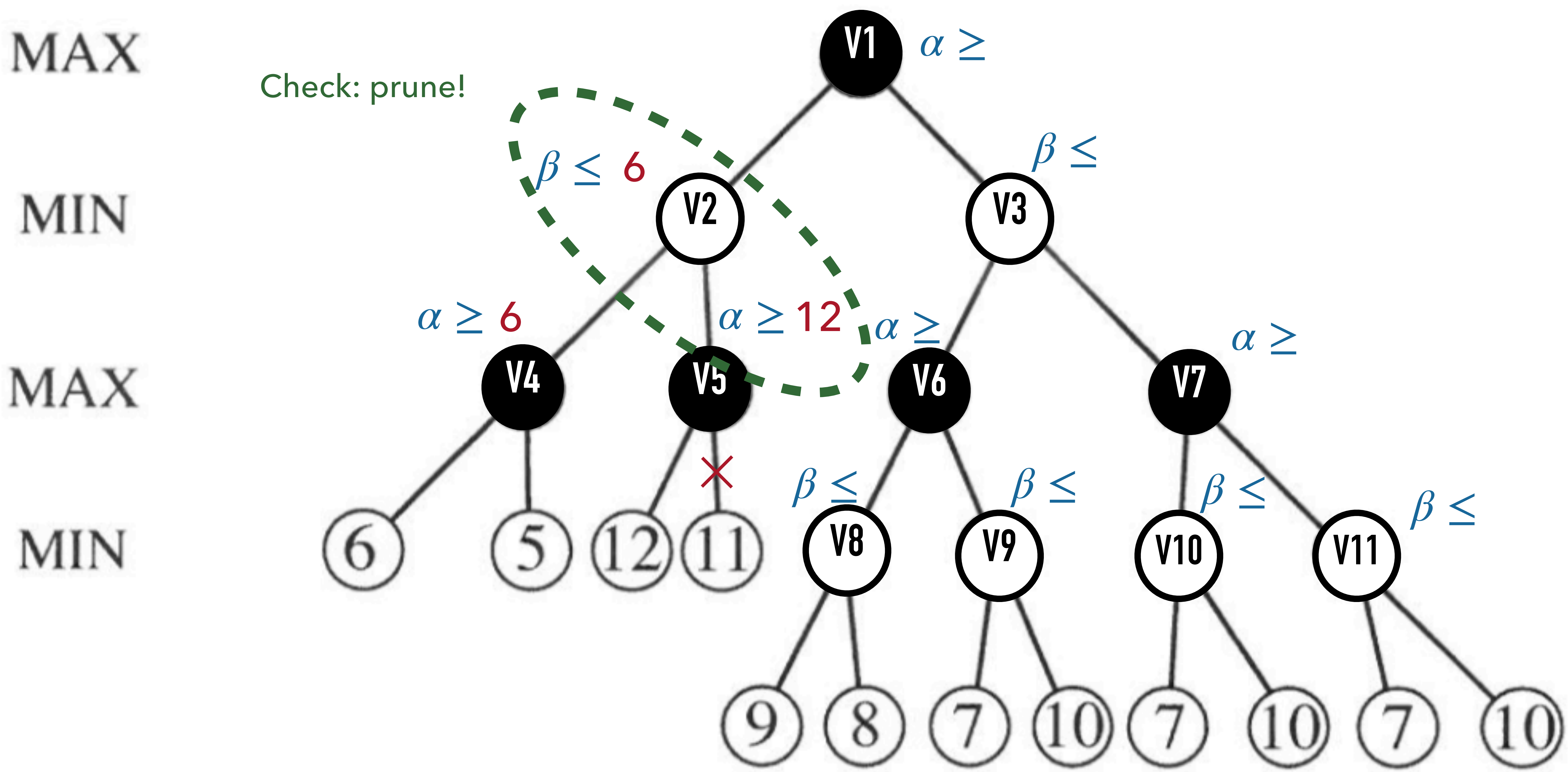

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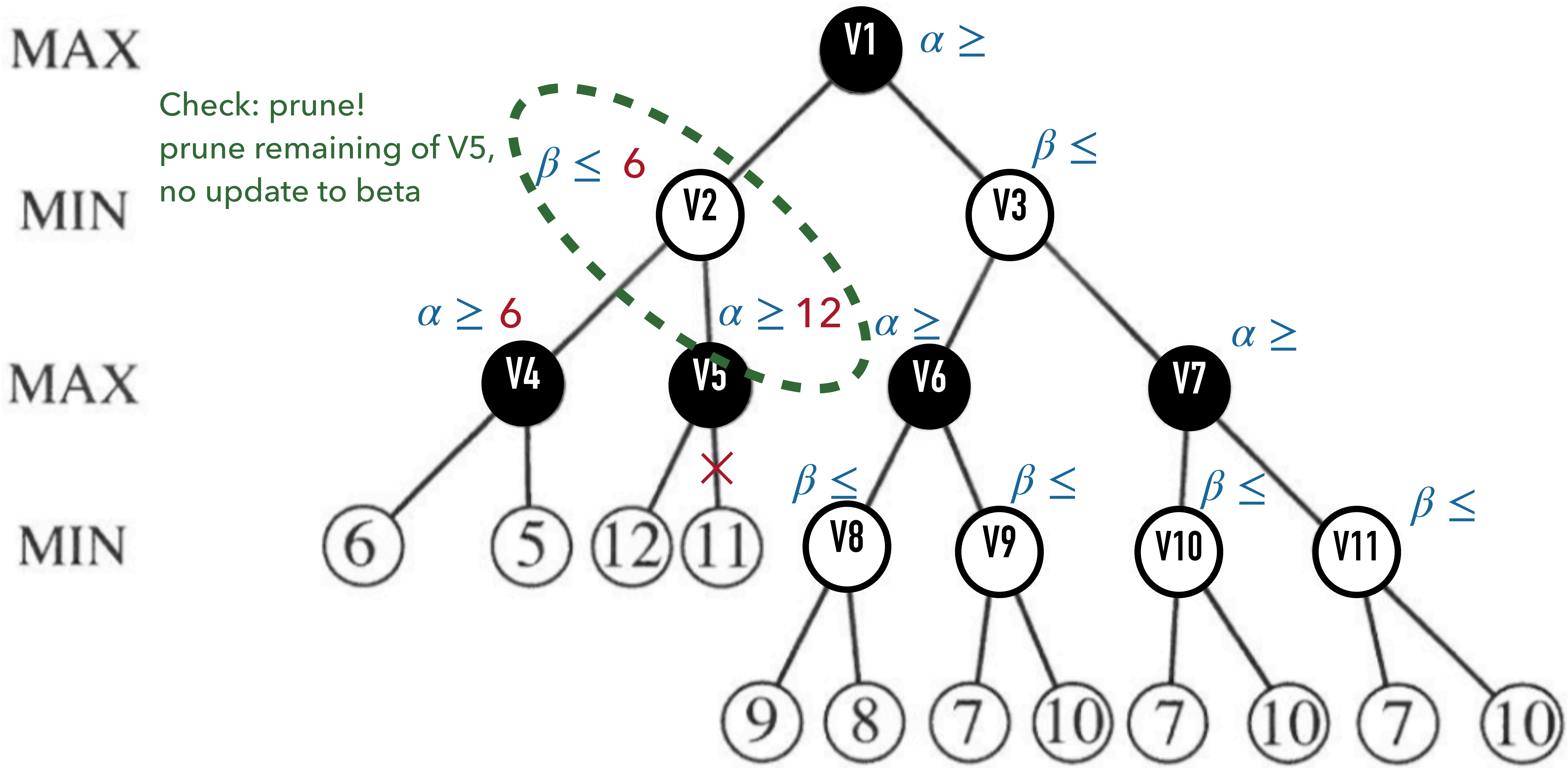

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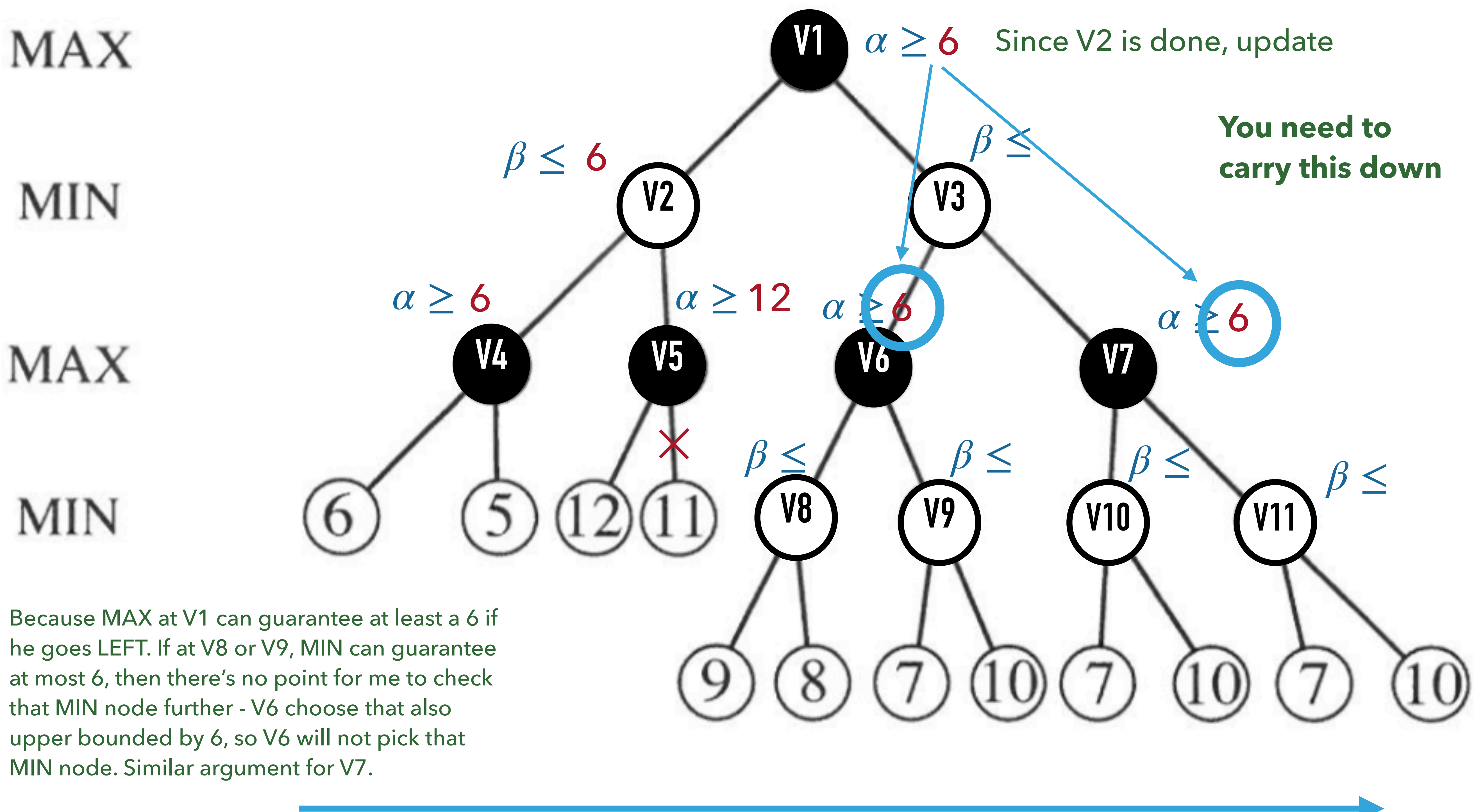

Because MAX at V1 can guarantee at least a 6 if he goes LEFT. If at V8 or V9, MIN can guarantee at most 6, then there's no point for me to check that MIN node further - V6 choose that also upper bounded by 6, so V6 will not pick that MIN node. Similar argument for V7.

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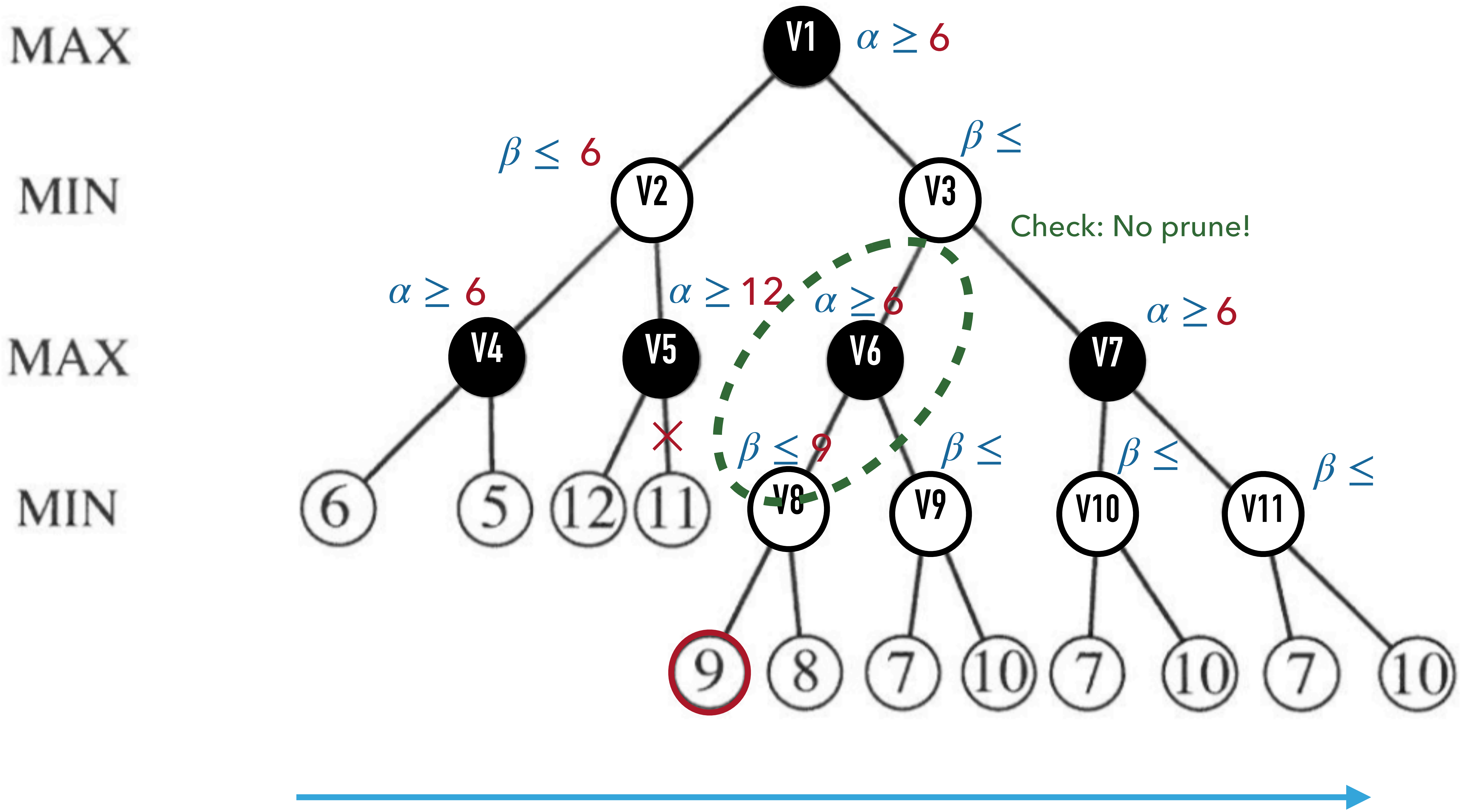

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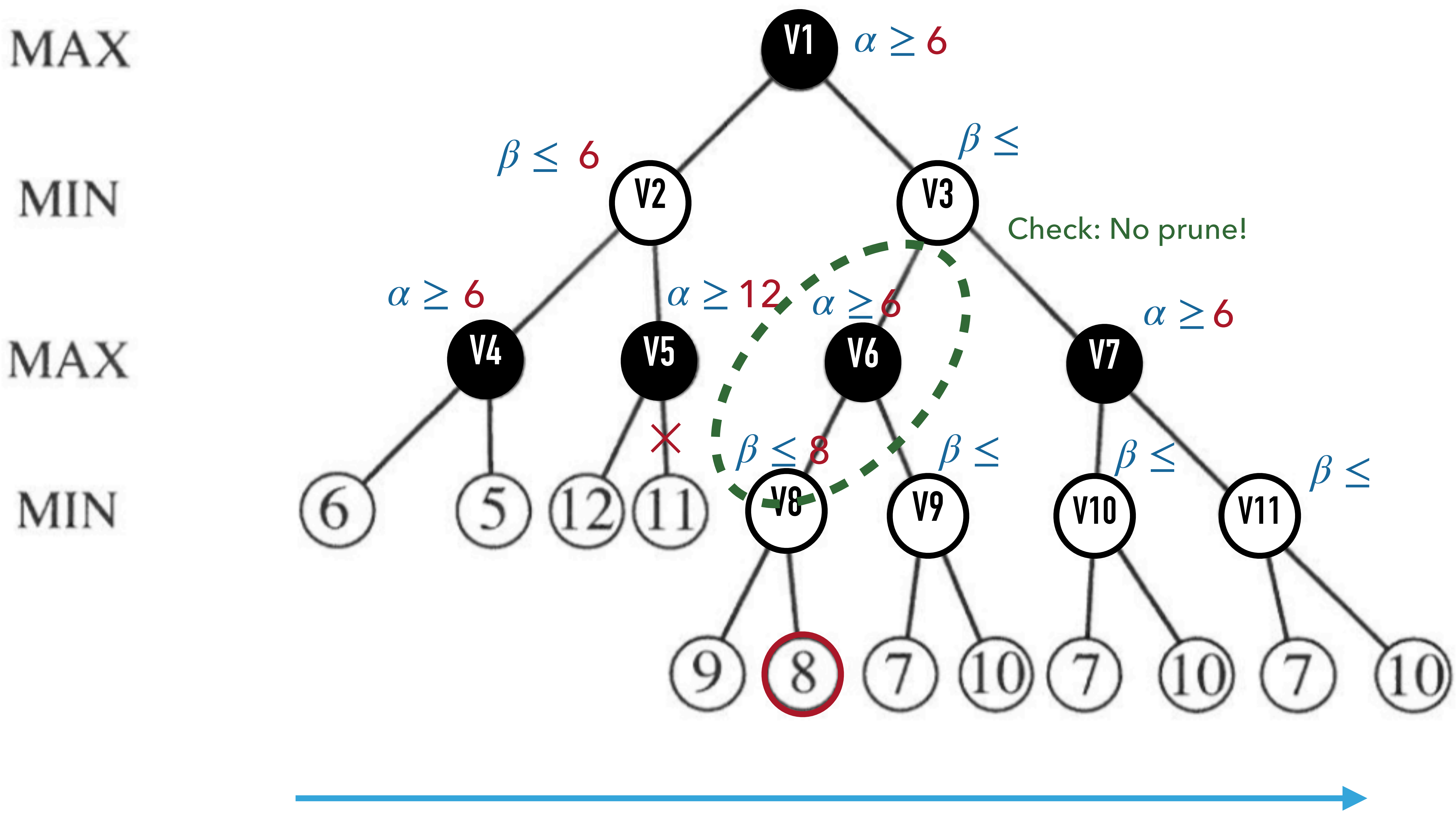

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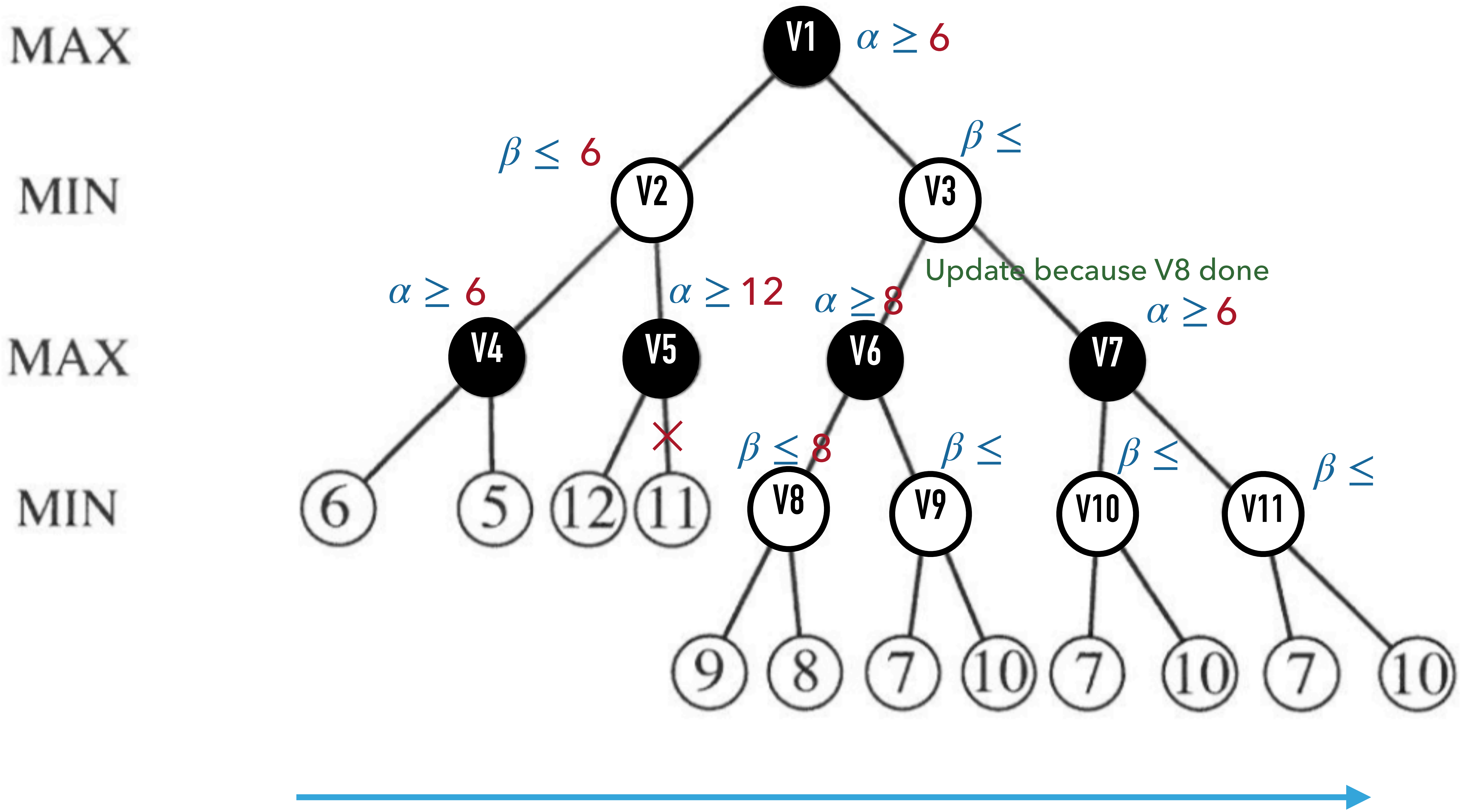

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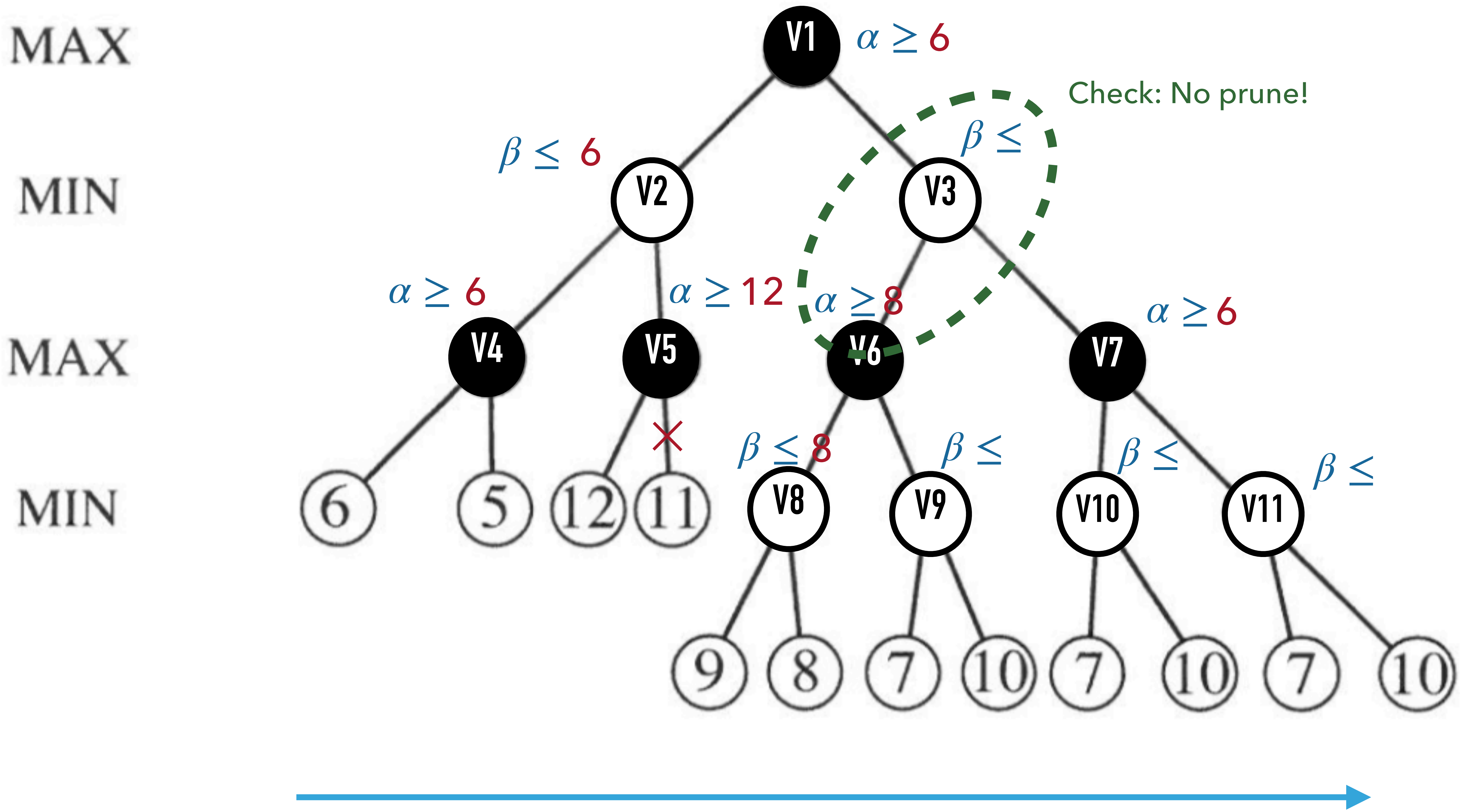

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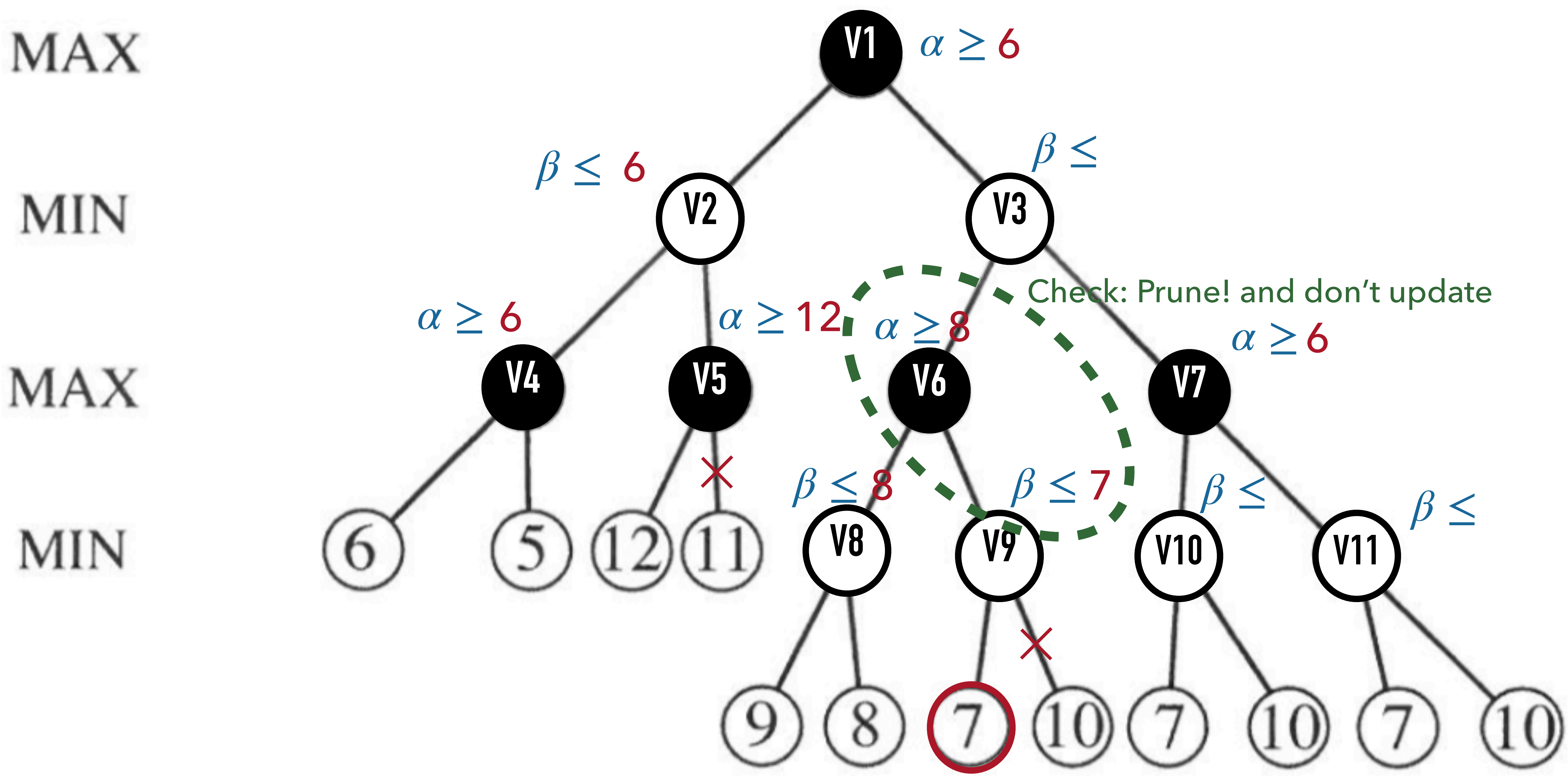

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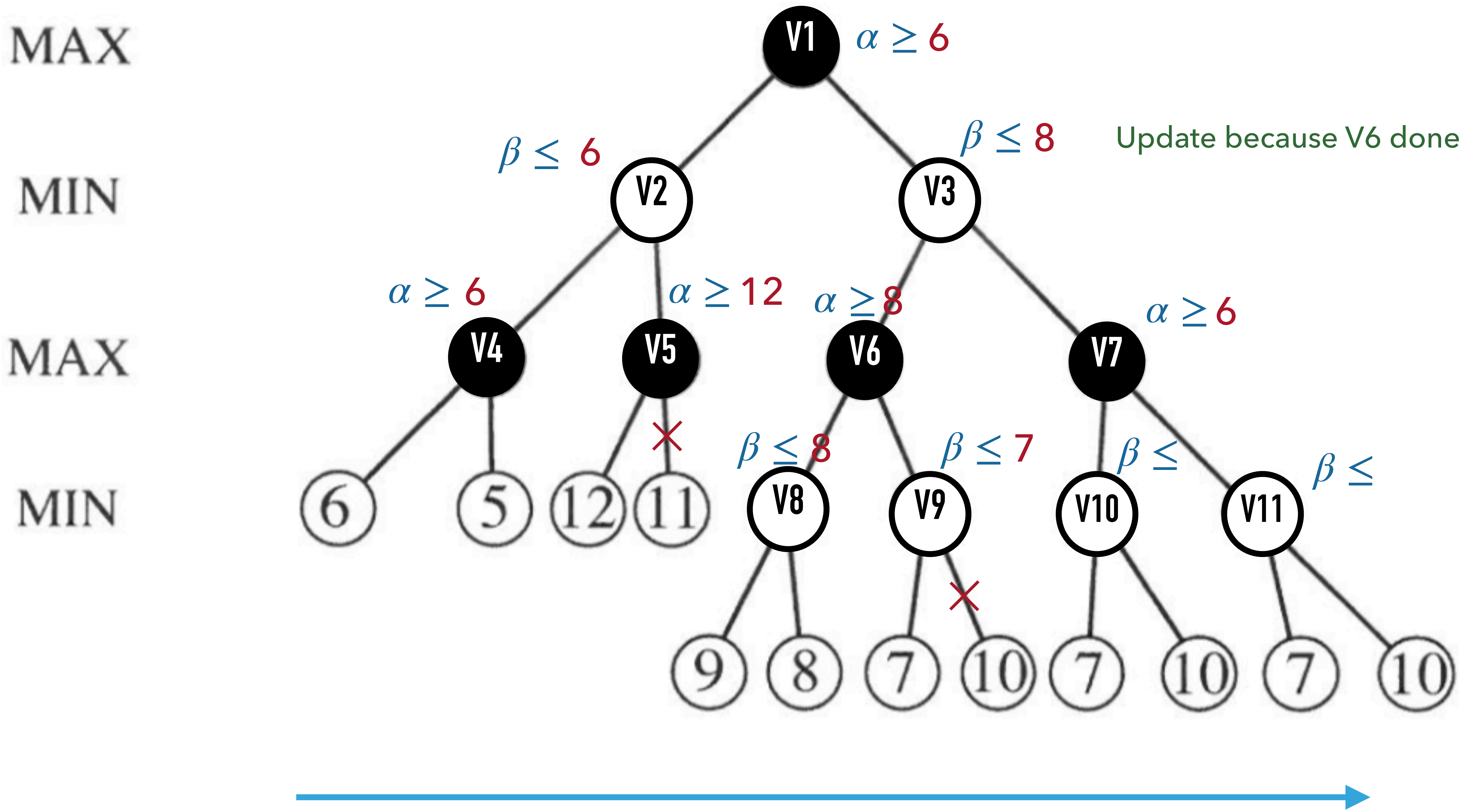

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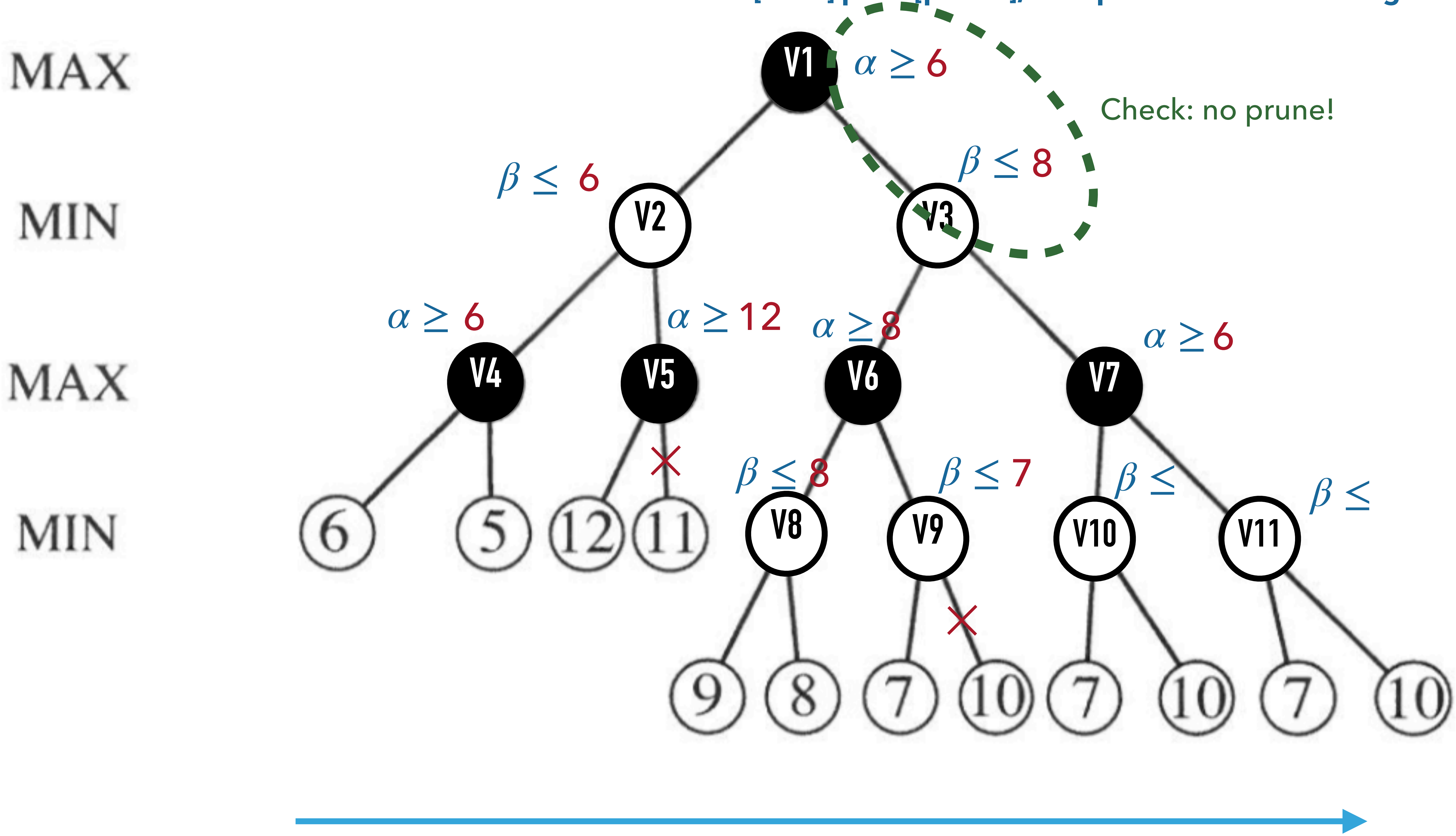

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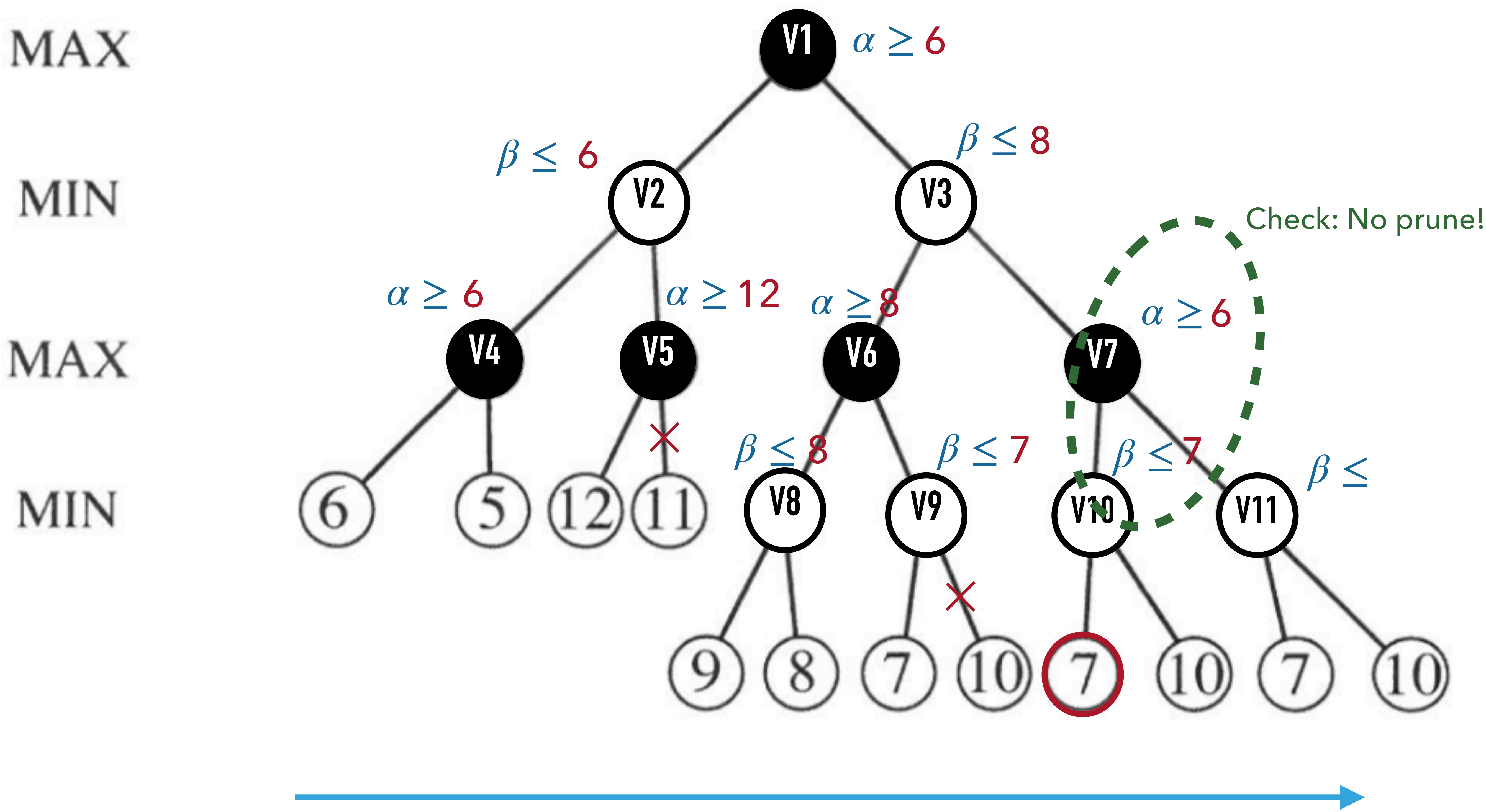

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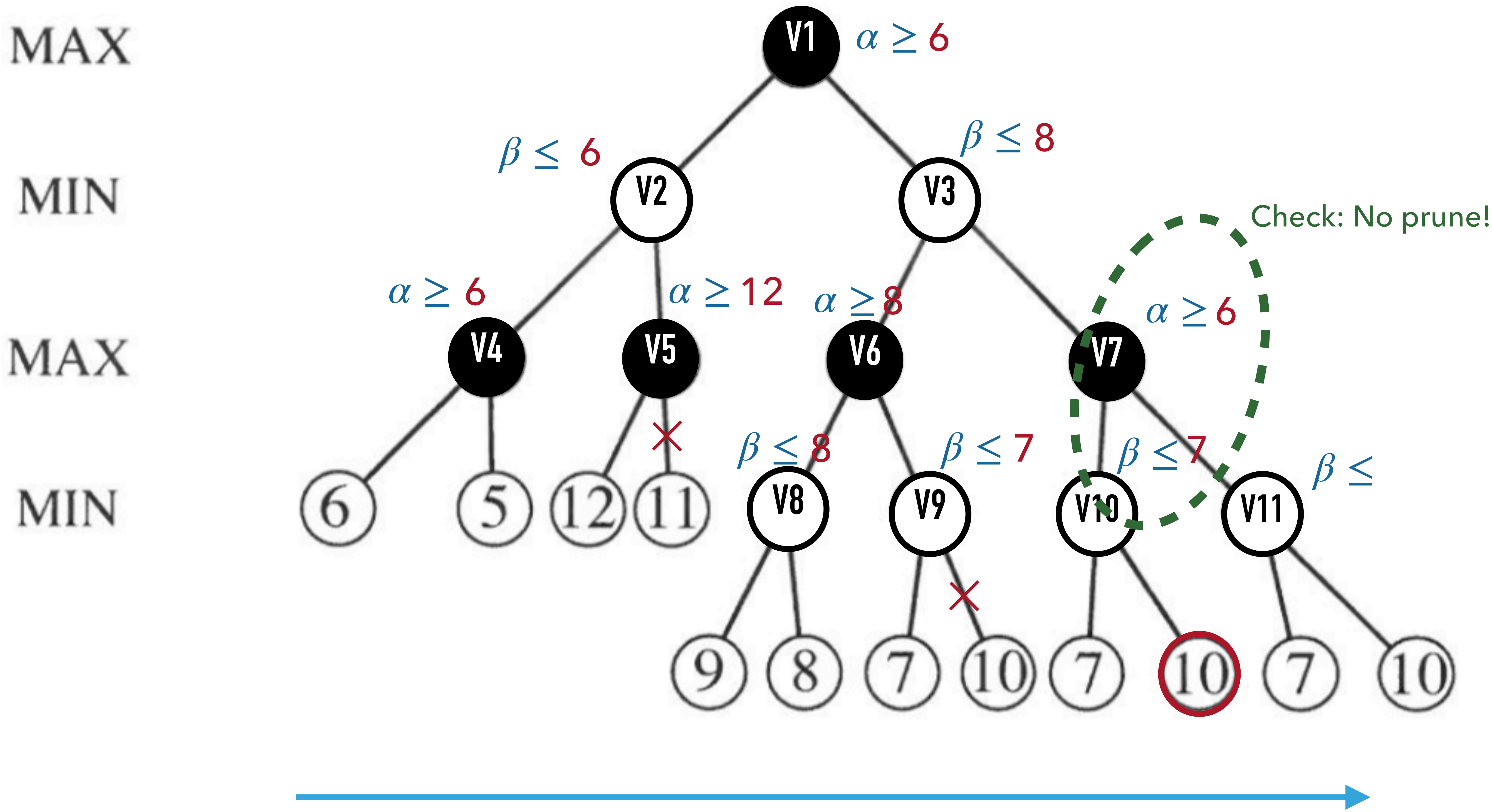

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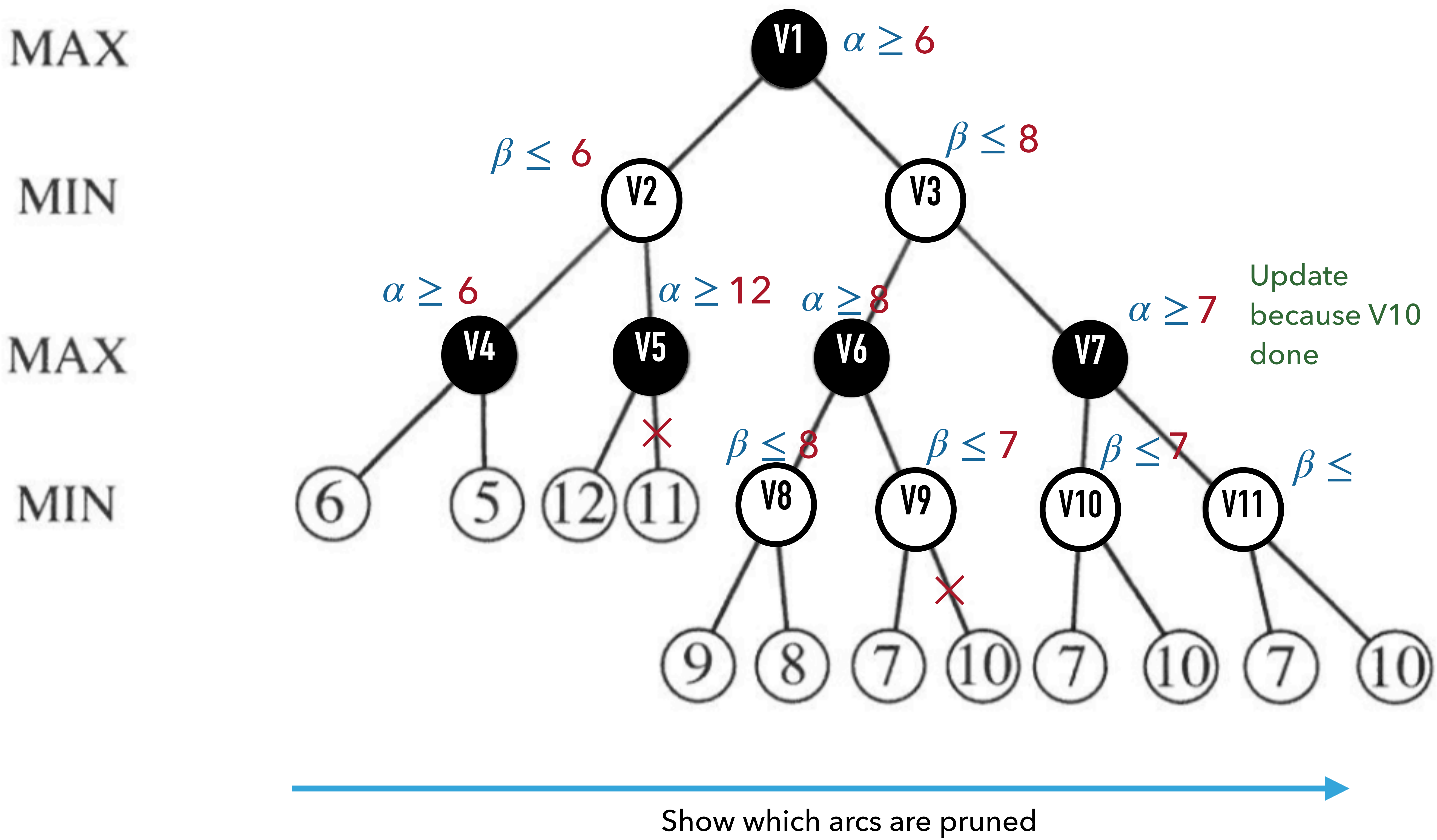

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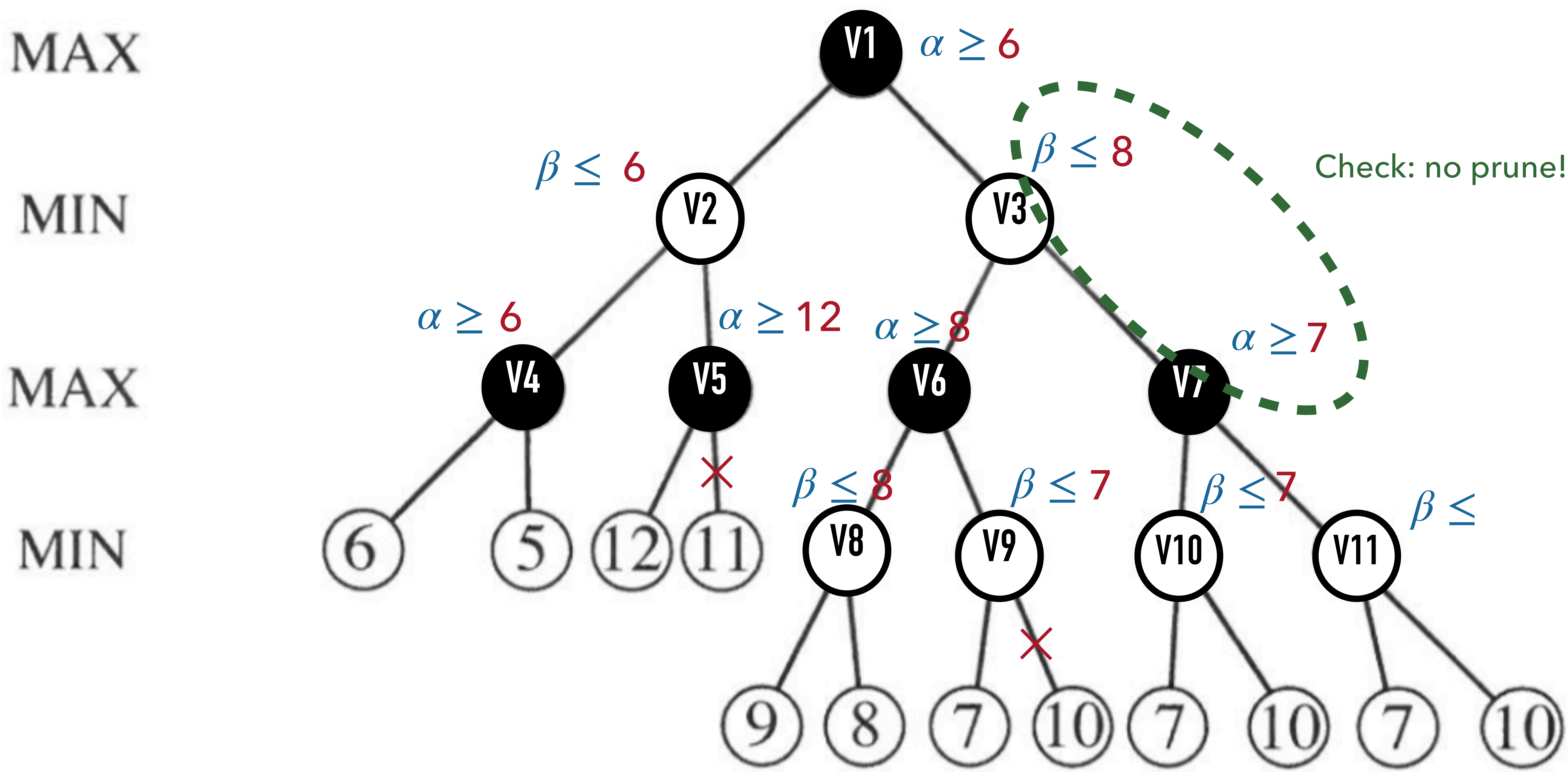

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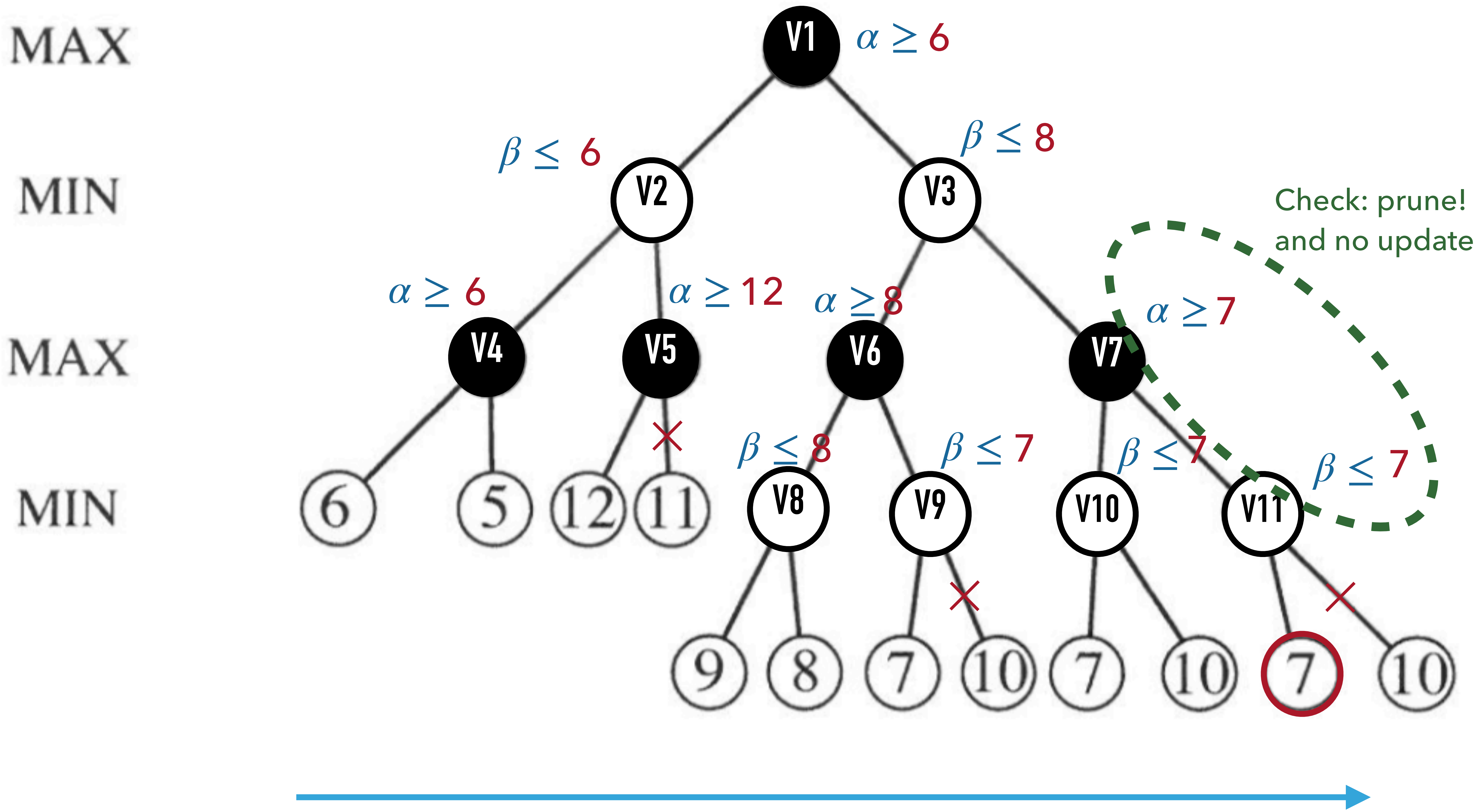

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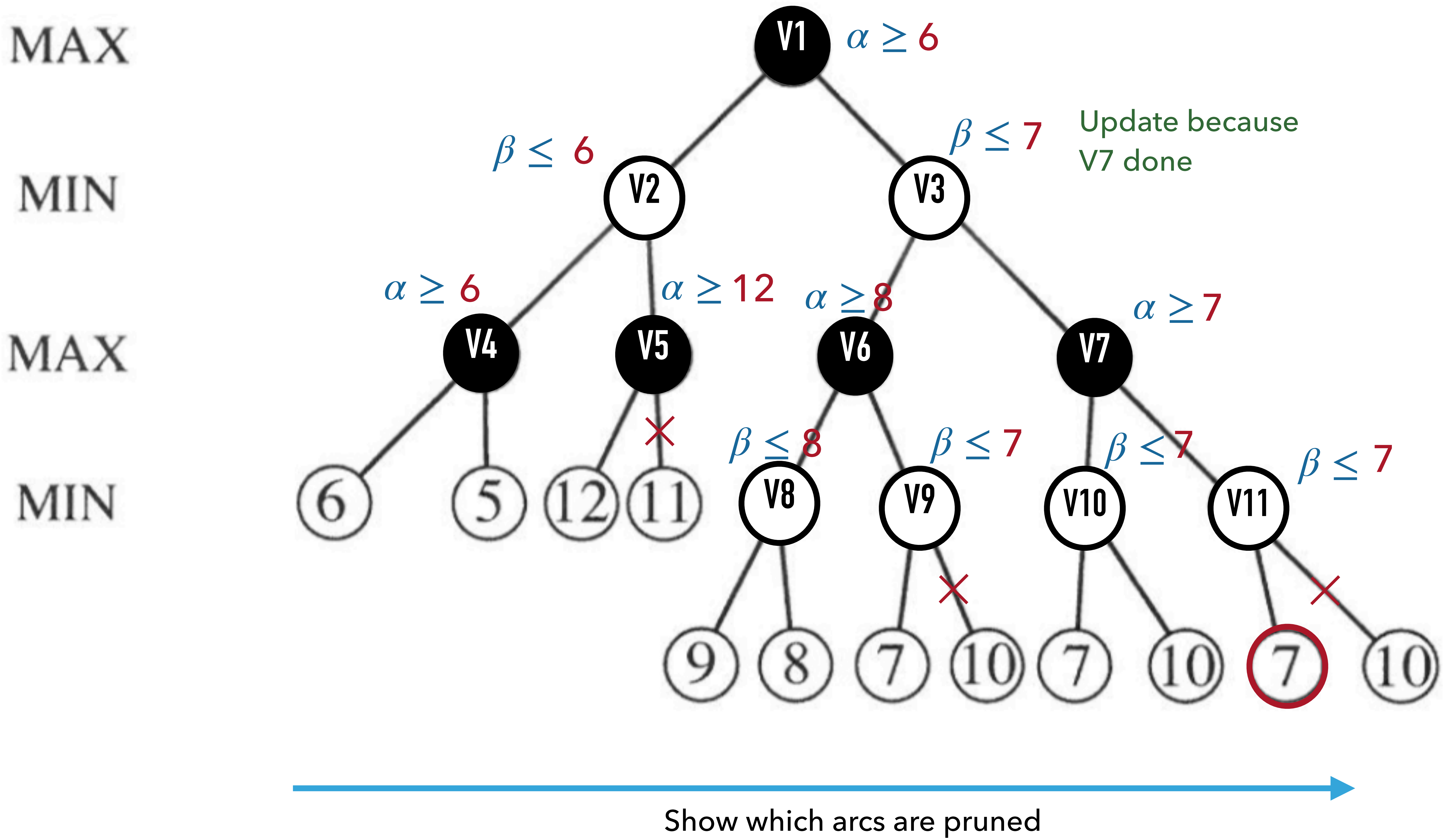

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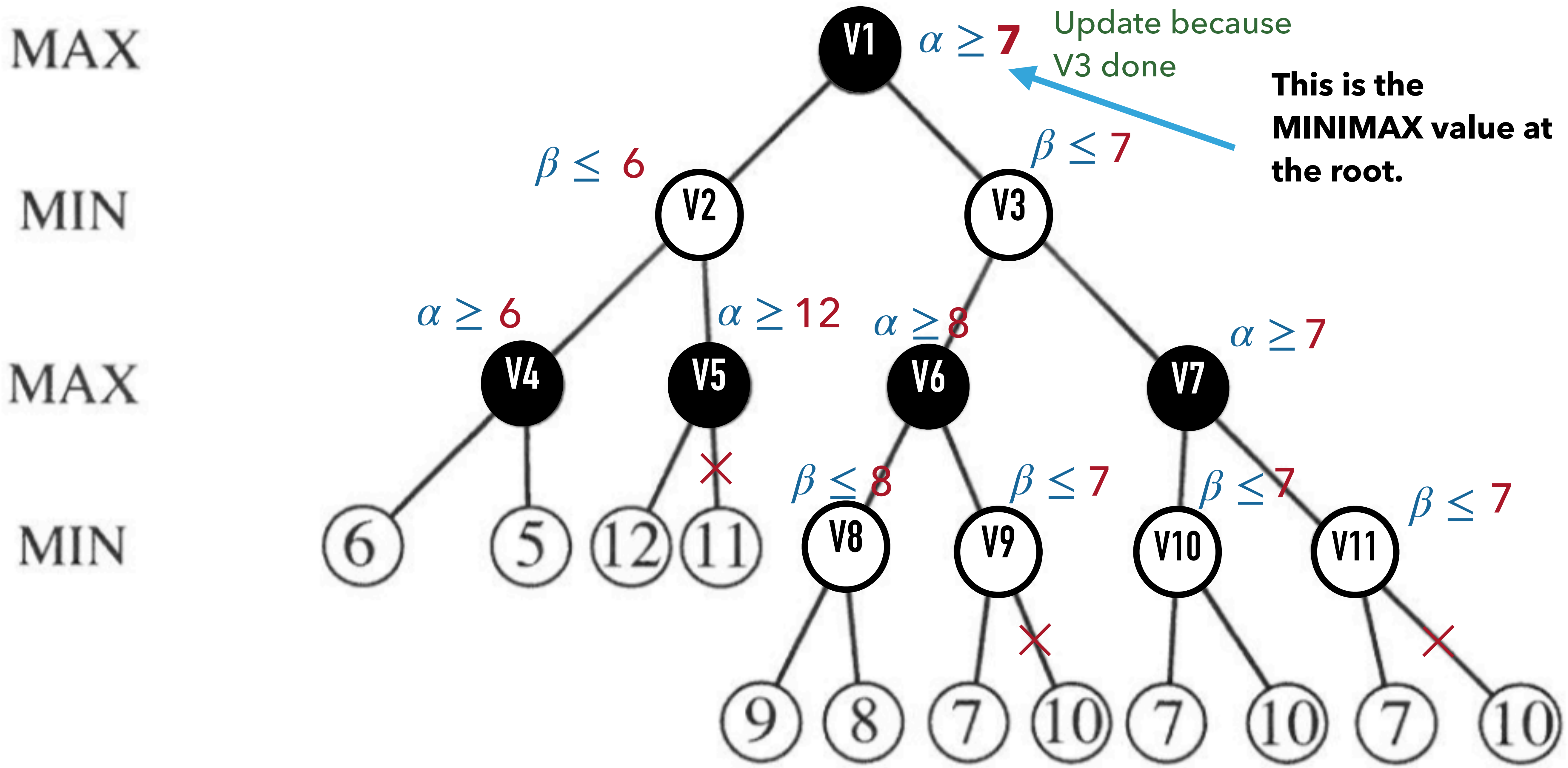


Show which arcs are pruned

TUTORIAL 5 QUESTION 2

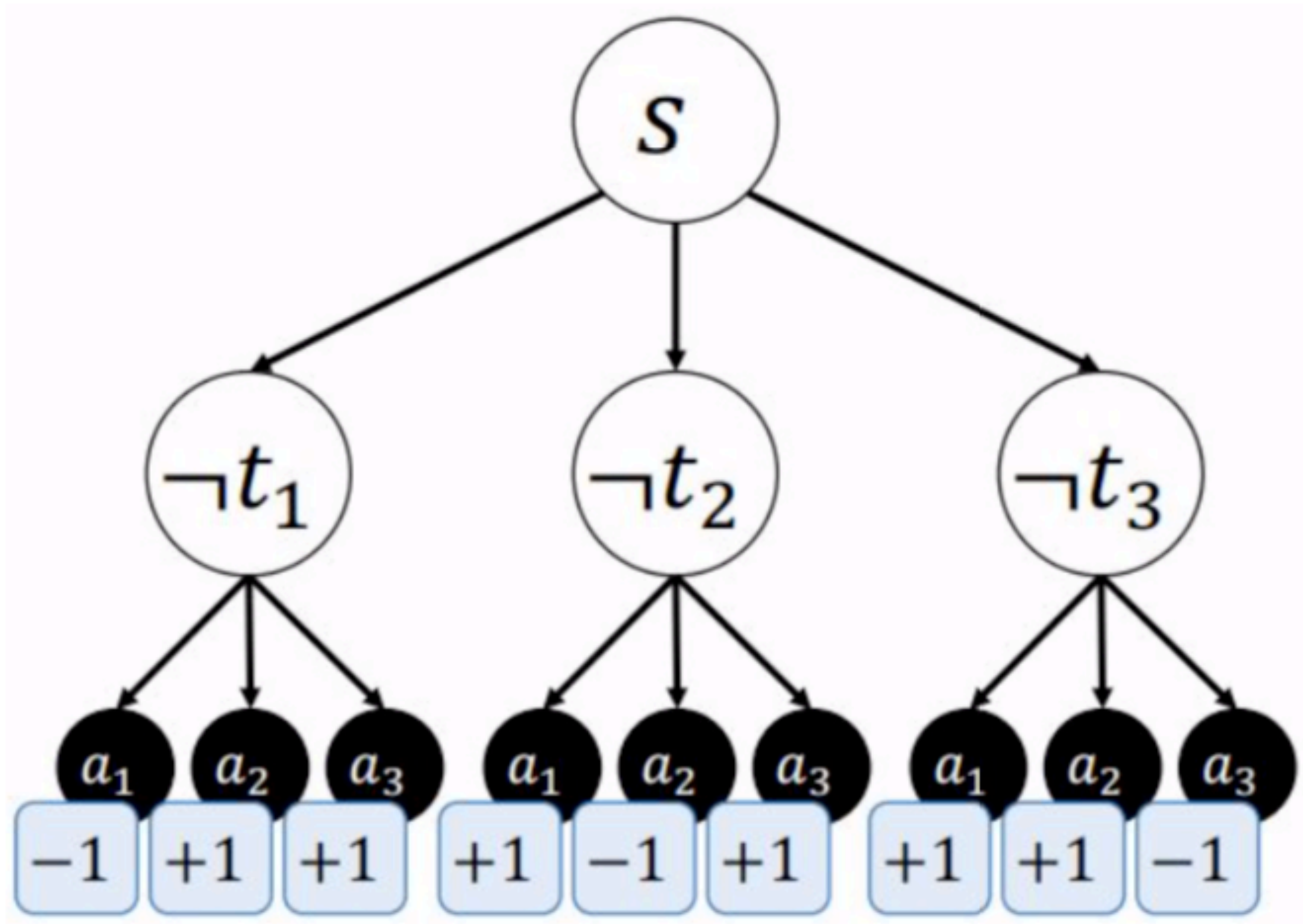
(Stackelberg Security Games)

- Consider the following game: we have an attacker looking at three targets: t_1, t_2 and t_3. A defender must choose which of the two targets it will guard; however, the attacker has an advantage: it can observe what the defender is doing before it chooses its move. If an attacker successfully attacks it receives a payoff of 1 and the defender gets a payoff of -1.

  (a) Model this problem as a minimax search problem. Draw out the search tree. What is the defender's payoff in this game?

TUTORIAL 5 QUESTION 2

- **Model this problem as a minimax search problem. Draw out the search tree. What is the defender's payoff in this game?**



TUTORIAL 5 QUESTION 3

- **Construct an example where, should the MIN player play sub-optimally, the MINIMAX algorithm makes a sub-optimal move.**