CSC208 0420 Lab: More Counting Practice

Havin Lim

April 20th, 2023

1 Problem: Alternative

The second formula is derived by removing the overcounting that occurs in the first formula. The first formula counts all possible sets of size k, without regard to order. However, some of these sets will contain the same elements, but in a different order. For example, if we are selecting two elements from the set 1,2,3, then the set 1,2 is counted the same as 2,1. Thus, we are overcounting the number of sets by a factor of (n-k)!. To correct for this overcounting, we divide by (n-k)! to get the second formula, which gives the number of combinations of k elements from n objects.

2 Problem: Poker Hands

because it counts each possible five-card hand multiple times. In a card game, the order in which the cards are drawn does not matter, so hands that contain the same set of cards should be considered equivalent.

3 Problem 2: Dress-down

a. 52!/5!(52-5)! = 2,598,960

b. To find two pairs in a poker game, we have to calculate the following.
$\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}$=123,552

We choose 2 ranks out of the 13 ranks, 2 suits out of the 4 suits, 1 rank out of the remaining 11 ranks, which will be the last card that is not included in the pairs, and 1 out of 4 suits for the last card.

c. To find the possible number of full houses in the game, we have to calculate the following.
$\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 3{,}744$

We choose 1 rank out of the 13 ranks and 2 3 suits out of the 4 suits for a three-of-a-kind, and 1 rank out of the 13 ranks with 2 suits out of the 4 suits for a pair, which makes a full-house.

d. Using the same strategy, the four-of-a-kind can be calculated as followed.
$\binom{13}{1}\binom{4}{4}\binom{12}{1}\binom{4}{1} = 624$

e. To calculate a one-pair hand without any additional combinations in it, first we need to calculate all the one-pair possible combinations and subtract all the other pairs that are available.
One-pair possible combinations will be $\binom{13}{1}\binom{4}{2}\binom{12}{3}\binom{4}{1}\binom{4}{1}\binom{4}{1}$, and we subtract all the other combinations from the previous questions which is 123,552 + 3744 + 624.
The answer for this would be 1,098,240.

f. Calculating a flush would be choosing one from four suits and five from 13 ranks, which makes $\binom{4}{1}\binom{13}{5}$. But this is also counting higher possible combinations, so we will subtract them from these flushes. There are 40 straight flushes and 4 royal flushes. So, the answer will be 5,148-40-4 = 5,104.

4 Problem 3: Deceptive

There are only 4 possible combinations for a royal flush because there are only one 10-J-K-Q-A set in every suit. There are totla 4 suits in the card deck, so there will be only 4 possible combinations.

5 Problem 4: Shades of Pre-registration

a. $\binom{k}{5}$
b. $\binom{k}{3}$
c. $\binom{k-2}{2}$
d. $\binom{k}{4}$ - $\binom{k-2}{2}$
e. $\binom{k}{2}$ * 5 * 3