

Lecture 3:

Loss functions and

Optimization

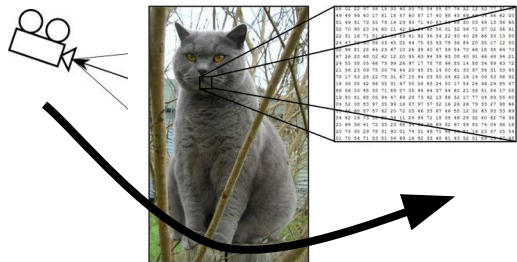
Administrative

A1 is due Jan 20 (Wednesday). ~9 days left

Warning: Jan 18 (Monday) is Holiday (no class/office hours)

Recall from last time... Challenges in Visual Recognition

Camera pose



Illumination



Deformation



Occlusion



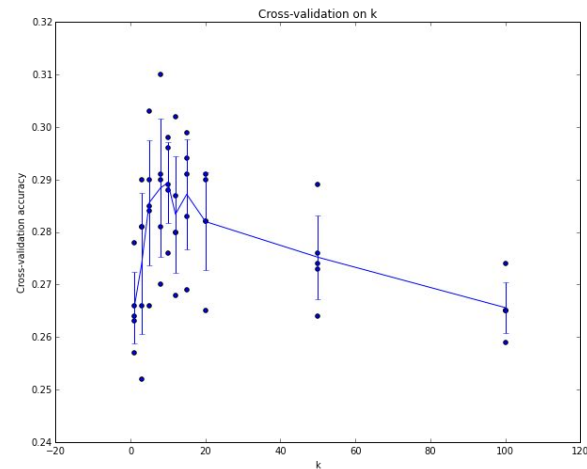
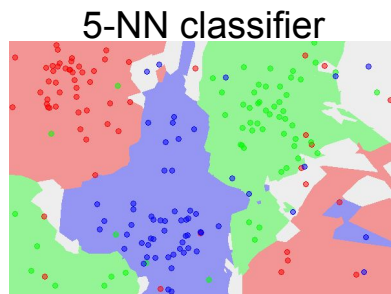
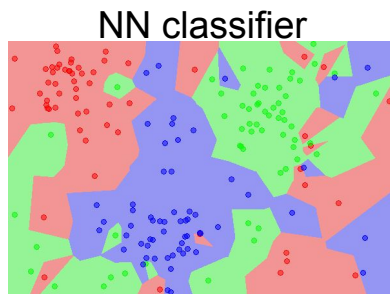
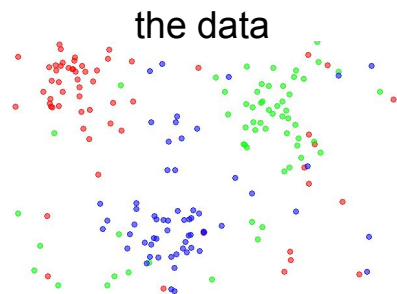
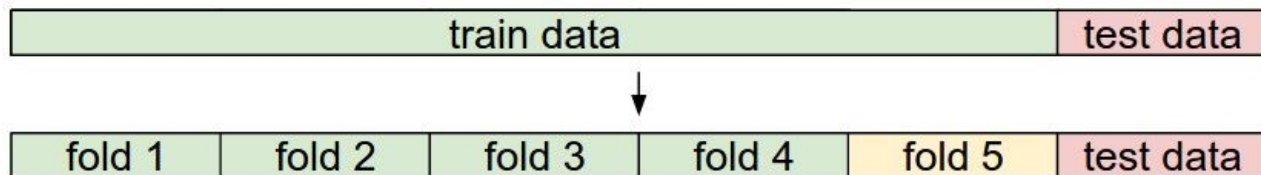
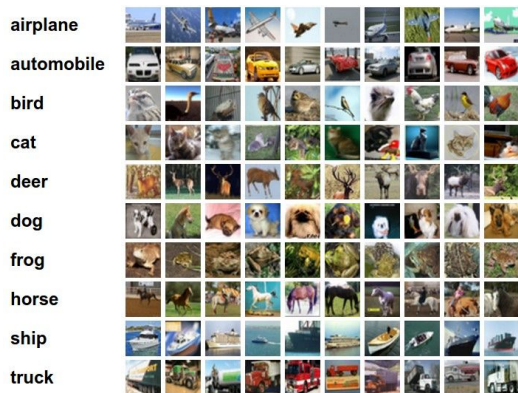
Background clutter



Intraclass variation



Recall from last time... data-driven approach, kNN



Recall from last time... Linear classifier

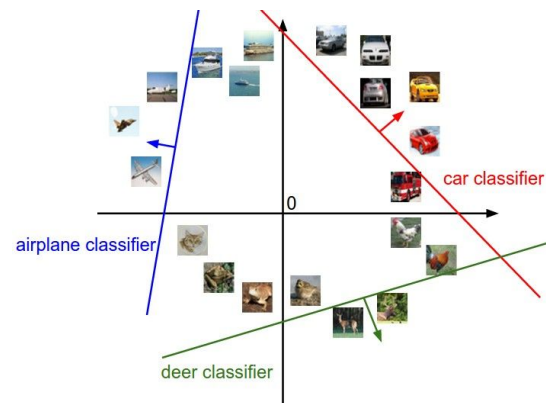
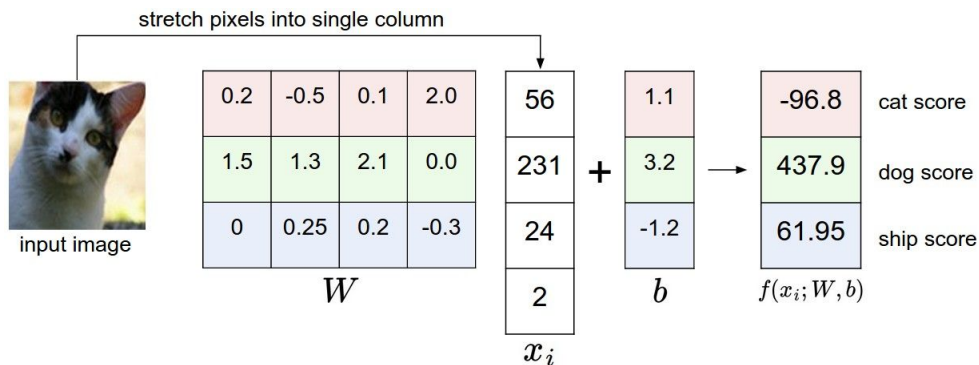


[32x32x3]

array of numbers 0...1
(3072 numbers total)

image parameters
 $f(\mathbf{x}, \mathbf{W})$

10 numbers, indicating
class scores



Recall from last time... Going forward: Loss function/Optimization



airplane	-3.45	-0.51	3.42
automobile	-8.87	6.04	4.64
bird	0.09	5.31	2.65
cat	2.9	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	-4.34
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

TODO:

1. Define a **loss function** that quantifies our unhappiness with the scores across the training data.
2. Come up with a way of efficiently finding the parameters that minimize the loss function.
(optimization)

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1

Losses: **2.9**

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\begin{aligned} L_i &= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \\ &= \max(0, 5.1 - 3.2 + 1) \\ &\quad + \max(0, -1.7 - 3.2 + 1) \\ &= \max(0, 2.9) + \max(0, -3.9) \\ &= 2.9 + 0 \\ &= 2.9 \end{aligned}$$

Suppose: 3 training examples, 3 classes.
 With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1
Losses:	2.9	0	

Multiclass SVM loss:

Given an example (x_i, y_i)
 where x_i is the image and
 where y_i is the (integer) label,

and using the shorthand for the
 scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\begin{aligned}
 L_i &= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \\
 &= \max(0, 1.3 - 4.9 + 1) \\
 &\quad + \max(0, 2.0 - 4.9 + 1) \\
 &= \max(0, -2.6) + \max(0, -1.9) \\
 &= 0 + 0 \\
 &= 0
 \end{aligned}$$

Suppose: 3 training examples, 3 classes.
 With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1
Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
 where x_i is the image and
 where y_i is the (integer) label,

and using the shorthand for the
 scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$\begin{aligned}
 L_i &= \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \\
 &= \max(0, 2.2 - (-3.1) + 1) \\
 &\quad + \max(0, 2.5 - (-3.1) + 1) \\
 &= \max(0, 5.3) + \max(0, 5.6) \\
 &= 5.3 + 5.6 \\
 &= 10.9
 \end{aligned}$$

Suppose: 3 training examples, 3 classes.
 With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1
Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
 where x_i is the image and
 where y_i is the (integer) label,

and using the shorthand for the
 scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

and the full training loss is the mean
 over all examples in the training data:

$$L = \frac{1}{N} \sum_{i=1}^N L_i$$

$$L = (2.9 + 0 + 10.9)/3 \\ = 4.6$$

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
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Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q: what if the sum
was instead over all
classes?
(including $j = y_i$)

Suppose: 3 training examples, 3 classes.
With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
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Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
where x_i is the image and
where y_i is the (integer) label,

and using the shorthand for the
scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q2: what if we used a
mean instead of a
sum here?

Suppose: 3 training examples, 3 classes.
 With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
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Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
 where x_i is the image and
 where y_i is the (integer) label,

and using the shorthand for the
 scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q3: what if we used

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)^2$$

Suppose: 3 training examples, 3 classes.
 With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
car	5.1	4.9	2.5
frog	-1.7	2.0	-3.1
Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
 where x_i is the image and
 where y_i is the (integer) label,

and using the shorthand for the
 scores vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q4: what is the
 min/max possible
 loss?

Suppose: 3 training examples, 3 classes.
 With some W the scores $f(x, W) = Wx$ are:



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Losses:	2.9	0	10.9

Multiclass SVM loss:

Given an example (x_i, y_i)
 where x_i the image and
 where y_i the (integer) label,

and using the shorthand for the scores
 vector: $s = f(x_i, W)$

the SVM loss has the form:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Q5: usually at
 initialization W are small
 numbers, so all $s \approx 0$.
 What is the loss?

Example numpy code:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

```
def L_i_vectorized(x, y, W):  
    scores = W.dot(x)  
    margins = np.maximum(0, scores - scores[y] + 1)  
    margins[y] = 0  
    loss_i = np.sum(margins)  
    return loss_i
```

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$

There is a bug with the loss:

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$



There is a bug with the loss:

$$f(x, W) = Wx$$

$$L = \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1)$$



E.g. Suppose that we found a W such that $L = 0$.
Is this W unique?

Suppose: 3 training examples, 3 classes.
 With some W the scores $f(x, W) = Wx$ are:



cat	3.2	1.3	2.2
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frog	-1.7	2.0	-3.1
Losses:	2.9	0	

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Before:

$$\begin{aligned}
 &= \max(0, 1.3 - 4.9 + 1) \\
 &\quad + \max(0, 2.0 - 4.9 + 1) \\
 &= \max(0, -2.6) + \max(0, -1.9) \\
 &= 0 + 0 \\
 &= 0
 \end{aligned}$$

With W twice as large:

$$\begin{aligned}
 &= \max(0, 2.6 - 9.8 + 1) \\
 &\quad + \max(0, 4.0 - 9.8 + 1) \\
 &= \max(0, -6.2) + \max(0, -4.8) \\
 &= 0 + 0 \\
 &= 0
 \end{aligned}$$

Weight Regularization

λ = regularization strength
(hyperparameter)

$$L = \frac{1}{N} \sum_{i=1}^N \sum_{j \neq y_i} \max(0, f(x_i; W)_j - f(x_i; W)_{y_i} + 1) + \lambda R(W)$$

In common use:

L2 regularization

$$R(W) = \sum_k \sum_l W_{k,l}^2$$

L1 regularization

$$R(W) = \sum_k \sum_l |W_{k,l}|$$

Elastic net (L1 + L2)

$$R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$$

Max norm regularization (might see later)

Dropout (will see later)

L2 regularization: motivation

$$x = [1, 1, 1, 1]$$

$$w_1 = [1, 0, 0, 0]$$

$$w_2 = [0.25, 0.25, 0.25, 0.25]$$

$$w_1^T x = w_2^T x = 1$$

Softmax Classifier (Multinomial Logistic Regression)



cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)



scores = unnormalized log probabilities of the classes.

$$s = f(x_i; W)$$

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)



scores = unnormalized log probabilities of the classes.

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}} \quad \text{where} \quad s = f(x_i; W)$$

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)



scores = unnormalized log probabilities of the classes.

$$P(Y = k | X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$

where

$$s = f(x_i; W)$$

cat	3.2
car	5.1
frog	-1.7

Softmax function

Softmax Classifier (Multinomial Logistic Regression)



scores = unnormalized log probabilities of the classes.

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}} \quad \text{where} \quad s = f(x_i; W)$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i|X = x_i)$$

cat	3.2
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)



cat	3.2
car	5.1
frog	-1.7

scores = unnormalized log probabilities of the classes.

$$P(Y = k|X = x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}} \quad \text{where} \quad s = f(x_i; W)$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y = y_i|X = x_i)$$

in summary:
$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

cat

3.2

car

5.1

frog

-1.7

unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat
car
frog

3.2

5.1

-1.7

exp

24.5

164.0

0.18

unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat
car
frog

3.2
5.1
-1.7

exp

24.5
164.0
0.18

normalize

0.13
0.87
0.00

unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat
car
frog

3.2
5.1
-1.7

exp

24.5
164.0
0.18

normalize

0.13
0.87
0.00

$$L_i = -\log(0.13) = 0.89$$

unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

Q: What is the min/max possible loss L_i ?

cat
car
frog

3.2
5.1
-1.7

exp

24.5
164.0
0.18

normalize

0.13
0.87
0.00

$$L_i = -\log(0.13) = 0.89$$

unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)



$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

Q5: usually at initialization W are small numbers, so all $s \approx 0$. What is the loss?

cat
car
frog

3.2

5.1

-1.7

exp

24.5

164.0

0.18

normalize

0.13

0.87

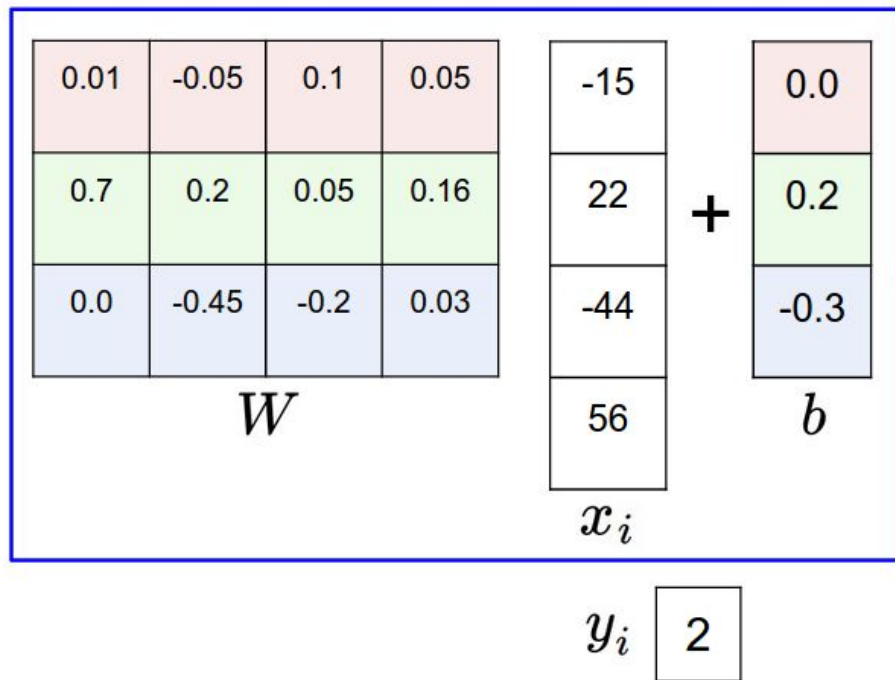
0.00

$$L_i = -\log(0.13) = 0.89$$

unnormalized log probabilities

probabilities

matrix multiply + bias offset



hinge loss (SVM)

-2.85
0.86
0.28

$$\begin{aligned} &\max(0, -2.85 - 0.28 + 1) + \\ &\max(0, 0.86 - 0.28 + 1) \\ &= \\ &\mathbf{1.58} \end{aligned}$$

cross-entropy loss (Softmax)

-2.85
0.86
0.28

$\xrightarrow{\text{exp}}$

0.058
2.36
1.32

$\xrightarrow{\text{normalize}}$
(to sum to one)

0.016
0.631
0.353

$$\begin{aligned} &-\log(0.353) \\ &= \\ &\mathbf{0.452} \end{aligned}$$

Softmax vs. SVM

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

Softmax vs. SVM

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

assume scores:

[10, -2, 3]

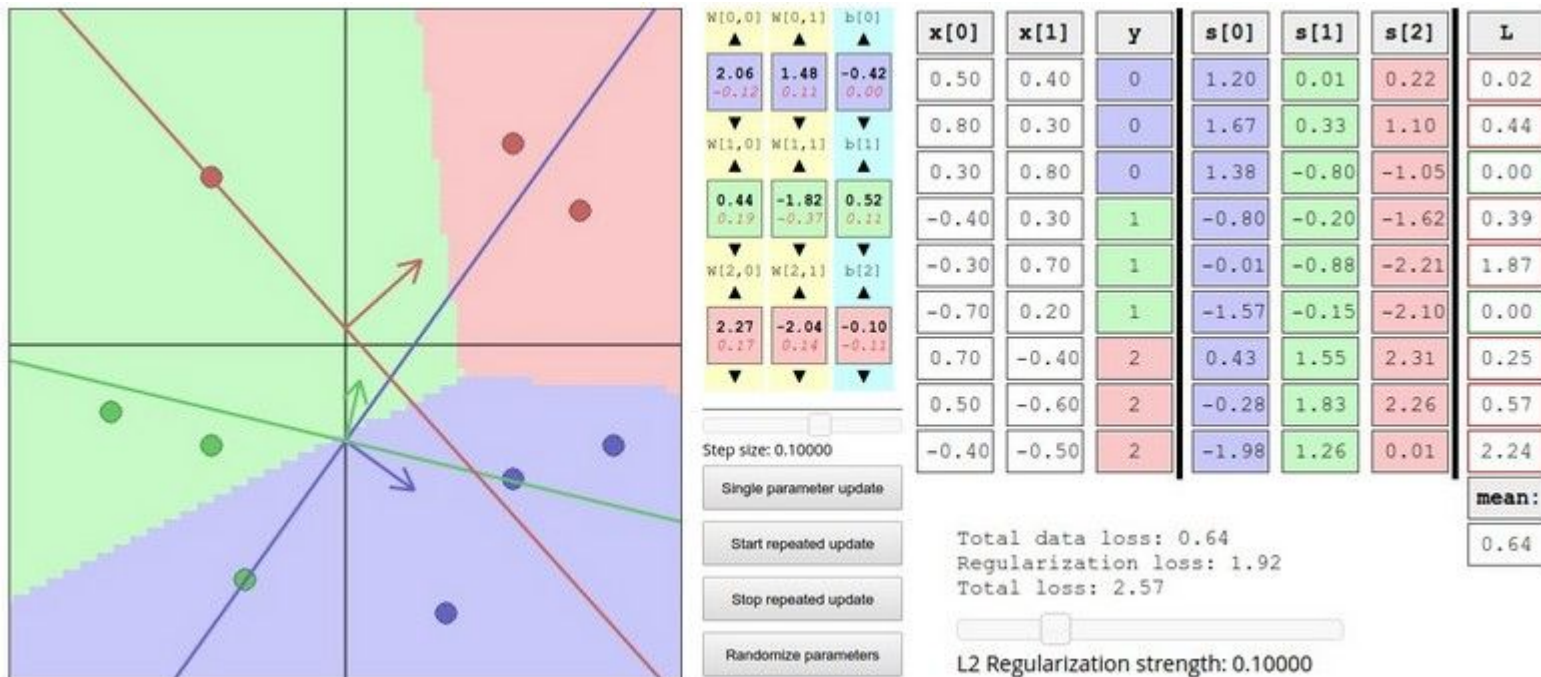
[10, 9, 9]

[10, -100, -100]

and $y_i = 0$

Q: Suppose I take a datapoint and I jiggle a bit (changing its score slightly). What happens to the loss in both cases?

Interactive Web Demo time....



<http://vision.stanford.edu/teaching/cs231n/linear-classify-demo/>

Optimization

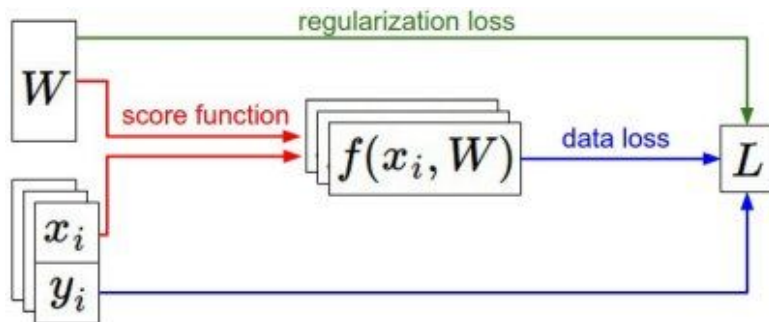
Recap

- We have some dataset of (x,y)
- We have a **score function**: $s = f(x; W) \stackrel{\text{e.g.}}{=} Wx$
- We have a **loss function**:

$$L_i = -\log\left(\frac{e^{sy_i}}{\sum_j e^{sj}}\right) \quad \text{Softmax}$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1) \quad \text{SVM}$$

$$L = \frac{1}{N} \sum_{i=1}^N L_i + R(W) \quad \text{Full loss}$$



Strategy #1: A first very bad idea solution: **Random search**

```
# assume X_train is the data where each column is an example (e.g. 3073 x 50,000)
# assume Y_train are the labels (e.g. 1D array of 50,000)
# assume the function L evaluates the loss function

bestloss = float("inf") # Python assigns the highest possible float value
for num in xrange(1000):
    W = np.random.randn(10, 3073) * 0.0001 # generate random parameters
    loss = L(X_train, Y_train, W) # get the loss over the entire training set
    if loss < bestloss: # keep track of the best solution
        bestloss = loss
        bestW = W
    print 'in attempt %d the loss was %f, best %f' % (num, loss, bestloss)

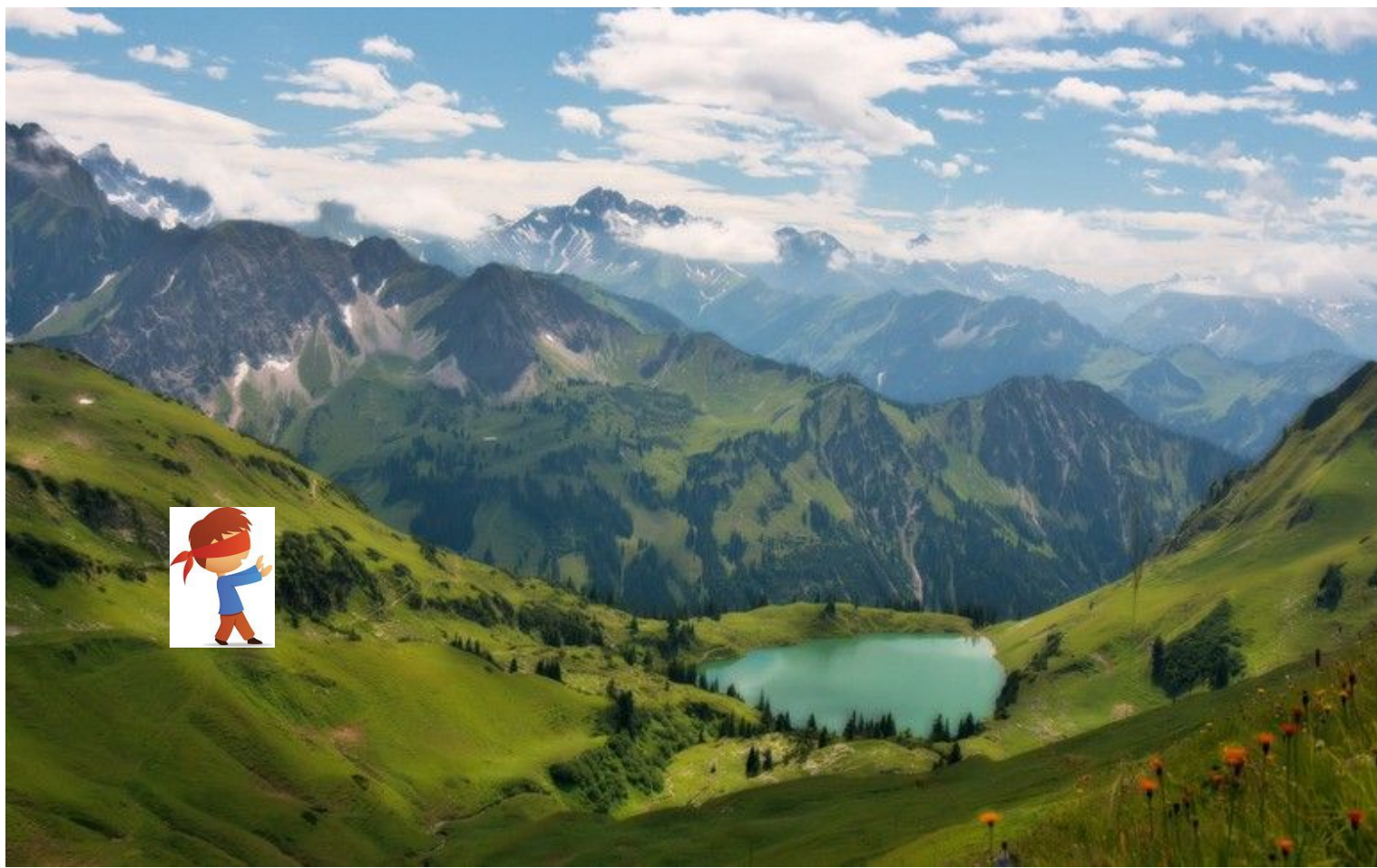
# prints:
# in attempt 0 the loss was 9.401632, best 9.401632
# in attempt 1 the loss was 8.959668, best 8.959668
# in attempt 2 the loss was 9.044034, best 8.959668
# in attempt 3 the loss was 9.278948, best 8.959668
# in attempt 4 the loss was 8.857370, best 8.857370
# in attempt 5 the loss was 8.943151, best 8.857370
# in attempt 6 the loss was 8.605604, best 8.605604
# ... (truncated: continues for 1000 lines)
```

Lets see how well this works on the test set...

```
# Assume X_test is [3073 x 10000], Y_test [10000 x 1]  
scores = Wbest.dot(Xte_cols) # 10 x 10000, the class scores for all test examples  
# find the index with max score in each column (the predicted class)  
Yte_predict = np.argmax(scores, axis = 0)  
# and calculate accuracy (fraction of predictions that are correct)  
np.mean(Yte_predict == Yte)  
# returns 0.1555
```

15.5% accuracy! not bad!
(SOTA is ~95%)





Strategy #2: **Follow the slope**

In 1-dimension, the derivative of a function:

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

In multiple dimensions, the **gradient** is the vector of (partial derivatives).

current W:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

gradient dW:

[?,
?,
?,
?,
?,
?,
?,
?,
?,...]

current W:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

W + h (first dim):

[0.34 + **0.0001**,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25322

gradient dW:

[?,
?,
?,
?,
?,
?,
?,
?,
?,...]

current **W**:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

W + h (first dim):

[0.34 + **0.0001**,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25322

gradient dW:

[-2.5,
?,
?,

$$(1.25322 - 1.25347)/0.0001 = -2.5$$

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

?,
?,...]

current W:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

W + h (second dim):

[0.34,
-1.11 + **0.0001**,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25353

gradient dW:

[-2.5,
?,
?,
?,
?,
?,
?,
?,
?,...]

current **W**:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

W + h (second dim):

[0.34,
-1.11 + **0.0001**,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25353

gradient **dW**:

[-2.5,
0.6,
?,
?,

$$(1.25353 - 1.25347)/0.0001 = 0.6$$

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

?,...]

current W:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

W + h (third dim):

[0.34,
-1.11,
0.78 + **0.0001**,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

gradient dW:

[-2.5,
0.6,
?,
?,
?,
?,
?,
?,
?,...]

current **W**:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

W + h (third dim):

[0.34,
-1.11,
0.78 + **0.0001**,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

gradient **dW**:

[-2.5,
0.6,
0,
?,
?

$$(1.25347 - 1.25347)/0.0001 = 0$$

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

f,...]

Evaluation the gradient numerically

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

```
def eval_numerical_gradient(f, x):  
    """  
    a naive implementation of numerical gradient of f at x  
    - f should be a function that takes a single argument  
    - x is the point (numpy array) to evaluate the gradient at  
    """  
  
    fx = f(x) # evaluate function value at original point  
    grad = np.zeros(x.shape)  
    h = 0.00001  
  
    # iterate over all indexes in x  
    it = np.nditer(x, flags=['multi_index'], op_flags=['readwrite'])  
    while not it.finished:  
  
        # evaluate function at x+h  
        ix = it.multi_index  
        old_value = x[ix]  
        x[ix] = old_value + h # increment by h  
        fxh = f(x) # evaluate f(x + h)  
        x[ix] = old_value # restore to previous value (very important!)  
  
        # compute the partial derivative  
        grad[ix] = (fxh - fx) / h # the slope  
        it.iternext() # step to next dimension  
  
    return grad
```

Evaluation the gradient numerically

$$\frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- approximate
- very slow to evaluate

```
def eval_numerical_gradient(f, x):  
    """  
    a naive implementation of numerical gradient of f at x  
    - f should be a function that takes a single argument  
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    while not it.finished:  
  
        # evaluate function at x+h  
        ix = it.multi_index  
        old_value = x[ix]  
        x[ix] = old_value + h # increment by h  
        fxh = f(x) # evaluate f(x + h)  
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        # compute the partial derivative  
        grad[ix] = (fxh - fx) / h # the slope  
        it.iternext() # step to next dimension  
  
    return grad
```


This is silly. The loss is just a function of W :

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \sum_k W_k^2$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$s = f(x; W) = Wx$$

want $\nabla_W L$

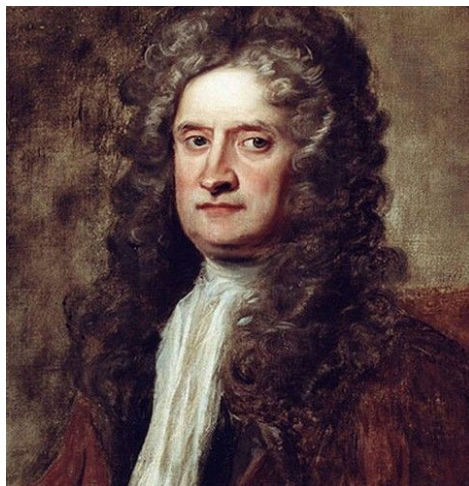
This is silly. The loss is just a function of W :

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \sum_k W_k^2$$

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want $\nabla_W L$



This is silly. The loss is just a function of W :

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \sum_k W_k^2$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$s = f(x; W) = Wx$$

want $\nabla_W L$

Calculus



This is silly. The loss is just a function of W :

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \sum_k W_k^2$$

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

$$s = f(x; W) = Wx$$


$$\nabla_W L = \dots$$

current W:

[0.34,
-1.11,
0.78,
0.12,
0.55,
2.81,
-3.1,
-1.5,
0.33,...]

loss 1.25347

$dW = \dots$
(some function
data and W)



gradient dW:

[-2.5,
0.6,
0,
0.2,
0.7,
-0.5,
1.1,
1.3,
-2.1,...]

In summary:

- Numerical gradient: approximate, slow, easy to write
- Analytic gradient: exact, fast, error-prone

=>

In practice: Always use analytic gradient, but check implementation with numerical gradient. This is called a **gradient check**.

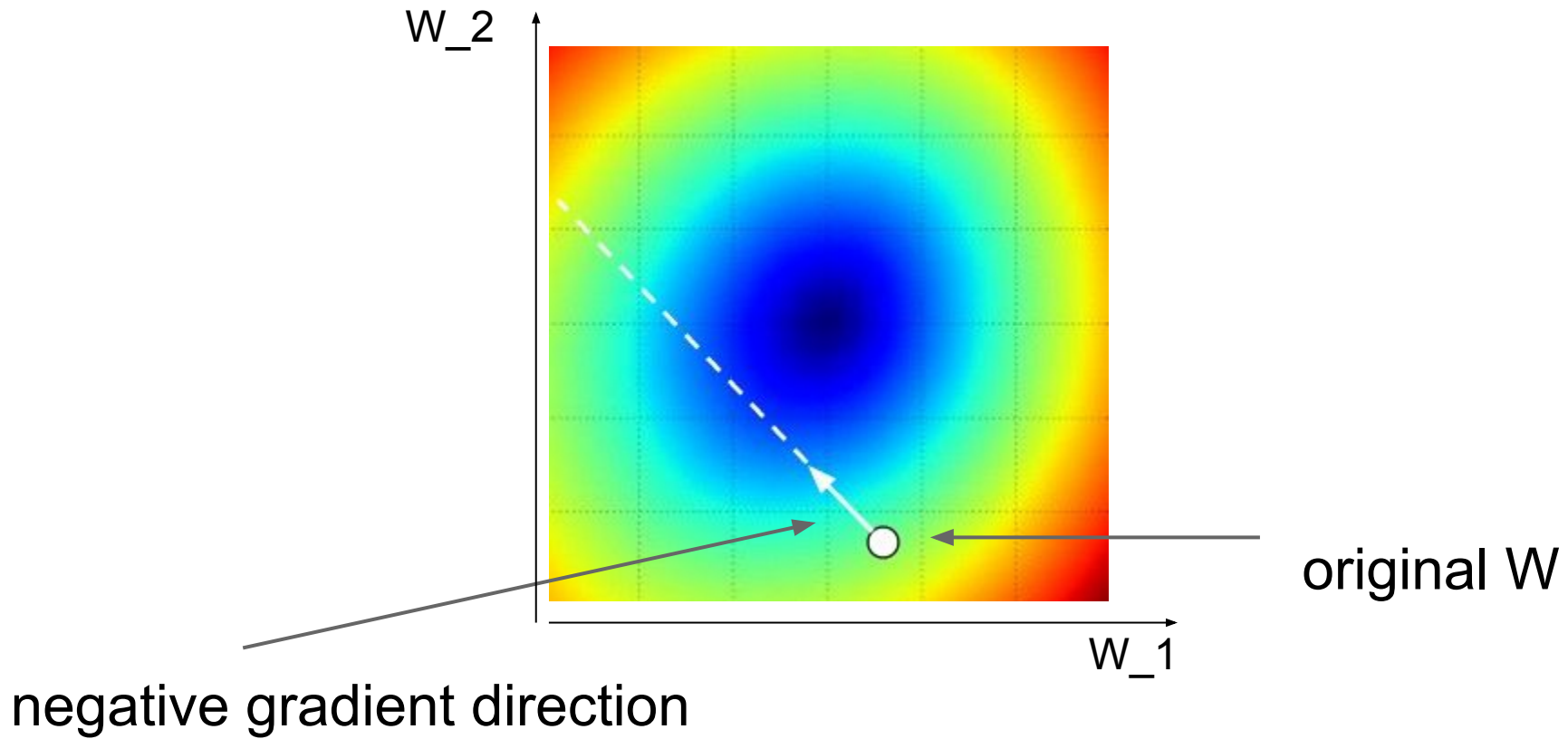
Gradient Descent

```
# Vanilla Gradient Descent
```

```
while True:
```

```
    weights_grad = evaluate_gradient(loss_fun, data, weights)
```

```
    weights += - step_size * weights_grad # perform parameter update
```



Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

```
# Vanilla Minibatch Gradient Descent
```

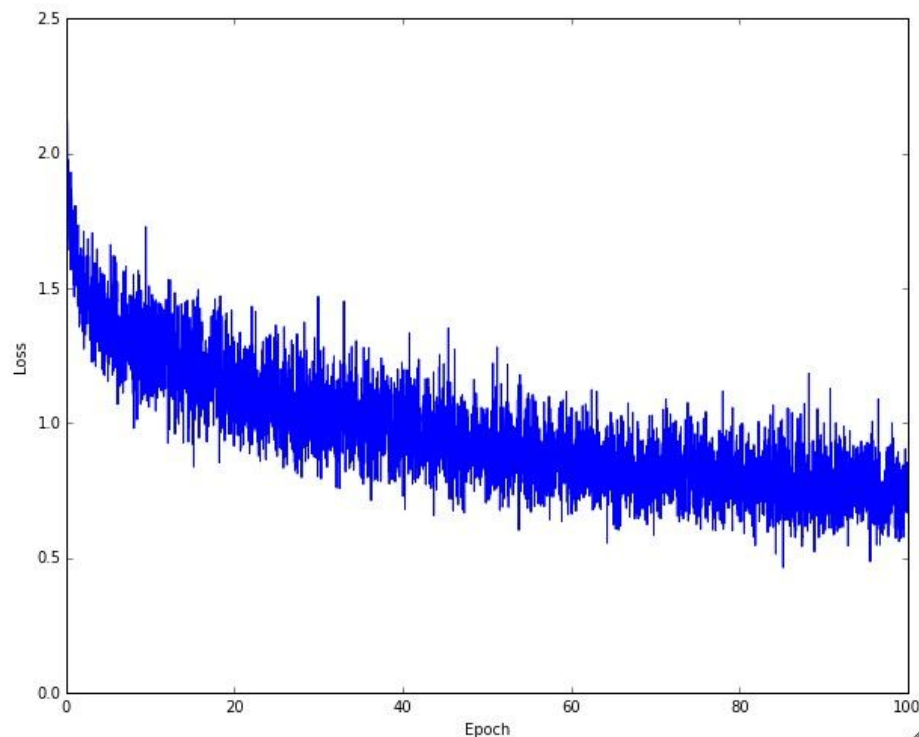
```
while True:
```

```
    data_batch = sample_training_data(data, 256) # sample 256 examples
```

```
    weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
```

```
    weights += - step_size * weights_grad # perform parameter update
```

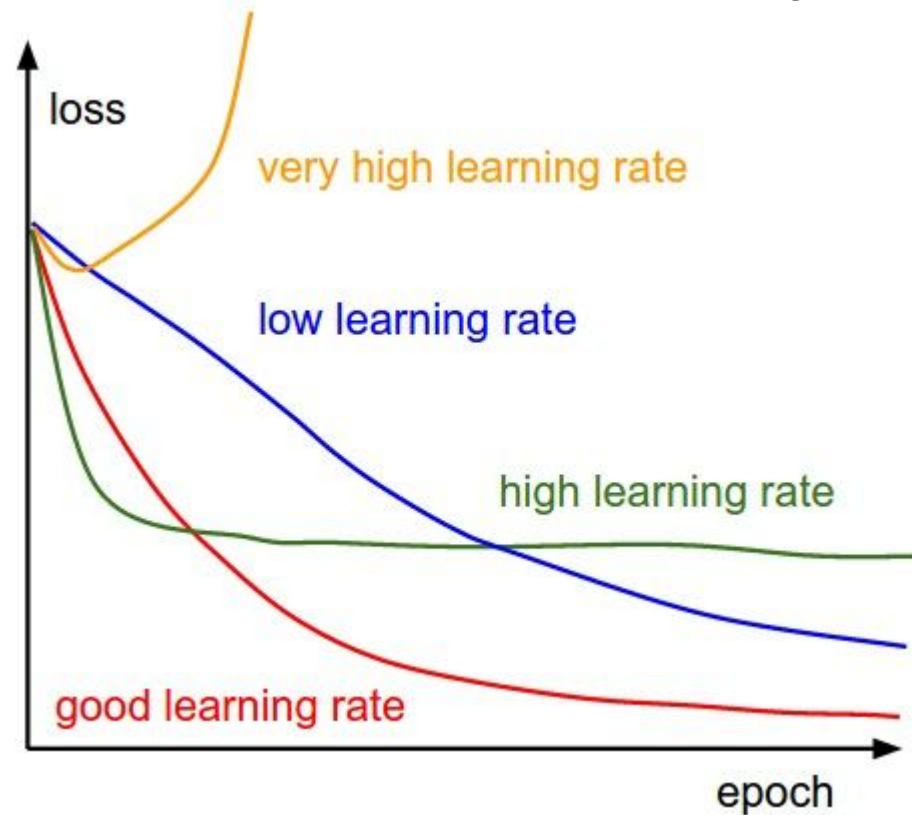
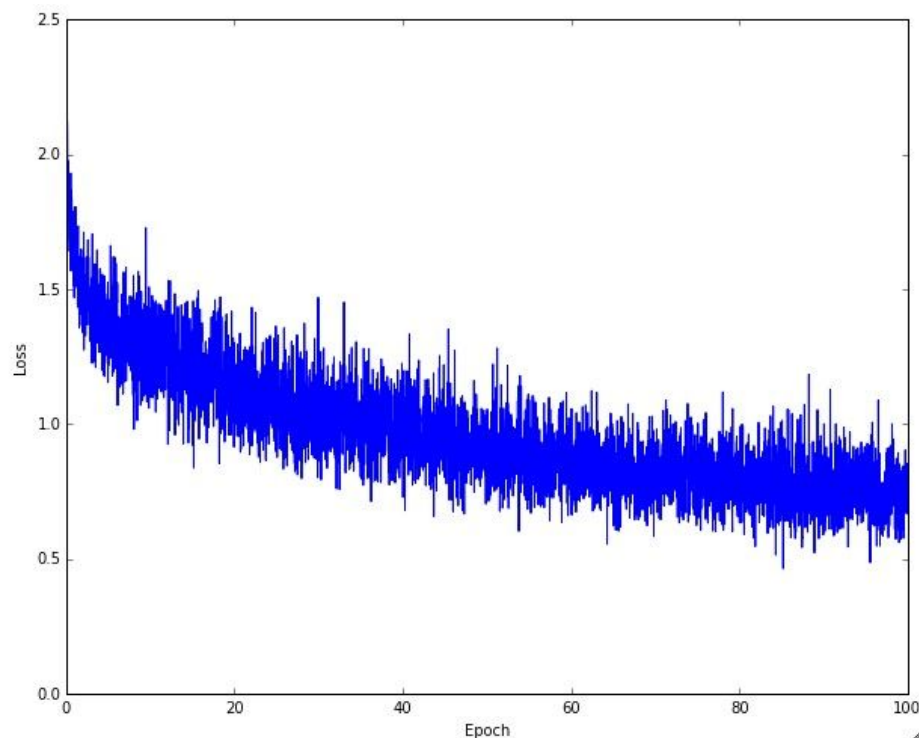
Common mini-batch sizes are 32/64/128 examples
e.g. Krizhevsky ILSVRC ConvNet used 256 examples



Example of optimization progress while training a neural network.

(Loss over mini-batches goes down over time.)

The effects of step size (or “learning rate”)



Mini-batch Gradient Descent

- only use a small portion of the training set to compute the gradient.

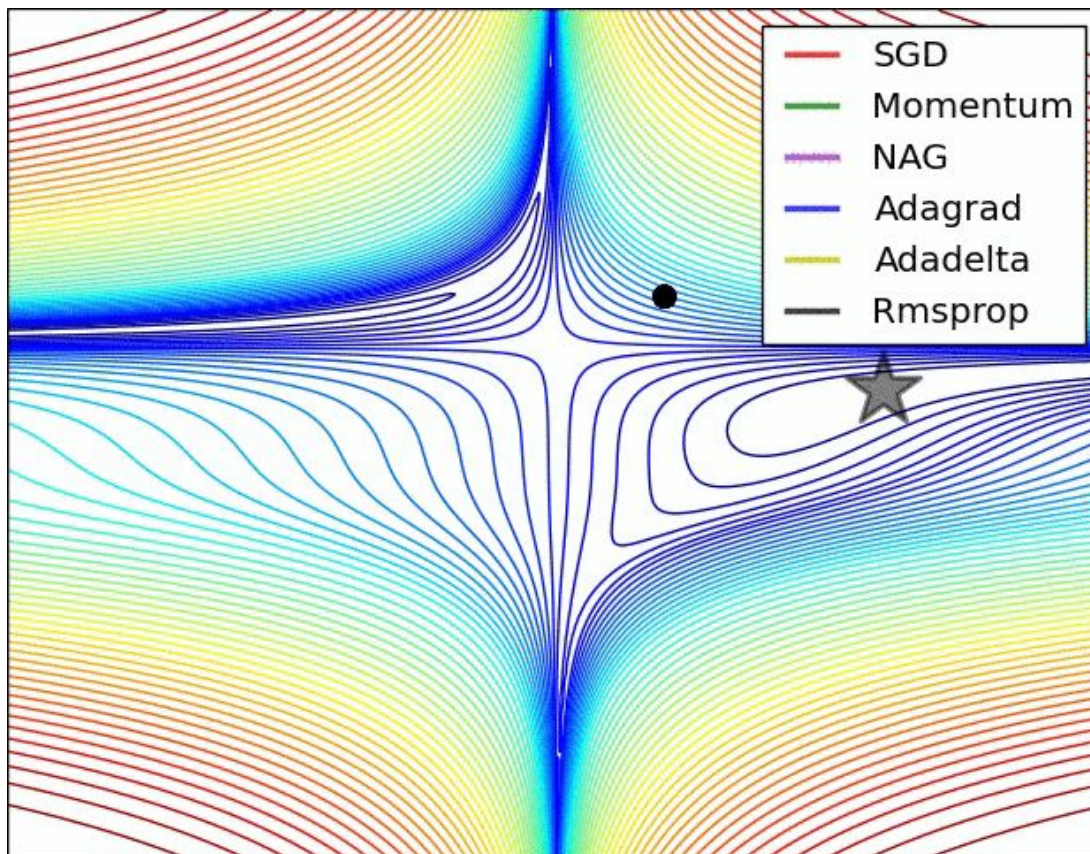
```
# Vanilla Minibatch Gradient Descent

while True:
    data_batch = sample_training_data(data, 256) # sample 256 examples
    weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
    weights += - step_size * weights_grad # perform parameter update
```

Common mini-batch sizes are 32/64/128 examples
e.g. Krizhevsky ILSVRC ConvNet used 256 examples

we will look at more
fancy update formulas
(momentum, Adagrad,
RMSProp, Adam, ...)

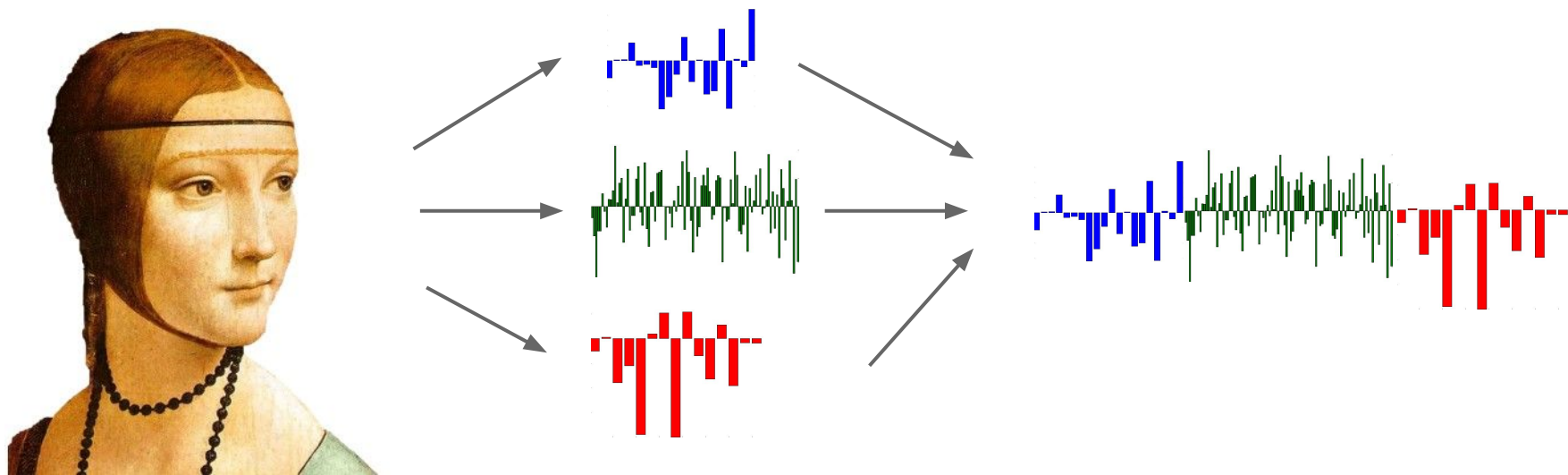
The effects of different update formulas



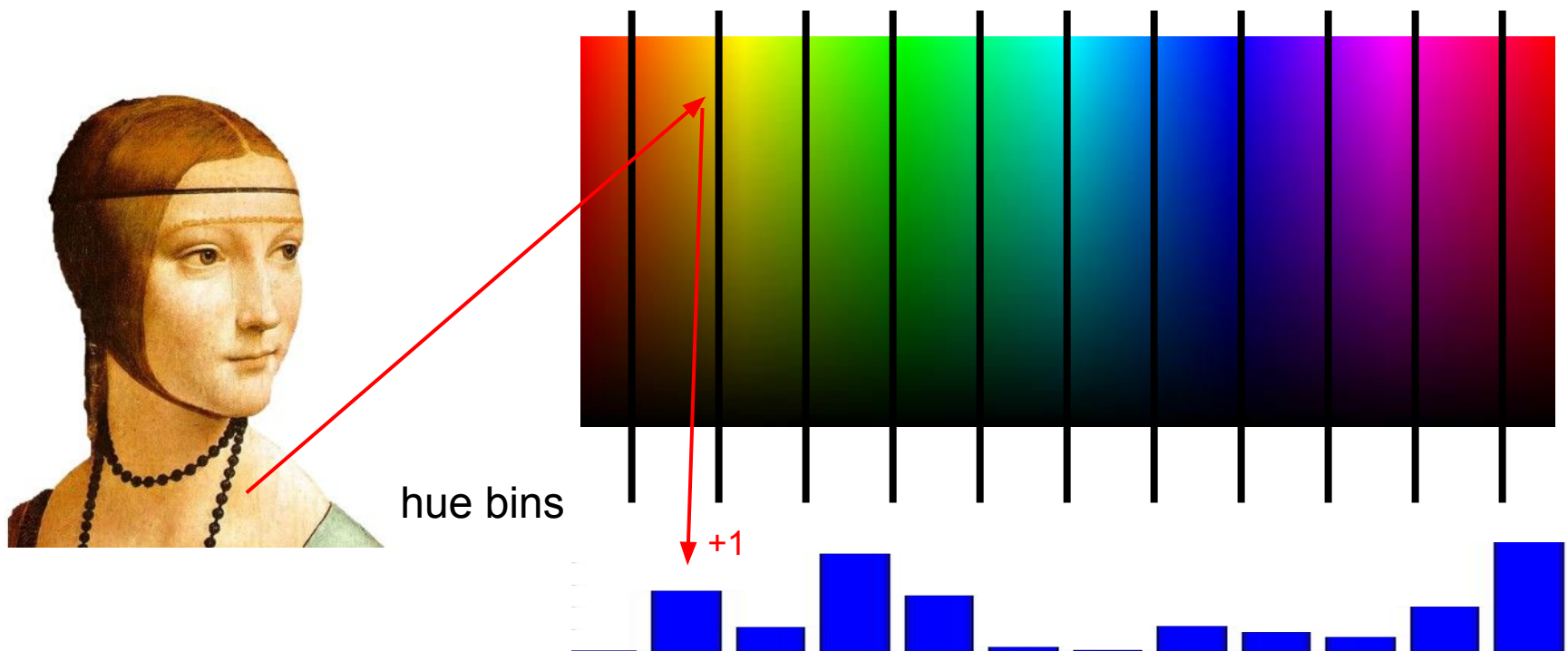
<http://www.denizyuret.com/2015/03/alec-radfords-animations-for.html>

(image credits to Alec Radford)

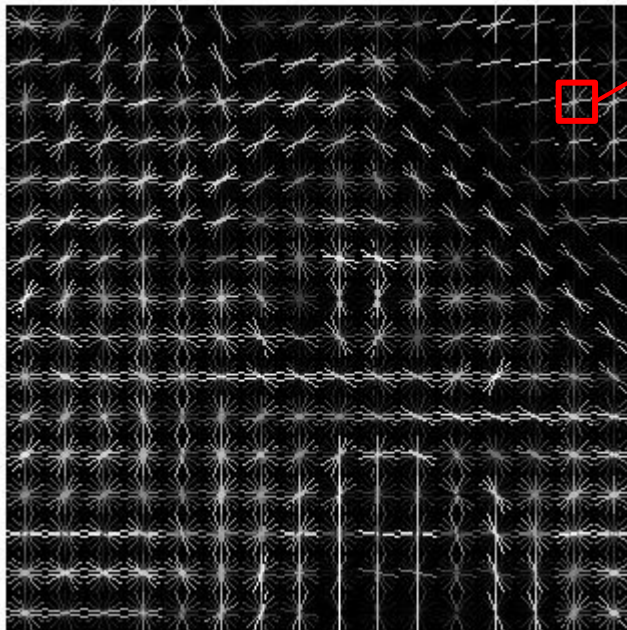
Aside: Image Features



Example: Color (Hue) Histogram



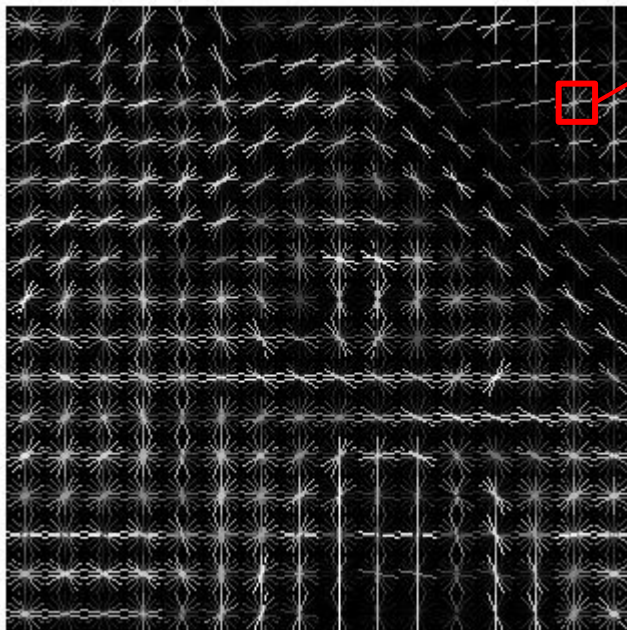
Example: HOG/SIFT features



8x8 pixel region,
quantize the edge
orientation into 9 bins

(image from vlfeat.org)

Example: HOG/SIFT features

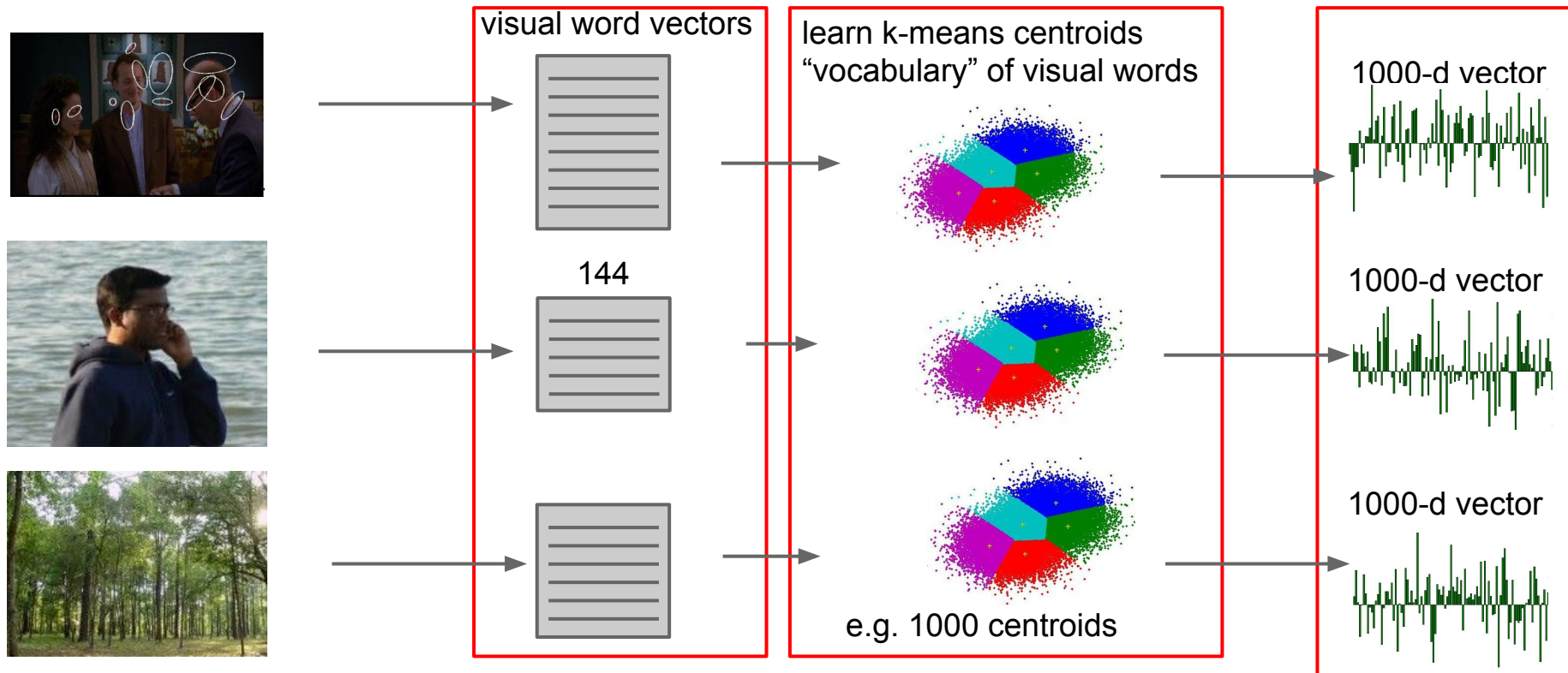


8x8 pixel region,
quantize the edge
orientation into 9 bins

Many more:
GIST, LBP,
Texton,
SSIM, ...

(image from vlfeat.org)

Example: Bag of Words





[32x32x3]



vector describing various
image statistics



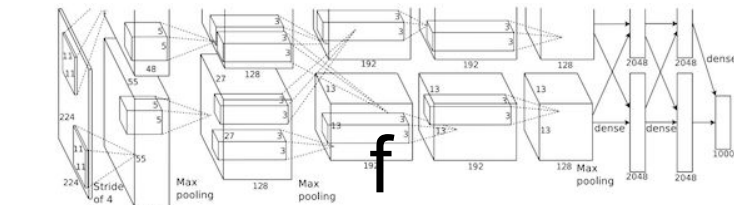
f

training

10 numbers, indicating
class scores



[32x32x3]



training

10 numbers, indicating
class scores

Next class:

Becoming a backprop ninja
and
Neural Networks (part 1)