# CSC341 HW4

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## Academic Honesty

### Written Sources Used:

Michael Sipser - Introduction to the Theory of Computation

### Help Obtained:

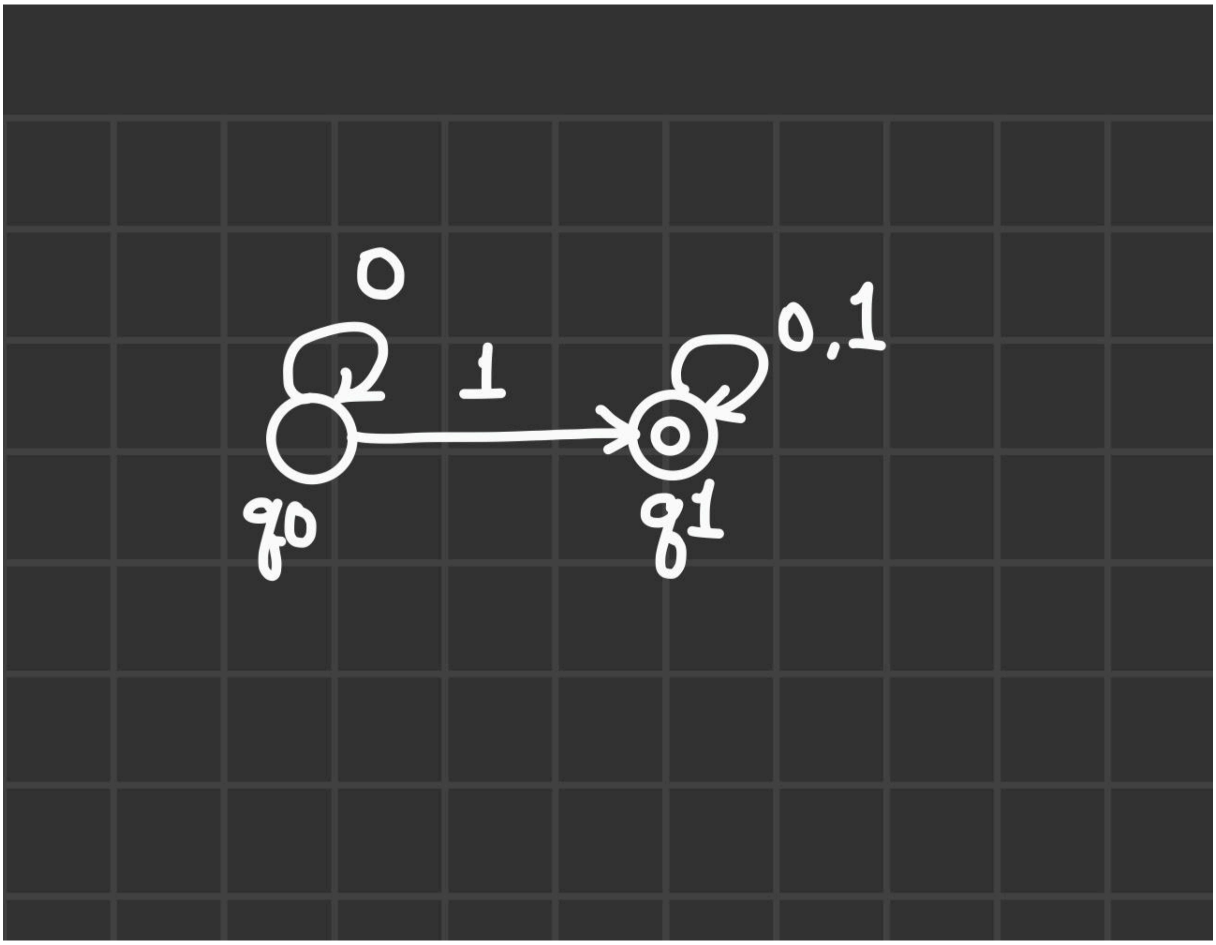
None

## Problem 1

### Number 1

In the proof we only consider when $y$ consists of only 0s, 1s, or substring of 0s followed by a substring of 1s. In this case analysis we are leaving out the possible situation when 0s and 1s are in random order.

### Number 2




Creating the following DFA allows to accept any string that has zero or more 0s followed by any number of 1s more than zero, which satisfies 0*1*

# Problem 2

## Number 1

Assume that $A$ is regular. Through pumping lemma since $A$ is regular, there exists a pumping length $p$ such that any string $s$ in $A$ with length at least $p$ can be decomposed as $s = xyz$ with the following conditions.

1. $\mid xy \mid \leq p$
2. $\mid y \mid > 0$
3. for all $i \geq 0, xy^i z \in A$

In the string $s = 0^p 1^p 2^p$, $3p \geq p$ so pumping lemma applies for the language $A$.
When $y$ contains only 0 or 1, if we pump $y$, the result string will consist of more 0s or 1s that y is pumped which breaks the condition that equal 0s and 1s are in $A$. Also, if y contains both 0s and 1s, if we pump $y$ it might create unequal numbers of 0s and 1s that break the condition.
Therefore, pumping in this case will create strings that are not in language $A$ which contradicts the pumping lemma.
Pumping lemma's failure means that $A$ is irregular.

## Number 2

Like the first question, we assume $B$ is regular. Then the pumping lemma applies for a pumping length $p$.
Assume $s = a^p b^p$, $k = i + j = 2p$
We make $s$ as $xyz$ that follows the pumping lemma conditions.

When $y$ contains only $a$s or $b$s, pumping y violates the condition $k = i + j$ by creating an unequal number of $a$ and $b$. Even when y contains both $a$ and $b$, it is possible to change $x$ and $z$ which violates the definition $k = i + j$ easily.

Through this, we show that the pumping lemma fails for $B$ showing that $B$ is irregular.

# Problem 3

## Number 1

$UTV \Rightarrow < -TV \Rightarrow < --TV \Rightarrow < ---V \Rightarrow < --- >$
Through the following step we can prove that $< --- >$ is in the language $L(G)$.

## Number 2

$< (-)^* >$

# Problem 4

## Number 1

$V = \{S, A, B\}$
$\Sigma = \{0, 1\}$
$R = S \rightarrow AB \mid 1AB1 \mid 1AS1 \mid 1S1 \mid 1SB1$
$\quad A \rightarrow 1 \mid 0A \mid 1A$

$B \to 1 \mid 0B \mid 1B$

$S \to 1A \mid 0A \mid 1B \mid 0B \mid AB \mid 1S1 \mid 1AB1 \mid 1AS1 \mid 1SB1$

## Number 2

$V = \{S, A, B\}$

$\Sigma = \{0, 1\}$

$R = S \to A \mid B$

$A \to 00S \mid 11S$

$B \to 0 \mid 1$

$S \to 0 \mid 1 \mid A \mid B$

## Number 3

$V = \{S, A, B\}$

$\Sigma = \{0, 1\}$

$R = S \to A \mid B$

$A \to 0A0 \mid 1A1 \mid 0B0 \mid 1B1$

$B \to 0 \mid 1$

$S \to 0 \mid 1 \mid A \mid B \mid 0A0 \mid 1A1 \mid 0B0 \mid 1B1$