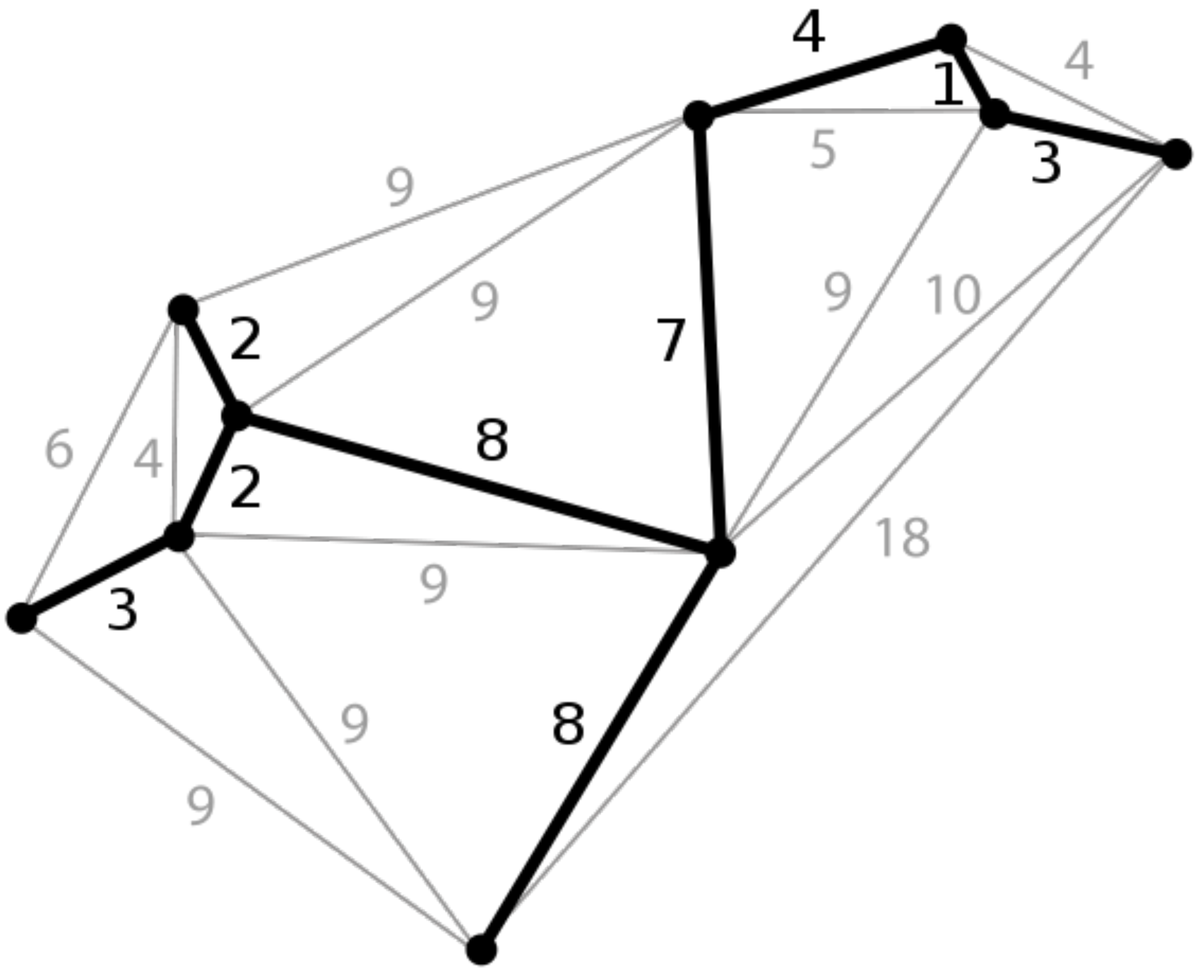
CS 106B

Lecture 23: Graphs II

Tuesday, August 7, 2017



Programming Abstractions
Summer 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:
Programming Abstractions in C++, Chapter 18

Today's Topics

- Logistics:
  - Remember, Huffman is the last assignment that you can use late days on.
  - Final review preparation will be early next week.
  - If you need accommodations for the final exam, let Chris and Jason know now.
  - The Final will be using BlueBook — you'll need 3 hours of battery life, or find an outlet (we will try to provide enough outlets for those who need it)

- Real Graphs:
  - Internet routers and traceroute
- Topological Sort
- Minimum Spanning Trees
  - Kruskal's algorithm

Real Graphs!

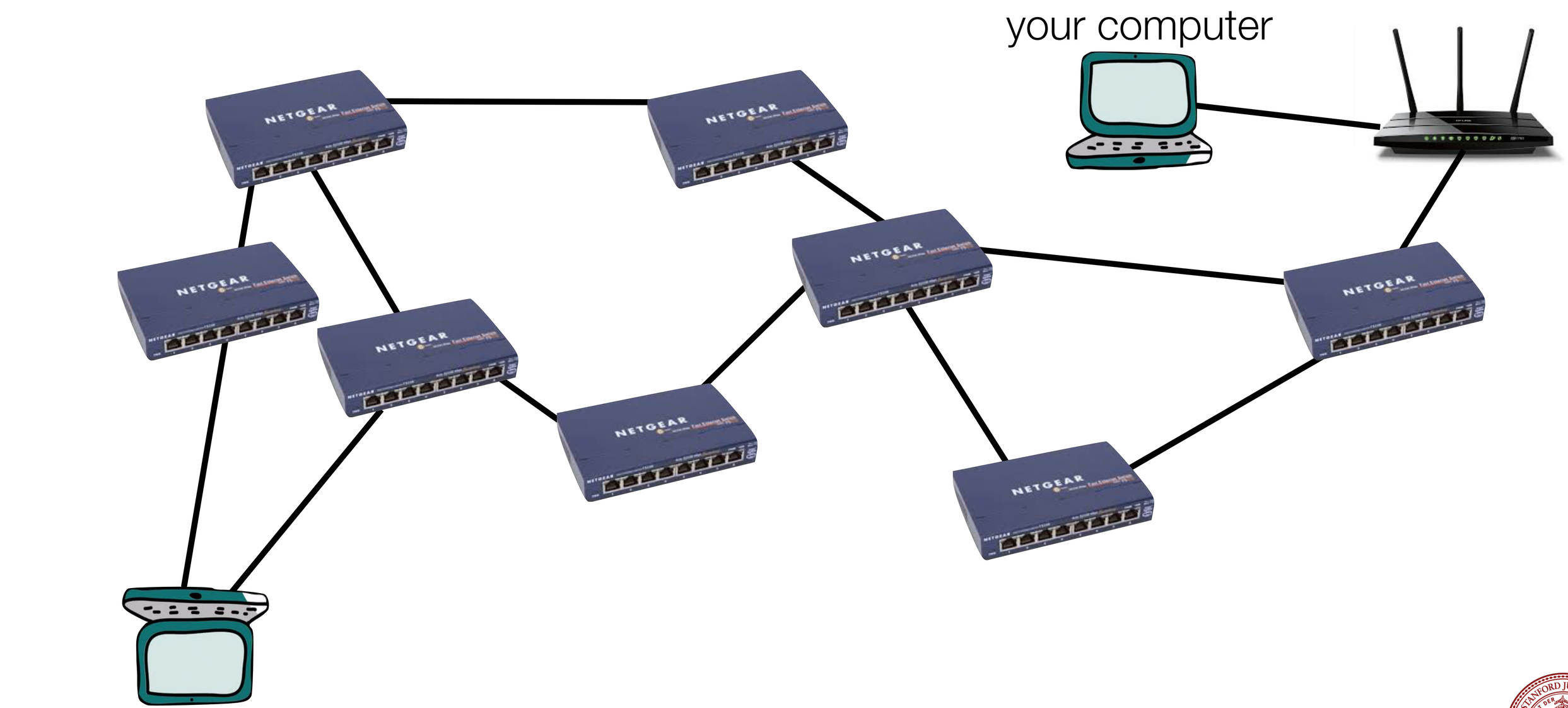
I received some feedback from last quarter that said,

> *“I would give more examples of how graphs are used, or, in general, give a wider variety of examples of applications to the methods being used. I think it would be interesting to give examples of how certain topics can be used in cross-sections of CS and other majors/departments.”*

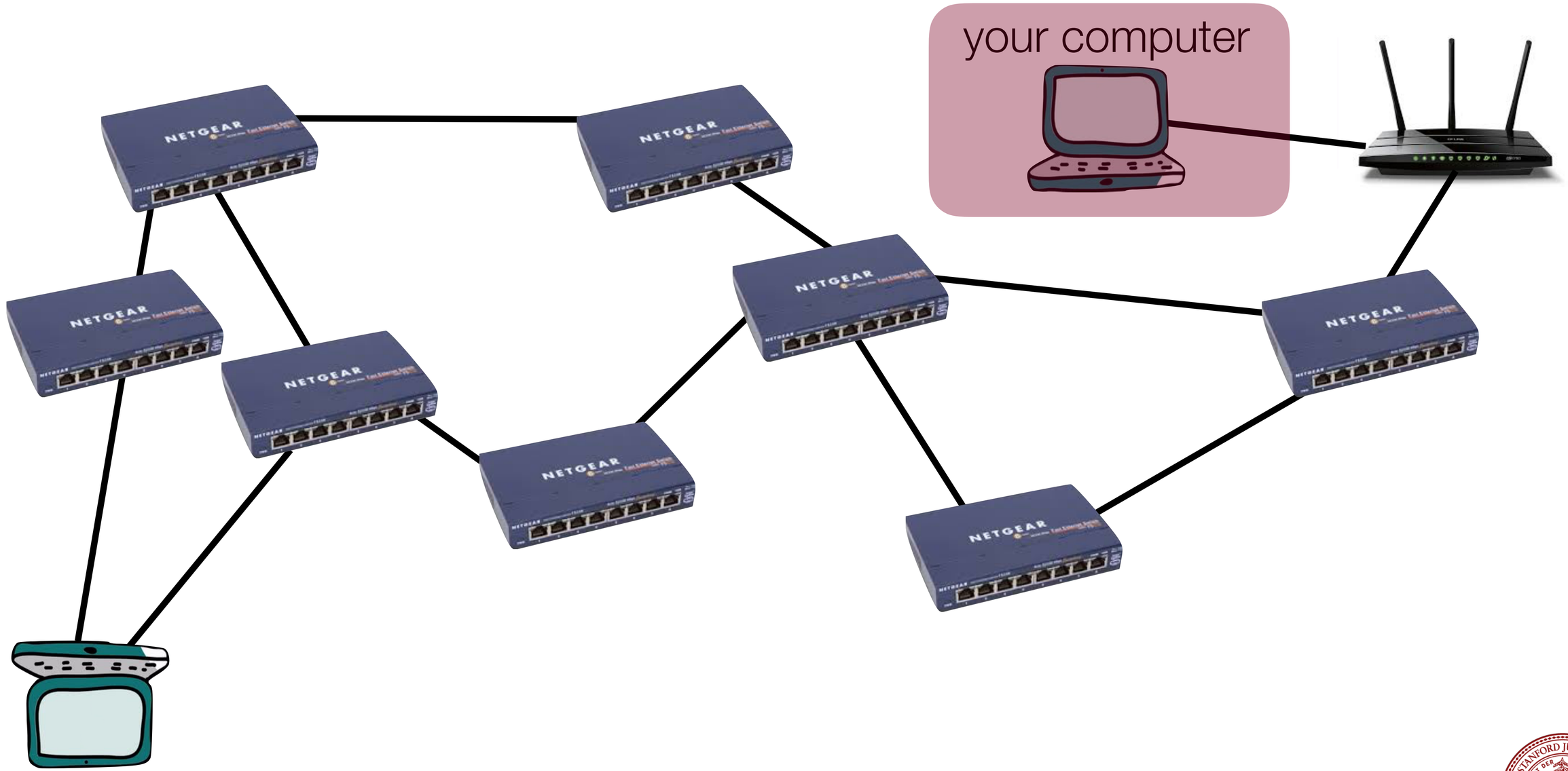
Let's dig a bit deeper into how the Internet is a real graph by analyzing internet routers, or:

How does a message get sent from your computer to another computer on the Internet, say in Australia?

The Internet: Computers connected through routers



The Internet: Computers connected through routers



The Internet: Let's simplify a bit

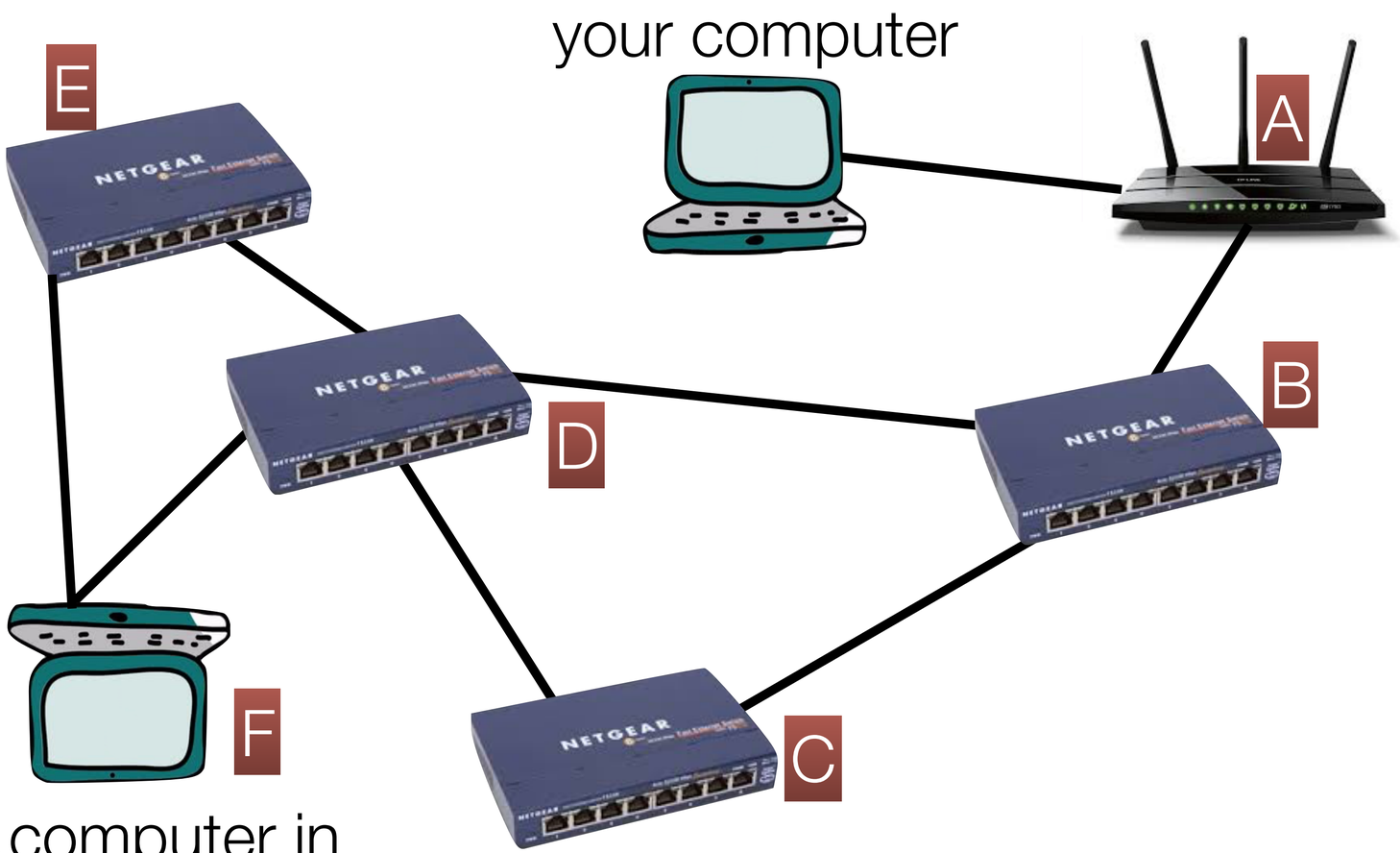
The destination computer has a name and an IP address, like this:

`www.engineering.unsw.edu.au`
`IP address: 149.171.158.109`

The first number denotes the "network address" and routers continually pass around information about how many "hops" they think it will take for them to get to all the networks.

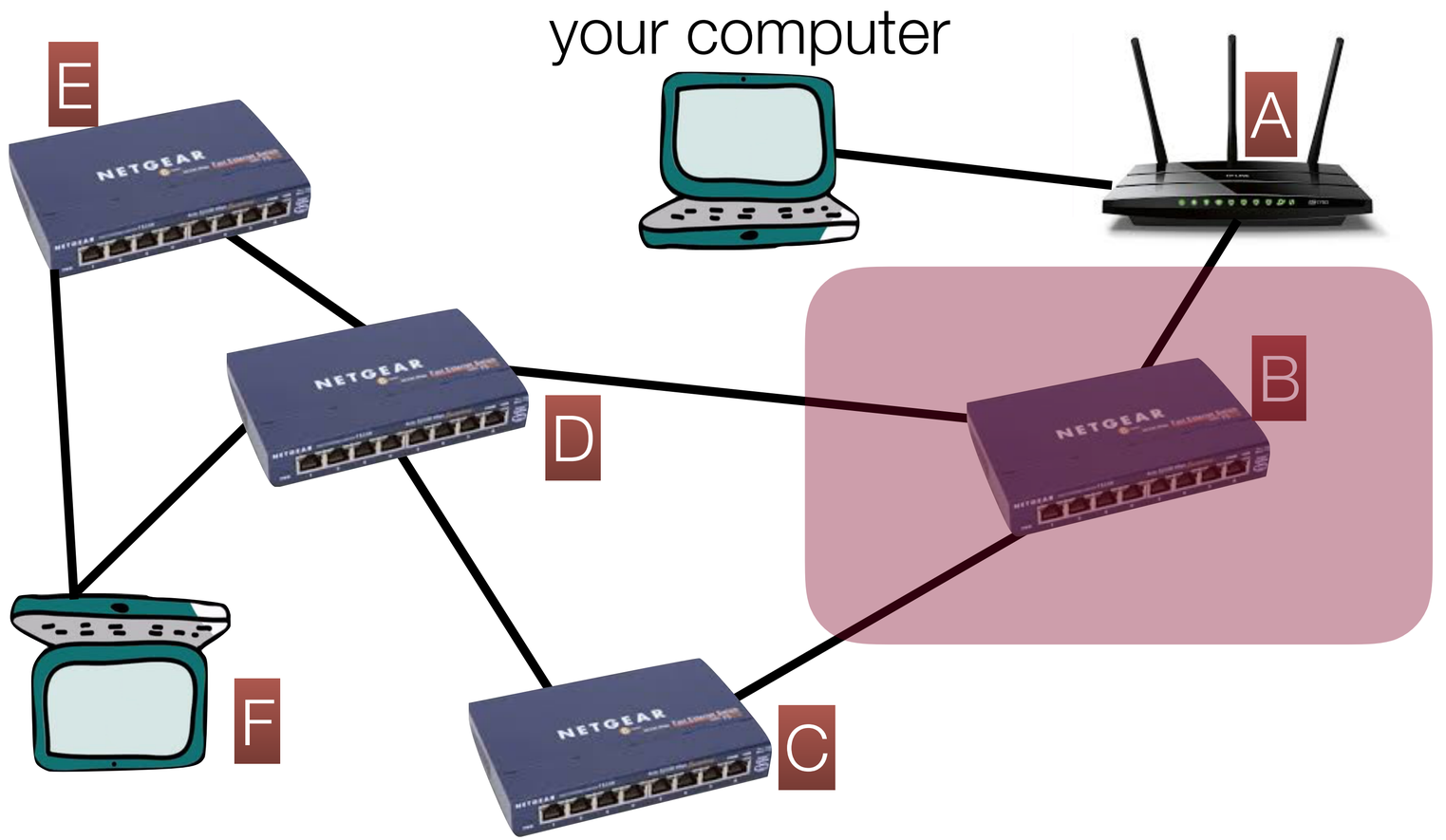
E.g., for router **`C`**:

router	hops
A	2
B	1
C	-
D	1
E	2
F	2



The Internet: Let's simplify a bit

Each router knows its neighbors, and it has a copy of its neighbors' tables. So, `B` would have the following tables:



`A`

router	hops
A	-
B	1
C	3
D	2
E	3
F	3

`C`

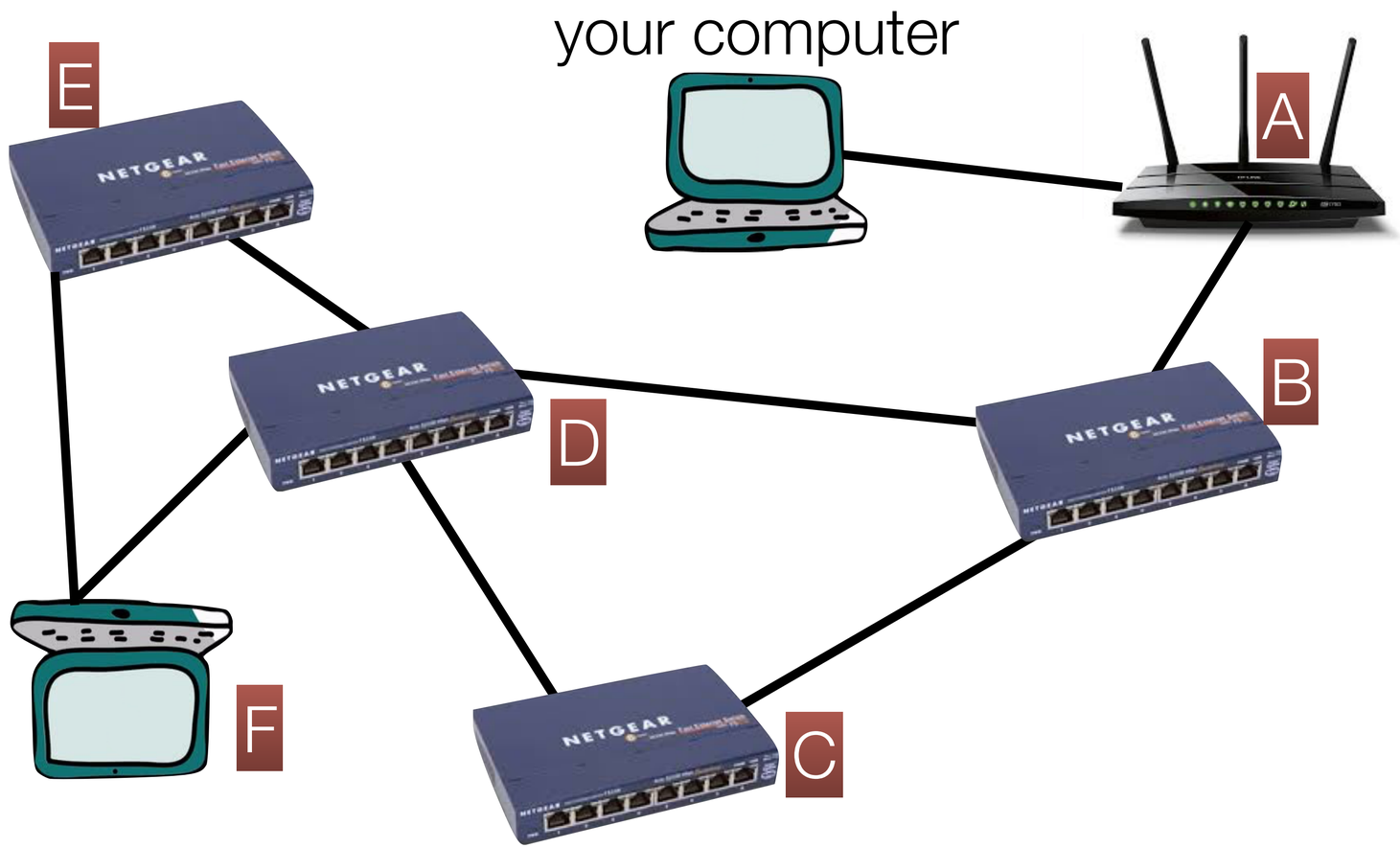
router	hops
A	2
B	1
C	-
D	1
E	2
F	2

`D`

router	hops
A	2
B	1
C	1
D	-
E	1
F	1

The Internet: Let's simplify a bit

If B wants to connect to F, it connects through its neighbor that reports the shortest path to F. Which router would it choose?



A

router	hops
A	-
B	1
C	3
D	2
E	3
F	3

C

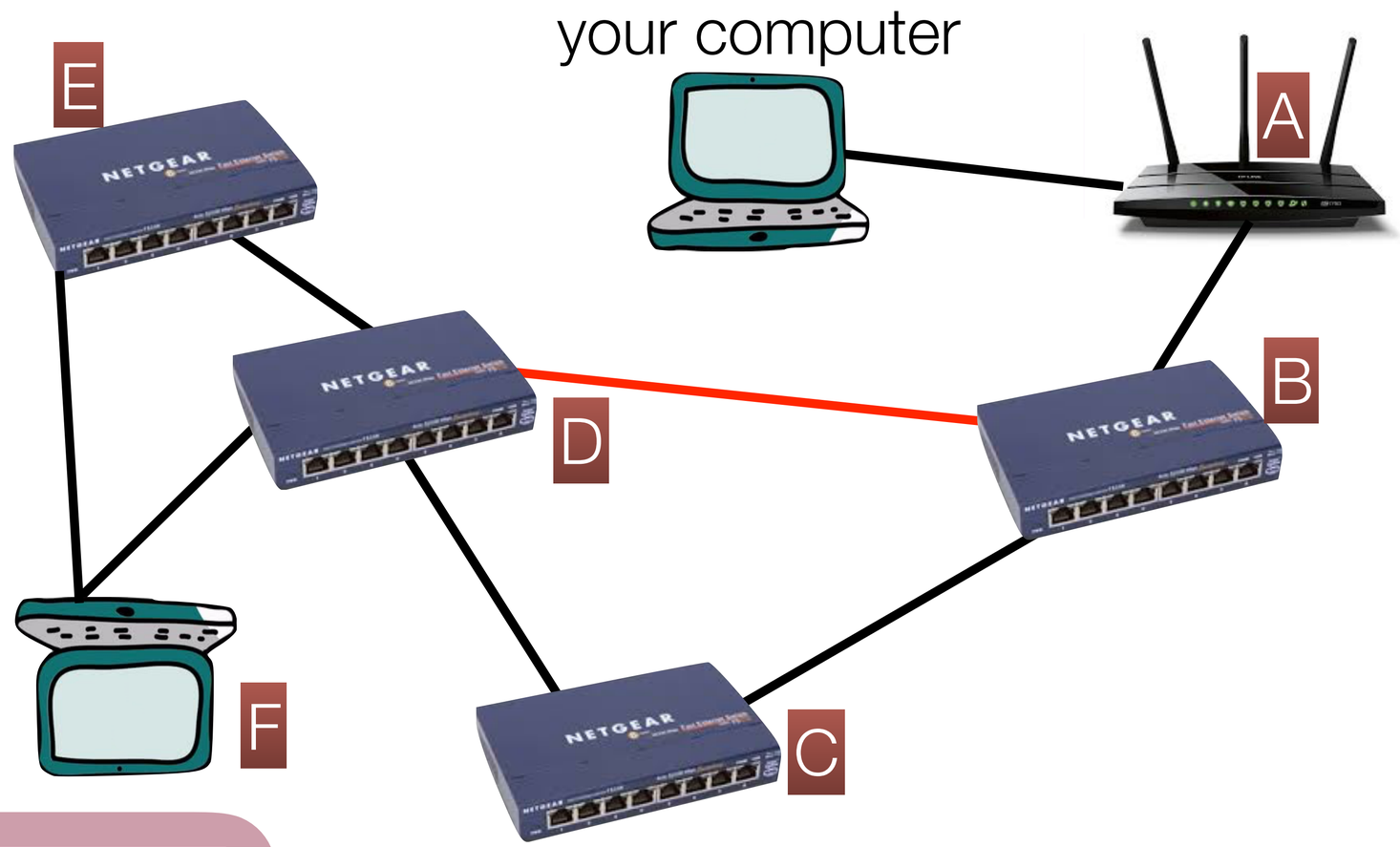
router	hops
A	2
B	1
C	-
D	1
E	2
F	2

D

router	hops
A	2
B	1
C	1
D	-
E	1
F	1

The Internet: Let's simplify a bit

If B wants to connect to F, it connects through its neighbor that reports the shortest path to F. Which router would it choose? D.



A

router	hops
A	-
B	1
C	3
D	2
E	3
F	3

C

router	hops
A	2
B	1
C	-
D	1
E	2
F	2

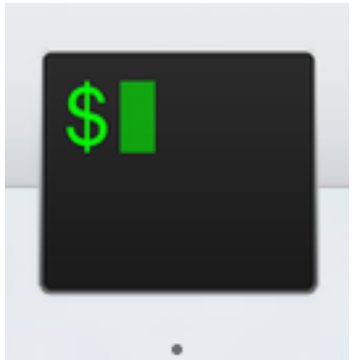
D

router	hops
A	2
B	1
C	1
D	-
E	1
F	1

Traceroute

We can use a program called "traceroute" to tell us the path between our computer and a different computer:

`traceroute -I -e www.engineering.unsw.edu.au`



Traceroute: Stanford Hops

```
traceroute -I -e www.engineering.unsw.edu.au
traceroute to www.engineering.unsw.edu.au (149.171.158.109), 64 hops max, 72 byte packets
 1  csmx-west-rtr.sunet (171.67.64.2)  7.414 ms  9.155 ms  8.288 ms
 2  gnat-2.sunet (172.24.70.12)  0.339 ms  1.532 ms  0.423 ms
 3  csmx-west-rtr-vl3866.sunet (171.64.66.2)  38.916 ms  10.506 ms  8.402 ms
 4  dca-rtr-vlan8.sunet (171.64.255.204)  0.530 ms  0.521 ms  0.713 ms
 5  dc-svl-agg4--stanford-10ge.cenic.net (137.164.50.157)  1.554 ms  1.653 ms  2.828 ms
 6  hpr-svl-hpr2--svl-agg4-10ge.cenic.net (137.164.26.249)  1.212 ms  1.161 ms  1.204 ms
 7  aarnet-2-is-jmb-778.sttlwa.pacificwave.net (207.231.245.4)  17.994 ms  17.998 ms  18.319 ms
 8  et-2-0-0.pe2.brwy.nsw.aarnet.net.au (113.197.15.98)  160.020 ms  160.234 ms  159.922 ms
 9  et-3-3-0.pe1.brwy.nsw.aarnet.net.au (113.197.15.148)  160.285 ms  160.076 ms  160.118 ms
10  138.44.5.1 (138.44.5.1)  160.124 ms  160.138 ms  160.068 ms
11  ombcr1-te-1-5.gw.unsw.edu.au (149.171.255.106)  160.090 ms  160.381 ms  160.185 ms
12  r1dcdnex1-po-2.gw.unsw.edu.au (149.171.255.178)  160.909 ms  160.847 ms  160.921 ms
13  dcfw1-ae-1-3049.gw.unsw.edu.au (129.94.254.60)  160.592 ms  160.558 ms  160.949 ms
14  www.engineering.unsw.edu.au (149.171.158.109)  160.978 ms  161.184 ms  160.987 ms
```

Traceroute: CENIC

```
traceroute -I -e www.engineering.unsw.edu.au
traceroute to www.engineering.unsw.edu.au (149.171.158.109), 64 hops max, 72 byte packets
 1  csmx-west-rtr.sunet (171.67.64.2)  7.414 ms  9.155 ms  8.288 ms
 2  gnat-2.sunet (172.24.70.12)  0.339 ms  1.532 ms  0.423 ms
 3  csmx-west-rtr-vl3866.sunet (171.64.66.2)  38.916 ms  10.506 ms  8.402 ms
 4  dca-rtr-vlan8.sunet (171.64.255.204)  0.530 ms  0.521 ms  0.713 ms
 5  dc-svl-agg4--stanford-10ge.cenic.net (137.164.50.157)  1.554 ms  1.653 ms  2.828 ms
 6  hpr-svl-hpr2--svl-agg4-10ge.cenic.net (137.164.26.249)  1.212 ms  1.161 ms  1.204 ms
 7  aarnet-2-is-jmb-778.sttlwa.pacificwave.net (207.231.245.4)  17.994 ms  17.998 ms  18.319 ms
 8  et-2-0-0.pe2.brwy.nsw.aarnet.net.au (113.197.15.98)  160.020 ms  160.234 ms  159.922 ms
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14  www.engineering.unsw.edu.au (149.171.158.109)  160.978 ms  161.184 ms  160.987 ms
```

The **Corporation for Education Network Initiatives in California** (**CENIC**) is a nonprofit corporation formed in 1996 to provide high-performance, high-bandwidth networking services to California universities and research institutions (source: Wikipedia)

Traceroute: Pacificwave (Seattle)

```
traceroute -I -e www.engineering.unsw.edu.au
traceroute to www.engineering.unsw.edu.au (149.171.158.109), 64 hops max, 72 byte packets
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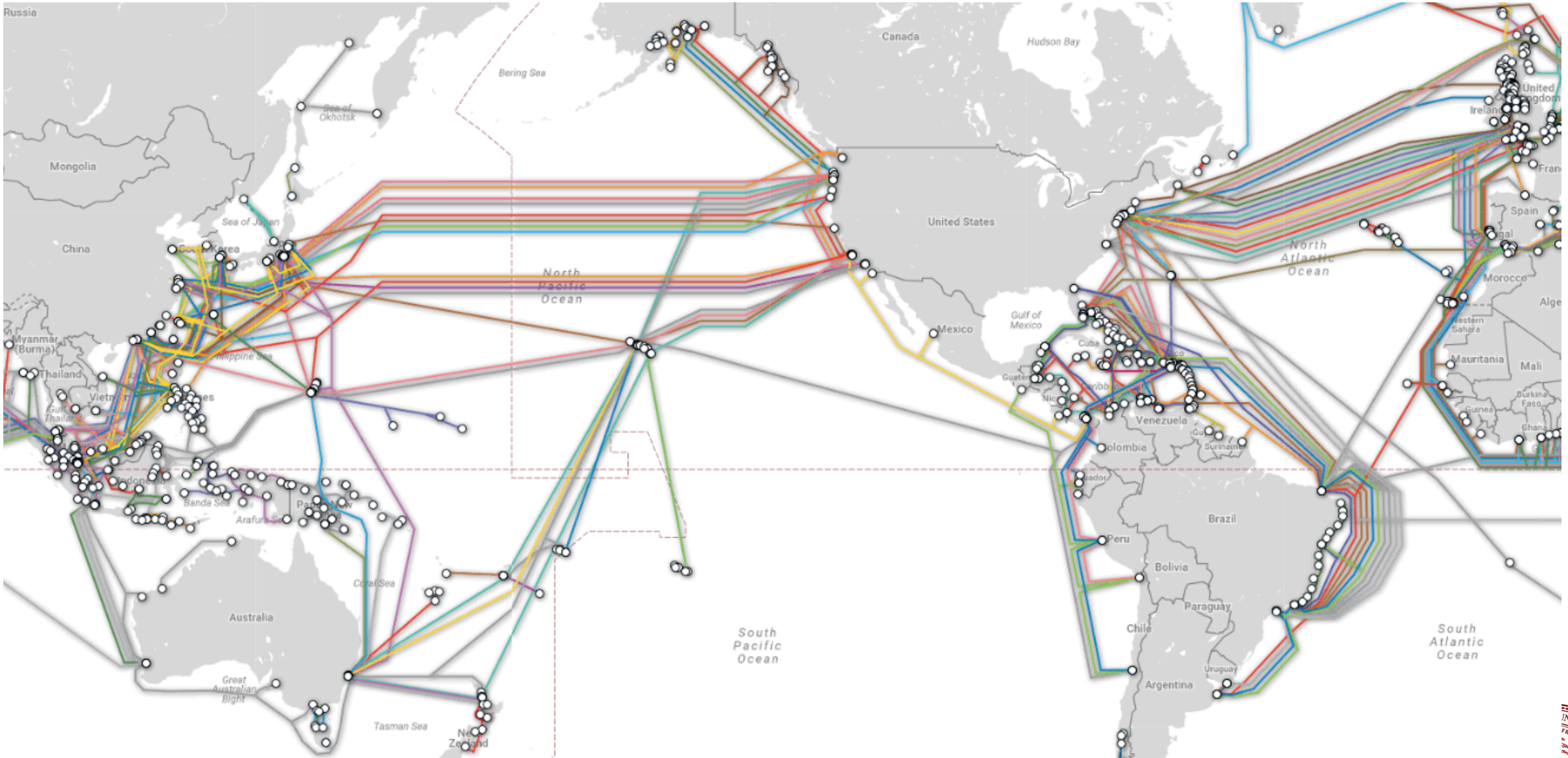
```

PACIFIC NORTHWEST GIGAPOP

Pass Internet traffic directly with other major national and international networks, including U.S. federal agencies and many Pacific Rim R&E networks (source: http://www.pnwgp.net/services/pacific-wave-peering-exchange/)

LELAND STANFORD JUNIOR UNIVERSITY
DIE LUFT DER FREIHEIT WEHT
1891

Traceroute: Oregon to Australia - underwater!



http://www.submarinecablemap.com

Traceroute: Australia

```
traceroute -I -e www.engineering.unsw.edu.au
traceroute to www.engineering.unsw.edu.au (149.171.158.109), 64 hops max, 72 byte packets
 1  csmx-west-rtr.sunet (171.67.64.2)  7.414 ms  9.155 ms  8.288 ms
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```

Traceroute: University of New South Wales

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traceroute to www.engineering.unsw.edu.au (149.171.158.109), 64 hops max, 72 byte packets
 1  csmx-west-rtr.sunet (171.67.64.2)  7.414 ms  9.155 ms  8.288 ms
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14  www.engineering.unsw.edu.au (149.171.158.109)  160.978 ms  161.184 ms  160.987 ms
```

161 milliseconds to get to the final computer

Other Real Life Uses for Graphs

- Amazon.com -- Product relationships are graph-based

Sponsored products related to this item (What's this?)

Page 1 of 18

Go Pet Club IQ Busy Box Cat Tree, 26" x 21" x 45"

4.5 stars 62

$58.75 Prime

Go Pet Club F3019 Cat Scratcher Condo Furniture

4.0 stars 56

$36.99 Prime

Kurgo Heather Dog Booster Seat for Cars with Seat Belt Tether, Charcoal

4.0 stars 15

$58.49 Prime

Favorite Medium Cat Tunnel Foldable Lightweight Fun Dangling Ball Toy,...

4.5 stars 124

$9.99 Prime

PET SHINEWINGS- Basic Cat Scratcher Scratching Board Pad

5.0 stars 1

$23.99 Prime

- What product might this be related to?

Other Real Life Uses for Graphs

Roll over image to zoom in

Cat Scratching DJ Deck

248 customer reviews | 3 answered questions

Price: $24.00 Prime | FREE Same-Day

Delivered today for FREE with qualifying orders over $35. Details

In Stock.

Get it TODAY, May 24. Order within **2 hrs 53 mins** and choose **Same-Day Delivery** at checkout. Details

Ships from and sold by Amazon.com. Gift-wrap available.

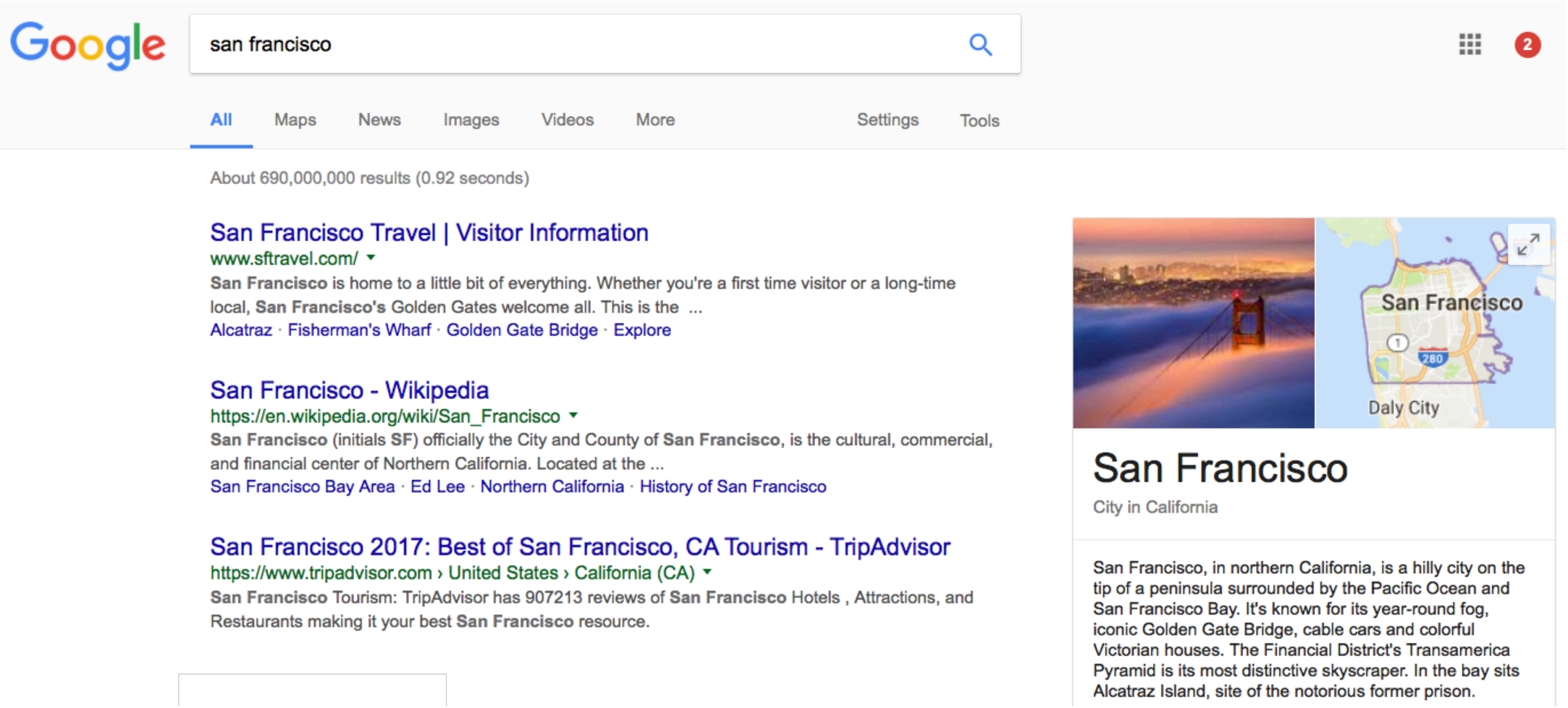
Style: **DJ Deck**

Cabin $33.83 Prime	Catillac $31.23 Prime	**DJ Deck $24.00 Prime**	Fire Engine $33.86 Prime
Laptop $24.09 Prime	Plane $44.95 Prime	Tank - -	Teepee - -

- Cardboard scratching deck for cats
- Easy to assemble - comes with full instructions
- A fun distraction for your cats!
- Comes with spinable deck and posable arm
- Measures 35cm x 38.8cm x 14.4cm

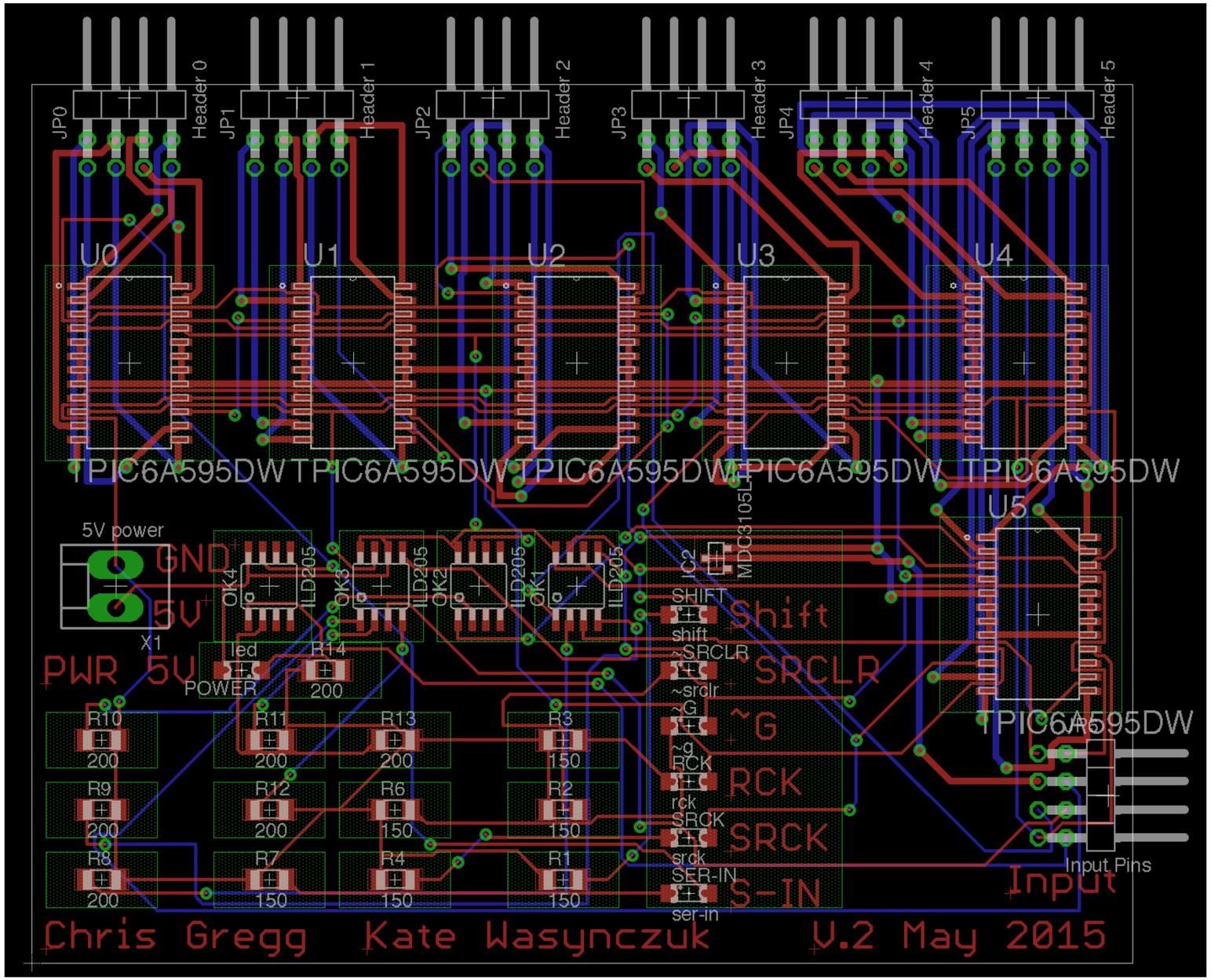
Other Real Life Uses for Graphs

- Web page searching (we will discuss this on Friday!)
- Google Maps (your final assignment, Trailblazer!)



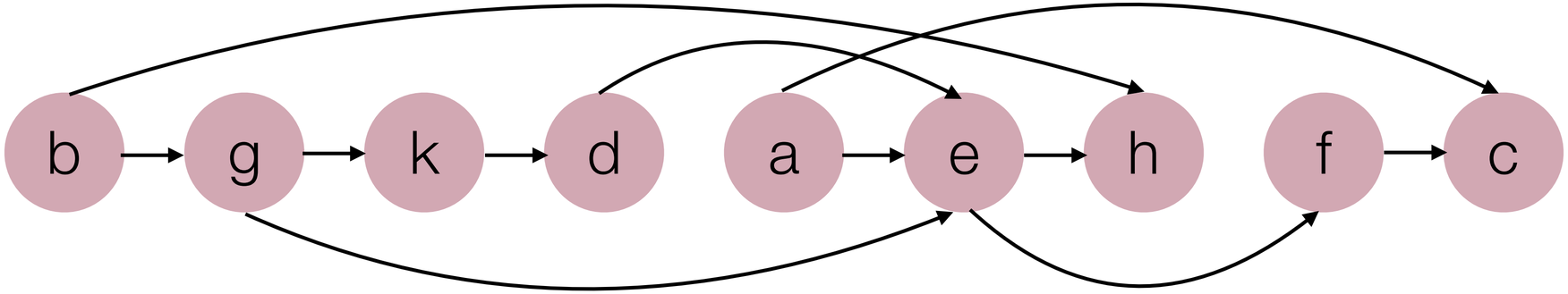
Other Real Life Uses for Graphs

- Routing circuits:



Other Real Life Uses for Graphs

- Scheduling work based on dependencies (e.g., when doing laundry, the washer must finish before the dryer, and before folding) -- this is called a "topological sort")

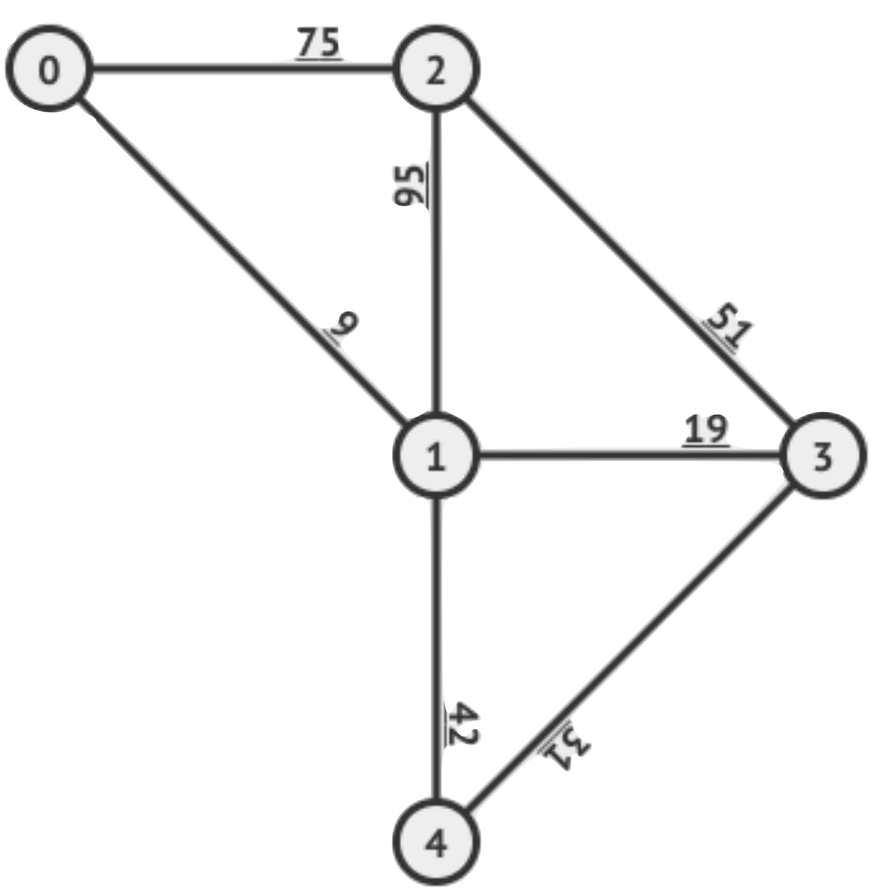


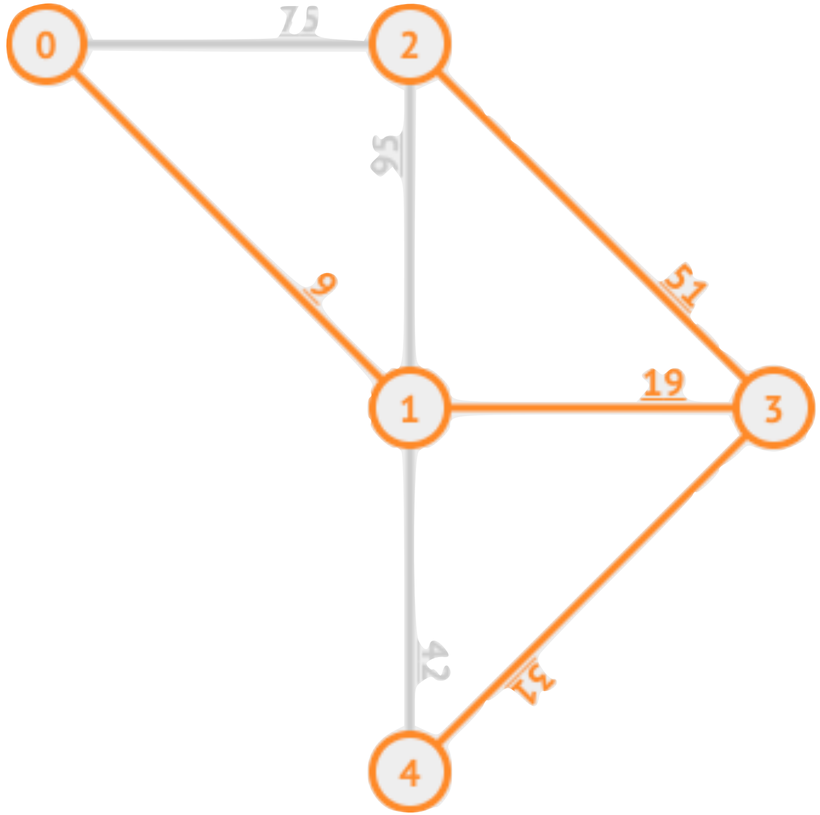
Other Real Life Uses for Graphs

- The "Oracle of Bacon": https://oracleofbacon.org (we will cover graph searching on Friday)

Other Real Life Uses for Graphs

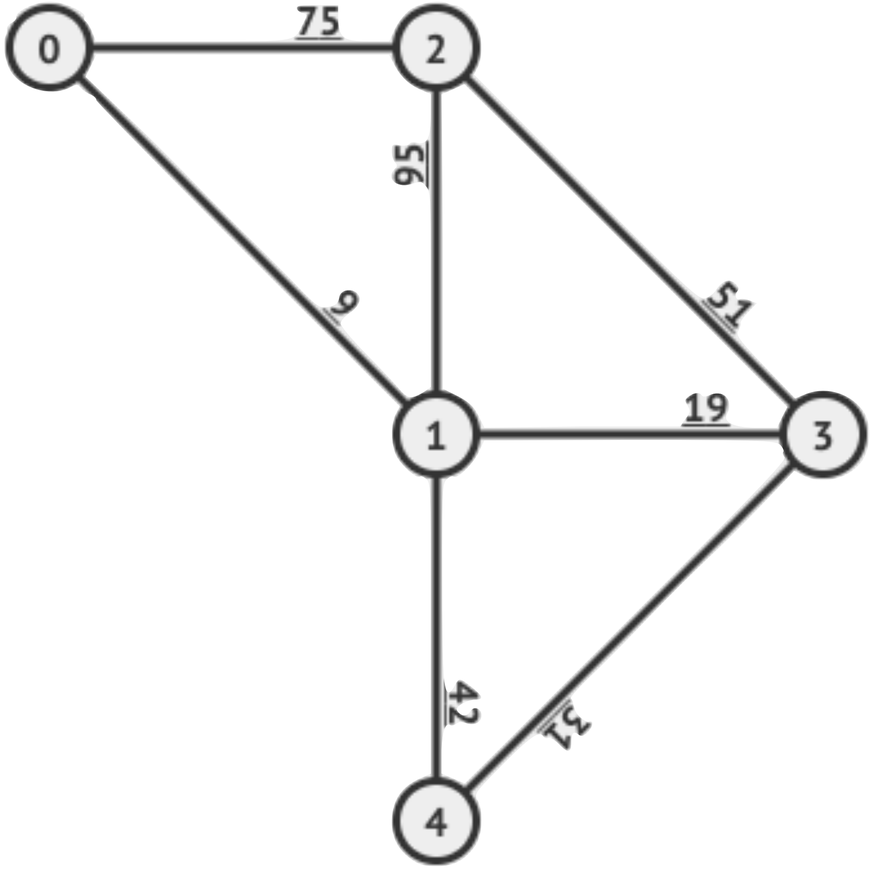
- Telecommunications: find the least expensive way to lay out a set of cables for a telephone or cable TV system ("a Minimum Spanning Tree")

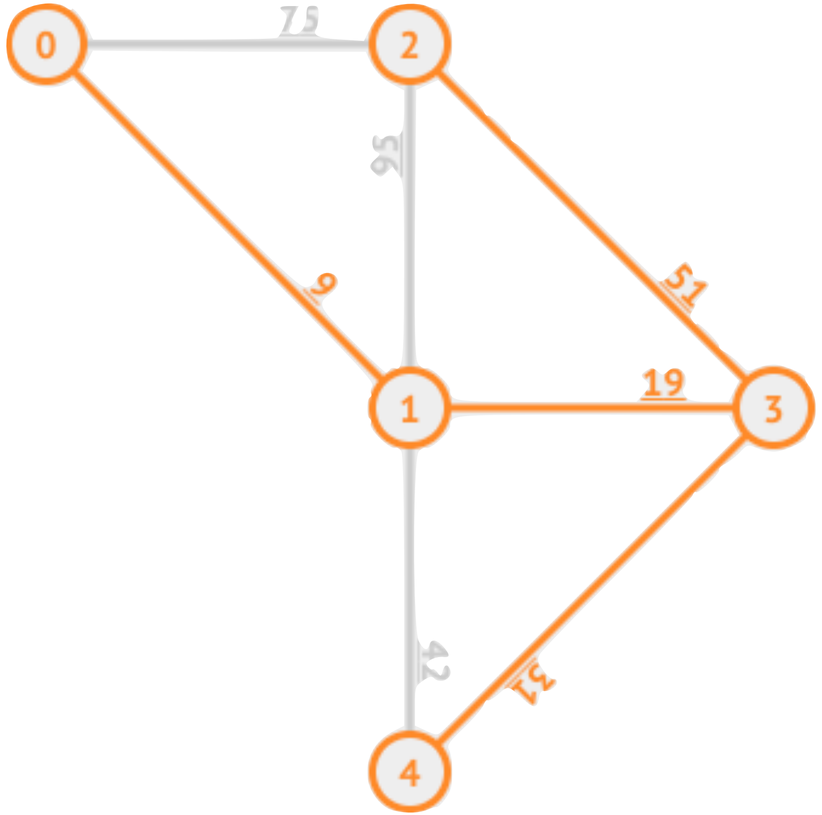




Spanning Trees and Minimum Spanning Trees

Definition: A **Spanning Tree (ST)** of a connected undirected weighted graph **G** is a subgraph of **G** that is a **tree** and **connects (spans) all vertices of G**. A graph **G** can have multiple STs. A **Minimum Spanning Tree (MST)** of **G** is a ST of **G** that has the **smallest total weight** among the various STs. A graph **G** can have multiple MSTs but the MST weight is unique.





Minimum Spanning Tree

Kruskal's Algorithm to find a Minimum Spanning Tree

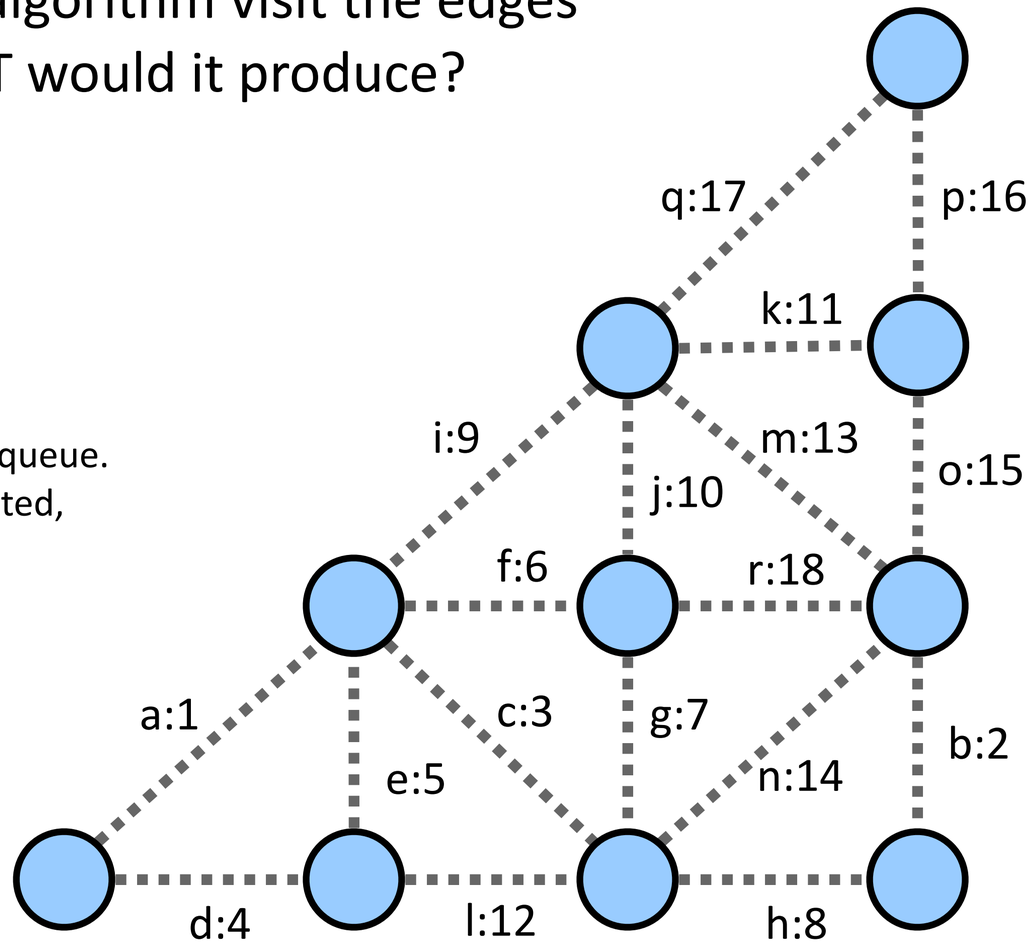
- **Kruskal's algorithm**: Finds a MST in a given graph.

function **kruskal**(graph):
Remove all edges from the graph.
Place all edges into a **priority queue** based on their weight (cost).
While the priority queue is not empty:
    Dequeue an edge *e* from the priority queue.
    If *e*'s endpoints aren't already connected to one another,
        add that edge into the graph.
    Otherwise, skip the edge.

Kruskal Example

- In what order would Kruskal's algorithm visit the edges in the graph below? What MST would it produce?

function **kruskal**(graph):
 Remove all edges from the graph.
 Place all edges into a priority queue
 based on their weight (cost).
 While the priority queue is not empty:
 Dequeue an edge *e* from the priority queue.
 If *e*'s endpoints aren't already connected,
 add that edge into the graph.
 Otherwise, skip the edge.



pq = {a:1, b:2, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

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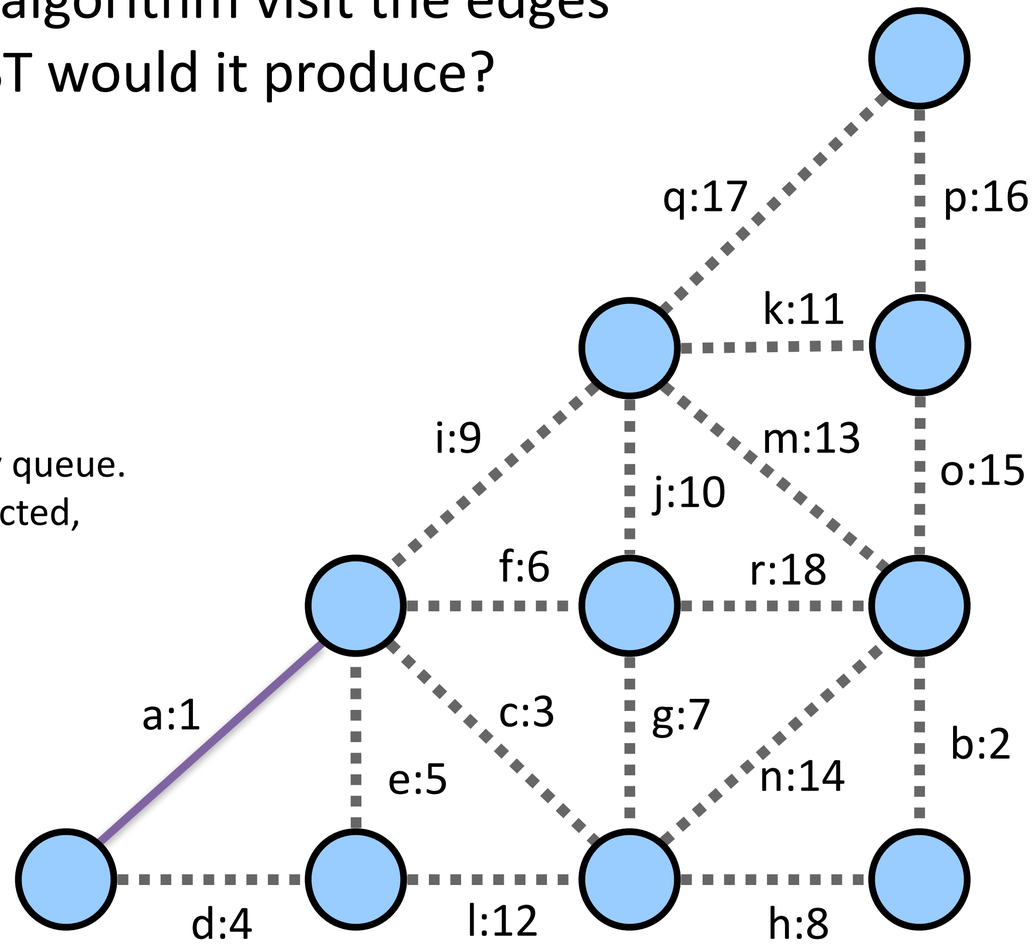

pq = {**a:1**, b:2, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

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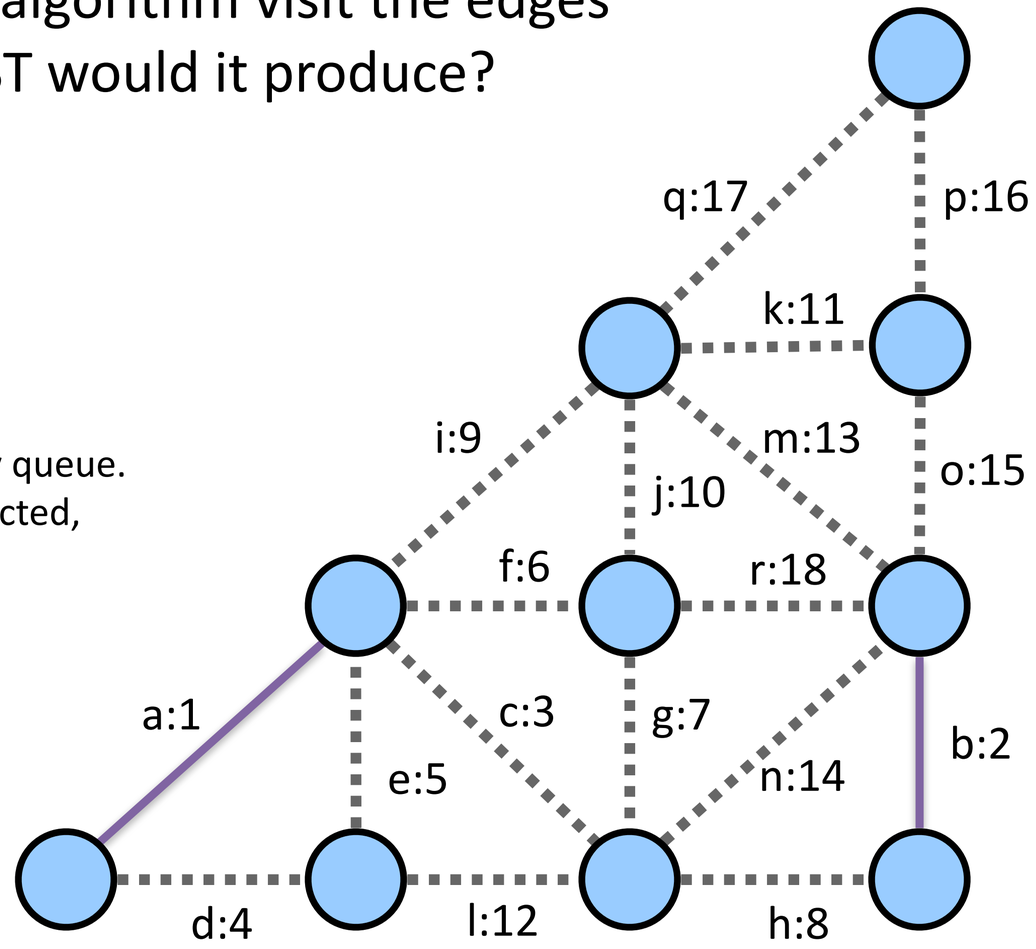

pq = {**b:2**, c:3, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

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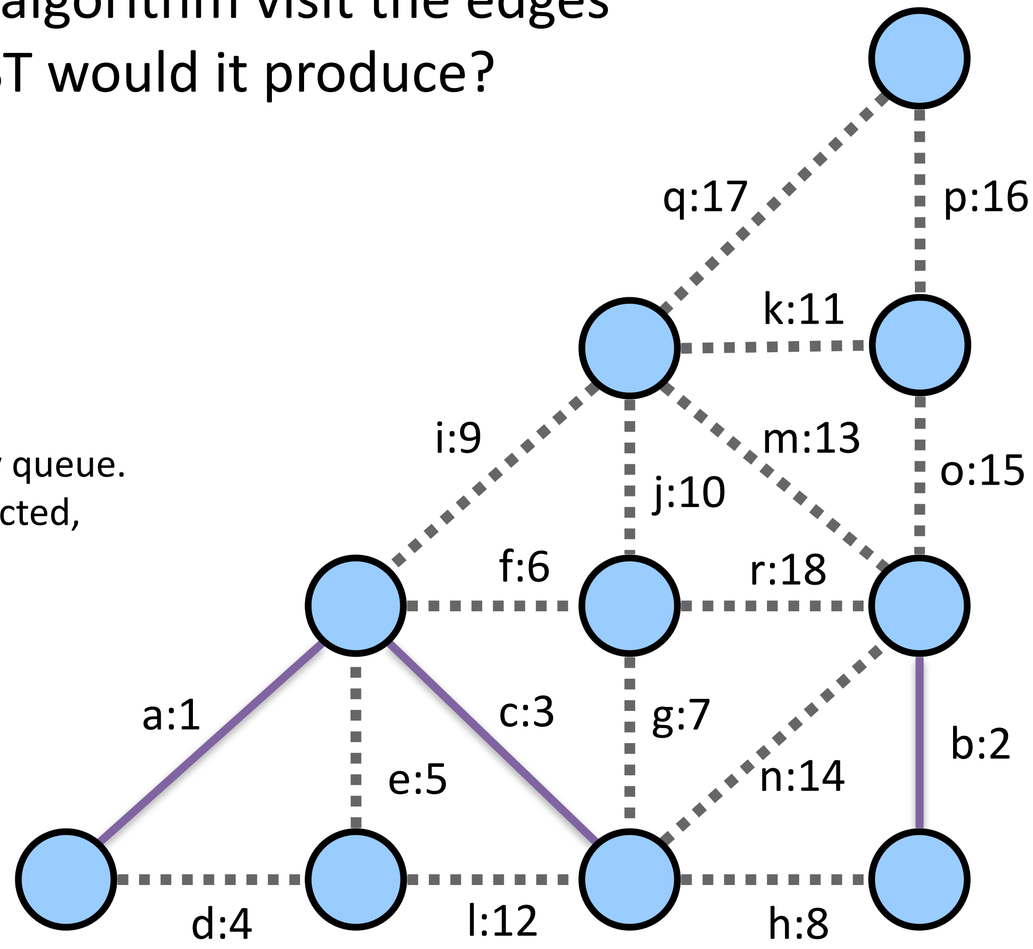

pq = {**c:3**, d:4, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

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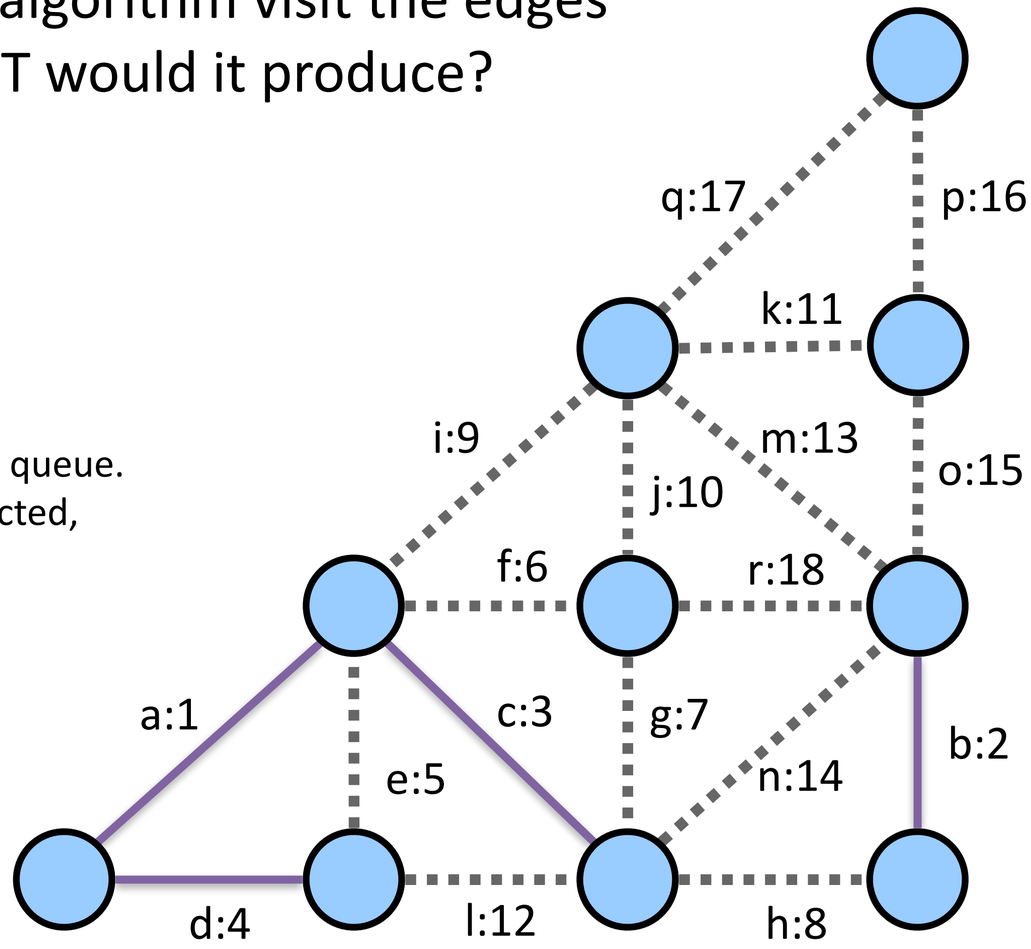

pq = {**d:4**, e:5, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

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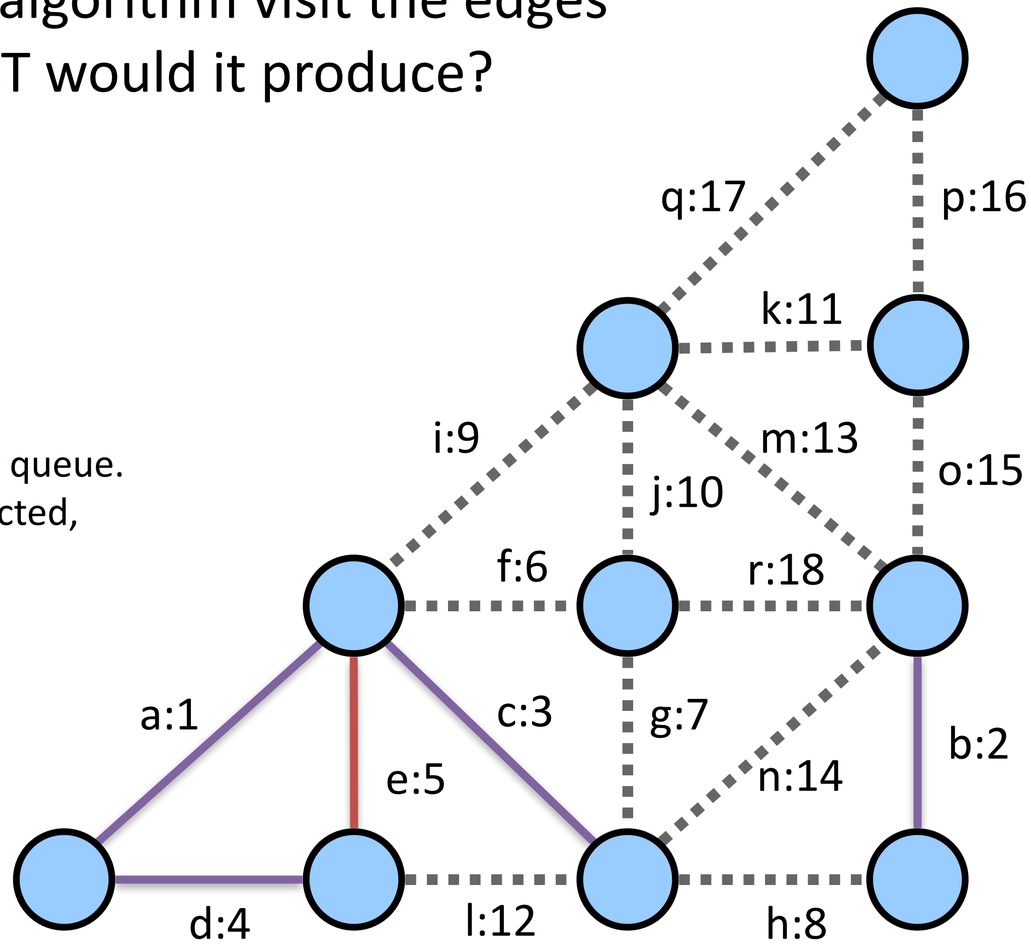

pq = {**e:5**, f:6, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

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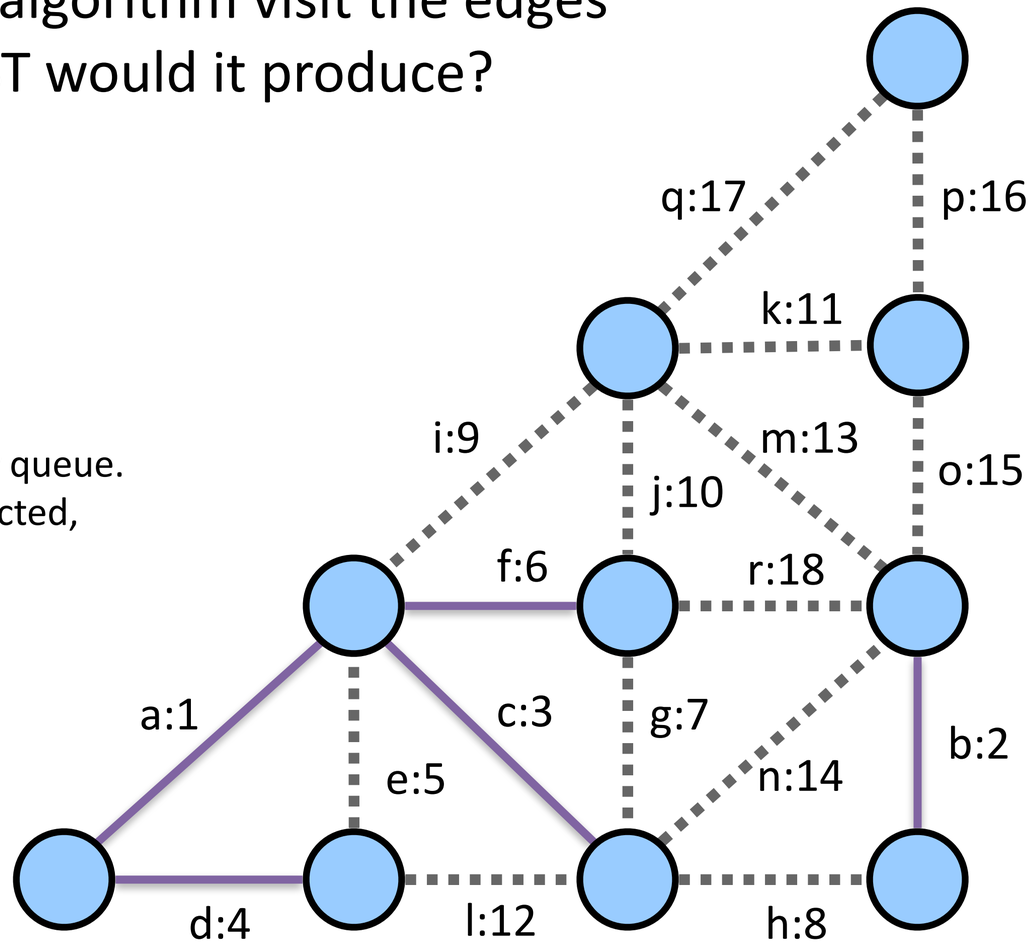

pq = {**f:6**, g:7, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

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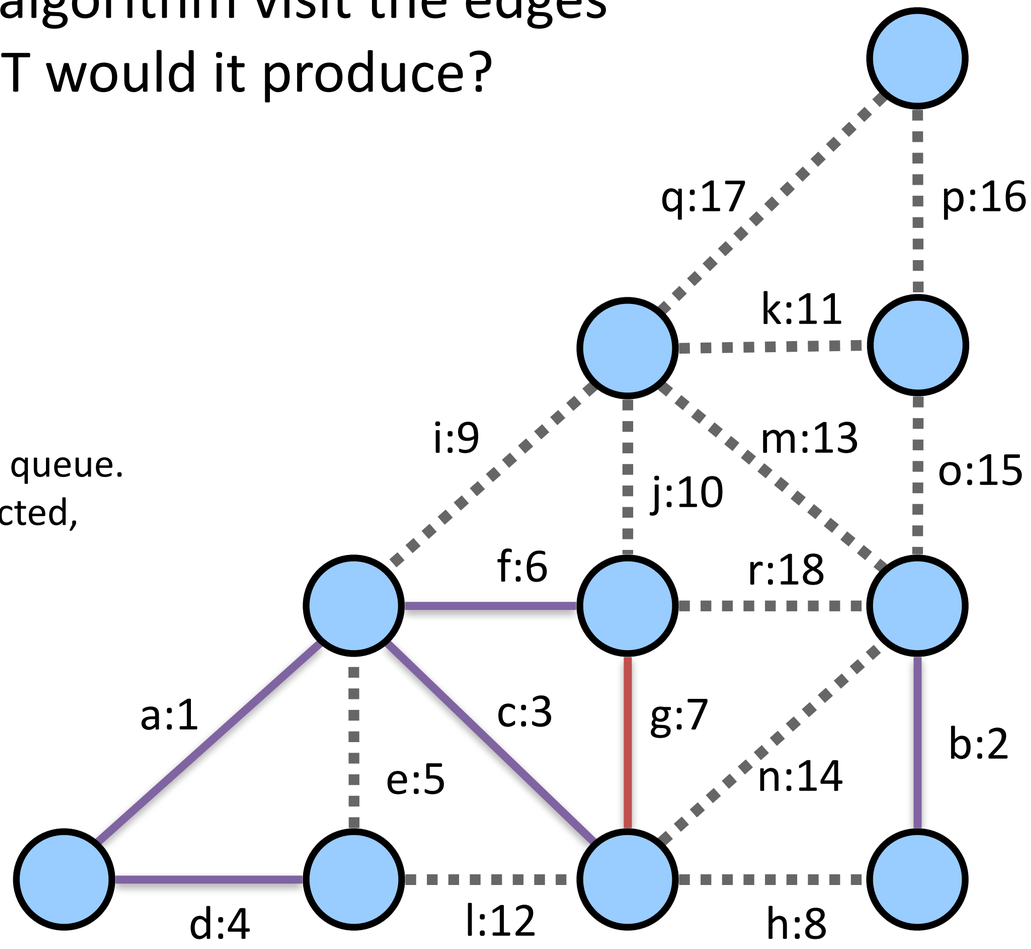

pq = {**g:7**, h:8, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

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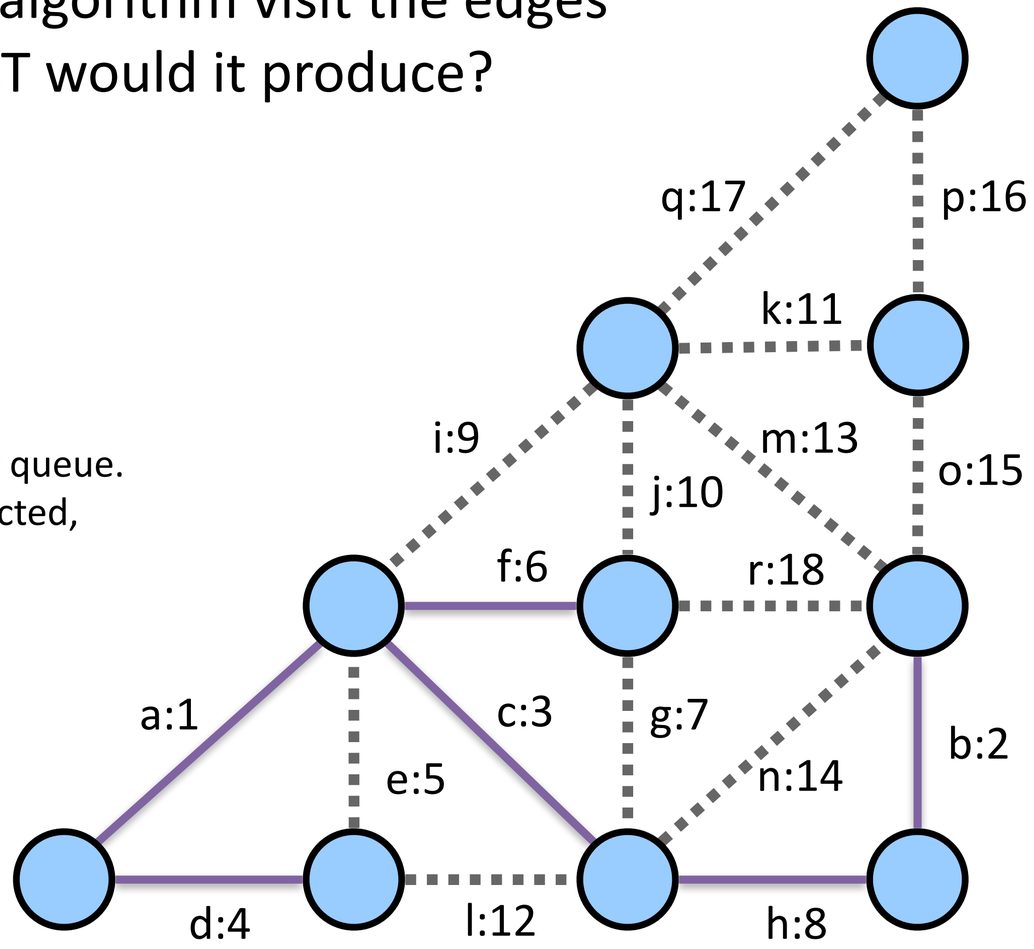

pq = {**h:8**, i:9, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

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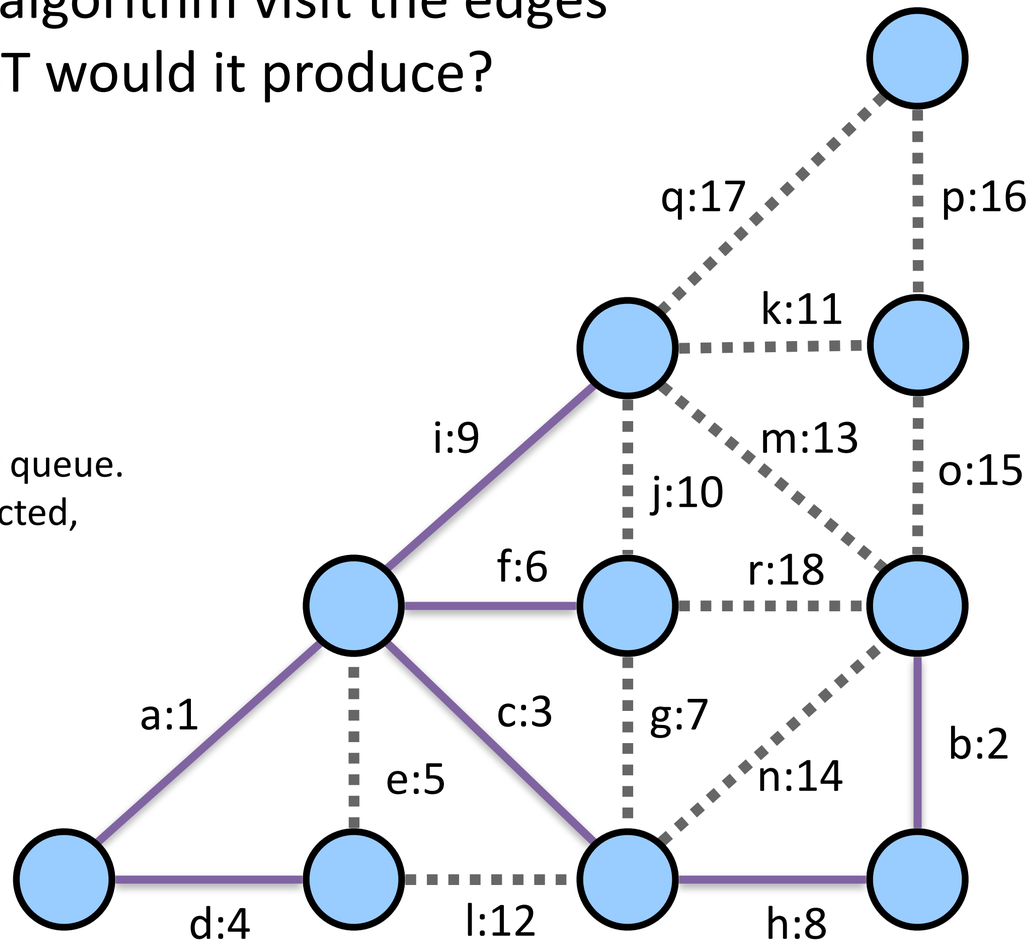

pq = {**i:9**, j:10, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

Kruskal Example

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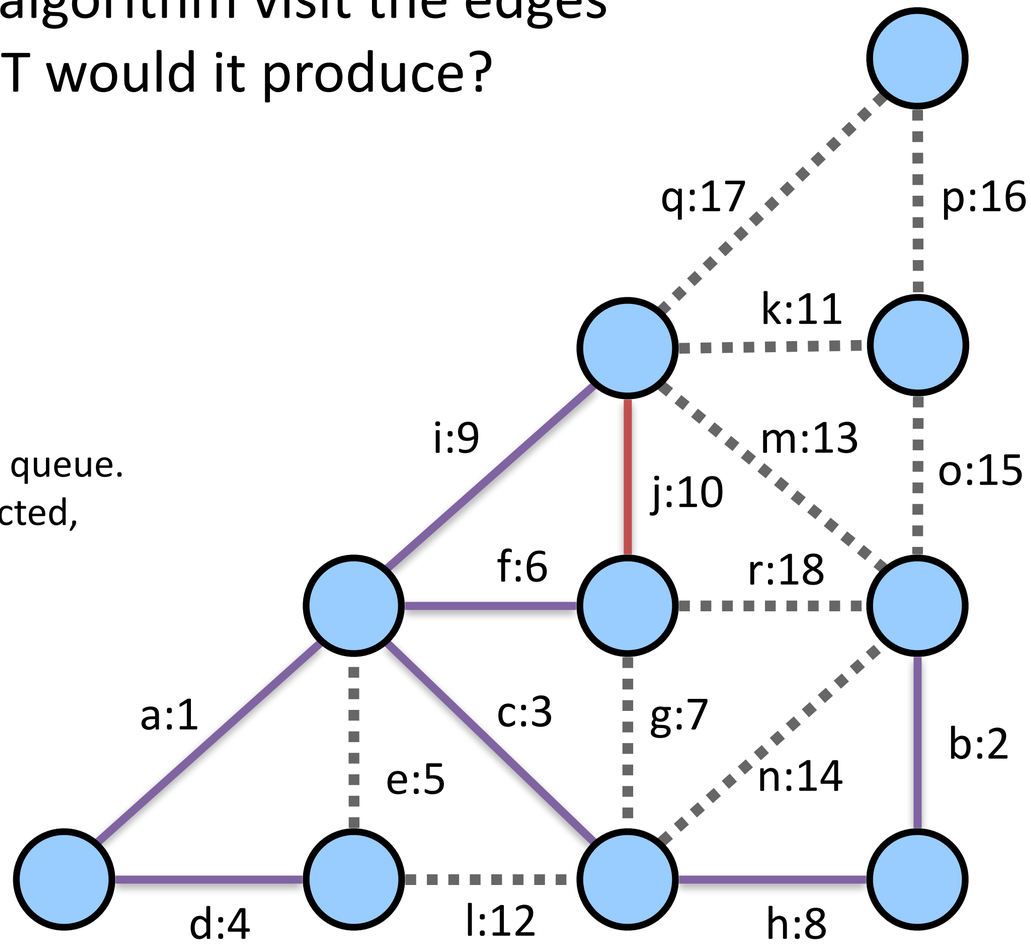

pq = {**j:10**, k:11, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

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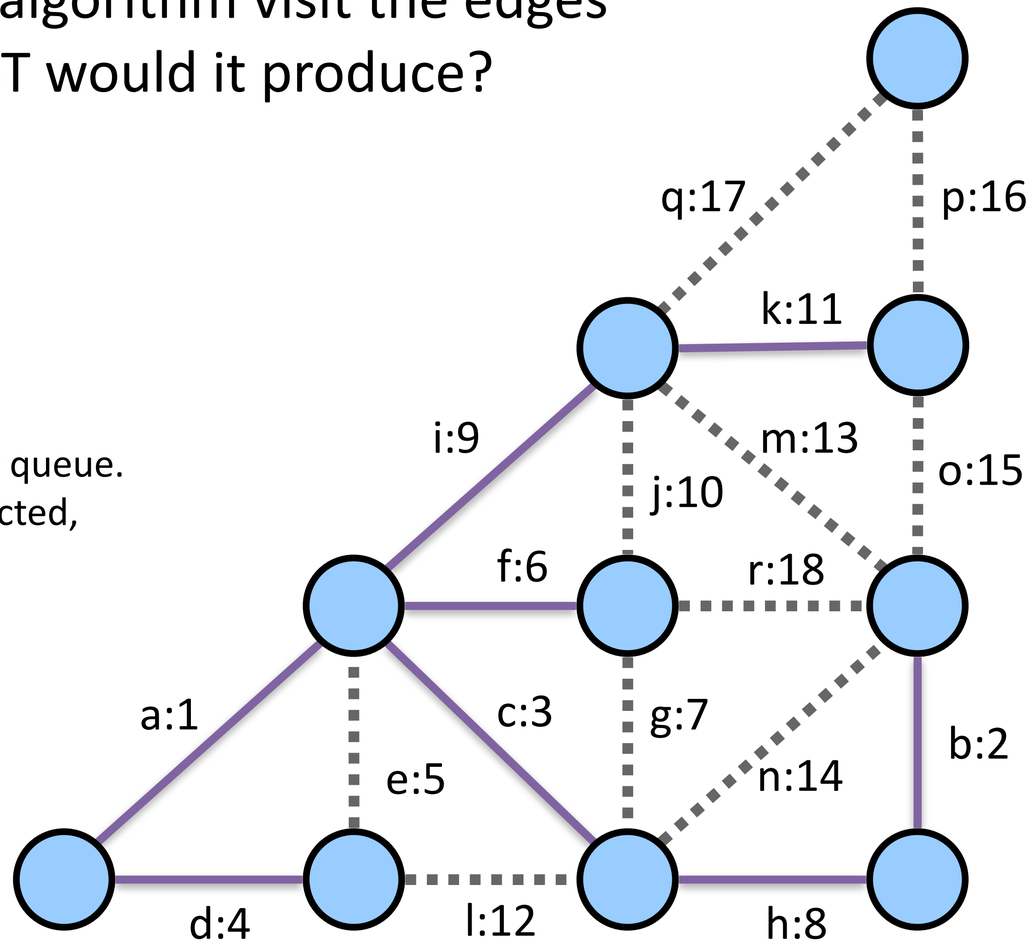

pq = {**k:11**, l:12, m:13, n:14, o:15, p:16, q:17, r:18}

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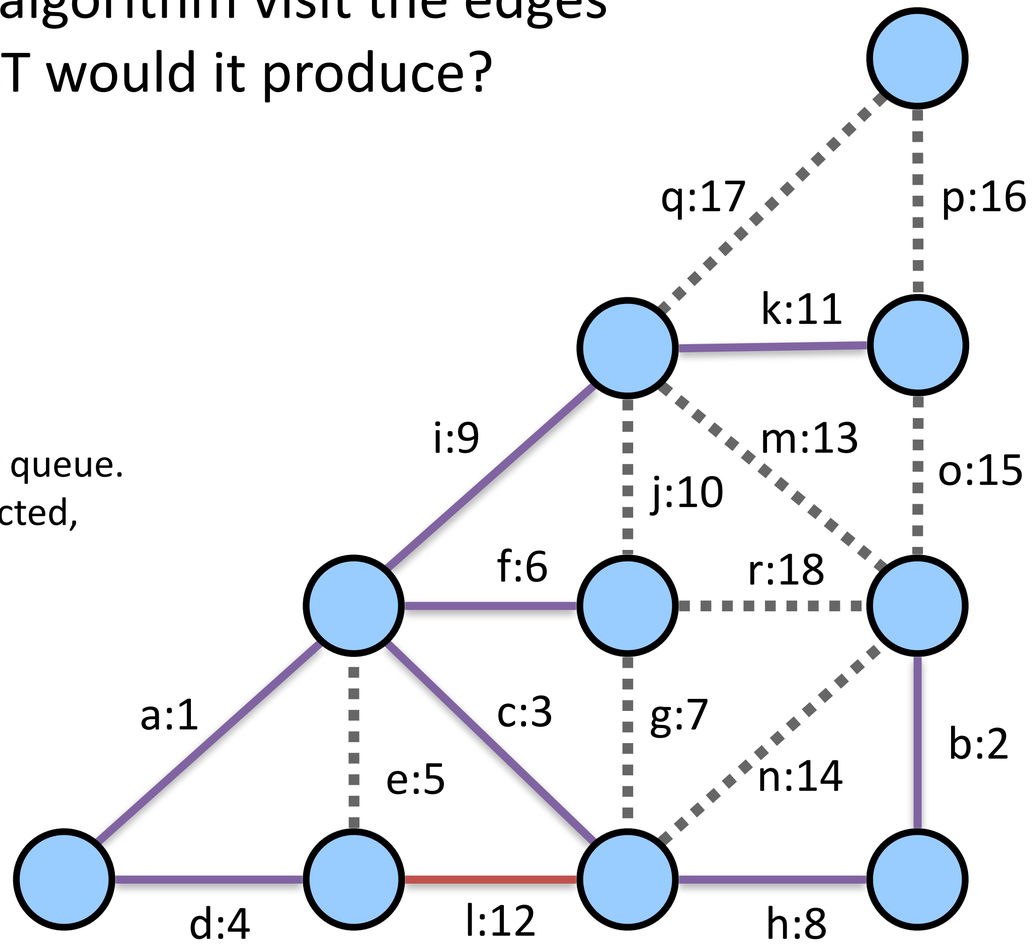

pq = {**l:12**, m:13, n:14, o:15, p:16, q:17, r:18}

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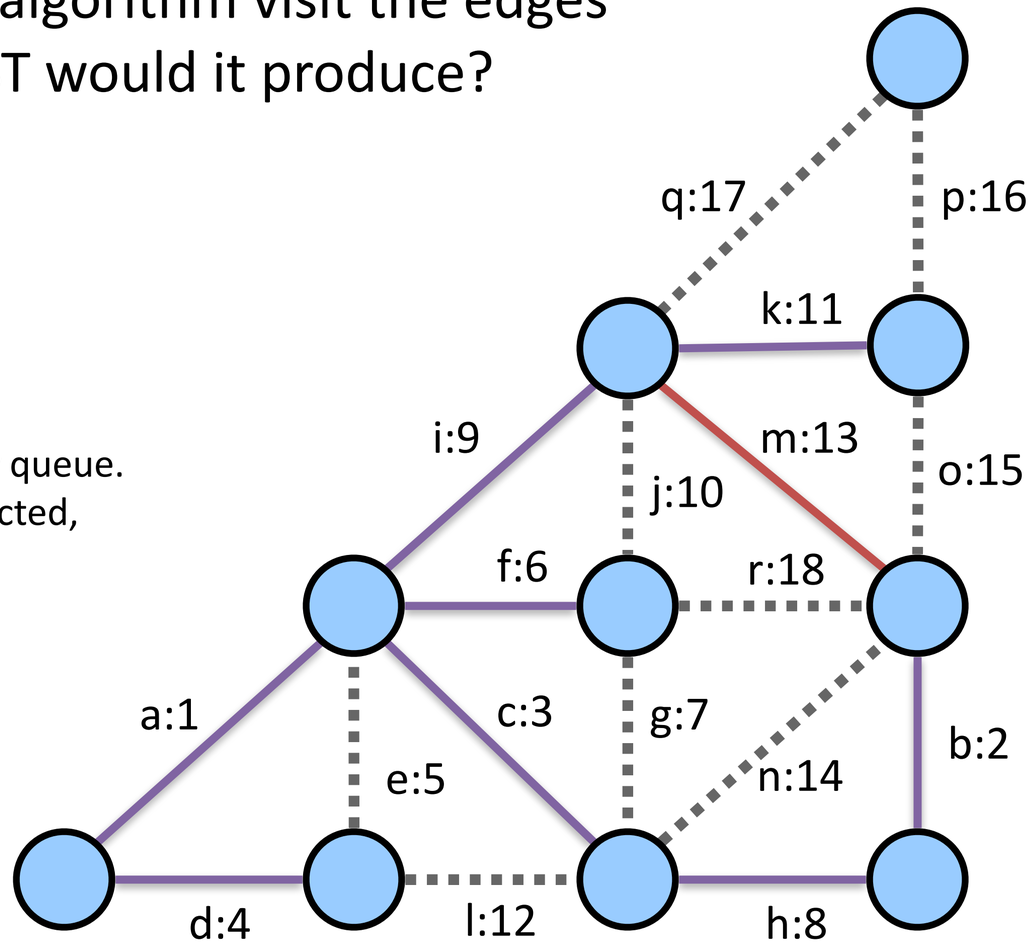

pq = {**m:13**, n:14, o:15, p:16, q:17, r:18}

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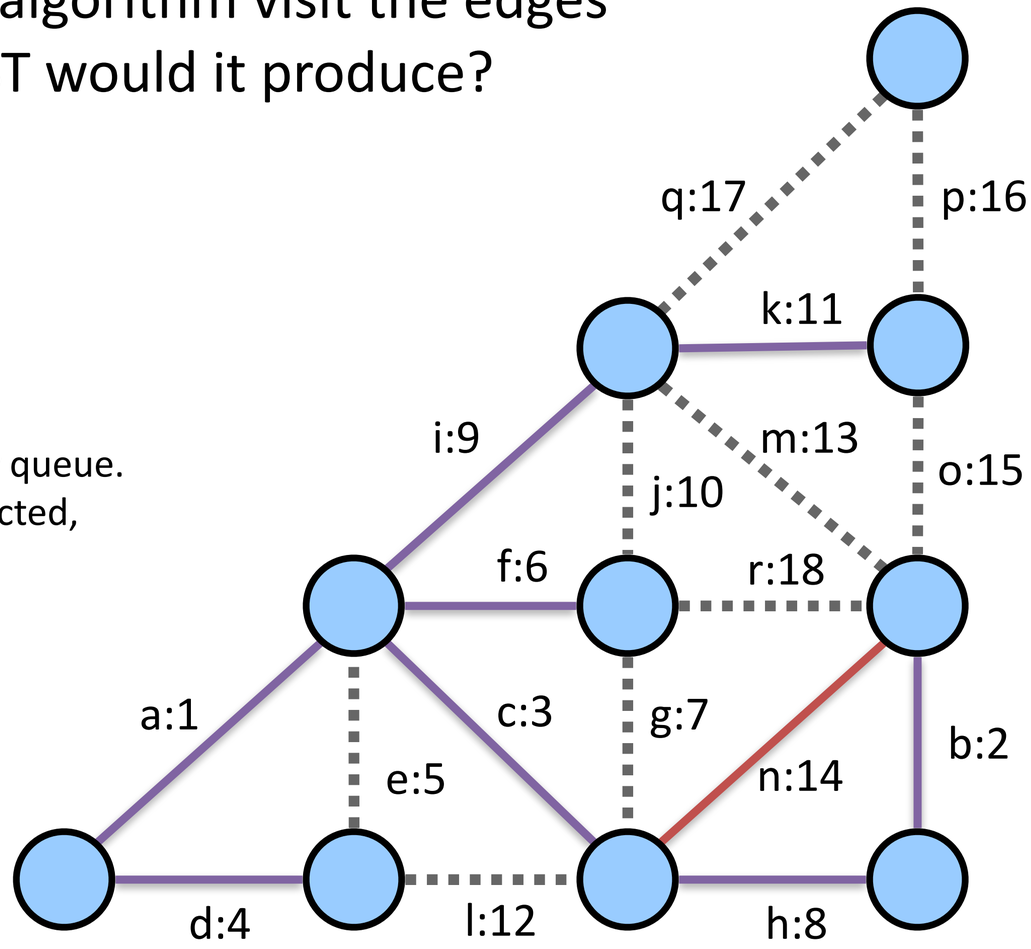

pq = {**n:14**, o:15, p:16, q:17, r:18}

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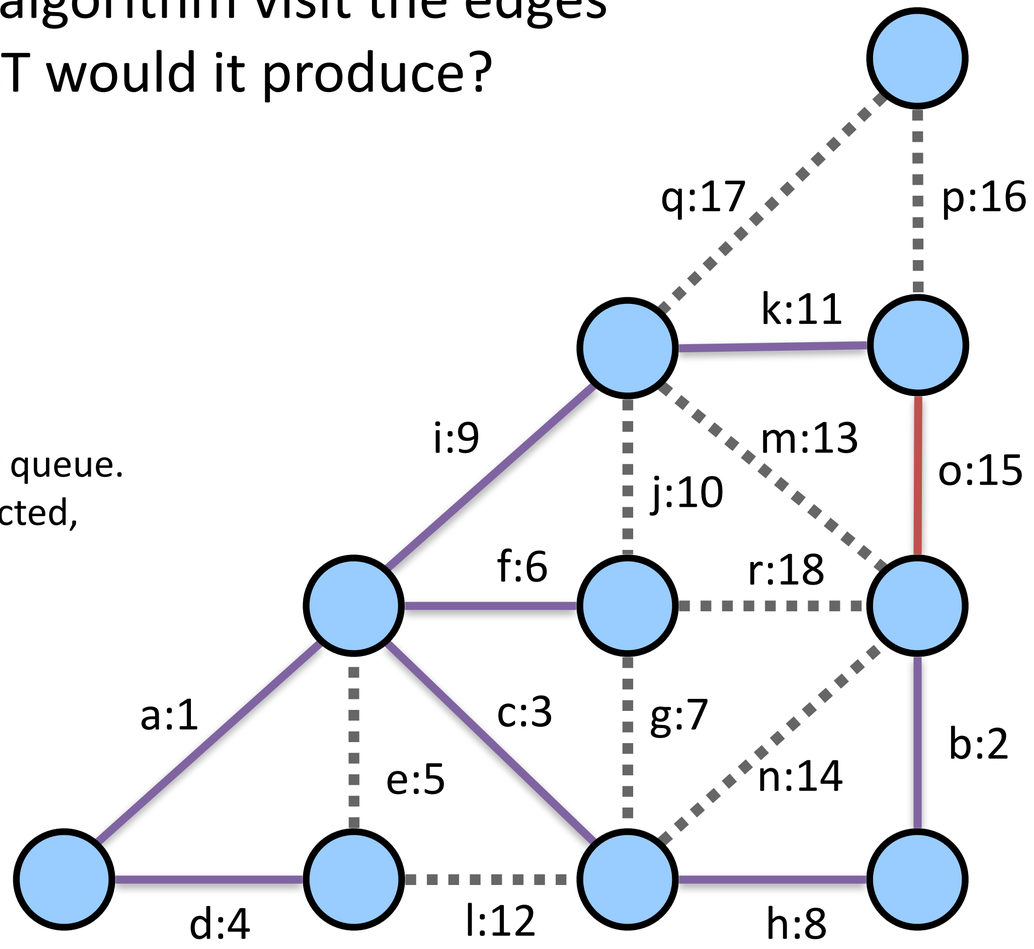

pq = {**o:15**, p:16, q:17, r:18}

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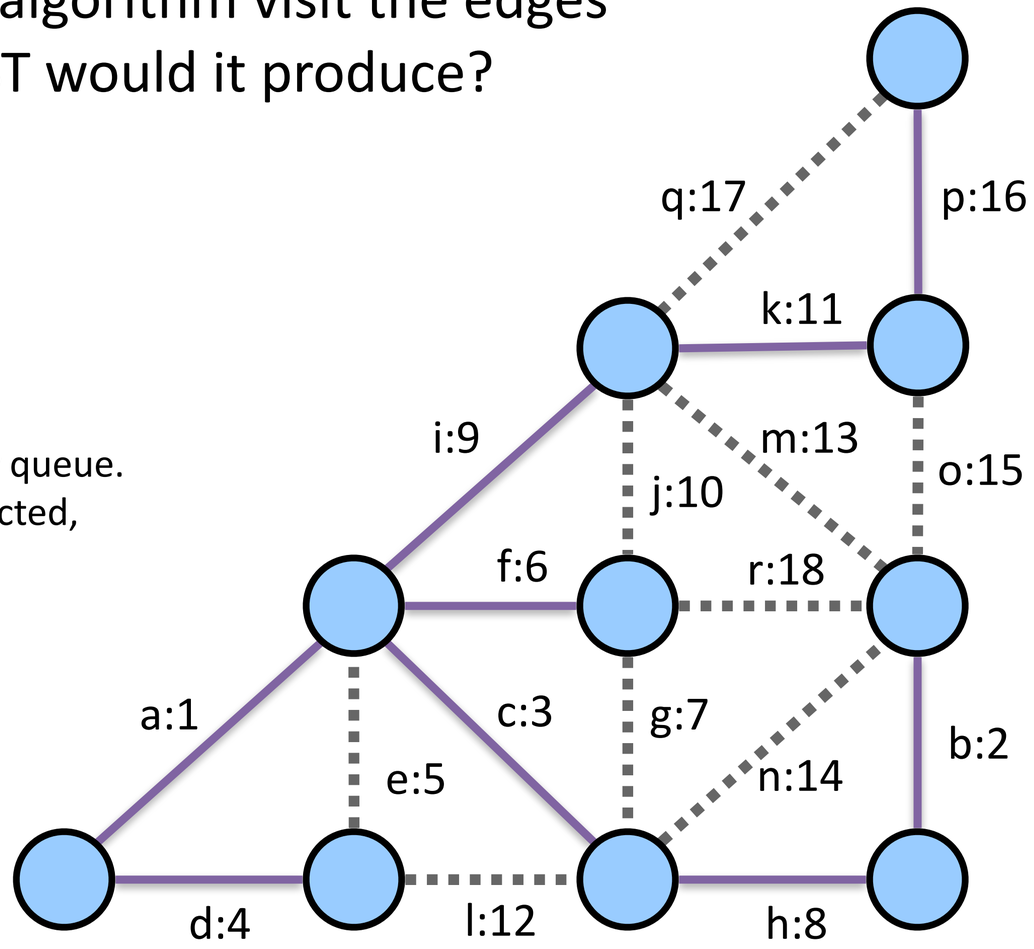

pq = {**p:16**, q:17, r:18}

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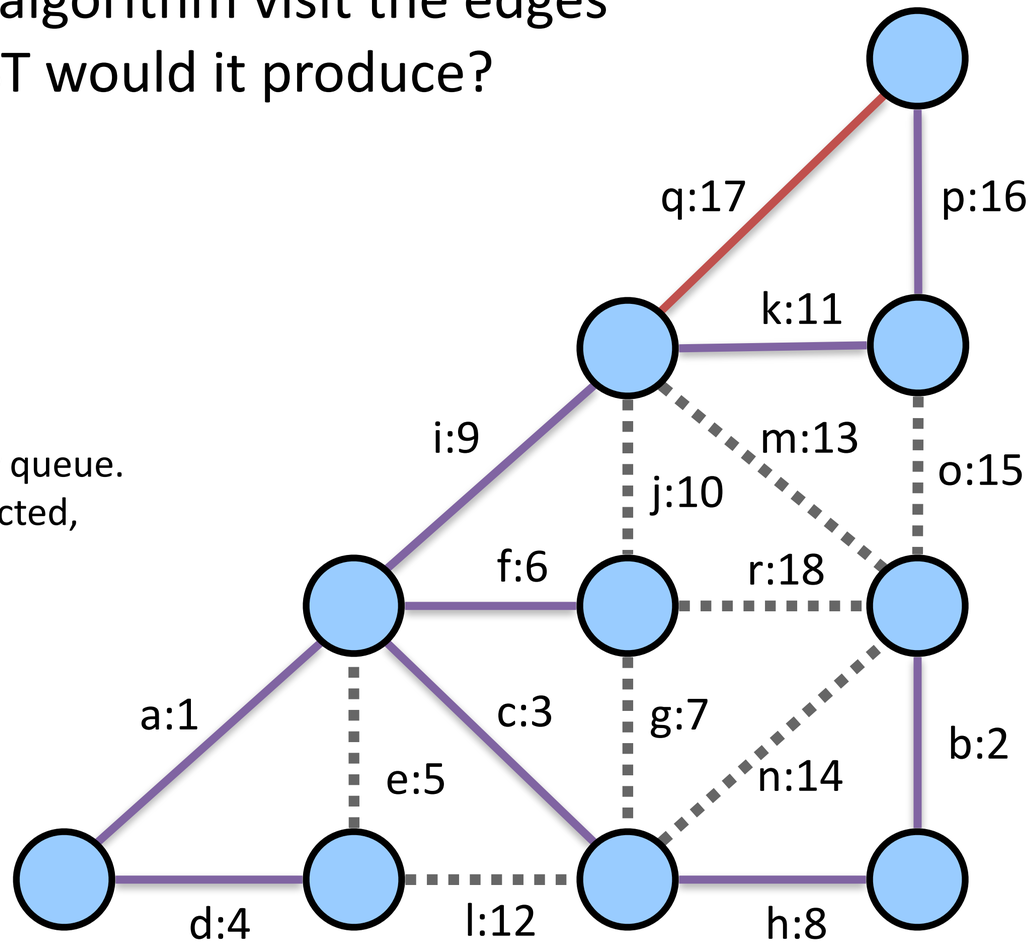

pq = {**q:17**, r:18}

Kruskal Example

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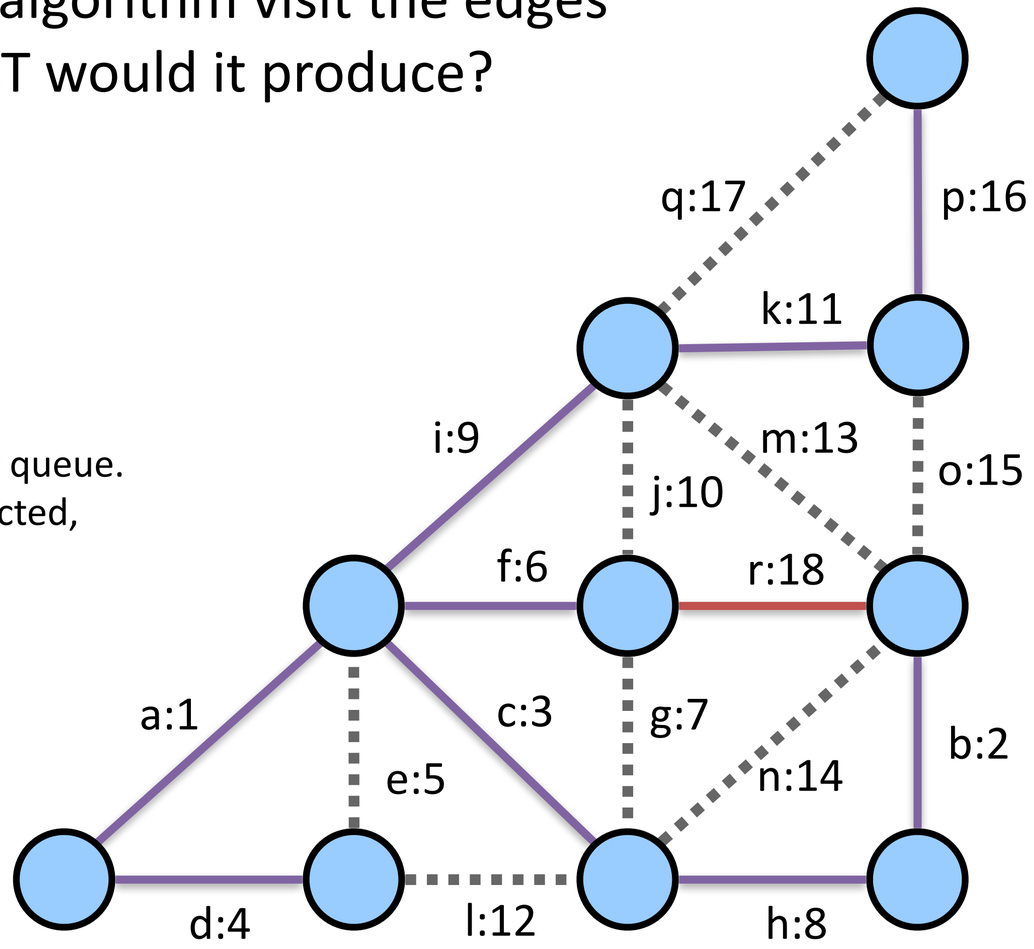

pq = {**r:18**}

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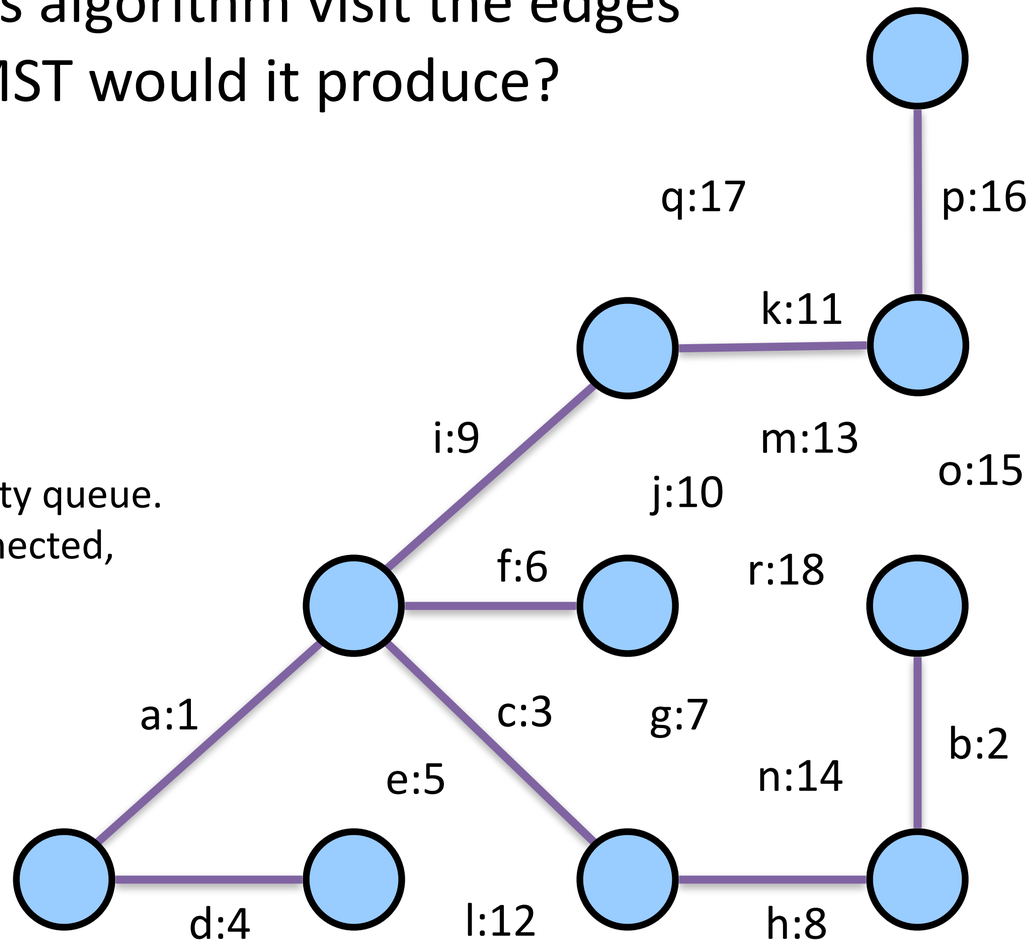


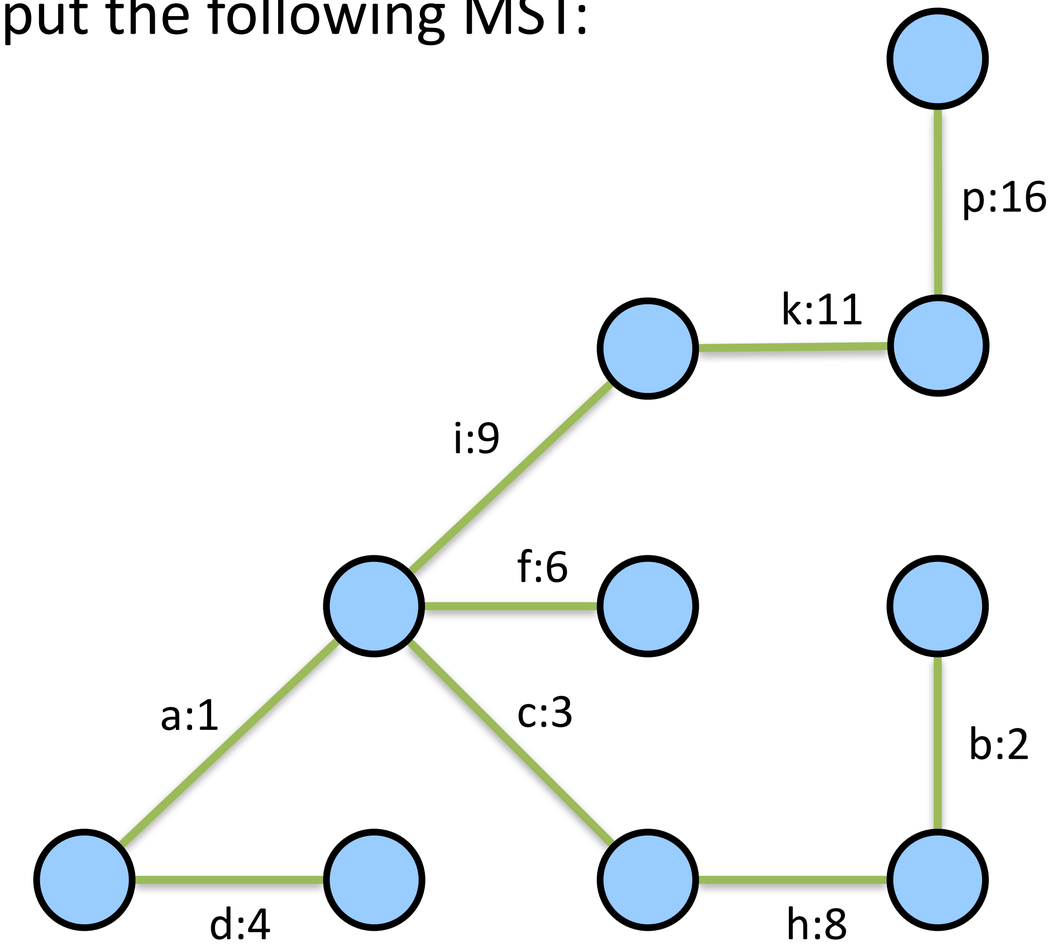
pq = {}

Kruskal Example

- Kruskal's algorithm would output the following MST:
  - {a, b, c, d, f, h, i, k, p}

- The MST's total cost is:

  1+2+3+4+6+8+9+11+16 = 60



- What data structures should we use to implement this algorithm?

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Remove all edges from the graph.
Place all edges into a **priority queue**
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While the priority queue is not empty:
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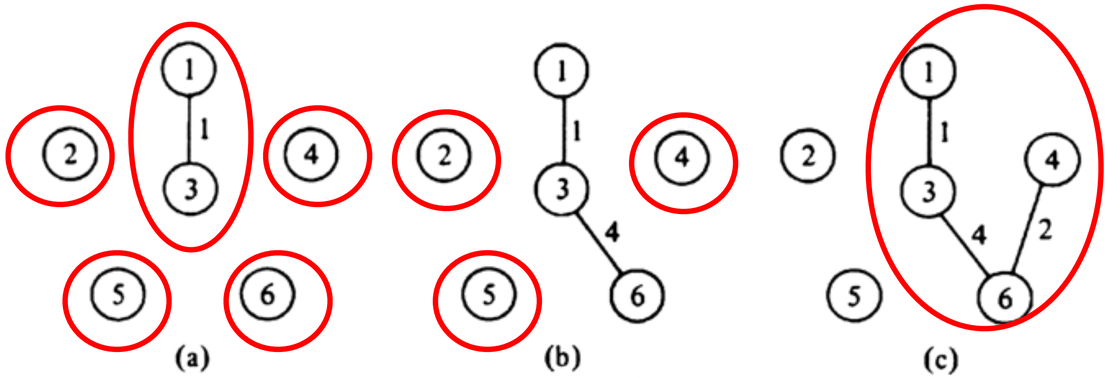
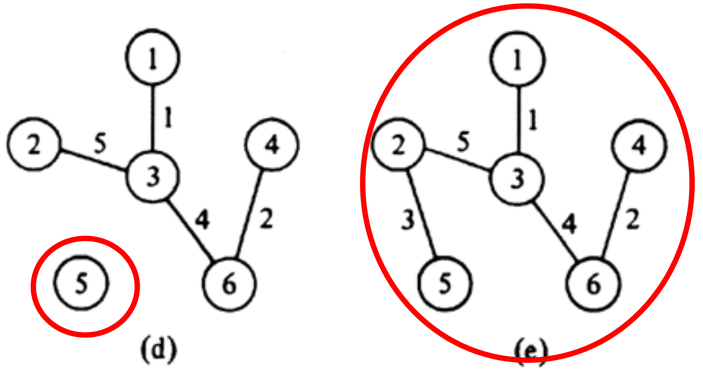
add that edge into the graph.
Otherwise, skip the edge.

- Need some way to identify which vertexes are "connected" to which other ones
  - we call these "**clusters**" of vertices

- Also need an efficient way to figure out which cluster a given vertex is in.



- Also need to **merge clusters** when adding an edge.



References and Advanced Reading

- **References:**
  - Minimum Spanning Tree visualization: https://visualgo.net/mst
  - Kruskal's Algorithm: https://en.wikipedia.org/wiki/Kruskal's_algorithm

- **Advanced Reading:**
  - How Internet Routing works: https://web.stanford.edu/class/msande91si/www-spr04/readings/week1/InternetWhitepaper.htm
  - http://www.explainthatstuff.com/internet.html

Extra Slides