Compactness Measures for Gerrymandering

Geometry of Redistricting Workshop: Educator Track

Metric Geometry and Gerrymandering Group

Squaretopia

For each of the following measures, calculate its *average* over all the districts on your four maps of Squaretopia. For the first four measures, also record its *minimum* value on each map (i.e. its value for the worst district).

- **Skew measure:** W/L, where W is the district's shorter dimension (length or width) and L is its longer one.
- **Isoperimetric measure:** $16A/P^2$, where A is the district's area and P is its perimeter.
- **Square Reock measure:** A/S, where A is the district's area S is the area of the smallest square containing the district.
- **Convex Hull measure:** A/H, where A is the district's area and H is the area of its convex hull.
- **Total perimeter:** Total perimeter of all districts. (No need to average!)

	Skew	Isoperimetric	Reock	Convex Hull	Total Perimeter
Compact districts Average: Minimum:					
Proportional outcomes Average: Minimum:					
Gerrymandered for Gray Average: Minimum:					
Gerrymandered against Gray Average: Minimum:					

Which measure do you think is best for detecting gerrymandered districts?

The Real World

All of these have been proposed as compactness measures for detecting gerrymandering. In some cases, the versions we used above were simplified for ease of use in Squaretopia.

- **Harris:** W/L, where is L is its longest axis and W is its width perpendicular to that axis.
- **Polsby-Popper:** $4\pi A/P^2$, where P is the district's perimeter and and A is its area.
- **Reock:** A/C, where A is the district's area C is the area of the smallest circle containing the district.
- **Convex Hull:** A/H, where A is the district's area and H is the area of its convex hull.
- **Total perimeter:** Total perimeter of all districts.

Problems

1. Calculate the Polsby-Popper measure of a regular hexagon.
2. Find two shapes with identical Reock measures but different Convex Hull measures.
3. Show that the Polsby-Popper measure is equal to the ratio of a district's area to the area of a circle with the same perimeter as that district.
4. The *Schwarzberg measure* of a district is P/P_C, where P is the district's perimeter and P_C is the circumference of a circle with the same area as the district. What is the relationship between Schwarzberg measure and Polsby-Popper?
5. Find a shape with Convex Hull measure less than 0.5 but Harris measure more than 0.8.
6. Find a shape with Harris measure less than 0.5 but Convex Hull measure more than 0.8.
7. What problems do you think total perimeter has as a measure of gerrymandering? (These problems may be mathematical, practical, or legal.)
8. Find a family of shapes with Polsby-Popper measures arbitrarily close to 0 and Reock measures arbitrarily close to 1.
9. Prove the "isosquarimetric inequality" for contiguous regions made of unit squares: for a given perimeter P divisible by 4, $16A \leq P^2$, with equality only for squares. [1]
10. CHALLENGE: Find a shape with Polsby-Popper measure at least 0.5 and Reock measure as low as possible.

[1] Hint: first prove the inequality holds for rectangles with perimeter P. Then, if your shape is non-rectangular, draw the smallest possible rectangle surrounding it. What must be true of the areas and perimeters of your shape and its surrounding rectangle?