Chapter 13
Calculus Applications

In this Chapter we discuss applications of the calculator's functions to operations related to Calculus, e.g., limits, derivatives, integrals, power series, etc.

The CALC (Calculus) menu

Many of the functions presented in this Chapter are contained in the calculator's CALC menu, available through the keystroke sequence ← CALC (associated with the 4 key). The CALC menu shows the following entries:

The first four options in this menu are actually sub-menus that apply to (1) derivatives and integrals, (2) limits and power series, (3) differential equations, and (4) graphics. The functions in entries (1) and (2) will be presented in this Chapter. Differential equations, the subject of item (3), are presented in Chapter 16. Graphic functions, the subject of item (4), were presented at the end of Chapter 12. Finally, entries 5. DERVX and 6.INTVX are the functions to obtain a derivative and a indefinite integral for a function of the default CAS variable (typically, 'X'). Functions DERVX and INTVX are discussed in detail later.

Limits and derivatives

Differential calculus deals with derivatives, or rates of change, of functions and their applications in mathematical analysis. The derivative of a function is defined as a limit of the difference of a function as the increment in the independent variable tends to zero. Limits are used also to check the continuity of functions.

Function lim

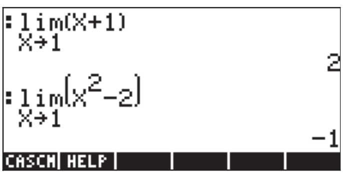
The calculator provides function *lim* to calculate limits of functions. This function uses as input an expression representing a function and the value where the limit is to be calculated. Function *lim* is available through the command catalog (→ CAT ALPHA ← L) or through option 2. LIMITS & SERIES… of the CALC menu (see above).

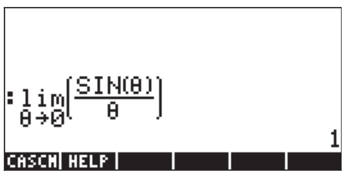
Note: The functions available in the LIMITS & SERIES menu are shown next:

Function DIVPC is used to divide two polynomials producing a series expansion. Functions DIVPC, SERIES, TAYLOR0, and TAYLOR are used in series expansions of functions and discussed in more detail in this Chapter.

Function *lim* is entered in ALG mode as `lim(f(x),x=a)` to calculate the limit $\lim_{x \to a} f(x)$. In RPN mode, enter the function first, then the expression 'x=a', and finally function lim. Examples in ALG mode are shown next, including some limits to infinity. The keystrokes for the first example are as follows (using Algebraic mode, and system flag 117 set to CHOOSE boxes):

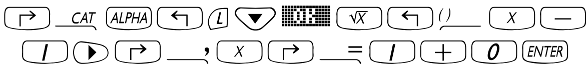
← CALC 2 OK 2 OK X + 1 → , X → = 1 ENTER



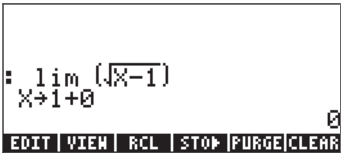


The infinity symbol is associated with the 0 key, i.e.., ← ∞ .

To calculate one-sided limits, add +0 or -0 to the value to the variable. A "+0" means limit from the right, while a "-0" means limit from the left. For example, the limit of $\sqrt{x-1}$ as x approaches 1 from the left can be determined with the following keystrokes (ALG mode):



The result is as follows:

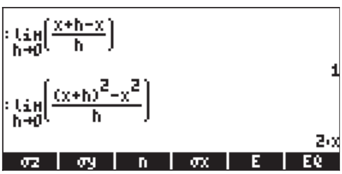


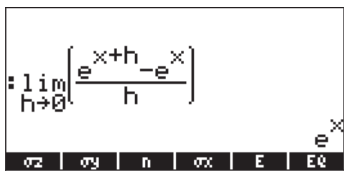
Derivatives

The derivative of a function f(x) at x = a is defined as the limit

$$\frac{df}{dx} = f'(x) = \lim_{h->0} \frac{f(x+h) - f(x)}{h}$$

Some examples of derivatives using this limit are shown in the following screen shots:

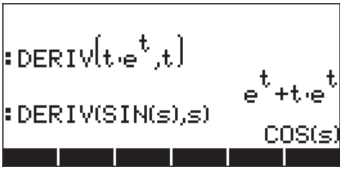


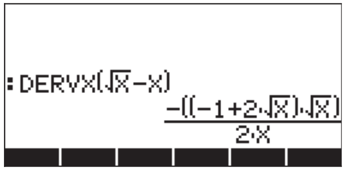


Functions DERIV and DERVX

The function DERIV is used to take derivatives in terms of any independent variable, while the function DERVX takes derivatives with respect to the CAS default variable VX (typically 'X'). While function DERVX is available directly in the CALC menu, both functions are available in the DERIV.&INTEG sub-menu within the CALCL menu (← CALC).

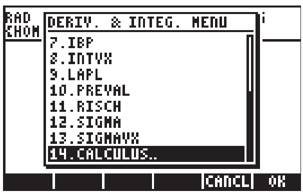
Function DERIV requires a function, say f(t), and an independent variable, say, t, while function DERVX requires only a function of VX. Examples are shown next in ALG mode. Recall that in RPN mode the arguments must be entered before the function is applied.





The DERIV&INTEG menu

The functions available in this sub-menu are listed below:



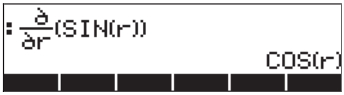
Out of these functions DERIV and DERVX are used for derivatives. The other functions include functions related to anti-derivatives and integrals (IBP, INTVX, PREVAL, RISCH, SIGMA, and SIGMAVX), to Fourier series (FOURIER),and to vector analysis (CURL, DIV, HESS, LAPL). Next we discuss functions DERIV and DERVX, the remaining functions are presented either later in this Chapter or in subsequent Chapters.

Calculating derivatives with ∂

The symbol is available as ↱ ∂ (the COS key). This symbol can be used to enter a derivative in the stack or in the Equation Writer (see Chapter 2). If you use the symbol to write a derivative into the stack, follow it immediately with the independent variable, then by a pair of parentheses enclosing the function to

be differentiated. Thus, to calculate the derivative d(sin(r),r), use, in ALG mode:
[→] ∂ [ALPHA] [←] [R] [←] () [SIN] [ALPHA] [←] [R] [ENTER]

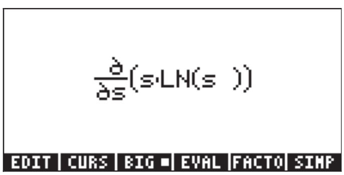
In RPN mode, this expression must be enclosed in quotes before entering it into the stack. The result in ALG mode is:



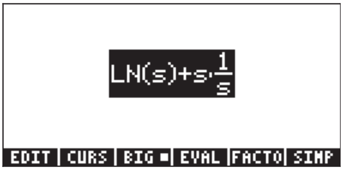
In the Equation Writer, when you press [→] ∂, the calculator provides the following expression:

The insert cursor (◆) will be located right at the denominator awaiting for the user to enter an independent variable, say, s: [ALPHA] [←] [S]. Then, press the right-arrow key (▷) to move to the placeholder between parentheses:

Next, enter the function to be differentiated, say, s*ln(s):



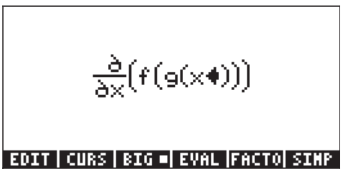
To evaluate the derivative in the Equation Writer, press the up-arrow key ⬆, four times, to select the entire expression, then, press EVAL. The derivative will be evaluated in the Equation Writer as:

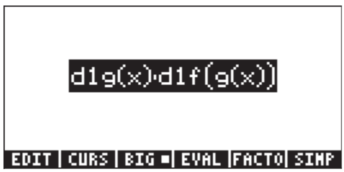


Note: The symbol ∂ is used formally in mathematics to indicate a partial derivative, i.e., the derivative of a function with more than one variable. However, the calculator does not distinguish between ordinary and partial derivatives, utilizing the same symbol for both. The user must keep this distinction in mind when translating results from the calculator to paper.

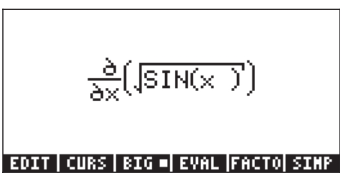
The chain rule

The chain rule for derivatives applies to derivatives of composite functions. A general expression for the chain-rule is d{f[g(x)]}/dx = (df/dg)· (dg/dx). Using the calculator, this formula results in:



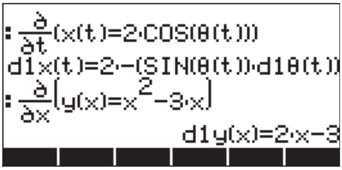


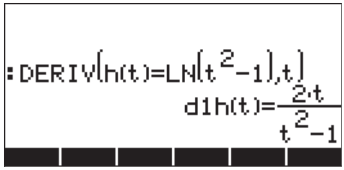
The terms d1 in front of g(x) and f(g(x)) in the expression above are abbreviations the calculator uses to indicate a first derivative when the independent variable, in this case x, is clearly defined. Thus, the latter result is interpreted as in the formula for the chain rule shown above. Here is another example of a chain rule application:

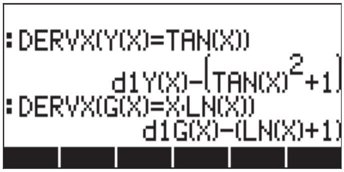


Derivatives of equations

You can use the calculator to calculate derivatives of equations, i.e., expressions in which derivatives will exist in both sides of the equal sign. Some examples are shown below:



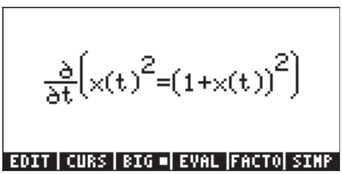


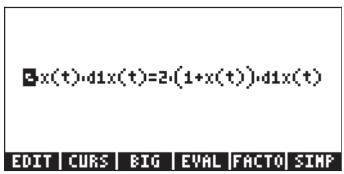


Notice that in the expressions where the derivative sign (∂) or function DERIV was used, the equal sign is preserved in the equation, but not in the cases where function DERVX was used. In these cases, the equation was re-written with all its terms moved to the left-hand side of the equal sign. Also, the equal sign was removed, but it is understood that the resulting expression is equal to zero.

Implicit derivatives

Implicit derivatives are possible in expressions such as:





Application of derivatives

Derivatives can be used for analyzing the graphs of functions and for optimizing functions of one variable (i.e., finding maxima and minima). Some applications of derivatives are shown next.

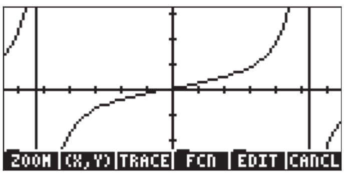
Analyzing graphics of functions

In Chapter 11 we presented some functions that are available in the graphics screen for analyzing graphics of functions of the form y = f(x). These functions include (X,Y) and TRACE for determining points on the graph, as well as functions in the ZOOM and FCN menu. The functions in the ZOOM menu allow the user to zoom in into a graph to analyze it in more detail. These functions are described in detail in Chapter 12. Within the functions of the FCN menu, we can use the functions SLOPE, EXTR, F', and TANL to determine the slope of a tangent to the graph, the extrema (minima and maxima) of the function, to plot the derivative, and to find the equation of the tangent line.

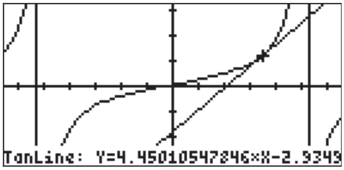
Try the following example for the function y = tan(x).

- Press ← 2D/3D , simultaneously in RPN mode, to access to the PLOT SETUP window.
- Change `TYPE` to `FUNCTION`, if needed, by using [CHOOS].
- Press ▼ and type in the equation 'TAN(X)'.
- Make sure the independent variable is set to 'X'.
- Press NXT OK to return to normal calculator display.
- Press ← WIN , simultaneously, to access the PLOT window
- Change H-VIEW range to –2 to 2, and V-VIEW range to –5 to 5.
- Press ERASE DRAW to plot the function in polar coordinates.

The resulting plot looks as follows:



- Notice that there are vertical lines that represent asymptotes. These are not part of the graph, but show points where TAN(X) goes to ± ∞ at certain values of X.
- Press TRACE (X,Y), and move the cursor to the point X: 1.08E0, Y: 1.86E0. Next, press NXT FCN SLOPE. The result is Slope: 4.45010547846.
- Press NXT NXT TANL. This operation produces the equation of the tangent line, and plots its graph in the same figure. The result is shown in the figure below:



- Press NXT EXIT CANCL ON to return to normal calculator display. Notice that the slope and tangent line that you requested are listed in the stack.

Function DOMAIN

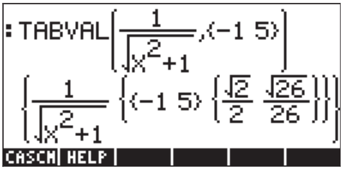
Function DOMAIN, available through the command catalog (→ CAT), provides the domain of definition of a function as a list of numbers and specifications. For example,

indicates that between –∞ and 0, the function LN(X) is not defined (?), while from 0 to +∞, the function is defined (+). On the other hand,

indicates that the function is not defined between –∞ and -1, nor between 1 and +∞. The domain of this function is, therefore, -1<X<1.

Function TABVAL

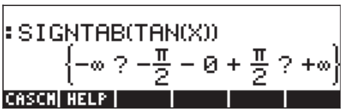
This function is accessed through the command catalog or through the GRAPH sub-menu in the CALC menu. Function TABVAL takes as arguments a function of the CAS variable, f(X), and a list of two numbers representing a domain of interest for the function f(X). Function TABVAL returns the input values plus the range of the function corresponding to the domain used as input. For example,



This result indicates that the range of the function $f(X) = \frac{1}{\sqrt{X^2 + 1}}$ corresponding to the domain D = { -1,5 } is R = $\left\{ \frac{\sqrt{2}}{2}, \frac{\sqrt{26}}{26} \right\}$.

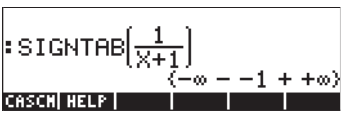
Function SIGNTAB

Function SIGNTAB, available through the command catalog (‹right-shift› _CAT_), provides information on the sign of a function through its domain. For example, for the TAN(X) function,



SIGNTAB indicates that TAN(X) is negative between $-\pi/2$ and 0, and positive between 0 and $\pi/2$. For this case, SIGNTAB does not provide information (?) in the intervals between $-\infty$ and $-\pi/2$, nor between $+\pi/2$ and ∞. Thus, SIGNTAB, for this case, provides information only on the main domain of TAN(X), namely $-\pi/2 < X < +\pi/2$.

A second example of function SIGNTAB is shown below:



For this case, the function is negative for X<-1 and positive for X> -1.

Function TABVAR

This function is accessed through the command catalog or through the GRAPH sub-menu in the CALC menu. It uses as input the function f(VX), where VX is the default CAS variable. The function returns the following, in RPN mode:

- Level 3: the function f(VX)

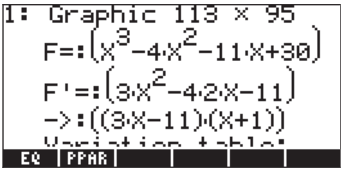
- Two lists, the first one indicates the variation of the function (i.e., where it increases or decreases) in terms of the independent variable VX, the second one indicates the variation of the function in terms of the dependent variable.

- A graphic object showing how the variation table was computed.

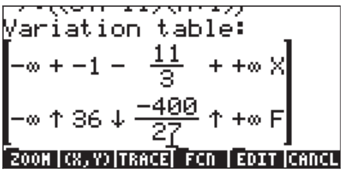
Example: Analyze the function Y = X^3-4X^2-11X+30, using the function TABVAR. Use the following keystrokes, in RPN mode:

`'X^3-4*X^2-11*X+30'` ENTER → CAT ALPHA T (select TABVAR) OK

This is what the calculator shows in stack level 1:



This is a graphic object. To be able to the result in its entirety, press ▽. The variation table of the function is shown as follows:



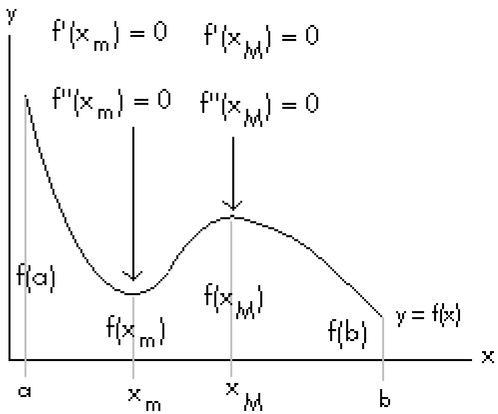
Press ON to recover normal calculator display. Press ◄ to drop this last result from the stack.

Two lists, corresponding to the top and bottom rows of the graphics matrix shown earlier, now occupy level 1. These lists may be useful for programming purposes. Press ◄ to drop this last result from the stack.

The interpretation of the variation table shown above is as follows: the function F(X) increases for X in the interval (-∞, -1), reaching a maximum equal to 36 at X = -1. Then, F(X) decreases until X = 11/3, reaching a minimum of –400/27. After that F(X) increases until reaching +∞. Also, at X = ±∞, F(X) = ±∞.

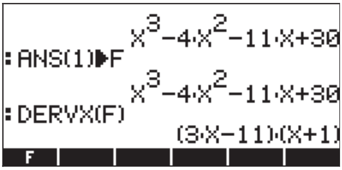
Using derivatives to calculate extreme points

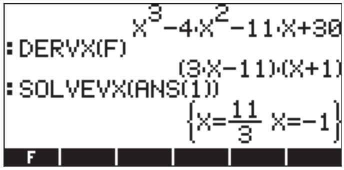
"Extreme points," or extrema, is the general designation for maximum and minimum values of a function in a given interval. Since the derivative of a function at a given point represents the slope of a line tangent to the curve at that point, then values of x for which f'(x) =0 represent points where the graph of the function reaches a maximum or minimum. Furthermore, the value of the second derivative of the function, f"(x), at those points determines whether the point is a *relative or local maximum* [f"(x)<0] or *minimum* [f"(x)>0]. These ideas are illustrated in the figure below.



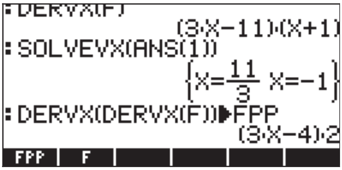
In this figure we limit ourselves to determining extreme points of the function y = f(x) in the x-interval [a,b]. Within this interval we find two points, x = x_m and x = x_M, where f'(x)=0. The point x = x_m, where f"(x)>0, represents a local minimum, while the point x = x_M, where f"(x)<0, represents a local maximum. From the graph of y = f(x) it follows that the absolute maximum in the interval [a,b] occurs at x = a, while the absolute minimum occurs at x = b.

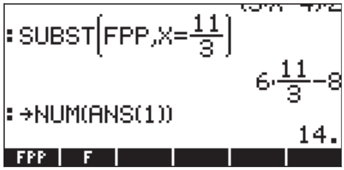
For example, to determine where the critical points of function 'X^3-4*X^2-11*X+30' occur, we can use the following entries in ALG mode:



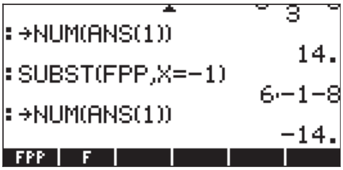


We find two critical points, one at x = 11/3 and one at x = -1. To evaluate the second derivative at each point use:

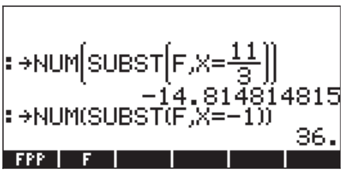




The last screen shows that f"(11/3) = 14, thus, x = 11/3 is a relative minimum. For x = -1, we have the following:

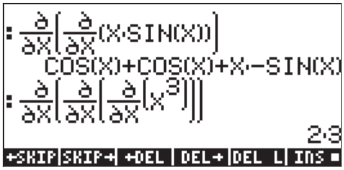


This result indicates that f"(-1) = -14, thus, x = -1 is a relative maximum. Evaluate the function at those points to verify that indeed f(-1) > f(11/3).



Higher order derivatives

Higher order derivatives can be calculated by applying a derivative function several times, e.g.,

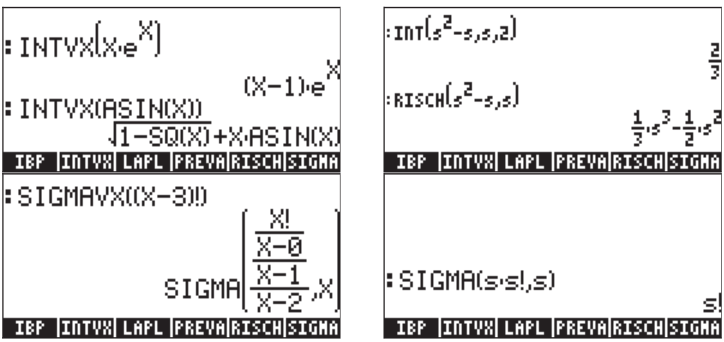


Anti-derivatives and integrals

An anti-derivative of a function f(x) is a function F(x) such that f(x) = dF/dx. For example, since $d(x^3)/dx = 3x^2$, an anti-derivative of $f(x) = 3x^2$ is $F(x) = x^3 + C$, where C is a constant. One way to represent an anti-derivative is as a indefinite integral, i.e., $\int f(x)dx = F(x) + C$, if and only if, f(x) = dF/dx, and C = constant.

Functions INT, INTVX, RISCH, SIGMA and SIGMAVX

The calculator provides functions INT, INTVX, RISCH, SIGMA and SIGMAVX to calculate anti-derivatives of functions. Functions INT, RISCH, and SIGMA work with functions of any variable, while functions INTVX, and SIGMAVX utilize functions of the CAS variable VX (typically, 'x'). Functions INT and RISCH require, therefore, not only the expression for the function being integrated, but also the independent variable name. Function INT, requires also a value of x where the anti-derivative will be evaluated. Functions INTVX and SIGMAVX require only the expression of the function to integrate in terms of VX. Some examples are shown next in ALG mode:

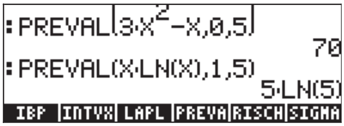


Please notice that functions SIGMAVX and SIGMA are designed for integrands that involve some sort of integer function like the factorial (!) function shown above. Their result is the so-called discrete derivative, i.e., one defined for integer numbers only.

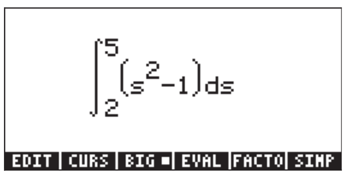
Definite integrals

In a definite integral of a function, the resulting anti-derivative is evaluated at the upper and lower limit of an interval (a,b) and the evaluated values subtracted. Symbolically, $\int_a^b f(x)dx = F(b) - F(a),$ where f(x) = dF/dx.

The PREVAL(f(x),a,b) function of the CAS can simplify such calculation by returning f(b)-f(a) with x being the CAS variable VX.



To calculate definite integrals the calculator also provides the integral symbol as the keystroke combination ↱ ∫ (associated with the TAN key). The simplest way to build an integral is by using the Equation Writer (see Chapter 2 for an example). Within the Equation Writer, the symbol ↱ ∫ produces the integral sign and provides placeholders for the integration limits (a,b), for the function, f(x), and for the variable of integration (x). The following screen shots show how to build a particular integral. The insert cursor is first located in the lower limit of integration, enter a value and press the right-arrow key (▶) to move to the upper limit of integration. Enter a value in that location and press ▶ again to move to the integrand location. Type the integrand expression, and press once more to move to the differential place holder, type the variable of integration in that location and the integral is ready to be calculated.



At this point, you can press ENTER to return the integral to the stack, which will show the following entry (ALG mode shown):

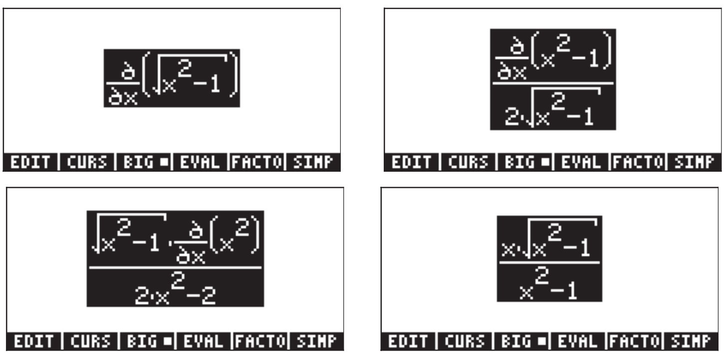
This is the general format for the definite integral when typed directly into the stack, i.e., ∫(lower limit, upper limit, integrand, variable of integration)

Pressing ENTER at this point will evaluate the integral in the stack:

The integral can be evaluated also in the Equation Writer by selecting the entire expression an using the soft menu key EVAL.

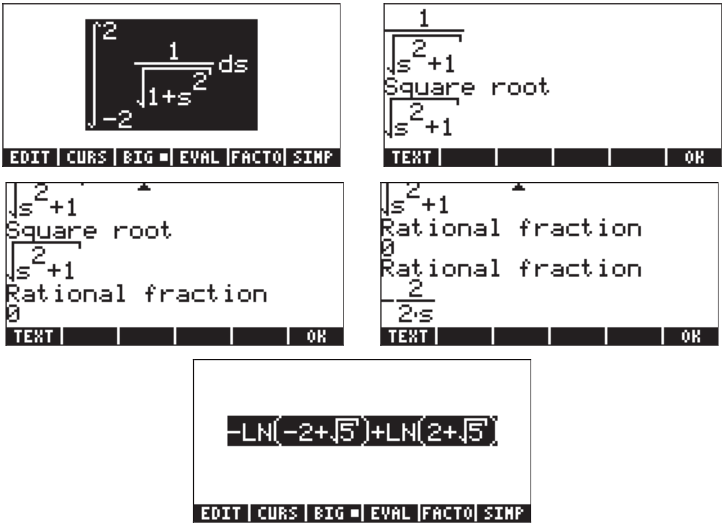
Step-by-step evaluation of derivatives and integrals

With the Step/Step option in the CAS MODES windows selected (see Chapter 1), the evaluation of derivatives and integrals will be shown step by step. For example, here is the evaluation of a derivative in the Equation Writer:



Notice the application of the chain rule in the first step, leaving the derivative of the function under the integral explicitly in the numerator. In the second step, the resulting fraction is rationalized (eliminating the square root from the denominator), and simplified. The final version is shown in the third step. Each step is shown by pressing the EVAL menu key, until reaching the point where further application of function EVAL produce no more changes in the expression.

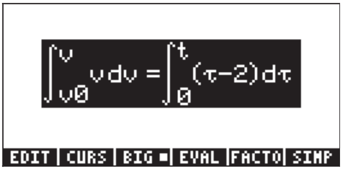
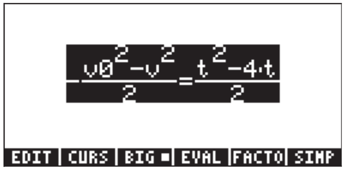
The following example shows the evaluation of a definite integral in the Equation Writer, step-by-step:



Notice that the step-by-step process provides information on the intermediate steps followed by the CAS to solve this integral. First, CAS identifies a square root integral, next, a rational fraction, and a second rational expression, to come up with the final result. Notice that these steps make a lot of sense to the calculator, although not enough information is provided to the user on the individual steps.

Integrating an equation

Integrating an equation is straightforward, the calculator simply integrates both sides of the equation simultaneously, e.g.,

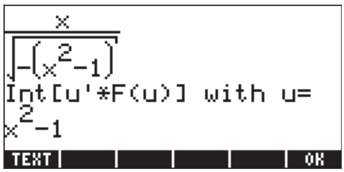



Techniques of integration

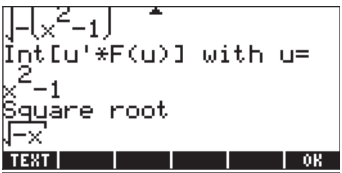
Several techniques of integration can be implemented in the calculators, as shown in the following examples.

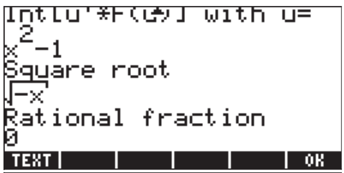
Substitution or change of variables

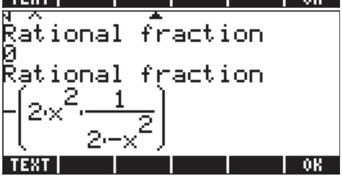
Suppose we want to calculate the integral $\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}dx$. If we use step-by-step calculation in the Equation Writer, this is the sequence of variable substitutions:

This second step shows the proper substitution to use, $u = x^2-1$.







The last four steps show the progression of the solution: a square root, followed by a fraction, a second fraction, and the final result. This result can be simplified by using function SIMP, to read:

Integration by parts and differentials

A differential of a function y = f(x), is defined as dy = f'(x) dx, where f'(x) is the derivative of f(x). Differentials are used to represent small increments in the variables. The differential of a product of two functions, y = u(x)v(x), is given by dy = u(x)dv(x) +du(x)v(x), or, simply, d(uv) = udv - vdu. Thus, the integral of udv = d(uv) - vdu, is written as $\int udv = \int d(uv) - \int vdu$. Since by the definition of a differential, ∫dy = y, we write the previous expression as

$$\int udv = uv - \int vdu .$$

This formulation, known as integration by parts, can be used to find an integral if dv is easily integrable. For example, the integral $\int xe^x dx$ can be solved by integration by parts if we use u = x, dv = e^xdx, since, v = e^x. With du = dx, the integral becomes $\int xe^x dx = \int udv = uv - \int vdu = xe^x - \int e^x dx = xe^x - e^x$.

The calculator provides function IBP, under the CALC/DERIV&INTG menu, that takes as arguments the original function to integrate, namely, u(X)*v'(X), and the function v(X), and returns u(X)*v(X) and -v(X)*u'(X). In other words, function IBP returns the two terms of the right-hand side in the integration by parts equation. For the example used above, we can write in ALG mode:

Thus, we can use function IBP to provide the components of an integration by parts. The next step will have to be carried out separately.

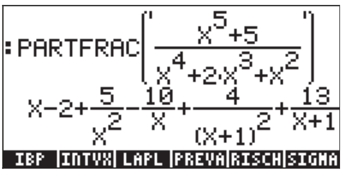
It is important to mention that the integral can be calculated directly by using, for example,

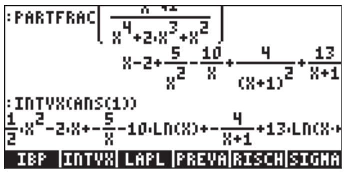
Integration by partial fractions

Function PARTFRAC, presented in Chapter 5, provides the decomposition of a fraction into partial fractions. This technique is useful to reduce a complicated fraction into a sum of simple fractions that can then be integrated term by term. For example, to integrate

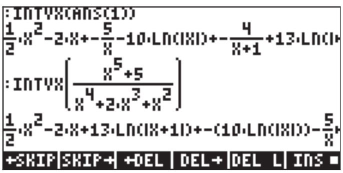
$$\int \frac{X^5+5}{X^4+2X^3+X}dX$$

we can decompose the fraction into its partial component fractions, as follows:





The direct integration produces the same result, with some switching of the terms (Rigorous mode set in the CAS – see Chapter 2):



Improper integrals

These are integrals with infinite limits of integration. Typically, an improper integral is dealt with by first calculating the integral as a limit to infinity, e.g.,

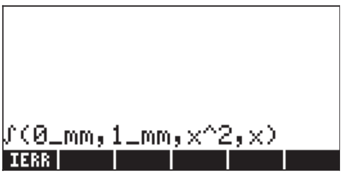
$$\int_1^{\infty}\frac{dx}{x^2} = \lim_{\varepsilon\to\infty}\int_1^{\varepsilon}\frac{dx}{x^2}.$$

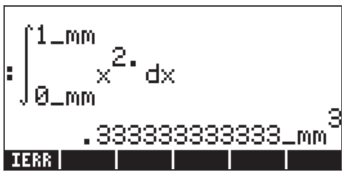
Using the calculator, we proceed as follows:

Alternatively, you can evaluate the integral to infinity from the start, e.g.,

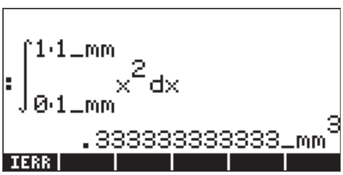
Integration with units

An integral can be calculated with units incorporated into the limits of integration, as in the example shown below that uses ALG mode, with the CAS set to Approx mode. The left-hand side figure shows the integral typed in the line editor before pressing ENTER. The right-hand figure shows the result after pressing ENTER.



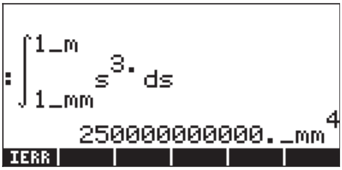


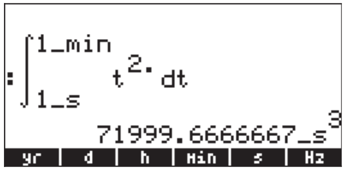
If you enter the integral with the CAS set to Exact mode, you will be asked to change to Approx mode, however, the limits of the integral will be shown in a different format as shown here:



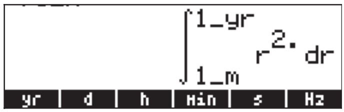
These limits represent 1×1_mm and 0×1_mm, which is the same as 1_mm and 0_mm, as before. Just be aware of the different formats in the output.

Some notes in the use of units in the limits of integrations:
1 – The units of the lower limit of integration will be the ones used in the final result, as illustrated in the two examples below:

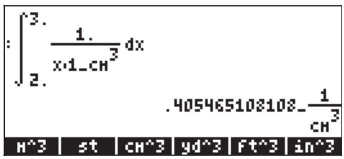




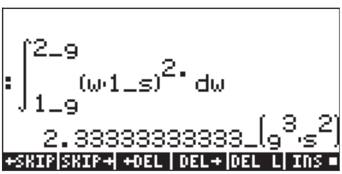
2 - Upper limit units must be consistent with lower limit units. Otherwise, the calculator simply returns the unevaluated integral. For example,

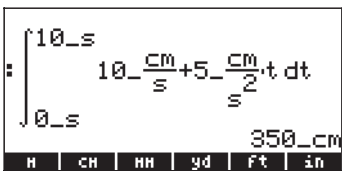


3 – The integrand may have units too. For example:



4 – If both the limits of integration and the integrand have units, the resulting units are combined according to the rules of integration. For example,





Infinite series

An infinite series has the form $\sum_{n=0,1}^{\infty} h(n)(x-a)^n$. The infinite series typically starts with indices n = 0 or n = 1. Each term in the series has a coefficient h(n) that depends on the index n.

Taylor and Maclaurin's series

A function f(x) can be expanded into an infinite series around a point $x=x_0$ by using a Taylor's series, namely,

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_o)}{n!} \cdot (x - x_o)^n \ ,$$

where $f^{(n)}(x)$ represents the n-th derivative of f(x) with respect to x, $f^{(0)}(x) = f(x)$.

If the value x_0 is zero, the series is referred to as a Maclaurin's series, i.e.,

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} \cdot x^n$$

Taylor polynomial and reminder

In practice, we cannot evaluate all terms in an infinite series, instead, we approximate the series by a polynomial of order k, $P_k(x)$, and estimate the order of a residual, $R_k(x)$, such that

$$f(x) = \sum_{n=0}^{k} \frac{f^{(n)}(x_o)}{n!} \cdot (x - x_o)^n + \sum_{n=k+1}^{\infty} \frac{f^{(n)}(x_o)}{n!} \cdot (x - x_o)^n \ ,$$

i.e.,

$$f(x) = P_k(x) + R_k(x).$$

The polynomial $P_k(x)$ is referred to as Taylor's polynomial. The order of the residual is estimated in terms of a small quantity $h = x-x_0$, i.e., evaluating the polynomial at a value of x very close to x_0. The residual if given by

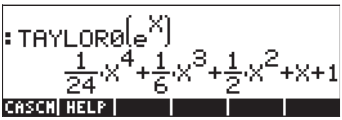
$$R_k(x) = \frac{f^{(k+1)}(\xi)}{k!} \cdot h^{k+1} \ ,$$

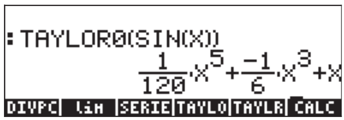
where ξ is a number near $x = x_0$. Since ξ is typically unknown, instead of an estimate of the residual, we provide an estimate of the order of the residual in reference to h, i.e., we say that $R_k(x)$ has an error of order h^{n+1}, or $R \approx O(h^{k+1})$. If h is a small number, say, h<<1, then h^{k+1} will be typically very small, i.e., $h^{k+1} << h^k << \ldots << h << 1$. Thus, for x close to x_0, the larger the number of elements in the Taylor polynomial, the smaller the order of the residual.

Functions TAYLR, TAYLR0, and SERIES

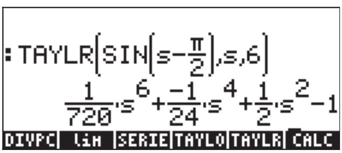
Functions TAYLR, TAYLR0, and SERIES are used to generate Taylor polynomials, as well as Taylor series with residuals. These functions are available in the CALC/LIMITS&SERIES menu described earlier in this Chapter.

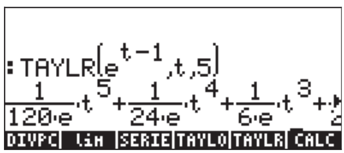
Function TAYLOR0 performs a Maclaurin series expansion, i.e., about X = 0, of an expression in the default independent variable, VX (typically 'X'). The expansion uses a 4-th order relative power, i.e., the difference between the highest and lowest power in the expansion is 4. For example,





Function TAYLR produces a Taylor series expansion of a function of any variable x about a point x = a for the order k specified by the user. Thus, the function has the format TAYLR(f(x-a),x,k). For example,





Function SERIES produces a Taylor polynomial using as arguments the function f(x) to be expanded, a variable name alone (for Maclaurin's series) or an expression of the form 'variable = value' indicating the point of expansion of a Taylor series, and the order of the series to be produced. Function SERIES returns two output items a list with four items, and an expression for h = x - a, if the second argument in the function call is 'x=a', i.e., an expression for the