CSC341 HW8

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Written Sources Used:

Michael Sipser - Introduction to the Theory of Computation

Help Obtained:

None

Question 1

To prove the cardinality of E is equal to O, we have to show the existence of bijection between E and O. This can be done by showing both sets are injectie and surjectie to one another.

To prove this, we define a function $f : E \to O$, such that for every even number $e \in E, f(e) = e + 1$. This maps all even number to a single odd number after that number, for example 2 will be mapped with the odd number 3.

To prove function f's injectivity we assume that $f(e_1) = f(e_2)$. This means also $e_1 + 1 = e_2 + 1$, which is same as $e_1 = e_2$, therefore f is injective because it assigns to different even numbers to distinct odd numbers.

To prove functiono f's surjectivity we pick out an odd number o from O. For o, there will be an even numberf $o - 1 \in E$. $f(o - 1) = (o - 1) + 1 = o$. This means that for any odd number $o \in O$ there exists an even number $e \in E$.

Question 2

If we assume B is countable then there would be list of elements in sequence which can be listed with natural numbers.
$b_1 = b_{11}, b_{12}, ...$
$b_2 = b_{21}, b_{22}, ...$

In which b_{ij}, the i and j represents the j-th element of the i-th binary sqeuence in the list above.

To use the diagonalization, we make a new binary sequence c in which $c_i = 1 - b_{ii}$ for each $i \in N$, meaning that the i-th bit in c will be the opposite of the i-th bit in the sequence. With this, $c \in B$ differs from all the n-th sequence in the n-th bit, which makes $c \neq b_n$ for any n proving that B is uncountable.

Question 3

1)

$EQ_{DR} = \{< D, R > \mid L(D) = LR)\}$

2)

We convert the regular expression R to an equivalent DFA D_R.

Then we also have $\overline{D}$ and $\overline{D_R}$, which accepts all strings that are not accepted by D and D_R.

Using these, we create intersections $D \cap \overline{D_R}$ and $\overline{D} \cap D_R$. These intersections of two DFAs can recognize the intersection of their languages. After this we check if the accepted language is empty.

If both intersections $D \cap \overline{D_R}$ and $\overline{D} \cap D_R$ are found to recognize the empty language, this signifies that there is no string accepted by one machine that is rejected by the other. As a result, D and R are equivalent as they accept precisely the same set of strings. Thus, given any DFA D and regular expression R, we can decide whether they are equivalent by performing these steps, proving that the language EQ_{DR} is decidable.