10 10

Chapter 10

Faraday's Law of Induction

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10.1 Faraday's Law of Induction

The electric fields and magnetic fields considered up to now have been produced by stationary charges and moving charges (currents), respectively. Imposing an electric field on a conductor gives rise to a current that in turn generates a magnetic field. One could then inquire whether or not an electric field could be produced by a magnetic field. In 1831, Michael Faraday discovered that, by varying a magnetic field with time, an electric field could be generated. The phenomenon is known as electromagnetic induction. Figure 10.1.1 illustrates one of Faraday's experiments, and Figure 10.1.1 shows one frame from a movie of the actual experiment.

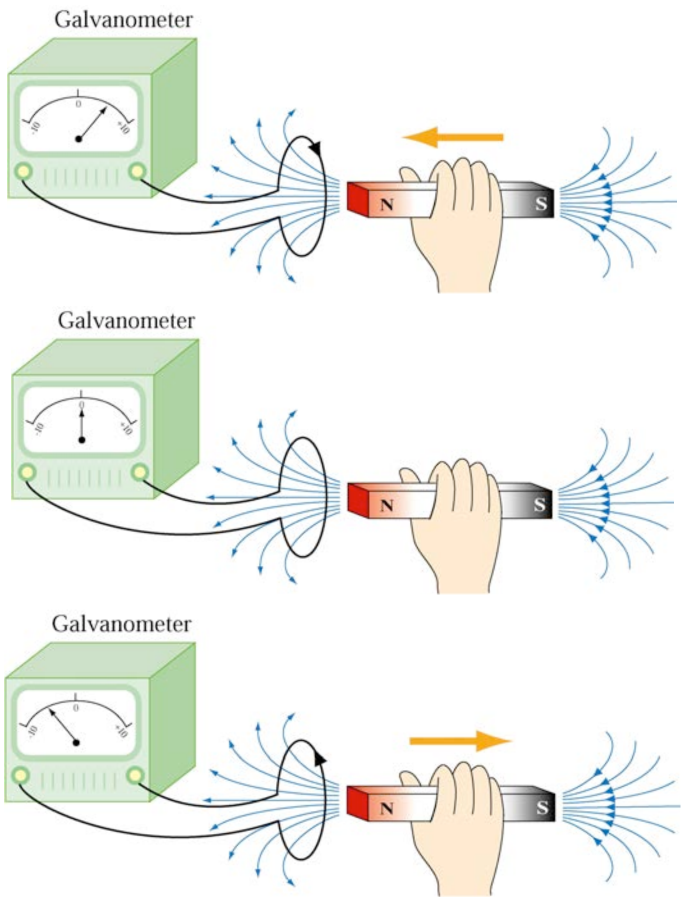

Figure 10.1.1 Electromagnetic induction

Faraday showed that no current is registered in the galvanometer when bar magnet is stationary with respect to the loop. However, a current is induced in the loop when a relative motion exists between the bar magnet and the loop. In particular, the galvanometer deflects in one direction as the magnet approaches the loop, and the opposite direction as it moves away. There is an interactive simulation of this aspect of Faraday's Law in Section 10.13 below.

Faraday's experiment demonstrates that an electric current is induced in the loop by changing the magnetic field. The coil behaves as if it were connected to an emf source. Experimentally it is found that the induced emf depends on the rate of change of magnetic flux through the coil.

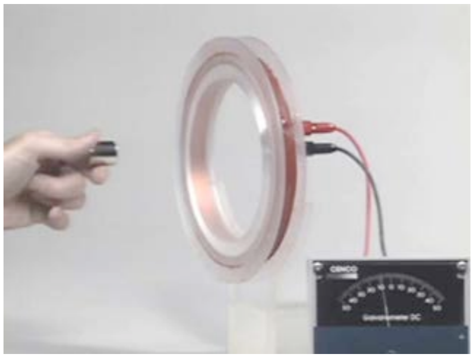

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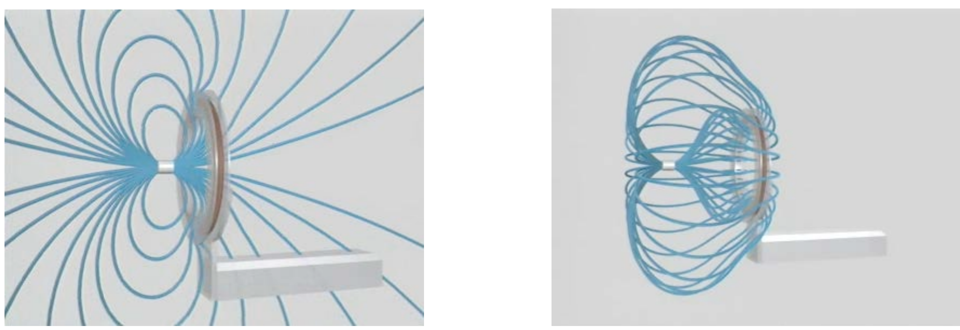
Figure 10.1.2 Electromagnetic induction experiment



(a) http://youtu.be/vcG2wv6IZ8k (b) http://youtu.be/NWE9SCRgBv0

Figure 10.1.3 Visualizations of the total magnetic field when (a) the magnet is moving toward the coil and (b) when the magnet is moving away from the coil.

Figure 10.1.3 shows a visualization of the total magnetic field in this experiment, that is the field due the magnet itself *and* to the magnetic field generated by the induced currents in the coil. Whether we are moving the magnet into the coil or out of the coil, we see that the total magnetic field is such that the magnetic field lines tend to get "hung up" momentarily in trying to move through the coil. This is an example of Lenz's Law, as discussed in Section 10.4 below.

We now explore how we can quantitatively describe this phenomena in mathematical terms. To do this we must introduce the concept of magnetic flux.

10.1.1 Magnetic Flux

Consider a uniform magnetic field passing through a surface *S*, as shown in Figure 10.1.4(a). Let the area vector be $\vec{\mathbf{A}} = A\hat{\mathbf{n}}$, where A is the area of the surface and $\hat{\mathbf{n}}$ its unit normal.

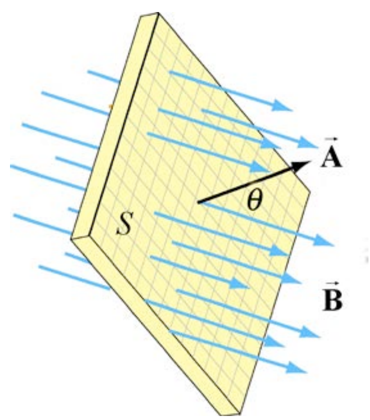


Figure 10.1.4(a) Magnetic flux through a planar surface with uniform magnetic field

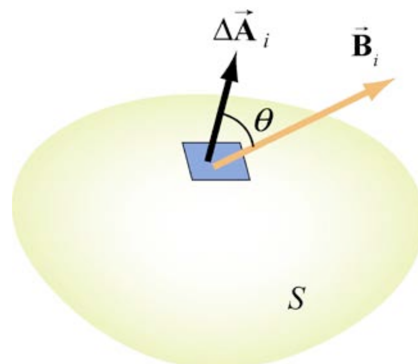


Figure 10.1.4(b) Magnetic flux through a non-planar surface

The magnetic flux through the surface is given by

$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta\,, \tag{10.1.1}$$

where θ is the angle between $\vec{\mathbf{B}}$ and $\hat{\mathbf{n}}$.

If the field is non-uniform or the surface is non-planar, we divide the surface into N small area elements with area vector $\Delta\vec{\mathbf{A}}_{\mathbf{i}} = \Delta A_i \hat{\mathbf{n}}_{\mathbf{i}}$ as shown in Figure 10.1.4(b). We calculate the magnetic flux through each element $\Delta\Phi_{B,i} = \vec{\mathbf{B}}_i \cdot \Delta\vec{\mathbf{A}}_i$, and then sum up over all the elements, $\sum_{i=1}^{i=N} \Delta\Phi_{B,i} = \sum_{i=1}^{i=N} \vec{\mathbf{B}}_i \cdot \Delta\vec{\mathbf{A}}_i$. We then take the limit as $N \to \infty$, to find the magnetic flux as a surface integral

$$\Phi_B = \lim_{N\to\infty} \sum_{i=1}^{i=N} \Delta\Phi_{B,i} \equiv \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}\,. \tag{10.1.2}$$

The SI unit of magnetic flux is the weber [Wb] with

$$1\ \text{Wb} = 1\ \text{T} \cdot \text{m}^2\,.$$

10.2 Motional EMF

Consider a conducting bar of length l moving through a uniform magnetic field which points into the page, as shown in Figure 10.2.1. Particles with charge $q > 0$ inside experience a magnetic force $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ that tends to push them upward, leaving negative charges on the lower end.

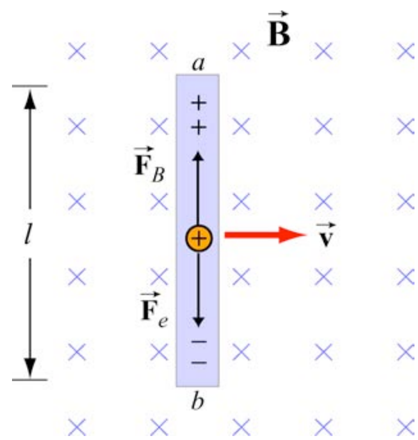


Figure 10.2.1 A conducting bar moving through a uniform magnetic field

The separation of charge gives rise to an electric field $\vec{\mathbf{E}}$ inside the bar, which in turn produces a downward electric force $\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$. At equilibrium where the two forces cancel, we have $qvB = qE$, or $E = vB$. Between the two ends of the conductor, there exists a potential difference given by

$$V_{ab} = V_a - V_b = \varepsilon = El = Blv \tag{10.2.1}$$

Since ε arises from the motion of the conductor, this potential difference is called the motional emf.

Now suppose the conducting bar moves through a region of uniform magnetic field $\vec{\mathbf{B}} = -B\hat{\mathbf{k}}$ (pointing into the page) by sliding along two frictionless conducting rails that are at a distance w apart and connected together by a resistor with resistance R, as shown in Figure 10.2.2.

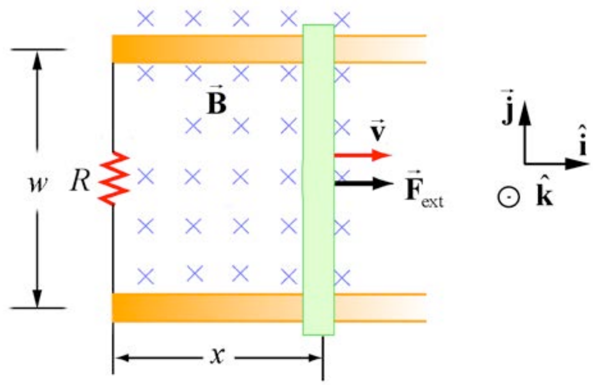


Figure 10.2.2 A conducting bar sliding along two conducting rails

Let an external force $\vec{\mathbf{F}}_{\text{ext}}$ be applied so that the conductor moves to the right with a constant velocity $\vec{\mathbf{v}} = v\hat{\mathbf{i}}$. Choose the area element $\vec{\mathbf{A}} = A\hat{\mathbf{k}}$. The magnetic flux through the closed loop formed by the bar and the rails is given by

$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = (-B\hat{\mathbf{k}}) \cdot (A\hat{\mathbf{k}}) = -BA = -Bwx \tag{10.2.2}$$

The changing magnetic flux is then

$$\frac{d\Phi_B}{dt} = -\frac{d}{dt}(Bwx) = -Bw\frac{dx}{dt} = -Bwv\,, \tag{10.2.3}$$

where $dx/dt = v$ is the speed of the bar.

A charge carrier with charge q in the moving bar therefore experiences a magnetic force given by

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = qv\hat{\mathbf{i}} \times B(-\hat{\mathbf{k}}) = qvB\hat{\mathbf{j}}\,. \tag{10.2.4}$$

We orient the closed path formed by the bar and the rail clockwise so that the unit normal $\hat{\mathbf{n}} = \hat{\mathbf{k}}$, (pointing out the page), in order to be consistent with our choice of orientation for the surface integral in our calculation of magnetic flux. The only contribution to the integral is upward along the moving bar. The electromotive force is then

$$\varepsilon = \oint \frac{\vec{\mathbf{F}}_B}{q} \cdot d\vec{\mathbf{s}} = \int_{bar} vB\hat{\mathbf{j}} \cdot (dy\hat{\mathbf{j}}) = wvB\,, \tag{10.2.5}$$

where the integral is taken at one instant in time, so the element $d\vec{\mathbf{s}} = dy\hat{\mathbf{j}}$ points upward even though the bar is moving to the right. Comparing Eq. (10.2.3) and Eq. (10.2.5), we conclude that

$$\varepsilon = -\frac{d\Phi_B}{dt}. \tag{10.2.6}$$

The magnetic field is not actually doing the work; the external force that is pulling the bar is doing the work. In order to see why, we first observe that the emf is causing charge carriers to move upward in the bar. The charge carrier has an additional vertical component of velocity

$$\vec{\mathbf{v}} = v\hat{\mathbf{i}} + u\hat{\mathbf{j}}. \tag{10.2.7}$$

where u is the y-component of the velocity of the charge carriers. Therefore the magnetic force on the charge carrier is given by

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = q(v\hat{\mathbf{i}} + u\hat{\mathbf{j}}) \times B(-\hat{\mathbf{k}}) = qvB\hat{\mathbf{j}} + quB(-\hat{\mathbf{i}}). \tag{10.2.8}$$

The external force must exactly oppose the x-component of the magnetic force in order to keep the bar moving at a constant speed,

$$\vec{\mathbf{F}}_{ext} = quB\hat{\mathbf{i}}. \tag{10.2.9}$$

If a charge carrier moves from the bottom of the bar to the top of the bar in time Δt, then $w = u\Delta t$. The bar is also moving in the positive x-direction by an amount $\Delta x = v\Delta t = vw/u$. The displacement of the moving charge carrier, $\Delta\vec{\mathbf{s}} = (vw/u)\hat{\mathbf{i}} + w\hat{\mathbf{j}}$, and the magnetic force are perpendicular because their scalar product is zero,

$$\vec{\mathbf{F}}_B \cdot \Delta\vec{\mathbf{s}} = qvB\hat{\mathbf{j}} + quB(-\hat{\mathbf{i}}) \cdot ((vw/u)\hat{\mathbf{i}} + w\hat{\mathbf{j}}) = qvBw - quB(vw/u) = 0. \tag{10.2.10}$$

This is not surprising because magnetic forces do no work.

The work done by the external force per charge is equal to the emf,

$$\int \frac{\vec{\mathbf{F}}_{ext}}{q} \cdot d\vec{\mathbf{s}} = uB\Delta x = uB\frac{vw}{u} = vwB = \varepsilon. \tag{10.2.11}$$

In general, motional emf around a closed conducting loop can be written as

$$\varepsilon = \oint (\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot d\vec{\mathbf{s}} \tag{10.2.12}$$

where $d\vec{\mathbf{s}}$ is a differential length element.

10.3 Faraday's Law (see also Faraday's Law Simulation in Section 10.13)

In our previous section, the loop formed by the moving bar and rails was not stationary and so pulling the loop produced motional emf. Suppose we consider a stationary closed conducting path (for example a coil) as shown in Figure 10.3.1.

Figure 10.3.1 Inducing emf by varying the magnetic field strength

Now decrease the magnetic flux through the loop by decreasing the strength of the magnetic field. Then experimentally, while the magnetic field is changing, a current is observed in the loop in the clockwise direction. This cannot be a motional emf because the loop is a rest. What causes the charges to move? Faraday conjectured that it is an electric field that causes the charge carriers to move. The emf done must be the result of electric fields.

$$\varepsilon = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \,. \tag{10.3.1}$$

The electric field in this situation cannot be electrostatic because the line integral of an electrostatic field must vanish. Therefore, **Faraday's Law** is the statement that there is a non-electrostatic electric field $\vec{\mathbf{E}}$ associated with the emf such that

$$\boxed{\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt}} \tag{10.3.2}$$

The above expression implies that a changing magnetic flux will induce a non-electrostatic electric field that can vary with time. It is important to distinguish between the induced, non-electrostatic electric field and the electrostatic field that arises from electric charges.

The direction of integration for the emf is chosen according to the right hand rule with respect to the choice of unit normal in the definition of magnetic flux. For example, if we choose unit normal pointing into the page in Figure 10.3.1, then we must integrate in the clockwise direction.

In Chapter 4, we defined the electric potential difference between two points A and B in an electrostatic electric field $\vec{\mathbf{E}}$ as

$$\Delta V = V_B - V_A = -\int_A^B \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}, \qquad (\vec{\mathbf{E}} \text{ electrostatic}). \qquad (10.3.3)$$

When the electric field is non-electrostatic, $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \neq 0$, and therefore the potential difference between two points is no longer a well-defined concept.

10.3.1 Example Induced Electric Field

Let's consider a uniform magnetic field that points *into* the page and is confined to a circular region with radius R, as shown in Figure 10.3.2. Suppose the magnitude of $\vec{\mathbf{B}}$ increases with time, *i.e.*, $dB/dt > 0$. Let's find the induced electric field everywhere due to the changing magnetic field.

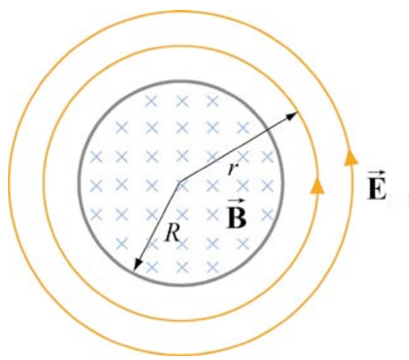


Figure 10.3.2 Induced electric field due to changing magnetic flux

We begin by considering the region $r < R$. The magnetic field is confined to a circular region, so we choose the integration path to be a circle of radius r. Choose an area vector $\vec{\mathbf{A}}$ pointing *into* the plane of the figure, so that the magnetic flux is positive.

$$\vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA = B\pi r^2 . \qquad (10.3.4)$$

The rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(\vec{\mathbf{B}} \cdot \vec{\mathbf{A}}) = \frac{d}{dt}(BA) = \frac{dB}{dt}\pi r^2 . \qquad (10.3.5)$$

With $dB/dt > 0$, the inward magnetic flux is increasing. Therefore according to Faraday's Law there is emf given by

$$\varepsilon = \oint_{\substack{circle \\ radius\ r}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} = -\frac{dB}{dt}\pi r^2, \tag{10.3.6}$$

where we integrated in the clockwise direction. By symmetry, we assume that the component of the electric field, E, tangent to the circle (a positive component points in the clockwise direction), must be uniform on the circle of radius r. Therefore

$$\oint_{\substack{circle \\ radius\ r}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = E 2\pi r\,. \tag{10.3.7}$$

We can now combine Eqs. (10.3.6) and (10.3.7), and solve for the component of the electric field tangent to the circle

$$E = -\frac{1}{2}\frac{dB}{dt}r, \qquad r < R\,. \tag{10.3.8}$$

The negative sign means that the electric field points counterclockwise. The direction of $\vec{\mathbf{E}}$ is shown in Figure 10.3.2.

For $r > R$, the magnetic flux is $\Phi_B = B\pi R^2$ and Faraday's Law requires that

$$E 2\pi r = -\frac{d\Phi_B}{dt} = -\frac{dB}{dt}\pi R^2\,. \tag{10.3.9}$$

Therefore the component of the electric field tangent to the circle is

$$E_{\mathrm{nc}} = \frac{R^2}{2r}\frac{dB}{dt}, \qquad r > R\,. \tag{10.3.10}$$

A plot of $|E|$ as a function of r is shown in Figure 10.3.3.

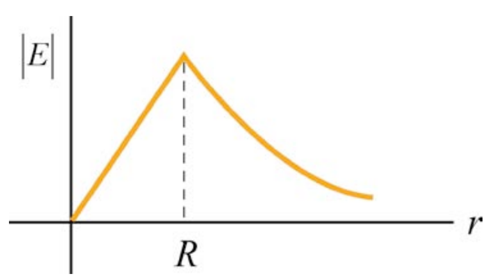


Figure 10.3.3 Plot of magnitude of electric field as a function of *r*

The relationship between emf and changing magnetic flux,

$$\varepsilon = -\frac{d\Phi_B}{dt}, \tag{10.3.11}$$

applies to both motional emf, and emf generated by induced electric fields. Therefore without analyzing its origin, we can determine emf just by considering how magnetic flux is changing. We shall consider the following ways that magnetic flux can change:

(i) by varying the magnitude of $\vec{\mathbf{B}}$ with time (illustrated in Figure 10.3.1.)

(ii) by varying the magnitude of $\vec{\mathbf{A}}$, i.e., the area enclosed by the loop with time (illustrated in Figure 10.3.4.)

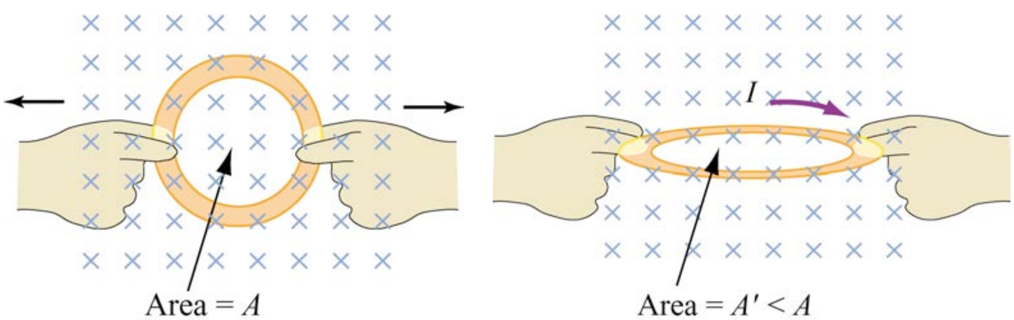


Figure 10.3.4 Inducing emf by changing the area of the loop by stretching it

(iii) varying the angle between $\vec{\mathbf{B}}$ and the area vector $\vec{\mathbf{A}}$ with time (illustrated in Figure 10.3.5).

Figure 10.3.5 Inducing emf by varying the angle between $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$.

Note that in cases (ii) and (iii), the closed loop is not fixed in time and so we are not always considering induced electric fields.

10.4 Lenz's Law

Let's return to our example of the bar moving on the fixed rails, Figure 10.4.1.

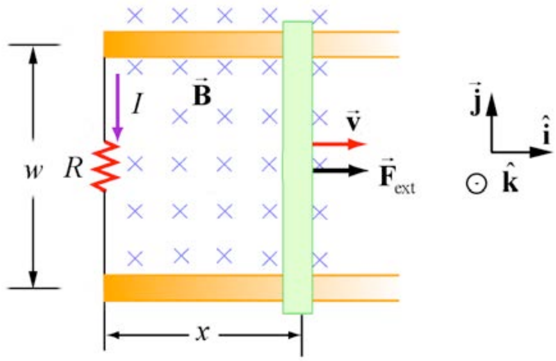


Figure 10.4.1 A conducting bar sliding along two conducting rails

The electromotive force is responsible for an induced current in the bar in the positive y-direction given by

$$I = \frac{\varepsilon}{R} = \frac{Bwv}{R}. \tag{10.4.1}$$

and its direction is counterclockwise. The magnetic flux through the loop increased as the bar moved to the right. A counterclockwise current will generate its own magnetic field pointing out of the loop. This will generate magnetic flux that opposes the change of flux due to the increasing area of the loop. This is an example of **Lenz's law**.

The induced current produces magnetic fields that tend to oppose the change in magnetic flux that induces such currents.

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector $\vec{\mathbf{A}}$.

2. Assuming that $\vec{\mathbf{B}}$ is uniform, take the dot product of $\vec{\mathbf{B}}$ and $\vec{\mathbf{A}}$. This allows for the determination of the sign of the magnetic flux Φ_B.

3. Obtain the rate of flux change $d\Phi_B / dt$ by differentiation. There are three possibilities:

$$\frac{d\Phi_B}{dt} \begin{cases} >0 & \Rightarrow \text{ induced emf } \varepsilon < 0 \\ <0 & \Rightarrow \text{ induced emf } \varepsilon > 0 \\ =0 & \Rightarrow \text{ induced emf } \varepsilon = 0 \end{cases}.$$

4. Determine the direction of the induced current using the right-hand rule. With your right-hand thumb pointing in the direction of $\vec{\mathbf{A}}$, curl the fingers of your right-hand around the closed loop. The induced current is in the same direction as the way your fingers curl if $\varepsilon > 0$, and the opposite direction if $\varepsilon < 0$, as shown in Figure 10.4.2.

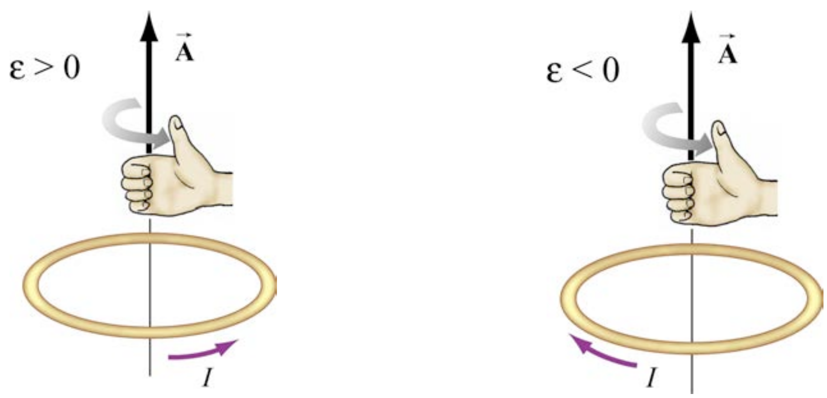


Figure 10.4.2 Determination of the direction of induced current by the right-hand rule

In Figure 10.4.3 we illustrate the four possible scenarios of time-varying magnetic flux and show how Lenz's law is used to determine the direction of the induced current I. The situations can be summarized with the following sign convention:

Φ_B	$d\Phi_B/dt$	ε	I
+	+	−	−
	−	+	+
−	+	−	−
	−	+	+

The positive and negative signs of I correspond to counterclockwise and clockwise currents, respectively.

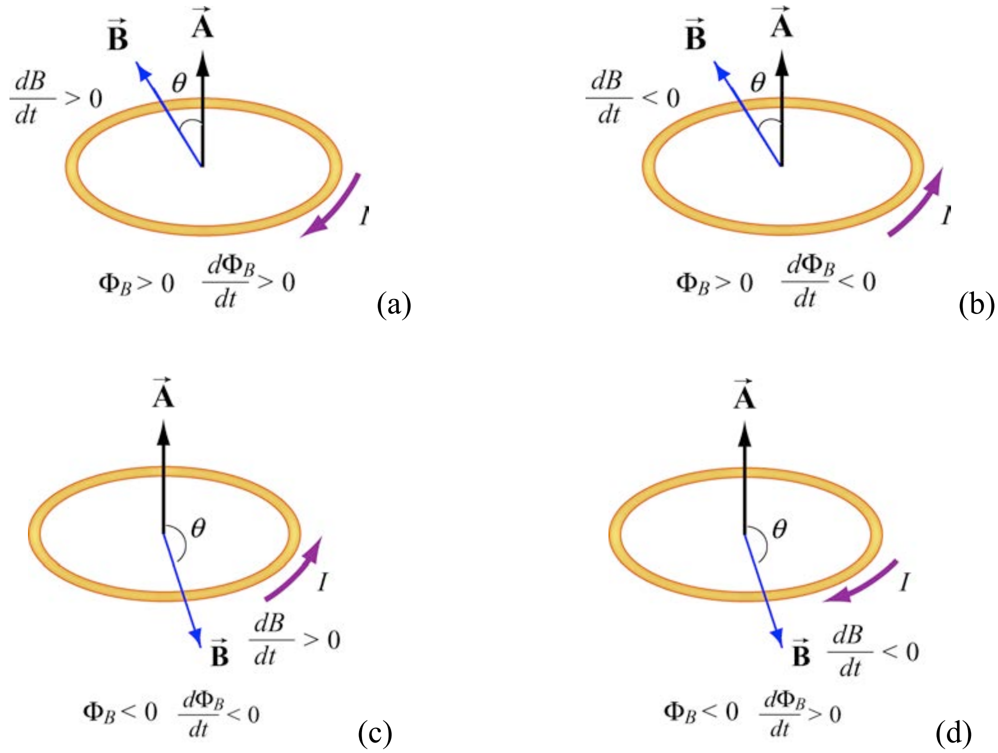


Figure 10.4.3 Direction of the induced current using Lenz's law

As an example to illustrate how Lenz's law may be applied, consider the situation where a bar magnet is moving toward a conducting loop with its north pole down, as shown in Figure 10.4.4(a).

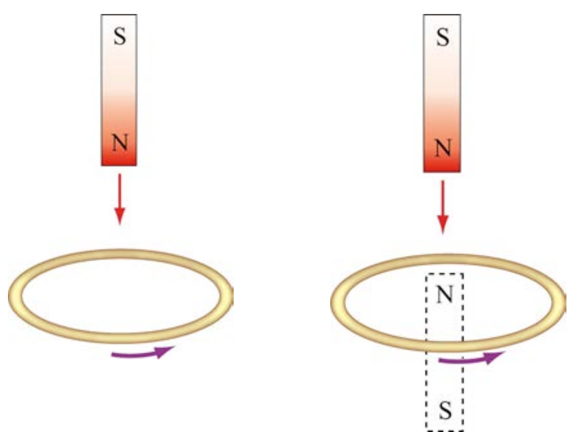


Figure 10.4.4 (a) A bar magnet moving toward a current loop. (b) Determination of the direction of induced current by considering the magnetic force between the bar magnet and the loop. The dotted rectangle represents the magnetic dipole induced in the loop.

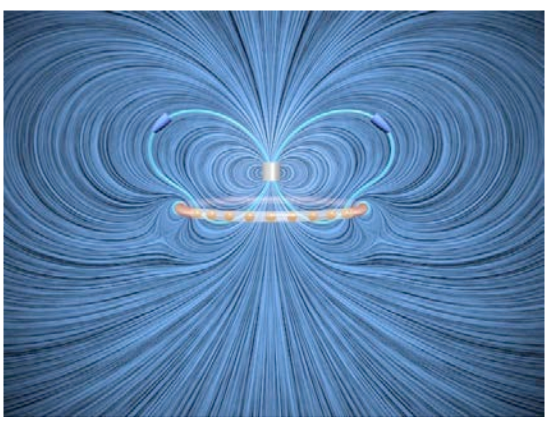
With the magnetic field pointing downward and the area vector $\vec{\mathbf{A}}$ pointing upward, the magnetic flux is negative, *i.e.*, $\Phi_B = -BA < 0$, where A is the area of the loop. As the magnet moves closer to the loop, the magnetic field at a point on the loop increases ($dB/dt > 0$), producing more flux through the plane of the loop. Therefore, $d\Phi_B/dt = -A(dB/dt) < 0$, implying a positive induced emf, $\varepsilon > 0$, and the induced current flows in the counterclockwise direction. The current then sets up an induced magnetic field and produces a *positive* flux to counteract the change. The situation described here corresponds to that illustrated in Figure 10.4.3(c).

Alternatively, the direction of the induced current can also be determined from the point of view of magnetic force. Lenz's law states that the induced emf must be in the direction that opposes the change. Therefore, as the bar magnet approaches the loop, it experiences a repulsive force due to the induced emf. Since like poles repel, the loop must behave as if it were a bar magnet with its north pole pointing up. Using the right-hand rule, the direction of the induced current is counterclockwise, as viewed from above. Figure 10.4.4(b) illustrates how this alternative approach is used.

More generally, Lenz's law states that the direction of induced currents, forces, or torques in a system is always in a direction to resist how the system changes.

10.4.1 Magnets and Conducting Loop Movies

To get another perspective on this, let us look at a visualization of the total magnetic field in a scenario similar to that of Figure 10.4.4, that is a magnet falling on the axis of a stationary conducting loop



http://youtu.be/LSMdsabj0aQ

Figure 10.4.5 A magnet falls on the axis of a stationary conducting loop

In Figure 10.4.5, we show a conducting loop that is fixed at the origin. The magnet is sitting some distance above the loop on its axis and has its north pole pointed upwards (opposite the direction assumed for Figure 10.4.4). The magnet is released from rest and falls under gravity toward the stationary loop. As the magnet falls toward the loop, the sense of the current induced in the loop is such as to try to keep the magnetic flux through the loop from changing, with the result that the field lines appear to get "hung up" when crossing the loop. The result is a build of magnetic field just above the loop, that is a magnetic pressure pushing upward from below on the magnet and slowing its fall. When the magnet falls below the loop, again the field lines appear to get "hung up" when trying the exit the loop, which now stretches them instead of compressing them. This results in a tension that is pulling upward on the magnet from above, again slowing its fall.

10.4.2 Example Bar Moving Along Rails in a Constant Magnetic Field

Let's return to our example of the bar moving on the fixed rails, Figure 10.4.5.

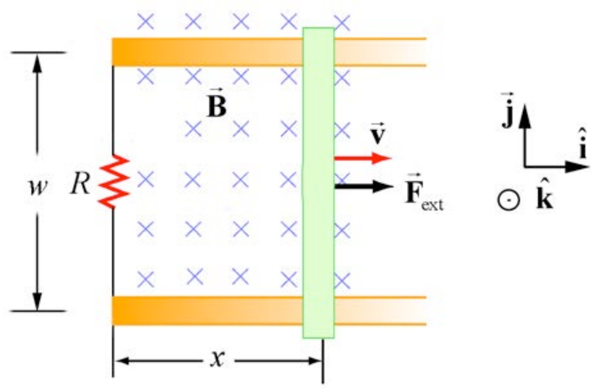


Figure 10.4.5 A conducting bar sliding along two conducting rails

We have already determined that there is a counterclockwise current $I = Bwv / R$ in the bar and rails. Therefore there is an induced magnetic force, $\vec{\mathbf{F}}_{\text{ind}}$, experienced by current in the bar as the bar moves to the right given by

$$\vec{\mathbf{F}}_{\text{ind}} = I(w\hat{\mathbf{j}}) \times (-B\,\hat{\mathbf{k}}) = -IwB\hat{\mathbf{i}} = -\left(\frac{B^2w^2v}{R}\right)\hat{\mathbf{i}}\,, \tag{10.4.2}$$

that is in the opposite direction of $\vec{\mathbf{v}}$. For the bar to move at a constant velocity, the net force acting on it must be zero. This means that the external agent must supply a force

$$\vec{\mathbf{F}}_{\text{ext}} = -\vec{\mathbf{F}}_{\text{ind}} = +\left(\frac{B^2l^2v}{R}\right)\hat{\mathbf{i}}\,. \tag{10.4.3}$$

The power delivered by $\vec{\mathbf{F}}_{\text{ext}}$ is equal to the power dissipated in the resistor:

$$P = \vec{\mathbf{F}}_{\text{ext}} \cdot \vec{\mathbf{v}} = F_{\text{ext}}v = \left(\frac{B^2l^2v}{R}\right)v = \frac{(Blv)^2}{R} = \frac{\varepsilon^2}{R} = I^2R\,, \tag{10.4.4}$$

as required by energy conservation.

From the analysis above, in order for the bar to move at a constant speed, an external agent must constantly supply a force $\vec{\mathbf{F}}_{\text{ext}}$. What happens if at $t=0$, the speed of the rod is v_0, and the external agent stops pushing? In this case, the bar will slow down because of the magnetic force directed to the left. From Newton's Second Law, we have

$$F_B = -\frac{B^2 l^2 v}{R} = ma = m\frac{dv}{dt}. \tag{10.4.5}$$

We can rewrite this differential equation as

$$\frac{dv}{v} = -\frac{B^2 l^2}{mR} dt = -\frac{dt}{\tau}, \tag{10.4.6}$$

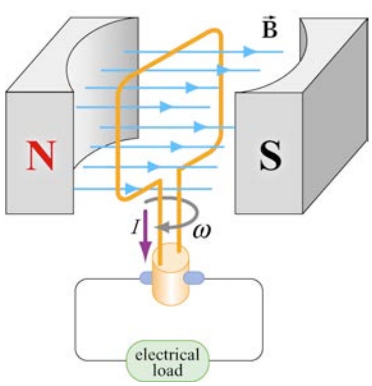
where $\tau = mR / B^2 l^2$. Upon integration, we obtain

$$v(t) = v_0 e^{-t/\tau} \tag{10.4.7}$$

Thus, we see that the speed decreases exponentially in the absence of an external agent doing work. In principle, the bar never stops moving. However, one may verify that the total distance traveled is finite.

10.5 Generators

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.



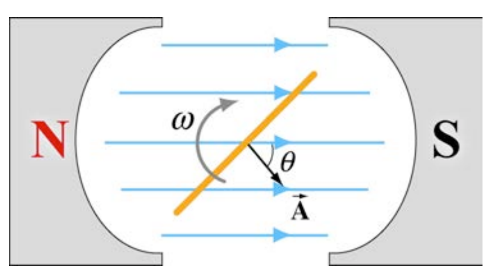


Figure 10.5.1 (a) A simple generator. (b) The rotating loop, as seen from above.

Figure 10.5.1(a) is a simple illustration of a generator. It consists of an *N*-turn loop rotating in a magnetic field that is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Figure 10.5.1(b), we see that the magnetic flux through the loop is

$$\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA\cos\theta = BA\cos\omega t\,. \tag{10.5.1}$$

The rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = -BA\omega\sin\omega t\,. \tag{10.5.2}$$

Because there are *N* turns in the loop, the total induced emf across the two ends of the loop is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = NBA\omega\sin\omega t\,. \tag{10.5.3}$$

If we connect the generator to a circuit that has resistance *R*, then the current generated in the circuit is given by

$$I = \frac{|\varepsilon|}{R} = \frac{NBA\omega}{R}\sin\omega t\,. \tag{10.5.4}$$

The current is an alternating current that oscillates in sign and has amplitude $I_0 = NBA\omega / R$. The power delivered to this circuit is

$$P = I\,|\varepsilon| = \frac{(NBA\omega)^2}{R}\sin^2\omega t\,. \tag{10.5.5}$$

The current loop acts like a magnetic dipole, with the dipole moment for the *N*-turn current loop given by

$$\mu = NIA = \frac{N^2A^2B\omega}{R}\sin\omega t\,. \tag{10.5.6}$$

The torque exerted on the loop is

$$\tau = \mu B\sin\theta = \frac{N^2A^2B^2\omega}{R}\sin^2\omega t\,. \tag{10.5.7}$$

Thus, the mechanical power supplied to rotate the loop is

$$P_m = \tau\omega = \frac{(NAB\omega)^2}{R}\sin^2\omega t\,. \tag{10.5.8}$$

As expected, the mechanical power input is equal to the electrical power output.

10.6 Eddy Currents

We have seen that when a conducting loop moves through a magnetic field, current is induced as the result of changing magnetic flux. If a solid conductor were used instead of a loop, as shown in Figure 10.6.1, current can also be induced. The induced current appears form current loops and is called an *eddy current*.

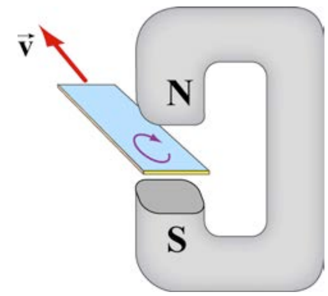


Figure 10.6.1 Appearance of an eddy current when a solid conductor moves through a magnetic field.

The induced eddy currents also generate a magnetic force that opposes the motion, making it more difficult to move the conductor across the magnetic field (Figure 10.6.2).

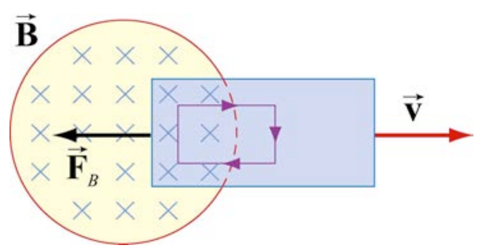


Figure 10.6.2 Magnetic force arising from the eddy current that opposes the motion of the conducting slab.

Because the conductor has non-vanishing resistance R, Joule heating causes a loss of power by an amount $P = \varepsilon^2 / R$. Therefore, by increasing the value of R, power loss can be reduced. One way to increase R is to laminate the conducting slab, or construct the slab by gluing together thin strips that are insulated from one another (see Figure 10.6.3a). Another way is to make cuts in the slab, thereby disrupting the conducting path (Figure 10.6.3b).

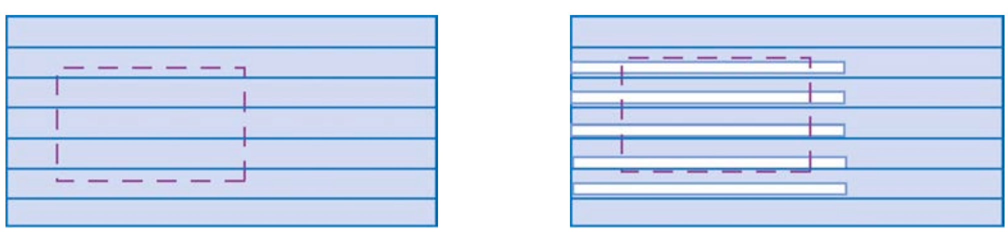


Figure 10.6.3 Eddy currents can be reduced by (a) laminating the slab, or (b) making cuts on the slab.

There are important applications of eddy currents. For example, the currents can be used to suppress unwanted mechanical oscillations. Another application is the magnetic braking systems in high-speed transit cars.

10.7 Summary

- The **magnetic flux** through a surface *S* is given by

$$\Phi_B = \iint_S \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} \, .$$

- A **motional emf** ε is induced if a conductor moves in a magnetic field. The general expression for ε is

$$\varepsilon = \oint (\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot d\,\vec{\mathbf{s}} \, .$$

  In the case of a conducting bar of length w moving with constant velocity $\vec{\mathbf{v}}$ through a magnetic field that points in the direction perpendicular to the bar and $\vec{\mathbf{v}}$, the induced emf is $\varepsilon = Bvw$.

- **Faraday's law** of induction states that changing magnetic flux is associated with a **non-electrostatic electric field** $\vec{\mathbf{E}}$ according to

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = -\frac{d\Phi_B}{dt} \, .$$

- **A generalized law of induction** states that the induced emf ε through a conducting loop is proportional to the negative of the rate of change of magnetic flux through the loop,

$$\varepsilon = -\frac{d\Phi_B}{dt} \, .$$

- **Lenz's law** states that the direction of induced currents, forces, or torques in a system is always in a direction to resist how the system changes.

10.8 Appendix: Induced Emf and Reference Frames

In Section 10.2, we have stated that the general equation of motional emf is given by

$$\varepsilon = \oint (\vec{\mathbf{v}} \times \vec{\mathbf{B}}) \cdot d\,\vec{\mathbf{s}} \, ,$$

where $\vec{\mathbf{v}}$ is the velocity of the length element $d\vec{\mathbf{s}}$ of the *moving* conductor. In addition, we have also shown in Section 10.3 that induced emf associated with a *stationary* conductor may be written as the line integral of the non-electrostatic electric field

$$\varepsilon = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} \,.$$

However, whether an object is moving or stationary actually depends on the reference frame.

As an example, let's examine the situation where a bar magnet is approaching a conducting loop (Figure 10.8.1). An observer O in the rest frame of the loop sees the bar magnet moving toward the loop (Figure 10.8.1(a)). A non-electrostatic electric field $\vec{\mathbf{E}}$ is induced to drive the current around the loop, and a charge on the loop experiences an electric force $\vec{\mathbf{F}}_e = q\vec{\mathbf{E}}$. Because the charge is at rest according to observer O, no magnetic force is present. An observer O' in the rest frame of the bar magnet sees the loop moving toward the magnet (Figure 10.8.1(b)). Because the conducting loop is moving with velocity $\vec{\mathbf{v}}$, a motional emf is induced. In this frame, O' sees the charge carrier with charge q move with velocity $\vec{\mathbf{v}}$, and concludes that the charge experiences a magnetic force $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}'$.

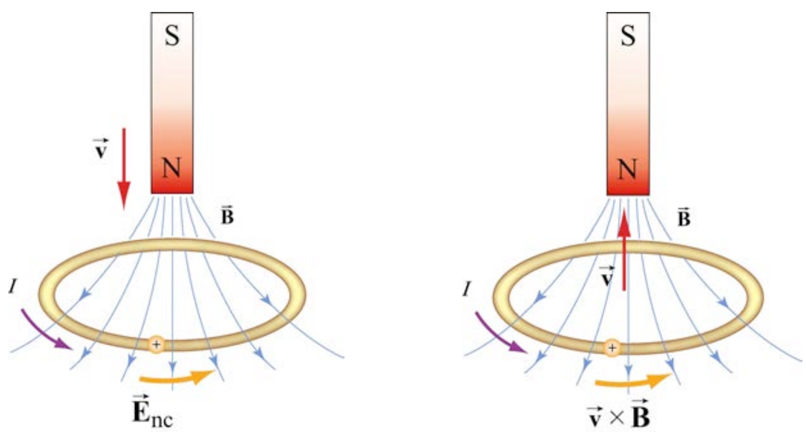


Figure 10.8.1 Induction observed in different reference frames. In (a) the bar magnet is moving, while in (b) the conducting loop is moving.

Because the two observers are moving a constant velocity relative to each other, the force acting on the charge carrier must be the same, $\vec{\mathbf{F}}_e = \vec{\mathbf{F}}_B$, which implies

$$\vec{\mathbf{E}} = \vec{\mathbf{v}} \times \vec{\mathbf{B}}' \,. \tag{10.8.1}$$

In general, as a consequence of relativity, an electric phenomenon observed in a reference frame O may appear to be a magnetic phenomenon in a frame O'. However the electric and magnetic fields in one frame are different than the electric and magnetic fields in the other reference frame.

10.9 Problem-Solving Tips: Faraday's Law and Lenz's Law

In this chapter we have seen that according to Faraday's law of induction a changing magnetic flux induces an emf

$$\varepsilon = -\frac{d\Phi_B}{dt},$$

For a conductor that forms a closed loop, the emf sets up an induced current $I = |\varepsilon|/R$, where R is the resistance of the loop. To compute the induced current and its direction, we follow the procedure outlined below.

1. For a planar closed loop of area A, define an area vector $\vec{\mathbf{A}}$. Compute the magnetic flux through the loop using

$$\Phi_B = \begin{cases} \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} & (\vec{\mathbf{B}} \text{ is uniform}) \\ \iint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} & (\vec{\mathbf{B}} \text{ is non-uniform}) \end{cases}$$

Determine the sign of Φ_B.

2. Evaluate the rate of change of magnetic flux $d\Phi_B/dt$. Keep in mind that the change could be caused by

(i) changing the magnetic field $dB/dt \neq 0$,
(ii) changing the loop area if the conductor is moving ($dA/dt \neq 0$), or
(iii) changing the orientation of the loop with respect to the magnetic field ($d\vec{\mathbf{A}}/dt \neq 0$).

Determine the sign of $d\Phi_B/dt$.

3. The sign of the induced emf is opposite of $d\Phi_B/dt$. The direction of the induced current can be found by using Lenz's law discussed in Section 10.4.

10.10 Solved Problems

10.10.1 Rectangular Loop Near a Wire

An infinite straight wire that carries a current *I,* is placed to the left of a rectangular loop of wire with width w and length *l*, as shown in the Figure 10.10.1.

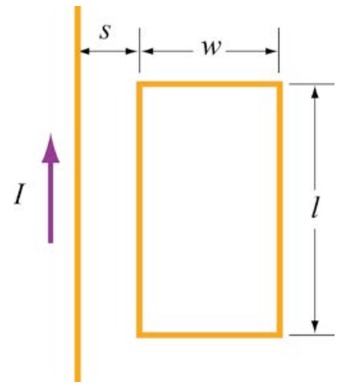


Figure 10.10.1 Rectangular loop near a wire

(a) Determine the magnetic flux through the rectangular loop due to the current *I*.

(b) Suppose that the current, $I(t) = a + bt$, is a function of time, where a and b are positive constants. What is the induced emf in the loop and the direction of the induced current?

Solutions:

(a) Using Ampere's law, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{enc}}$, the magnitude of the magnetic field due to the current-carrying wire at a distance r away is given by

$$B = \frac{\mu_0 I}{2\pi r}. \tag{10.10.1}$$

The magnetic flux Φ_B through the loop can be obtained by summing over contributions from all the differential area elements $dA = l\,dr$

$$\Phi_B = \int d\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \frac{\mu_0 I l}{2\pi} \int_s^{s+w} \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln\left(\frac{s+w}{s}\right), \tag{10.10.2}$$

where we have chosen the area vector to point *into* the page, so that $\Phi_B > 0$.

(b) According to Faraday's law, the induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 I l}{2\pi} \ln\left(\frac{s+w}{s}\right)\right] = -\frac{\mu_0 l}{2\pi} \ln\left(\frac{s+w}{s}\right) \cdot \frac{dI}{dt} = -\frac{\mu_0 b l}{2\pi} \ln\left(\frac{s+w}{s}\right), \tag{10.10.3}$$

where we have used $dI/dt = b$.

The straight wire, carrying a current I, produces a magnetic flux into the page through the rectangular loop. By Lenz's law, the induced current in the loop must be directed *counterclockwise* in order to produce a magnetic field out of the page to counteract the increase in inward flux.

10.10.2 Loop Changing Area

A square loop with length l on each side is placed in a uniform magnetic field pointing into the page. During a time interval Δt, the loop is pulled from its two edges and turned into a rhombus, as shown in the Figure 10.10.2. Assuming that the resistance of the loop is R, find the average induced current in the loop and its direction.

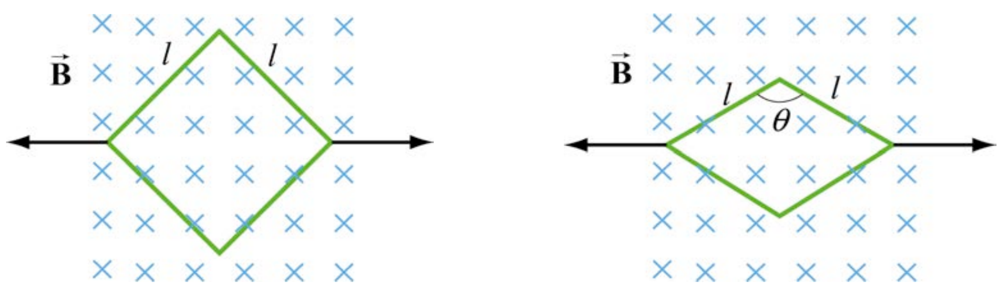


Figure 10.10.2 Conducting loop changing area

Solution: Using Faraday's law, we have that the emf is

$$\varepsilon = -\frac{\Delta \Phi_B}{\Delta t} = -B\left(\frac{\Delta A}{\Delta t}\right). \tag{10.10.4}$$

Because the initial and the final areas of the loop are $A_i = l^2$ and $A_f = l^2 \sin\theta$, respectively (recall that the area of a parallelogram defined by two vectors $\vec{l}_1$ and $\vec{l}_2$ is $A = |\vec{l}_1 \times \vec{l}_2| = l_1 l_2 \sin\theta$), the average rate of change of area is

$$\frac{\Delta A}{\Delta t} = \frac{A_f - A_i}{\Delta t} = -\frac{l^2(1-\sin\theta)}{\Delta t} < 0. \tag{10.10.5}$$

We apply Faraday's Law and find that the emf is

$$\varepsilon = \frac{Bl^2(1-\sin\theta)}{\Delta t} > 0. \tag{10.10.6}$$

The average induced current is

$$I = \frac{\varepsilon}{R} = \frac{Bl^2(1-\sin\theta)}{\Delta t R}. \tag{10.10.7}$$

Because $(\Delta A / \Delta t) < 0$, the magnetic flux into the page decreases. Hence, the current is in the clockwise direction to compensate the loss of flux.

10.10.3 Sliding Rod

A conducting bar of length l is free to slide on two parallel conducting rails as in Figure 10.10.3.

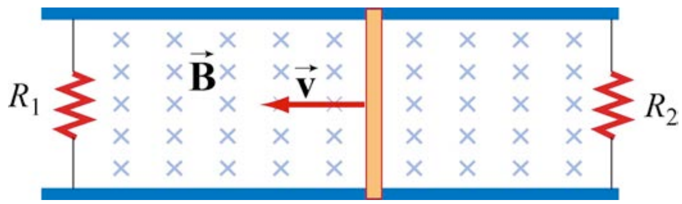


Figure 10.10.3 Sliding bar

In addition, two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field pointing into the page. Suppose an external agent pulls the bar to the left at a constant speed v. Evaluate the following quantities.

(a) The currents through both resistors;

(b) The total power delivered to the resistors;

(c) The applied force needed for the bar to maintain a constant velocity.

Solutions:

(a) The emf induced between the ends of the moving bar is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -Blv \,. \tag{10.10.8}$$

The currents through the resistors are

$$I_1 = \frac{|\varepsilon|}{R_1}, \quad I_2 = \frac{|\varepsilon|}{R_2} \quad . \tag{10.10.9}$$

We have chosen positive unit normal in the direction of the magnetic field. In the left loop, the area is decreasing and so the flux is positive and decreasing. Therefore the current I_1 is in the clockwise direction producing a positive magnetic flux (into the plane of Figure 10.10.3), opposing the positive decreasing flux due to area change.

In the right loop, the area is increasing, and so the flux is positive and increasing (into the plane of Figure 10.10.3). In order to oppose this change, I_2 is in the counterclockwise direction, producing a magnetic field pointing out of the plane of Figure 10.10.3. Hence the induced flux is negative opposing the positive increasing flux due to area change.

(b) The total power dissipated in the two resistors is

$$P_R = I_1 |\varepsilon| + I_2 |\varepsilon| = (I_1 + I_2)|\varepsilon| = \varepsilon^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = B^2 l^2 v^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right). \quad (10.10.10)$$

(c) The total current flowing through the rod is $I = I_1 + I_2$. Thus, the magnetic force acting on the rod is

$$F_B = IlB = |\varepsilon| lB \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = B^2 l^2 v \left(\frac{1}{R_1} + \frac{1}{R_2} \right), \quad (10.10.11)$$

and the direction is to the right. Thus, an external agent must apply an equal but opposite force $\vec{\mathbf{F}}_{\text{ext}} = -\vec{\mathbf{F}}_B$ to the left in order to maintain a constant speed. Therefore the mechanical power supplied by the external agent

$$P_{\text{ext}} = \vec{\mathbf{F}}_{\text{ext}} \cdot \vec{\mathbf{v}} = F_{\text{ext}} v = B^2 l^2 v^2 \left(\frac{1}{R_1} + \frac{1}{R_2} \right). \quad (10.10.12)$$

Therefore the power dissipated in the resistors is equal to the mechanical power supplied by the external agent, as we expect from energy conservation.

10.10.4 Moving Bar

A conducting rod of length l moves with a constant velocity $\vec{\mathbf{v}}$ perpendicular to an infinitely long, straight wire carrying a current I, as shown in the Figure 10.10.4. What is the emf generated between the ends of the rod?

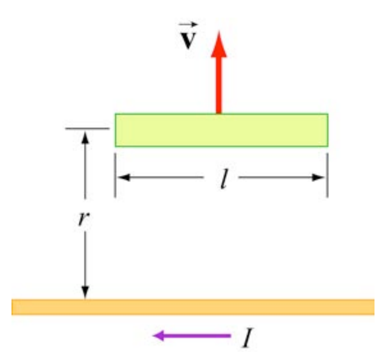


Figure 10.10.4 A bar moving away from a current-carrying wire

Solution: From Faraday's law, the motional emf is

$$|\varepsilon| = Blv \tag{10.10.13}$$

where v is the speed of the rod. However, using Ampere's law we determine that the magnetic field due to the straight current-carrying wire at a distance r away is

$$B = \frac{\mu_0 I}{2\pi r}. \tag{10.10.14}$$

Thus, the emf between the ends of the rod is given by

$$|\varepsilon| = \left(\frac{\mu_0 I}{2\pi r}\right) lv. \tag{10.10.15}$$

10.10.5 Time-Varying Magnetic Field

A circular loop of wire of radius a is placed in a uniform magnetic field, with the plane of the loop perpendicular to the direction of the field, as shown in Figure 10.10.5.

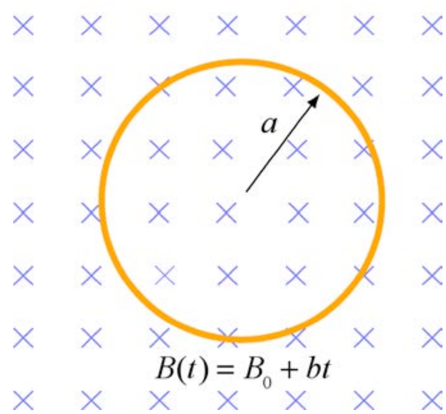


Figure 10.10.5 Circular loop in a time-varying magnetic field

The magnetic field varies with time according to $B(t) = B_0 + bt$, where B_0 and b are positive constants.

(a) Calculate the magnetic flux through the loop at $t = 0$.

(b) Calculate the induced emf in the loop.

(c) What are the induced current and its direction if the overall resistance of the loop is R?

(d) Find the power dissipated due to the resistance of the loop.

Solution:

(a) The magnetic flux at time t is given by

$$\Phi_B = BA = (B_0 + bt)(\pi a^2) = \pi(B_0 + bt)a^2 . \tag{10.10.16}$$

where we have chosen the area vector to point *into* the page, so that $\Phi_B > 0$. At $t = 0$, we have

$$\Phi_B = \pi B_0 a^2 . \tag{10.10.17}$$

(b) Using Faraday's Law, the induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -(\pi a^2)\frac{d\left(B_0 + bt\right)}{dt} = -\pi b a^2 . \tag{10.10.18}$$

(c) The induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{\pi b a^2}{R}, \tag{10.10.19}$$

and its direction is counterclockwise by Lenz's law.

(d) The power dissipated due to the resistance R is

$$P = I^2 R = \left(\frac{\pi b a^2}{R}\right)^2 R = \frac{(\pi b a^2)^2}{R} . \tag{10.10.20}$$

10.10.6 Moving Loop

A rectangular loop of dimensions l and w moves with a constant velocity $\vec{\mathbf{v}}$ away from an infinitely long straight wire that lies in the plane of the loop and is carrying a current I in the direction shown in Figure 10.10.6. Let the total resistance of the loop be R. What is the current in the loop at the instant the near side is a distance r from the wire?

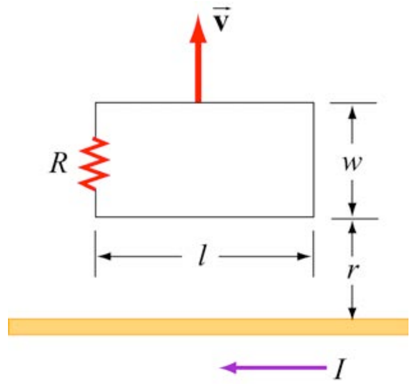


Figure 10.10.6 A rectangular loop moving away from a current-carrying wire

Solution: Using Ampere's law we find that the magnetic field at a distance r' from the straight wire is

$$B(r') = \frac{\mu_0 I}{2\pi r'}. \qquad (10.10.21)$$

The magnetic flux through a differential area element $dA = l\,dr'$ of the loop is

$$d\Phi_B = \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \frac{\mu_0 I}{2\pi r'} l\,dr', \qquad (10.10.22)$$

where we have chosen the area vector to point *into* the page, so that $\Phi_B > 0$. Integrating over the entire area of the loop, the total flux is

$$\Phi_B = \frac{\mu_0 I l}{2\pi}\int_r^{r+w}\frac{dr'}{r'} = \frac{\mu_0 I l}{2\pi}\ln\left(\frac{r+w}{r}\right). \qquad (10.10.23)$$

Differentiating with respect to t, we obtain the induced emf

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{\mu_0 I l}{2\pi}\frac{d}{dt}\left(\ln\frac{r+w}{r}\right) = -\frac{\mu_0 I l}{2\pi}\left(\frac{1}{r+w}-\frac{1}{r}\right)\frac{dr}{dt} = \frac{\mu_0 I l}{2\pi}\frac{wv}{r(r+w)}, \qquad (10.10.24)$$

where $v = dr/dt$. Notice that the induced emf can also be obtained by using Eq. (10.2.2),

$$\begin{aligned}\varepsilon &= \oint(\vec{\mathbf{v}}\times\vec{\mathbf{B}})\cdot d\vec{\mathbf{s}} = vl\left[B(r) - B(r+w)\right] = vl\left[\frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi(r+w)}\right] \\ &= \frac{\mu_0 I l}{2\pi}\frac{vw}{r(r+w)}.\end{aligned} \qquad (10.10.25)$$

The induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{\mu_0 Il}{2\pi R}\frac{vw}{r(r+w)}\,. \qquad (10.10.26)$$

10.11 Conceptual Questions

1. A bar magnet falls through a circular loop, as shown in Figure 10.11.1

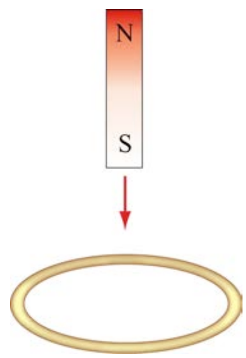


Figure 10.11.1

(a) Describe qualitatively the change in magnetic flux through the loop when the bar magnet is above and below the loop.

(b) Make a qualitative sketch of the graph of the induced current in the loop as a function of time, choosing I to be positive when its direction is counterclockwise as viewed from above.

2. Two circular loops A and B have their planes parallel to each other, as shown in Figure 10.11.2.

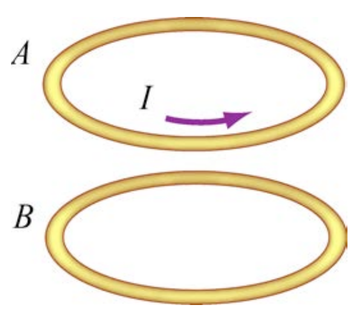


Figure 10.11.2

Loop A has a current moving in the counterclockwise direction, viewed from above.

(a) If the current in loop A decreases with time, what is the direction of the induced current in loop B? Will the two loops attract or repel each other?

(b) If the current in loop A increases with time, what is the direction of the induced current in loop B? Will the two loops attract or repel each other?

3. A spherical conducting shell is placed in a time-varying magnetic field. Is there an induced current along the equator?

4. A rectangular loop moves across a uniform magnetic field but the induced current is zero. How is this possible?

10.12 Additional Problems

10.12.1 Sliding Bar

A conducting bar of mass m and resistance R slides on two frictionless parallel rails that are separated by a distance ℓ and connected by a battery which maintains a constant emf ε, as shown in Figure 10.12.1.

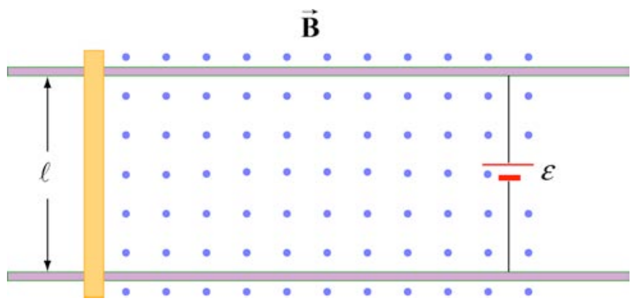


Figure 10.12.1 Sliding bar

A uniform magnetic field $\vec{\mathbf{B}}$ is directed out of the page. The bar is initially at rest. Show that at a later time t, the speed of the bar is

$$v = \frac{\varepsilon}{B\ell}(1 - e^{-t/\tau}),$$

where $\tau = mR/B^2\ell^2$.

10.12.2 Sliding Bar on Wedges

A conducting bar of mass m and resistance R slides down two frictionless conducting rails which make an angle θ with the horizontal, and are separated by a distance ℓ, as shown in Figure 10.12.2. In addition, a uniform magnetic field $\vec{\mathbf{B}}$ is applied vertically upward. The bar is released from rest and slides down.

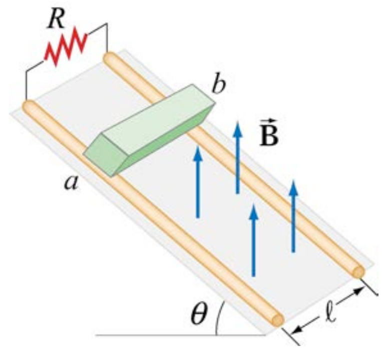


Figure 10.12.2 Sliding bar on wedges

(a) Find the induced current in the bar. Which way does the current flow, from *a* to b or *b* to *a*?

(b) Find the terminal speed v_t of the bar.

After the terminal speed has been reached,

(c) What is the induced current in the bar?

(d) What is the rate at which electrical energy has been dissipated through the resistor?

(e) What is the rate of work done by gravity on the bar?

10.12.3 RC Circuit in a Magnetic Field

Consider a circular loop of wire of radius *r* lying in the *xy* plane, as shown in Figure 10.12.3. The loop contains a resistor *R* and a capacitor *C*, and is placed in a uniform magnetic field which points into the page and decreases at a rate $dB/dt = -\alpha$, with $\alpha > 0$.

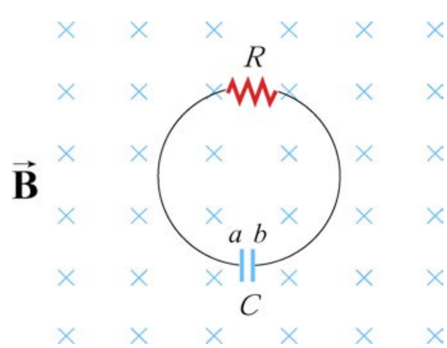


Figure 10.12.3 *RC* circuit in a changing magnetic field

(a) Find the maximum amount of charge on the capacitor.

(b) Which plate, *a* or *b*, has a higher potential? What causes charges to separate?

10.12.4 Sliding Bar

A conducting bar of mass m and resistance R is pulled in the horizontal direction across two frictionless parallel rails a distance ℓ apart by a massless string which passes over a frictionless pulley and is connected to a block of mass M, as shown in Figure 10.12.4. A uniform magnetic field $\vec{\mathbf{B}}$ is applied vertically upward. The bar is released from rest.

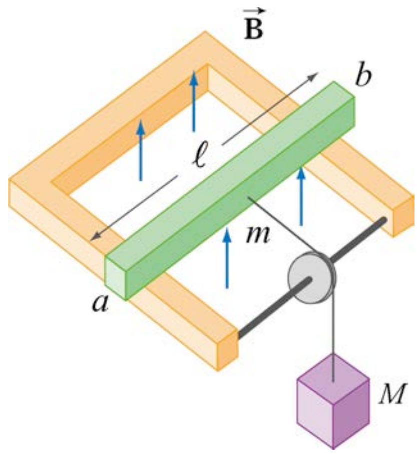


Figure 10.12.4 Sliding bar

(a) Let the speed of the bar at some instant be v. Find an expression for the induced current. Which direction does it flow, from *a* to *b* or *b* to *a*? You may ignore the friction between the bar and the rails.

(b) Solve the differential equation and find the speed of the bar as a function of time.

10.12.5 Rotating Bar

A conducting bar of length l with one end fixed, rotates at a constant angular speed ω, in a plane perpendicular to a uniform magnetic field $\vec{\mathbf{B}}$, as shown in Figure 10.12.5.

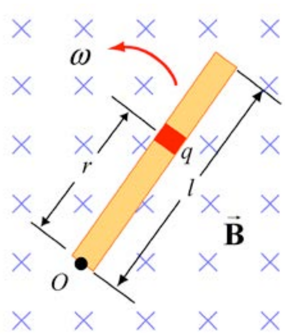


Figure 10.12.5 Rotating bar

(a) A small element carrying charge q is located at a distance r away from the pivot point O. Show that the magnetic force on the element is $F_B = qBr\omega$.

(b) Show that the potential difference between the two ends of the bar is $\Delta V = (1/2)B\omega l^2$.

10.12.6 Rectangular Loop Moving Through Magnetic Field

A small rectangular loop of length $l = 10$ cm and width $w = 8.0$ cm with resistance $R = 2.0\ \Omega$ is pulled at a constant speed $v = 2.0$ cm/s through a region of uniform magnetic field $B = 2.0$ T, pointing into the page, as shown in Figure 10.12.6.

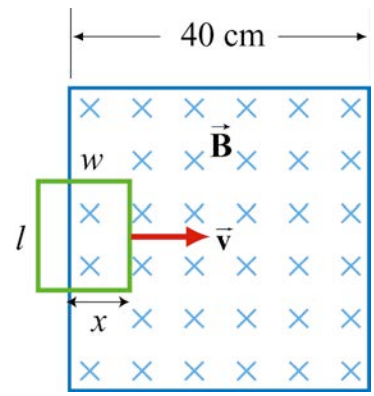


Figure 10.12.6

At $t = 0$, the front of the rectangular loop enters the region non-zero magnetic field.

(a) Find the magnetic flux and plot it as a function of time (from $t = 0$ till the loop leaves the region of magnetic field.)

(b) Find the emf and plot it as a function of time.

(c) Which way does the induced current flow?

10.12.7 Magnet Moving Through a Coil of Wire

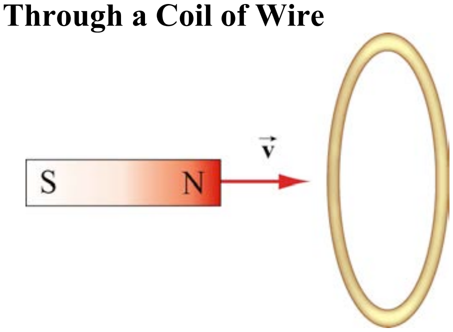


Figure 10.12.7

Suppose a bar magnet is pulled through a stationary conducting loop of wire at constant speed, as shown in Figure 10.12.7. Assume that the north pole of the magnet enters the loop of wire first, and that the center of the magnet is at the center of the loop at time $t = 0$.

(a) Sketch qualitatively a graph of the magnetic flux Φ_B through the loop as a function of time.

(b) Sketch qualitatively a graph of the current I in the loop as a function of time. Take the direction of positive current to be clockwise in the loop as viewed from the left.

(c) What is the direction of the force on the permanent magnet due to the current in the coil of wire just before the magnet enters the loop?

(d) What is the direction of the force on the magnet just after it has exited the loop?

(e) Do your answers in (c) and (d) agree with Lenz's law?

(f) Where does the energy come from that is dissipated in ohmic heating in the wire?

10.12.8 Alternating-Current Generator

An N-turn rectangular loop of length a and width b is rotated at a frequency f in a uniform magnetic field $\vec{\mathbf{B}}$ which points into the page, as shown in Figure 10.12.8. At time $t = 0$, the loop is vertical as shown in the sketch, and it rotates counterclockwise when viewed along the axis of rotation from the left.

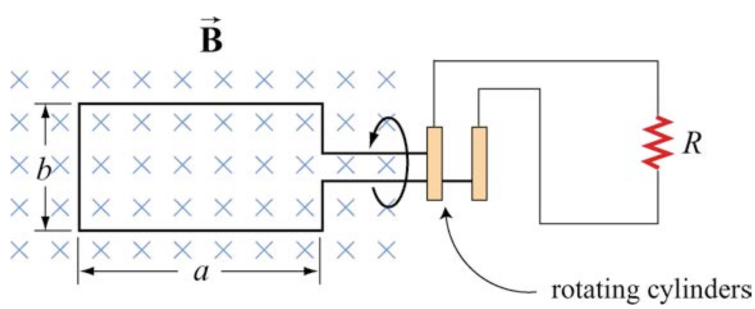


Figure 10.12.8

(a) Make a sketch depicting this "generator" as viewed from the left along the axis of rotation at a time Δt shortly after $t = 0$, when it has rotated an angle θ from the vertical. Show clearly the vector $\vec{\mathbf{B}}$, the plane of the loop, and the direction of the induced current.

(b) Write an expression for the magnetic flux Φ_B passing through the loop as a function of time for the given parameters.

(c) Show that an induced emf ε appears in the loop, given by

$$\varepsilon = 2\pi fNbaB\sin(2\pi ft) = \varepsilon_0 \sin(2\pi ft) .$$

(d) Design a loop that will produce an emf with $\varepsilon_0 =$ 120 V when rotated at 60 Hz in a magnetic field of 0.40 T.

10.12.9 EMF Due to a Time-Varying Magnetic Field

A uniform magnetic field $\vec{\mathbf{B}}$ is perpendicular to a one-turn circular loop of wire of negligible resistance, as shown in Figure 10.12.9. The field changes with time as shown (the positive z-direction is out of the page). The loop has radius r = 50 cm and is connected in series with a resistor of resistance R = 20 Ω. The "+" direction around the circuit is indicated in the figure.

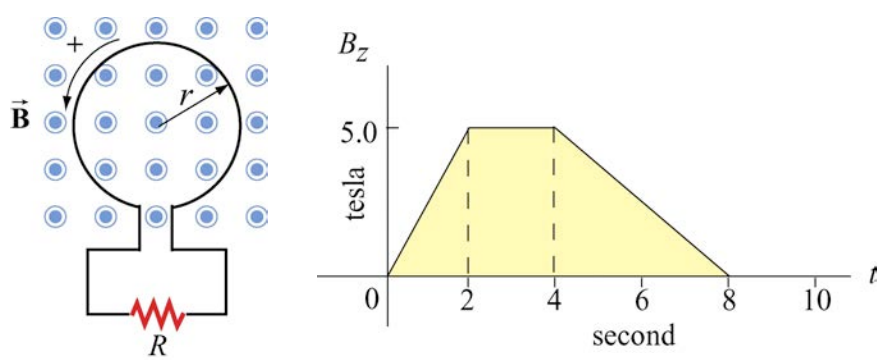


Figure 10.12.9

(a) What is the expression for emf in this circuit in terms of $B_z(t)$ for this arrangement?

(b) Plot the emf in the circuit as a function of time. Label the axes quantitatively (numbers and units). Watch the signs. Note that we have labeled the positive direction of the emf in the left sketch consistent with the assumption that positive $\vec{\mathbf{B}}$ is out of the paper.

(c) Plot the current I through the resistor R. Label the axes quantitatively (numbers and units). Indicate with arrows on the sketch the direction of the current through R during each time interval. [Partial Ans. Values of current are 98 mA, 0.0 mA, 49 mA].

(d) Plot the rate of thermal energy production in the resistor. Square Loop Moving Through Magnetic Field

An external force is applied to move a square loop of dimension $l \times l$ and resistance R at a constant speed across a region of uniform magnetic field. The sides of the square loop make an angle $\theta = 45°$ with the boundary of the field region, as shown in Figure 10.12.10. At $t = 0$, the loop is completely inside the field region, with its right edge at the boundary. Calculate the power delivered by the external force as a function of time.

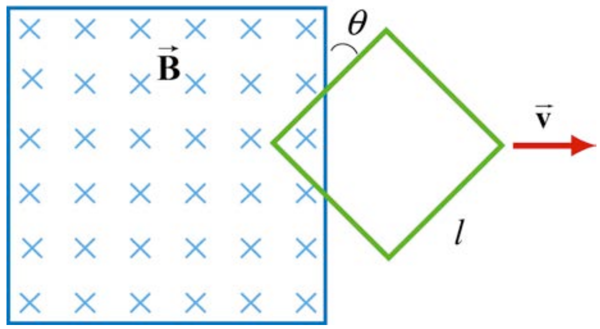


Figure 10.12.10

10.12.10 Falling Loop

A rectangular loop of wire with mass m, width w, vertical length l, and resistance R falls out of a magnetic field under the influence of gravity, as shown in Figure 10.12.11. The magnetic field is uniform and out of plane of the figure ($\vec{\mathbf{B}} = B\hat{\mathbf{i}}$) within the area shown and zero outside of that area. At the time shown in the sketch, the loop is exiting the magnetic field at speed $\vec{\mathbf{v}} = -v\hat{\mathbf{k}}$.

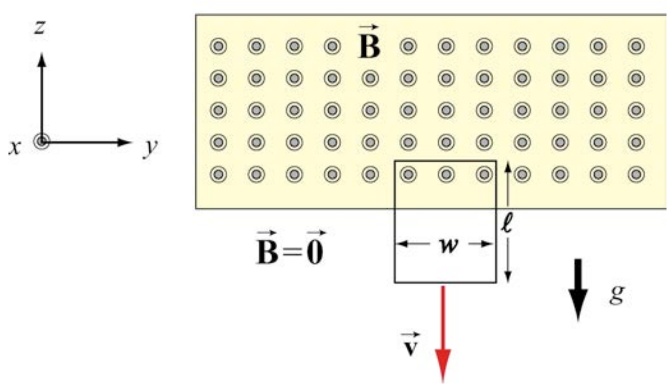


Figure 10.12.11

(a) What is the direction of the current flowing in the circuit at the time shown, clockwise or counterclockwise? Why did you pick this direction?

(b) Using Faraday's law, find an expression for the magnitude of the emf in this circuit in terms of the quantities given. What is the magnitude of the current flowing in the circuit at the time shown?

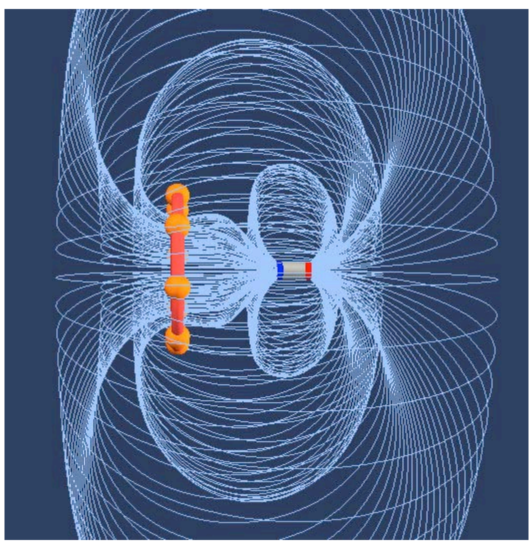
(c) Besides gravity, what other force acts on the loop in the $\pm\hat{\mathbf{k}}$ direction? Give its magnitude and direction in terms of the quantities given.

(d) Assume that the loop has reached a "terminal velocity" and is no longer accelerating. What is the magnitude of that terminal velocity in terms of given quantities?

(e) Show that at terminal velocity, the rate at which gravity is doing work on the loop is equal to the rate at which energy is being dissipated in the loop through Joule heating.

10.13 Faraday's Law Simulation

In this section we explore the meaning of Faraday's Law using a 3D simulation that creates in which you can move a magnet in the presence of a conducting ring or vice versa.



Link to simulation

Figure 9.14.1 Screen Shot of Faraday's Law Simulation

This simulation illustrates the electromagnetic interaction between a conducting non-magnetic ring and a magnet, both constrained on a horizontal axis. If the "Motion on" box unchecked, you can move the magnet or the ring (by left clicking and dragging on either), and the changing magnetic flux through the ring gives rise to a current which is in a direction such as to oppose the change in flux, as described by Lenz's Law. You can vary the resistance of the ring and the strength of the magnetic dipole moment to see how these parameters affect the resulting field, flux, and current. If the resistance is zero, the change in induced flux in the ring will exactly counter the change in external flux due to the magnet, thus keeping the total flux constant. Increasing the resistance hinders the flow of induced current, resulting in a delay in the response of the ring to the change in external flux, and a corresponding change in total flux. This can be seen in the flux graph as you manipulate the objects. There are two graphs to the right of the simulation window. The top graph shows two different measures of the magnetic flux through the ring, and the bottom graph shows the current induced in the ring.

If the "Motion on" box is checked, the magnet is automatically moved to simulate a simple AC (Alternating Current) generator. The physics is unchanged, but the position of the magnet oscillates sinusoidally, giving rise to a near-sinusoidal current in the ring.