Lab: Artificial Examples and Sets

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1 Problem 1: Foundations

a. {1,2,3} $\in$ S

b. ({1,2,4,5} $\in$ S $\wedge$ {3,4,5} $\in$ S) $\vee$ {4,8} $\in$ S

c. (x * 3 + 1) $\in$ U

d. x $\in$ U $\vee$ y $\in$ U $\vee$ (x+y) $\in$ U

e. $\overline{S_1} \vee S_2$

2 Problem 2: Corners

a. The following claim is not always true.
If we assume if S_1 is {1,2,3,4} and S_2 is {2,3,4,5}, the union of these two sets would be {1,2,3,4,5}, not {1,2,3,4,2,3,4,5}.

b. The empty set would be considered as the subset of any other set, including itself. If we assume a random set, let's say S = {1,2,3}, when we consider the power set of this set it would show $\mathcal{P}$(S) = $\{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}\}$. Since the power set shows all the subsets of a set, $\varnothing$ would be also considered the subset of any other set including itself.

c. No, there would be no such thing as a "negative" set. Consider the two sets S_1 = {1,2,3} and S_2 = {1,2,3,4}. When we do the subtraction S_1 - S_2, the result would be an empty set ($\varnothing$), not a negative set, because all the elements present in S_1 is also present in S_2.

d. Yes. In set theory for Cartesian Product the $\varnothing$ would correspond to the multiplication of zero. Consider doing the Cartesian Product on a set S_1 = 1,2,3,4 with $\varnothing$. The Cartesian Product is considering set of all ordered pairs (a,b), but in this case since the second set is an empty set there would be no ordered pair, resulting in an empty set.

e. Yes. In the arithmetic perspective the exponential, for example x^y would be considered as an arithmetic analog to the power set operation. For example, if we try to construct a power set of 3 elements, it produces a total of 8 subsets. Whereas in the mathematical analog, the 2^3 would produce the result of 8, which is the number of subsets from the power set. In addition, the power set of an empty set would be itself ($\varnothing$), and nothing else.

3 Problem 3: Trickiness

1. Let's say T = {1,2,3,4}. The partition of this set, S_1 and S_2, can exist in many forms, but as an example we would define it as S_1 = {1,2} and S_2 = {3,4}.

2. In the real world an example of a partition could be distributing items among different group of people. Let's say that a new school semester is beginning and we are trying to distribute pencil, pen, crayon, and eraser to new students. The set T would be T = {pencil, pen, crayon, eraser} To make this more efficient we would use partition set by saying S_1 = {pencil, pen}, and S_2 = crayon, eraser. Two subsets are disjoint and each category is only considered in one of the subsets, so we can call it a partition set. By using subset in this case we can easily distribute the items equally among the students.

3. a. The two partition sets should not overlap each other's element, meaning that same elements can't exist in the pair of sets twice.
b. After creating the two partition sets, they need to cover the entire set (the set these partition sets were created from) without any of the elements left.

4 Problem 4: The Other Side

To prove the De Morgan's Law right-to-left direction, it means that when $\overline{A} \cap \overline{B}$ is true, then $\overline{A \cup B}$ is also true. Let's assume that the left-hand-side is true, this means that $\overline{A}$ and $\overline{B}$ are both true. This again means that A and B are false, because if one of them were true, the result for the claim $\overline{A} \cap \overline{B}$ is false. Since A and B are both false, $A \cup B$ would be also false because for this to be true at least one of the values should be true, but we already know that both A and B are false. By proving that $A \cup B$ is false, this implies that $\overline{A \cup B}$ is true.

5 Problem 5: Pivots

a. An artificial example of a S would be $S = \{1, 2, 3\}$, and $\{a\}$ will be 1. Following this choice the elements in the partition T_1 and T_2 can be $T_1 = \{0,\ 2,\ 3,\ \{2{,}3\}\}$ and $T_2 = \{1,\ \{1{,}2\},\ \{1{,}3\},\ \{1{,}2{,}3\})$. These two are considered the partition sets of S because they have at least one element in each set, and disjoint. In addition, the union of the two sets are equal to the definition defined in the problem, $T_1, T_2 \subseteq \mathcal{P}(S)$.

b.