

# Chapter 12

## Graphics

In this chapter we introduce some of the graphics capabilities of the calculator. We will present graphics of functions in Cartesian coordinates and polar coordinates, parametric plots, graphics of conics, bar plots, scatterplots, and a variety of three-dimensional graphs.

### Graphs options in the calculator

To access the list of graphic formats available in the calculator, use the keystroke sequence  $\leftarrow 2D/3D \left( F4 \right)$  Please notice that if you are using the RPN mode these two keys must be pressed simultaneously to activate any of the graph functions. After activating the 2D/3D function, the calculator will produce the PLOT SETUP window, which includes the TYPE field as illustrated below.



Right in front of the TYPE field you will, most likely, see the option *Function* highlighted. This is the default type of graph for the calculator. To see the list of available graph types, press the soft menu key labeled  $\left[ \text{MENU} \right]$ . This will produce a drop down menu with the following options (use the up- and down- arrow keys to see all the options):



These graph options are described briefly next.

*Function*: for equations of the form  $y = f(x)$  in plane Cartesian coordinates

*Polar*: for equations of the form  $r = f(\theta)$  in polar coordinates in the plane

*Parametric*: for plotting equations of the form  $x = x(t)$ ,  $y = y(t)$  in the plane

*Diff Eq*: for plotting the numerical solution of a linear differential equation

*Conic*: for plotting conic equations (circles, ellipses, hyperbolas, parabolas)

*Truth*: for plotting inequalities in the plane

*Histogram*: for plotting frequency histograms (statistical applications)

*Bar*: for plotting simple bar charts

*Scatter*: for plotting scatter plots of discrete data sets (statistical applications)

*Slopefield*: for plotting traces of the slopes of a function  $f(x,y) = 0$ .

*Fast3D*: for plotting curved surfaces in space

*Wireframe*: for plotting curved surfaces in space showing wireframe grids

*Ps-Contour*: for plotting contour plots of surfaces

*Y-Slice*: for plotting a slicing view of a function  $f(x,y)$ .

*Gridmap*: for plotting real and imaginary part traces of a complex function




*Pr-Surface*: for parametric surfaces given by  $x = x(u,v)$ ,  $y = y(u,v)$ ,  $z = z(u,v)$ .

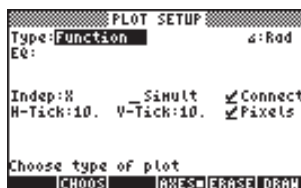
## Plotting an expression of the form $y = f(x)$

In this section we present an example of a plot of a function of the form  $y = f(x)$ . In order to proceed with the plot, first, purge the variable  $x$ , if it is defined in the current directory ( $x$  will be the independent variable in the calculator's PLOT feature, therefore, you don't want to have it pre-defined). Create a sub-directory called 'TPLOT' (for test plot), or other meaningful name, to perform the following exercise.

As an example, let's plot the function,

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right)$$

- First, enter the PLOT SETUP environment by pressing,  **2D/3D**. Make sure that the option Function is selected as the TYPE, and that 'X' is selected as the independent variable (INDEP). Press   to return to normal calculator display. The PLOT SET UP window should look similar to this:



• **Note:** You will notice that a new variable, called PPAR, shows up in your soft menu key labels. This stands for Plot PARAmeters. To see its contents, press  $\rightarrow$   $\boxed{\text{PPAR}}$ . A detailed explanation of the contents of PPAR is provided later in this Chapter. Press  $\leftarrow$  to drop this line from the stack.


- Enter the PLOT environment by pressing  $\leftarrow$   $\boxed{\text{Y=}}$  (press them simultaneously if in RPN mode). Press  $\boxed{\text{EQ}}$  to get you into the equation writer. You will be prompted to fill the right-hand side of an equation  $Y1(X) = \blacksquare$ . Type the function to be plotted so that the Equation Writer shows the following:




- Press  $\boxed{\text{ENTER}}$  to return to the PLOT - FUNCTION window. The expression ' $Y1(X) = \text{EXP}(-X^2/2)/\sqrt{(2*\pi)}$ ' will be highlighted. Press  $\boxed{\text{NXT}}$   $\boxed{\text{EQ}}$  to return to normal calculator display.

**Note:** Two new variables show up in your soft menu key labels, namely EQ and Y1. To see the contents of EQ, use  $\rightarrow$   $\boxed{\text{EQ}}$ . The content of EQ is simply the function name 'Y1(X)'. The variable EQ is used by the calculator to store the equation, or equations, to plot.














To see the contents of Y1 press  $\rightarrow$   $\boxed{\text{Y1}}$ . You will get the function Y1(X) defined as the program:

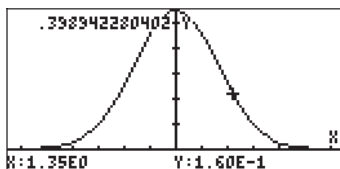
<< →X 'EXP(-X^2/2)/√(2\*π)' >>.



Press , twice, to drop the contents of the stack.

- Enter the PLOT WINDOW environment by entering   (press them simultaneously if in RPN mode). Use a range of -4 to 4 for H-VIEW, then press  to generate the V-VIEW automatically. The PLOT WINDOW screen looks as follows:



- Plot the graph:   (wait till the calculator finishes the graphs)
- To see labels:    
- To recover the first graphics menu:   
- To trace the curve:  . Then use the right- and left-arrow keys ( ) to move about the curve. The coordinates of the points you trace will be shown at the bottom of the screen. Check that for  $x = 1.05$ ,  $y = 0.231$ . Also, check that for  $x = -1.48$ ,  $y = 0.134$ . Here is picture of the graph in tracing mode:



- To recover the menu, and return to the PLOT WINDOW environment, press  .

## Some useful PLOT operations for FUNCTION plots

In order to discuss these PLOT options, we'll modify the function to force it to have some real roots (Since the current curve is totally contained above the x axis, it has no real roots.) Press  $\rightarrow$   $\text{VARS}$  to list the contents of the function Y1 on the stack:  $\ll \rightarrow X \text{'EXP(-X^2/2)}/\sqrt{(2*\pi)} \text{' } \gg$ . To edit this expression use:

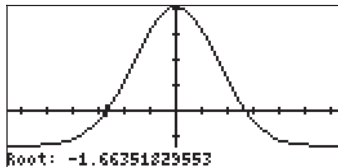
$\nabla$	Launches the line editor
$\rightarrow$ $\nabla$	Moves cursor to the end of the line
$\leftarrow$ $\leftarrow$ $\leftarrow$ $-$ $0$ $.$ $ $	Modifies the expression
ENTER	Returns to calculator display

Next, store the modified expression into variable y by using  $\leftarrow$   $\text{VARS}$  if in RPN mode, or  $\leftarrow$   $\text{ANS}$   $\text{STO}$   $\text{VARS}$  in ALG mode.

The function to be plotted is now,  $f(x) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{x^2}{2}) - 0.1$

Enter the PLOT WINDOW environment by entering  $\leftarrow$   $\text{WIN}$  (press them simultaneously if in RPN mode.) Keep the range of -4 to 4 for H-VIEW, press  $\nabla$   $\text{VIEWS}$  to generate the V-VIEW. To plot the graph, press  $\text{F6}$   $\text{F6}$

- Once the graph is plotted, press  $\text{F7}$  to access the *function* menu. With this menu you can obtain additional information about the plot such as intersects with the x-axis, roots, slopes of the tangent line, area under the curve, etc.
- For example, to find the root on the left side of the curve, move the cursor near that point, and press  $\text{F8}$ . You will get the result: ROOT: -1.6635.... Press  $\text{NXT}$  to recover the menu. Here is the result of ROOT in the current plot:



- If you move the cursor towards the right-hand side of the curve, by pressing the right-arrow key ( $\rightarrow$ ), and press  $\text{F8}$ , the result now is

ROOT: 1.6635... The calculator indicated, before showing the root, that it was found through *SIGN REVERSAL*. Press **NXT** to recover the menu.

- Pressing **□□□□** will give you the intersection of the curve with the x-axis, which is essentially the root. Place the cursor exactly at the root and press **□□□□**. You will get the same message as before, namely *SIGN REVERSAL*, before getting the result I-SECT: 1.6635.... The **□□□□** function is intended to determine the intersection of any two curves closest to the location of the cursor. In this case, where only one curve, namely,  $Y1(X)$ , is involved, the intersection sought is that of  $f(x)$  with the x-axis, however, you must place the cursor right at the root to produce the same result. Press **NXT** to recover the menu.
- Place the cursor on the curve at any point and press **□□□□** to get the value of the slope at that point. For example, at the negative root, SLOPE: 0.16670.... Press **NXT** to recover the menu.
- To determine the highest point in the curve, place the cursor near the vertex and press **□□□□**. The result is EXTRM: 0.. Press **NXT** to recover the menu.
- Other buttons available in the first menu are **□□□□** to calculate the area under the curve, and **□□□□** to shade an area under the curve. Press **NXT** to see more options. The second menu includes one button called **□□□□** that flashes for a few seconds the equation plotted. Press **□□□□**. Alternatively, you can press the button **□□□□** (NEXT eQuation) to see the name of the function  $Y1(x)$ . Press **NXT** to recover the menu.
- The button **□□□□** gives the value of  $f(x)$  corresponding to the cursor position. Place the cursor anywhere in the curve and press **□□□□**. The value will be shown in the lower left corner of the display. Press **NXT** to recover the menu.
- Place the cursor in any given point of the trajectory and press **TANL** to obtain the equation of the tangent line to the curve at that point. The equation will be displayed on the lower left corner of the display. Press **NXT** to recover the menu.
- If you press **□□□□** the calculator will plot the derivative function,  $f'(x) = df/dx$ , as well as the original function,  $f(x)$ . Notice that the two curves intercept at two points. Move the cursor near the left intercept point and press **□□□□ □□□□**, to get I-SECT: (-0.6834...,0.21585). Press **NXT** to recover the menu.
- To leave the FCN environment, press **□□□□** (or **NXT □□□□**).
- Press **□□□□** to return to the PLOT WINDOW environment. Then, press **NXT □□□□** to return to normal calculator display.

**Note:** the stack will show all the graph operations performed, properly identified.

- Enter the PLOT environment by pressing, simultaneously if in RPN mode,  $\leftarrow$   $\frac{Y}{X}$  . Notice that the highlighted field in the PLOT environment now contains the derivative of  $Y1(X)$ . Press  $\text{NXT}$   $\text{QUIT}$  to return to return to normal calculator display.
- Press  $\rightarrow$   $\text{EQ}$  to check the contents of EQ. You will notice that it contains a list instead of a single expression. The list has as elements an expression for the derivative of  $Y1(X)$  and  $Y1(X)$  itself. Originally, EQ contained only  $Y1(x)$ . After we pressed  $\text{F}'$  in the  $\text{PLOT}$  environment, the calculator automatically added the derivative of  $Y1(x)$  to the list of equations in EQ.

## Saving a graph for future use

If you want to save your graph to a variable, get into the PICTURE environment by pressing  $\leftarrow$  . Then, press  $\text{QUIT}$   $\text{NXT}$   $\text{NXT}$   $\text{QUIT}$   $\rightarrow$  . This captures the current picture into a graphics object. To return to the stack, press  $\text{QUIT}$   $\text{QUIT}$  .

In level 1 of the stack you will see a graphics object described as Graphic  $131 \times 64$ . This can be stored into a variable name, say, PIC1.

To display your figure again, recall the contents of variable PIC1 to the stack. The stack will show the line: Graphic  $131 \times 64$ . To see the graph, enter the PICTURE environment, by pressing  $\leftarrow$  .

Clear the current picture,  $\text{QUIT}$   $\text{NXT}$   $\text{QUIT}$  .

Move the cursor to the upper left corner of the display, by using the  $\leftarrow$  and  $\triangleup$  keys.

To display the figure currently in level 1 of the stack press  $\text{NXT}$  REPL .

To return to normal calculator function, press **2ND** **MODE**.

**Note:** To save printing space, we will not include more graphs produced by following the instructions in this Chapter. The user is invited to produce those graphics on his or her own.

## Graphics of transcendental functions

In this section we use some of the graphics features of the calculator to show the typical behavior of the natural log, exponential, trigonometric and hyperbolic functions. You will not see more graphs in this chapter, instead the user should see them in the calculator.

### Graph of $\ln(X)$

Press, simultaneously if in RPN mode, the left-shift key **⇐** and the **2D/3D** (**F4**) key to produce the PLOT SETUP window. The field labeled **Type** will be highlighted. If the option **Function** is not already selected press the soft key labeled **MODE**, use the up and down keys to select **Function**, and press **OK** to complete the selection. Check that the field labeled **Indep:** contains the variable 'X'. If that is not so, press the down arrow key twice until the **Indep** field is highlighted, press the soft key labeled **MODE** and modify the value of the independent variable to read 'X'. Press **OK** when done. Press **NEXT** **OK** to return to normal calculator display.

Next, we'll resize the plot window. First, press, simultaneously if in RPN mode, the left-shift key **⇐** and the **Y=** (**F1**) key to produce the PLOT-FUNCTION window. If there is any equation highlighted in this window, press **DEL** as needed to clear the window completely. When the PLOT-FUNCTION window is empty you will get a prompt message that reads: **No Equ., Press ADD**. Press the soft key labeled **MODE**. This will trigger the equation writer with the expression **Y1(X)=**. Type **LN(X)**. Press **ENTER** to return to the PLOT-FUNCTION window. Press **NEXT** **OK** to return to normal calculator display.

The next step is to press, simultaneously if in RPN mode, the left-shift key **⇐** and the **WIN** (**F2**) key to produce the PLOT WINDOW - FUNCTION window. Most likely, the display will show the horizontal (H-View) and vertical (V-View) ranges as: H-View: -6.5    6.5, V-View: -3.9    4.0

These are the default values for the x- and y-range, respectively, of the current graphics display window. Next, change the H-View values to read: H-View: -1



10 by using  $\boxed{\div} \boxed{+/-} \boxed{0.00} \boxed{\div} \boxed{0} \boxed{.00}$ . Next, press the soft key labeled  $\boxed{\text{PLOT}}$  to let the calculator determine the corresponding vertical range. After a couple of seconds this range will be shown in the PLOT WINDOW-FUNCTION window. At this point we are ready to produce the graph of  $\ln(X)$ . Press  $\boxed{\text{GRAPH}}$  to plot the natural logarithm function.

To add labels to the graph press  $\boxed{\text{F1}} \boxed{\text{NXT}} \boxed{\text{F2}}$ . Press  $\boxed{\text{F1}}$  to remove the menu labels, and get a full view of the graph. Press  $\boxed{\text{NXT}}$  to recover the current graphic menu. Press  $\boxed{\text{NXT}} \boxed{\text{F1}}$  to recover the original graphical menu.

To determine the coordinates of points on the curve press  $\boxed{\text{F2}}$  (the cursor moves on top of the curve at a point located near the center of the horizontal range). Next, press  $\boxed{\text{X,Y}}$  to see the coordinates of the current cursor location. These coordinates will be shown at the bottom of the screen. Use the right- and left-arrow keys to move the cursor along the curve. As you move the cursor along the curve the coordinates of the curve are displayed at the bottom of the screen. Check that when  $Y:1.00E0$ ,  $X:2.72E0$ . This is the point  $(e, 1)$ , since  $\ln(e) = 1$ . Press  $\boxed{\text{NXT}}$  to recover the graphics menu.

Next, we will find the intersection of the curve with the x-axis by pressing  $\boxed{\text{F1}} \boxed{\text{F1}}$ . The calculator returns the value  $\text{Root: } 1$ , confirming that  $\ln(1) = 0$ . Press  $\boxed{\text{NXT}} \boxed{\text{NXT}} \boxed{\text{F1}} \boxed{\text{F1}}$  to return to the PLOT WINDOW – FUNCTION. Press  $\boxed{\text{ENTER}}$  to return to normal calculator display. You will notice that the root found in the graphics environment was copied to the calculator stack.

**Note:** When you press  $\boxed{\text{VAR}}$ , your variables list will show new variables called  $\boxed{\text{X}}$  and  $\boxed{\text{Y}}$ . Press  $\boxed{\text{F1}} \boxed{\text{F1}}$  to see the contents of this variable. You will get the program  $\ll \rightarrow X \text{ 'LN(X)'} \gg$ , which you will recognize as the program that may result from defining the function 'Y1(X) = LN(X)' by using  $\boxed{\leftarrow} \boxed{\text{DEF}}$ . This is basically what happens when you  $\boxed{\text{PLOT}}$  a function in the PLOT – FUNCTION window (the window that results from pressing  $\boxed{\leftarrow} \boxed{\text{Y=}}$ , simultaneously if in RPN mode), i.e., the function gets defined and added to your variable list.

Next, press  $\rightarrow$   $\boxed{X}$  to see the contents of this variable. A value of 10.275 is placed in the stack. This value is determined by our selection for the horizontal display range. We selected a range between -1 and 10 for X. To produce the graph, the calculator generates values between the range limits using a constant increment, and storing the values generated, one at a time, in the variable  $\boxed{X}$  as the graph is drawn. For the horizontal range  $(-1, 10)$ , the increment used seems to be 0.275. When the value of X becomes larger than the maximum value in the range (in this case, when  $X = 10.275$ ), the drawing of the graph stops. The last value of X for the graphic under consideration is kept in variable X. Delete X and Y1 before continuing.

## Graph of the exponential function

First, load the function  $\exp(X)$ , by pressing, simultaneously if in RPN mode, the left-shift key  $\leftarrow$  and the  $\underline{Y=}$  ( $\boxed{EEX}$ ) key to access the PLOT-FUNCTION window. Press  $\boxed{DEL}$  to remove the function  $\ln(X)$ , if you didn't delete Y1 as suggested in the previous note. Press  $\boxed{EXP}$  and type  $\leftarrow$   $e^x$  ( $\boxed{ALPHA}$   $\boxed{X}$   $\boxed{ENTER}$ ) to enter  $\exp(X)$  and return to the PLOT-FUNCTION window. Press  $\boxed{NXT}$   $\boxed{OFF}$  to return to normal calculator display.

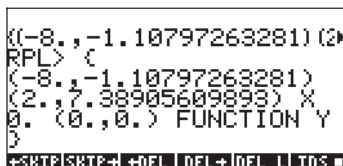
Next, press, simultaneously if in RPN mode, the left-shift key  $\leftarrow$  and the  $\underline{WIN}$  ( $\boxed{F2}$ ) key to produce the PLOT WINDOW - FUNCTION window. Change the H-View values to read: H-View: -8 2

by using  $\boxed{8}$   $\boxed{+/-}$   $\boxed{OK}$   $\boxed{2}$   $\boxed{OK}$ . Next, press  $\boxed{AUTO}$ . After the vertical range is calculated, press  $\boxed{GRAPH}$   $\boxed{GRAPH}$  to plot the exponential function.

To add labels to the graph press  $\boxed{EDIT}$   $\boxed{NXT}$   $\boxed{LABELS}$ . Press  $\boxed{EDIT}$  to remove the menu labels, and get a full view of the graph. Press  $\boxed{NXT}$   $\boxed{NXT}$   $\boxed{EDIT}$   $\boxed{EDIT}$  to return to the PLOT WINDOW - FUNCTION. Press  $\boxed{ENTER}$  to return to normal calculator display.

## The PPAR variable

Press  $\boxed{\text{VAR}}$  to recover your variables menu, if needed. In your variables menu you should have a variable labeled PPAR. Press  $\boxed{\rightarrow} \boxed{\text{PPAR}}$  to get the contents of this variable in the stack. Press the down-arrow key,  $\downarrow$ , to launch the stack editor, and use the up- and down-arrow keys to view the full contents of PPAR. The screen will show the following values:



```
{(-8.,-1.10797263281)} (2)
RPL> {
(-8.,-1.10797263281)
(2.,7.38905609893) X
0. (0.,0.) FUNCTION Y
}
*SKIP SKIP+ DEL DEL+ DEL L INS
```

PPAR stands for *Plot PARameters*, and its contents include two ordered pairs of real numbers,  $(-8., -1.10797263281)$  and  $(2., 7.38905609893)$ ,

which represent the *coordinates of the lower left corner and the upper right corner* of the plot, respectively. Next, PPAR lists the *name of the independent variable*, X, followed by a number that specifies the *increment of the independent variable* in the generation of the plot. The value shown here is the default value, zero (0.), which specifies increments in X corresponding to 1 pixel in the graphics display. The next element in PPAR is a *list containing first the coordinates of the point of intersection of the plot axes*, i.e.,  $(0., 0.)$ , followed by a list that specifies the tick mark annotation on the x- and y-axes, respectively  $\{\# 10d \# 10d\}$ . Next, PPAR lists the *type of plot that is to be generated*, i.e., FUNCTION, and, finally, the *y-axis label*, i.e., Y.

The variable PPAR, if non-existent, is generated every time you create a plot. The contents of the function will change depending on the type of plot and on the options that you select in the PLOT window (the window generated by the simultaneous activation of the  $\boxed{\leftarrow}$  and  $\boxed{\text{WIN}} (\boxed{F2})$  keys.

## Inverse functions and their graphs

Let  $y = f(x)$ , if we can find a function  $y = g(x)$ , such that,  $g(f(x)) = x$ , then we say that  $g(x)$  is the *inverse function* of  $f(x)$ . Typically, the notation  $g(x) = f^{-1}(x)$  is used to denote an inverse function. Using this notation we can write: if  $y = f(x)$ , then  $x = f^{-1}(y)$ . Also,  $f(f^{-1}(x)) = x$ , and  $f^{-1}(f(x)) = x$ .

As indicated earlier, the  $\ln(x)$  and  $\exp(x)$  functions are inverse of each other, i.e.,  $\ln(\exp(x)) = x$ , and  $\exp(\ln(x)) = x$ . This can be verified in the calculator by typing and evaluating the following expressions in the Equation Writer:  $\text{LN}(\text{EXP}(X))$  and  $\text{EXP}(\text{LN}(X))$ . They should both evaluate to  $X$ .

When a function  $f(x)$  and its inverse  $f^{-1}(x)$  are plotted simultaneously in the same set of axes, their graphs are reflections of each other about the line  $y = x$ . Let's check this fact with the calculator for the functions  $\text{LN}(X)$  and  $\text{EXP}(X)$  by following this procedure:

Press, simultaneously if in RPN mode,  $\leftarrow$   $\frac{Y=}{Y=}$ . The function  $Y1(X) = \text{EXP}(X)$  should be available in the PLOT - FUNCTION window from the previous exercise. Press  $\text{F1}$ , and type the function  $Y2(X) = \text{LN}(X)$ . Also, load the function  $Y3(X) = X$ . Press  $\text{NEXT}$   $\text{F1}$  to return to normal calculator display.

Press, simultaneously if in RPN mode,  $\leftarrow$   $\frac{\text{WIN}}{\text{WIN}}$ , and change the H-View range to read: H-View: -8 8

Press  $\text{F2}$  to generate the vertical range. Press  $\text{F3}$   $\text{F1}$  to produce the graph of  $y = \ln(x)$ ,  $y = \exp(x)$ , and  $y = x$ , simultaneously if in RPN mode.

You will notice that only the graph of  $y = \exp(x)$  is clearly visible. Something went wrong with the  $\text{F2}$  selection of the vertical range. What happens is that, when you press  $\text{F2}$  in the PLOT FUNCTION – WINDOW screen, the calculator produces the vertical range corresponding to the first function in the list of functions to be plotted. Which, in this case, happens to be  $Y1(X) = \text{EXP}(X)$ . We will have to enter the vertical range ourselves in order to display the other two functions in the same plot.

Press  $\text{F3}$  to return to the PLOT FUNCTION – WINDOW screen. Modify the vertical and horizontal ranges to read: H-View: -8 8, V-View: -4 4

By selecting these ranges we ensure that the scale of the graph is kept 1 vertical to 1 horizontal. Press  $\text{F3}$   $\text{F1}$  and you will get the plots of the natural logarithm, exponential, and  $y = x$  functions. It will be evident from the graph that  $\text{LN}(X)$  and  $\text{EXP}(X)$  are reflections of each other about the line  $y = X$ . Press  $\text{F3}$  to return to the PLOT WINDOW – FUNCTION. Press  $\text{ENTER}$  to return to normal calculator display.

## Summary of FUNCTION plot operation

In this section we present information regarding the PLOT SETUP, PLOT-FUNCTION, and PLOT WINDOW screens accessible through the left-shift key combined with the soft-menu keys  $\boxed{F1}$  through  $\boxed{F4}$ . Based on the graphing examples presented above, the procedure to follow to produce a FUNCTION plot (i.e., one that plots one or more functions of the form  $Y = F(X)$ ), is the following:

$\boxed{\leftarrow} \boxed{2D/3D}$ , simultaneously if in RPN mode: Access to the PLOT SETUP window. If needed, change TYPE to FUNCTION, and enter the name of the independent variable.

### Settings:

- A check on `_Simult` means that if you have two or more plots in the same graph, they will be plotted simultaneously when producing the graph.
- A check on `_Connect` means that the curve will be a continuous curve rather than a set of individual points.
- A check on `_Pixels` means that the marks indicated by H-Tick and V-Tick will be separated by that many pixels.
- The default value for both by H-Tick and V-Tick is 10.

### Soft key menu options:

- Use  $\boxed{F01}$  to edit functions of values in the selected field.
- Use  $\boxed{F02}$  to select the type of plot to use when the TYPE: field is highlighted. For the current exercises, we want this field set to FUNCTION.

**Note:** the soft menu keys  $\boxed{F01}$  and  $\boxed{F02}$  are not available at the same time. One or the other will be selected depending on which input field is highlighted.

- Press the AXES soft menu key to select or deselect the plotting of axes in the graph. If the option 'plot axes' is selected, a square dot will appear in the key label:  $\boxed{AXES} \blacksquare$ . Absence of the square dot indicates that axes will not be plotted in the graph.
- Use  $\boxed{F03}$  to erase any graph currently existing in the graphics display window.
- Use  $\boxed{F04}$  to produce the graph according to the current contents of PPAR for the equations listed in the PLOT-FUNCTION window.
- Press  $\boxed{NXT}$  to access the second set of soft menu keys in this screen.
- Use  $\boxed{F05}$  to reset any selected field to its default value.

- Use **QUIT** to cancel any changes to the PLOT SETUP window and return to normal calculator display.
- Press **OK** to save changes to the options in the PLOT SETUP window and return to normal calculator display.

**←** **Y=**, simultaneously if in RPN mode: Access to the PLOT window (in this case it will be called PLOT –FUNCTION window).

### Soft menu key options:

- Use **EDIT** to edit the highlighted equation.
- Use **ADD** to add new equations to the plot.

**Note:** **ADD** or **EDIT** will trigger the equation writer EQW that you can use to write new equations or edit old equations.

- Use **DEL** to remove the highlighted equation.
- Use **IMPORT** to add an equation that is already defined in your variables menu, but not listed in the PLOT – FUNCTION window.
- Use **GRAPH** to erase any graph currently existing in the graphics display window.
- Use **VIEW** to produce the graph according to the current contents of PPAR for the equations listed in the PLOT-FUNCTION window.
- Press **NEXT** to activate the second menu list.
- Use **UP** and **DOWN** to move the selected equation one location up or down, respectively.
- Use **CLEAR** if you want to clear all the equations currently active in the PLOT – FUNCTION window. The calculator will verify whether or not you want to clear all the functions before erasing all of them. Select YES, and press **OK** to proceed with clearing all functions. Select NO, and press **OK** to de-activate the option CLEAR.
- Press **OK** when done to return to normal calculator display.

**←** **WIN**, simultaneously if in RPN mode: Access to the PLOT WINDOW screen.



### Settings:

- Enter lower and upper limits for horizontal view (H-View) and vertical view (V-View) ranges in the plot window. Or,

- Enter lower and upper limits for horizontal view (H-View), and press **VIEW**, while the cursor is in one of the V-View fields, to generate the vertical view (V-View) range automatically. Or,
- Enter lower and upper limits for vertical view (V-View), and press **VIEW**, while the cursor is in one of the H-View fields, to generate the horizontal view (H-View) range automatically.
- The calculator will use the horizontal view (H-View) range to generate data values for the graph, unless you change the options Indep Low, (Indep) High, and (Indep) Step. These values determine, respectively, the minimum, maximum, and increment values of the independent variable to be used in the plot. If the option Default is listed in the fields Indep Low, (Indep) High, and (Indep) Step, the calculator will use the minimum and maximum values determined by H-View.
- A check on **\_Pixels** means that the values of the independent variable increments (Step:) are given in pixels rather than in plot coordinates.

#### Soft menu key options:

- Use **EDIT** to edit any entry in the window.
- Use **VIEW** as explained in Settings, above.
- Use **EEEE** to erase any graph currently existing in the graphics display window.
- Use **GRAPH** to produce the graph according to the current contents of PPAR for the equations listed in the PLOT-FUNCTION window.
- Press **NEXT** to activate the second menu list.
- Use **EEEE** to reset the field selected (i.e., where the cursor is positioned) to its default value.
- Use **QUIT** to access calculator stack to perform calculations that may be necessary to obtain a value for one of the options in this window. When the calculator stack is made available to you, you will also have the soft menu key options **QUIT** and **QUIT**.
- Use **QUIT** in case you want to cancel the current calculation and return to the PLOT WINDOW screen. Or,
- Use **QUIT** to accept the results of your calculation and return to the PLOT WINDOW screen.
- Use **QUIT** to get information on the type of objects that can be used in the selected option field.
- Use **QUIT** to cancel any changes to the PLOT WINDOW screen and return to normal calculator display.
- Press **QUIT** to accept changes to the PLOT WINDOW screen and return to normal calculator display.

 **GRAPH**, simultaneously if in RPN mode: Plots the graph based on the settings stored in variable PPAR and the current functions defined in the PLOT – FUNCTION screen. If a graph, different from the one you are plotting, already exists in the graphic display screen, the new plot will be superimposed on the existing plot. This may not be the result you desire, therefore, I recommend to use the  soft menu keys available in the PLOT SETUP, PLOT-FUNCTION or PLOT WINDOW screens.

## Plots of trigonometric and hyperbolic functions

The procedures used above to plot  $\text{LN}(X)$  and  $\text{EXP}(X)$ , separately or simultaneously, can be used to plot any function of the form  $y = f(x)$ . It is left as an exercise to the reader to produce the plots of trigonometric and hyperbolic functions and their inverses. The table below suggests the values to use for the vertical and horizontal ranges in each case. You can include the function  $Y=X$  when plotting simultaneously a function and its inverse to verify their 'reflection' about the line  $Y = X$ .

Function	H-View range		V-View range	
	Minimum	Maximum	Minimum	Maximum
SIN(X)	-3.15	3.15	AUTO	
ASIN(X)	-1.2	1.2	AUTO	
SIN & ASIN	-3.2	3.2	-1.6	1.6
COS(X)	-3.15	3.15	AUTO	
ACOS(X)	-1.2	1.2	AUTO	
COS & ACOS	-3.2	3.2	-1.6	1.6
TAN(X)	-3.15	3.15	-10	10
ATAN(X)	-10	10	-1.8	1.8
TAN & ATAN	-2	-2	-2	-2
SINH(X)	-2	2	AUTO	
ASINH(X)	-5	5	AUTO	
SINH & ASINH	-5	5	-5	5
COSH(X)	-2	2	AUTO	
ACOSH(X)	-1	5	AUTO	
COS & ACOS	-5	5	-1	5



TANH(X)	-5	5	AUTO	
ATANH(X)	-1.2	1.2	AUTO	
TAN & ATAN	-5	5	-2.5	2.5

## Generating a table of values for a function

The combinations  $\left(\leftarrow\right) \text{ TBLSET } \left(\leftarrow\right) \text{ F5 } \left(\leftarrow\right)$  and  $\left(\leftarrow\right) \text{ TABLE } \left(\leftarrow\right) \text{ F6 } \left(\leftarrow\right)$ , pressed simultaneously if in RPN mode, let's the user produce a table of values of functions. For example, we will produce a table of the function  $Y(X) = X/(X+10)$ , in the range  $-5 < X < 5$  following these instructions:

- We will generate values of the function  $f(x)$ , defined above, for values of  $x$  from  $-5$  to  $5$ , in increments of  $0.5$ . First, we need to ensure that the graph type is set to **FUNCTION** in the PLOT SETUP screen ( $\left(\leftarrow\right) \text{ 2D/3D } \left(\leftarrow\right)$ , press them simultaneously, if in RPN mode). The field in front of the *Type* option will be highlighted. If this field is not already set to **FUNCTION**, press the soft key  $\left[\text{F10}\right]$  and select the **FUNCTION** option, then press  $\left[\text{OK}\right]$ .
- Next, press  $\left[\nabla\right]$  to highlight the field in front of the option EQ, type the function expression ' $X/(X+10)$ ' and press  $\left[\text{OK}\right]$ .
- To accept the changes made to the PLOT SETUP screen press  $\left(\text{NEXT}\right) \left[\text{OK}\right]$ . You will be returned to normal calculator display.
- The next step is to access the Table Set-up screen by using the keystroke combination  $\left(\leftarrow\right) \text{ TBLSET } \left(\leftarrow\right) \text{ F5 } \left(\leftarrow\right)$  – simultaneously if in RPN mode. This will produce a screen where you can select the starting value (*Start*) and the increment (*Step*). Enter the following:  $\left[5\right] \left[\text{+/-}\right] \left[\text{OK}\right] \left[0\right] \left[\cdot\right] \left[5\right] \left[\text{OK}\right] \left[0\right] \left[\cdot\right] \left[5\right] \left[\text{OK}\right]$  (i.e., Zoom factor =  $0.5$ ). Toggle the  $\left[\text{I/O}\right]$  soft menu key until a check mark appears in front of the option *Small Font* if you so desire. Then press  $\left[\text{OK}\right]$ . This will return you to normal calculator display.

## The TPAR variable

After finishing the table set up, your calculator will create a variable called TPAR (Table PARAmeters) that store information relevant to the table that is to be generated. To see the contents of this variable, press  $\left(\rightarrow\right) \left[\text{I/O}\right]$ .

- To see the table, press  $\left(\leftarrow\right) \text{ TABLE } \left(\leftarrow\right) \text{ F6 } \left(\leftarrow\right)$  – simultaneously if in RPN mode. This will produce a table of values of  $x = -5, -4.5, \dots$ , and

the corresponding values of  $f(x)$ , listed as Y1 by default. You can use the up and down arrow keys to move about in the table. You will notice that we did not have to indicate an ending value for the independent variable  $x$ . Thus, the table continues beyond the maximum value for  $x$  suggested early, namely  $x = 5$ .

Some options available while the table is visible are **FOOT**, **SIZE**, and **DEFN**:

- The **DEFN**, when selected, shows the definition of the independent variable.
- The **SIZE** key simply changes the font in the table from small to big, and vice versa. Try it.
- The **FOOT** key, when pressed, produces a menu with the options: *In*, *Out*, *Decimal*, *Integer*, and *Trig*. Try the following exercises:
  - With the option *In* highlighted, press **0.5**. The table is expanded so that the  $x$ -increment is now 0.25 rather than 0.5. Simply, what the calculator does is to multiply the original increment, 0.5, by the zoom factor, 0.5, to produce the new increment of 0.25. Thus, the *zoom in* option is useful when you want more resolution for the values of  $x$  in your table.
  - To increase the resolution by an additional factor of 0.5 press **FOOT**, select *In* once more, and press **OK**. The  $x$ -increment is now 0.0125.
  - To recover the previous  $x$ -increment, press **FOOT** **▲** **0.5** to select the option *Un-zoom*. The  $x$ -increment is increased to 0.25.
  - To recover the original  $x$ -increment of 0.5 you can do an *un-zoom* again, or use the *option zoom out* by pressing **FOOT** **0.5**.
  - The option *Decimal* in **FOOT** produces  $x$ -increments of 0.10.
  - The option *Integer* in **FOOT** produces  $x$ -increments of 1.
  - The option *Trig* in produces increments related to fractions of  $\pi$ , thus being useful when plotting trigonometric functions.
  - To return to normal calculator display press **ENTER**.

## Plots in polar coordinates

First of all, you may want to delete the variables used in previous examples (e.g.,  $X$ ,  $EQ$ ,  $Y1$ ,  $PPAR$ ) using function **PURGE** (**TOOL** **PURGE**). By doing this, all parameters related to graphics will be cleared. Press **VAR** to check that the variables were indeed purged.

We will try to plot the function  $f(\theta) = 2(1 - \sin(\theta))$ , as follows:

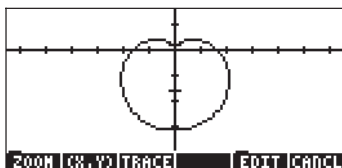
- First, make sure that your calculator's angle measure is set to radians.
- Press  $\left[ \leftarrow \right] \left[ 2D/3D \right]$ , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Polar, by pressing  $\left[ \text{MODE} \right] \left[ \nabla \right] \left[ \text{POLAR} \right]$ .
- Press  $\left[ \nabla \right]$  and type:

$\left[ ' \right] \left[ 2 \right] \left[ \times \right] \left[ \leftarrow \right] \left[ ( ) \right] \left[ / \right] \left[ - \right] \left[ \text{SIN} \right] \left[ \text{ALPHA} \right] \left[ \rightarrow \right] \left[ \text{T} \right] \left[ \text{OK} \right]$ .

- The cursor is now in the Indep field. Press  $\left[ ' \right] \left[ \text{ALPHA} \right] \left[ \rightarrow \right] \left[ \text{T} \right] \left[ \text{OK} \right]$  to change the independent variable to  $\theta$ .
- Press  $\left[ \text{NXT} \right] \left[ \text{OK} \right]$  to return to normal calculator display.
- Press  $\left[ \leftarrow \right] \left[ \text{WIN} \right]$ , simultaneously if in RPN mode, to access the PLOT window (in this case it will be called PLOT –POLAR window).
- Change the H-VIEW range to  $-8$  to  $8$ , by using  $\left[ 8 \right] \left[ +/- \right] \left[ \text{OK} \right] \left[ 8 \right] \left[ \text{OK} \right]$ , and the V-VIEW range to  $-6$  to  $2$  by using  $\left[ 6 \right] \left[ +/- \right] \left[ \text{OK} \right] \left[ 2 \right] \left[ \text{OK} \right]$ .

**Note:** the H-VIEW and V-VIEW determine the scales of the display window only, and their ranges are not related to the range of values of the independent variable in this case.

- Change the Indep Low value to 0, and the High value to  $6.28 (\approx 2\pi)$ , by using:  $\left[ 0 \right] \left[ \text{OK} \right] \left[ 6 \right] \left[ . \right] \left[ 2 \right] \left[ 8 \right] \left[ \text{OK} \right]$ .
- Press  $\left[ \text{F5} \right] \left[ \text{F5} \right]$  to plot the function in polar coordinates. The result is a curve shaped like a hearth. This curve is known as a *cardioid* (cardios, Greek for heart).

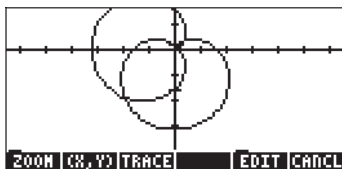


- Press  $\left[ \text{F5} \right] \left[ \text{NXT} \right] \left[ \text{F5} \right] \left[ \text{F5} \right]$  to see the graph with labels. Press  $\left[ \text{NXT} \right]$  to recover the menu. Press  $\left[ \text{NXT} \right] \left[ \text{F5} \right]$  to recover the original graphics menu.
- Press  $\left[ \text{F5} \right] \left[ \text{OK} \right]$  to trace the curve. The data shown at the bottom of the display is the angle  $\theta$  and the radius  $r$ , although the latter is labeled  $Y$  (default name of dependent variable).

- Press **NXT** **MODE** to return to the PLOT WINDOW screen. Press **NXT** **MODE** to return to normal calculator display.

In this exercise we entered the equation to be plotted directly in the PLOT SETUP window. We can also enter equations for plotting using the PLOT window, i.e., simultaneously if in RPN mode, pressing **←** **Y=**. For example, when you press **←** **Y=** after finishing the previous exercise, you will get the equation '2\*(1-SIN(θ))' highlighted. Let's say, we want to plot also the function '2\*(1-COS(θ))' along with the previous equation.

- Press **MODE**, and type **(2) (×) (←) (/) (-) (COS) (ALPHA) (→) (T) (ENTER)**, to enter the new equation.
- Press **MODE** **MODE** to see the two equations plotted in the same figure. The result is two intersecting *cardioids*. Press **MODE** **ON** to return to normal calculator display.



## Plotting conic curves

The most general form of a conic curve in the x-y plane is:

$Ax^2 + By^2 + Cxy + Dx + Ey + F = 0$ . We also recognize as conic equations those given in the canonical form for the following figures:

- circle:  $(x-x_0)^2 + (y-y_0)^2 = r^2$
- ellipse:  $(x-x_0)^2/a^2 + (y-y_0)^2/b^2 = 1$
- parabola:  $(y-b)^2 = K(x-a)$  or  $(x-a)^2 = K(y-b)$
- hyperbola:  $(x-x_0)^2/a^2 - (y-y_0)^2/b^2 = 1$  or  $xy = K$ ,

where  $x_0$ ,  $y_0$ ,  $a$ ,  $b$ , and  $K$  are constant.

The name *conic curves* follows because these figures (circles, ellipses, parabolas or hyperbolas) result from the intersection of a plane with a cone. For example, a circle is the intersection of a cone with a plane perpendicular to the cone's main axis.

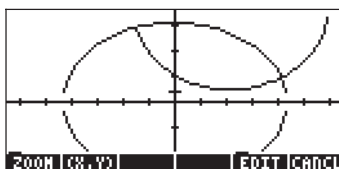
The calculator has the ability of plotting one or more conic curves by selecting Conic as the function TYPE in the PLOT environment. Make sure to delete the variables PPAR and EQ before continuing. For example, let's store the list of equations

$$\{ '(X-1)^2+(Y-2)^2=3', 'X^2/4+Y^2/3=1' \}$$

into the variable EQ.

These equations we recognize as those of a circle centered at (1,2) with radius  $\sqrt{3}$ , and of an ellipse centered at (0,0) with semi-axis lengths  $a = 2$  and  $b = \sqrt{3}$ .

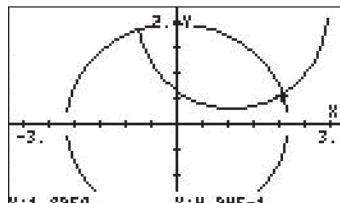
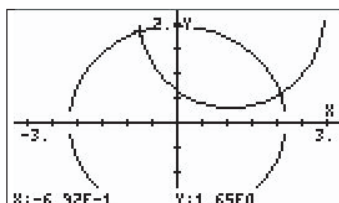
- Enter the PLOT environment, by pressing  $\leftarrow$  **2D/3D**, simultaneously if in RPN mode, and select **Conic** as the TYPE. The list of equations will be listed in the EQ field.
- Make sure that the independent variable (Indep) is set to 'X' and the dependent variable (Depnd) to 'Y'.
- Press **NXT** **03** to return to normal calculator display.
- Enter the PLOT WINDOW environment, by pressing  $\leftarrow$  **WIN**, simultaneously if in RPN mode.
- Change the range for H-VIEW to -3 to 3, by using **3** **+/-** **08** **3** **08**. Also, change the V-VIEW range to -1.5 to 2 by using **1** **.** **5** **+/-** **08** **2** **08**.
- Change the *Indep Low:* and *High:* fields to Default by using **NXT** **7777** while each of those fields is highlighted. Select the option *Reset value* after pressing **7777**. Press **08** to complete the resetting of values. Press **NXT** to return to the main menu.
- Plot the graph: **GRAPH** **NEXT**



**Note:** The H-View and V-View ranges were selected to show the intersection of the two curves. There is no general rule to select those ranges, except based on what we know about the curves. For example, for the equations shown above, we know that the circle will extend from  $-3+1 = -2$  to  $3+1 = 4$  in  $x$ , and from  $-3+2=-1$  to  $3+2=5$  in  $y$ . In addition, the ellipse, which is centered at the origin  $(0,0)$ , will extend from  $-2$  to  $2$  in  $x$ , and from  $-\sqrt{3}$  to  $\sqrt{3}$  in  $y$ .

Notice that for the circle and the ellipse the region corresponding to the left and right extremes of the curves are not plotted. This is the case with all circles or ellipses plotted using `Conic` as the `TYPE`.

- To see labels: `EDIT` `NXT` `MODE` `EDIT`
- To recover the menu: `NXT` `NXT` `MODE`
- To estimate the coordinates of the point of intersection, press the `2ND` `F1` menu key and move the cursor as close as possible to those points using the arrow keys. The coordinates of the cursor are shown in the display. For example, the left point of intersection is close to  $(-0.692, 1.67)$ , while the right intersection is near  $(1.89, 0.5)$ .



- To recover the menu and return to the PLOT environment, press `NXT` `MODE`.
- To return to normal calculator display, press `NXT` `OK`.

## Parametric plots

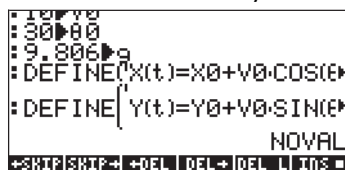
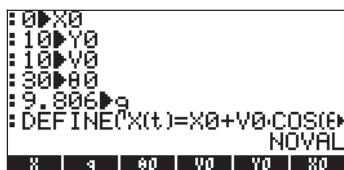
Parametric plots in the plane are those plots whose coordinates are generated through the system of equations  $x = x(t)$  and  $y = y(t)$ , where  $t$  is known as the parameter. An example of such graph is the trajectory of a projectile,  $x(t) = x_0 + v_0 \cdot \cos \theta_0 \cdot t$ ,  $y(t) = y_0 + v_0 \cdot \sin \theta_0 \cdot t - \frac{1}{2} \cdot g \cdot t^2$ . To plot equations like these,

which involve constant values  $x_0$ ,  $y_0$ ,  $v_0$ , and  $\theta_0$ , we need to store the values of those parameters in variables. To develop this example, create a sub-directory called 'PROJM' for PROjectile Motion, and within that sub-directory store the following variables:  $X_0 = 0$ ,  $Y_0 = 10$ ,  $V_0 = 10$ ,  $\theta_0 = 30$ , and  $g = 9.806$ . Make sure that the calculator's angle measure is set to DEG. Next, define the functions (use  $\leftarrow$  DEF  $\rightarrow$ ):

$$X(t) = X_0 + V_0 \cdot \cos(\theta_0) \cdot t$$

$$Y(t) = Y_0 + V_0 \cdot \sin(\theta_0) \cdot t - 0.5 \cdot g \cdot t^2$$

which will add the variables  $\boxed{Y}$  and  $\boxed{X}$  to the soft menu key labels.



To produce the graph itself, follow these steps:

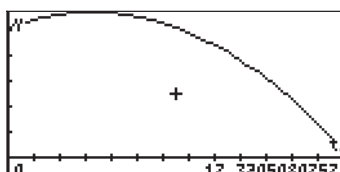
- Press  $\leftarrow$  2D/3D  $\rightarrow$ , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Parametric, by pressing  $\boxed{MODE}$   $\downarrow$   $\downarrow$   $\boxed{08}$ .
- Press  $\downarrow$  and type 'X(t) + i\*Y(t)'  $\boxed{08}$  to define the parametric plot as that of a complex variable. (The real and imaginary parts of the complex variable correspond to the x- and y-coordinates of the curve.)
- The cursor is now in the Indep field. Press  $\boxed{}$  ALPHA  $\leftarrow$  T  $\boxed{08}$  to change the independent variable to  $t$ .
- Press  $\boxed{NXT}$   $\boxed{08}$  to return to normal calculator display.
- Press  $\leftarrow$  WIN  $\rightarrow$ , simultaneously if in RPN mode, to access the PLOT window (in this case it will be called PLOT –PARAMETRIC window). Instead of modifying the horizontal and vertical views first, as done for other types of plot, we will set the lower and upper values of the independent variable first as follows:
- Select the Indep Low field by pressing  $\downarrow$   $\downarrow$ . Change this value to  $\boxed{0} \boxed{08}$ . Then, change the value of High to  $\boxed{2} \boxed{08}$ . Enter  $\boxed{0} \boxed{.}$   $\boxed{1} \boxed{08}$  for the Step value (i.e., step = 0.1).

**Note:** Through these settings we are indicating that the parameter  $t$  will take values of  $t = 0, 0.1, 0.2, \dots$ , until reaching the value of 2.0.

- Press **||||**. This will generate automatic values of the H-View and V-View ranges based on the values of the independent variable  $t$  and the definitions of  $X(t)$  and  $Y(t)$  used. The result will be:



- Press **||||** **NXT** to draw the parametric plot.
- Press **||||** **NXT** **||||** **NXT** to see the graph with labels. The window parameters are such that you only see half of the labels in the x-axis.



- Press **NXT** to recover the menu. Press **NXT** **||||** to recover the original graphics menu.
- Press **||||** **||||** to determine coordinates of any point on the graph. Use **▶** and **◀** to move the cursor about the curve. At the bottom of the screen you will see the value of the parameter  $t$  and coordinates of the cursor as  $(X,Y)$ .
- Press **NXT** **||||** to return to the PLOT WINDOW environment. Then, press **ON** , or **NXT** **||||**, to return to normal calculator display.

A review of your soft menu key labels shows that you now have the following variables:  $t$ , EQ, PPAR,  $Y$ ,  $X$ ,  $g$ ,  $\theta_0$ ,  $V_0$ ,  $Y_0$ ,  $X_0$ . Variables  $t$ , EQ, and PPAR are generated by the calculator to store the current values of the parameter,  $t$ , of the equation to be plotted EQ (which contains ' $X(t) + I*Y(t)$ '), and the plot



parameters. The other variables contain the values of constants used in the definitions of  $X(t)$  and  $Y(t)$ .

You can store different values in the variables and produce new parametric plots of the projectile equations used in this example. If you want to erase the current picture contents before producing a new plot, you need to access either the PLOT, PLOT WINDOW, or PLOT SETUP screens, by pressing,  $\leftarrow$   $Y=$ ,  $\leftarrow$   $WIN$ , or  $\leftarrow$   $2D/3D$  (the two keys must be pressed simultaneously if in RPN mode). Then, press  $\leftarrow$   $TABLE$ . Press  $\leftarrow$   $QUIT$  to return to the PLOT, PLOT WINDOW, or PLOT SETUP screen. Press  $\leftarrow$   $ON$ , or  $\leftarrow$   $NXT$   $\leftarrow$   $OFF$ , to return to normal calculator display.

## Generating a table for parametric equations

In an earlier example we generated a table of values  $(X,Y)$  for an expression of the form  $Y=f(X)$ , i.e., a Function type of graph. In this section, we present the procedure for generating a table corresponding to a parametric plot. For this purpose, we'll take advantage of the parametric equations defined in the example above.

- First, let's access the TABLE SETUP window by pressing  $\leftarrow$   $TBLSET$ , simultaneously if in RPN mode. For the independent variable change the Starting value to 0.0, and the Step value to 0.1. Press  $\leftarrow$   $OFF$ .
- Generate the table by pressing, simultaneously if in RPN mode,  $\leftarrow$   $TABLE$ . The resulting table has three columns representing the parameter  $t$ , and the coordinates of the corresponding points. For this table the coordinates are labeled  $X1$  and  $Y1$ .

t	X1	Y1
0	0	10
.1	.8660254	10.45097
.2	1.732051	10.80388
.3	2.598076	11.05383
.4	3.464102	11.21552
.5	4.330127	11.27425

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- Use the arrow keys,  $\leftarrow$   $\rightarrow$   $\uparrow$   $\downarrow$ , to move about the table.
- Press  $\leftarrow$   $ON$  to return to normal calculator display.

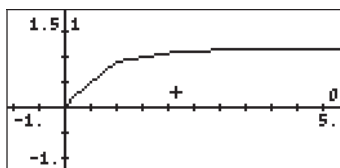
This procedure for creating a table corresponding to the current type of plot can be applied to other plot types.

## Plotting the solution to simple differential equations

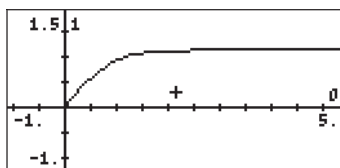
The plot of a simple differential equation can be obtained by selecting **Diff Eq** in the **TYPE** field of the **PLOT SETUP** environment as follows: suppose that we want to plot  $x(t)$  from the differential equation  $dx/dt = \exp(-t^2)$ , with initial conditions:  $x = 0$  at  $t = 0$ . The calculator allows for the plotting of the solution of differential equations of the form  $Y'(T) = F(T,Y)$ . For our case, we let  $Y \rightarrow x$  and  $T \rightarrow t$ , therefore,  $F(T,Y) \rightarrow f(t,x) = \exp(-t^2)$ .

Before plotting the solution,  $x(t)$ , for  $t = 0$  to  $5$ , delete the variables **EQ** and **PPAR**.

- Press  $\leftarrow$  **2D/3D**, simultaneously if in RPN mode, to access to the **PLOT SETUP** window.
- Change **TYPE** to **Diff Eq**.
- Press  $\nabla$  and type  $\leftarrow$   $e^x$   $\leftarrow$  **ALPHA**  $\leftarrow$  **T**  $\leftarrow$   $y^x$   $\leftarrow$  **2** **OK**.
- The cursor is now in the **H-Var** field. It should show **H-Var:0** and also **V-Var:1**. This is the code used by the calculator to identify the variables to be plotted. **H-Var:0** means the independent variable (to be selected later) will be plotted in the horizontal axis. Also, **V-Var:1** means the dependent variable (default name 'Y') will be plotted in the vertical axis.
- Press  $\nabla$ . The cursor is now in the **Indep** field. Press  $\leftarrow$  **ALPHA**  $\leftarrow$  **T** **OK** to change the independent variable to  $t$ .
- Press **NXT** **OK** to return to normal calculator display.
- Press  $\leftarrow$  **WIN**, simultaneously if in RPN mode, to access the **PLOT** window (in this case it will be called **PLOT WINDOW – DIFF EQ**).
- Change the **H-VIEW** and **V-VIEW** parameters to read: **H-VIEW: -15,V-VIEW: -11.5**
- Change the **Init** value to **0**, and the **Final** value to **5** by using:  $\leftarrow$  **0** **OK**  $\leftarrow$  **5** **OK**.
- The values **Step** and **Tol** represent the step in the independent variable and the tolerance for convergence to be used by the numerical solution. Let's leave those values with their default settings (if the word *default* is not shown in the **Step:** field, use **NXT** **OK** to reset that value to its default value. Press **NXT** to return to the main menu.) Press  $\nabla$ .
- The **Init-Soln** value represents the initial value of the solution to start the numerical result. For the present case, we have for initial conditions  $x(0) = 0$ , thus, we need to change this value to **0.0**, by using  $\leftarrow$  **0** **OK**.
- Press **EEEE** **OK** to plot the solution to the differential equation.
- Press **EEEE** **NXT** **EEEE** **EEEE** to see the graph with labels.



- Press **NXT** to recover the menu. Press **NXT** **GRAPH** to recover the original graphics menu.
- When we observed the graph being plotted, you'll notice that the graph is not very smooth. That is because the plotter is using a time step that is too large. To refine the graph and make it smoother, use a step of 0.1. Try the following keystrokes: **MODE** **▼** **▼** **▼** **0.** **1** **MODE** **GRAPH** **GRAPH** **GRAPH**. The plot will take longer to be completed, but the shape is definitely smoother than before.
- Press **EDIT** **NXT** **GRAPH** **EDIT**, to see axes labels and range. Notice that the labels for the axes are shown as 0 (horizontal) and 1 (vertical). These are the definitions for the axes as given in the PLOT WINDOW screen (see above), i.e., H-VAR (t): 0, and V-VAR(x): 1.



- Press **NXT** **NXT** **GRAPH** to recover menu and return to PICT environment.
- Press **COORD** to determine coordinates of any point on the graph. Use **▶** and **◀** to move the cursor in the plot area. At the bottom of the screen you will see the coordinates of the cursor as (X,Y). The calculator uses X and Y as the default names for the horizontal and vertical axes, respectively.
- Press **NXT** **GRAPH** to return to the PLOT WINDOW environment. Then, press **ON** to return to normal calculator display.

More details on using graphical solutions of differential equations are presented in Chapter 16.

## Truth plots

Truth plots are used to produce two-dimensional plots of regions that satisfy a certain mathematical condition that can be either true or false. For example, suppose that you want to plot the region for  $X^2/36 + Y^2/9 < 1$ ,

proceed as follows:

- Press  $\leftarrow$   $2D/3D$ , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Truth.
- Press  $\nabla$  and type  $\{(X^2/36+Y^2/9 < 1)',(X^2/16+Y^2/9 > 1)'\}$   $\leftarrow$  to define the conditions to be plotted.
- The cursor is now in the *Indep* field. Leave that as 'X' if already set to that variable, or change it to 'X' if needed.
- Press  $\leftarrow$   $NXT$   $\leftarrow$  to return to normal calculator display.
- Press  $\leftarrow$   $WIN$ , simultaneously if in RPN mode, to access the PLOT window (in this case it will be called PLOT WINDOW – TRUTH window). Let's keep the default value for the window's ranges: H-View: -6.5 6.5, V-View: -3.9 4.0 (To reset them use  $\leftarrow$   $NXT$   $\leftarrow$  (select Reset all)  $\leftarrow$   $NXT$ ).

**Note:** if the window's ranges are not set to default values, the quickest way to reset them is by using  $\leftarrow$   $NXT$   $\leftarrow$  (select *Reset all*)  $\leftarrow$   $NXT$ .

- Press  $\leftarrow$   $\leftarrow$  to draw the truth plot. Because the calculator samples the entire plotting domain, point by point, it takes a few minutes to produce a truth plot. The present plot should produce a shaded ellipse of semi-axes 6 and 3 (in x and y, respectively), centered at the origin.
- Press  $\leftarrow$   $NXT$   $\leftarrow$  to see the graph with labels. The window parameters are such that you only see half of the labels in the x-axis. Press  $\leftarrow$   $NXT$  to recover the menu. Press  $\leftarrow$   $NXT$   $\leftarrow$  to recover the original graphics menu.
- Press  $\leftarrow$  to determine coordinates of any point on the graph. Use the arrow keys to move the cursor about the region plotted. At the bottom of the screen you will see the value of the coordinates of the cursor as (X,Y).
- Press  $\leftarrow$   $NXT$   $\leftarrow$  to return to the PLOT WINDOW environment. Then, press  $\leftarrow$   $ON$ , or  $\leftarrow$   $NXT$   $\leftarrow$ , to return to normal calculator display.

You can have more than one condition plotted at the same time if you multiply the conditions. For example, to plot the graph of the points for which  $X^2/36 + Y^2/9 < 1$ , and  $X^2/16 + Y^2/9 > 1$ , use the following:

- Press  $\leftarrow$   $\frac{2D}{3D}$  , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Press  $\nabla$  and type ' $(X^2/36+Y^2/9 < 1) \cdot (X^2/16+Y^2/9 > 1)$ '  $\left[\frac{\square\square\square\square}{\square\square\square\square}\right]$  to define the conditions to be plotted.
- Press  $\left[\frac{\square\square\square\square}{\square\square\square\square}\right]$  to draw the truth plot. Again, you have to be patient while the calculator produces the graph. If you want to interrupt the plot, press  $\text{ON}$  , once. Then press  $\left[\frac{\square\square\square\square}{\square\square\square\square}\right]$  .

## Plotting histograms, bar plots, and scatter plots

Histograms, bar plots and scatter plots are used to plot discrete data stored in the reserved variable  $\Sigma\text{DAT}$ . This variable is used not only for these types of plots, but also for all kind of statistical applications as will be shown in Chapter 18. As a matter of fact, the use of histogram plots is postponed until we get to that chapter, for the plotting of a histogram requires to perform a grouping of data and a frequency analysis before the actual plot. In this section we will show how to load data in the variable  $\Sigma\text{DAT}$  and how to plot bar plots and scatter plots.

We will use the following data for plotting bar plots and scatter plots:

x	y	z
3.1	2.1	1.1
3.6	3.2	2.2
4.2	4.5	3.3
4.5	5.6	4.4
4.9	3.8	5.5
5.2	2.2	6.6

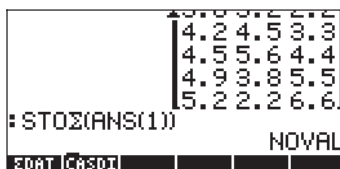
### Bar plots

First, make sure your calculator's CAS is in **Exact** mode. Next, enter the data shown above as a matrix, i.e.,

$$[[3.1,2.1,1.1],[3.6,3.2,2.2],[4.2,4.5,3.3],$$

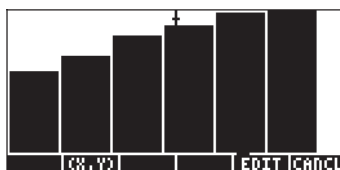
[4.5,5.6,4.4],[4.9,3.8,5.5],[5.2,2.2,6.6]] **ENTER**

to store it in  $\Sigma$ DAT, use the function **STO $\Sigma$**  (available in the function catalog, **↵** **CAT** ). Press **VAR** to recover your variables menu. A soft menu key labeled  $\Sigma$ DAT should be available in the stack. The figure below shows the storage of this matrix in ALG mode:



To produce the graph:

- Press **↵** **2D/3D** , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change **TYPE** to **Bar**.
- A matrix will be shown at the  $\Sigma$ DAT field. This is the matrix we stored earlier into  $\Sigma$ DAT.
- Highlight the **Col:** field. This field lets you choose the column of  $\Sigma$ DAT that is to be plotted. The default value is 1. Keep it to plot column 1 in  $\Sigma$ DAT.
- Press **NXT** **||||** to return to normal calculator display.
- Press **↵** **WIN** , simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Change the V-View to read, V-View: 0 5.
- Press **||||** **||||** to draw the bar plot.



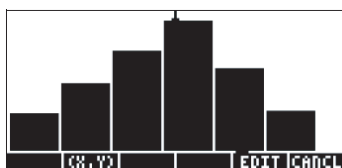
- Press **CANCEL** to return to the PLOT WINDOW environment. Then, press **ON** , or **NXT** **||||** , to return to normal calculator display.

The number of bars to be plotted determines the width of the bar. The H- and V-VIEW are set to 10, by default. We changed the V-VIEW to better

accommodate the maximum value in column 1 of  $\Sigma$ DAT. Bar plots are useful when plotting categorical (i.e., non-numerical) data.

Suppose that you want to plot the data in column 2 of the  $\Sigma$ DAT matrix:

- Press  $\leftarrow$   $2D/3D$ , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Press  $\nabla \nabla$  to highlight the **Col:** field and type 2  $\blacksquare \blacksquare$ , followed by  $\text{NXT} \blacksquare \blacksquare$ .
- Press  $\leftarrow$   $WIN$ , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change V-View to read V-View: 0 6
- Press  $\blacksquare \blacksquare \blacksquare \blacksquare$



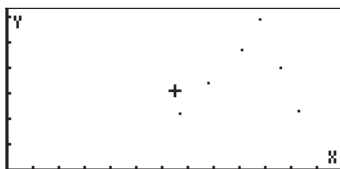
- Press  $\blacksquare \blacksquare \blacksquare$  to return to the PLOT WINDOW screen, then  $\text{ON}$  to return to normal calculator display.

## Scatter plots

We will use the same  $\Sigma$ DAT matrix to produce scatter plots. First, we will plot the values of y vs. x, then those of y vs. z, as follows:

- Press  $\leftarrow$   $2D/3D$ , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Scatter.
- Press  $\nabla \nabla$  to highlight the **Cols:** field. Enter  $\text{1} \blacksquare \blacksquare$   $\text{2} \blacksquare \blacksquare$  to select column 1 as X and column 2 as Y in the Y-vs.-X scatter plot.
- Press  $\text{NXT} \blacksquare \blacksquare$  to return to normal calculator display.
- Press  $\leftarrow$   $WIN$ , simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Change the plot window ranges to read: H-View: 0 6, V-View: 0 6.

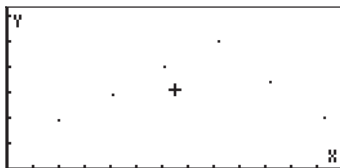
- Press **EDIT** **EDIT** to draw the bar plot. Press **QUIT** **NXT** **QUIT** **QUIT** to see the plot unencumbered by the menu and with identifying labels (the cursor will be in the middle of the plot, however):



- Press **NXT** **NXT** **EDIT** to leave the EDIT environment.
- Press **EDIT** to return to the PLOT WINDOW environment. Then, press **ON** , or **NXT** **OFF** , to return to normal calculator display.

To plot  $y$  vs.  $z$ , use:

- Press **←** **2D/3D** , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Press **▼** **▼** to highlight the **COLs:** field. Enter **3** **OFF** **2** **OFF** to select column 3 as X and column 2 as Y in the Y-vs.-X scatter plot.
- Press **NXT** **OFF** to return to normal calculator display.
- Press **←** **WIN** , simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Change the plot window ranges to read: H-View: 0 7, V-View: 0 7.
- Press **EDIT** **EDIT** to draw the bar plot. Press **QUIT** **NXT** **QUIT** **QUIT** to see the plot unencumbered by the menu and with identifying labels.



- Press **NXT** **NXT** **EDIT** to leave the EDIT environment.
- Press **EDIT** to return to the PLOT WINDOW environment. Then, press **ON** , or **NXT** **OFF** , to return to normal calculator display.

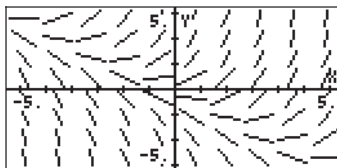


## Slope fields

Slope fields are used to visualize the solutions to a differential equation of the form  $y' = f(x,y)$ . Basically, what is presented in the plot are segments tangential to the solution curves, since  $y' = dy/dx$ , evaluated at any point  $(x,y)$ , represents the slope of the tangent line at point  $(x,y)$ .

For example, to visualize the solution to the differential equation  $y' = f(x,y) = x+y$ , use the following:

- Press  $\leftarrow$  **2D/3D**, simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Slopefield.
- Press  $\nabla$  and type 'X+Y' **OK**.
- Make sure that 'X' is selected as the Indep: and 'Y' as the Depnd: variables.
- Press **NXT** **||||** to return to normal calculator display.
- Press  $\leftarrow$  **WIN**, simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Change the plot window ranges to read: X-Left:-5, X-Right:5, Y-Near:-5, Y-Far:5
- Press **||||** **||||** to draw the slope field plot. Press **||||** **NXT** **||||** **||||** to see the plot unencumbered by the menu and with identifying labels.



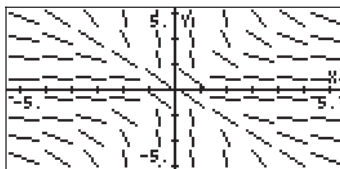
- Press **NXT** **NXT** **||||** to leave the EDIT environment.
- Press **||||** to return to the PLOT WINDOW environment. Then, press **ON**, or **NXT** **OK**, to return to normal calculator display.

If you could reproduce the slope field plot in paper, you can trace by hand lines that are tangential to the line segments shown in the plot. These lines constitute lines

of  $y(x,y) = \text{constant}$ , for the solution of  $y' = f(x,y)$ . Thus, slope fields are useful tools for visualizing particularly difficult equations to solve.

Try also a slope field plot for the function  $y' = f(x,y) = -(y/x)^2$ , by using:

- Press  $\leftarrow$  **2D/3D** , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Slopefield.
- Press  $\nabla$  and type ' $-(Y/X)^2$ '  $\blacksquare$ .
- Press  $\blacksquare$   $\blacksquare$  to draw the slope field plot. Press  $\blacksquare$  **NXT**  $\blacksquare$   $\blacksquare$  to see the plot unencumbered by the menu and with identifying labels.



- Press **NXT** **NXT**  $\blacksquare$  to leave the EDIT environment.
- Press  $\blacksquare$  to return to the PLOT WINDOW environment. Then, press **ON** , or **NXT**  $\blacksquare$ , to return to normal calculator display.

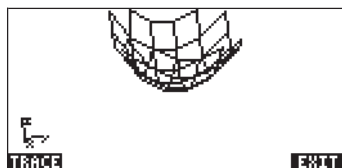
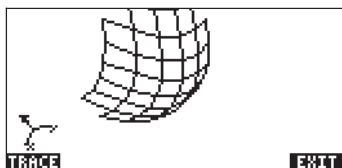
## Fast 3D plots

Fast 3D plots are used to visualize three-dimensional surfaces represented by equations of the form  $z = f(x,y)$ . For example, if you want to visualize  $z = f(x,y) = x^2 + y^2$ , we can use the following:

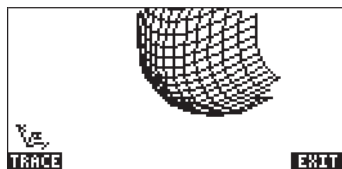
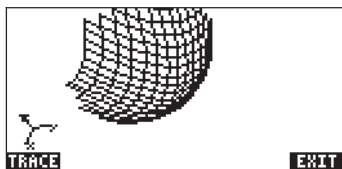
- Press  $\leftarrow$  **2D/3D** , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Fast3D.
- Press  $\nabla$  and type ' $X^2+Y^2$ '  $\blacksquare$ .
- Make sure that 'X' is selected as the Indep: and 'Y' as the Depnd: variables.
- Press **NXT**  $\blacksquare$  to return to normal calculator display.
- Press  $\leftarrow$  **WIN** , simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Keep the default plot window ranges to read: X-Left:-1, X-Right:1, Y-Near:-1, Y-Far: 1, Z-Low:-1, Z-High: 1, Step Indep: 10, Depnd: 8

**Note:** The Step Indep: and Depnd: values represent the number of gridlines to be used in the plot. The larger these number, the slower it is to produce the graph, although, the times utilized for graphic generation are relatively fast. For the time being we'll keep the default values of 10 and 8 for the Step data.

- Press **ERASE DATA** to draw the three-dimensional surface. The result is a wireframe picture of the surface with the reference coordinate system shown at the lower left corner of the screen. By using the arrow keys (**◀ ▶ ▲ ▼**) you can change the orientation of the surface. The orientation of the reference coordinate system will change accordingly. Try changing the surface orientation on your own. The following figures show a couple of views of the graph:



- When done, press **EXIT**.
- Press **MODE** to return to the PLOT WINDOW environment.
- Change the Step data to read: Step Indep: 20 Depnd: 16
- Press **ERASE DATA** to see the surface plot. Sample views:



- When done, press **EXIT**.
- Press **MODE** to return to PLOT WINDOW.
- Press **ON** , or **NXT MODE** , to return to normal calculator display.

Try also a Fast 3D plot for the surface  $z = f(x,y) = \sin(x^2+y^2)$

- Press  $\leftarrow$  2D/3D , simultaneously if in RPN mode, to access the PLOT SETUP window.
- Press  $\nabla$  and type 'SIN(X^2+Y^2)'  $\text{00}$ .
- Press  $\text{F1}$   $\text{F2}$  to draw the plot.
- When done, press  $\text{F1}$ .
- Press  $\text{F1}$  to return to PLOT WINDOW.
- Press  $\text{ON}$  , or  $\text{NXT}$   $\text{00}$  , to return to normal calculator display.

## Wireframe plots

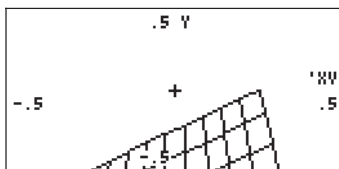
Wireframe plots are plots of three-dimensional surfaces described by  $z = f(x,y)$ . Unlike *Fast 3D* plots, wireframe plots are static plots. The user can choose the viewpoint for the plot, i.e., the point from which the surface is seen. For example, to produce a wireframe plot for the surface  $z = x + 2y - 3$ , use the following:

- Press  $\leftarrow$  2D/3D , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Wireframe.
- Press  $\nabla$  and type 'X+2\*Y-3'  $\text{00}$ .
- Make sure that 'X' is selected as the Indep: and 'Y' as the Depnd: variables.
- Press  $\text{NXT}$   $\text{00}$  to return to normal calculator display.
- Press  $\leftarrow$  WIN , simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Keep the default plot window ranges to read: X-Left:-1, X-Right:1, Y-Near:-1, Y-Far: 1, Z-Low: -1, Z-High: 1, XE:0,YE:-3, ZE:0, Step Indep: 10 Depnd: 8

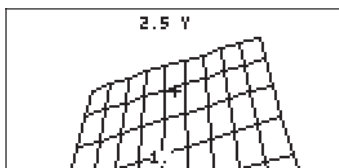
The coordinates XE, YE, ZE, stand for "eye coordinates," i.e., the coordinates from which an observer sees the plot. The values shown are the default values. The Step Indep: and Depnd: values represent the number of gridlines to be used in the plot. The larger these number, the slower it is to produce the graph. For the time being we'll keep the default values of 10 and 8 for the Step

- Press  $\text{F1}$   $\text{F2}$  to draw the three-dimensional surface. The result is a wireframe picture of the surface.

- Press **QUIT** **NXT** **QUIT** **QUIT** to see the graph with labels and ranges. This particular version of the graph is limited to the lower part of the display. We can change the viewpoint to see a different version of the graph.

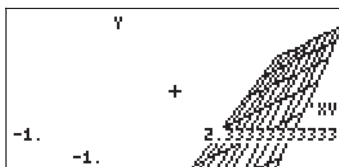


- Press **NXT** **NXT** **QUIT** **QUIT** to return to the PLOT WINDOW environment.
- Change the eye coordinate data to read :  $XE:0$   $YE:-3$   $ZE:3$
- Press **QUIT** **QUIT** to see the surface plot.
- Press **QUIT** **NXT** **QUIT** **QUIT** to see the graph with labels and ranges.



This version of the graph occupies more area in the display than the previous one. We can change the viewpoint, once more, to see another version of the graph.

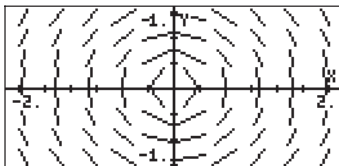
- Press **NXT** **NXT** **QUIT** **QUIT** to return to the PLOT WINDOW environment.
- Change the eye coordinate data to read :  $XE:3$   $YE:3$   $ZE:3$
- Press **QUIT** **QUIT** to see the surface plot. This time the bulk of the plot is located towards the right-hand side of the display.



- Press **QUIT** to return to the PLOT WINDOW environment.
- Press **ON** , or **NXT** **ON** , to return to normal calculator display.



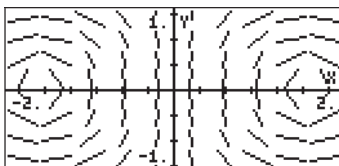
- Press **EDIT** **NXT** **EDIT** **EDIT** to see the graph with labels and ranges.



- Press **NEXT** **NEXT** **EDIT** **EDIT** to return to the PLOT WINDOW environment.
- Press **ON** , or **NEXT** **OK** , to return to normal calculator display.

Try also a Ps-Contour plot for the surface  $z = f(x,y) = \sin x \cos y$ .

- Press **←** **2D/3D** , simultaneously if in RPN mode, to access the PLOT SETUP window.
- Press **▼** and type 'SIN(X)\*COS(Y)' **OK**.
- Press **EDIT** **EDIT** to draw the slope field plot. Press **EDIT** **NEXT** **EDIT** **EDIT** to see the plot unencumbered by the menu and with identifying labels.



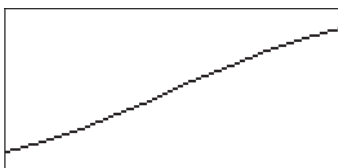
- Press **NEXT** **NEXT** **EDIT** to leave the EDIT environment.
- Press **EDIT** to return to the PLOT WINDOW environment. Then, press **ON** , or **NEXT** **OK** , to return to normal calculator display.

## Y-Slice plots

Y-Slice plots are animated plots of  $z$ -vs.- $y$  for different values of  $x$  from the function  $z = f(x,y)$ . For example, to produce a Y-Slice plot for the surface  $z = x^3 - xy^3$ , use the following:

- Press **←** **2D/3D** , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Y-Slice.
- Press **▼** and type ' $X^3 + X*Y^3$ ' **OK**.

- Make sure that 'X' is selected as the Indep: and 'Y' as the Depnd: variables.
- Press  $\boxed{\text{NXT}} \boxed{\text{01}}$  to return to normal calculator display.
- Press  $\boxed{\leftarrow} \boxed{\text{WIN}}$ , simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Change the default plot window ranges to read: X-Left:-1, X-Right:1, Y-Near:-1, Y-Far: 1, Z-Low:-1, Z-High:1, Step Indep: 10 Depnd: 8
- Press  $\boxed{\text{F5}} \boxed{\text{F6}}$  to draw the three-dimensional surface. You will see the calculator produce a series of curves on the screen, that will immediately disappear. When the calculator finishes producing all the y-slice curves, then it will automatically go into animating the different curves. One of the curves is shown below.



- Press  $\boxed{\text{ON}}$  to stop the animation. Press  $\boxed{\text{END}}$  to return to the PLOT WINDOW environment.
- Press  $\boxed{\text{ON}}$ , or  $\boxed{\text{NXT}} \boxed{\text{01}}$ , to return to normal calculator display.

Try also a Ps-Contour plot for the surface  $z = f(x,y) = (x+y) \sin y$ .

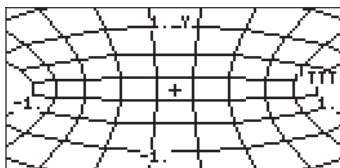
- Press  $\boxed{\leftarrow} \boxed{\text{2D/3D}}$ , simultaneously if in RPN mode, to access the PLOT SETUP window.
- Press  $\boxed{\nabla}$  and type '(X+Y)\*SIN(Y)'  $\boxed{\text{01}}$ .
- Press  $\boxed{\text{F5}} \boxed{\text{F6}}$  to produce the Y-Slice animation.
- Press  $\boxed{\text{ON}}$  to stop the animation.
- Press  $\boxed{\text{END}}$  to return to the PLOT WINDOW environment. Then, press  $\boxed{\text{ON}}$ , or  $\boxed{\text{NXT}} \boxed{\text{01}}$ , to return to normal calculator display.

## Gridmap plots

Gridmap plots produce a grid of orthogonal curves describing a function of a complex variable of the form  $w = f(z) = f(x+iy)$ , where  $z = x+iy$  is a complex variable. The functions plotted correspond to the real and imaginary part of  $w = \Phi(x,y) + i\Psi(x,y)$ , i.e., they represent curves  $\Phi(x,y) = \text{constant}$ , and  $\Psi(x,y) = \text{constant}$ . For example, to produce a Gridmap plot for the function  $w = \sin(z)$ , use the following:



- Press  $\leftarrow$   $2D/3D$ , simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Gridmap.
- Press  $\nabla$  and type 'SIN(X+i\*Y)'  $\boxed{08}$ .
- Make sure that 'X' is selected as the Indep: and 'Y' as the Depnd: variables.
- Press  $\boxed{NXT}$   $\boxed{03}$  to return to normal calculator display.
- Press  $\leftarrow$   $WIN$ , simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Keep the default plot window ranges to read: X-Left:-1, X-Right:1, Y-Near:-1 Y-Far: 1, XXLeft:-1 XXRight:1, YYNear:-1, yyFar: 1, Step Indep: 10 Depnd: 8
- Press  $\boxed{F1}$   $\boxed{F2}$  to draw the gridmap plot. The result is a grid of functions corresponding to the real and imaginary parts of the complex function.
- Press  $\boxed{F1}$   $\boxed{NXT}$   $\boxed{F1}$   $\boxed{F2}$  to see the graph with labels and ranges.



- Press  $\boxed{NXT}$   $\boxed{NXT}$   $\boxed{F1}$   $\boxed{F2}$  to return to the PLOT WINDOW environment.
- Press  $\boxed{ON}$ , or  $\boxed{NXT}$   $\boxed{03}$ , to return to normal calculator display.

Other functions of a complex variable worth trying for Gridmap plots are:

- |                                |                        |                 |                             |
|--------------------------------|------------------------|-----------------|-----------------------------|
| (1) SIN((X,Y))                 | i.e., $F(z) = \sin(z)$ | (2) (X,Y)^2     | i.e., $F(z) = z^2$          |
| (3) EXP((X,Y))                 | i.e., $F(z) = e^z$     | (4) SINH((X,Y)) | i.e., $F(z) = \sinh(z)$     |
| (5) TAN((X,Y))                 | i.e., $F(z) = \tan(z)$ | (6) ATAN((X,Y)) | i.e., $F(z) = \tan^{-1}(z)$ |
| (7) (X,Y)^3                    | i.e., $F(z) = z^3$     | (8) 1/(X,Y)     | i.e., $F(z) = 1/z$          |
| (9) $\sqrt{\phantom{x}}$ (X,Y) | i.e., $F(z) = z^{1/2}$ |                 |                             |

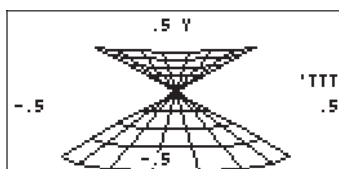
## Pr-Surface plots

*Pr-Surface* (parametric surface) plots are used to plot a three-dimensional surface whose coordinates (x,y,z) are described by  $x = x(X,Y)$ ,  $y = y(X,Y)$ ,  $z = z(X,Y)$ , where X and Y are independent parameters.

**Note:** The equations  $x = x(X,Y)$ ,  $y = y(X,Y)$ ,  $z = z(X,Y)$  represent a parametric description of a surface.  $X$  and  $Y$  are the independent parameters. Most textbooks will use  $(u,v)$  as the parameters, rather than  $(X,Y)$ . Thus, the parametric description of a surface is given as  $x = x(u,v)$ ,  $y = y(u,v)$ ,  $z = z(u,v)$ .

For example, to produce a Pr-Surface plot for the surface  $x = x(X,Y) = X \sin Y$ ,  $y = y(X,Y) = x \cos Y$ ,  $z = z(X,Y) = X$ , use the following:

- Press  $\leftarrow$  **2D/3D**, simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Pr-Surface.
- Press  $\nabla$  and type '{X\*SIN(Y), X\*COS(Y), X}' **OK**.
- Make sure that 'X' is selected as the Indep: and 'Y' as the Depnd: variables.
- Press **NXT** **OK** to return to normal calculator display.
- Press  $\leftarrow$  **WIN**, simultaneously if in RPN mode, to access the PLOT WINDOW screen.
- Keep the default plot window ranges to read: X-Left:-1, X-Right:1, Y-Near:-1, Y-Far: 1, Z-Low: -1, Z-High:1, XE: 0, YE:-3, zE:0, Step Indep: 10, Depnd: 8
- Press **EEEE EEEE** to draw the three-dimensional surface.
- Press **EEEE** **NXT** **EEEE EEEE** to see the graph with labels and ranges.



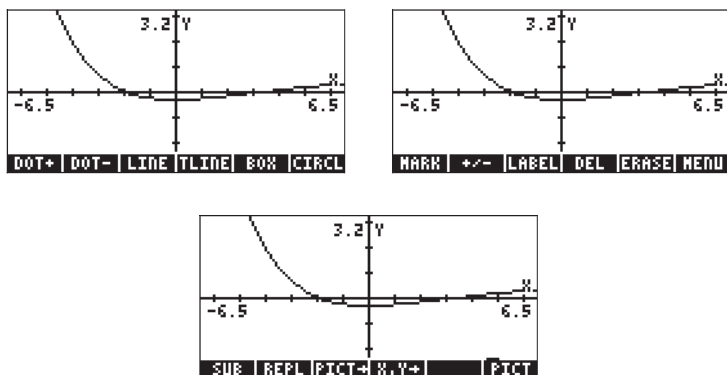
- Press **NXT** **NXT** **EEEE EEEE** to return to the PLOT WINDOW environment.
- Press **ON**, or **NXT** **OK**, to return to normal calculator display.

## The VPAR variable

The VPAR (Volume Parameter) variable contains information regarding the "volume" used to produce a three dimensional graph. Therefore, you will see it produced whenever you create a three dimensional plot such as Fast3D, Wireframe, or Pr-Surface.

## Interactive drawing

Whenever we produce a two-dimensional graph, we find in the graphics screen a soft menu key labeled **EDIT**. Pressing **EDIT** produces a menu that include the following options (press **NXT** to see additional functions):



Through the examples above, you have the opportunity to try out functions LABEL, MENU, PICT→, and REPL. Many of the remaining functions, such as DOT+, DOT-, LINE, BOX, CIRCL, MARK, DEL, etc., can be used to draw points, lines, circles, etc. on the graphics screen, as described below. To see how to use these functions we will try the following exercise:






First, we get the graphics screen corresponding to the following instructions:

- Press **2D/3D**, simultaneously if in RPN mode, to access to the PLOT SETUP window.
- Change TYPE to Function, if needed
- Change EQ to 'X'
- Make sure that Indep: is set to 'X' also
- Press **NXT** **OK** to return to normal calculator display.
- Press **WIN**, simultaneously if in RPN mode, to access the PLOT window (in this case it will be called PLOT –POLAR window).
- Change the H-VIEW range to -10 to 10, by using **1** **0** **+/-** **OK** **1** **0** **OK**, and the V-VIEW range to -5 to 5 by using **5** **+/-** **OK** **5** **OK**.
- Press **GRAPH** **EDIT** to plot the function.
- Press **EDIT** **NXT** **EDIT** to add labels to the graph. Press **NXT** **NXT** (or **PREV**) to recover the original EDIT menu.

Next, we illustrate the use of the different drawing functions on the resulting graphics screen. They require use of the cursor and the arrow keys (◀ ▶ ▲ ▼) to move the cursor about the graphics screen.

## DOT+ and DOT-



When DOT+ is selected, pixels will be activated wherever the cursor moves leaving behind a trace of the cursor position. When DOT- is selected, the opposite effect occurs, i.e., as you move the cursor, pixels will be deleted.

For example, use the ▶ ▲ keys to move the cursor somewhere in the middle of the first quadrant of the x-y plane, then press . The label will be selected (). Press and hold the ▶ key to see a horizontal line being traced. Now, press , to select this option (). Press and hold the ◀ key to see the line you just traced being erased. Press , when done, to deselect this option.





## MARK


This command allows the user to set a mark point which can be used for a number of purposes, such as:

- Start of line with the LINE or TLINE command
- Corner for a BOX command
- Center for a CIRCLE command

Using the MARK command by itself simply leaves an x in the location of the mark (Press   to see it in action).

## LINE

This command is used to draw a line between two points in the graph. To see it in action, position the cursor somewhere in the first quadrant, and press   . A MARK is placed over the cursor indicating the origin of the line. Use the ▶ key to move the cursor to the right of the current position, say about 1 cm to the right, and press . A line is draw between the first and the last points.

Notice that the cursor at the end of this line is still active indicating that the calculator is ready to plot a line starting at that point. Press ▼ to move the cursor downwards, say about another cm, and press  again. Now you

should have a straight angle traced by a horizontal and a vertical segments. The cursor is still active. To deactivate it, without moving it at all, press **LINE**. The cursor returns to its normal shape (a cross) and the LINE function is no longer active.

## TLINE

(Toggle LINE) Move the cursor to the second quadrant to see this function in action. Press **TLINE**. A MARK is placed at the start of the toggle line. Move the cursor with the arrow keys away from this point, and press **TLINE**. A line is drawn from the current cursor position to the reference point selected earlier. Pixels that are on in the line path will be turned off, and vice versa. To remove the most recent line traced, press **TLINE** again. To deactivate TLINE, move the cursor to the original point where TLINE was activated, and press **TLIN** **TLIN**.

## BOX

This command is used to draw a box in the graph. Move the cursor to a clear area of the graph, and press **BOX**. This highlights the cursor. Move the cursor with the arrow keys to a point away, and in a diagonal direction, from the current cursor position. Press **BOX** again. A rectangle is drawn whose diagonal joins the initial and ending cursor positions. The initial position of the box is still marked with an x. Moving the cursor to another position and pressing **BOX** will generate a new box containing the initial point. To deselect BOX, move the cursor to the original point where BOX was activated, then press **BOX** **BOX**.

## CIRCL

This command produces a circle. Mark the center of the circle with a MARK command, then move the cursor to a point that will be part of the periphery of the circle, and press **CIRCL**. To deactivate CIRCL, return the cursor to the MARK position and press **LINE**.

Try this command by moving the cursor to a clear part of the graph, press **MARK**. Move the cursor to another point, then press **CIRCL**. A circle centered at the MARK, and passing through the last point will be drawn.

## LABEL

Pressing **LABEL** places the labels in the x- and y-axes of the current plot. This feature has been used extensively through this chapter.

## DEL

This command is used to remove parts of the graph between two MARK positions. Move the cursor to a point in the graph, and press **MARK**. Move the cursor to a different point, press **MARK** again. Then, press **DEL**. The section of the graph boxed between the two marks will be deleted.

## ERASE

The function ERASE clears the entire graphics window. This command is available in the PLOT menu, as well as in the plotting windows accessible through the soft menu keys.

## MENU

Pressing **MENU** will remove the soft key menu labels to show the graphic unencumbered by those labels. To recover the labels, press **NXT**.

## SUB

Use this command to extract a subset of a graphics object. The extracted object is automatically placed in the stack. Select the subset you want to extract by placing a MARK at a point in the graph, moving the cursor to the diagonal corner of the rectangle enclosing the graphics subset, and press **SUB**. This feature can be used to move parts of a graphics object around the graph.

## REPL

This command places the contents of a graphic object currently in stack level 1 at the cursor location in the graphics window. The upper left corner of the graphic object being inserted in the graph will be placed at the cursor position. Thus, if you want a graph from the stack to completely fill the graphic window, make sure that the cursor is placed at the upper left corner of the display.

## PICT→

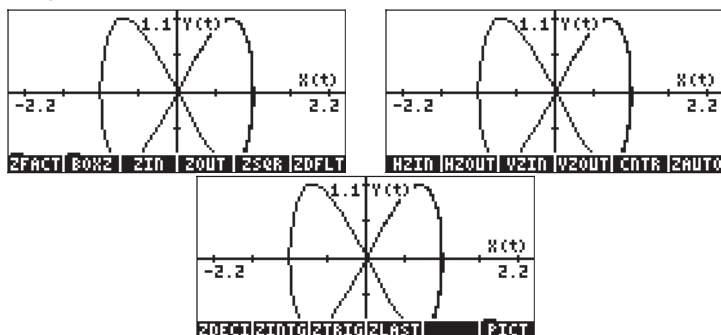
This command places a copy of the graph currently in the graphics window on to the stack as a graphic object. The graphic object placed in the stack can be saved into a variable name for storage or other type of manipulation.

## X,Y→

This command copies the coordinates of the current cursor position, in user coordinates, in the stack.

## Zooming in and out in the graphics display

Whenever you produce a two-dimensional FUNCTION graphic interactively, the first soft-menu key, labeled **EDIT**, lets you access functions that can be used to zoom in and out in the current graphics display. The ZOOM menu includes the following functions (press **NEXT** to move to the next menu):



We present each of these functions following. You just need to produce a graph as indicated in Chapter 12, or with one of the programs listed earlier in this Chapter.

### ZFACT, ZIN, ZOUT, and ZLAST

Pressing **EDIT** produces an input screen that allows you to change the current X- and Y-Factors. The X- and Y-Factors relate the horizontal and vertical user-defined unit ranges to their corresponding pixel ranges. Change the H-Factor to read 8., and press **OK**, then change the V-Factor to read 2., and press **OK**. Check off the option ☒ Recenter on cursor, and press **OK**.

Back in the graphics display, press **EDIT**. The graphic is re-drawn with the new vertical and horizontal scale factors, centered at the position where the cursor was located, while maintaining the original PICT size (i.e., the original number of pixels in both directions). Using the arrow keys, scroll horizontally or vertically as far as you can of the zoomed-in graph.

To zoom out, subjected to the H- and V-Factors set with ZFACT, press **EDIT** **EDIT**. The resulting graph will provide more detail than the zoomed-in graph.

You can always return to the very last zoom window by using **EXIT**.

## **BOXZ**

Zooming in and out of a given graph can be performed by using the soft-menu key BOXZ. With BOXZ you select the rectangular sector (the “box”) that you want to zoom in into. Move the cursor to one of the corners of the box (using the arrow keys), and press **BOXZ**. Using the arrow keys once more, move the cursor to the opposite corner of the desired zoom box. The cursor will trace the zoom box in the screen. When desired zoom box is selected, press **BOXZ**. The calculator will zoom in the contents of the zoom box that you selected to fill the entire screen.

If you now press **EXIT**, the calculator will zoom out of the current box using the H- and V-Factors, which may not recover the graph view from which you started the zoom box operation.

## **ZDFLT, ZAUTO**

Pressing **ZDFLT** re-draws the current plot using the default x- and y-ranges, i.e., -6.5 to 6.5 in x, and -3.1 to 3.1 in y. The command **ZAUTO**, on the other hand, creates a zoom window using the current independent variable (x) range, but adjusting the dependent variable (y) range to fit the curve (as when you use the function **YMIN** in the PLOT WINDOW input form (**YMIN** **WIN**), simultaneously in RPN mode).

## **HZIN, HZOUT, VZIN and VZOUT**

These functions zoom in and out the graphics screen in the horizontal or vertical direction according to the current H- and V-Factors.

## **CNTR**

Zooms in with the center of the zoom window in the current cursor location. The zooming factors used are the current H- and V-Factors.

## **ZDECI**

Zooms the graph so as to round off the limits of the x-interval to a decimal value.

## **ZINTG**

Zooms the graph so that the pixel units become user-define units. For example, the minimum PICT window has 131 pixels. When you use ZINTG, with the




cursor at the center of the screen, the window gets zoomed so that the x-axis extends from  $-64.5$  to  $65.5$ .

**ZSQR**


Zooms the graph so that the plotting scale is maintained at 1:1 by adjusting the x scale, keeping the y scale fixed, if the window is wider than taller. This forces a proportional zooming.

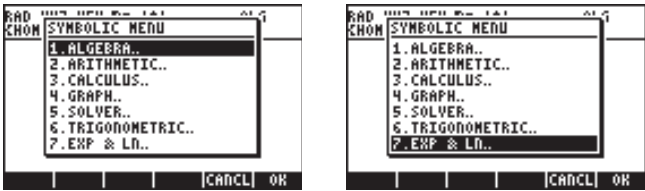
**ZTRIG**

Zooms the graph so that the x scale incorporates a range from about  $-3\pi$  to  $+3\pi$ , the preferred range for trigonometric functions.







**Note:** None of these functions are programmable. They are only useful in an interactive way. Do not confuse the command  in the ZOOM menu with the function ZFACTOR, which is used for gas dynamic and chemistry applications (see Chapter 3).

**The SYMBOLIC menu and graphs**

The SYMBOLIC menu is activated by pressing the  key (fourth key from the left in fourth row from the top of the keyboard). This menu provides a list of menus related to the Computer Algebraic System or CAS, these are:



All but one of these menus are available directly in the keyboard by pressing the appropriate keystroke combination as follows. The Chapter of the user manual where the menus are described is also listed:

ALGEBRA..	 <u>ALG</u> (the  key)	Ch. 5
ARITHMETIC..	 <u>ARITH</u> (the  key)	Ch. 5
CALCULUS..	 <u>CALC</u> (the  key)	Ch. 13

SOLVER..	<u>S.SLV</u> (the <b>7</b> key)	Ch. 6
TRIGONOMETRIC..	<u>TRIG</u> (the <b>8</b> key)	Ch. 5
EXP&LN..	<u>EXP&amp;LN</u> (the <b>8</b> key)	Ch. 5

## The SYMB/GRAPH menu

The GRAPH sub-menu within the SYMB menu includes the following functions:



DEFINE: same as the keystroke sequence DEF (the **2** key)

GROBADD: pastes two GROBs first over the second (See Chapter 22)

PLOT(function): plots a function, similar to 2D/3D

PLOTADD(function): adds this function to the list of functions to plot, similar to 2D/3D

Plot setup...: same as 2D/3D

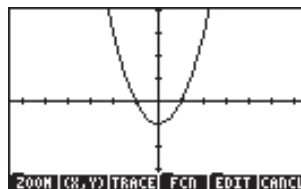
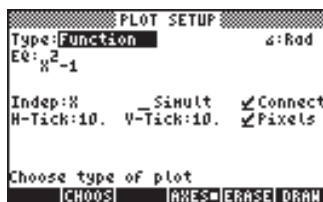
SIGNTAB(function): sign table of given function showing intervals of positive and negative variation, zero points and infinite asymptotes

TABVAL: table of values for a function

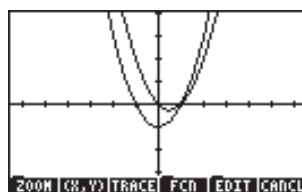
TABVAR: variation table of a function

Examples of some of these functions are provided next.

PLOT( $X^2-1$ ) is similar to 2D/3D with EQ:  $X^2-1$ . Using produces the plot:



PLOTADD( $X^2-X$ ) is similar to 2D/3D but adding this function to EQ:  $X^2-1$ . Using produces the plot:



TABVAL( $X^2-1$ , {1, 3}) produces a list of {min max} values of the function in the interval {1,3}, while SIGNTAB( $X^2-1$ ) shows the sign of the function in the interval  $(-\infty, +\infty)$ , with  $f(x) > 0$  in  $(-\infty, -1)$ ,  $f(x) < 0$  in  $(-1, 1)$ , and  $f(x) > 0$  in  $(1, +\infty)$ .

```

:HELP
: TABVAL(X^2-1,{1 3})
      {X^2-1 {{1 3} {0 8}}}
: SIGNTAB(X^2-1)
      { -∞ + -1 - 1 + +∞ }
CASCM HELP

```

TABVAR(LN(X)/X) produces the following table of variation:

```

: TABVAR LN(X)/X
{ 'LN(X)/X' { ' -∞ ' '
? '0+0' + 'EXP(1)' -
+∞ } { '?' '1' '∞' ↑
'1/EXP(1)' ↓ +: 0 } }
Graphic 113 x 131
+SHIP+SHIP+ +DEL DEL+DEL L INS

```

A detailed interpretation of the table of variation is easier to follow in RPN mode:

$$F = \frac{\ln(X)}{X}$$

$$F' = \frac{X \cdot \frac{1}{X} - \ln(X)}{SQ(X)}$$

$$F' = \frac{X \cdot \frac{1}{X} - \ln(X)}{SQ(X)}$$

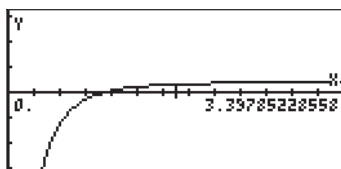
$$\rightarrow = \frac{-(\ln(X)-1)}{X^2}$$

Variation table:  

$$\begin{array}{c} -\infty ? 0+0 + e^1 - +\infty X \\ ? ? \infty \uparrow \frac{1}{e} \downarrow +: 0 F \end{array}$$

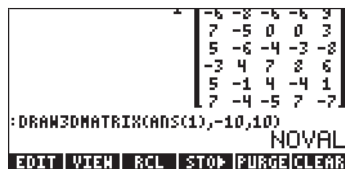
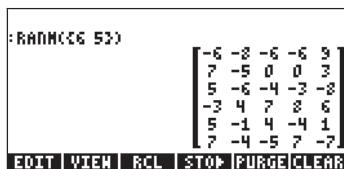
The output is in a graphical format, showing the original function,  $F(X)$ , the derivative  $F'(X)$  right after derivation and after simplification, and finally a table of variation. The table consists of two rows, labeled in the right-hand side. Thus, the top row represents values of  $X$  and the second row represents values

of  $F$ . The question marks indicate uncertainty or non-definition. For example, for  $X < 0$ ,  $\text{LN}(X)$  is not defined, thus the  $X$  lines show a question mark in that interval. Right at zero ( $0+0$ )  $F$  is infinite, for  $X = e$ ,  $F = 1/e$ .  $F$  increases before reaching this value, as indicated by the upward arrow, and decreases after this value ( $X=e$ ) becoming slightly larger than zero ( $+0$ ) as  $X$  goes to infinity. A plot of the graph is shown below to illustrate these observations:



## Function DRAW3DMATRIX

This function takes as argument a  $n \times m$  matrix,  $\mathbf{Z}$ ,  $\mathbf{Z} = [z_{ij}]$ , and minimum and maximum values for the plot. You want to select the values of  $v_{\min}$  and  $v_{\max}$  so that they contain the values listed in  $\mathbf{Z}$ . The general call to the function is, therefore,  $\text{DRAW3DMATRIX}(\mathbf{Z}, v_{\min}, v_{\max})$ . To illustrate the use of this function we first generate a  $6 \times 5$  matrix using  $\text{RANM}(\{6,5\})$ , and then call function  $\text{DRAW3DMATRIX}$ , as shown below:



The plot is in the style of a FAST3D PLOT. Different views of the plot are shown below:

