CSC208 0404 Lab: Graph Problems

Havin Lim

April 4th, 2023

1 Problem 1: Bipartiteness

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a. Bipartite : G = (V,E) where V = \{a,b,c,d,e\}

E = \{(a,d), (a,e), (b,d), (b,e), (c,d), (c,e)\}
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Non-Bipartite : G = (V,E) where V = \{a,b,c,d,e\}

E = \{(a,d), (a,e), (b,d), (b,e), (c,d), (c,e), (d,e)\}
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b. In the bipartite case, we can divide the vertices in to two groups, customers and products, where an edge connects a customer to a product if that customer has purchased that product before.

By connecting and analyzing one's purchasing pattern, we can easily recommend products that are favorable.

c.
$$G = (V,E)$$
 Where $V = \{a,b,c,d,e,f\}$
 $E = \{(a,b), (b,c), (d,e), (e,f)\}$

This graph is a perfect-matching graph, but if the nodes are connected like this would connect the customers to each other which doesn't make a great example for the one above.

2 Problem 2: Cliques

a. Graph with a 4-clique : G = (V,E) where $V = \{a,b,c,d\}$

$$\begin{split} E &= \{(a,b),\, (a,c),\, (a,d),\, (b,c),\, (b,d),\, (c,d)\} \\ Graph \ without \ a \ 4\text{-clique}: \ G &= (V,E) \ where \ V = \{a,b,c,d,e\} \end{split}$$

$$E = \{(a,b), (b,c), (b,d), (c,e), (d,e)\}$$

- b. The 4-clique example can be interpreted as the friendship between four friends. They are connected to one another showing that they think of one another as friends.
- c. In the friendship example, a complete graph would represent a scenario in which all friends are connected to each other. This means that all friends share

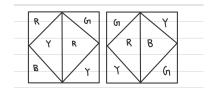
a close relationship with each other and form a single clique. However, in real-world scenarios is is very rare to have equal friendships with one another. //

3 Problem 3: Coloring

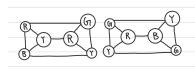
a. Graph with a 3-coloring : G = (V,E) where $V = \{a,b,c,d,e,f\}$

 $E = \{(a,d), (a,b), (b,c), (d,e), (b,e), (c,f), (e,f)\}$

Where the colors are a,e: red / b,d: blue / c,f: green

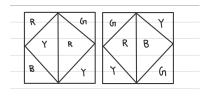


b.



c.

Nodes represent the regions and edges represent the connection between the regions and the borders.



d.

Working as a Petersen graph, which cannot be colored with 3-colors, since it cannot be colored with colors of 3, it is impossible to be colored with 4 colors as well.