CS 106B

Lecture 11: Sorting

Thursday, July 13, 2017

Programming Abstractions
Summer 2017
Stanford University
Computer Science Department

Lecturer: Chris Gregg

reading:
Programming Abstractions in C++, Section 10.2

Today's Topics

- Logistics
  - Midterm Review -- Monday (details on Piazza)
  - Midterm: Next **Wednesday/Thursday**, in class
  - Test BlueBook before coming to class — a good way is to do the practice exams

- Sorting
  - Insertion Sort
  - Selection Sort
  - Merge Sort
  - Quicksort
  - Other sorts you might want to look at:
    - Radix Sort
    - Shell Sort
    - Tim Sort
    - Heap Sort (we will cover heaps later in the course)
    - Bogosort

Sorting!

- In general, sorting consists of putting elements into a particular order, most often the order is numerical or lexicographical (i.e., alphabetic).
- In order for a list to be sorted, it must:
  - be in nondecreasing order (each element must be no smaller than the previous element)
  - be a permutation of the input

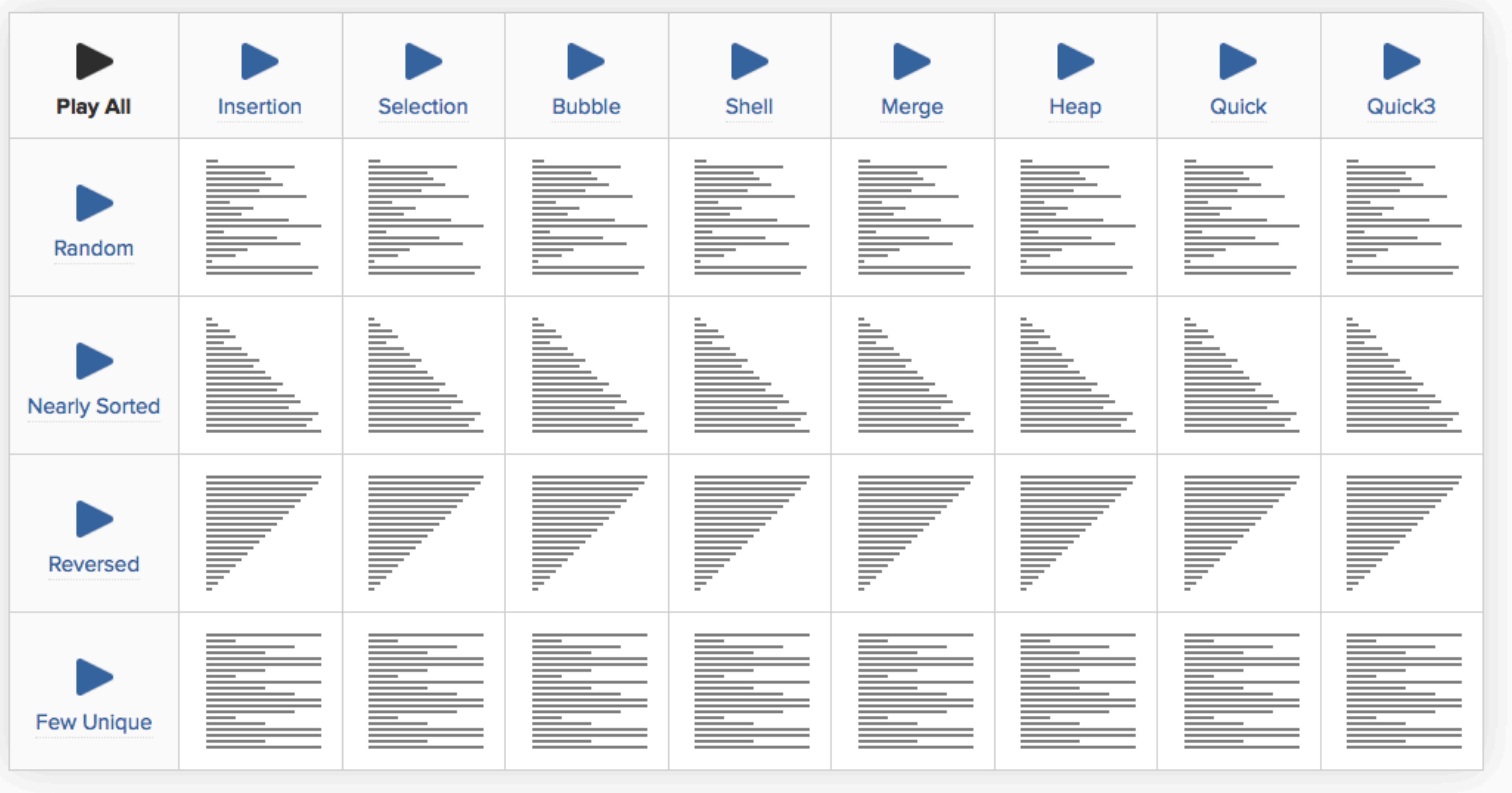
Sorting!

- Sorting is a well-researched subject, although new algorithms do arise (see Timsort, from 2002)
- Fundamentally, *comparison* sorts at best have a complexity of **O(n log n)**.
- We also need to consider the space complexity: some sorts can be done in place, meaning the sorting does not take extra memory. This can be an important factor when choosing a sorting algorithm!

(must sort)

Sorting!

- In-place sorting can be "stable" or "unstable": a stable sort retains the order of elements with the same key, from the original unsorted list to the final, sorted, list
- There are some phenomenal online sorting demonstrations: see the "Sorting Algorithm Animations" website:



- http://www.sorting-algorithms.com, or the animation site at: http://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html or the cool "15 sorts in 6 minutes" video on YouTube: https://www.youtube.com/watch?v=kPRA0W1kECg

Sorts

- There are many, many different ways to sort elements in a list. We will look at the following:

Insertion Sort
Selection Sort
Merge Sort
Quicksort

Sorts

Insertion Sort

Selection Sort

Merge Sort

Quicksort

Insertion Sort

Insertion sort: orders a list of values by repetitively inserting a particular value into a sorted subset of the list

More specifically:
– consider the first item to be a sorted sublist of length 1
– insert second item into sorted sublist, shifting first item if needed
– insert third item into sorted sublist, shifting items 1-2 as needed
– ...
– repeat until all values have been inserted into their proper positions

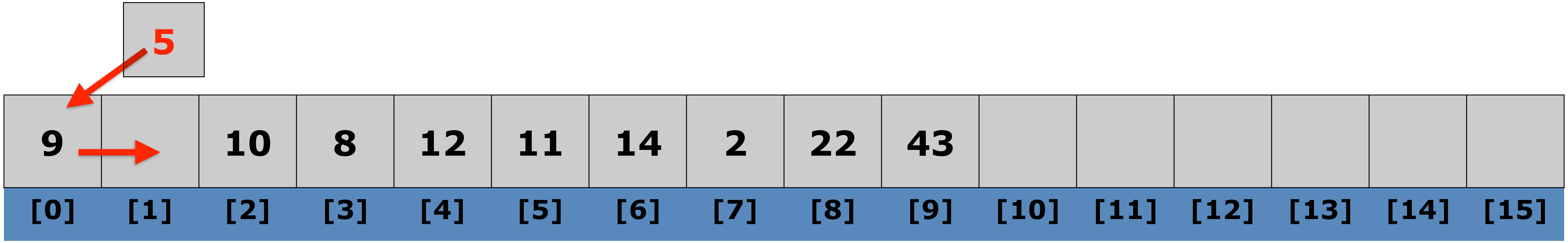
Insertion Sort

9	5	10	8	12	11	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

Algorithm:

- iterate through the list (starting with the second element)
- at each element, shuffle the neighbors below that element up until the proper place is found for the element, and place it there.

Insertion Sort



5

9		10	8	12	11	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

Algorithm:

- iterate through the list (starting with the second element)
- at each element, shuffle the neighbors below that element up until the proper place is found for the element, and place it there.

Insertion Sort

in place already (i.e., already bigger than 9)

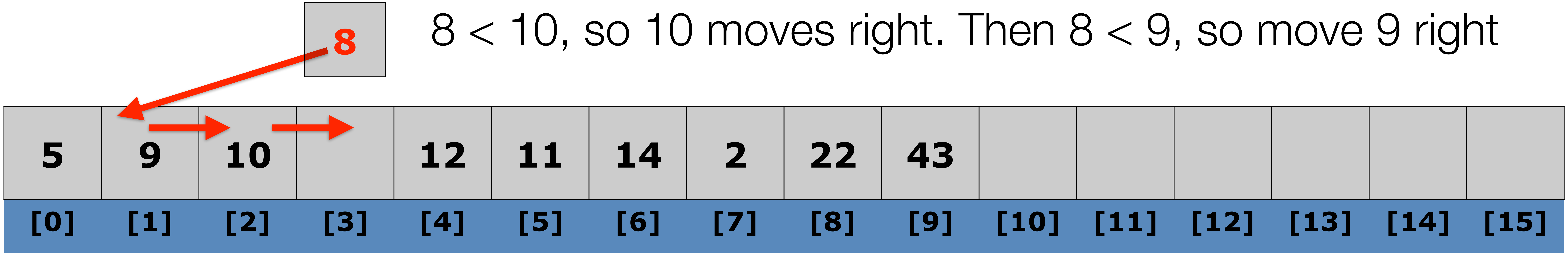
5	9	10	8	12	11	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

Algorithm:

- iterate through the list (starting with the second element)
- at each element, shuffle the neighbors below that element up until the proper place is found for the element, and place it there.

Insertion Sort

8 < 10, so 10 moves right. Then 8 < 9, so move 9 right



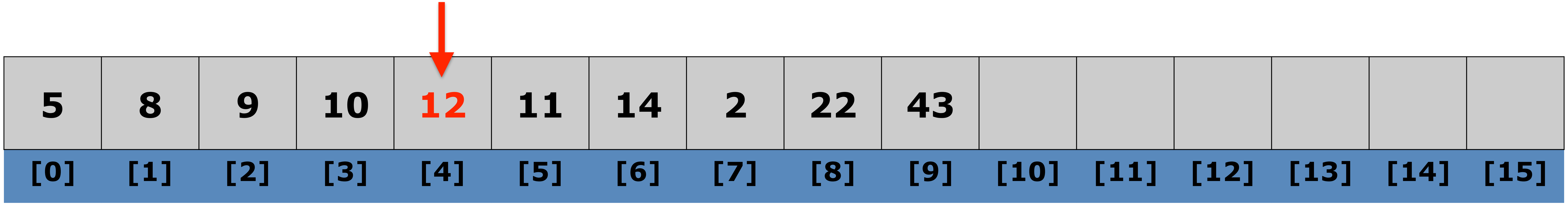
5	9	10		12	11	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

Algorithm:

- iterate through the list (starting with the second element)
- at each element, shuffle the neighbors below that element up until the proper place is found for the element, and place it there.

Insertion Sort

in place already (i.e., already bigger than 10)



5	8	9	10	12	11	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

Algorithm:

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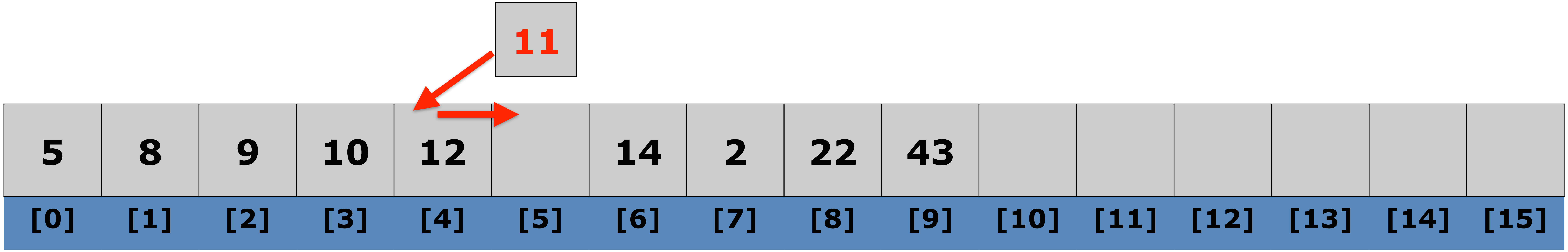
Insertion Sort



11

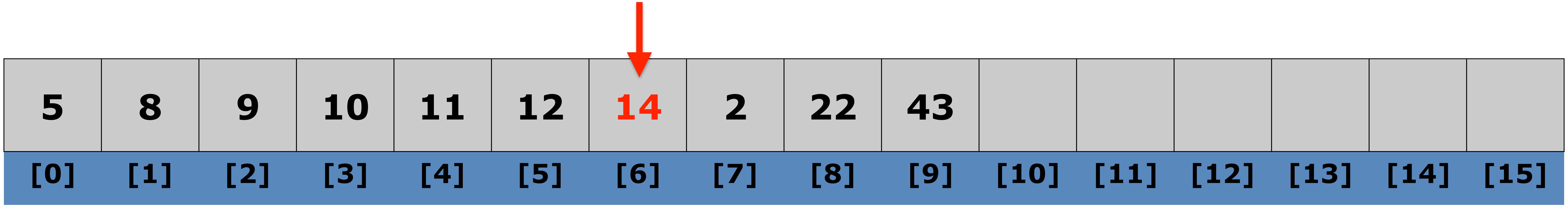
5	8	9	10	12		14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

Algorithm:

- iterate through the list (starting with the second element)
- at each element, shuffle the neighbors below that element up until the proper place is found for the element, and place it there.

Insertion Sort

in place already (i.e., already bigger than 12)



5	8	9	10	11	12	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

Algorithm:

- iterate through the list (starting with the second element)
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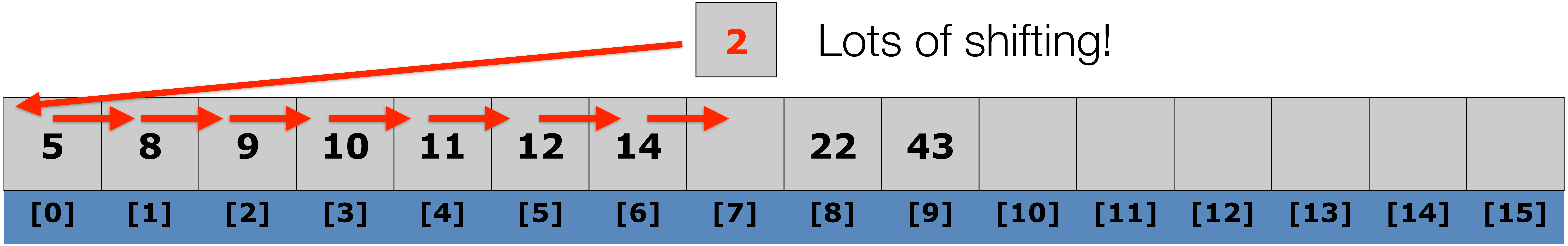
Insertion Sort



2

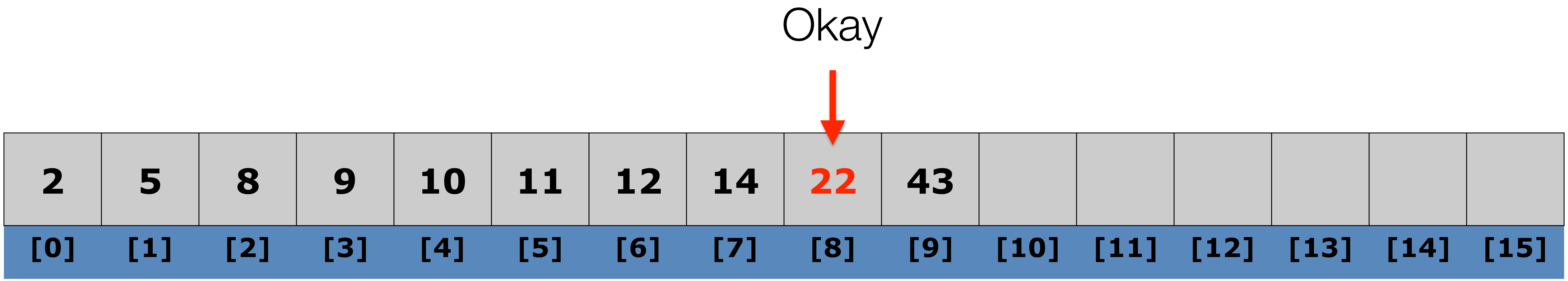
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
5	8	9	10	11	12	14		22	43						

Lots of shifting!

Algorithm:

- iterate through the list (starting with the second element)
- at each element, shuffle the neighbors below that element up until the proper place is found for the element, and place it there.

Insertion Sort



Okay

2	5	8	9	10	11	12	14	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

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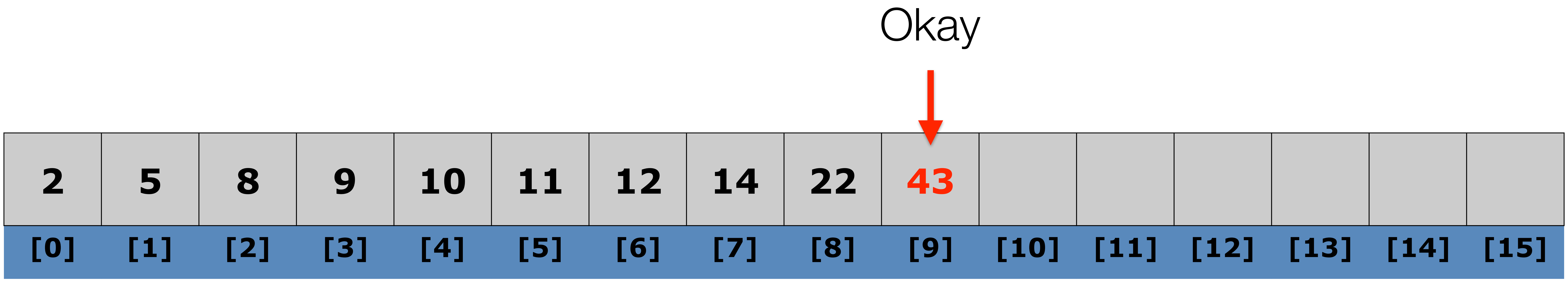
Insertion Sort



Okay

2	5	8	9	10	11	12	14	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

Complexity:

Worst performance: $O(n^2)$ (why? -- see extra slide!)

Best performance: $O(n)$

–Average performance: $O(n^2)$ (but very fast for small arrays!)

–Worst case space complexity: $O(n)$ total (plus one for swapping)

Insertion Sort Code

```
// Rearranges the elements of v into sorted order.
void insertionSort(Vector<int>& v) {
    for (int i = 1; i < v.size(); i++) {
        int temp = v[i];
        // slide elements right to make room for v[i]
        int j = i;
        while (j >= 1 && v[j - 1] > temp) {
            v[j] = v[j - 1];
            j--;
        }
        v[j] = temp;
    }
}
```

Sorts

Insertion Sort

Selection Sort

Merge Sort

Quicksort

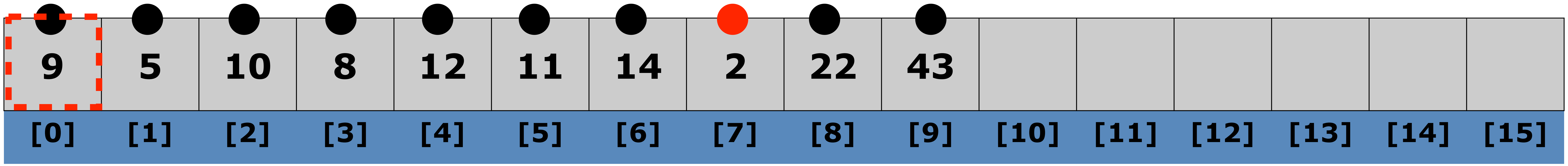
Selection Sort

9	5	10	8	12	11	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

- Selection Sort is another in-place sort that has a simple algorithm:
  - Find the smallest item in the list, and exchange it with the left-most unsorted element.
  - Repeat the process from the first unsorted element.

- See animation at: http://www.cs.usfca.edu/~galles/visualization/ComparisonSort.html

Selection Sort

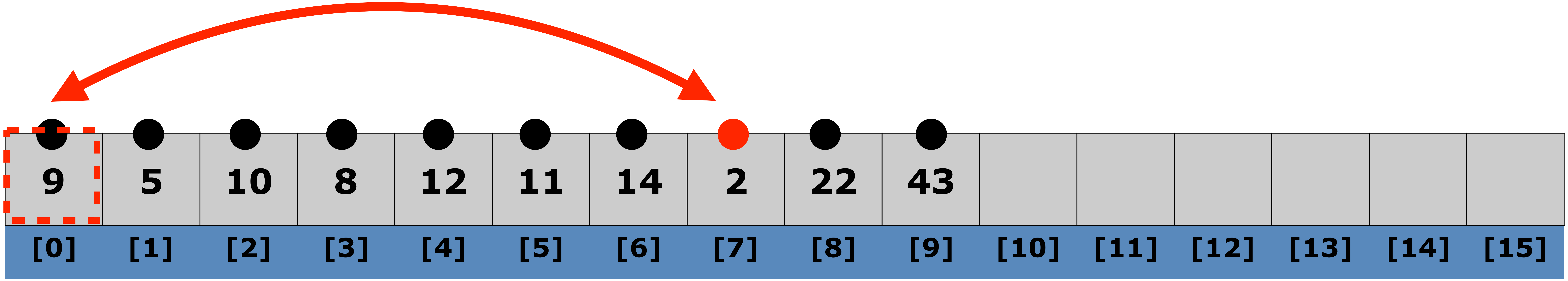


9	5	10	8	12	11	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

- Algorithm
  - Find the **smallest item in the list**, and exchange it with the left-most unsorted element.
  - Repeat the process from the first unsorted element.

- Selection sort is particularly slow, because it needs to go through **the entire list** each time to find the smallest item.

Selection Sort



9	5	10	8	12	11	14	2	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

- Algorithm
  - Find the **smallest item in the list**, and exchange it with the left-most unsorted element.
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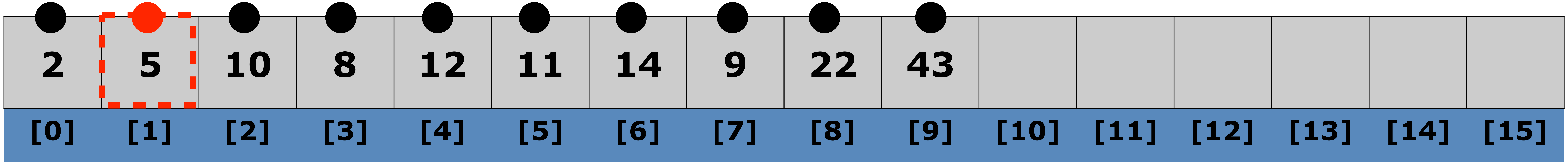
- Selection sort is particularly slow, because it needs to go through **the entire list** each time to find the smallest item.

Selection Sort

(no swap necessary)



2	5	10	8	12	11	14	9	22	43						
[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]

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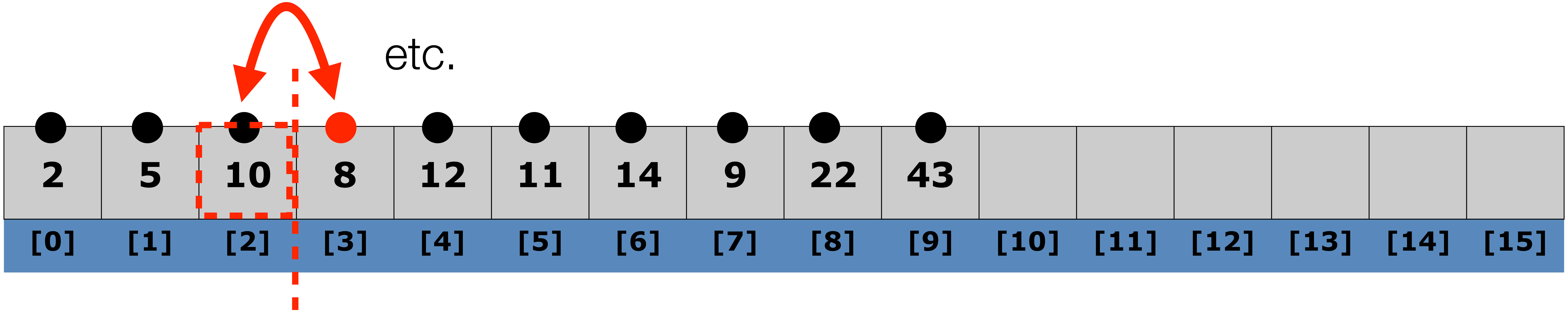
Selection Sort



- Complexity:
  - Worst performance: O(n^2)
  - Best performance: O(n^2)
  - Average performance: O(n^2)
  - Worst case space complexity: O(n) total (plus one for swapping)

Selection Sort Code

```
// Rearranges elements of v into sorted order
// using selection sort algorithm
void selectionSort(Vector<int>& v) {
    for (int i = 0; i < v.size() - 1; i++) {
        // find index of smallest remaining value
        int min = i;
        for (int j = i + 1; j < v.size(); j++) {
            if (v[j] < v[min]) {
                min = j;
            }
        }
        // swap smallest value to proper place, v[i]
        if (i != min) {
            int temp = v[i];
            v[i] = v[min];
            v[min] = temp;
        }
    }
}
```

Sorts

Insertion Sort

Selection Sort

Merge Sort

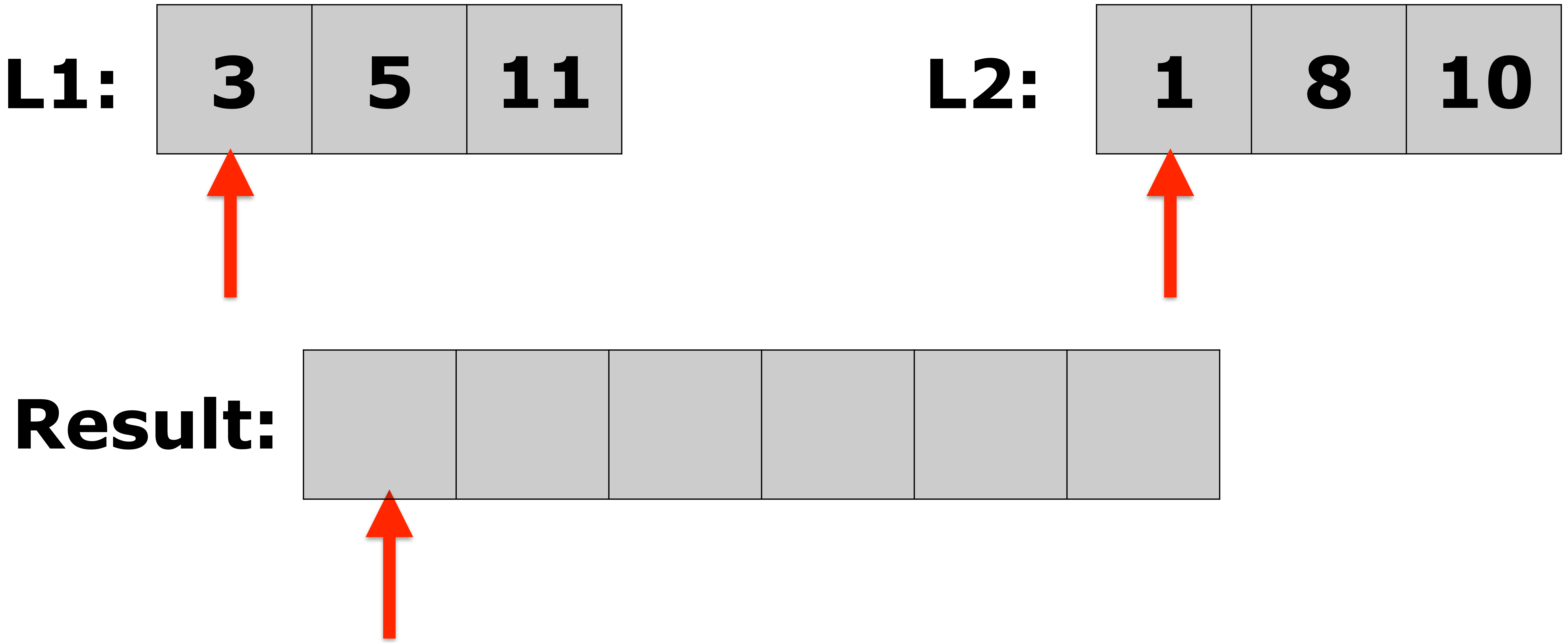
Quicksort

Merge Sort

- Merge Sort is another comparison-based sorting algorithm and it is a *divide-and-conquer* sort.
- Merge Sort can be coded recursively
- In essence, you are merging sorted lists, e.g.,
- L1 = {3,5,11}   L2 = {1,8,10}
- merge(L1,L2)={1,3,5,8,10,11}

Merge Sort

- Merging two sorted lists is easy:



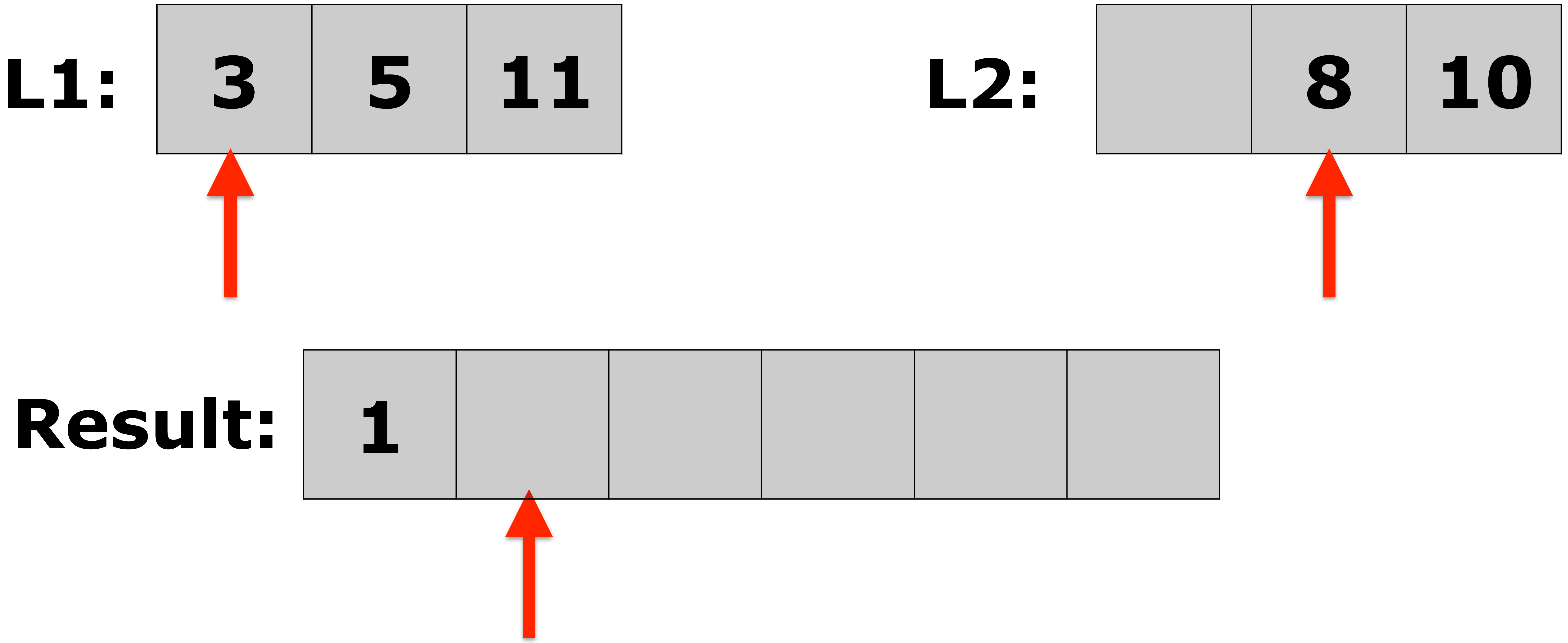
L1: 3 5 11

L2: 1 8 10

Result:

Merge Sort

- Merging two sorted lists is easy:



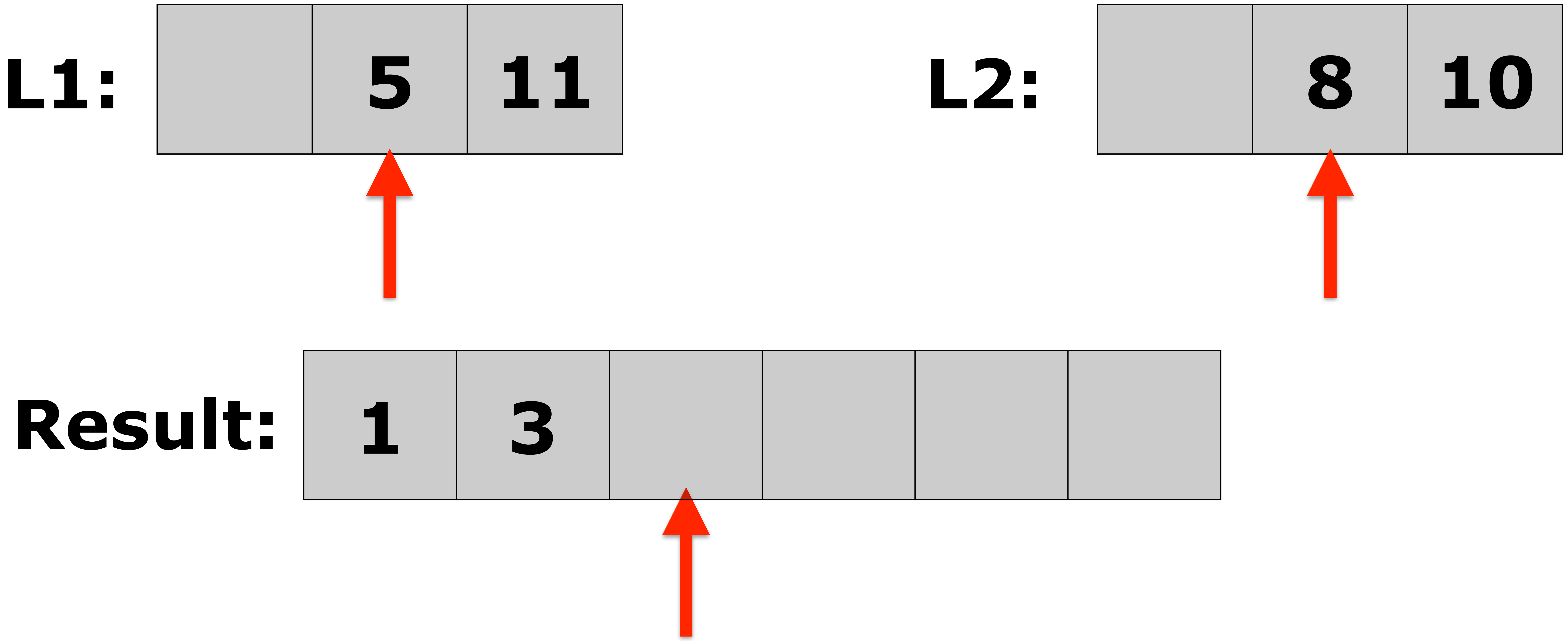
L1: 3 5 11

L2: 8 10

Result: 1

Merge Sort

- Merging two sorted lists is easy:



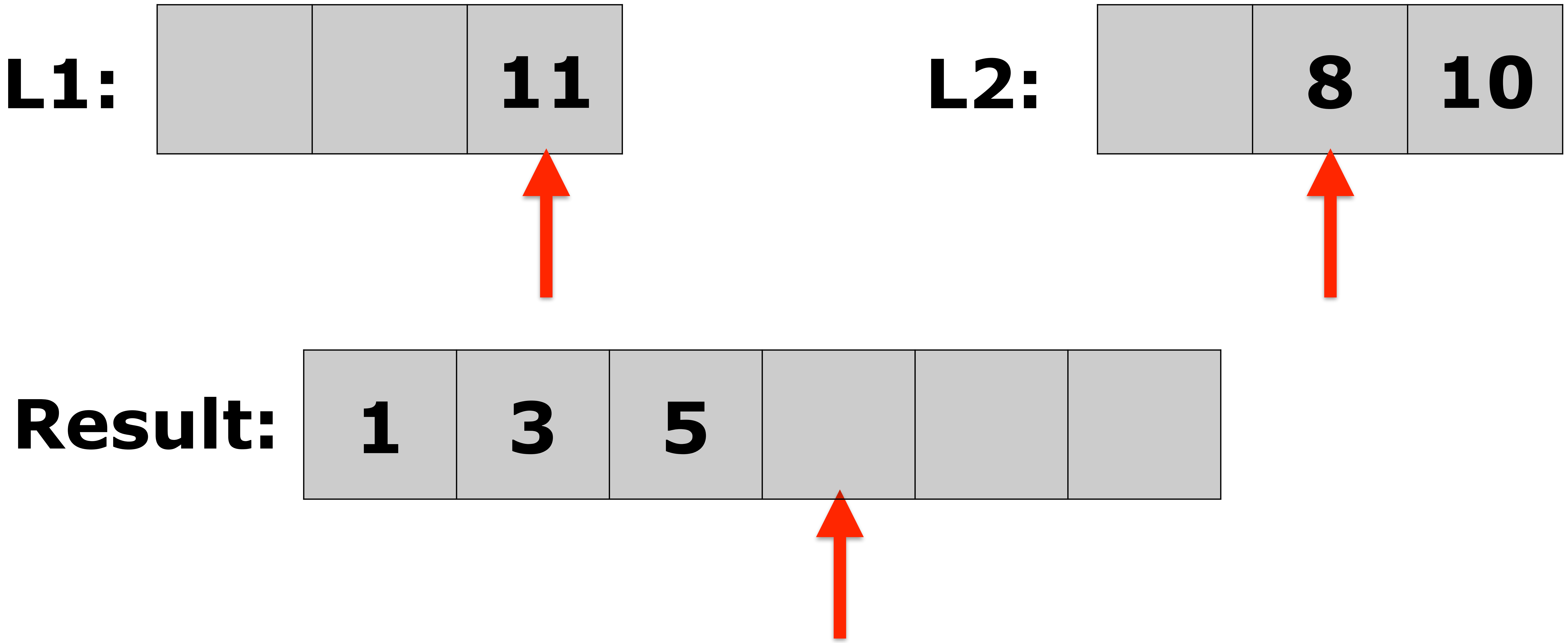
L1:		5	11

L2:		8	10

Result:	1	3				

Merge Sort

- Merging two sorted lists is easy:



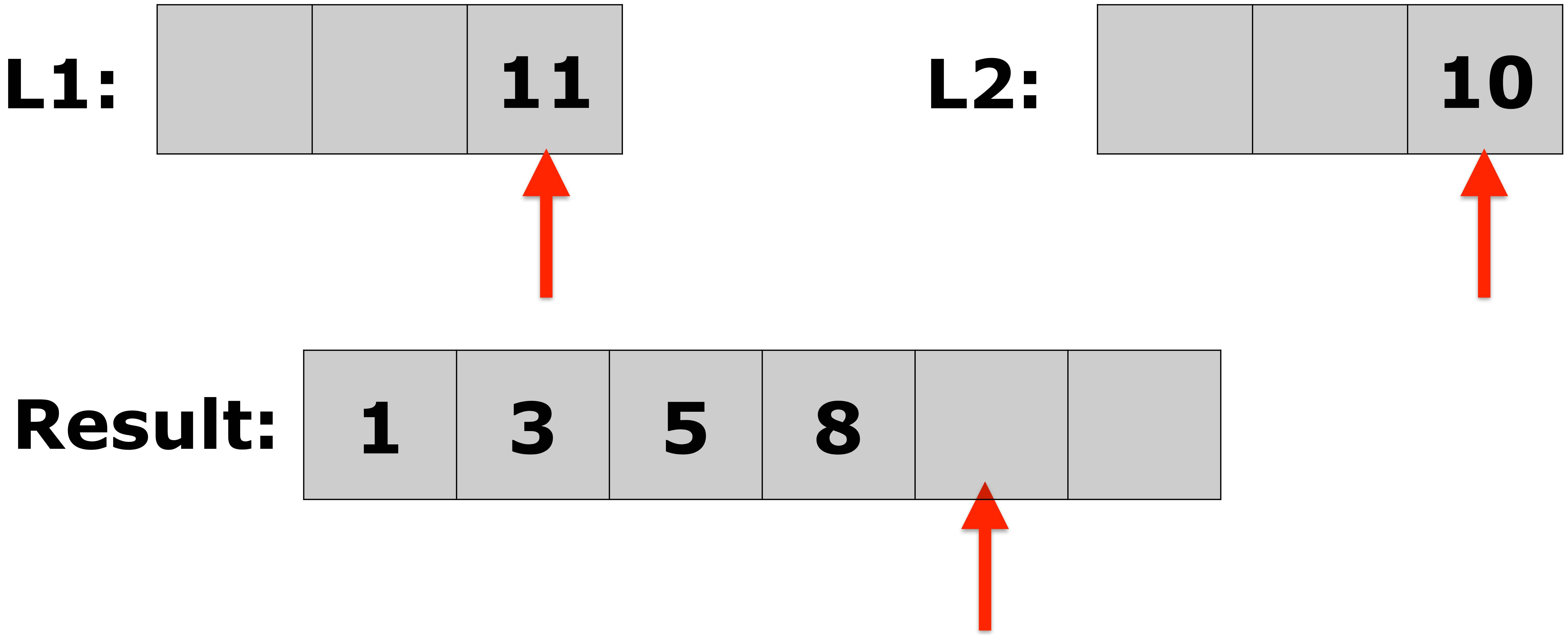
L1: | | | 11 |

L2: | | 8 | 10 |

Result: | 1 | 3 | 5 | | | |

Merge Sort

- Merging two sorted lists is easy:



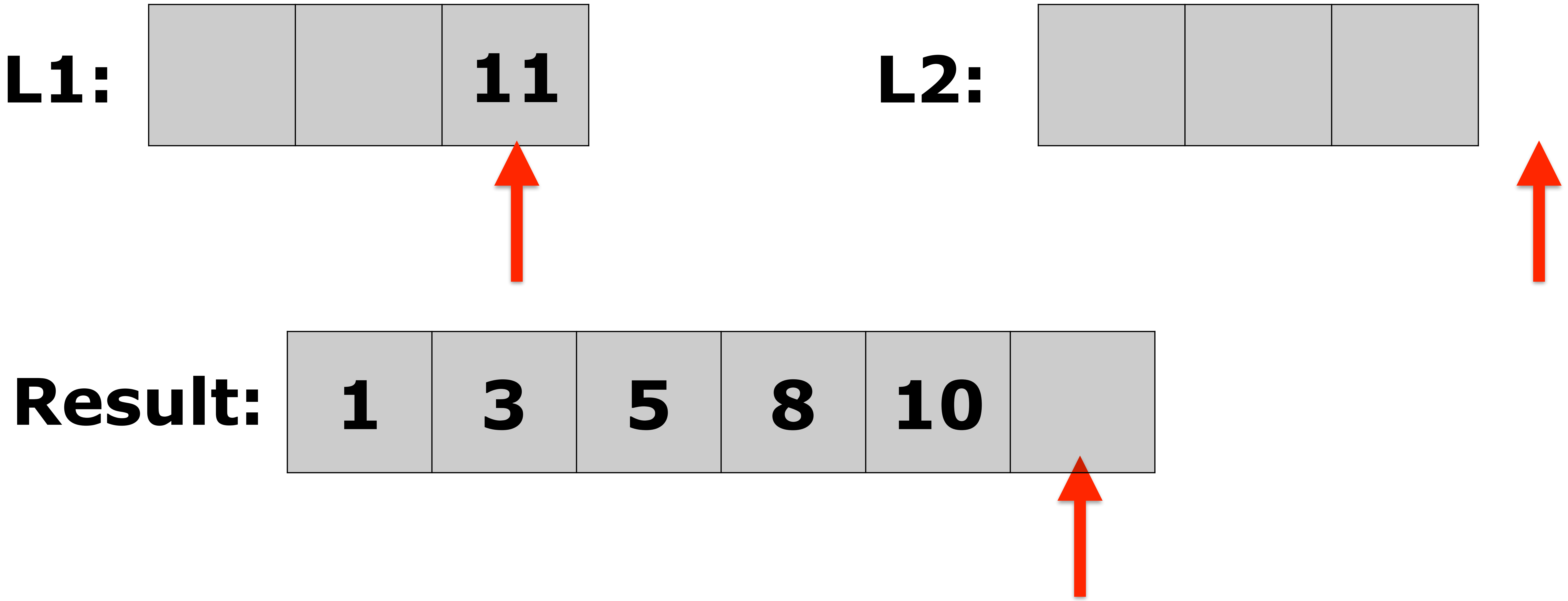
L1: | | | 11

L2: | | | 10

Result: 1 | 3 | 5 | 8 | |

Merge Sort

- Merging two sorted lists is easy:



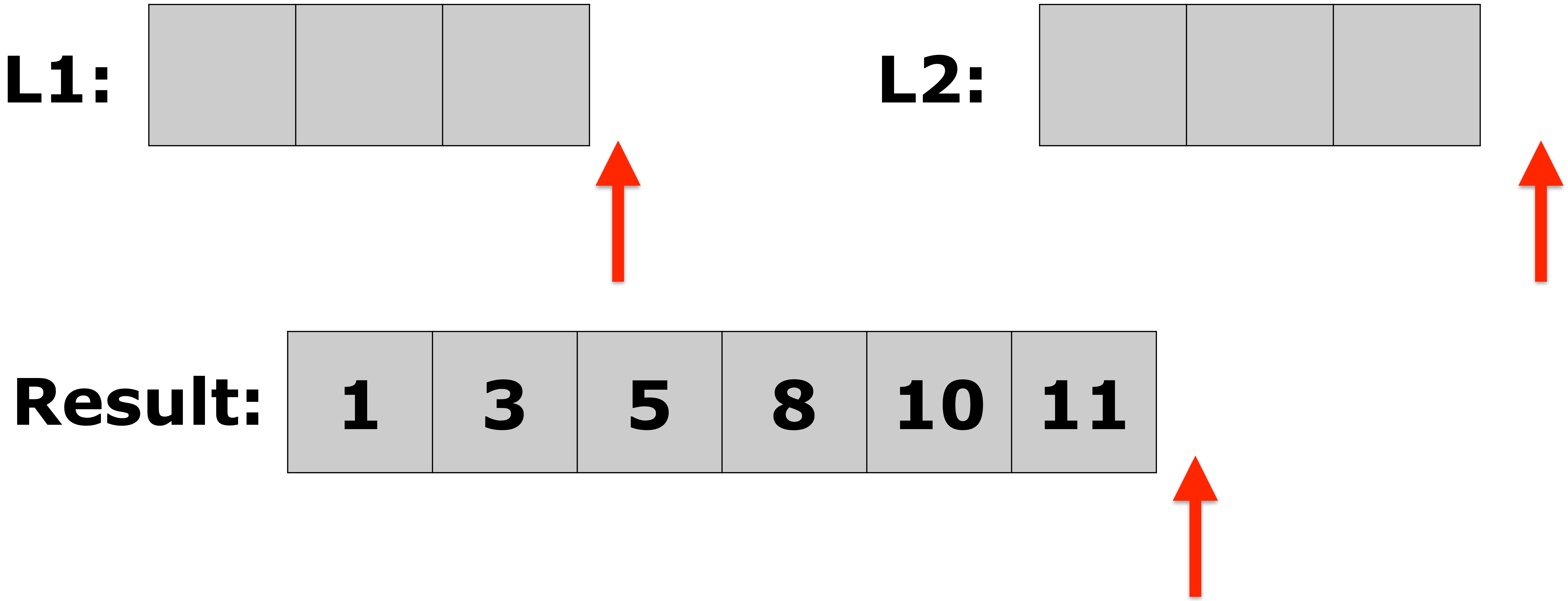
L1: | | | 11

L2: | | |

Result: 1 | 3 | 5 | 8 | 10 |

Merge Sort

- Merging two sorted lists is easy:



L1:

L2:

Result:	**1**	**3**	**5**	**8**	**10**	**11**

Merge Sort

- Full algorithm:
  - Divide the unsorted list into n sublists, each containing 1 element (a list of 1 element is considered sorted).
  - Repeatedly merge sublists to produce new sorted sublists until there is only 1 sublist remaining. This will be the sorted list.

Merge Sort Code (Recursive!)

```
// Rearranges the elements of v into sorted order using
// the merge sort algorithm.
void mergeSort(Vector<int> &vec) {
    int n = vec.size();
    if (n <= 1) return;
    Vector<int> v1;
    Vector<int> v2;
    for (int i=0; i < n; i++) {
        if (i < n / 2) {
            v1.add(vec[i]);
        } else {
            v2.add(vec[i]);
        }
    }
    mergeSort(v1);
    mergeSort(v2);
    vec.clear();
    merge(vec, v1, v2);
}
```

Merge Halves Code

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted, and vec is empty
void merge(Vector<int> &vec, Vector<int> &v1, Vector<int> &v2) {
    int n1 = v1.size();
    int n2 = v2.size();
    int p1 = 0;
    int p2 = 0;
    while (p1 < n1 && p2 < n2) {
        if (v1[p1] < v2[p2]) {
            vec.add(v1[p1++]);
        } else {
            vec.add(v2[p2++]);
        }
    }
    while (p1 < n1) {
        vec.add(v1[p1++]);
    }
    while (p2 < n2) {
        vec.add(v2[p2++]);
    }
}
```

Merge Sort: Full Example

99	6	86	15	58	35	86	4	0

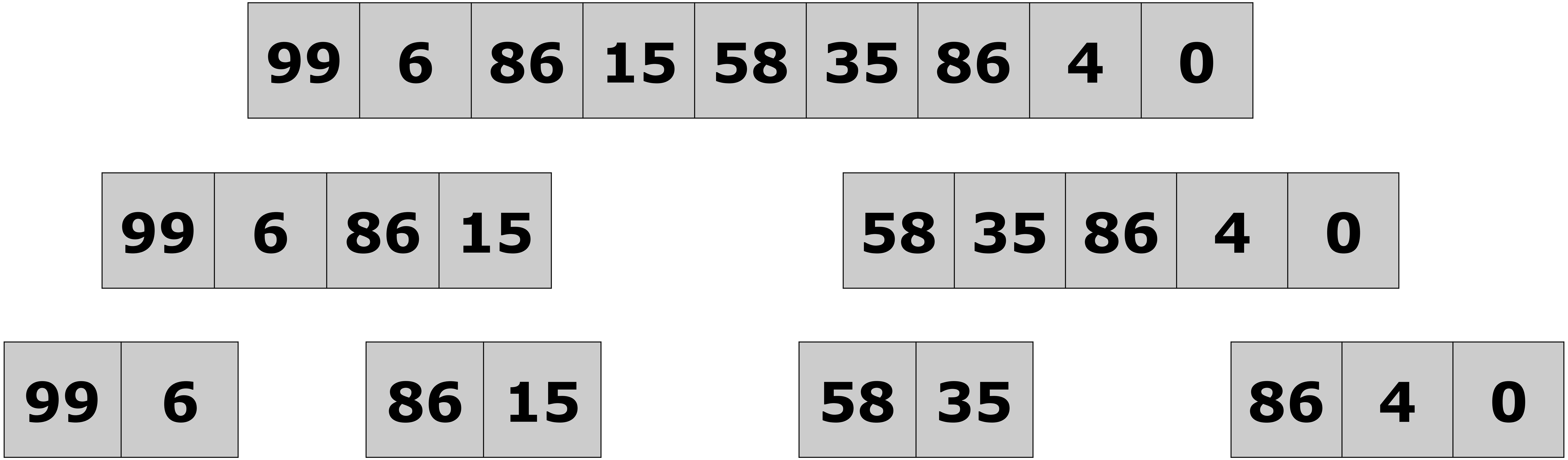
Merge Sort: Full Example

99	6	86	15	58	35	86	4	0

99	6	86	15

58	35	86	4	0

Merge Sort: Full Example



99	6	86	15	58	35	86	4	0

99	6	86	15

58	35	86	4	0

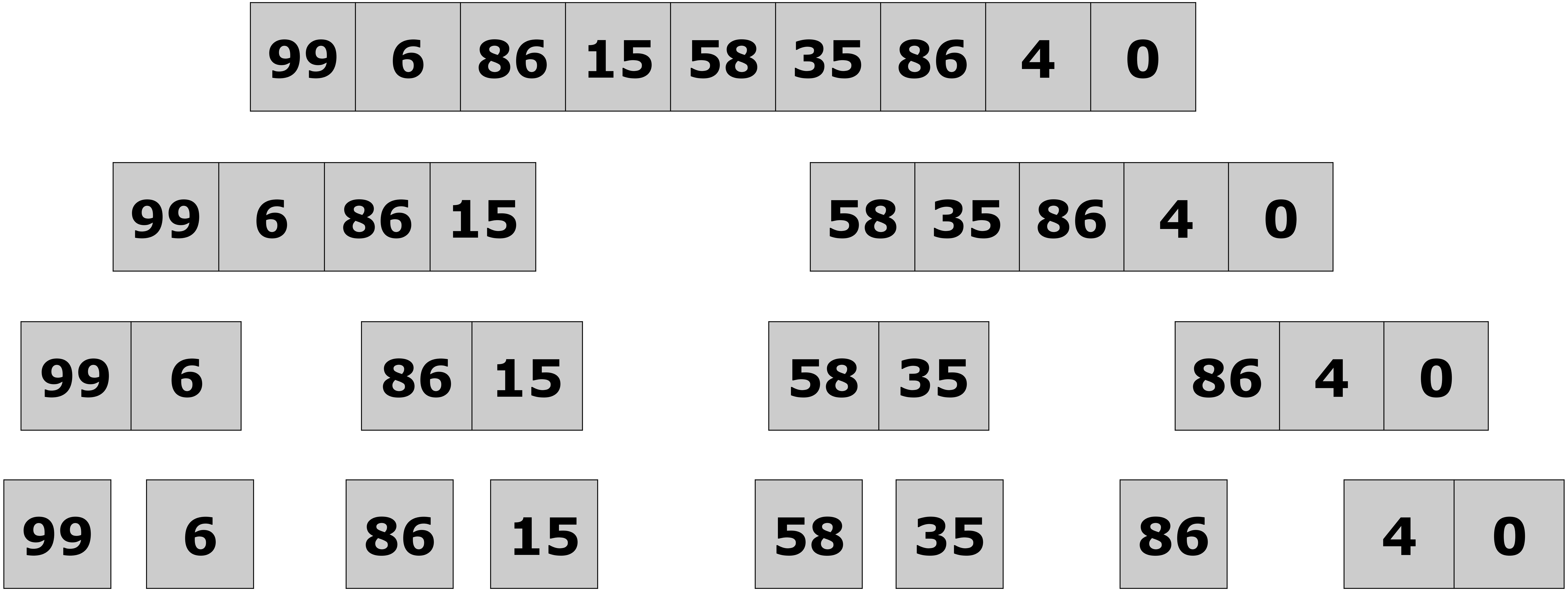
99	6

86	15

58	35

86	4	0

Merge Sort: Full Example



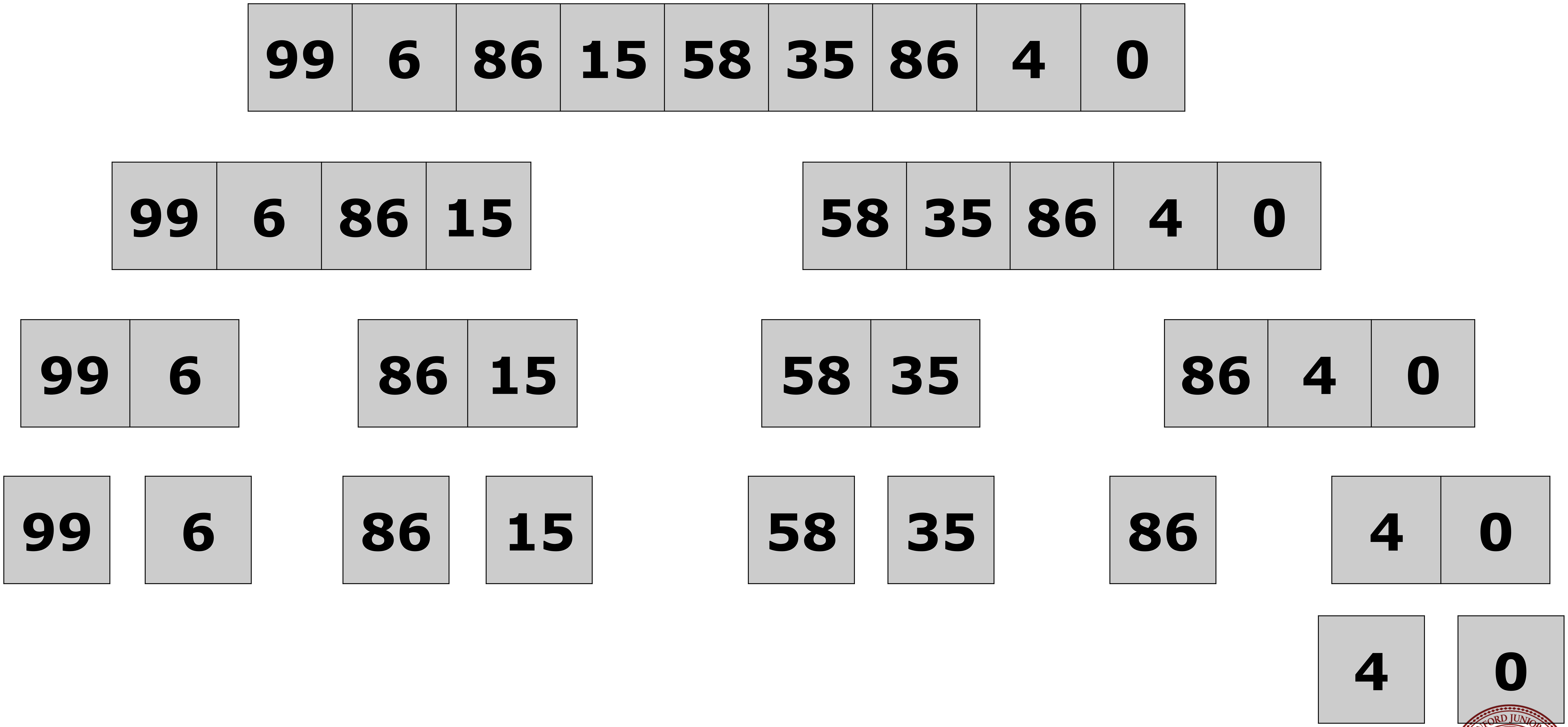
99 6 86 15 58 35 86 4 0

99 6 86 15 | 58 35 86 4 0

99 6 | 86 15 | 58 35 | 86 4 0

99 | 6 | 86 | 15 | 58 | 35 | 86 | 4 0

Merge Sort: Full Example



99 6 86 15 58 35 86 4 0

99 6 86 15

58 35 86 4 0

99 6

86 15

58 35

86 4 0

99

6

86

15

58

35

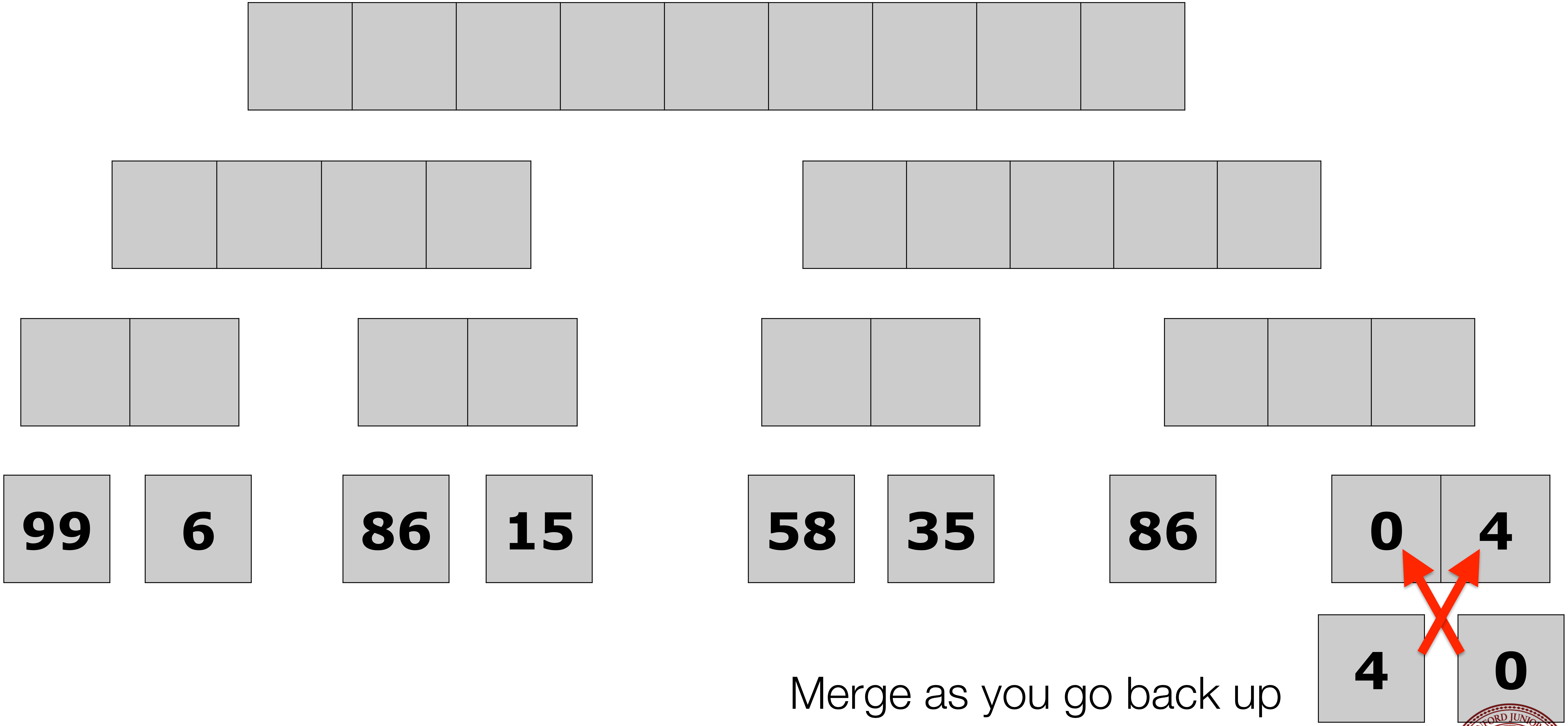
86

4 0

4

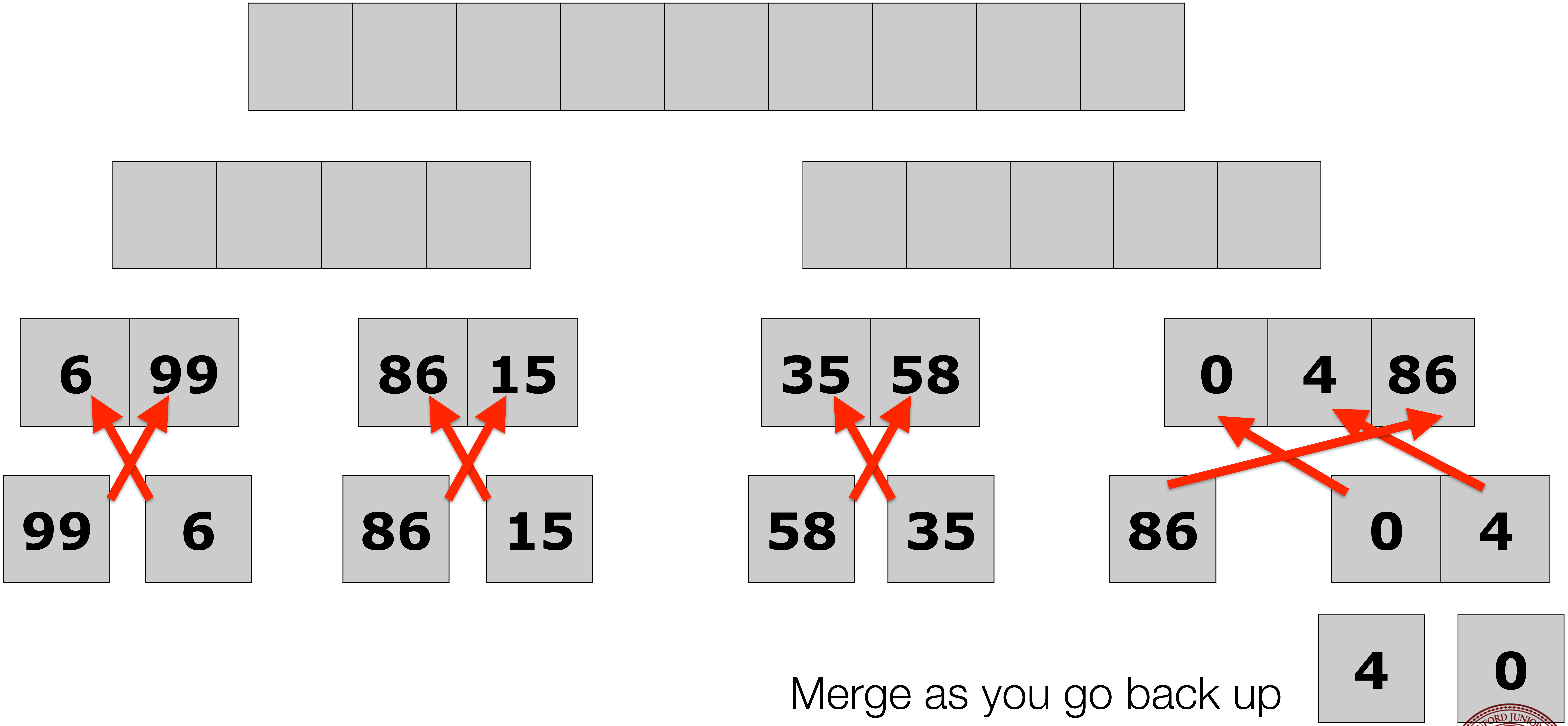
0

Merge Sort: Full Example



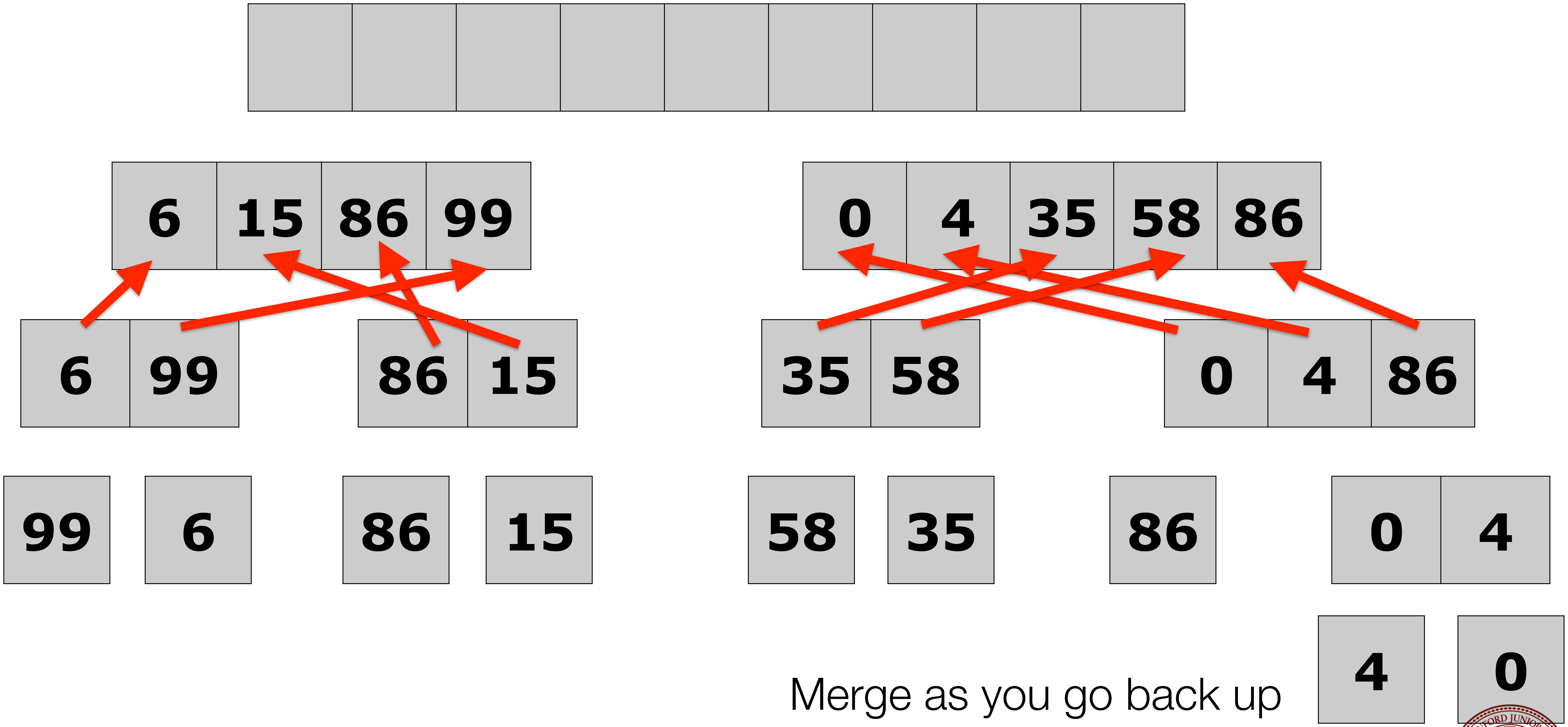
Merge as you go back up

Merge Sort: Full Example



Merge as you go back up

Merge Sort: Full Example



6 15 86 99

0 4 35 58 86

6 99

86 15

35 58

0 4 86

99

6

86

15

58

35

86

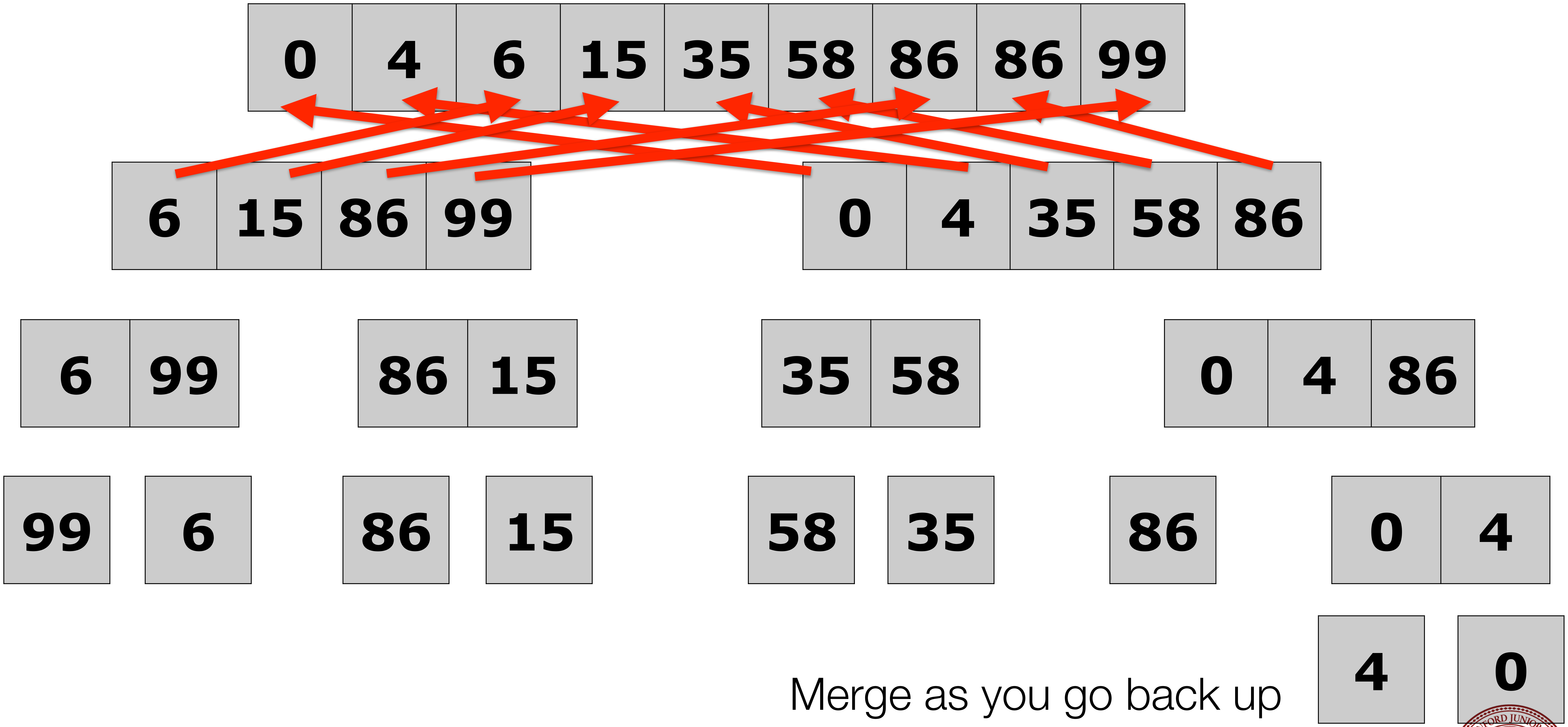
0 4

4

0

Merge as you go back up

Merge Sort: Full Example



0 4 6 15 35 58 86 86 99

6 15 86 99

0 4 35 58 86

6 99

86 15

35 58

0 4 86

99

6

86

15

58

35

86

0 4

4

0

Merge as you go back up

Merge Sort: Space Complexity

0	4	6	15	35	58	86	86	99

- Merge Sort can be completed in place, but
  - It takes more time because elements may have to be shifted often
- It can also use “double storage” with a temporary array.
  - This is fast, because no elements need to be shifted
  - It takes double the memory, which makes it inefficient for in-memory sorts.

Merge Sort: Time Complexity

0	**4**	**6**	**15**	**35**	**58**	**86**	**86**	**99**

- The Double Memory merge sort has a worst-case time complexity of **O(n log n)** (this is great!)
- Best case is also **O(n log n)**
- Average case is **O(n log n)**

- *Note*: We would like you to understand this analysis (and know the outcomes above), but it is not something we will expect you to reinvent on the midterm.

Is our Merge Sort "stable"?

- A "stable sort" keeps the original order of the same values in the same order. E.g.,

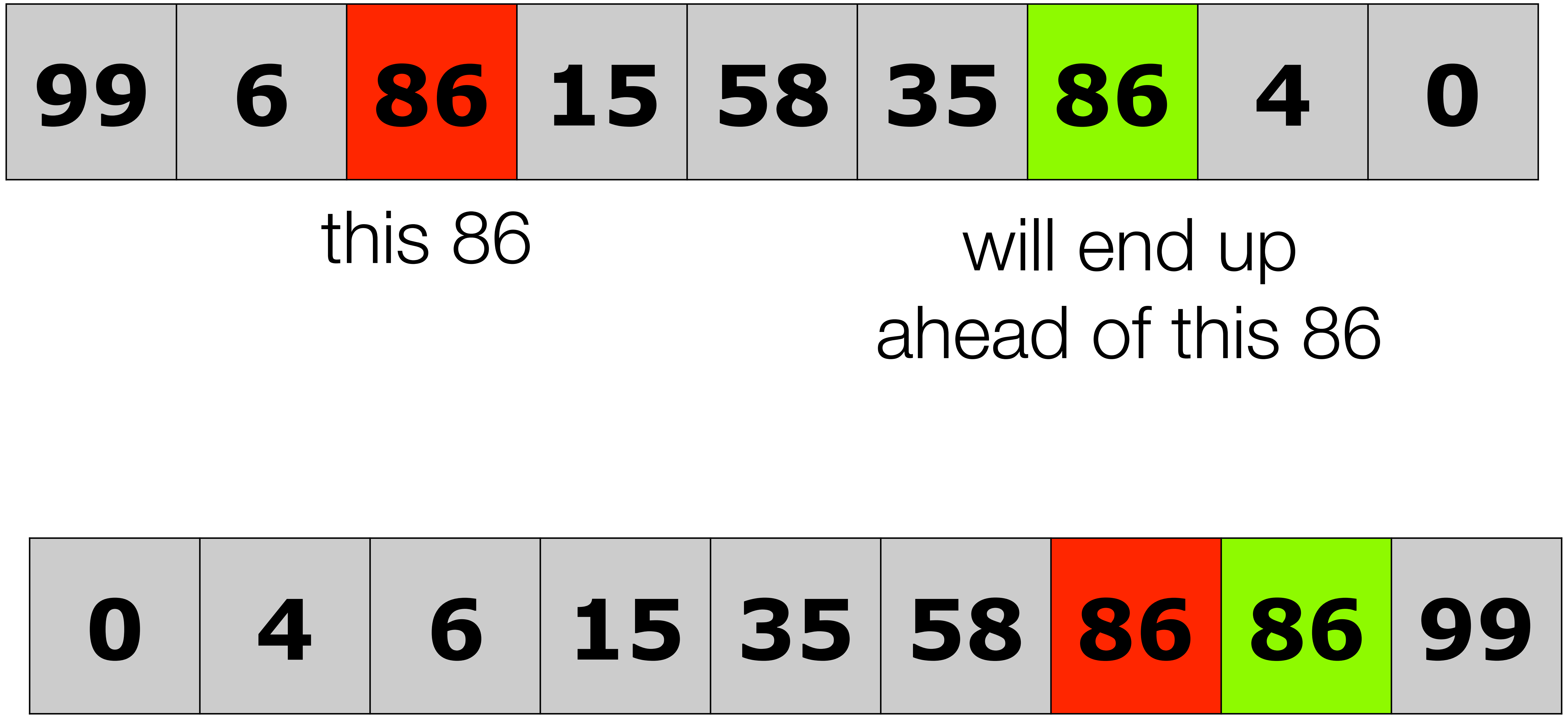
99	6	86	15	58	35	86	4	0

this 86

will end up ahead of this 86

Is our Merge Sort "stable"?

- A "stable sort" keeps the original order of the same values in the same order. E.g.,



99	6	86	15	58	35	86	4	0

this 86

will end up ahead of this 86

0	4	6	15	35	58	86	86	99

Who cares? It's just a number!

Is our Merge Sort "stable"?

What if we were sorting linked vectors? What if we first sorted by alpha below, then by num...

num	99	6	86	15	58	35	86	4	0
alpha	A	B	C	D	E	F	G	H	I

We might care! If we are sorting first names with last names, maybe we want all the last names to be in order.

num	0	4	6	15	35	58	86	86	99
alpha	I	H	B	D	F	E	C	G	A

num	0	4	6	15	35	58	86	86	99
alpha	I	H	B	D	F	E	G	C	A

Is our Merge Sort "stable"?

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted, and vec is empty
void merge(Vector<int> &vec, Vector<int> &v1, Vector<int> &v2) {
    int n1 = v1.size();
    int n2 = v2.size();
    int p1 = 0;
    int p2 = 0;
    while (p1 < n1 && p2 < n2) {
        if (v1[p1] < v2[p2]) {
            vec.add(v1[p1++]);
        } else {
            vec.add(v2[p2++]);
        }
    }
    while (p1 < n1) {
        vec.add(v1[p1++]);
    }
    while (p2 < n2) {
        vec.add(v2[p2++]);
    }
}
```

Is our Merge Sort "stable"?

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted, and vec is empty
void merge(Vector<int> &vec, Vector<int> &v1, Vector<int> &v2) {
    int n1 = v1.size();
    int n2 = v2.size();
    int p1 = 0;
    int p2 = 0;
    while (p1 < n1 && p2 < n2) {
        if (v1[p1] < v2[p2]) {
            vec.add(v1[p1++]);
        } else {
            vec.add(v2[p2++]);
        }
    }
    while (p1 < n1) {
        vec.add(v1[p1++]);
    }
    while (p2 < n2) {
        vec.add(v2[p2++]);
    }
}
```

Nope, not stable!

Is our Merge Sort "stable"?

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted, and vec is empty
void merge(Vector<int> &vec, Vector<int> &v1, Vector<int> &v2) {
    int n1 = v1.size();
    int n2 = v2.size();
    int p1 = 0;
    int p2 = 0;
    while (p1 < n1 && p2 < n2) {
        if (v1[p1] <= v2[p2]) {
            vec.add(v1[p1++]);
        } else {
            vec.add(v2[p2++]);
        }
    }
    while (p1 < n1) {
        vec.add(v1[p1++]);
    }
    while (p2 < n2) {
        vec.add(v2[p2++]);
    }
}
```

But we can make it stable

Sorts

Insertion Sort

Selection Sort

Merge Sort

Quicksort

Quicksort

- Quicksort is a sorting algorithm that is often faster than most other types of sorts.
- However, although it has an average **$O(n \log n)$** time complexity, it also has a worst-case **$O(n^2)$** time complexity, though this rarely occurs.

Quicksort

- Quicksort is another divide-and-conquer algorithm.

- The basic idea is to **divide** a list into two smaller sub-lists: **the low elements and the high elements**. Then, the algorithm can recursively sort the sub-lists.

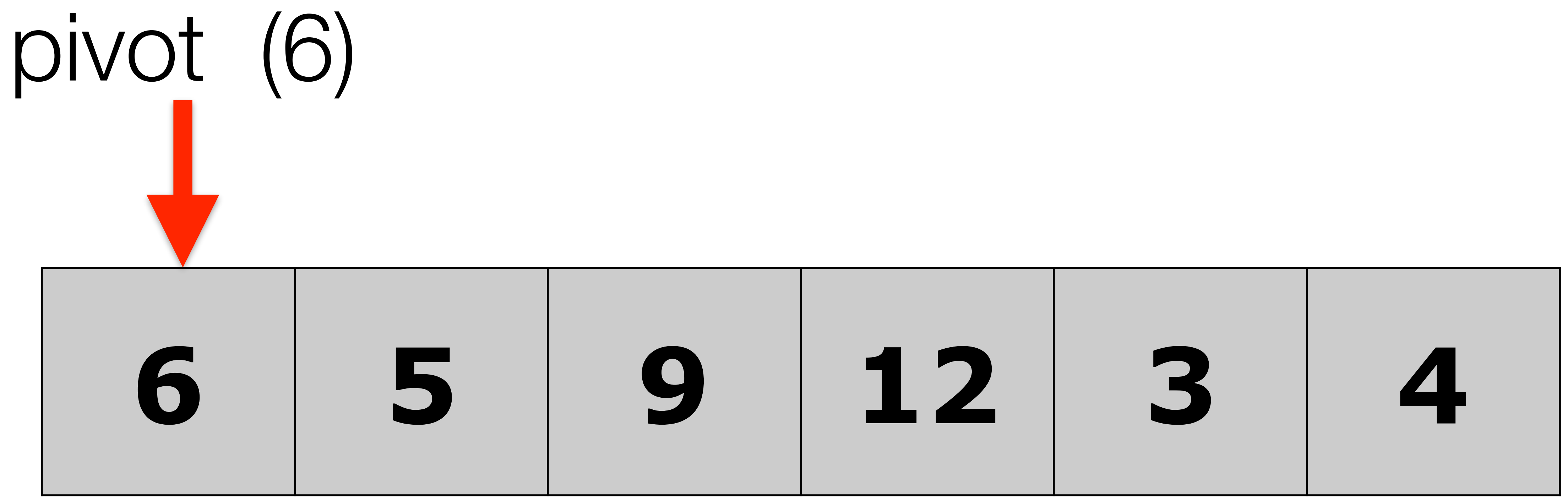
Quicksort Algorithm

- **Pick an element**, called a **pivot**, from the list
- **Reorder** the list so that all elements with **values less than the pivot come before the pivot**, while all elements with values **greater than the pivot come after it**. After this partitioning, the pivot is in its final position. This is called the partition operation.
- **Recursively apply the above steps to the sub-list of elements** with smaller values and separately to the sub-list of elements with greater values.
- The **base case** of the recursion is for **lists of 0 or 1** elements, which do not need to be sorted.

Quicksort Algorithm

- We have two ways to perform quicksort:
  - The **naive** algorithm: create new lists for each sub-sort, leading to an overhead of *n* additional memory.
  - The **in-place** algorithm, which swaps elements.

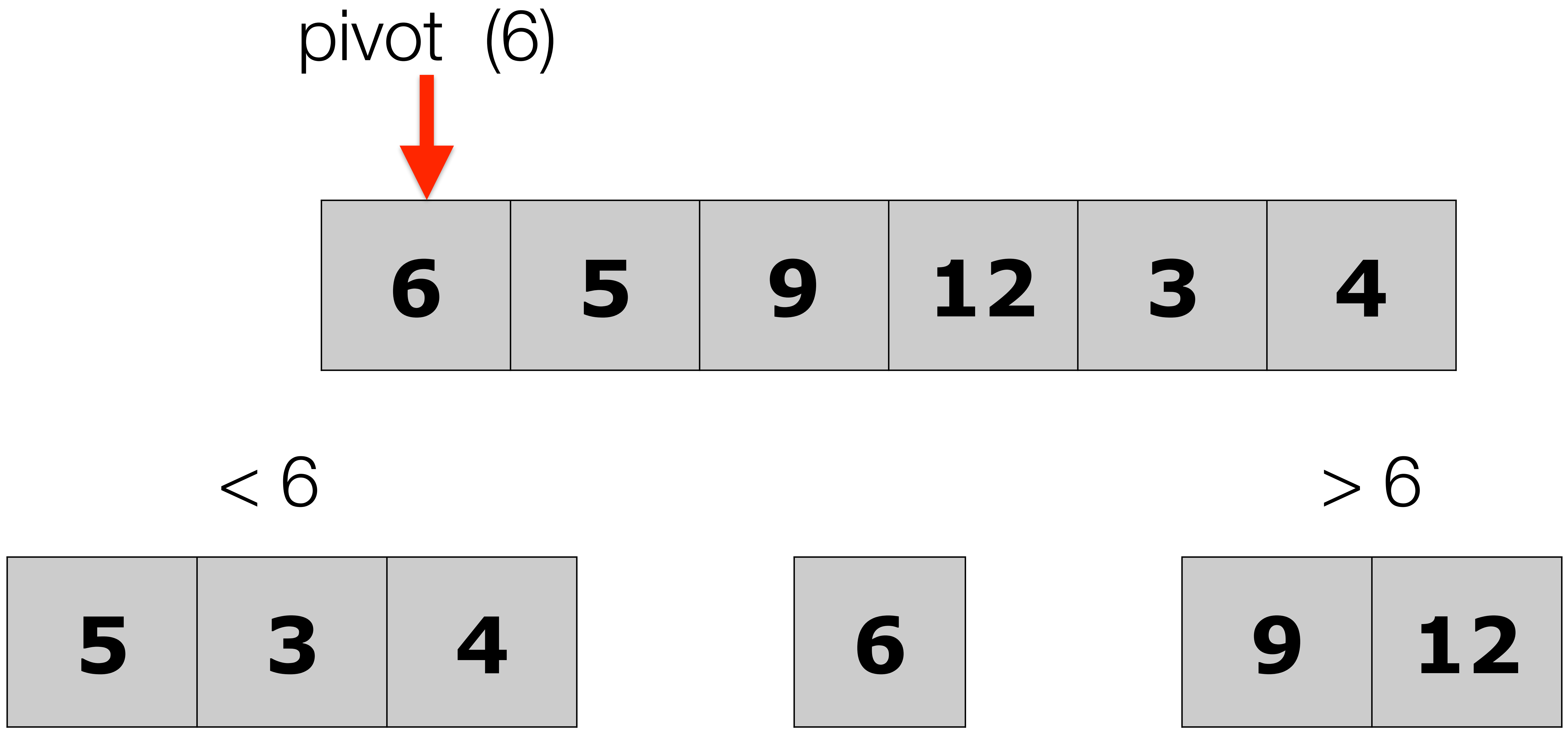
Quicksort Algorithm: Naive



pivot (6)

6	5	9	12	3	4

Quicksort Algorithm: Naive



pivot (6)

6	5	9	12	3	4

< 6

5	3	4

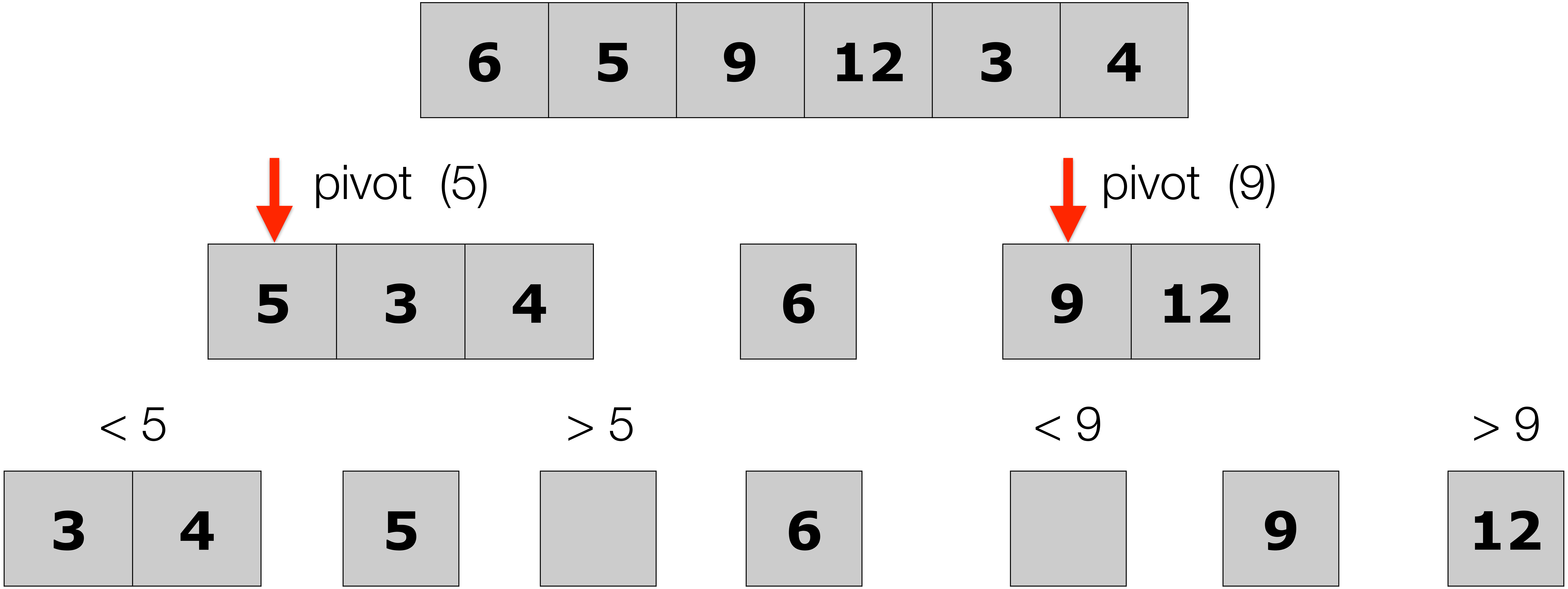
6

> 6

9	12

Partition into two new lists -- less than the pivot on the left, and greater than the pivot on the right. Even if all elements go into one list, that was just a poor partition.

Quicksort Algorithm: Naive



6 5 9 12 3 4

pivot (5)

5 3 4

6

pivot (9)

9 12

< 5

3 4

5

> 5

6

< 9

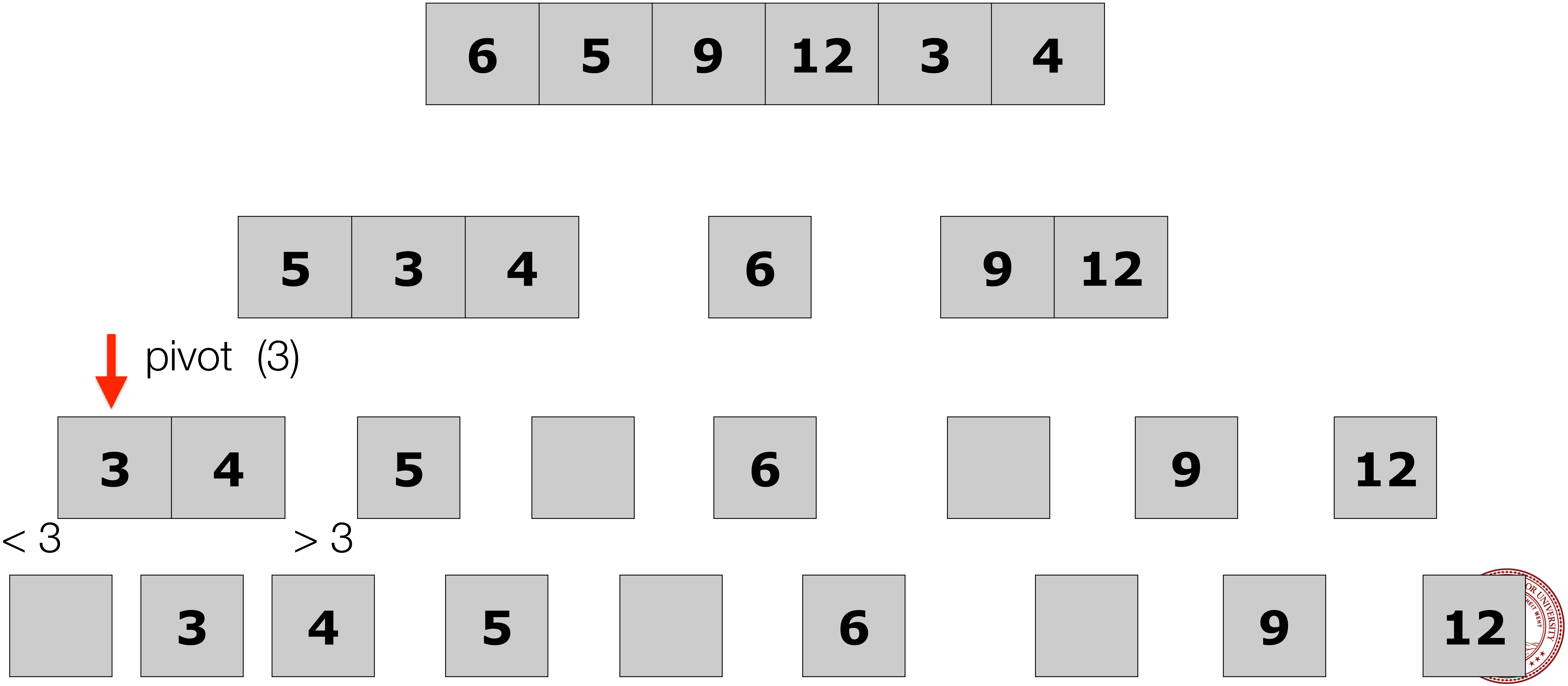
9

> 9

12

Keep partitioning the sub-lists

Quicksort Algorithm: Naive



6 5 9 12 3 4

5 3 4   6   9 12

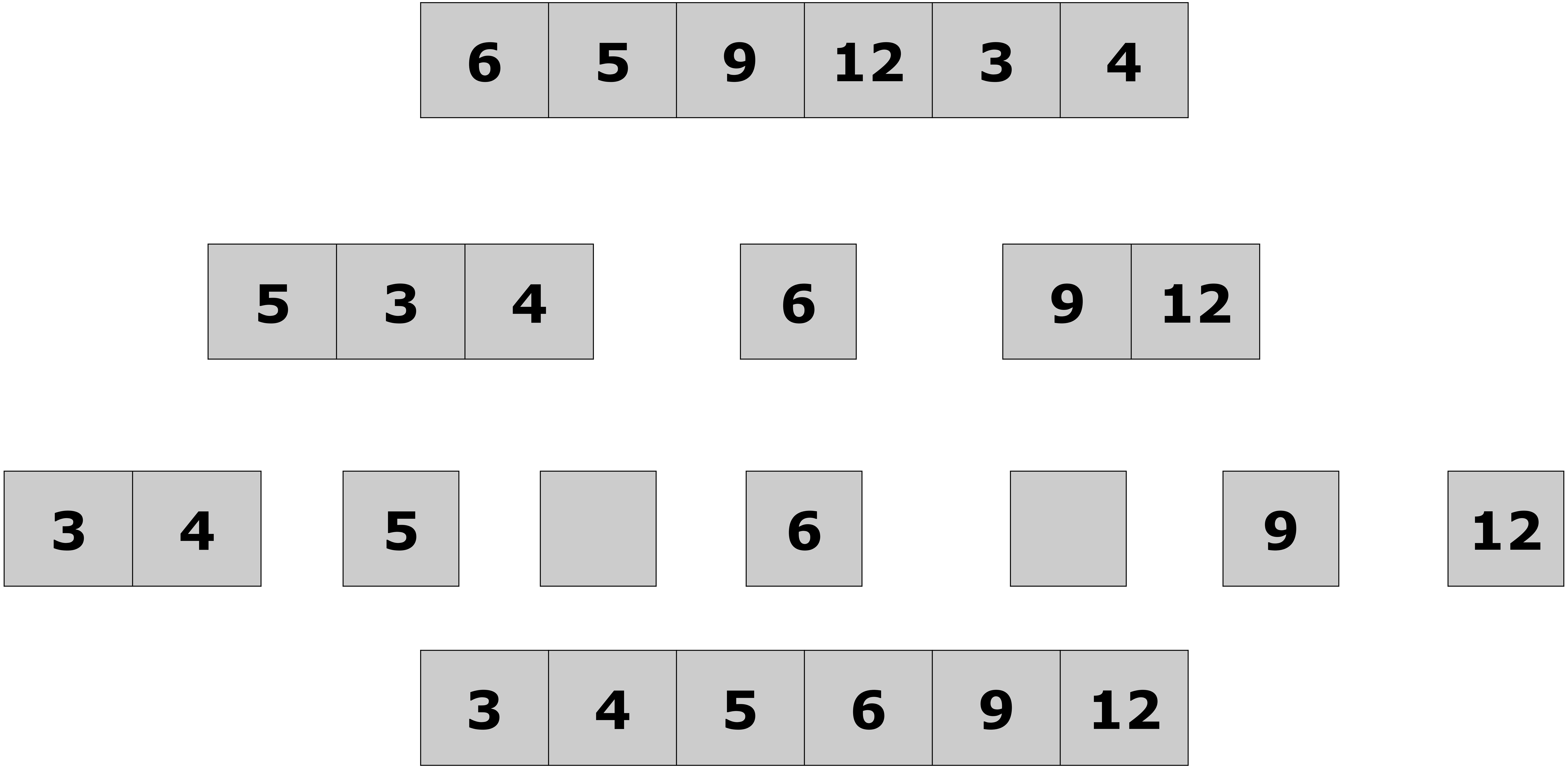
pivot (3)

3 4   5   6   9   12

< 3   > 3

3   4   5   6   9   12

Quicksort Algorithm: Naive



6 5 9 12 3 4

5 3 4　6　9 12

3 4　5　　6　　9　12

3 4 5 6 9 12

Quicksort Algorithm: Naive Code

```
Vector<int> naiveQuickSort(Vector<int> v) { // not passed by reference!
    // base case: list of 0 or 1
    if (v.size() < 2) {
        return v;
    }
    int pivot = v[0];    // choose pivot to be left-most element

    // create two new vectors to partition into
    Vector<int> left, right;

    // put all elements <= pivot into left, and all elements > pivot into right
    for (int i=1; i<v.size(); i++) {
        if (v[i] <= pivot) {
            left.add(v[i]);
        }
        else {
            right.add(v[i]);
        }
    }
    left = naiveQuickSortHelper(left); // recursively handle the left
    right = naiveQuickSortHelper(right); // recursively handle the right

    left.add(pivot); // put the pivot at the end of the left

    return left + right; // return the combination of left and right
}
```

Quicksort Algorithm: In-Place

0	1	2	3	4	5	6	7
56	25	37	58	95	19	73	30

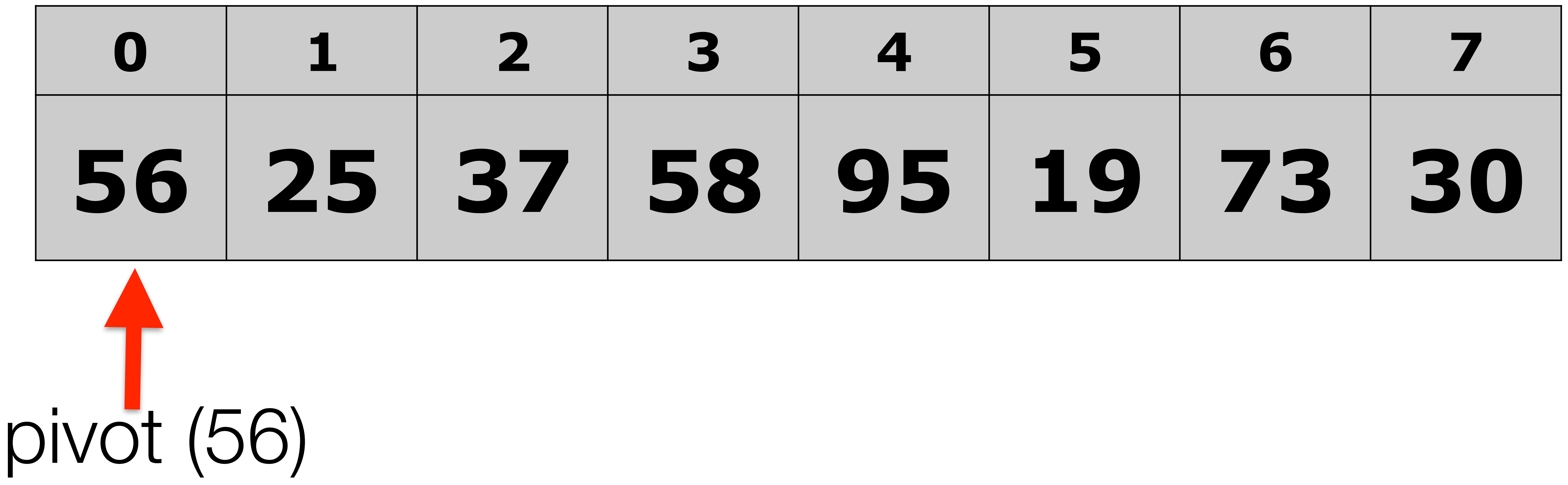
In-place, recursive algorithm:

```
        int quickSort(Vector<int> &v, int start, int finish);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the pivot with the element where the left/right cross, unless it happens to be the pivot.

This is best described with a detailed example...

Quicksort Algorithm: In-Place



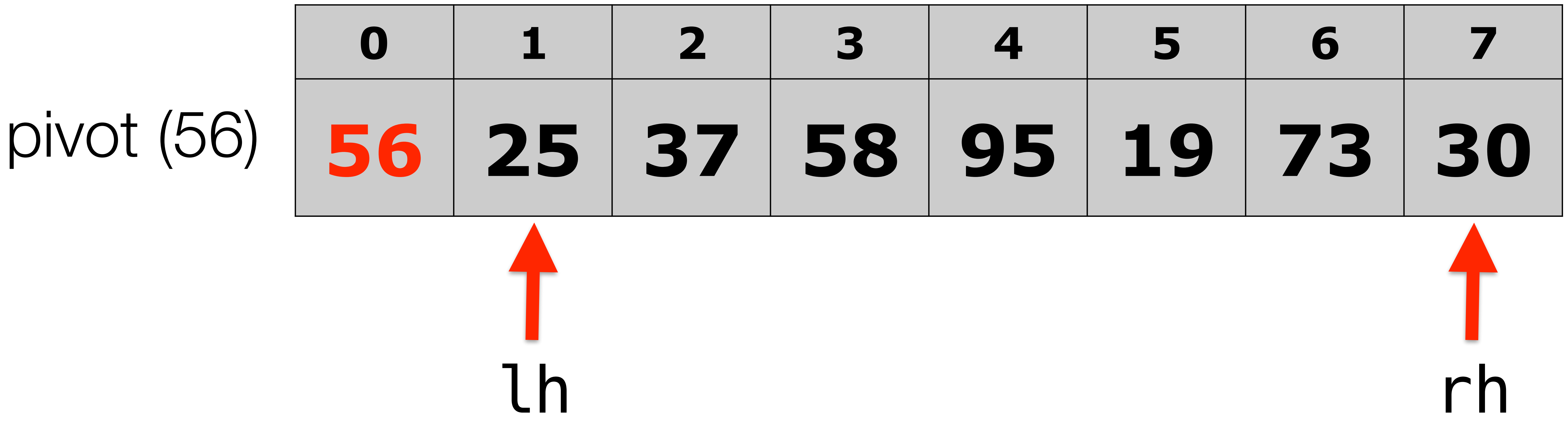
0	1	2	3	4	5	6	7
56	25	37	58	95	19	73	30

pivot (56)

```
quickSort(vec, 0, 7);
```

- **Pick your pivot as the left element (might not be a good choice...)**
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



pivot (56)

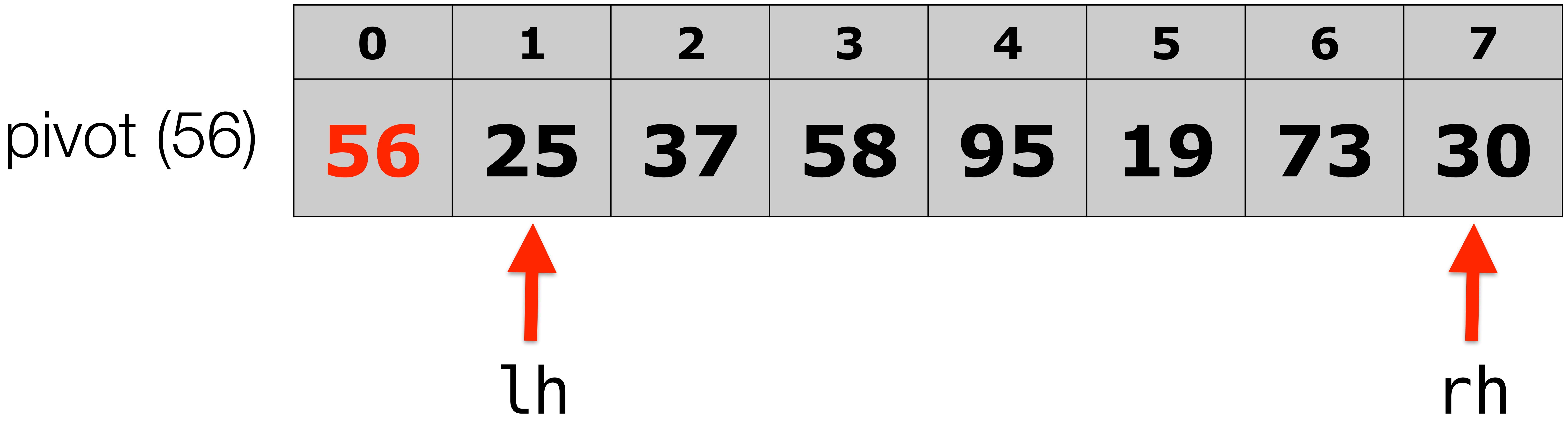
0	1	2	3	4	5	6	7
56	25	37	58	95	19	73	30

30 is already smaller than 56

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- **Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.**
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



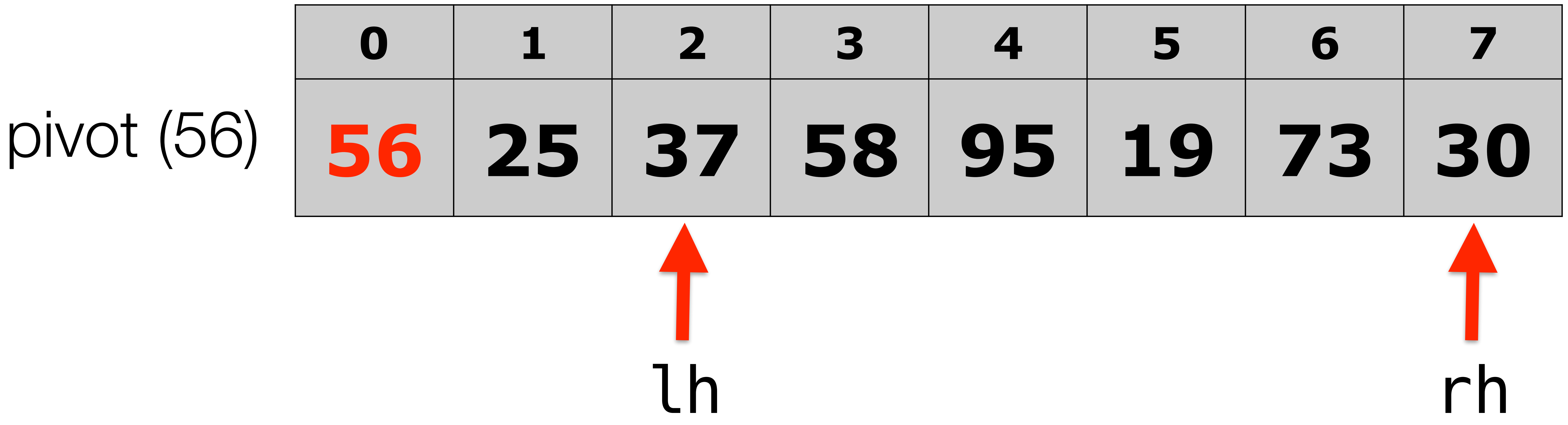
pivot (56)

0	1	2	3	4	5	6	7
56	25	37	58	95	19	73	30

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- **Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.**
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



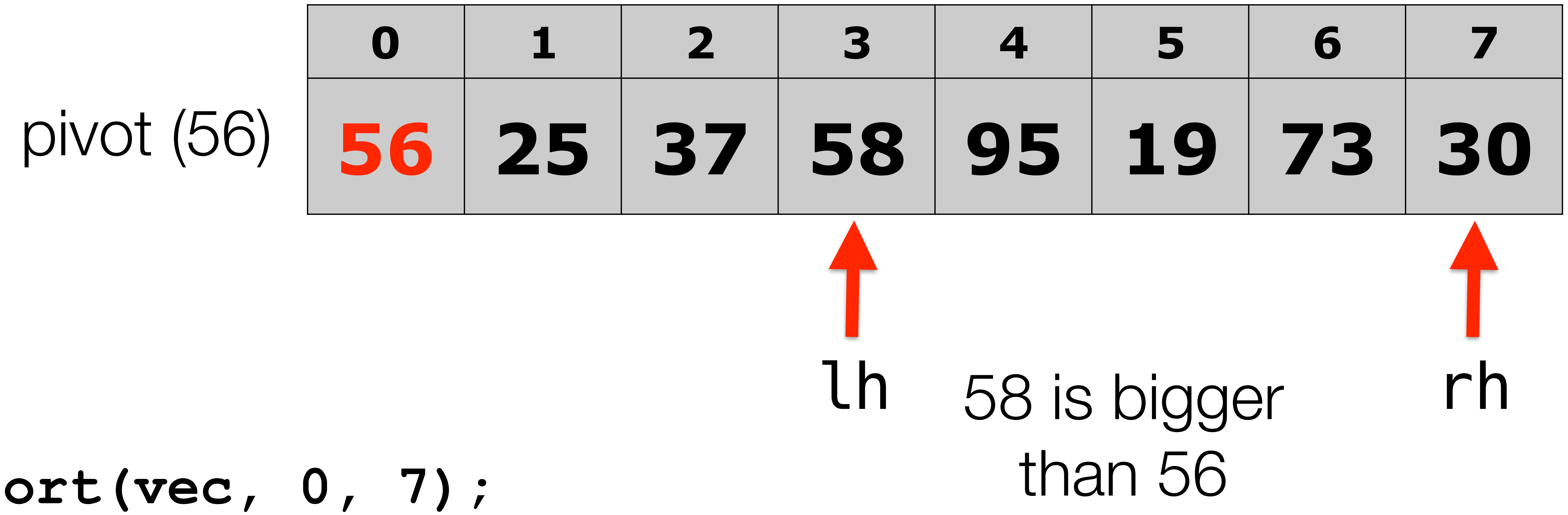
pivot (56)

0	1	2	3	4	5	6	7
56	25	37	58	95	19	73	30

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- **Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.**
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



pivot (56)

0	1	2	3	4	5	6	7
56	25	37	58	95	19	73	30

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
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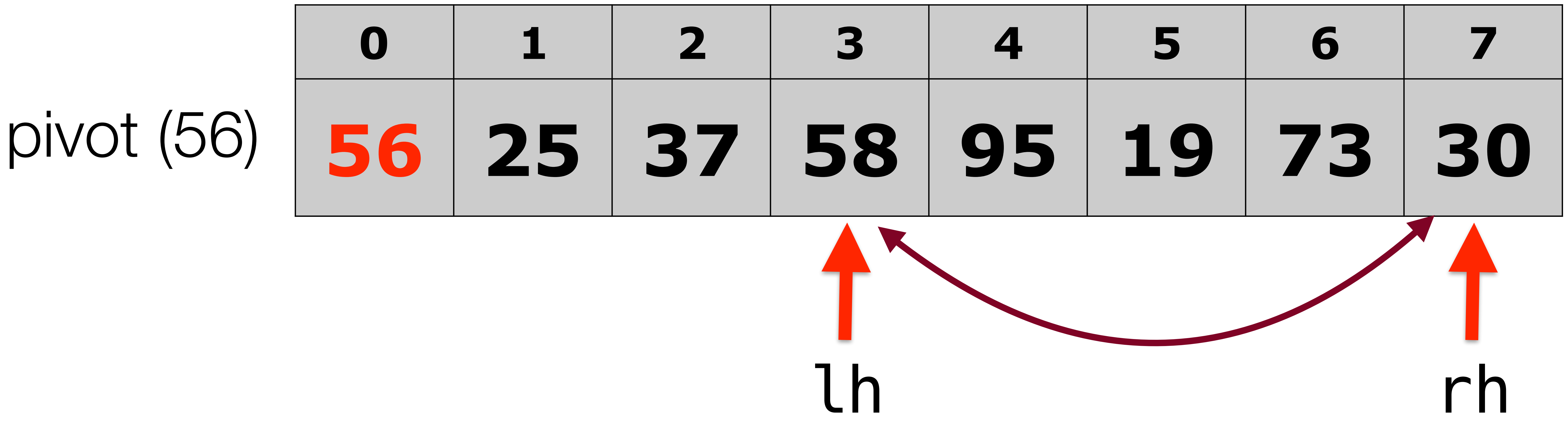
Quicksort Algorithm: In-Place



pivot (56)

0	1	2	3	4	5	6	7
56	25	37	58	95	19	73	30

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
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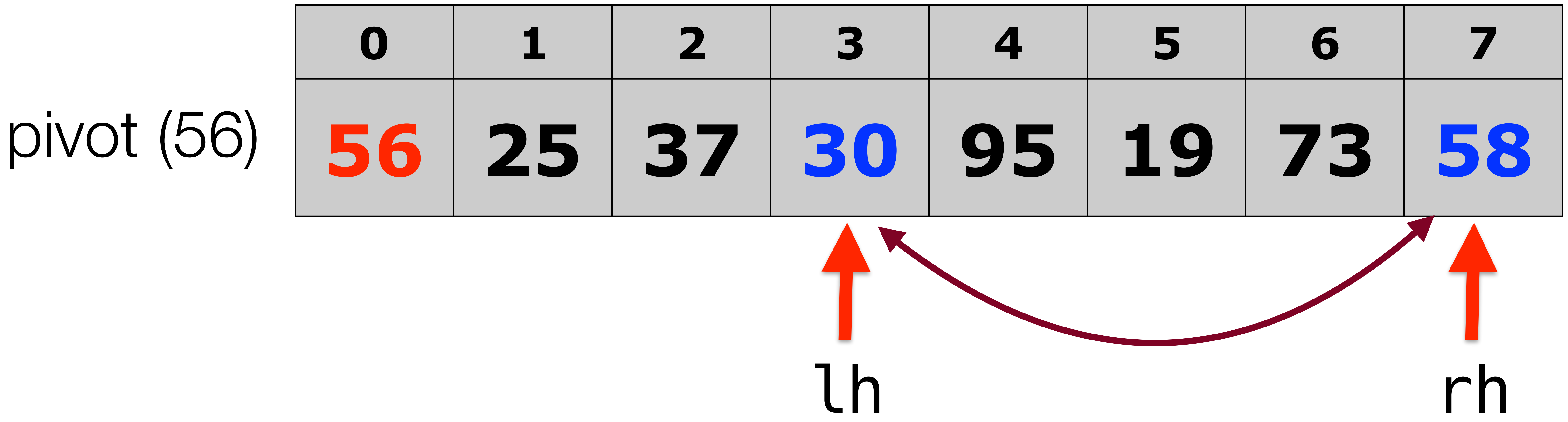
Quicksort Algorithm: In-Place

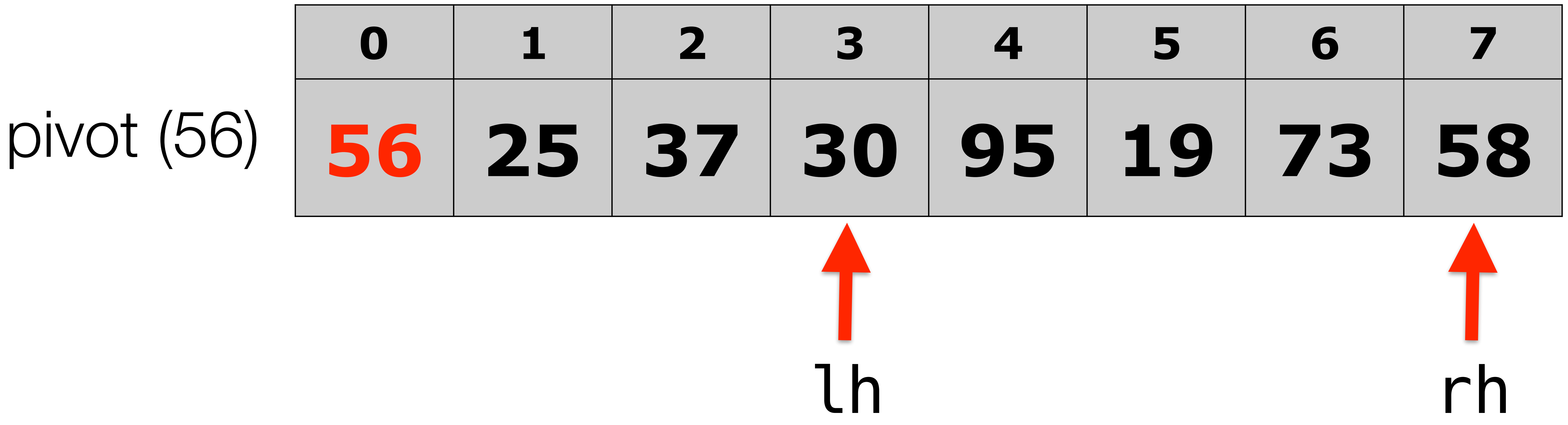


0	1	2	3	4	5	6	7
56	25	37	30	95	19	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- **Swap the two elements where the left/right cross, unless the pivot is the smallest.**
- Repeat the traversals until they cross, at which point you swap that element with the pivot

Quicksort Algorithm: In-Place

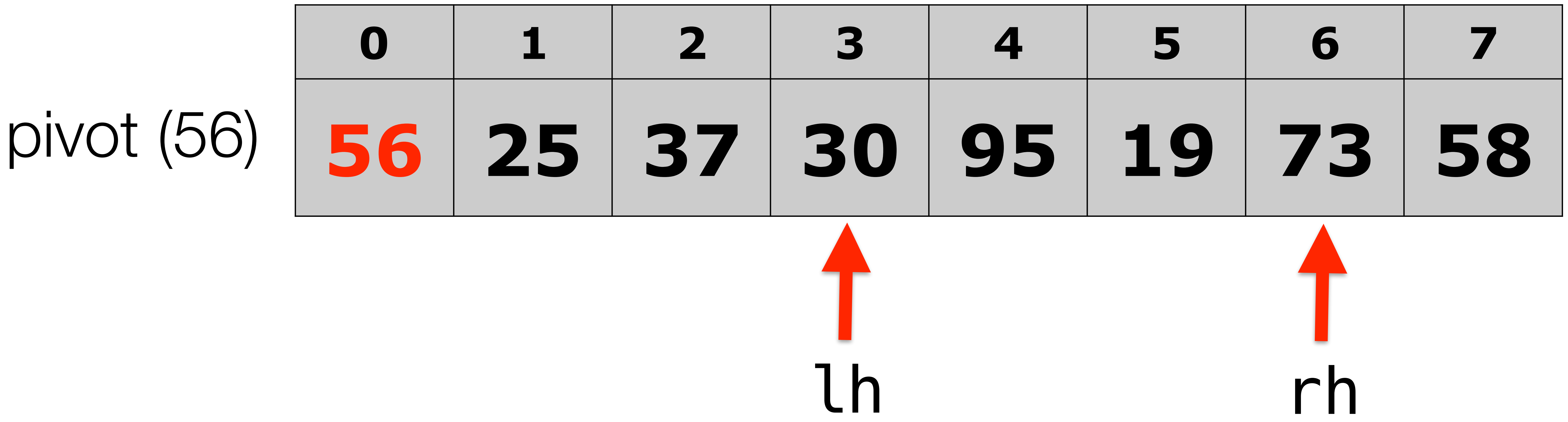


	0	1	2	3	4	5	6	7
pivot (56)	56	25	37	30	95	19	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- **Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.**
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



pivot (56)

0	1	2	3	4	5	6	7
56	25	37	30	95	19	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- **Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.**
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
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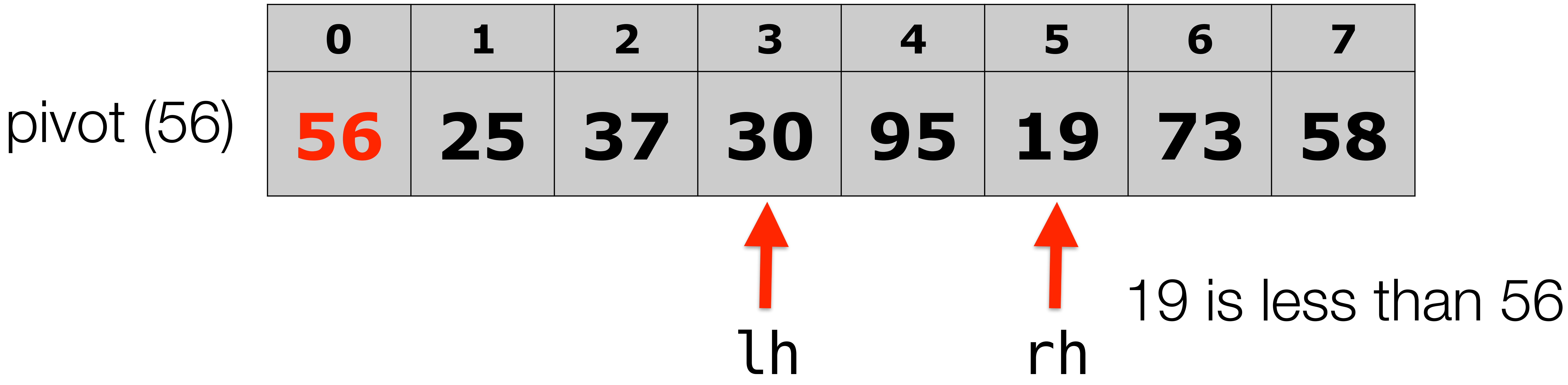
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



pivot (56)

0	1	2	3	4	5	6	7
56	25	37	30	95	19	73	58

lh

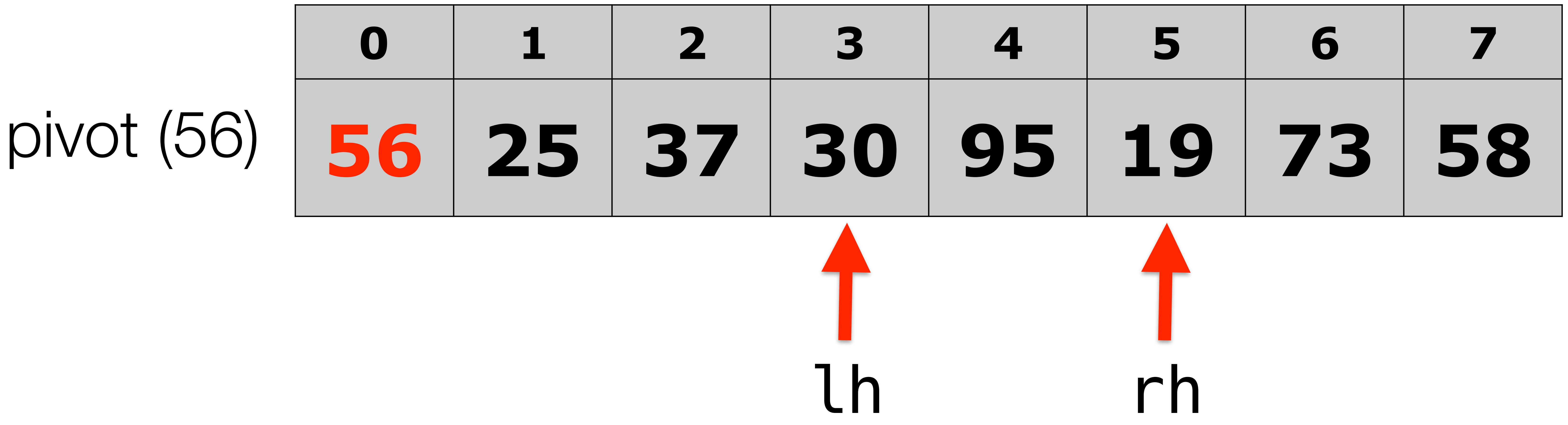
rh

19 is less than 56

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- **Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.**
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



pivot (56)

0	1	2	3	4	5	6	7
56	25	37	30	95	19	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- **Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.**
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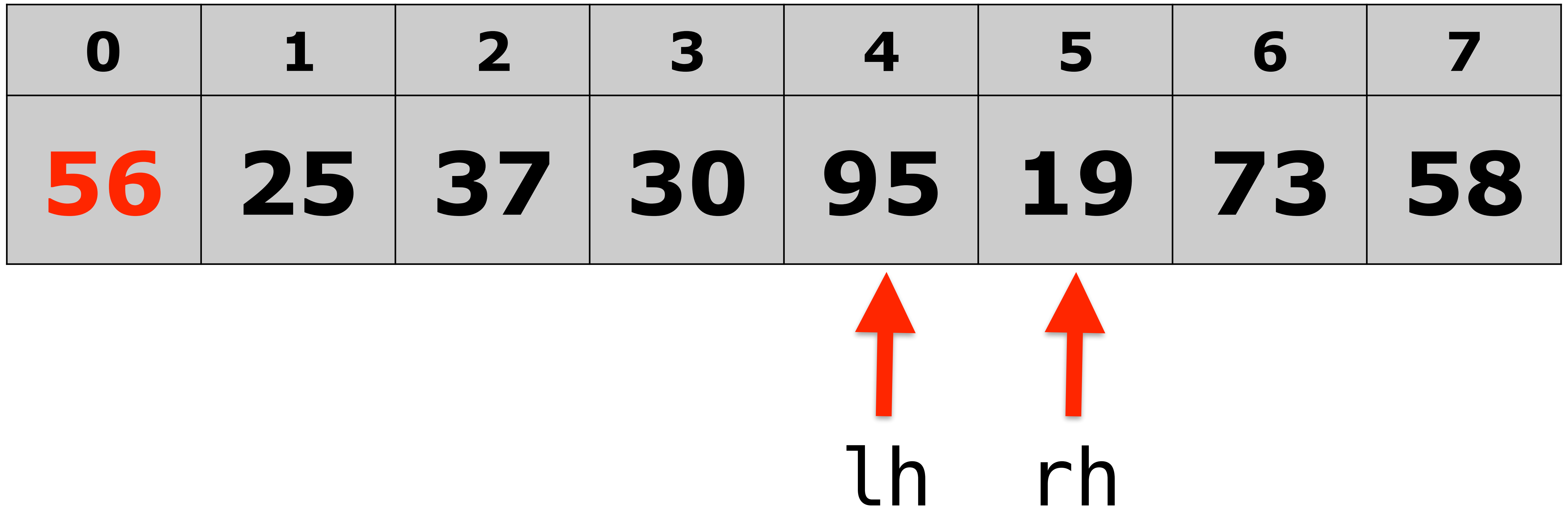
Quicksort Algorithm: In-Place

pivot (56)



0	1	2	3	4	5	6	7
56	25	37	30	95	19	73	58

95 is greater than 56

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- **Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.**
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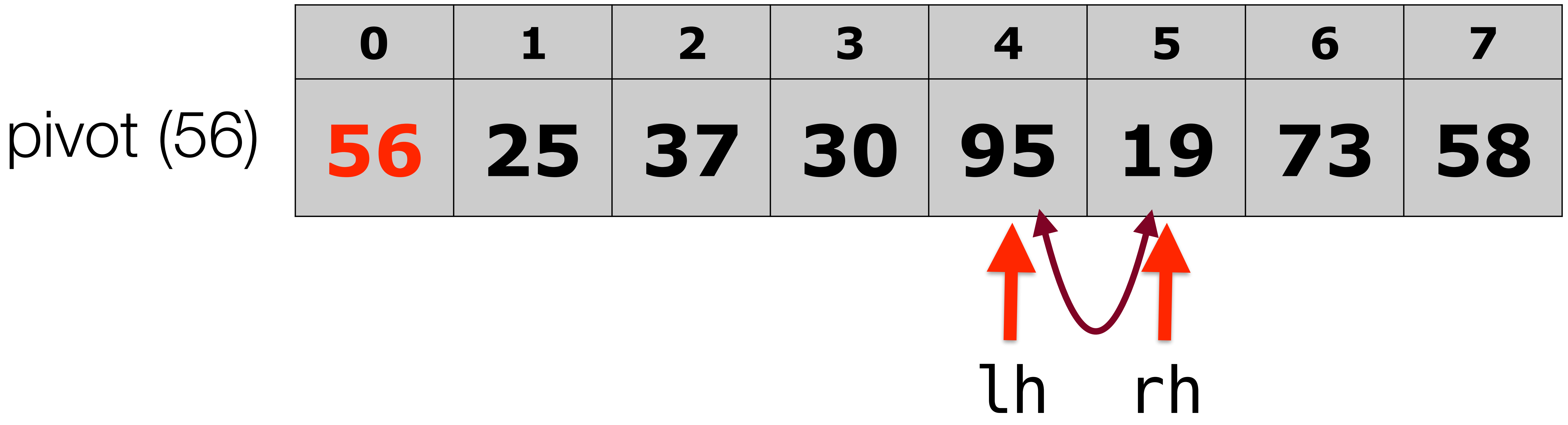
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



pivot (56)

0	1	2	3	4	5	6	7
56	25	37	30	95	19	73	58

lh rh

95 is greater than 56

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
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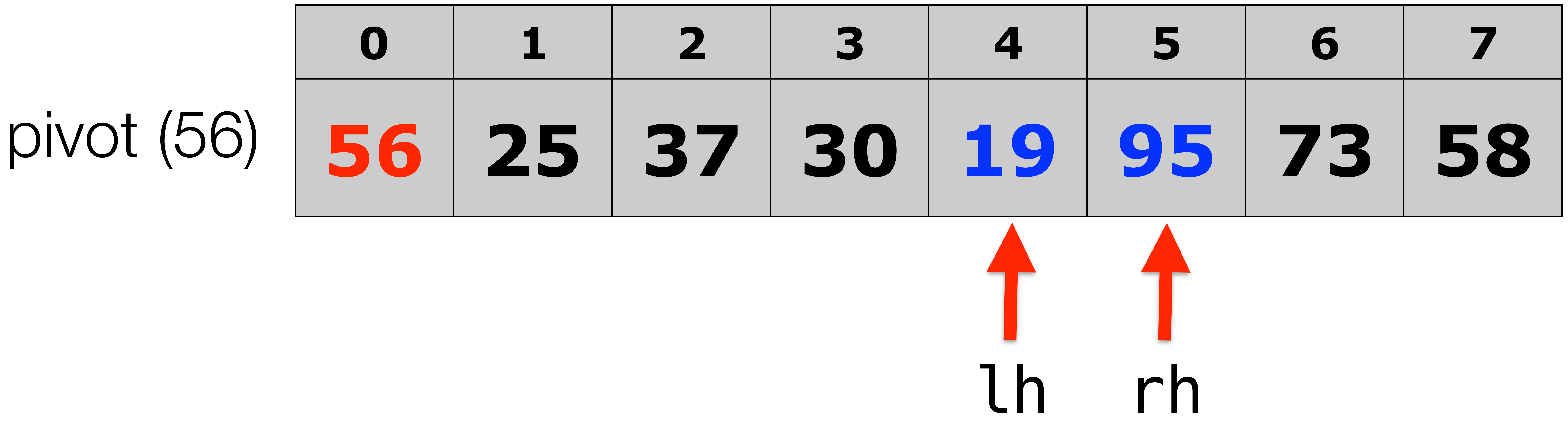
Quicksort Algorithm: In-Place



pivot (56)

0	1	2	3	4	5	6	7
56	25	37	30	19	95	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
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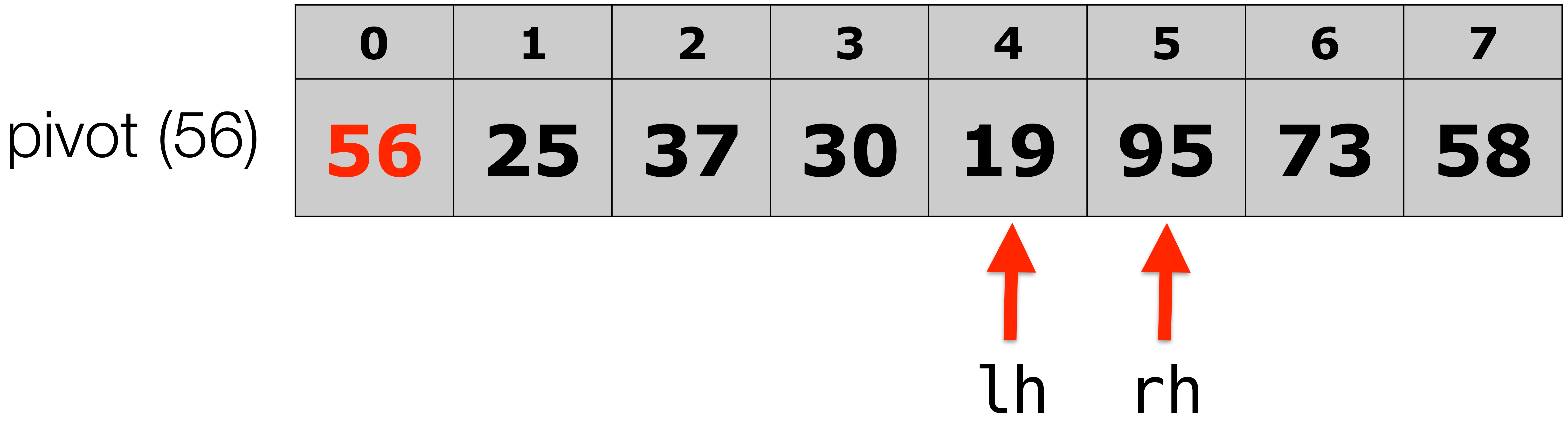
Quicksort Algorithm: In-Place



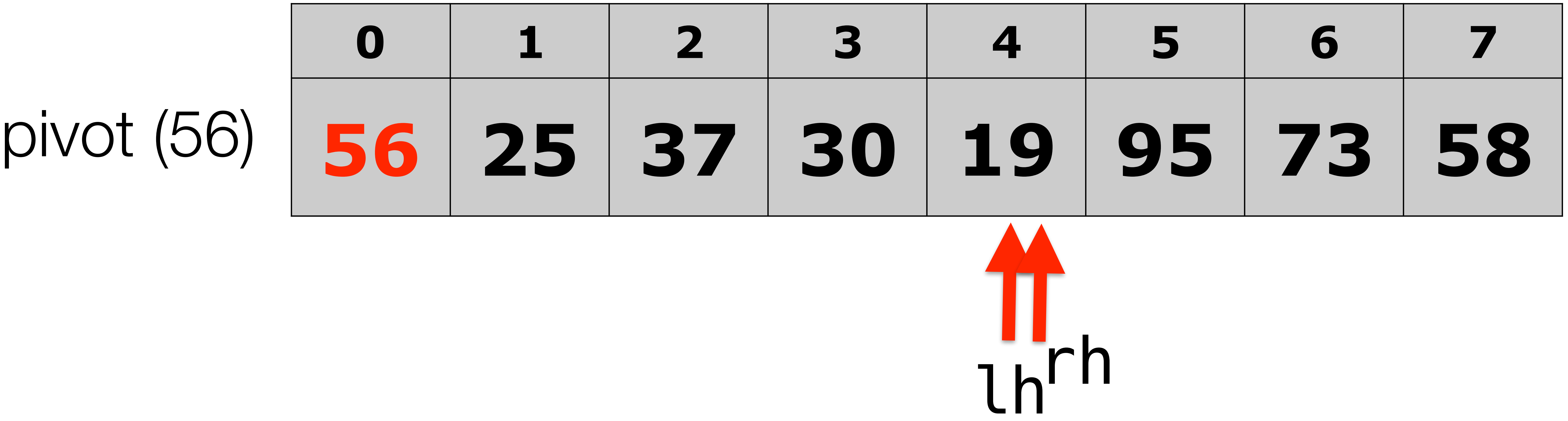
pivot (56)

0	1	2	3	4	5	6	7
56	25	37	30	19	95	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- **Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.**
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place



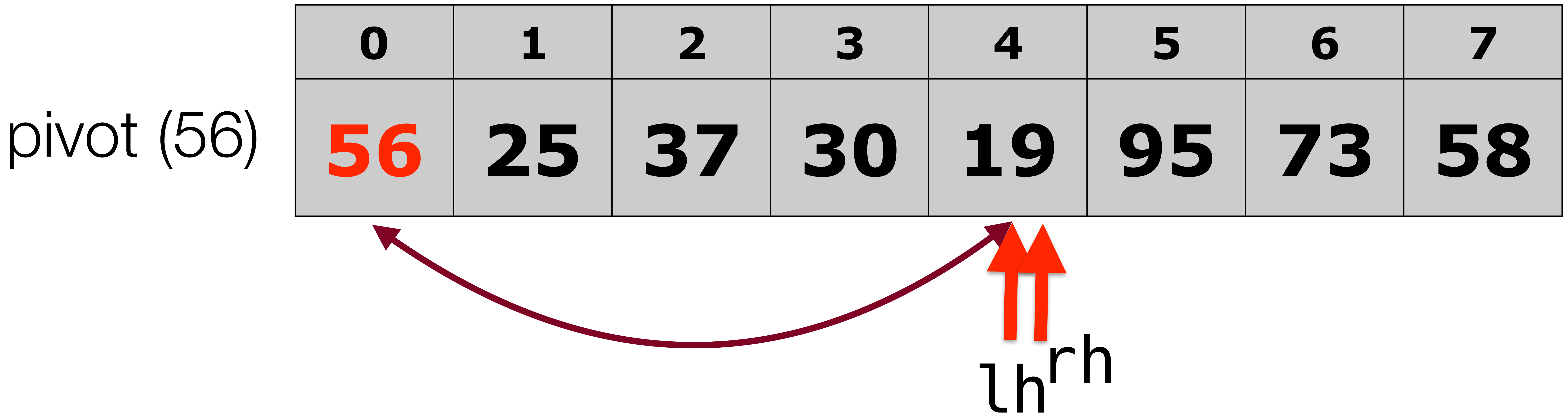
pivot (56)

0	1	2	3	4	5	6	7
56	25	37	30	19	95	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- **Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.**
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- Repeat the traversals until they cross, at which point you swap that element with the pivot.

Quicksort Algorithm: In-Place

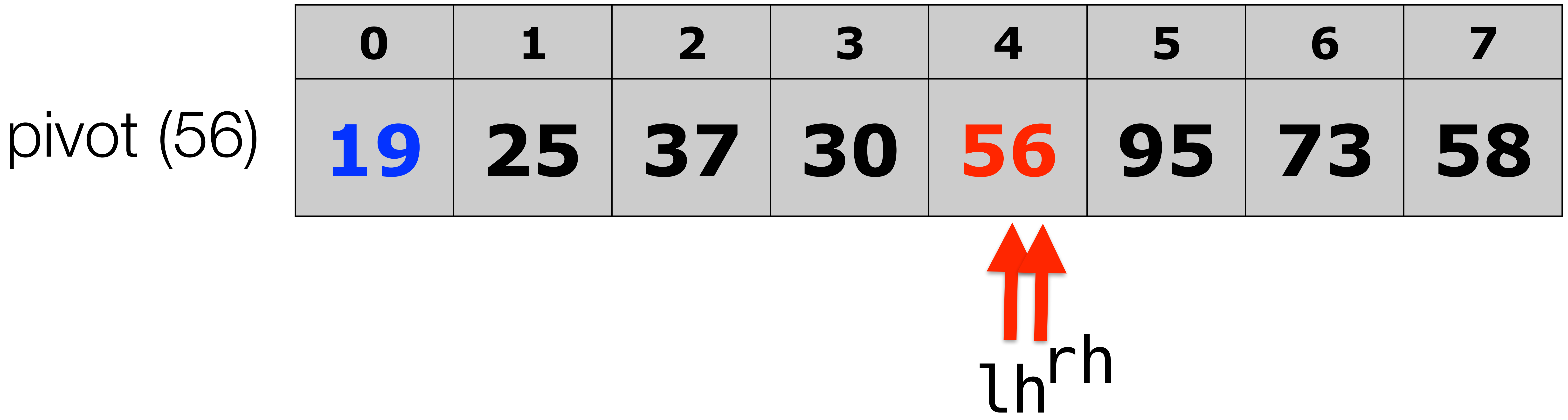


	0	1	2	3	4	5	6	7
pivot (56)	56	25	37	30	19	95	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- **Repeat the traversals until they cross, at which point you swap that element with the pivot.**

Quicksort Algorithm: In-Place



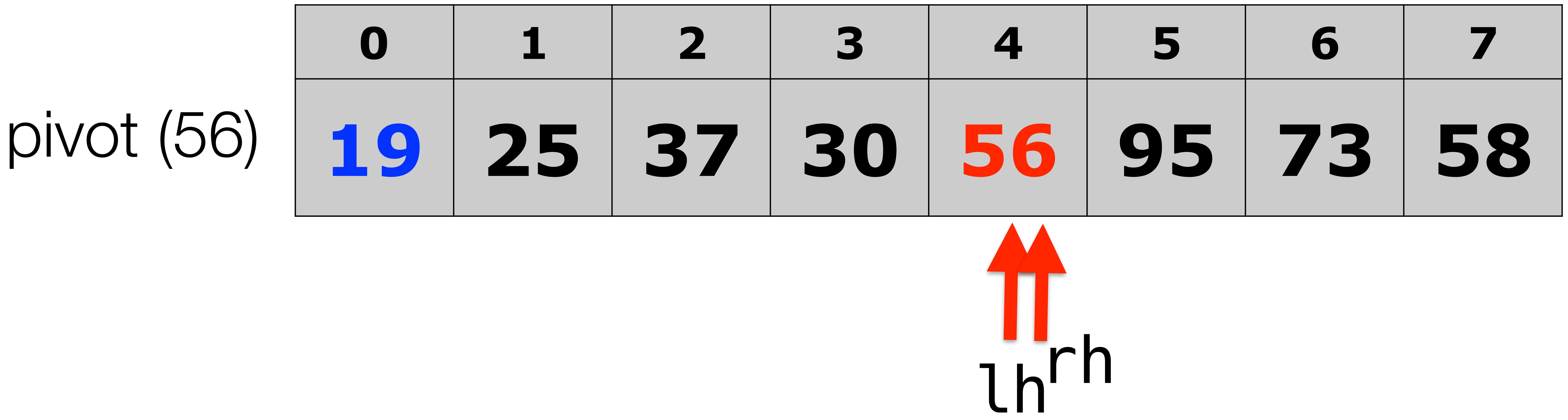
pivot (56)

0	1	2	3	4	5	6	7
19	25	37	30	56	95	73	58

```
quickSort(vec, 0, 7);
```

- Pick your pivot as the left element (might not be a good choice...)
- Traverse the list from the end (right) backwards until the value should be to the *left* of the pivot, or it hits the left.
- Traverse the list from the beginning (left, after pivot) forwards until the value should be to the *right* of the pivot, or until it hits the right.
- Swap the two elements where the left/right cross, unless the pivot is the smallest.
- **Repeat the traversals until they cross, at which point you swap that element with the pivot.**

Quicksort Algorithm: In-Place



pivot (56)

0	1	2	3	4	5	6	7
19	25	37	30	56	95	73	58

```
quickSort(vec, 0, 7);
```

- The partitioning step has completed! The elements to the left of 56 are smaller, and the elements to the right are bigger!
- The partitioning step returns the "boundary" value (index 4, in this case), and we can now sort each sub-part of the vector:

```
quickSort(vec, 0, 3);
quickSort(vec, 4, 7);
```

If start is ever bigger than finish, we just return!

Quicksort Algorithm: Big-O

0	1	2	3	4	5	6	7
19	25	37	30	56	95	73	58

- Best-case time complexity: O(n log n)
- Worst-case time complexity: $O(n^2)$
- Average time complexity: O(n log n)
- Space complexity: naive: O(n) extra, in-place: O(log n) extra (because of recursion)
- Stable?

Quicksort In-place Code

```
/*
 * Rearranges the elements of v into sorted order using
 * a recursive quick sort algorithm.
 */
void quicksort(Vector<int> &vec) {
    quicksort(vec, 0, vec.size() - 1);
}

void quicksort(Vector<int> &vec, int start, int finish) {
    if (start >= finish) return;
    int boundary = partition(vec, start, finish);
    quicksort(vec, start, boundary - 1);
    quicksort(vec, boundary + 1, finish);
}
```

We need a helper function to pass along left and right.

Quicksort In-place Code: Partition

```cpp
int partition(Vector<int> &vec, int start, int finish) {
    int pivot = vec[start];
    int lh = start + 1;
    int rh = finish;

    while (true) {
        while (lh < rh && vec[rh] >= pivot) rh--;
        while (lh < rh && vec[lh] < pivot) lh++;
        if (lh == rh) break;

        // swap
        int tmp = vec[lh];
        vec[lh] = vec[rh];
        vec[rh] = tmp;
    }

    if (vec[lh] >= pivot) return start;
    vec[start] = vec[lh];
    vec[lh] = pivot;
    return lh;
}
```

Recap

Sorting Big-O Cheat Sheet			
Sort	**Worst Case**	**Best Case**	**Average Case**
Insertion	$O(n^2)$	$O(n)$	$O(n^2)$
Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Merge	$O(n \log n)$	$O(n \log n)$	$O(n \log n)$
Quicksort	$O(n^2)$	$O(n \log n)$	$O(n \log n)$

References and Advanced Reading

- **References:**
  - http://en.wikipedia.org/wiki/Sorting_algorithm (excellent)
  - http://www.sorting-algorithms.com (fantastic visualization)
  - More online visualizations: http://www.cs.usfca.edu/~galles/visualization/Algorithms.html (excellent)
  - Excellent mergesort video: https://www.youtube.com/watch?v=GCae1WNvnZM
  - Excellent quicksort video: https://www.youtube.com/watch?v=XE4VP_8Y0BU
  - Full quicksort trace: http://goo.gl/vOgaT5

- **Advanced Reading:**
  - YouTube video, 15 sorts in 6 minutes: https://www.youtube.com/watch?v=kPRA0W1kECg (fun, with sound!)
  - Amazing folk dance sorts: https://www.youtube.com/channel/UCIqiLefbVHsOAXDAxQJH7Xw
  - Radix Sort: https://en.wikipedia.org/wiki/Radix_sort
  - Good radix animation: https://www.cs.auckland.ac.nz/software/AlgAnim/radixsort.html
  - Shell Sort: https://en.wikipedia.org/wiki/Shellsort
  - Bogosort: https://en.wikipedia.org/wiki/Bogosort

Extra Slides

Why is the following nested loop O(n^2)?

```
for (int i=0; i < n; i++) {
    for (int j=i; j < n; j++) {
        // do stuff...
    }
}
```

The first time through the outer loop, there are n steps.
The second time through the outer loop, there are n-1 steps.
The third time through the outer loop, there are n-2 steps.
...
The last time through the outer loop, there is 1 step.

Why is the following nested loop O(n^2)?

```
for (int i=0; i < n; i++) {
    for (int j=i; j < n; j++) {
        // do stuff...
    }
}
```

In other words, the number of total steps is:

$$n + (n-1) + (n-2) + \ldots + 2 + 1 = (n + 1) * n/2 = \mathbf{n^2/2 + n/2}$$

which, by our normal rules of simplifying Big O:

$$\mathbf{n^2/2 + n/2 = O(n^2/2) = O(n^2)}$$