CSC341 HW9

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Havin Lim

Academic Honesty

Written Sources Used:

Michael Sipser - Introduction to the Theory of Computation

Help Obtained:

None

Question 1

It is not necessarily true.

To give a counter-example, we can assume A and B to be the following.

A = $\{0^n 1^n \mid n \geq 0\}$
$B = \{1\}$

This shows that B is just a simple language that is regular, and A is a language that is non-regular because a finite automaton is unable to remember and count the number of 0s and 1s and know if they are equal or not.

For the computable function $f : \Sigma^* \rightarrow \Sigma^*$, we can define it as
$f(w) = 1$ if $w \in A$
$f(w) = 0$ if $w \notin A$

This function maps A to B but still does not imply that A is regular when B is.

Question 2

To prove that USELESS is undecidable we will do a reduction from the A_{TM}, which means that if USELESS is decidable that makes A_{TM} also decidable, but since A_{TM} is known to be undecidable it gives a contradiction.

We create a TM M' from any instance $\langle M, w \rangle$ of A_{TM} such that if we decide whether M' has a useless state or not will also tell us that if M accepts w or not which solves A_{TM} as well.

We assume that there exists a decider D for USELESS. We run D on M'. With this, if D shows that M' has a useless state it means that M accepts w. On the other hand, if D is unable to find a useless state, it menas that M does not accept w. This way of proof is used to show that a decider for USELESS can be used to decide A_{TM}, which is impossible because A_{TM} is undecidable.

Question 3

Unary alphabet of $\Sigma = \{1\}$ means that the dominoes have difference in the number of 1s that consist the top and bottom of a domino. There can be a several conditions that have to meet in order for the TM to accept.

First, we calculate the total length of the top part of the dominoes and the bottom part of the dominoes, if they are equal two each other, accept.

Next, since repetition is available, we check if there is at least one pair of dominoes where their greatest common divisor is 1. Then we the TM can accept because this means that repeating these dominoes can make the number of 1s equal at the end.

If none of this is possible, TM rejects.

This way we can show that the PCP is decidable over the unary alphabet $\Sigma = \{1\}$.

Question 4

We can do a reduction from the A_{TM} to L_1 to prove it to be undecidable. We create a function $f \to \Sigma^* \to \Sigma^*$ that from any instance $\langle M, w \rangle$, M' is infinite if M accepts w, but finite if M rejects.

If M accepts w, M' checks if x is in the form of $0^n 1^n$. If it is, accept. This makes sure that if M accepts w, all the languages accepts by M' are in the form of $0^n 1^n$ which is infinite, and otherwise finite.