CSC341 Lab 9A

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Academic Honesty

Written Sources Used:

Michael Sipser - Introduction to the Theory of Computation

Help Obtained:

None

Question 2

(1)

(a)

It should either accept or reject, but we don't know which one since M' is looping forever.

(b)

We could run H with the inputs M and w as the TM and its input. By doing this it will always accept or reject making a TM into a decider by using H.

(2)

$D(\langle M_1 \rangle)$ = reject
$D(\langle M_2 \rangle)$ = accept
$D(\langle M_3 \rangle)$ = unknown

D outputs the opposite of H, so for M_1 and M_2 they will be the opposite results of H's. For M_3, since it loops forever on H, it would be impossible for us to assume what kind of result it would make.

(3)

If we run D on D itself, the result will contradict itself because it needs to accept when it rejects and reject when it accepts, which is a contradiction.

Question 3

(4)

Since a Turing Machine can be explained with set of states, a transition function, and a tape alphabet, which are all finite, it is possible to list all Turing Machines that could ever exist.

(5)

D is created to give an opposite outcome of machine M_i when it has the input of $\langle M_i \rangle$. This is similar to the uncountability of real numbers in a way that both create a new entry that does not exist in the original list. For example, a real number that does not exist in the list or the TM D that has a different behavior than the ones that have existed. The difference would be that the case of the real number scenario is used to prove accountability when the TM example is to show undecidability.

Question 4

Assume that $HALT_{TM}$ is decidable.

We create a new Turing Machine D that takes M as input. For D it should loop forever when H accepts the input $\langle M, \langle M \rangle \rangle$ and halt when H rejects $\langle M, \langle M \rangle \rangle$.

If we run D on $\langle D \rangle$, whether $H(\langle D, \langle D \rangle \rangle)$ accepts or rejects it would contradict H's result because D needs to perform the opposite result of H.

Since D on $\langle D \rangle$ results in a contradiction, $HALT_{TM}$ should be undecidable.

Question 5

We assume that $HALT_{TM}$ is decidable. There exists a TM R that decides it.
Then we create a new TM S that decides A_{TM}. S must accept if M accepts w and reject in all other cases. However, we already know that A_{TM} is undecidable, so it also leads to the fact that R can decide $HALT_{TM}$ is also false.