Chapter 3
Calculation with real numbers

This chapter demonstrates the use of the calculator for operations and functions related to real numbers. Operations along these lines are useful for most common calculations in the physical sciences and engineering. The user should be acquainted with the keyboard to identify certain functions available in the keyboard (e.g., SIN, COS, TAN, etc.). Also, it is assumed that the reader knows how to adjust the calculator's operation, i.e., select operating mode (see Chapter 1), use menus and choose boxes (see Chapter 1), and operate with variables (see Chapter 2).

Checking calculators settings

To check the current calculator and CAS settings you need to just look at the top line in the calculator display in normal operation. For example, you may see the following setting: RAD XYZ DEC **R** = 'X'

This stands for RADians for angular measurements, XYZ for Rectangular (Cartesian) coordinates, DECimal number base, **R**eal numbers preferred, = means "exact" results, and 'X' is the value of the default independent variable.

Another possible listing of options could be DEG R∠Z HEX C ~ 't'

This stands for DEGrees as angular measurements, R∠Z for Polar coordinates, HEXagesimal number base, Complex numbers allowed, ~ stands for "approximate" results, and 't' as the default independent variable.

In general, this part of the display contains seven elements. Each element is identified next under numbers 1 through 7. The possible values for each element are shown between parentheses following the element description. The explanation of each of those values is also shown:

1. Angle measure specification (DEG, RAD, GRD)
   DEG: degrees, 360 degrees in a complete circle
   RAD: radians, 2π radians in a complete circle
   GRD: grades, 400 grades in a complete circle

2. Coordinate system specification (XYZ, R∠Z, R∠∠). The symbol ∠ stands for an angular coordinate.
   XYZ: Cartesian or rectangular (x,y,z)
   R∠Z: cylindrical Polar coordinates (r,θ,z)
   R∠∠: Spherical coordinates (ρ,θ,ϕ)
3. Number base specification (HEX, DEC, OCT, BIN)
   HEX: hexadecimal numbers (base 16)
   DEC: decimal numbers (base 10)
   OCT: octal numbers (base 8)
   BIN: binary numbers (base 2)
4. Real or complex mode specification (**R**, **C**)
   R: real numbers
   C: complex numbers
5. Exact or approximate mode specification (=, ~)
   = exact (symbolic) mode
   ~ approximate (numerical) mode
6. Default CAS independent variable (e.g., 'X', 't', etc.)

Checking calculator mode

When in RPN mode the different levels of the stack are listed in the left-hand side of the screen. When the ALGEBRAIC mode is selected there are no numbered stack levels, and the word ALG is listed in the top line of the display towards the right-hand side. The difference between these operating modes was described in detail in Chapter 1.

Real number calculations

To perform real number calculations it is preferred to have the CAS set to *Real* (as opposite to *Complex*) mode. In some cases, a complex result may show up, and a request to change the mode to *Complex* will be made by the calculator. *Exact* mode is the default mode for most operations. Therefore, you may want to start your calculations in this mode. Any change to *Approx* mode required to complete an operation will be requested by the calculator. There is no preferred selection for the angle measure or for the number base specification.

Real number calculations will be demonstrated in both the Algebraic (ALG) and Reverse Polish Notation (RPN) modes.

Changing sign of a number, variable, or expression

Use the [+/-] key. In ALG mode, you can press [+/-] before entering the number, e.g., [+/-] [2] [.] [5] [ENTER]. Result = -2.5. In RPN mode, you need to enter at least part of the number first, and then use the [+/-] key, e.g., [2] [.] [5] [+/-]. Result = -2.5. If you use the [+/-] function while there is no command line, the calculator will apply the NEG function (inverse of sign) to the object on the first level of the stack.

The inverse function

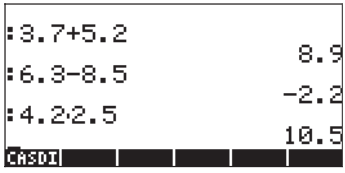
Use the [1/x] key. In ALG mode, press [1/x] first, followed by a number or algebraic expression, e.g., [1/x] [2]. Result = ½ or 0.5. In RPN mode, enter the number first, then use the key, e.g., [4] [ENTER] [1/x]. Result = ¼ or 0.25.

Addition, subtraction, multiplication, division

Use the proper operation key, namely, [+] [−] [×] [÷]. In ALG mode, press an operand, then an operator, then an operand, followed by an [ENTER] to obtain a result. Examples:

[3] [.] [7] [+] [5] [.] [2] [ENTER]
[6] [.] [3] [−] [8] [.] [5] [ENTER]
[4] [.] [2] [×] [2] [.] [5] [ENTER]
[2] [.] [3] [÷] [4] [.] [5] [ENTER]

The first three operations above are shown in the following screen shot:



In RPN mode, enter the operands one after the other, separated by an [ENTER], then press the operator key. Examples:

[3] [.] [7] [ENTER] [5] [.] [2] [+]
[6] [.] [3] [ENTER] [8] [.] [5] [−]
[4] [.] [2] [ENTER] [2] [.] [5] [×]
[2] [.] [3] [ENTER] [4] [.] [5] [÷]

Alternatively, in RPN mode, you can separate the operands with a space (SPC) before pressing the operator key. Examples:

3 . 7 SPC 5 . 2 +
6 . 3 SPC 8 . 5 −
4 . 2 SPC 2 . 5 ×
2 . 3 SPC 4 . 5 ÷

Using parentheses

Parentheses can be used to group operations, as well as to enclose arguments of functions. The parentheses are available through the keystroke combination ↰ ( ) . Parentheses are always entered in pairs. For example, to calculate (5+3.2)/(7-2.2):

In ALG mode:

↰ ( ) 5 + 3 . 2 ▶ ÷ ↰ ( ) 7 − 2 . 2 ENTER

In RPN mode, you do not need the parenthesis, calculation is done directly on the stack:

5 ENTER 3 . 2 + 7 ENTER 2 . 2 − ÷

In RPN mode, typing the expression between quotes will allow you to enter the expression like in algebraic mode:

' ↰ ( ) 5 + 3 . 2 ▶ ÷
↰ ( ) 7 − 2 . 2 ENTER EVAL

For both, ALG and RPN modes, using the Equation Writer:

↱ EQW 5 + 3 . 2 ▶ ÷ 7 − 2 . 2

The expression can be evaluated within the Equation writer, by using:

▲ ▲ ▲ ▲ EVAL or, ↱ ▲ EVAL

Absolute value function

The absolute value function, ABS, is available through the keystroke combination: ↰ ABS . When calculating in the stack in ALG mode, enter the function before the argument, e.g., ↰ ABS +/− 2 . 3 2 ENTER

In RPN mode, enter the number first, then the function, e.g.,

2 . 3 2 +/− ↰ ABS

Squares and square roots

The square function, SQ, is available through the keystroke combination: ◀ x^2 . When calculating in the stack in ALG mode, enter the function before the argument, e.g., ◀ x^2 +/– 2 . 3 ENTER

In RPN mode, enter the number first, then the function, e.g.,

2 . 3 +/– ◀ x^2

The square root function, √, is available through the R key. When calculating in the stack in ALG mode, enter the function before the argument, e.g.,

√x 1 2 3 . 4 ENTER

In RPN mode, enter the number first, then the function, e.g.,

1 2 3 . 4 √x

Powers and roots

The power function, ^, is available through the y^x key. When calculating in the stack in ALG mode, enter the base (*y*) followed by the y^x key, and then the exponent (*x*), e.g., 5 . 2 y^x 1 . 2 5

In RPN mode, enter the number first, then the function, e.g.,

5 . 2 ENTER 1 . 2 5 ENTER y^x

The root function, XROOT*(y,x)*, is available through the keystroke combination ▶ $\sqrt[x]{y}$. When calculating in the stack in ALG mode, enter the function XROOT followed by the arguments (*y,x*), separated by commas, e.g.,

▶ $\sqrt[x]{y}$ 3 ▶ , 2 7 ENTER

In RPN mode, enter the argument *y*, first, then, *x*, and finally the function call, e.g., 2 7 ENTER 3 ENTER ▶ $\sqrt[x]{y}$

Base-10 logarithms and powers of 10

Logarithms of base 10 are calculated by the keystroke combination ▶ LOG (function LOG) while its inverse function (ALOG, or antilogarithm) is calculated by using ◀ 10^x . In ALG mode, the function is entered before the argument:

▶ LOG 2 . 4 5 ENTER
◀ 10^x +/– 2 . 3 ENTER

In RPN mode, the argument is entered before the function

2 . 4 5 ENTER ▶ LOG
2 . 3 +/– ENTER ◀ 10^x

Using powers of 10 in entering data

Powers of ten, i.e., numbers of the form -4.5×10^{-2}, etc., are entered by using the EEX key. For example, in ALG mode:

+/– 4 • 5 EEX +/– 2 ENTER

Or, in RPN mode:

4 • 5 +/– EEX 2 +/– ENTER

Natural logarithms and exponential function

Natural logarithms (i.e., logarithms of base *e = 2.7182818282*) are calculated by the keystroke combination → LN (function LN) while its inverse function, the exponential function (function EXP) is calculated by using ← e^x . In ALG mode, the function is entered before the argument:

→ LN 2 • 4 5 ENTER
← e^x +/– 2 • 3 ENTER

In RPN mode, the argument is entered before the function

2 • 4 5 ENTER → LN
2 • 3 +/– ENTER ← e^x

Trigonometric functions

Three trigonometric functions are readily available in the keyboard: sine (SIN), cosine (COS), and tangent (TAN). The arguments of these functions are angles, therefore, they can be entered in any system of angular measure (degrees, radians, grades). For example, with the DEG option selected, we can calculate the following trigonometric functions:

In ALG mode:

SIN 3 0 ENTER
COS 4 5 ENTER
TAN 1 3 5 ENTER

In RPN mode:

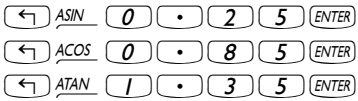
3 0 ENTER SIN
4 5 ENTER COS
1 3 5 ENTER TAN

Inverse trigonometric functions

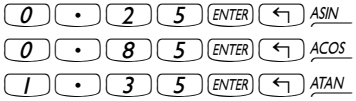
The inverse trigonometric functions available in the keyboard are the arcsine (ASIN), arccosine (ACOS), and arctangent (ATAN), available through the keystroke combinations ← ASIN , ← ACOS , and ← ATAN , respectively. Since

the inverse trigonometric functions represent angles, the answer from these functions will be given in the selected angular measure (DEG, RAD, GRD). Some examples are shown next:

In ALG mode:



In RPN mode:



All the functions described above, namely, ABS, SQ, √, ^, XROOT, LOG, ALOG, LN, EXP, SIN, COS, TAN, ASIN, ACOS, ATAN, can be combined with the fundamental operations (+ − × ÷) to form more complex expressions. The Equation Writer, whose operations is described in Chapter 2, is ideal for building such expressions, regardless of the calculator operation mode.

Differences between functions and operators

Functions like ABS, SQ, √, LOG, ALOG, LN, EXP, SIN, COS, TAN, ASIN, ACOS, ATAN require a single argument. Thus, their application is ALG mode is straightforward, e.g., ABS(x). Some functions like XROOT require two arguments, e.g., XROOT(x,y). This function has the equivalent keystroke sequence → $\sqrt[x]{y}$.

Operators, on the other hand, are placed after a single argument or between two arguments. The factorial operator (!), for example, is placed after a number, e.g., 5 ALPHA → 2 ENTER. Since this operator requires a single argument it is referred to as a unary operator. Operators that require two arguments, such as + − × ÷ y^x, are binary operators, e.g., 3 × 5, or 4 y^x 2.

Real number functions in the MTH menu

The MTH (MaTHematics) menu include a number of mathematical functions mostly applicable to real numbers. To access the MTH menu, use the keystroke

combination ↰ MTH . With the default setting of *CHOOSE boxes* for system flag 117 (see Chapter 2), the MTH menu is shown as the following menu list:

As they are a great number of mathematic functions available in the calculator, the MTH menu is sorted by the type of object the functions apply on. For example, options 1. *VECTOR..*, *2. MATRIX.*, and *3. LIST..* apply to those data types (i.e., vectors, matrices, and lists) and will discussed in more detail in subsequent chapters. Options *4. HYPERBOLIC..* and *5. REAL..* apply to real numbers and will be discussed in detailed herein. Option *6. BASE..* is used for conversion of numbers in different bases, and is also to be discussed in a separate chapter. Option *7. PROBABILITY..* is used for probability applications and will be discussed in an upcoming chapter. Option *8. FFT..* (Fast Fourier Transform) is an application of signal processing and will be discussed in a different chapter. Option *9. COMPLEX..* contains functions appropriate for complex numbers, which will be discussed in the next chapter. Option *10. CONSTANTS* provides access to the constants in the calculator. This option will be presented later in this section. Finally, option *11. SPECIAL FUNCTIONS..* includes functions of advanced mathematics that will be discussed in this section also.

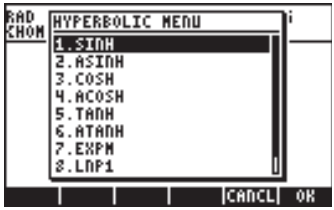
In general, to apply any of these functions you need to be aware of the number and order of the arguments required, and keep in mind that, in ALG mode you should select first the function and then enter the argument, while in RPN mode, you should enter the argument in the stack first, and then select the function.

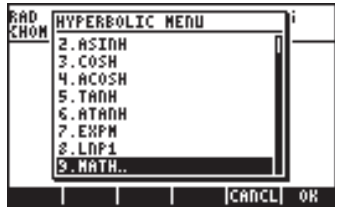
Using calculator menus:

1. Since the operation of *MTH* functions (and of many other calculator menus) is very similar, we will describe in detail the use of the *4. HYPERBOLIC..* menu in this section, with the intention of describing the general operation of calculator menus. Pay close attention to the process for selecting different options.
2. To quickly select one of the numbered options in a menu list (or CHOOSE box), simply press the number for the option in the keyboard. For example, to select option *4. HYPERBOLIC..* in the MTH menu, simply press 4 .

Hyperbolic functions and their inverses

Selecting Option *4. HYPERBOLIC..* , in the *MTH* menu, and pressing OK, produces the hyperbolic function menu:





The hyperbolic functions are:

Hyperbolic sine, SINH, and its inverse, ASINH or $\sinh^{-1}$

Hyperbolic cosine, COSH, and its inverse, ACOSH or $\cosh^{-1}$

Hyperbolic tangent, TANH, and its inverse, ATANH or $\tanh^{-1}$

This menu contains also the functions:

$$EXPM(x) = \exp(x) - 1,$$

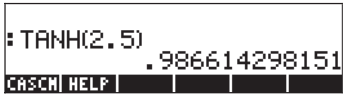
$$LNP1(x) = \ln(x+1).$$

Finally, option *9. MATH*, returns the user to the MTH menu.

For example, in ALG mode, the keystroke sequence to calculate, say, *tanh(2.5),* is the following:

↰ MTH	Select *MTH* menu
4 OK	Select the *4. HYPERBOLIC..* menu
5 OK	Select the *5. TANH* function
2 . 5 ENTER	Evaluate *tanh(2.5)*

The screen shows the following output:



In the RPN mode, the keystrokes to perform this calculation are the following:

2 . 5 ENTER	Enter the argument in the stack
↰ MTH	Select *MTH* menu
4 OK	Select the *4. HYPERBOLIC..* menu
5 OK	Select the *5. TANH* function

The result is:

The operations shown above assume that you are using the default setting for system flag 117 (*CHOOSE boxes*). If you have changed the setting of this flag (see Chapter 2) to *SOFT menu*, the MTH menu will show as labels of the soft menu keys, as follows (left-hand side in ALG mode, right –hand side in RPN mode):

Pressing NXT shows the remaining options:

Note: Pressing ← PREV will return to the first set of *MTH* options. Also, using the combination → ▼ will list all menu functions in the screen, e.g.

Thus, to select, for example, the hyperbolic functions menu, with this menu format press MTH , to produce:

Finally, in order to select, for example, the hyperbolic tangent (tanh) function, simply press TANH.

Note: To see additional options in these soft menus, press the NXT key or the ◀ PREV keystroke sequence.

For example, to calculate *tanh(2.5),* in the ALG mode, when using *SOFT menus* over *CHOOSE boxes*, follow this procedure:

◀ MTH	Select *MTH* menu
HYP	Select the *HYPERBOLIC..* menu
TANH	Select the *TANH* function
2 . 5 ENTER	Evaluate *tanh(2.5)*

In RPN mode, the same value is calculated using:

2 . 5 ENTER	Enter argument in the stack
◀ MTH	Select *MTH* menu
HYP	Select the *HYPERBOLIC..* menu
TANH	Select the *TANH* function

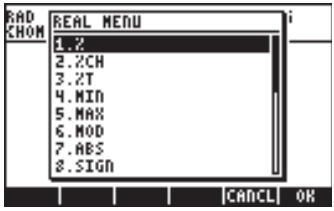
As an exercise of applications of hyperbolic functions, verify the following values:

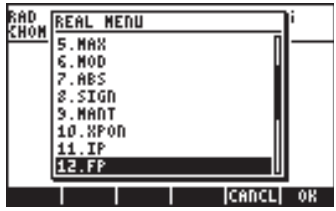
SINH (2.5) = 6.05020..	ASINH(2.0) = 1.4436…
COSH (2.5) = 6.13228..	ACOSH (2.0) = 1.3169…
TANH(2.5) = 0.98661..	ATANH(0.2) = 0.2027…
EXPM(2.0) = 6.38905….	LNP1(1.0) = 0.69314….

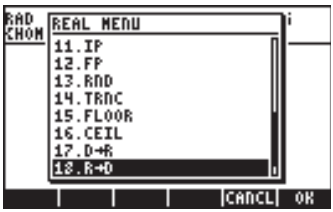
Once again, the general procedure shown in this section can be applied for selecting options in any calculator menu.

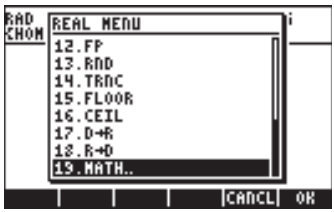
Real number functions

Selecting option *5. REAL..* in the MTH menu, with system flag 117 set to *CHOOSE boxes*, generates the following menu list:



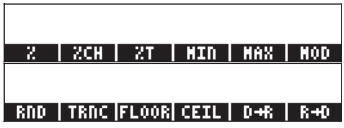






Option *19. MATH..* returns the user to the MTH menu. The remaining functions are grouped into six different groups described below.

If system flag 117 is set to *SOFT menus*, the *REAL* functions menu will look like this (ALG mode used, the same soft menu keys will be available in RPN mode):



The very last option, NXT, returns the user to the *MTH* menu.

Percentage functions

These functions are used to calculate percentages and related values as follows:

% (y,x) : calculates the x percentage of y

%CH(y,x) : calculates 100(y-x)/x, i.e., the percentage change, the difference between two numbers.

%T(y,x) : calculates 100 x/y, i.e., the percentage total, the portion that one number (x) is of another (y).

These functions require two arguments, we illustrate the calculation of *%T(15,45)*, i.e., calculation 15% of 45, next. We assume that the calculator is set to ALG mode, and that system flag 117 is set to *CHOOSE boxes*. The procedure is as follows:

↰ MTH	Select *MTH* menu
5 NXT	Select the *5. REAL..* menu
3 NXT	Select the *5. %T* function
1 5	Enter first argument
↱ ,	Enter a comma to separate arguments
4 5	Enter second argument
ENTER	Calculate function

The result is shown next:

In RPN mode, recall that argument *y* is located in the second level of the stack, while argument *x* is located in the first level of the stack. This means, you should enter *x* first, and then, *y*, just as in ALG mode. Thus, the calculation of *%T(15,45)*, in RPN mode, and with system flag 117 set to *CHOOSE boxes*, we proceed as follows:

1 5 ENTER	Enter first argument
4 5 ENTER	Enter second argument
↰ MTH	Select *MTH* menu
5 NXT	Select the *5. REAL..* menu
3 NXT	Select the *5. %T* function

Note: The exercises in this section illustrate the general use of calculator functions having 2 arguments. The operation of functions having 3 or more arguments can be generalized from these examples.

As an exercise for percentage-related functions, verify the following values:
%(5,20) = 1, %CH(22,25) = 13.6363.., %T(500,20) = 4

Minimum and maximum

Use these functions to determine the minimum or maximum value of two arguments.

MIN(x,y) : minimum value of x and y

MAX(x,y) : maximum value of x and y

As an exercise, verify that *MIN(-2,2) = -2, MAX(-2,2) = 2*

Modulo

MOD: y mod x = residual of y/x, i.e., if x and y are integer numbers, y/x = d + r/x, where d = quotient, r = residual. In this case, r = y mod x.

Please notice that MOD is not a function, but rather an operator, i.e., in ALG mode, MOD should be used as y MOD x, and not as MOD(y,x). Thus, the operation of MOD is similar to that of +, −, ×, ÷.

As an exercise, verify that *15 MOD 4 = 15 mod 4 = residual of 15/4 = 3*

Absolute value, sign, mantissa, exponent, integer and fractional parts

ABS(x) : calculates the absolute value, |x|

SIGN(x) : determines the sign of x, i.e., -1, 0, or 1.

MANT(x) : determines the mantissa of a number based on $\log_{10}$.

XPON(x) : determines the power of 10 in the number

IP(x) : determines the integer part of a real number

FP(x) : determines the fractional part of a real number

As an exercise, verify that *ABS(-3) = |-3| = 3, SIGN(-5) = -1, MANT(2540) = 2.540, XPON(2540) = 3, IP(2.35) = 2, FP(2.35) = 0.35.*

Rounding, truncating, floor, and ceiling functions

RND(x,y) : rounds up y to x decimal places

TRNC(x,y) : truncate y to x decimal places

FLOOR(x) : closest integer that is less than or equal to x

CEIL(x) : closest integer that is greater than or equal to x

As an exercise, verify that *RND(1.4567,2) = 1.46, TRNC(1.4567,2) = 1.45, FLOOR(2.3) = 2, CEIL(2,3) = 3*

Radians-to-degrees and degrees-to-radians functions

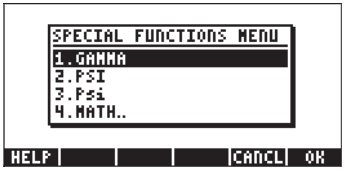
D→R (x) : converts degrees to radians

R→D (x) : converts radians to degrees.

As an exercise, verify that *D→R(45) = 0.78539* (i.e., 45° = 0.78539^{rad}), *R→D(1.5) = 85.943669..* (i.e., 1.5^{rad} = 85.943669..°).

Special functions

Option *11. Special functions…* in the MTH menu includes the following functions:



GAMMA: The Gamma function $\Gamma(\alpha)$

PSI: N-th derivative of the digamma function

Psi: Digamma function, derivative of the ln(Gamma)

The Gamma function is defined by $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}dx$. This function has applications in applied mathematics for science and engineering, as well as in probability and statistics.

Factorial of a number

The factorial of a positive integer number *n* is defined as *n!=n·(n-1)×(n-2)…3×2×1*, with *0! = 1*. The factorial function is available in the calculator by using [ALPHA] [→] [2]. In both ALG and RPN modes, enter the number first, followed by the sequence [ALPHA] [→] [2]. Example: [5] [ALPHA] [→] [2] [ENTER]. The Gamma function, defined above, has the property that

$$\Gamma(\alpha) = (\alpha-1)\,\Gamma(\alpha-1), \text{ for } \alpha > 1.$$

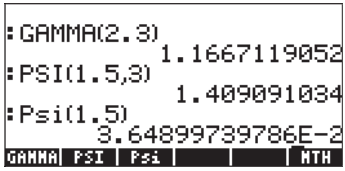
Therefore, it can be related to the factorial of a number, i.e., $\Gamma(\alpha) = (\alpha-1)!$, when α is a positive integer. We can also use the factorial function to calculate the Gamma function, and vice versa. For example, $\Gamma(5) = 4!$ or, [4] [ALPHA] [→] [2] [ENTER]. The factorial function is available in the MTH menu, through the *7. PROBABILITY..* menu.

The PSI function, $\Psi(x,y)$, represents the y-th derivative of the digamma function, i.e., $\Psi(n,x) = \frac{d^n}{dx^n}\psi(x)$, where $\Psi(x)$ is known as the digamma function, or Psi function. For this function, *y* must be a positive integer.

The Psi function, $\Psi(x)$, or digamma function, is defined as $\psi(x) = \ln[\Gamma(x)]$.

Examples of these special functions are shown here using both the ALG and RPN modes. As an exercise, verify that *GAMMA(2.3) = 1.166711…*, *PSI(1.5,3) = 1.40909..*, and *Psi(1.5) = 3.64899739..E-2*.

These calculations are shown in the following screen shot:



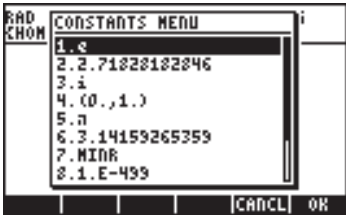
Calculator constants

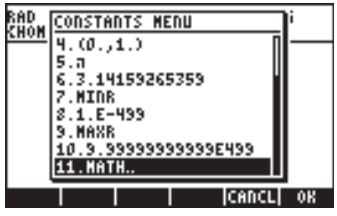
The following are the mathematical constants used by your calculator:

- *e*: the base of natural logarithms.
- *i*: the imaginary unit, $i^2 = -1$.
- π: the ratio of the length of the circle to its diameter.
- MINR: the minimum real number available to the calculator.
- MAXR: the maximum real number available to the calculator.

To have access to these constants, select option *11. CONSTANTS..* in the *MTH* menu,

The constants are listed as follows:





Selecting any of these entries will place the value selected, whether a symbol (e.g., *e*, *i*, *π*, *MINR*, or *MAXR*) or a value (*2.71.., (0,1), 3.14.., 1E-499, 9.99..E499*) in the stack.

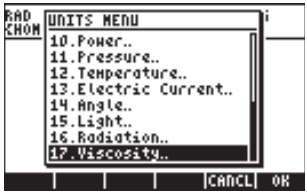
Please notice that *e* is available from the keyboard as *exp(1)*, i.e., ◁ e^x 1 ENTER, in ALG mode, or 1 ENTER ◁ e^x, in RPN mode. Also, π is available directly from the keyboard as ◁ π. Finally, *i* is available by using ◁ *i*.

Operations with units

Numbers in the calculator can have units associated with them. Thus, it is possible to calculate results involving a consistent system of units and produce a result with the appropriate combination of units.

The UNITS menu

The units menu is launched by the keystroke combination ▷ UNITS (associated with the 6 key). With system flag 117 set to *CHOOSE boxes*, the result is the following menu:

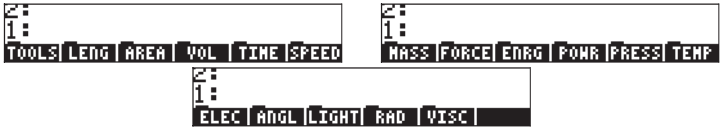


Option *1. Tools..* contains functions used to operate on units (discussed later). Options *3. Length..* through *17.Viscosity..* contain menus with a number of units for each of the quantities described. For example, selecting option *8. Force..* shows the following units menu:

The user will recognize most of these units (some, e.g., dyne, are not used very often nowadays) from his or her physics classes: *N* = newtons, *dyn* = dynes, *gf* = grams – force (to distinguish from gram-mass, or plainly gram, a unit of mass), *kip* = kilo-poundal (1000 pounds), *lbf* = pound-force (to distinguish from pound-mass), *pdl* = poundal.

To attach a unit object to a number, the number must be followed by an underscore. Thus, a force of 5 N will be entered as 5_N.

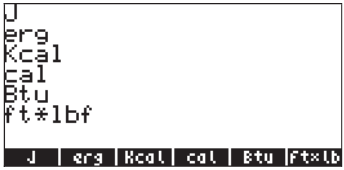
For extensive operations with units SOFT menus provide a more convenient way of attaching units. Change system flag 117 to SOFT menus (see Chapter 1), and use the keystroke combination ↱ UNITS to get the following menus. Press NXT to move to the next menu page.



Pressing on the appropriate soft menu key will open the sub-menu of units for that particular selection. For example, for the SPEED sub-menu, the following units are available:

Pressing the soft menu key UNITS will take you back to the UNITS menu.

Recall that you can always list the full menu labels in the screen by using ↱ ▼, e.g., for the LENG set of units the following labels will be listed:



Note: Use the NXT key or the ↰ PREV keystroke sequence to navigate through the menus.

Available units

The following is a list of the units available in the UNITS menu. The unit symbol is shown first followed by the unit name in parentheses:

LENGTH

m (meter), cm (centimeter), mm (millimeter), yd (yard), ft (feet), in (inch), Mpc (Mega parsec), pc (parsec), lyr (light-year), au (astronomical unit), km (kilometer), mi (international mile), nmi (nautical mile), miUS (US statute mile), chain (chain), rd (rod), fath (fathom), ftUS (survey foot), Mil (Mil), µ (micron), Å (Angstrom), fermi (fermi)

AREA

m^2 (square meter), cm^2 (square centimeter), b (barn), yd^2 (square yard), ft^2 (square feet), in^2 (square inch), km^2 (square kilometer), ha (hectare), a (are), mi^2 (square mile), miUS^2 (square statute mile), acre (acre)

VOLUME

m^3 (cubic meter), st (stere), cm^3 (cubic centimeter), yd^3 (cubic yard), ft^3 (cubic feet), in^3 (cubic inch), l (liter), galUK (UK gallon), galC (Canadian gallon), gal (US gallon), qt (quart), pt (pint), ml (mililiter), cu (US cup), ozfl (US fluid ounce), ozUK (UK fluid ounce), tbsp (tablespoon), tsp (teaspoon), bbl (barrel), bu (bushel), pk (peck), fbm (board foot)

TIME

yr (year), d (day), h (hour), min (minute), s (second), Hz (hertz)

SPEED

m/s (meter per second), cm/s (centimeter per second), ft/s (feet per second), kph (kilometer per hour), mph (mile per hour), knot (nautical miles per hour), c (speed of light), ga (acceleration of gravity)

MASS

kg (kilogram), g (gram), Lb (avoirdupois pound), oz (ounce), slug (slug), lbt (Troy pound), ton (short ton), tonUK (long ton), t (metric ton), ozt (Troy ounce), ct (carat), grain (grain), u (unified atomic mass), mol (mole)

FORCE

N (newton), dyn (dyne), gf (gram-force), kip (kilopound-force), lbf (pound-force), pdl (poundal)

ENERGY

J (joule), erg (erg), Kcal (kilocalorie), Cal (calorie), Btu (International table btu), ft×lbf (foot-pound), therm (EEC therm), MeV (mega electron-volt), eV (electron-volt)

POWER

W (watt), hp (horse power),

PRESSURE

Pa (pascal), atm (atmosphere), bar (bar), psi (pounds per square inch), torr (torr), mmHg (millimeters of mercury), inHg (inches of mercury), inH20 (inches of water),

TEMPERATURE

° C (degree Celsius), ° F (degree Fahrenheit), K (Kelvin), ° R (degree Rankine),

ELECTRIC CURRENT (Electric measurements)

V (volt), A (ampere), C (coulomb), Ω (ohm), F (farad), W (watt), Fdy (faraday), H (henry), mho (mho), S (siemens), T (tesla), Wb (weber)

ANGLE (planar and solid angle measurements)

° (sexagesimal degree), r (radian), grad (grade), arcmin (minute of arc), arcs (second of arc), sr (steradian)

LIGHT (Illumination measurements)

fc (footcandle), flam (footlambert), lx (lux), ph (phot), sb (stilb), lm (lumem), cd (candela), lam (lambert)

RADIATION

Gy (gray), rad (rad), rem (rem), Sv (sievert), Bq (becquerel), Ci (curie), R (roentgen)

VISCOSITY

P (poise), St (stokes)

Units not listed

Units not listed in the Units menu, but available in the calculator include: gmol (gram-mole), lbmol (pound-mole), rpm (revolutions per minute), dB (decibels). These units are accessible through menu 117.02, triggered by using MENU(117.02) in ALG mode, or 117.02 ENTER MENU in RPN mode. The menu will show in the screen as follows (use ↱ ▼ to show labels in display):

These units are also accessible through the catalog, for example:

gmol: ↱ CAT ALPHA ↰ G

lbmol: ↱ CAT ALPHA ↰ L

rpm: ↱ CAT ALPHA ↰ R

dB: ↱ CAT ALPHA ↰ D

Converting to base units

To convert any of these units to the default units in the SI system, use the function UBASE. For example, to find out what is the value of *1 poise* (unit of viscosity) in the SI units, use the following:

In ALG mode, system flag 117 set to *CHOOSE boxes*:

[→] UNITS	Select the UNITS menu
[OK]	Select the TOOLS menu
[▼] [OK]	Select the UBASE function
[1] [→] _	Enter 1 and underline
[→] UNITS	Select the UNITS menu
[▲] [OK]	Select the VISCOSITY option
[OK]	Select the UNITS menu
[ENTER]	Convert the units

This results in the following screen (i.e., *1 poise = 0.1 kg/(m·s)*):

In RPN mode, system flag 117 set to *CHOOSE boxes*:

[1]	Enter 1 (no underline)
[→] UNITS	Select the UNITS menu
[▲] [OK]	Select the VISCOSITY option
[OK]	Select the unit P (poise)
[→] UNITS	Select the UNITS menu
[OK]	Select the TOOLS menu
[▼] [OK]	Select the UBASE function

In ALG mode, system flag 117 set to *SOFT menus*:

[→] UNITS	Select the UNITS menu
TOOLS	Select the TOOLS menu
UBASE	Select the UBASE function
[1] [→] _	Enter 1 and underline
[→] UNITS	Select the UNITS menu
[←] PREV VISC	Select the VISCOSITY option
P	Select the unit P (poise)

ENTER	Convert the units

In RPN mode, system flag 117 set to *SOFT menus*:

1	Enter 1 (no underline)
→ UNITS	Select the UNITS menu
← PREV VISC	Select the VISCOSITY option
P	Select the unit P (poise)
→ UNITS	Select the UNITS menu
TOOLS	Select the TOOLS menu
UBASE	Select the UBASE function

Attaching units to numbers

To attach a unit object to a number, the number must be followed by an underscore (→ _ , key(8,5)). Thus, a force of 5 N will be entered as 5_N. Here is the sequence of steps to enter this number in ALG mode, system flag 117 set to *CHOOSE boxes*:

5 → _	Enter number and underscore
→ UNITS	Access the UNITS menu
8 FORC	Select units of force (*8. Force..*)
NEWT	Select Newtons (*N*)
ENTER	Enter quantity with units in the stack

The screen will look like the following:

Note: If you forget the underscore, the result is the expression 5*N, where N here represents a possible variable name and not Newtons.

To enter this same quantity, with the calculator in RPN mode, use the following keystrokes:

5	Enter number (do not enter underscore)
→ UNITS	Access the UNITS menu
8 FORC	Select units of force (*8. Force..*)
NEWT	Select Newtons (*N*)

Notice that the underscore is entered automatically when the RPN mode is active. The result is the following screen:

As indicated earlier, if system flag 117 is set to *SOFT menus*, then the UNITS menu will show up as labels for the soft menu keys. This set up is very convenient for extensive operations with units.

The keystroke sequences to enter units when the *SOFT menu* option is selected, in both ALG and RPN modes, are illustrated next. For example, in ALG mode, to enter the quantity 5_N use:

5 → _	Enter number and underscore
→ UNITS	Access the UNITS menu
NXT FORCE	Select units of force
N	Select Newtons (*N*)
ENTER	Enter quantity with units in the stack

The same quantity, entered in RPN mode uses the following keystrokes:

5	Enter number (no underscore)
→ UNITS	Access the UNITS menu
NXT FORCE	Select units of force
N	Select Newtons (*N*)

Note: You can enter a quantity with units by typing the underline and units with the ALPHA keyboard, e.g., 5 → _ ALPHA N will produce the entry: 5_N

Unit prefixes

You can enter prefixes for units according to the following table of prefixes from the SI system.

The prefix abbreviation is shown first, followed by its name, and by the exponent x in the factor 10^x corresponding to each prefix:

Prefix	Name	x	Prefix	Name	x

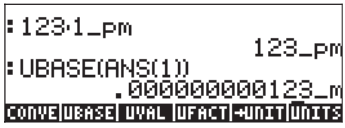
Y	yotta	+24	d	deci	-1
Z	zetta	+21	c	centi	-2
E	exa	+18	m	milli	-3
P	peta	+15	μ	micro	-6
T	tera	+12	n	nano	-9
G	giga	+9	p	pico	-12
M	mega	+6	f	femto	-15
k,K	kilo	+3	a	atto	-18
h,H	hecto	+2	z	zepto	-21
D(*)	deka	+1	y	yocto	-24

(*) In the SI system, this prefix is *da* rather than *D*. Use D for deka in the calculator, however.

To enter these prefixes, simply type the prefix using the ALPHA keyboard. For example, to enter 123 pm (1 picometer), use:

1 2 3 → _ ALPHA ← P ALPHA ← M

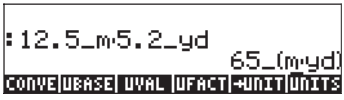
Using UBASE to convert to the default unit (1 m) results in:



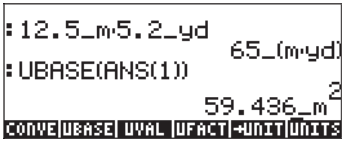
Operations with units

Once a quantity accompanied with units is entered into the stack, it can be used in operations similar to plain numbers, except that quantities with units cannot be used as arguments of functions (say, SQ or SIN). Thus, attempting to calculate LN(10_m) will produce an error message: *Error: Bad Argument Type.*

Here are some calculation examples using the ALG operating mode. Be warned that, when multiplying or dividing quantities with units, you must enclosed each quantity with its units between parentheses. Thus, to enter, for example, the product 12.5m × 5.2_yd, type it to read (12.5_m)*(5.2_yd) ENTER:

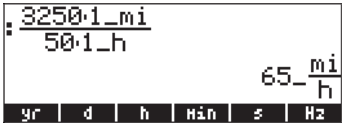


which shows as 65_(m·yd). To convert to units of the SI system, use function UBASE:

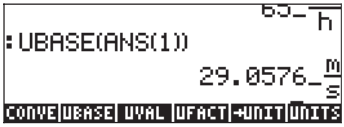


Note: Recall that the ANS(1) variable is available through the keystroke combination ↰ ANS (associated with the ENTER key).

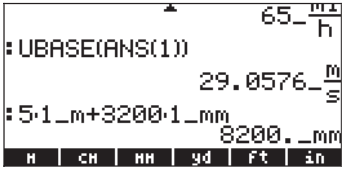
To calculate a division, say, 3250 mi / 50 h, enter it as (3250_mi)/(50_h) ENTER:



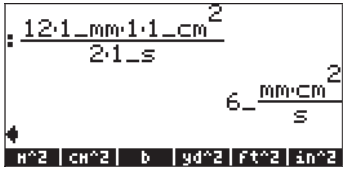
which transformed to SI units, with function UBASE, produces:



Addition and subtraction can be performed, in ALG mode, without using parentheses, e.g., 5 m + 3200 mm, can be entered simply as 5_m + 3200_mm ENTER:



More complicated expression require the use of parentheses, e.g., (12_mm)*(1_cm^2)/(2_s) ENTER:



Stack calculations in the RPN mode, do not require you to enclose the different terms in parentheses, e.g.,

12_m ENTER 1.5_yd ENTER ×

3250_mi ENTER 50_h ENTER ÷

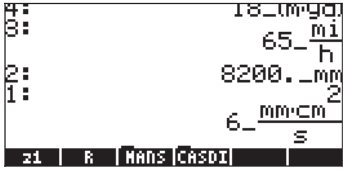
These operations produce the following output:

Also, try the following operations:

5_m ENTER 3200_mm ENTER +

12_mm ENTER 1_cm^2 ENTER × 2_s ENTER ÷

These last two operations produce the following output:



Note: Units are not allowed in expressions entered in the equation writer.

Units manipulation tools

The UNITS menu contains a TOOLS sub-menu, which provides the following functions:

CONVERT(x,y):	convert unit object x to units of object y
UBASE(x):	convert unit object x to SI units
UVAL(x):	extract the value from unit object x

UFACT(x,y): factors a unit y from unit object x
→UNIT(x,y): combines value of x with units of y

The UBASE function was discussed in detail in an earlier section in this chapter. To access any of these functions follow the examples provided earlier for UBASE. Notice that, while function UVAL requires only one argument, functions CONVERT, UFACT, and →UNIT require two arguments.

Try the following exercises. The output shown below was developed in ALG mode with system flat 117 set to *SOFT menu*:

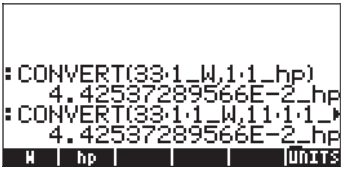
Examples of CONVERT

These examples produce the same result, i.e., to convert 33 watts to btus

CONVERT(33_W,1_hp) ENTER

CONVERT(33_W,11_hp) ENTER

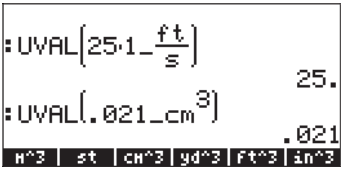
These operations are shown in the screen as:



Examples of UVAL:

UVAL(25_ft/s) ENTER

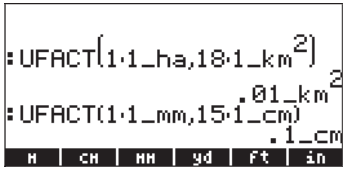
UVAL(0.021_cm^3) ENTER



Examples of UFACT

UFACT(1_ha,18_km^2) ENTER

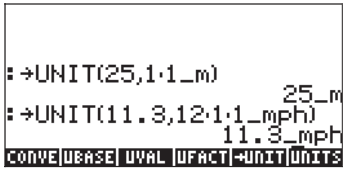
UFACT(1_mm,15.1_cm) ENTER



Examples of →UNIT

→UNIT(25,1_m) ENTER

→UNIT(11.3,1_mph) ENTER



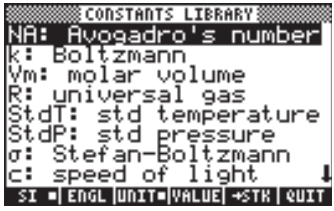
Physical constants in the calculator

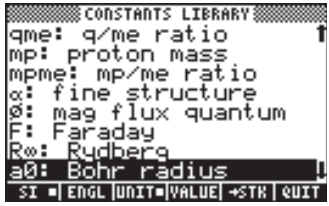
Following along the treatment of units, we discuss the use of physical constants that are available in the calculator's memory. These physical constants are contained in a *constants library* activated with the command CONLIB. To launch this command you could simply type it in the stack:

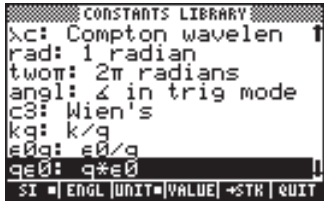
ALPHA ALPHA C O N L I B ALPHA ENTER

or, you can select the command CONLIB from the command catalog, as follows: First, launch the catalog by using: ↱ _CAT (ALPHA C). Next, use the up and down arrow keys ▲ ▼ to select CONLIB. Finally, press the F6 (▒▒) soft menu key. Press ENTER, if needed.

The constants library screen will look like the following (use the down arrow key to navigate through the library):







The soft menu keys corresponding to this CONSTANTS LIBRARY screen include the following functions:

SI	when selected, constants values are shown in SI units
ENGL	when selected, constants values are shown in English units (*)
UNIT	when selected, constants are shown with units attached (*)
VALUE	when selected, constants are shown without units
→STK	copies value (with or without units) to the stack
QUIT	exit constants library

(*) Active only if the function VALUE is active.

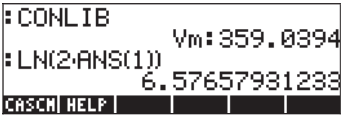
This is the way the top of the CONSTANTS LIBRARY screen looks when the option VALUE is selected (units in the SI system):

To see the values of the constants in the English (or Imperial) system, press the [ENGL] option:

If we de-select the UNITS option (press [UNITS]) only the values are shown (English units selected in this case):

To copy the value of Vm to the stack, select the variable name, and press ![SOLVE], then, press [QUIT]. For the calculator set to the ALG, the screen will look like this:

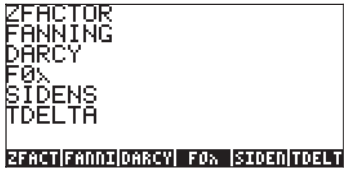
The display shows what is called a *tagged value*, `Vm:359.0394`. In here, Vm, is the *tag* of this result. Any arithmetic operation with this number will ignore the tag. Try, for example: ↱ LN 2 × ↰ ANS ENTER, which produces:



The same operation in RPN mode will require the following keystrokes (after the value of Vm was extracted from the constants library): 2 ENTER × ↱ LN

Special physical functions

Menu 117, triggered by using MENU(117) in ALG mode, or 117 ENTER MENU in RPN mode, produces the following menu (labels listed in the display by using ↱ ▼):



The functions include:

ZFACTOR: gas compressibility Z factor function

FANNING: Fanning friction factor for fluid flow

DARCY: Darcy-Weisbach friction factor for fluid flow

F0λ: Black body emissive power function

SIDENS: Silicon intrinsic density

TDELTA: Temperature delta function

In the second page of this menu (press NXT) we find the following items:

In this menu page, there is one function (TINC) and a number of units described in an earlier section on units (see above). The function of interest is:

TINC: temperature increment command

Out of all the functions available in this MENU (UTILITY menu), namely, ZFACTOR, FANNING, DARCY, F0λ, SIDENS, TDELTA, and TINC, functions FANNING and DARCY are described in Chapter 6 in the context of solving equations for pipeline flow. The remaining functions are described following.

Function ZFACTOR

Function ZFACTOR calculates the gas compressibility correction factor for nonideal behavior of hydrocarbon gas. The function is called by using

ZFACTOR(x_T, y_P), where x_T is the reduced temperature, i.e., the ratio of actual temperature to pseudo-critical temperature, and y_P is the reduced pressure, i.e., the ratio of the actual pressure to the pseudo-critical pressure. The value of x_T must be between 1.05 and 3.0, while the value of y_P must be between 0 and 30. Example, in ALG mode:

Function F0λ

Function F0λ (T, λ) calculates the fraction (dimensionless) of total black-body emissive power at temperature T between wavelengths 0 and λ. If no units are attached to T and λ, it is implied that T is in K and λ in m. Example, in ALG mode:

Function SIDENS

Function SIDENS(T) calculates the intrinsic density of silicon (in units of $1/cm^3$) as a function of temperature T (T in K), for T between 0 and 1685 K. For example,

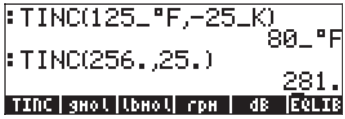
Function TDELTA

Function TDELTA(T_0,T_f) yields the temperature increment $T_f - T_0$. The result is returned with the same units as T_0, if any. Otherwise, it returns simply the difference in numbers. For example,

The purpose of this function is to facilitate the calculation of temperature differences given temperatures in different units. Otherwise, it's simply calculates a subtraction, e.g.,

Function TINC

Function TINC(T_0,ΔT) calculates T_0+DT. The operation of this function is similar to that of function TDELTA in the sense that it returns a result in the units of T_0. Otherwise, it returns a simple addition of values, e.g.,



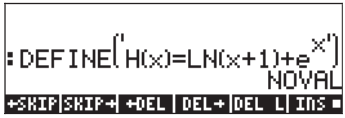
Defining and using functions

Users can define their own functions by using the DEF command available thought the keystroke sequence [↰] [DEF] (associated with the [2] key). The function must be entered in the following format:

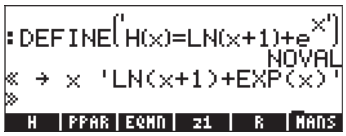
Function_name(arguments) = expression_containing_arguments

For example, we could define a simple function*H(x) = ln(x+1) + exp(-x)*. Suppose that you have a need to evaluate this function for a number of discrete values and, therefore, you want to be able to press a single button and get the result you want without having to type the expression in the right-hand side for each separate value. In the following example, we assume you have set your calculator to ALG mode. Enter the following sequence of keystrokes:

The screen will look like this:



Press the VAR key, and you will notice that there is a new variable in your soft menu key (▒▒). To see the contents of this variable press ↱ ▒▒. The screen will show now:



Thus, the variable H contains a program defined by:

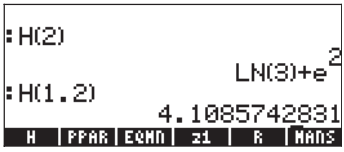
<< → x 'LN(x+1) + EXP(x)' >>

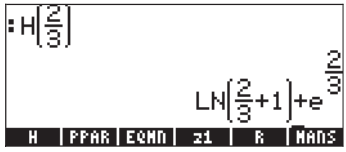
This is a simple program in the default programming language of the calculator. This programming language is called UserRPL. The program shown above is relatively simple and consists of two parts, contained between the program containers << >>:

- Input: → x → ⌅
- Process: 'LN(x+1) + EXP(x) '

This is to be interpreted as saying: enter a value that is temporarily assigned to the name *x* (referred to as a local variable), evaluate the expression between quotes that contain that local variable, and show the evaluated expression.

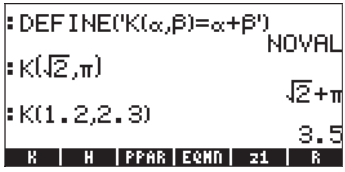
To activate the function in ALG mode, type the name of the function followed by the argument between parentheses, e.g., ▒▒ ↰ ( ) ⌴ 2 ENTER. Some examples are shown below:





In the RPN mode, to activate the function enter the argument first, then press the soft menu key corresponding to the variable name ▒▒. For example, you could try: 2 ▒▒. The other examples shown above can be entered by using: 1 . 2 ▒▒, 2 ENTER 3 ÷ ▒▒.

Functions can have more than 2 arguments. For example, the screen below shows the definition of the function $K(\alpha,\beta) = \alpha+\beta$, and its evaluation with the arguments $K(\sqrt{2},\pi)$, and $K(1.2,2.3)$:



The contents of the variable K are: << → α β 'α+β' >>.

Functions defined by more than one expression

In this section we discuss the treatment of functions that are defined by two or more expressions. An example of such functions would be

$$f(x) = \begin{cases} 2 \cdot x - 1, & x < 0 \\ x^2 - 1, & x > 0 \end{cases}$$

The function IFTE (IF-Then-Else) describes such functions.

The IFTE function

The IFTE function is written as IFTE(*condition, operation_if_true, operation_if_false*) If *condition* is true then *operation_if_true* is performed, else *operation_if_false* is performed. For example, we can write 'f(x) = IFTE(x>0, x^2-1, 2*x-1)', to describe the function listed above. Function IFTE is accessible from the function catalog (↱ CAT). The symbol '>' (greater than) is available as (associated with the 1/x key). To define this function in ALG mode use the command: DEF(f(x) = IFTE(x>0, x^2-1, 2*x-1)) then, press ENTER. In RPN mode, type the function definition between apostrophes:

'f(x) = IFTE(x>0, x^2-1, 2*x-1)'

then press ↰ DEF .

Press VAR to recover your variable menu. The function ▒▒ should be available in your soft key menu. Press ↱ ▒▒ to see the resulting program:

<< → x 'IFTE(x>0, x^2-1, 2*x-1)' >>

To evaluate the function in ALG mode, type the function name, f, followed by the number at which you want to evaluate the function, e.g., f(2), then press ENTER. In RPN mode, enter a number and press ▒▒. Check, for example, that *f(2) = 3*, while *f(-2) = -5*.

Combined IFTE functions

To program a more complicated function such as

$$g(x) = \begin{cases} -x, & x < -2 \\ x+1, & -2 \le x < 0 \\ x-1, & 0 \le x < 2 \\ x^2, & x \ge 2 \end{cases}$$

you can combine several levels of the IFTE function, i.e.,

'g(x) = IFTE(x<-2, -x, IFTE(x<0, x+1, IFTE(x<2, x-1, x^2)))',

Define this function by any of the means presented above, and check that g(-3) = 3, g(-1) = 0, g(1) = 0, g(3) = 9.