# ASSIGNMENT

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## INTRODUCTION

The Morris–Lecar model usually refers to a reduced two-variable model of action potential generation, originally formulated by Morris and Lecar in their study of barnacle muscle electrical activity. It was later popularized as a reduced model for neuronal excitability.

## PRINCIPAL ASSUMPTIONS

Generally, excitable systems have more than two relevant excitation variables, because there are often more than two species of gated channels and also because some channels have autonomous inactivation processes. Thus the primary assumption in using a two-dimensional model is that the true higher-order system can in fact be projected onto a two-dimensional phase space without altering the topological properties of the phase profile. This is true for the four-dimensional Hodgkin-Huxley system, which has a single singular point and exhibits excitation phenomena that can all be duplicated in two dimensions. There are other neural excitation phenomena such as bursting oscillations or chaotic firing which are intrinsically higher-dimensional, and cannot be duplicated in the phase plane.

The principal assumptions underlying the Morris-Lecar model include:

- Equations apply to a spatially iso-potential patch of membrane.
- There are two persistent (non-inactivating) voltage-gated currents with oppositely biased reversal potentials. The depolarizing current is carried by $Na^+$ or $Ca^{2+}$ ions (or both), depending on the system to be modeled, and the hyperpolarizing current is carried by $K^+$.
- Activation gates follow changes in membrane potential sufficiently rapidly that the activating conductance can instantaneously relax to its steady-state value at any voltage.
- The dynamics of the recovery variable can be approximated by a first-order linear differential equation for the probability of channel opening. This assumption is never exactly true, since channel proteins are composed of subunits, which must act in concert, to reach the open state. Despite missing delays in the onset of recovery, the model appears to be adequate for phase-plane considerations for many excitable systems.

# EQUATIONS

The Morris–Lecar model is a two-dimensional system of nonlinear differential equations. It is considered a simplified model compared to the four-dimensional Hodgkin–Huxley model.

Qualitatively, this system of equations describes the complex relationship between membrane potential and the activation of ion channels within the membrane: the potential depends on the activity of the ion channels, and the activity of the ion channels depends on the voltage. As bifurcation parameters are altered, different classes of neuron behavior are exhibited. $\tau_N$ is associated with the relative time scales of the firing dynamics, which varies broadly from cell to cell and exhibits significant temperature dependency.

It follows the following equations:

- $C(\frac{dV}{dt}) = I - g_L(V - V_L) - g_{Ca}M_{SS}(V - V_{Ca}) - g_K N(V - V_k)$
- $\frac{dN}{dt} = \frac{(N_{SS} - N)}{\tau_N}$

Where,

- $M_{SS} = \frac{1}{2}(1 + tanh\left[\frac{(V - V_1)}{V_2}\right])$
- $N_{SS} = \frac{1}{2}(1 + tanh\left[\frac{(V - V_3)}{V_4}\right])$
- $\tau_N = 1/(\varphi\, cosh\left[\frac{(V - V_3)}{2V_4}\right])$

**Note:** $M_{ss}$ and $N_{ss}$ equations may also be expressed as

$$M_{SS} = (1 + e^{\left[-2(V - V_1)/V_2\right]})^{-1} \text{ and } N_{SS} = (1 + e^{\left[-2(V - V_3)/V_4\right]})^{-1},$$

However most authors prefer the form using the hyperbolic functions.

## Variables

- $V$ : membrane potential
- $N$ : recovery variable: the probability that the K+ channel is conducting

## Parameters and constants

- $I$ : applied current
- $C$ : membrane capacitance
- $g_L$, $g_{Ca}$, $g_K$ : leak, $Ca^{++}$, and $K^+$ conductances through membranes channel
- $V_L$, $V_{Ca}$, $V_K$ : equilibrium potential of relevant ion channels
- $V_1$, $V_2$, $V_3$, $V_4$: tuning parameters for steady state and time constant
- $\varphi$: reference frequency

# SIMILARITY WITH HODGKIN-HUXLEY MODEL

The Morris–Lecar model is formulated in the Hodgkin–Huxley framework, with biophysically meaningful parameters and structure. It incorporates a slower hyperpolarizing potassium current and a fast non-inactivating calcium current (depolarizing and regenerative, like the Hodgkin–Huxley sodium current). In the simplest two-dimensional formulation, the activation of Ca2+ is assumed to be so fast that it is modeled as instantaneous.

In the Hodgkin-Huxley model repetitive firing emerges through a Hopf bifurcation with small amplitude and non-zero frequency. In fact, the Hodgkin-Huxley model can be tuned into parameter regimes that yield most of the dynamic behaviors that we described for the Morris-Lecar model. For increased $V_K$ steady state current becomes N-shaped, because $K^+$ now has a reversal potential that is not below resting $V$. If we also vary the temperature, the model will be tuned into plateauing behavior (coexistence of two steady states) or the type of behavior that we described for intermediate φ in Morris-Lecar model (resting state and a depolarized stable cycle)