CSC208 Lab: Mathematical Induction Practice

Patrick, Havin

February 9th 2023

1 Problem 1: Mathematical Induction Practice

Claim: for all values v and natural numbers n, $list\_length(replicate(v, n)) = n$.

Proof. By induction on n
Base case: n is zero, then the replicate definition returns 0, making the list_length also return 0.
When n is non-zero,

Induction Hypothesis : n + 1 > 0

The left-hand-side evaluates as follows

```
list_length(replicate(v,n+1)) = n+1
list_length(cons(v, replicate(v,n))) = n+1
1 + list_length(replicate(v,n)) = n+1
```

By subtracting 1 on both sides, it proves the above claim.

```
list_length(replicate(v,n)) = n
```

2 Problem 2: More Mathematical Induction Practice

Claim: for all natural numbers a,b, and n, $(ab)^n = a^n b^n$
Proof. Base Case: When n = 1 $(ab)^1$ = ab = $a^1 * b^1$
We start by assuming that the claim proves to be true for a natural number k. Then, we have to prove that the claim is also true for the natural number k+1.

$(ab)^{(k+1)}$ = (ab) * $(ab)^k$ = a * b * $(a^k * b^k)$
$-- > a^1 * b^1 * a^k * b^k = a^{(k+1)} * b^{(k+1)}$

Through this process we prove that the claim holds true for all natural number n.

3 Problem 3: Problem Proof

3.1 a. In a sentence of two, describe what the claim is saying an why it is incorrect.

The claim states that for all values of v and natural numbers n, the length of the list containing n replicates of v is equivalent to 0. This, however, is incorrect since if we have at least one replicate of v, this should return a list of one element, thus, length of 1.

3.2 b. In a sentence or two, describe the error in the "proof."

In the proof's second bullet point, where it proves the claim by assuming that n = k+1, it does not state anything about k, or k+1, being a natural number, also does not include enough explanation how it corresponds to the result 0.

3.3 c. Correct the error and attempt to finish the proof. You should get stuck, i.e., a point where you are unable to move further in the proof. Show your work to get to this point and describe in a sentence or two why you cannot proceed forward with the proof.

```
    list_length(replicate(v, n)) = 0

For n = 0
    Left-hand side
    --> list_length(replicate(v, 0)) = 0
    -->* list_length([])
    -->* 0
    0

For n = k + 1, being k a natural number and k + 1 > 0
    Left-hand side
    --> list_length(replicate(v, k+1))
    -->* list_length(cons(v, replicate(v, k)))
    --> 1 + list_length(replicate(v, k)
```

Here, we can't proceed because we lack on assumptions about the claim. However, list_length should never return a negative value, which means the claim might be wrong, since for a non-zero natural number n, we get list_length(replicate(v, k)) summed with 1.

4 Problem 4: Even More Mathematical Induction Practice

Claim: the sum of the first n odd numbers $d_1 + \cdots + d_n = n^2$
Proof. The base case would be when n = 0.
$d_1 = 1^2$
$-- > 1 = 1$
We start by assuming that the claim holds true for a natural number k.
$d_1 + \cdots + d_k = k^2$
Then, we will prove that it is also true for a natural number n = k+1.
$d_1 + \cdots + d_k + d_{k+1} = (k+1)^2$
The right side evaluates to
$d_1 + \cdots + d_k + d_{k+1} = k^2 + 2k + 1$
From the base assumption that the claim holds true for a natural number k, we can negate the parts on both the left and the right hand-side.
$d_{k+1} = 2k + 1$
Now we use the term that was defined before the claim, $d_i = 2i - 1$
$d_{k+1} = 2(k+1) - 1 -- > d_{k+1} = 2k + 2 - 1$
Finally, this equation becomes the same equation we ended up when we were proving the claim.
$-- > d_{k+1} = 2k + 1$
Which proves that the claim holds true.