CHAPTER

Fundamentals of electric motors

1

A moving object that has either a linear motion or rotary motion is powered by a prime mover. A prime mover is an equipment that produces mechanical power by using thermal power, electricity, hydraulic power, steam, gas, etc. Examples of the prime mover include a gas turbine, an internal combustion engine, and an electric motor. Among these, the electric motors have recently become one of the most important prime movers and their use is increasing rapidly. Nearly 70% of all the electricity used in the current industry is used to produce electric power in the motor-driven system [1].

Electric motors can be classified into two different kinds according to the type of the power source used as shown in Fig. 1.1: direct current (DC) motor and alternating current (AC) motor. The recently developed *brushless DC motor* is hard to be classified as either one of the motors since its configuration is similar to that of a permanent magnet synchronous motor (AC motor), while its electrical characteristics are similar to those of a DC motor.

The first electric motor built was inspired by Michael Faraday's discovery of electromagnetic induction. In 1831, Michael Faraday and Joseph Henry simultaneously succeeded in laboratory experiments in operating the motor for the first time. Later in 1834, M. Jacobi invented the first practical DC motor. The DC motor is the prototype of all motors from the viewpoint of torque production. In 1888, Nikola Tesla was granted a patent for his invention of AC motors, which include a synchronous motor, a reluctance motor, and an induction motor. By 1895, the three-phase power source, distributed stator winding, and the squirrel-cage rotor had been developed sequentially. Through these developments, the three-phase induction motors were finally made available for commercial use in 1896 [2].

Traditionally among the developed motors, DC motors have been widely used for speed and position control applications because of the ease of their torque control and excellent drive performance. On the other hand, induction motors have been widely used for a general purpose in constant-speed applications because of their low cost and rugged construction. Induction motors account for about 80% of all the electricity consumed by motors.

Until the early 1970s, major improvement efforts were made mainly toward reducing the cost, size, and weight of the motors. The improvement in magnetic material, insulation material, design and manufacturing technology has played a

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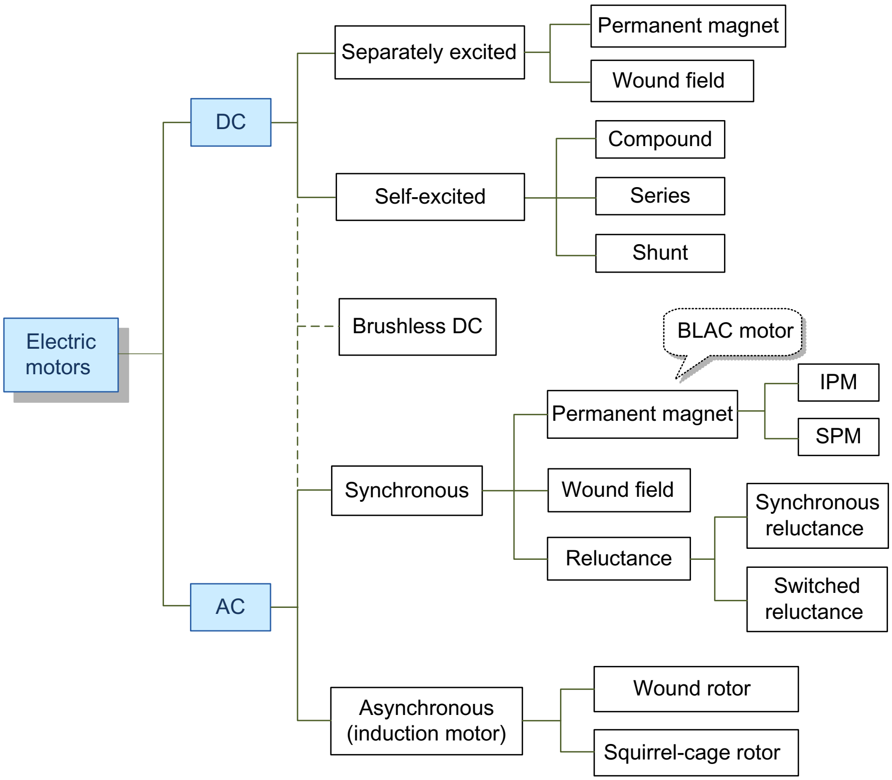


FIGURE 1.1

Classification of electric motors: AC motor and DC motor.

major role and made a big progress. As a result, a modern 100-hp motor is the same size as a 7.5-hp motor used in 1897. With the rising cost of oil price due to the oil crisis in 1973, saving the energy costs has become an especially important matter. Since then, major efforts have been made toward improving the efficiency of the motors. Recently, rapidly increasing energy costs and a strong global interest in reducing carbon dioxide emissions have been encouraging industries to pay more attention to high-efficiency motors and their drive systems [1].

Along with the improvement of motors, there have been many advances in their drive technology. In the 1960s, the advent of power electronic converters using power semiconductor devices enabled the making of motors with operation characteristics tailored to specific system applications. Moreover, using microcontrollers with high-performance digital signal processing features allowed the engineers to apply advanced control techniques to motors, greatly increasing the performance of motor-driven systems.

1.1 FUNDAMENTAL OPERATING PRINCIPLE OF ELECTRIC MOTORS

1.1.1 CONFIGURATION OF ELECTRIC MOTORS

An electric motor is composed of two main parts: a stationary part called the *stator* and a moving part called the *rotor* as shown in Fig. 1.2. The air gap between the stator and the rotor is needed to allow the rotor to spin, and the length of the air gap can vary depending on the kind of motors.

The stator and the rotor part each has both an electric and a magnetic circuit. The stator and the rotor are constructed with an iron core as shown in Fig. 1.3, through which the magnetic flux created by the winding currents will flow and which plays a role of supporting the conductors of windings. The current-carrying

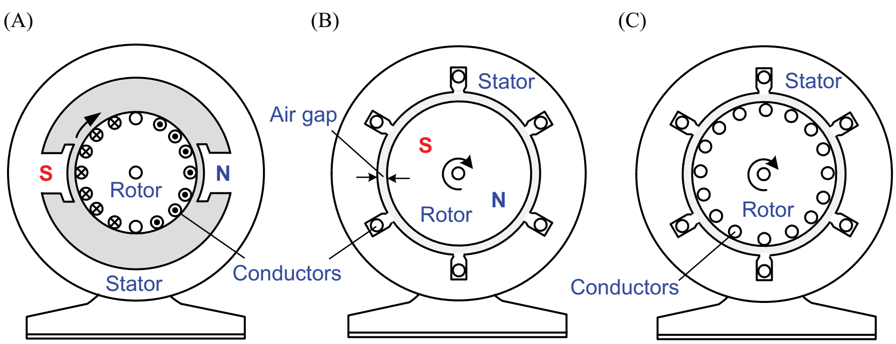


FIGURE 1.2

Configuration of electric motors. (A) DC motor, (B) AC synchronous motor, and (C) AC induction motor.

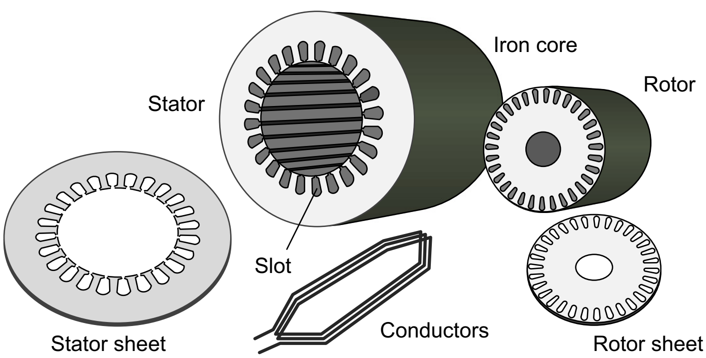


FIGURE 1.3

Electric and magnetic parts of electric motors.

conductors inserted into slots in the iron core form the electric circuit. When the current flows in these conductors, a magnetic field is created through the iron core, and the stator and the rotor each becomes an electromagnet.

To obtain a greater magnetic flux for a given current in the conductors, the iron core is usually made up of ferromagnetic material with high magnetic permeability, such as silicon steel. In some cases, the stator or the rotor creates a magnetic flux by using a permanent magnet.

1.1.2 BASIC OPERATING PRINCIPLE OF ELECTRIC MOTORS

All electric motors are understood to be rotating based on the same operating principle. As shown in Fig. 1.4A, there are generally two magnetic fields formed inside the motors. One of them is developed on the stationary stator and the other one on the rotating rotor. These magnetic fields are generated through either energized windings, use of permanent magnets, or induced currents. A force produced by the interaction between these two magnetic fields gives rise to a torque on the rotor and causes the rotor to turn. On the other hand, some motors, such as the reluctance motor, use the interaction between one magnet field and a magnetic material, such as iron, but they cannot produce a large torque (Fig. 1.4B). Most motors in commercial use today including DC, induction, and synchronous motors exploit the force produced through the interaction between two magnetic fields to produce a larger torque.

The torque developed in the motor must be produced continuously to function as a motor driving a mechanical load. Two motor types categorized according to the used power source, i.e., DC motor and AC motor, have different ways of achieving a continuous rotation. Now, we will take a closer look at these methods for achieving a continuous rotation.

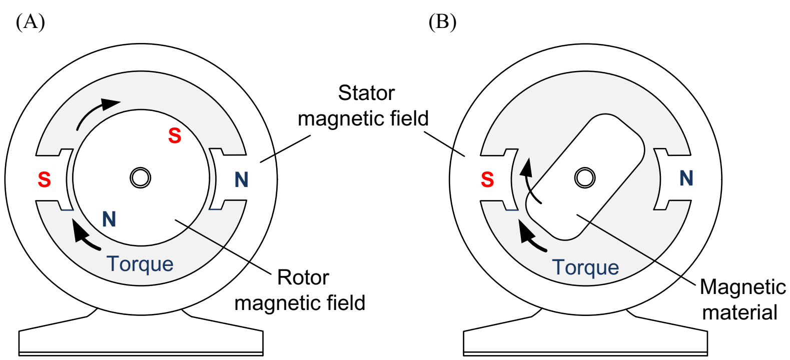


FIGURE 1.4

Rotation of electric motors. (A) Two magnetic fields and (B) one magnetic field and magnetic material.

1.1.2.1 Direct current motor

The simple concept for the rotation of a DC motor is that the rotor rotates by using the force produced on the current-carrying conductors placed in the magnetic field created by the stator as shown in Fig. 1.5A. Alternatively, we can consider the operating principle of a DC motor from the viewpoint of two magnetic fields as shown in Fig. 1.4A as follows.

There are two stationary magnetic fields in a DC motor as shown in Fig. 1.5B. One stationary magnetic field is the stator magnetic field produced by magnets or a field winding. The other is the rotor magnetic field produced by the current in the conductors of the rotor. It is important to note that the rotor magnetic field is also stationary despite the rotation of the rotor. This is due to the action of brushes and commutators, by which the current distribution in the rotor conductors is always made the same regardless of the rotor's rotation as shown in Fig. 1.5A. Thus the rotor magnetic field will not rotate along with the rotor. A consistent interaction between these two stationary magnetic fields produces a torque, which causes the rotor to turn continuously. We will study the DC motor in more detail in Chapter 2.

1.1.2.2 Alternating current motor

Unlike DC motors that rotate due to the force between two stationary magnetic fields, AC motors exploit the force between *two rotating magnetic fields*. In AC motors both the stator magnetic field and the rotor magnetic field rotate, as shown in Fig. 1.6.

As it will be described in more detail in Chapter 3, these two magnetic fields always rotate at the same speed and, thus, are at a standstill relative to each other and maintain a specific angle. As a result, a constant force is produced between them, making the AC motor is to run continually. The operating principle of the

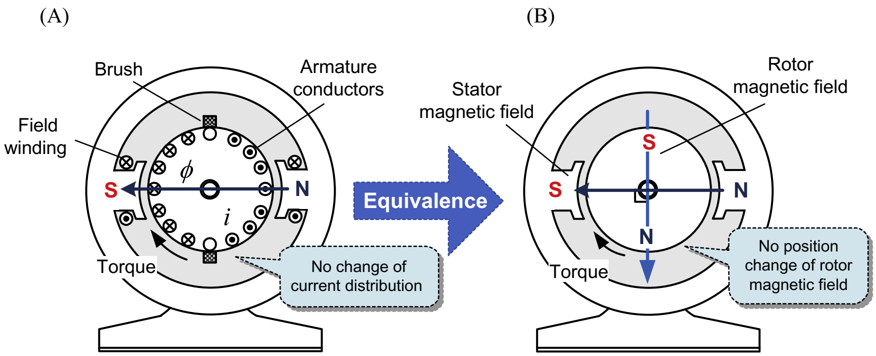


FIGURE 1.5

Operating principle of the DC motor. (A) Rotor current and stator magnetic field and (B) two magnetic fields.

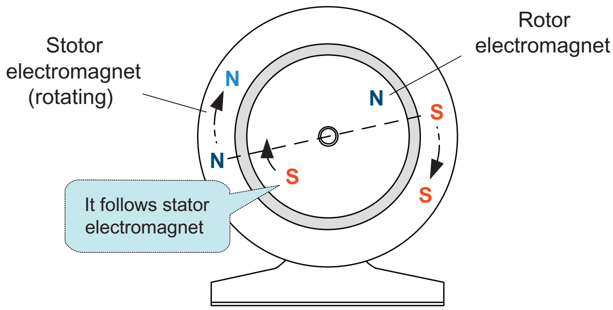


FIGURE 1.6

Operating principle of the AC motor.

AC motor is that the force produced by the interaction between the two rotating magnetic fields causes the rotor to turn.

In AC motors, the rotating magnetic field on the stator is created by three-phase currents. When a three-phase AC power source is applied to the three-phase stator windings of the AC motor, the three-phase currents flowing in these windings create a rotating magnetic field. We will examine the rotating magnetic field in more detail in Chapter 3.

There are two kinds of AC motors: a *synchronous motor* and an *induction (asynchronous) motor*. They generate the rotor magnetic field differently, whereas they generate the stator magnetic field in the same way. In a synchronous motor as depicted in Fig. 1.2B, the magnetic field on the rotor is generated either by a permanent magnet or by a field winding powered by a DC power supply separated from the stator AC power source. In this motor, the rotor magnetic field is stationary relative to the rotor. Hence, to produce a torque, the rotor should rotate at the same speed as the stator rotating magnetic field. This speed is called the *synchronous speed*. This is why this motor is referred to as the *synchronous motor*.

On the other hand, in an induction motor as shown in Fig. 1.2C, the rotor magnetic field is generated by the AC power. The AC power used in the rotor excitation is transferred from the stator by electromagnetic induction. Because of this crucial feature, this motor is referred to as the *induction motor*. In an induction motor, the rotor magnetic field rotates relative to the rotor at some speed. To produce a torque, the stator and the rotor rotating magnetic fields should rotate at the same speed. This requires that the rotor itself rotate at the speed difference between the stator and rotor rotating magnetic fields. More precisely, the rotor rotating magnetic field rotates at the speed difference between the stator rotating magnetic field and the rotor. To use the rotor excitation by electromagnetic induction, the rotor speed should always be less than the synchronous speed. Thus the induction motor is also called the *asynchronous motor*.

Among the motors, DC motors have largely been used for speed and torque control because of their simplicity. Their simplicity comes from the fact that the speed of a DC motor is proportional to the voltage, and its torque is proportional to the current. However, since DC motors require periodic maintenance of the brushes and commutators, the trend has recently moved toward employing maintenance-free AC motors as they can offer high performance at a reasonable price.

As mentioned earlier, electric motors can operate on the fundamental principle that the torque produced from the interaction between the magnetic fields generated in the stator and the rotor causes the motor to run. Now, we will take a look at the requirements that ensure continuous torque production by the motor.

1.2 REQUIREMENTS FOR CONTINUOUS TORQUE PRODUCTION [3]

An electric motor is a type of electromechanical energy conversion device that converts electric energy into mechanical energy. The principle of torque production in a motor can be understood by analyzing its energy conversion process. Electromechanical energy conversion devices normally use the magnetic field as an intermediate in their energy conversion processes. Thus an electromechanical energy conversion device is composed of three different parts: the *electric system*, *magnetic system*, and *mechanical system* as shown in Fig. 1.7.

Now, we will examine the mechanism of torque production of a motor by looking at how the force (or torque) is being produced inside the electromechanical energy conversion device.

To evaluate the force (or torque) produced inside the energy conversion devices, we will apply the law of conservation of energy: “within an isolated system, energy can be converted from one kind to another or transferred from one place to another, but it can neither be created nor destroyed.” Therefore the total amount of energy is constant. Fig. 1.8 shows an application of the law of conservation of energy when the electromechanical energy conversion device is acting as a motor.

Suppose that electric energy dW_e is supplied to the energy conversion device during the differential time interval dt. The supplied electric energy dW_e is converted to field energy dW_f in the magnetic system, and to mechanical energy dW_e in the mechanical system.

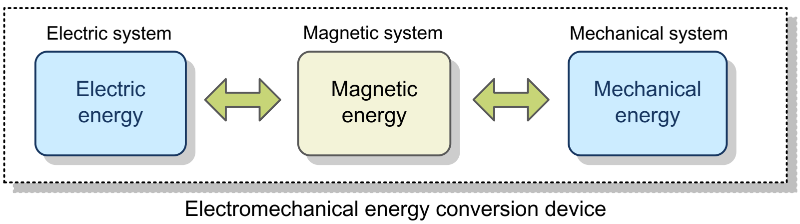


FIGURE 1.7

Electromechanical energy conversion.

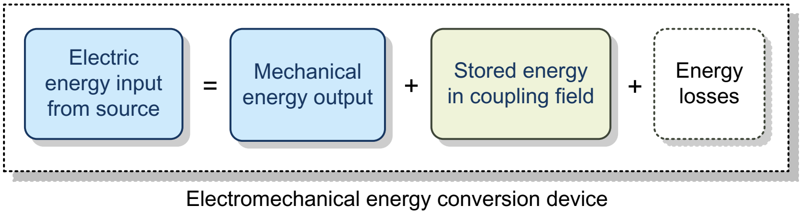


FIGURE 1.8

Equation of energy conservation for a motor.

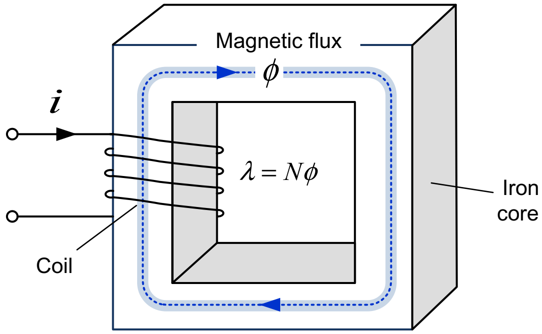


FIGURE 1.9

Coil wound on an iron core.

In addition a small amount of electric energy dW_e is converted into losses in each of the three parts: copper losses in an electric system, core losses such as hysteresis and eddy current losses in a magnetic system, and mechanical losses such as friction and windage losses. Since these losses may be dissipated in the form of heat or noise, we can leave out these losses in the process of evaluating the force or torque production in the energy conversion process. Therefore the energy conversion equation in Fig. 1.8 may be written as

$$dW_e = dW_f + dW_m \tag{1.1}$$

1.2.1 MAGNETIC ENERGY

In electric machines, including a motor, the magnetic flux is normally produced by the coil wound on an iron core as shown in Fig. 1.9. Because ferromagnetic materials such as an iron core have high permeability, much greater magnetic flux can be developed in the iron core for a given current than when the magnetic flux is developed in the air. Now, we will take a look at magnetic energy W_f stored in the magnetic system, where magnetic flux ϕ is developed in the iron core as shown in Fig. 1.9.

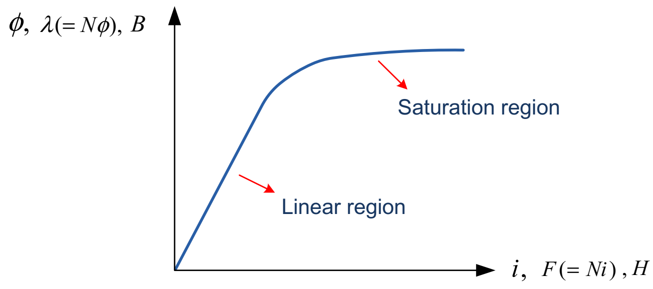


FIGURE 1.10

Flux–current characteristic.

First, we need to know the relationship between the magnetic flux ϕ produced in the core and the coil current i in this magnetic system. Fig. 1.10 demonstrates this relationship, which is the *B-H* characteristic called *magnetization curve*. In the beginning of the curve, the magnetic flux ϕ increases rapidly in proportion to the increase in the coil current i until it reaches a certain value. The region where the magnetic flux is linearly related to the applied current like this is called the *linear region* or *unsaturated region*.

However, above that certain value, further increases in the current produce relatively smaller increases in the magnetic flux. Eventually an increase in the current will produce almost no change at all in the magnetic flux. The region where the curve flattens out is called the *saturation region*, and the iron core is said to be saturated. When the saturation occurs, any further increase in the current will have little or no effect on the increase of the flux. In electric machines, the magnetic flux is designed to be produced as much as possible because their torque is directly proportional to the magnetic flux. Thus most electric machines normally operate near the knee of the magnetization curve, which is the transition range between the two regions.

Electric machines produce the magnetic flux by using coils, which normally consist of many turns. Thus, instead of the magnetic flux ϕ, the concept of flux linkage λ as a product of the number of turns N and the magnetic flux ϕ linking each turn is introduced. The flux linkage of N-turn coil can be given by

$$\lambda = N\phi \quad \text{(Wb-turns)} \tag{1.2}$$

The flux linkage λ can also be related to the coil current i by the definition of inductance L through the following relation.

$$L = \frac{\lambda}{i} = N\frac{\phi}{i} \quad \text{(H or Wb/A)} \tag{1.3}$$

A larger inductance value of the magnetic system implies that the same current can produce a larger flux linkage. Eq. (1.3) shows that the slope of the curve in Fig. 1.10 is the inductance of this magnetic system as shown in Fig. 1.11.

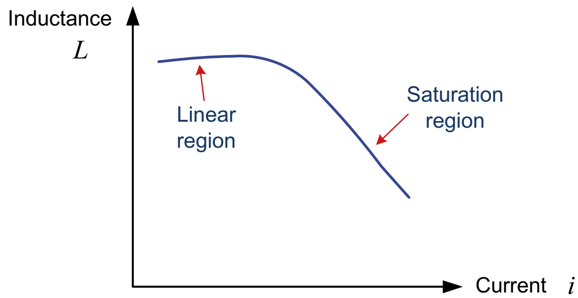


FIGURE 1.11

Inductance of the coil.

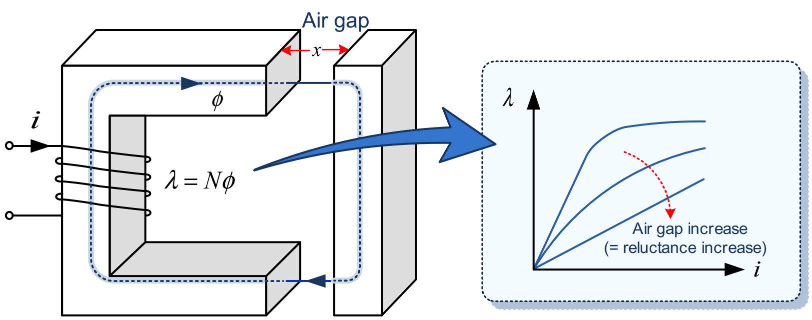


FIGURE 1.12

Flux–current characteristic according to the variation of an air-gap length.

The inductance is large and relatively constant in the unsaturated region but gradually drops to a very low value as the core becomes heavily saturated.

Now, we will take a look at magnetic energy W_f stored in the magnetic system, where magnetic flux ϕ is developed in the iron core as shown in Fig. 1.12.

The $\lambda - i$ characteristic of the magnetic system varies with the air-gap length x. Thus the inductance $L(x)$ of the coil is expressed as a function of the air-gap length x. As the air-gap length x increases, the slope of the $\lambda - i$ curve becomes smaller and results in smaller inductance as shown in Fig. 1.12.

The inductance L of the magnetic system is inversely related to its *reluctance* $\mathfrak{R}$, which is defined as the ratio of *magnetomotive force* (mmf) $F(=Ni)$ to magnetic flux ϕ, as in the following relation.

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{N^2}{\mathfrak{R}} \propto \frac{1}{\mathfrak{R}} \quad (F = \mathfrak{R}\phi = Ni) \tag{1.4}$$

Now, assume that the source voltage v is applied to the terminals of the N-turn coil on the iron core during dt as shown in Fig. 1.13. Then, the current i

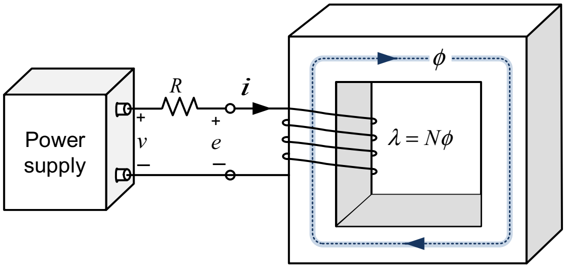


FIGURE 1.13

Magnetic system connected to source voltage.

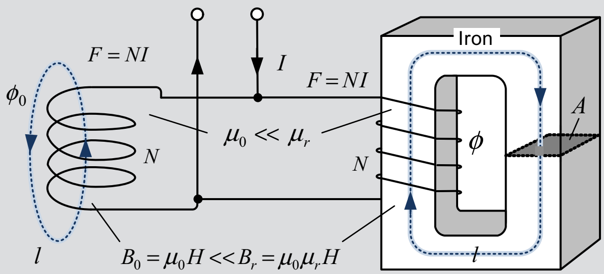
will flow in that coil. The resulting mmf $F(=Ni)$ will produce a magnetic flux ϕ in the core and establish flux linkage $\lambda(=N\phi)$ in the coil. Now, we will discuss the energy stored in the magnetic system.

MAGNETOMOTIVE FORCE (MMF) AND MAGNETIC FLUX

Magnetic flux ϕ is indispensable in producing torque in motors. The force that is capable of producing a magnetic flux is called magnetomotive force or mmf F. In general, mmf is generated by current, so the source of magnetic flux is the current. When the current I is flowing through an N-turn coil, the mmf F is expressed by NI (A-turn), i.e., $F = NI$. For a given mmf, the magnitude of the developed flux depends on the length of the closed-path through which the flux passes. The longer the length, the lower the flux density will be. Thus the magnetic field intensity H, which represents mmf per length, will be used instead of the mmf.

$$mmf = F = NI = \oint H dl \rightarrow H = \frac{mmf}{l} = \frac{NI}{l} \quad (\text{A/m})$$

Also, for a given magnetic field intensity, the flux density will be different depending on the material through which the flux passes. For example, high flux density can be created in iron or steel but not in air gap as in the following figure.



This magnetic characteristic can be described in terms of relative permeability μ_r in reference to permeability μ_0 of free space. Thus the magnetic flux density B produced by the current can be represented as

$$B = \mu_0 \mu_r H \quad (\text{Wb/m}^2)$$

(*Continued*)

MAGNETOMOTIVE FORCE (MMF) AND MAGNETIC FLUX (CONTINUED)

The total magnetic flux flowing through the cross section A of a magnetic circuit in an uniform magnetic flux density B can be expressed as

$$\phi = \int_A B \cdot ndA = BA\cos\theta \quad \text{(Wb)}$$

where θ is the angle between the magnetic field lines and the surface A. Similar to the relation between the current and the voltage by Ohm's law in the electric circuit, there is a following relationship between the mmf F and the flux ϕ in the magnetic circuit.

$$F = \Re\phi$$

where $\Re$ is called reluctance and is inversely proportional to permeability.

In the system of Fig. 1.13, the electric energy dW_e supplied during the differential time interval is expressed from input power $P(= vi)$ as

$$dW_e = Pdt = vidt \tag{1.5}$$

The supplied electric energy after subtracting the copper loss produced by current in the coil is stored as the magnetic field energy. i.e.

$$dW_f = (vi - Ri^2)dt = (vi - Ri)dt = eidt \tag{1.6}$$

where e is back-EMF voltage induced in the coil.

Since the back-EMF voltage e on the N-turn coil is proportional to the rate of change of flux linkage λ with time, i.e., $e = d\lambda/dt$, Eq. (1.5) can be given by

$$dW_f = eidt = id\lambda \tag{1.7}$$

When the flux linkage is increased from zero to λ, the total energy stored in the magnetic system is

$$dW_f = \int_0^\lambda id\lambda \tag{1.8}$$

This magnetic energy is the area to the left of the $\lambda - i$ curve as shown in Fig. 1.14. On the other hand, the area to the right of $\lambda - i$ curve is known as *coenergy* W_f', which is a useful quantity for the calculation of force and can be expressed as

$$W_f' = \int_0^\lambda \lambda di \tag{1.9}$$

For the linear system in which the $\lambda - i$ curve is a straight line, the magnetic energy is equal to the coenergy. The magnetic system that has an air gap in the flux path becomes a linear system.

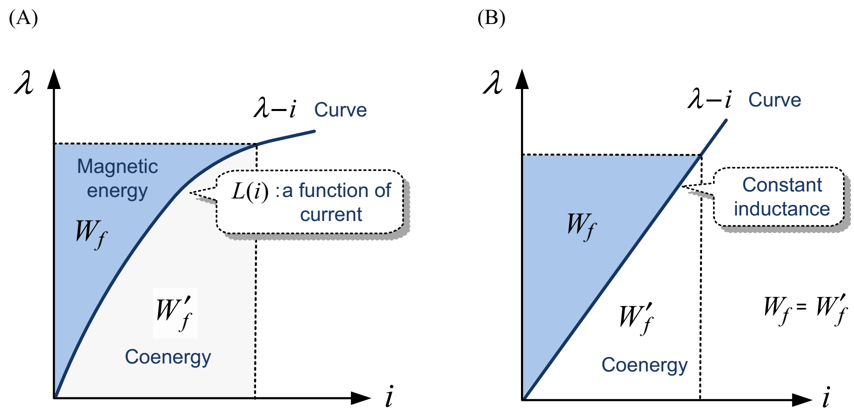


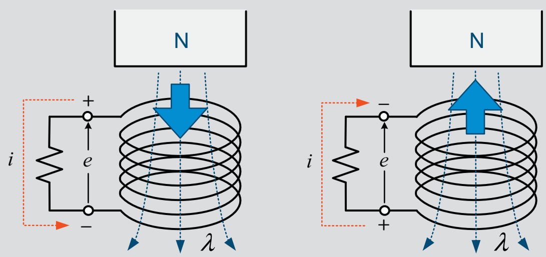
FIGURE 1.14

Magnetic energy in the flux–current characteristic. (A) Linear system and (B) nonlinear system.

BACK-ELECTROMOTIVE FORCE (BACK-EMF), OR INDUCED VOLTAGE *e*

Faraday's law states that the voltage e induced in the turn of a coil is directly proportional to the rate of change in the flux ϕ passing through that turn with respect to time. If there are N turns in the coil and the same flux passes through each turn of the coil, then the total voltage induced on the coil is given by

$$e = N\frac{d\phi}{dt} = \frac{d\lambda}{dt}$$



According to Lenz's law, the direction of the induced voltage opposes the change of the flux that causes it, and thus a minus sign is included in the equation above.

1.2.2 LINEAR MOTION DEVICE

Now, we will look at the force developed inside a movable energy conversion device. Before looking into rotating machines, we will first study the linear motion device, which consists of a fixed part (also called *stator*) and a moving part as shown in Fig. 1.15.

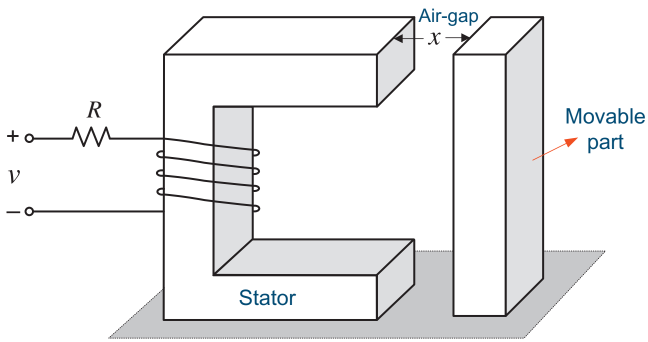


FIGURE 1.15

Linear motion device.

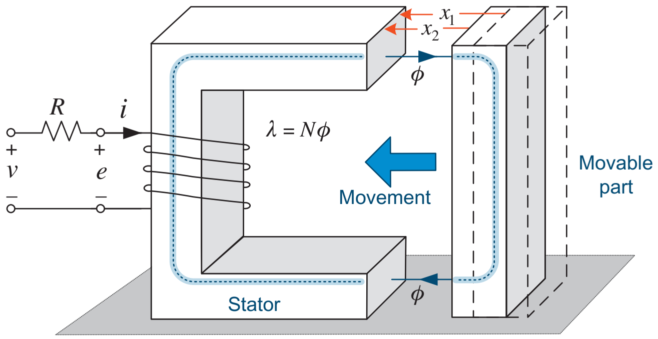


FIGURE 1.16

Movement of the movable part.

If a magnetic flux is produced by the current flowing through the coil of N turns in this device, then, as seen in everyday experiences, we can easily know that the movable part will move toward the stator as shown in Fig. 1.16. Now, we will discuss the force that causes this motion.

The current I of the coil is assumed to remain the same before and after the movable part moves. Suppose that the movable part moves from position x_1 to position x_2. The length of the air gap is changed as a result of the movement. This also changes the $\lambda - i$ characteristic of the system as shown in Fig. 1.17. The air gap decreases according to the movement, resulting in a decrease of reluctance so that the flux linkage increases from λ_1 to λ_2. The operating point will move from A to B. The trajectory of the movement depends on the moving speed. So, we will find the force acting on the movable part for the two extreme moving speeds.

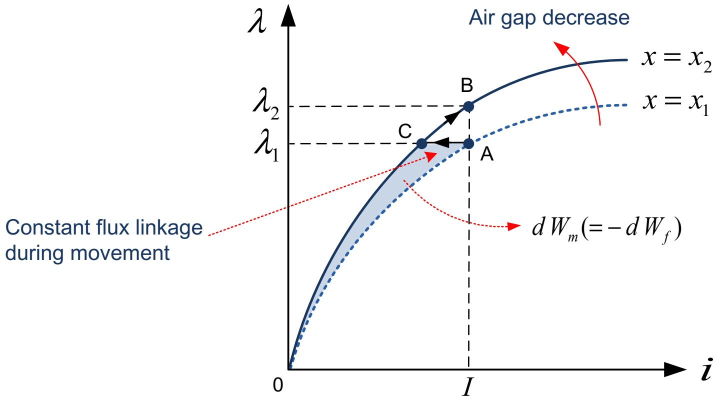


FIGURE 1.17

Flux linkage–current characteristic (rapid movement of the movable part).

First, suppose that the movement occurs very rapidly. In this case, the movement is so rapid that it may be completed before any significant change has occurred in flux linkage λ. Thus, during the movement, the flux linkage λ_1 remains constant so that the operating point moves from A to C as shown in Fig. 1.17.

After the completion of the movement, the flux linkage will be changed from λ_1 to λ_2 and the operating point will move from C to B. During the movement, the flux linkage does not change and this results in zero back-EMF, so the electrical energy input will be zero, i.e. $dW_e = eidt = id\lambda = 0$. Therefore Eq. (1.1) can be written as

$$dW_m = -dW_f \tag{1.10}$$

This equation indicates that the mechanical energy needed for the movement is supplied entirely by the magnetic energy. The amount of the reduced magnetic energy for the movement corresponds to the shaded area OAC of the $\lambda - i$ characteristic.

Considering the force to be defined as the mechanical work done dW_m per displacement dx, the developed force during this movement from Eq. (1.10) is

$$f_m = \frac{dW_m}{dx} = -\left.\frac{dW_f(i,x)}{dx}\right|_{\lambda=\text{constant}} \tag{1.11}$$

This implies that the force acts in the direction at which the magnetic energy of the system is decreasing.

Now, we will express the force of Eq. (1.11) in terms of the coil current. Assume for simplicity that the $\lambda - i$ characteristic is linear. The magnetic energy of Eq. (1.8) can be expressed as

$$W_f = \int i d\lambda = \int \frac{\lambda}{L(x)} d\lambda = \frac{\lambda^2}{2L(x)} = \frac{1}{2}L(x)i^2 \tag{1.12}$$

By substituting Eq. (1.12) into the force of Eq. (1.11), we obtain

$$\begin{aligned} f_m &= -\left.\frac{dW_f(i,x)}{dx}\right|_{\lambda=\text{constant}} = -\frac{d}{dx}\left(\frac{\lambda^2}{2L(x)}\right)\Bigg|_{\lambda=\text{constant}} \\ &= \frac{\lambda^2}{2L^2(x)}\frac{dL(x)}{dx} = \frac{1}{2}i^2\frac{dL(x)}{dx} \end{aligned} \tag{1.13}$$

Eq. (1.18) indicates that *the force acts in the direction that increases the inductance of the magnetic system*. Since the direction that increases the inductance is the direction that decreases the length of the air gap, the force is exerted on the movable part moving toward the stator.

From this, we can explain the attractive force between an electromagnet (or magnet) and magnetic materials. When an electromagnet and a magnetic material cling to each other without an air gap, the system has the greatest inductance. Thus an attractive force between the two objects is developed to move them toward each other.

Next, suppose that the movable part moves very slowly. In this case, the current $i=(v-e)/R$ remains constant during the movement. This is because the back-EMF $e(=d\lambda/dt)$ is negligibly small. Therefore the operating point on the $\lambda-i$ characteristic moves upward from A to B as shown in Fig. 1.18.

During this movement the change in the electric energy is given by

$$dW_e = eidt = id\lambda = i(\lambda_2 - \lambda_1) \tag{1.14}$$

This corresponds to the area *ABEF*. In this case, the amount of change in the stored magnetic energy from Eq. (1.8) is given by

$$dW_f = \int_0^{\lambda_2} id\lambda - \int_0^{\lambda_1} id\lambda = \text{Area } OBE - \text{Area } OAF \tag{1.15}$$

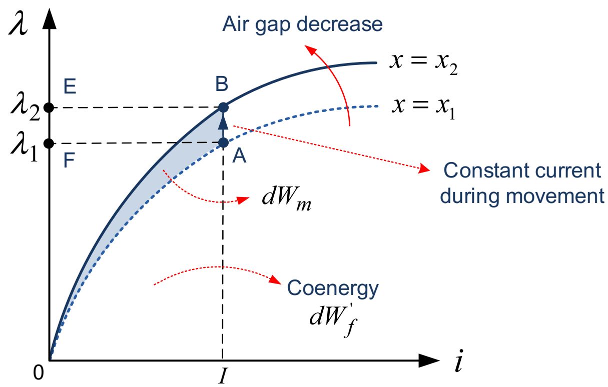


FIGURE 1.18

Flux linkage—current characteristic (very slow movement of the movable part).

Therefore, from Eq. (1.1), the mechanical energy required for the movement is

$$dW_m = dW_e - dW_f \tag{1.16}$$

By substituting Eqs. (1.14) and (1.15) into Eq. (1.16), it can be readily seen that the mechanical energy corresponds to the shaded area *OAB* of the $\lambda - i$ characteristic and is equal to the increments of coenergy dW'_f. Consequently, the used mechanical energy for the movement is equal to the increments of coenergy dW'_f during the movement as shown in the following.

$$dW_m = dW'_f \tag{1.17}$$

Therefore the developed force during this movement is

$$f_m = \frac{dW_m}{dx} = \left.\frac{dW'_f(i,x)}{dx}\right|_{i=\text{constant}} \tag{1.18}$$

This force acts in the direction of the increasing coenergy of the system.

The force for a slow movement is expressed in the same way as for a rapid movement. For a linear system $dW_f = dW'_f$, by substituting Eq. (1.12) into (1.18), we obtain

$$\begin{aligned} f_m &= \left.\frac{dW'_f(i,x)}{dx}\right|_{i=\text{constant}} = \left.\frac{dW_f(i,x)}{dx}\right|_{i=\text{constant}} \\ &= \left.\frac{d}{dx}\left(\frac{1}{2}L(x)i^2\right)\right|_{i=\text{constant}} = \frac{1}{2}i^2\frac{dL(x)}{dx} \end{aligned} \tag{1.19}$$

This is the same conclusion as was reached in Eq. (1.13), meaning that the force is the same regardless of the moving speed.

This force can also be expressed in terms of the *reluctance* $\mathfrak{R}$. The magnetic energy of Eq. (1.8) expressed as a function of reluctance $\mathfrak{R}$ is

$$W_f = \int_0^\lambda i\,d\lambda = \int_0^\phi F\,d\phi = \int_0^\phi \mathfrak{R}\phi\,d\phi = \frac{1}{2}\mathfrak{R}(x)\phi^2 \tag{1.20}$$

From Eq. (1.20), the force of Eq. (1.11) is

$$f_m = -\left.\frac{d}{dx}\left(\frac{1}{2}\mathfrak{R}(x)\phi^2\right)\right|_{\phi=\text{constant}} = -\frac{1}{2}\phi^2\frac{d\mathfrak{R}(x)}{dx} \tag{1.21}$$

This means that *the force acts in the direction that decreases the reluctance of the magnetic system*. Since the decrease in the air-gap length causes the reluctance to decrease, the force is exerted on the movable part moving toward the stator. Next, we will discuss the torque production in rotating machines.

1.2.3 ROTATING MACHINE

We will first start examining the machine in Fig. 1.19, which produces a rotational motion. The rotating machine consists of a fixed part (called *stator*) and a moving part (called *rotor*).

The rotor is mounted on a shaft and is free to rotate between the poles of the stator.

The force that causes a rotor to rotate can be expressed as torque, which is the mechanical work done per rotational distance or angle θ as follows:

$$T = \frac{dW_m}{d\theta} \tag{1.22}$$

In the machine shown in Fig. 1.18, when current i_s flows in the stator coil, a magnetic flux is produced, and the developed torque from Eqs. (1.12) and (1.22) acting on the rotor can be expressed as

$$T = \frac{1}{2} i_s^2 \frac{dL(\theta)}{d\theta} = -\frac{1}{2} \phi^2 \frac{d\Re(\theta)}{d\theta} \tag{1.23}$$

This torque results from the variation of reluctance (or inductance) with rotor position, and thus is called the *reluctance torque*. A machine using this torque is known as the *reluctance motor*. For motors of a cylindrical rotor configuration, since the reluctance does not vary with the rotor position, the reluctance torque cannot be produced.

Now, we will take a look at the necessary condition for ensuring that this machine continuously rotates to serve as a motor. First of all, it is not hard to consider that, if there is a DC current flowing in the coil, this machine never operates as a continuously rotating motor. Therefore we will consider the case in which the winding is excited from an AC current $i_s (= I_m \cos\omega_s t)$. In this case, the torque of Eq. (1.23) can be expressed in terms of the stator current i_s and the stator self-inductance L_{ss} as

$$T = \frac{1}{2} i_s^2 \frac{dL_{ss}(\theta)}{d\theta} \tag{1.24}$$

where the inductance $L_{ss} (= \lambda_s / i_s)$ is defined as the ratio of the total flux linkage λ_s in the stator coil to the stator current i_s generating the flux.

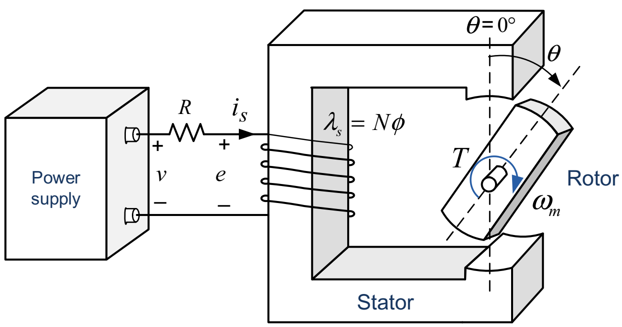


FIGURE 1.19

Rotating machine.

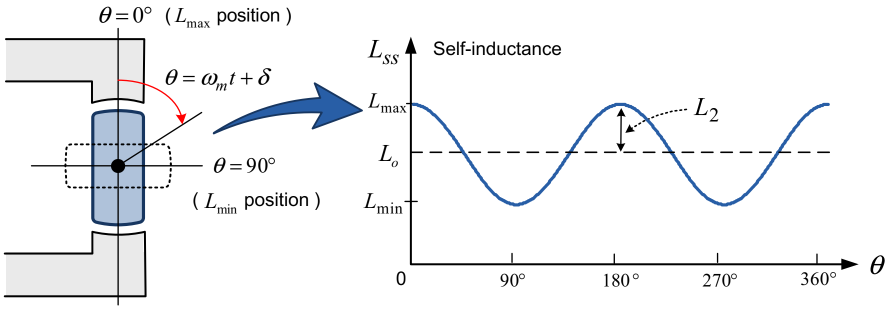


FIGURE 1.20

Self-inductance L_{ss} profile with respect to the rotor position.

The self-inductance L_{ss} in the machine of Fig. 1.19 varies with the angular position θ of the rotor as shown in Fig. 1.20.

There are two cycles of inductance during one revolution of a rotor. When $\theta = 0°$, the air gap is the smallest and the inductance L_{ss} becomes the maximum value $L_{\max}$. When $\theta = 90°$, the air gap is the largest and the inductance becomes the minimum value $L_{\min}$. In this way, the stator self-inductance L_{ss} varies with the rotor position θ in a sinusoidal manner and can be expressed as

$$L_{ss}(\theta) = L_0 + L_2 \cos 2\theta \tag{1.25}$$

where $L_0 = (L_{max} + L_{min})/2, \quad L_2 = (L_{max} - L_{min})/2$.

Substitution of Eq. (1.25) into Eq. (1.24) yields

$$T = \frac{1}{2} i_s^2 \frac{dL_{ss}(\theta)}{d\theta} = -I_m^2 L_2 \sin 2\theta \cos^2 \omega_s t \tag{1.26}$$

If the rotor is rotating at a constant angular velocity ω_m, then the angular position θ of the rotor can be given by

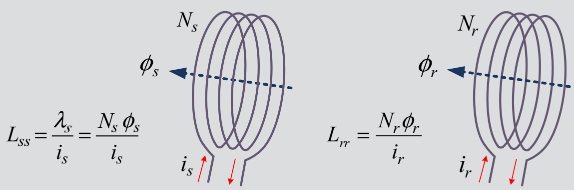
$$\theta = \omega_m t + \delta \tag{1.27}$$

where δ is the initial angular position of the rotor.

SELF-INDUCTANCE AND MUTUAL-INDUCTANCE

The inductance L of a coil is defined as the flux linkage per ampere of current in the coil.

- Self-inductance: Self-inductance is the flux linkage produced in the winding by the current in that same winding divided by that current.

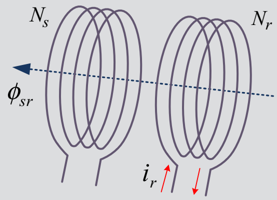


(Continued)

SELF-INDUCTANCE AND MUTUAL-INDUCTANCE (CONTINUED)

- Mutual-inductance: Mutual-inductance is the flux linkage produced in one winding by the current in the other winding divided by that current.

$$L_{sr} = \frac{N_s \phi_{sr}}{i_r}, \qquad L_{rs} = \frac{N_r \phi_{rs}}{i_s}$$



Also, we consider ω_m to be positive when the position θ is increasing (clockwise motion). Substitution of Eq. (1.27) into Eq. (1.26) yields

$$\begin{aligned} T &= -I_m^2 L_2 \sin 2(\omega_m t + \delta)\frac{1 + \cos 2\omega_s t}{2} \\ &= -\frac{1}{2} I_m^2 \left\{ L_2 \sin 2(\omega_m t + \delta) + \frac{1}{2}\sin 2([\omega_m + \omega_s]t + \delta) + \frac{1}{2}\sin 2([\omega_m - \omega_s]t + \delta \right\} \end{aligned} \tag{1.28}$$

The three sin terms on the right side of Eq. (1.28) are functions of time t whose average value is zero. The average value of this torque is, therefore, zero.

This means that a constant torque cannot be developed in this machine for any direction. Therefore this machine cannot operate as a continuously rotating motor. However, if $\omega_m = 0$ or $\omega_m = \pm\omega_s$, then a nonzero average torque can be produced. The case of $\omega_m = 0$ will be excluded since this indicates nonrotating. Consequently the requirement for a continuous rotation is $\omega_m = \omega_s$. This requires that the rotor must rotate at the speed equal to the angular frequency ω_s of the exciting current, which is also called *synchronous speed*. In this case this machine can operate as a continuously rotating motor and its average torque becomes

$$T_{avg} = -\frac{1}{4} I_m^2 L_2 \sin 2\delta = -\frac{1}{8} I_m^2 (L_{max} - L_{min}) \sin 2\delta \tag{1.29}$$

The average developed torque depends on the inductance difference $(L_{max} - L_{min})$ and the initial rotor position δ. The maximum torque occurs when $\delta = -45^\circ$. Since the torque in this motor caused by the variation of reluctance (or inductance) with rotor position can be developed only at the synchronous speed, this motor is called *synchronous reluctance motor*.

Next, we will examine the necessary condition for a continuous rotation of the *doubly fed machine* in which both the stator and the rotor have windings carrying currents as shown in Fig. 1.21. Typical electric motors such as a DC motor, an induction motor, and a synchronous motor belong to the doubly fed machine.

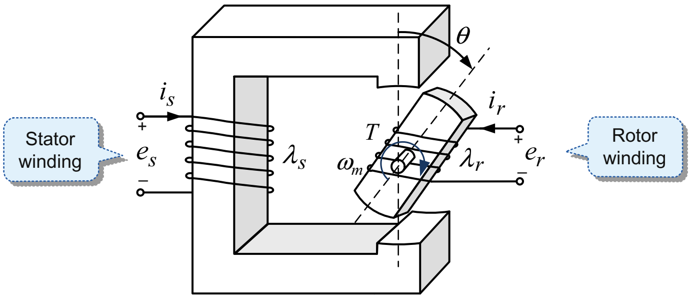


FIGURE 1.21

Doubly fed machine.

Modified from G.R. Slemon, A. Straughen, Electric Machines, Addison-Wesley, 1980 (Chapter 3); P.C. Sen, Principles of Electric Machines and Power Electronics, John Wiley & Sons, 1996.

Assume that this machine is a linear system, in which the magnetic energy and the coenergy are the same, i.e., $W_f = W_f'$. Similar to the force of Eq. (1.19), the torque of this machine can be obtained from

$$T = \left.\frac{dW_f'(i,\theta)}{d\theta}\right|_{i=\text{constant}} = \left.\frac{dW_f(i,\theta)}{d\theta}\right|_{i=\text{constant}} \tag{1.30}$$

To evaluate this torque, we need to obtain the magnetic energy W_f in this machine. When the magnetic field is established by winding currents i_s and i_r, the stored magnetic energy W_f can be easily derived by locking the rotor at an arbitrary position, which makes it produce no mechanical output. If there is no mechanical output, then the magnetic field energy increment during dt is

$$dW_f = dW_e = e_s i_s dt + e_r i_r dt = i_s d\lambda_s + i_r d\lambda_r \tag{1.31}$$

Here, the stator flux linkage λ_s consists of λ_{ss} produced by the stator current i_s and λ_{sr} produced by the rotor current i_r. This stator flux linkage can be expressed as functions of currents and inductances of windings as

$$\lambda_s = \lambda_{ss} + \lambda_{sr} = L_{ss} i_s + L_{sr} i_r \tag{1.32}$$

where $L_{ss}(=\lambda_{ss}/i_s)$ is the self-inductance of the stator winding, and $L_{sr}(=\lambda_{sr}/i_r)$ is the mutual-inductance between the stator and rotor windings.

The flux linkage λ_r in the rotor winding also consists of λ_{rr} produced by the rotor current i_r and λ_{rs} produced by the stator current i_s. This rotor flux linkage λ_r is expressed by

$$\lambda_r = \lambda_{rr} + \lambda_{rs} = L_{rr} i_r + L_{rs} i_s \tag{1.33}$$

where $L_{rr}(=\lambda_{rr}/i_r)$ is the self-inductance of the rotor winding, and $L_{rs}(=\lambda_{rs}/i_s)$ is the mutual-inductance between the stator and rotor windings. For a linear magnetic system, $L_{sr} = L_{rs}$.

In the doubly fed machine in Fig. 1.21, both self-inductances L_{ss}, L_{rr} and mutual-inductances L_{sr}, L_{rs} vary with the rotor position θ. For example, when $\theta = 0°$, at which the length of the air gap is the minimum, the reluctance of the system is the minimum and the inductance becomes the largest. Thus λ_s and λ_r are functions of rotor position θ.

Substitution of Eqs. (1.32) and (1.33) into Eq. (1.31) yields

$$\begin{aligned} dW_f &= i_s d(L_{ss} i_s + i_r dL_{sr} i_r) + i_r d(L_{sr} i_s + i_r dL_{rr} i_r) \\ &= L_{ss} i_s di_s + L_{rr} i_r di_r + L_{sr} d(i_s i_r) \end{aligned} \tag{1.34}$$

Assuming that the rotor current is increased from zero to i_r after the stator current is increased from zero to i_s, the total stored magnetic energy will be

$$\begin{aligned} W_f &= L_{ss} \int_o^{i_s} i_s di_s + L_{rr} \int_0^{i_r} i_r di_r + L_{sr} \int_0^{i_s, i_r} d(i_s i_r) \\ &= \frac{1}{2} L_{ss} i_s^2 + \frac{1}{2} L_{rr} i_r^2 + L_{sr} i_s i_r \end{aligned} \tag{1.35}$$

By substituting Eq. (1.35) into Eq. (1.30), the torque is expressed by

$$\begin{aligned} T &= -\left.\frac{dW_f(i,\theta)}{d\theta}\right|_{i=\text{constant}} \\ &= \frac{1}{2} i_s^2 \frac{dL_{ss}}{d\theta} + \frac{1}{2} i_r^2 \frac{dL_{rr}}{d\theta} + i_s i_r \frac{dL_{sr}}{d\theta} \end{aligned} \tag{1.36}$$

This torque expression is derived in the condition in which the rotor is locked to produce no mechanical output. Even if the rotor is rotating, we can obtain the same torque expression [3].

In Eq. (1.36), the first two terms on the right side represent reluctance torques involving a change of self-inductances according to the rotor position θ. The last term involving a change of mutual-inductance expresses the electromagnetic torque formed by the interaction of two fields produced by the stator and rotor currents i_s and i_r. Typical rotating motors such as a DC motor, an induction motor, and a synchronous motor are based on this torque.

In the previous section, we saw that the reluctance motor must run at the speed equal to the angular frequency of the exciting current for developing an average torque. In the doubly fed machine, if the stator and the rotor currents differ in their operating frequency, then the motor will need to be synchronized at two different speeds for producing an average torque. Only one of the two can be satisfied at a time, and the other torque term will be oscillatory. This will result in an unwanted speed oscillation. However, we can solve this problem by eliminating one or both of the reluctance torque terms in Eq. (1.36).

First, we will eliminate the reluctance torque term involving a change in the stator self-inductance. We can realize this by making the rotor configuration cylindrical as shown in Fig. 1.22.

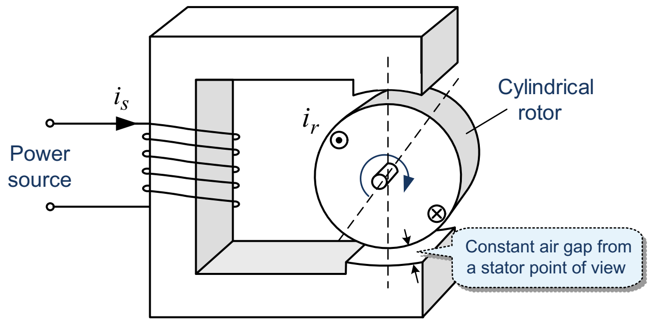


FIGURE 1.22

Cylindrical rotor configuration.

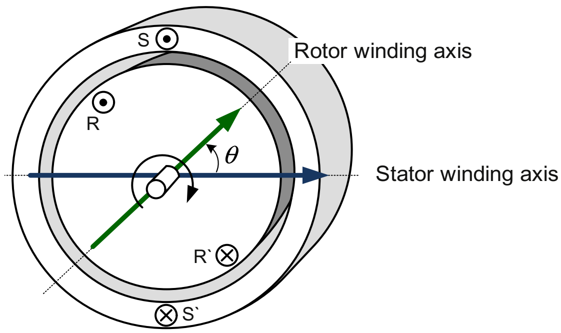


FIGURE 1.23

Cylindrical stator and rotor configurations.

In the cylindrical rotor configuration, the stator self-inductance L_{ss} becomes constant since the magnetic path becomes constant for the flux produced by the stator winding regardless of the rotor position. Thus

$$\frac{dL_{ss}}{d\theta} = 0 \tag{1.37}$$

Therefore the reluctance torque term due to the stator excitation will disappear from Eq. (1.36).

Furthermore, if we make the stator into a hollow cylinder coaxial with the rotor as shown in Fig. 1.23, then the rotor self-inductance L_{rr} becomes constant regardless of the rotor position. Thus

$$\frac{dL_{rr}}{d\theta} = 0 \tag{1.38}$$

This will lead to a disappearance of the second reluctance torque term due to rotor excitation.

We can see that no reluctance torque production results in a cylindrical machine that has both the stator and the rotor in the cylindrical configuration. This cylindrical machine produces only the torque involving a change of mutual-inductance between the stator and the rotor windings as given by

$$T = i_s i_r \frac{dL_{sr}}{d\theta} \tag{1.39}$$

Even in the cylindrical machine, this mutual-inductance can be varied by changing the relative positions of the stator and the rotor windings according to the rotor position.

Now, we will look at the necessary condition for the cylindrical machine to produce a torque continuously. Suppose that the currents in the stator and the rotor windings are

$$i_s = I_{sm}\cos\,\omega_s t \tag{1.40}$$

$$i_r = I_{rm}\cos\,(\omega_r t + \alpha) \tag{1.41}$$

If the axes of the two windings are aligned, i.e., when $\theta = 0°$ as in Fig. 1.23, then the linking flux of the two windings will become a maximum, which will make the mutual-inductance its maximum as well. Thus the mutual-inductance according to the position of the rotor can be expressed as

$$L_{sr} = L_m\cos\,\theta \tag{1.42}$$

where L_m is the maximum mutual-inductance and θ is the angle between the stator and the rotor winding axes. The position of the rotor at instant t with the angular velocity ω_m of the rotor is

$$\theta = \omega_m t + \delta \tag{1.43}$$

where δ is the initial rotor position at $t = 0$.

Substitution of Eqs. (1.40), (1.41) and (1.42) into Eq. (1.39) yields

$$\begin{aligned}
T &= i_s i_r \frac{dL_{sr}}{d\theta} \\
&= -I_{sm}I_{rm}L_m\cos\,\omega_s t\cdot\cos\,(\omega_r t+\alpha)\cdot\sin\,(\omega_m t+\delta) \\
&= -\frac{I_{sm}I_{rm}L_m}{4}\left[\sin\left\{(\omega_m+(\omega_s+\omega_r))t+\alpha+\delta\right\}\right. \\
&\quad +\sin\left\{(\omega_m-(\omega_s+\omega_r))t+\alpha+\delta\right\} \\
&\quad +\sin\left\{(\omega_m+(\omega_s-\omega_r))t+\alpha+\delta\right\} \\
&\quad \left.+\sin\left\{(\omega_m-(\omega_s-\omega_r))t+\alpha+\delta\right\}\right]
\end{aligned} \tag{1.44}$$

Since averaging over one period of the sinusoidal function is zero, the average torque of Eq. (1.44) will also be zero. However, an average torque can be developed if the coefficient of t in sinusoidal terms of Eq. (1.44) is zero. This needs to satisfy the following condition.

$$\boxed{|\omega_m| = |\omega_s \pm \omega_r| \qquad (1.45)}$$

Typical electric motors such as a DC motor, an induction motor, and a synchronous motor meet this condition in different ways, and thus, can rotate continuously.

Now, we will take a look at how each of these three motors meets this requirement for the average torque production. First, we will neglect the case in which both the stator and the rotor windings are excited by the DC ($\omega_s = \omega_r = 0$), because Eq. (1.45) results in $\omega_m = 0$, which indicates the state of nonrotating.

1.2.3.1 Direct current motor

If the stator is excited with a DC power ($\omega_s = 0$) and the rotor is excited with an AC power of an angular frequency ω_r, then the necessary condition of Eq. (1.45) for the development of an average torque is

$$|\omega_m| = |\omega_r| \tag{1.46}$$

This implies that the rotor needs to rotate at the same frequency as the frequency ω_r of the rotor current. A DC motor as shown in Fig. 1.24A is the one that satisfies this condition. In the DC motor, by the action of the commutator and brush, a rotor current becomes an AC, whose angular frequency is naturally equal to the rotor speed. Thus, the DC motor has a special mechanical structure, which allows it to always meet the condition of Eq. (1.46). From Eqs. (1.44) and (1.46), the average torque in this motor is

$$T_{avg} = -\frac{I_{sm} I_{rm}}{2} L_m \sin\delta \tag{1.47}$$

In the DC motor, δ, which indicates the angle between the stator field flux and the rotor mmf, is always 90° electric angle structurally, so the maximum torque per ampere can be obtained as

$$T_{avg} = -\frac{L_m}{2} I_{sm} I_{rm} \tag{1.48}$$

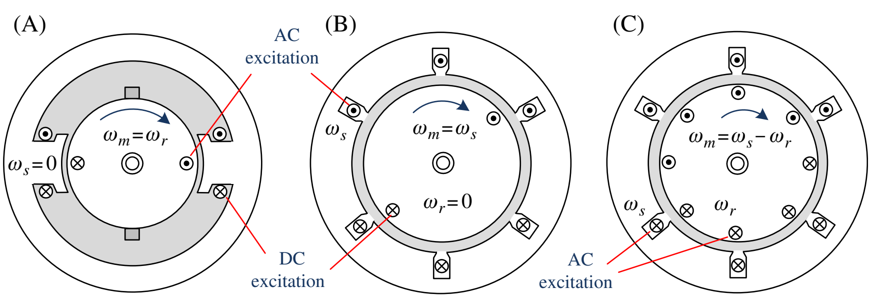


FIGURE 1.24

Doubly fed motors. (A) DC motor, (B) synchronous motor, and (C) induction motor.

1.2.3.2 Synchronous motor

If the stator is excited with an AC power source of an angular frequency ω_s and the rotor is excited with a DC power as shown in Fig. 1.24B, then Eq. (1.45) can only be satisfied if

$$|\omega_m| = |\omega_s| \tag{1.49}$$

To achieve this, the rotor must rotate at an angular frequency synchronized to the frequency ω_s of the stator current. A motor satisfying this condition becomes a synchronous motor. The name "*synchronous motor*" is due to this fact. From Eq. (1.44), the average torque in this motor can be given as

$$T_{avg} = -\frac{I_{sm}I_{rm}}{2}L_m \sin\delta \tag{1.47}$$

Unlike in the DC motor, the average torque in this motor varies according to the angle δ between the stator and the rotor field fluxes even though the same currents are flowing. We can easily see that a synchronous motor has no starting torque because it cannot meet the requirement of $\omega_m \neq \omega_s$ when starting.

1.2.3.3 Induction motor

When the stator and the rotor windings are excited with AC power of different frequencies ω_s and ω_r, respectively, a necessary condition should be

$$\omega_m = \omega_s - \omega_r \tag{1.50}$$

This means that the rotor has to rotate at the angular frequency of $\omega_s - \omega_r$. This can be achieved in an induction motor as shown in Fig. 1.24C. In an induction motor, the frequency ω_r of the currents induced in the rotor windings by electromagnetic induction becomes naturally $\omega_s - \omega_m$, which is the difference between the stator current and the rotor speed in the angular frequency. Thus, an induction motor always satisfies Eq. (1.51). From Eq. (1.44), the average torque in this motor can be given as

$$T_{avg} = -\frac{I_{sm}I_{rm}}{2}L_m \sin(\alpha + \delta) \tag{1.51}$$

Unlike in the DC motor, the average torque in the induction motor varies according to the angle δ between the stator and the rotor field fluxes even though the same currents are flowing.

As we saw previously, electric motors can be classified into cylindrical motors and reluctance motors. Cylindrical motors exploit the torque produced by varying the mutual-inductance between the windings as mentioned before. These cylindrical motors with windings on both the stator and the rotor can produce a larger torque even though they are more complex in construction. Therefore, most motors are of the cylindrical type. In comparison, reluctance motors of noncylindrical configuration utilize the torque produced by the variation of inductance (or reluctance) of the magnetic path. Reluctance motors are simple in construction, but the torque developed in these motors is small.

1.3 MECHANICAL LOAD SYSTEM

An electric motor is an electromechanical device that converts electrical energy into mechanical torque. When the torque developed by the motor is transferred to the load connected to it, the load variables such as speed, position, air-flow, pressure, and tension will be controlled. Now, we will discuss the characteristics of the load connected a motor and the mathematical representation that describes the motor-driven system.

1.3.1 DYNAMIC EQUATION OF MOTION

Consider that a driving force F_M acts on an object of mass M as shown in Fig. 1.25A, so that the object moves at a speed v[4]. From Newton's second law, which states that the rate of change of momentum of an object is directly proportional to the applied force, we have

$$F_M - F_L = \frac{d}{dt}(Mv) = M\frac{dv}{dt} + v\frac{dM}{dt} \tag{1.53}$$

where F_L is the load force, which acts in a direction opposite to F_M, and Mv is the object's momentum.

If the mass M of the load is constant, then the second term of the derivative in Eq. (1.53) is zero. Thus, Eq. (1.53) is given by a well-known equation derived from Newton's second law as

$$F_M - F_L = M\frac{dv}{dt} = Ma \tag{1.54}$$

where $a(=dv/dt)$ is the object's acceleration.

Next, for a rotational motion in Fig. 1.25B, the rotational analogue of Eq. (1.53) is given by

$$T_M - T_L = \frac{d}{dt}(J\omega) = J\frac{d\omega}{dt} + \omega\frac{dJ}{dt} \tag{1.55}$$

where T_M and T_L are driving and load torque, respectively, and ω is the angular speed of the rotating object. J denotes the moment of inertia of the rotating object and $J\omega$ is the object's angular momentum.

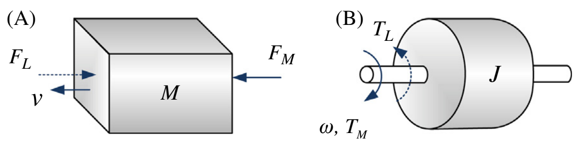


FIGURE 1.25

(A) Linear motion and (B) rotational motion objects.

When the moment of inertia J is assumed to be constant, we can rewrite Eq. (1.55) as

$$T_M - T_L = J\frac{d\omega}{dt} = J\alpha \tag{1.56}$$

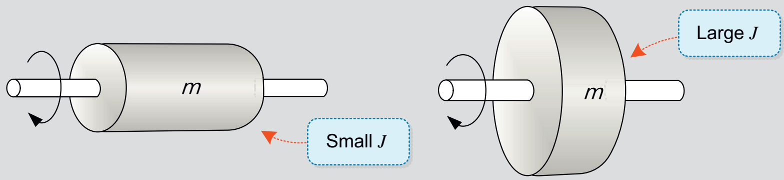
where $\alpha(=d\omega/dt)$ is the object's angular acceleration.

We can see that if $T_M > T_L$, then the object will accelerate, and if $T_M < T_L$, then it will decelerate. In the case of $T_M = T_L$, the speed will not be changed. Thus, when driving a load, to keep the load speed constant, the motor torque must be equal to the load torque.

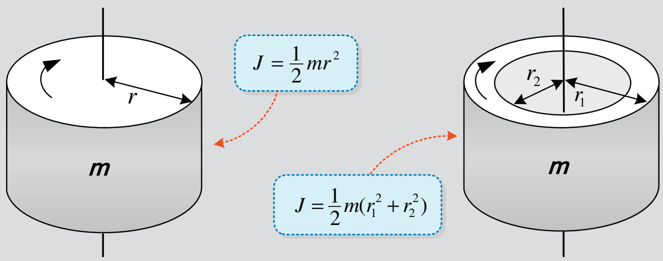
MOMENT OF INERTIA: J (kg m^2)

The tendency of a rotating object to continue in its original state of rotary motion is referred to as the *moment of inertia* J. The rotational kinetic energy of a rigid object with angular velocity ω is expressed as $\frac{1}{2}J\omega^2$ in terms of its moment of inertia. The moment of inertia of an object is also the measure of how fast the object can be accelerated or decelerated.

The moment of inertia in a rigid object is related to the distribution of mass as well as the mass m of the object. Therefore the moment of inertia of an object can depend on its shape. For example, if two objects have the same mass but are different in radius as shown in the following figure, then they may have different moments of inertia.



If the radius of an object is larger, then the moment of inertia is larger. The reason for this is as follows. A rotating object consists of many small particles at various distances from the axis of rotation. When the object is rotating, the particles which are far away from the axis of rotation will move faster than the ones which are located near the axis of rotation. Thus, the particles which are far away from the axis of rotation require more kinetic energy for a rotation. The moment of inertia I for each particle is defined as its mass multiplied by the square of the distance r from the axis of rotation to the particle, i.e., $I = mr^2$. Thus, for the same object mass, the further out from the axis of rotation its mass is distributed, the larger the moment of inertia of the object is. For solid or hollow cylinders, the moment of inertia can be calculated by the equations shown in the next figures.



(*Continued*)

MOMENT OF INERTIA: *J* (kg m^2) (Continued)

Instead of J, $GD^2(=4J)$, also known as *flywheel effect*, is generally used in industrial applications. Here, G is the mass (kg) and D is the diameter (m).

There is maximum permissible load inertia to a motor. It is important to ensure that the inertia of the motor matches the inertia of the driven load. Ideally, it is desirable to have a 1:1 inertia ratio between the load and the motor. For traction motors, the motor inertia should be large enough to drive the load of a large inertia. To have a large inertia, the motor will be designed with a rotor configuration of a large diameter and a short axial length. On the other hand, for servo motors operated in frequent acceleration/deceleration, the inertia of the rotor should be small. Thus, the rotor will be designed to have a small diameter and a long axial length.

Fig. 1.26 shows a simplified configuration of a motor drive system consisting of a driving motor and a driven mechanical load, in which the torque of the motor is transferred to the load through the shaft and coupling.

From the motor's viewpoint, a mechanical load can be seen as a load torque T_L connected to its shaft. Thus, from Eq. (1.56), the equation of motion of a motor drive system can be expressed as

$$T_M = (J_M + J_L)\frac{d\omega}{dt} + T_L \tag{1.57}$$

where J_M and J_L are the motor inertia and the load inertia, respectively.

When the motor is driving a mechanical load, the torque required to run the load is the load torque T_L, which varies with the type of load.

In addition to Eq. (1.57), when a load begins to move and is in motion, a friction force occurs, resisting the motion. Therefore, the equation of motion needs to include the friction force T_F as

$$T_M = (J_M + J_L)\frac{d\omega}{dt} + T_F + T_L \tag{1.58}$$

The friction force is usually proportional to the speed, given by $T_F = B\omega$, where B is the coefficient of friction. Finally, we have an equation of motion, which expresses the dynamic behavior of a motor drive system as

$$\boxed{T_M = J\frac{d\omega}{dt} + B\omega + T_L} \tag{1.59}$$

where $J = J_M + J_L$.

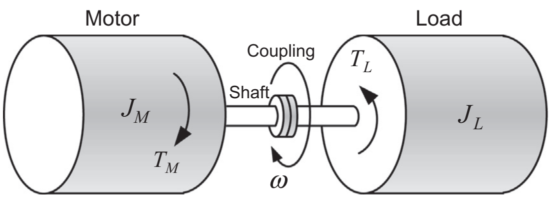


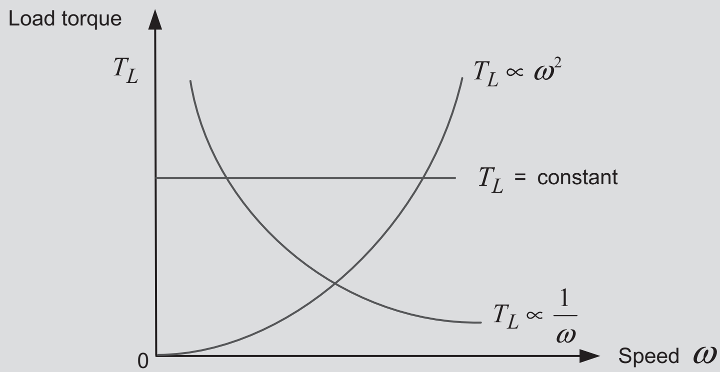
FIGURE 1.26

Simplified motor drive system.

A motor-driven system has mechanical transmission devices such as shaft, gearbox, or coupling to connect the motor and the load. In a real system, the stiffness of these components is limited, and the transmissions are flexible. Therefore, if the coupling or shaft between the motor and the load is long or elastic, then it may be subjected to torsion. In this case, we need to add the torque term $K_{sh}\theta$ produced by torsional deformation of the shaft to Eq. (1.59), where K_{sh} (Nm/rad) is the coefficient of stiffness.

LOAD TORQUE T_L

There are several types of loads driven by a motor. In most loads, the required torque is normally a function of speed. There are three typical loads whose speed-torque characteristics are shown in the following figure.



1. Constant torque load

   The torque required by this type of load remains constant regardless of the speed. Loads such as elevators, screw compressors, conveyors, and feeders have this type of characteristic.

2. Constant power load ($T_L \propto 1/\omega$)

   This load requires a torque, which is inversely proportional to the speed, so this load is considered a constant power load. This type is most often found in machine tool industry and drilling, milling industry.

3. Load proportional to the square of the speed ($T_L \propto \omega^2$)

   The torque of this load increases in proportion to the square of the speed. This is a typical characteristic of a fan, blower, and pump, which are the most commonly found in industrial drive applications.

The load in traction drives such as electric locomotives and electric vehicles shows the combined characteristic of a constant torque load in low speeds and a constant power load in high speeds.

1.3.1.1 Combination system of translational motion and rotational motion

Many motions in the industry such as an elevator, a conveyor, or electric vehicles are a combination of translational motion and rotational motion as shown in Fig. 1.27. In this case, we need to convert the load parameters such as load inertia to the motor shaft.

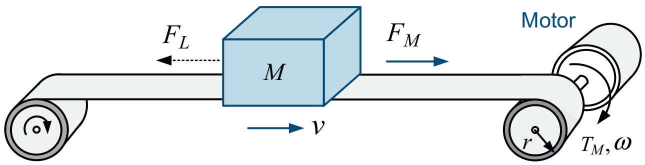


FIGURE 1.27

Combination system of translational motion and rotational motion.

The expression for the translational motion in Fig. 1.27 is given by

$$F_M - F_L = M\frac{dv}{dt} \tag{1.60}$$

The relationships between force and torque, and velocity and angular velocity are

$$T_M = rF_M \tag{1.61}$$

$$T_L = rF_L \tag{1.62}$$

$$\omega = \frac{v}{r} \tag{1.63}$$

By substituting Eqs. (1.61)–(1.63) into Eq. (1.60), we obtain

$$T_M - T_L = Mr^2\frac{d\omega}{dt} = J_e\frac{d\omega}{dt} \tag{1.64}$$

where $J_e = Mr^2$ represents the equivalent moment of inertia, which is reflected to the motor shaft side of the mass M in translational motion.

1.3.1.2 System with gears or pulleys

Often, the speed required by the load is too low compared to the nominal speed of the motor. Gears or pulleys between the motor and the load being driven are most often used to change the speed. In this case, we need to know how the load will be seen through the gears or pulleys at the motor side.

We will look at an example of two meshed gears as shown in Fig. 1.28.

The equation of motion at the load side is given by

$$T_L = J_L\frac{d\omega_L}{dt} + B\omega_L \tag{1.65}$$

Because the two gears will travel at an equal distance, we have

$$\omega_L = \frac{N_1}{N_2}\omega_M \tag{1.66}$$

where N_1 and N_2 are the number of teeth on the gears of the motor and load sides, respectively.

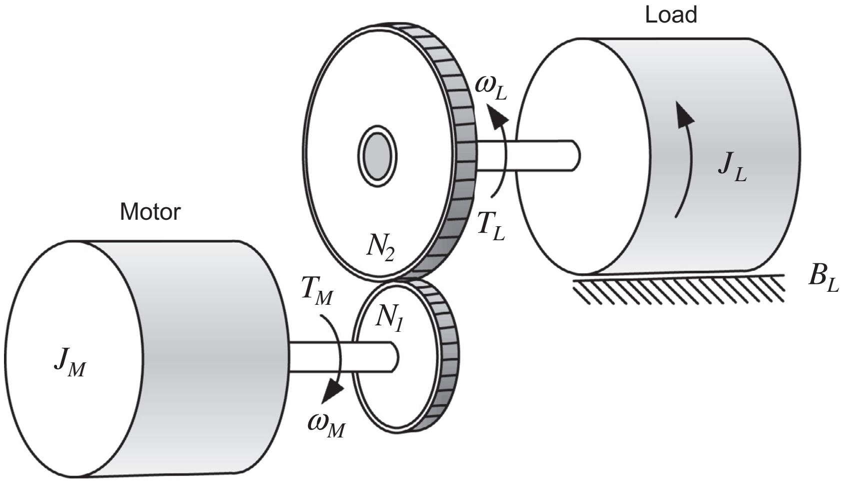


FIGURE 1.28

System with gears.

When neglecting friction and losses of the gears, the power is the same at the input and at the output of the gear, i.e.,

$$P = T_L \omega_L = T_M \omega_M \quad \rightarrow \quad T_L = \frac{N_2}{N_1} T_M \tag{1.67}$$

Substituting Eqs. (1.65) and (1.66) in Eq. (1.65) yields

$$T_L = J_L \left(\frac{N_1}{N_2} \right) \frac{d\omega_L}{dt} + B_L \left(\frac{N_1}{N_2} \right) \omega_L = \left(\frac{N_2}{N_1} \right) T_M \tag{1.68}$$

Finally, the equation of motion at the motor side is given by

$$\begin{aligned} T_M &= J_L \left(\frac{N_1}{N_2} \right)^2 \frac{d\omega_M}{dt} + B_L \left(\frac{N_1}{N_2} \right)^2 \omega_M \\ &= (J_M + J) \frac{d\omega_M}{dt} + B\omega_M \end{aligned} \tag{1.69}$$

where $J = J_L(N_1/N_2)^2$ and $B = B_L(N_1/N_2)^2$ are the load inertia J_L and the friction coefficient B_L converted to the motor side, respectively. It can be seen that the load parameters reflected back to the motor are a squared function of the gear ratio N_1/N_2.

1.3.2 OPERATING MODES OF AN ELECTRIC MOTOR

Operation of a motor can be divided into two modes according to the direction of the energy transfer.

When the motor rotates in the same direction as its developed torque in Fig. 1.29A, the motor will supply mechanical energy to the load. In this case, the motor operates as a typical *motoring mode*. On the other hand, when the

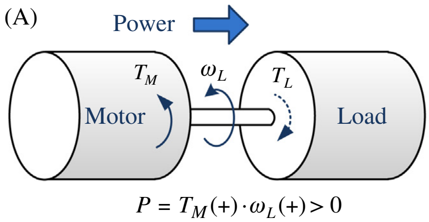


FIGURE 1.29

Operation mode of the motor. (A) Motoring mode and (B) generating (braking) mode.

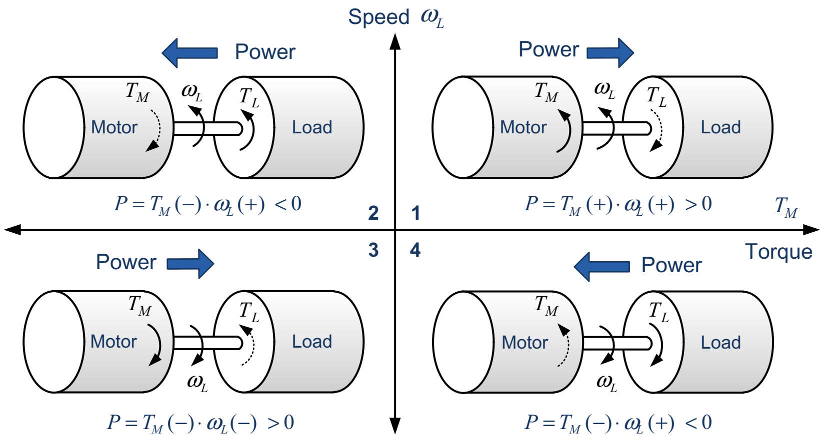


FIGURE 1.30

Four-quadrant operation modes.

motor is rotating in a direction opposite to its developed torque in Fig. 1.29B, the mechanical energy of the load is supplied to the motor. In this case, the motor is acting as a generator. It corresponds to the *generating mode*, in which the motor absorbs the load energy, providing a braking action.

When braking, the mechanical energy of the load is converted into electrical energy through the motor, and the generated electrical power can be returned to the power source. This method called *regenerative braking* is the most efficient braking method, but it requires a high-cost power converter. An alternative method for dealing with the generated electrical power is to dissipate it as heat in the resistors. This is called *dynamic braking*, which is inefficient but inexpensive.

Similarly, there are also operation modes of motoring and generating in the reverse driving. Consequently, four-quadrant operation modes are available in variable speed drive systems as shown in Fig. 1.30. Quadrants 1 and 2 are the motoring and generating operation modes, respectively, in the forward drive, while Quadrants 3 and 4 are the two operation modes in the reverse drive. Braking occurs in Quadrants 2 and 4. To achieve the four-quadrant operation modes in motor drive systems, the torque should be controlled in the positive and negative direction, and the speed should be controlled in the forward and reverse direction.

As an example of the four-quadrant operation, let us take a look at an elevator drive system as shown in Fig. 1.31. In an elevator system, the car carrying people or goods is connected to and is driven by a traction motor through ropes and gears. As a traction motor, induction motors are most widely used. In recent times, however, permanent magnet synchronous motors are used as well.

A counterweight attached to the opposite side of the ropes reduces the amount of power required to move the car. The weight of the counterweight is typically equal to the weight of the car plus 40–50% of the capacity of the elevator. When the car goes up or down, we can see the four-quadrant operation of the driving motor occurring due to the weight difference between the car and the counterweight as follows:

- Quadrant 1 operation: the forward motoring mode $(+T, +\omega)$

  When the car carrying people goes up and is heavier than the counterweight, the traction motor is needed to produce the forward direction torque, so it operates in the *forward motoring mode*.

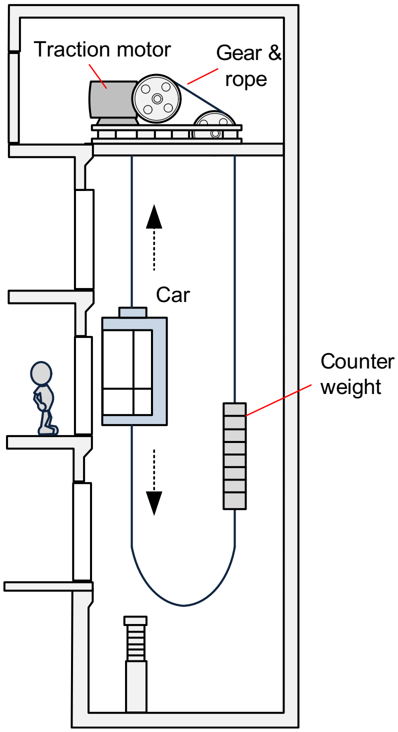


FIGURE 1.31

Four-quadrant operation in an elevator drive.

- Quadrant 2 operation: the forward braking mode $(-T, +\omega)$
  When an empty car goes up and is lighter than the counterweight, the traction motor is needed to produce the braking torque of the reverse direction, so it operates in the *forward braking (generating) mode*.
- Quadrant 3 operation: the reverse motoring mode $(-T, -\omega)$
  When an empty car goes down and is lighter than the counterweight, the traction motor is needed to produce the reverse direction torque, so it operates in the *reverse motoring mode*.
- Quadrant 4 operation: the reverse braking mode $(+T, -\omega)$
  When the car carrying people goes down and is heavier than the counterweight, the traction motor is needed to produce the braking torque of the forward direction, so it operates in the *reverse braking (generating) mode*.

Here, the going-up of the car is assumed as the forward direction.

1.4 COMPONENTS OF AN ELECTRIC DRIVE SYSTEM

For a given mechanical load, an adequate design of the motor drive system is essential to meet its required performance criteria such as speed-torque characteristic, operation speed range, speed regulation, efficiency, cost, duty cycle, etc.

A motor drive system consists normally of five parts as shown in Fig. 1.32. Within the available power supply, we have to choose these components appropriately to achieve the requirements of the drive performance. We will now briefly take a look at these five parts.

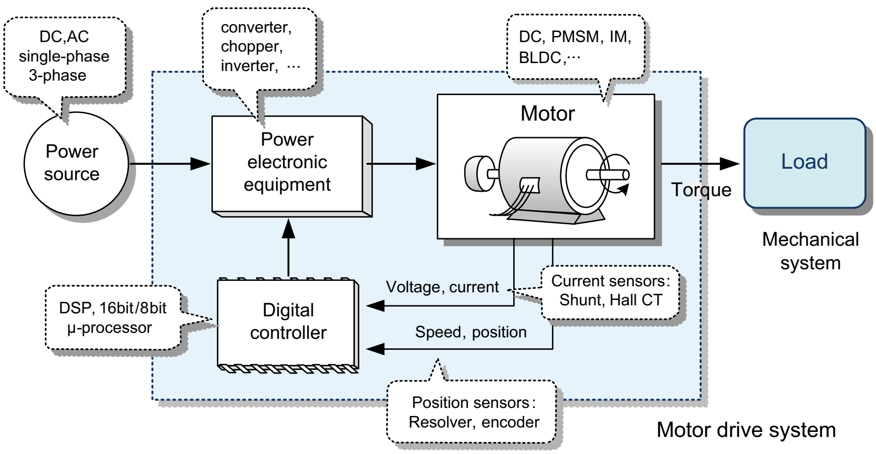


FIGURE 1.32

Configuration of a motor drive system.

1.4.1 POWER SUPPLY

Single-phase or three-phase AC voltage sources are most commonly used as a power supply that provides electric energy for motor drives. The three-phase AC voltage source is generally used for motors of several kilowatts or higher. In many cases, the AC voltage sources are converted into a DC source, which is converted into the form of power needed by a motor through using power electronic converters such as an inverter or a chopper. Recently, DC power has often been provided by renewable energy sources such as solar cells and fuel cells.

1.4.2 ELECTRIC MOTORS

For driving motors, we have to choose the one that meets the required speed-torque characteristic of the driven mechanical load. In addition, important performance criteria like efficiency, size, cost, duty cycle, and operating environment need to be considered. As mentioned earlier, there are two basic types of a driving motor: DC motor and AC motor. Some applications use special types such as a BLDC motor or a stepping motor, which cannot be classified into the basic types.

DC motors have an advantage of a simple speed/torque control for their excellent drive performance. However, due to the wear associated with their commutators and brushes, DC motors are less reliable and unsuitable for a maintenance-free operation. On the other hand, AC motors are smaller, lighter, and have more power density than DC motors. AC motors also require virtually no maintenance and thus are better suited for high speeds. However, AC drives with a high performance capability are more complex and expensive than DC drives.

1.4.3 POWER ELECTRONIC CONVERTERS

Power electronic converters play the role of taking electrical energy from the power system and turning it into a suitable form needed by a motor. The power electronic converter may be determined according to the given power source and the driving motor.

For DC drives, power electronic converters such as a controlled rectifier or a chopper can be used to adjust the DC power, which will be described in more detail in Chapter 2. In contrast, AC drives mostly use an inverter to adjust the voltage and frequency in the AC power, which will be also described in more detail in Chapter 7. In this case, a rectifier is often included to convert the AC power in the mains power system into the DC power.

These power electronic converters use power semiconductor devices such as gate turn-off (GTO) thyristor, integrated gate-commutated thyristor (IGCT), insulated gate bipolar transistor (IGBT), power metal oxide semiconductor field effect transistor (MOSFET), and power bipolar junction transistor (BJT). These switching devices are determined according to their power handling capability and their switching speed. The power handling capability can be ranked in an increasing order of

MOSFET, IGBT, and GTO thyristor, while the switching speed can be ranked in an increasing order of GTO thyristor, IGBT, and MOSFET. All these switching devices described above are based on the silicon (Si) semiconductor material. Recently, switching devices based on wide bandgap materials such as silicon carbide (SiC) or gallium nitride (GaN) are being recognized as a promising future device.

1.4.4 DIGITAL CONTROLLERS

The controller in charge of executing algorithms to control the motor is the most important part in the motor drive systems. This controller manipulates the operation of the power electronic converter to adjust the frequency, voltage, or current provided to the motor. Nowadays, digital controllers are usually used. A digital controller is based on microprocessor, microcontroller, or digital signal processor (DSP), which can be a 16-bit or 32-bit, and also either a fixed-point or floating-point.

There is a trade-off between the control performance and the cost. DC motor and BLDC motor drives requiring relatively simple controls may use 16-bit processors, while AC motor drives requiring complex controls such as a vector controlled induction or synchronous motors may need high-performance 32-bit processors or DSP.

1.4.5 SENSORS AND OTHER ANCILLARY CIRCUITS

To achieve a high-performance motor drive, a closed-loop control of position, speed or current is often adopted, and in this case, speed/position or current information is required.

Sensors are essential for obtaining this information. Shunt resistances, Hall effect sensors, or current transformers are mainly used as the current sensor. The shunt resistance is more cost effective, while the Hall effect sensors can usually give a high resolution and isolation between the controller and the system. Encoders and resolvers are widely used as speed/position sensors, which will also be discussed in Chapter 9.

In addition to these components, several circuits for protection, filtering, power factor correction, or harmonics reduction may be necessary to improve the reliability and quality of the drive system.

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