$\boxed{\text{IDENTITIES}}$

$$d\omega_i = \cos\theta_i \, d\theta_i \, d\phi$$

$$\int_\Omega f(\omega_i) \cdot d\omega_i = \int_\phi \int_{\theta_i} f(\omega_i) \cdot \cos\theta_i \, d\theta_i \, d\phi_i$$

$$d\theta_i = 2 d\theta_R$$

$$= \int_\phi \int_{\theta_R} 2 \cdot f(\omega_i) \cdot \cos\theta_i \, d\theta_R \, d\phi_i$$

$$\Downarrow$$

$$\int_\phi \int_{\theta_R} f(\omega_i) \cdot d\theta_R \, d\phi = 1$$

$$d\theta_R = \frac{1}{2} d\theta_i$$

$$\Leftrightarrow \int_\phi \int_{\theta_i} \frac{f(\omega_i)}{2} \cdot d\theta_i \, d\phi = 1$$

$$d\theta_i \, d\phi = \frac{d\omega_i}{\cos\theta_i}$$

$$\Leftrightarrow \int_\Omega \frac{f(\omega_i)}{2\cos\theta_i} \, d\omega_i = 1$$

$\boxed{\text{USING: } M(\omega_i) = \text{normalized gaussian}}$ | for R and TRT

$$M(\omega_i) = \frac{e^{-\frac{(\theta_h - \alpha)^2}{2\beta^2}}}{\beta \cdot \sqrt{2\pi}}$$

$$N(\phi) = \cos\left(\frac{\phi}{2}\right)$$

We want:

$$\int_{\Omega} S(\omega_i) \cdot \cos\theta_i \, d\omega = 1$$

$$\int_{\phi}\int_{\theta_i} S(\omega_i) \cdot \cos^2\theta_i \, d\theta_i \, d\phi = 1$$

$$\int_{\phi}\int_{\theta_h} 2 \, S(\omega_i) \cdot \cos^2\theta_i \, d\theta_h \, d\phi = 1$$

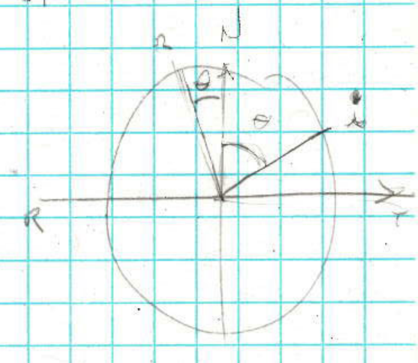
We know:

$$\int_{\phi}\int_{\theta_h} M(\omega_i) \cdot \frac{N(\phi)}{4} \, d\theta_h \, d\phi = 1$$

$$\int_{\phi}\int_{\theta_h} 2 \, M(\omega_i) \, \frac{N(\phi)}{8} \, d\theta_h \, d\phi = 1$$

$$\int_{\phi}\int_{\theta_h} 2 \, \frac{M(\omega_i) \cdot N(\phi)}{8 \cdot \cos^2\theta_i} \cdot \cos^2\theta_i \, d\theta_h \, d\phi = 1$$

$$\Rightarrow \boxed{S(\omega_i) = \frac{M(\omega_i) \cdot N(\phi)}{8 \cdot \cos^2\theta_i}}$$



USING : $M(\omega_i)$ and $N(\phi)$: normalized gaussian | for TT and Glint

TT lobe: ~~$S(\omega_i) = M(\omega_i) \cdot N(\phi)$~~

$$\int_{\phi=0}^{2\pi}\int_{\theta_h=-\frac{\pi}{2}}^{\frac{\pi}{2}} M(\omega_i) \cdot N(\phi) \; d\theta_h \, d\phi = 1$$

2 normalized gaussians (in $d\theta_h$ and $d\phi$)

$$\vdots$$

$$\boxed{S(\omega_i) = \frac{M(\omega_i) \cdot N(\phi)}{2 \cdot \cos^2\theta_i}}$$

Glint:

$$\int_{\phi=0}^{2\pi}\int_{\theta_h=-\frac{\pi}{2}}^{\frac{\pi}{2}} M(\omega_i) \cdot N(\phi) \; d\theta_h \cdot d\phi = 2$$

two lobes due to $|\phi|$

$$\vdots$$

$$\boxed{S(\omega_i) = \frac{M(\omega_i) \cdot N(\phi)}{4 \cdot \cos^2\theta_i}}$$

USING: $M(\omega_i)$ = normalized cauchy

We have:

$$\int_{\Omega} M(\omega_i) \cdot \frac{N(\phi)}{4\pi} \cdot d\omega = 1$$

$$\Leftrightarrow \int_{\Omega} \frac{M(\omega_i) \cdot N(\phi)}{\cos\theta_i \cdot 4} \cdot \cos\theta_i \, d\omega_i = 1$$

$$\Rightarrow \boxed{S(\omega_i) = \frac{M(\omega_i) \cdot N(\phi)}{4 \cdot \cos\theta_i}}$$