ORIE 4630 Project 2

Shelter from the Storm

Analyzing Optimal Portfolios throughout the Great Recession

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Abstract

On September 29th, 2008, the Dow Jones Industrial Average (DJIA) dropped 777.68 points, making it the largest point drop in the history of the stock market at the time. This marked the single worst day of returns during the Great Recession of 2007 – 2009. This 17-month bear market wiped out 54% of the value in the DJIA, and many Americans lost their life savings in this crash. Bear markets are difficult to combat for almost all investors, which is why alternative investments can be important for investors to manage the risk to their portfolios. In this project, we will apply risk management principals and portfolio analysis to create an optimal portfolio that will include index funds, real estate, and commodities.

1. Introduction:

For many people, a assets' meteoric rise in value leads many to wonder how their life could be different if they had just invested in that stock at the beginning, with recent notable examples include Amazon, Google, and Bitcoin. In a similar vein, a companies' collapse in stock value can be a traumatic event that can lead many to question why they ever invested to begin with. The Great Recession of 2008-2009 was that traumatic event, except one on such a large scale that it wiped out millions of Americans savings, and crushing the earning potential and asset accumulation of an entire generation. Even a decade after the recession, many Americans still have never quite recovered financially. It is with this traumatic backdrop that we decided to do our project. In this project, we wish to explore whether an optimal portfolio during the Great Recession would include commodities and real estate, and whether these portfolio weights would be close to optimal using 2020 data, or how generalizable this mixed portfolio could be.

2. Data Preparation:

We compiled our data for stock price, housing price, and commodity price from three different sources. For the monthly prices and returns of the S&P 500, we used Datahub.com, which gathered the full history of the benchmark. For real estate prices, we used the seasonally adjusted house price index, as provided by the Federal Housing Finance Agency. Finally, for commodities, we used IndexMundi.com, which drew their data from a variety of sources, such as the US Treasury Department, the World Bank, and the Chicago Mercantile Exchange. With all these datasets, we then combined these into Excel, and imported them into R to focus solely on 2007 – 2011. These years were chosen since the project focuses on the Great Recession. 2007 marked the beginning of the Recession, with the subprime loan crisis starting in April 2007. While 2011 may seem like a late endpoint for a project focused on the Recession, the DJIA had yet to reach peak pre-Recession values by December 2011, and this end-date gave us plenty of data, as our data was collected on a monthly basis.

3. Theory - Modern Portfolio Theory:

Harry Markowitz introduced Modern Portfolio Theory in 1952, creating the Markowitz, or Mean-Variance, Model. This paper focused on what Markowitz called the second stage of portfolio selection, with the first stage consisting of observation and past analysis on securities, and the second consisting of using these observations to create a portfolio. Since Markowitz posited that investors are generally risk-averse, they were missing out on portfolios with larger returns but larger risks. To find the "best" portfolio, he argued that portfolios should be optimized on expected return relative to volatility. Markowitz understood this tradeoff of high-risk, high-reward investments, noting that "there is a rate at which the investor can gain expected return by taking on variance, or reduce variance by giving up expected returns." Overall, these kind of optimized portfolios were shown to be less volatile than the total sum of their parts, and therefore their risk was less than what may be thought of by the risk-adverse investor.

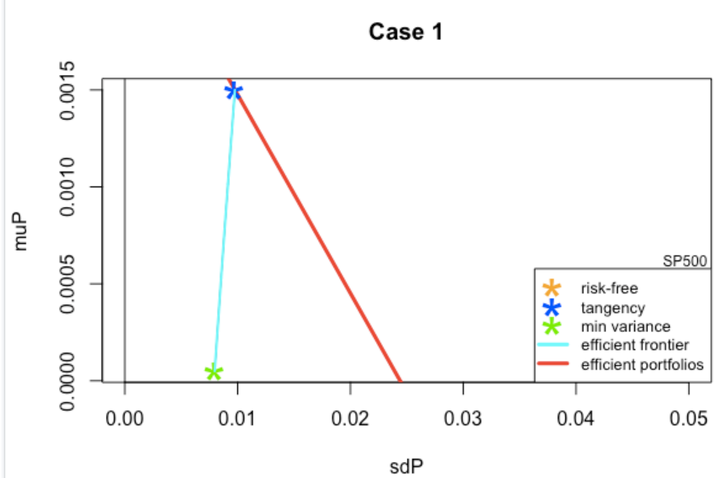
In order to create efficient portfolios, Markowitz advocated for "the 'right kind' of diversification for the 'right reason'." This "right kind" of diversification meant diversifying securities not just among companies, but among companies in different industries. This diversification across industries would reduce covariances between the individual securities, as hypothetically different industry economic characteristics would make them less susceptible to all do poorly at the same time.

As with many theories, Markowitz's Modern Portfolio Theory, and the Markowitz model, have their critics. MPT suffers from using historical data, and while this may be useful, it cannot predict previously unknown variables that can affect returns and volatility. These unknown risk factors have to be constantly identified and factored into a modeling in order to understand where and why losses may occur. Additionally, while diversification across many industries may reduce covariance between securities, these industries are linked together both intra- and internationally in the present day than when Markowitz proposed his theory in 1952. Therefore, large scale market risk, as described in below in Theory: Risk Management Perspective, would be more disruptive in a modern economy.

Despite the criticisms of MPT it remains a powerful tool in creating optimized portfolios that may be more resilient against the ebbing and flowing of economic forces. To understand the limitation of MPT is to understand the quote by statistician George Box: “All models are wrong, but some are useful”. MPT can certainly be useful, but understanding the simplifying assumptions of the model, such as a perfectly liquid market or no herd mentality, is critical to understanding when and where it may go wrong.

4. Optimized Portfolio:

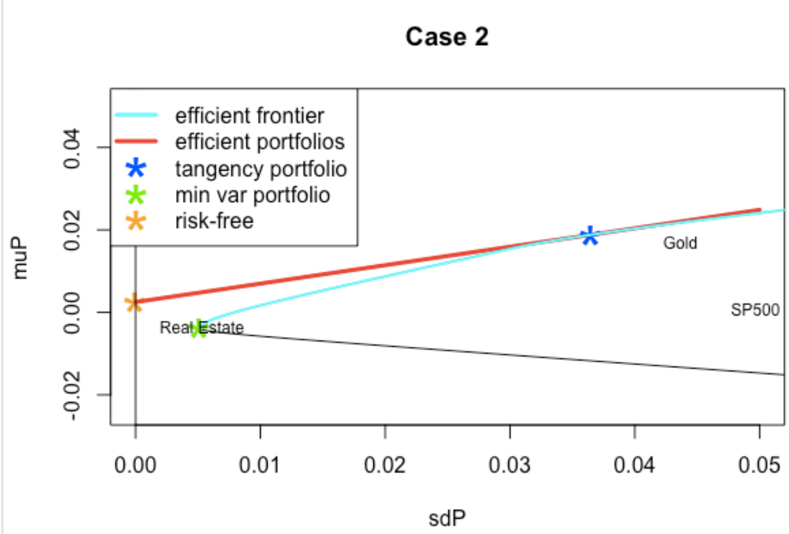
For the optimized portfolio, we used a variety of case to find an optimal portfolio consisting of S&P500 returns, gold futures, silver futures, oil futures, and real estate. Our first case prohibited short selling, while also not setting a cap on either the stock or the asset as a percentage of your portfolio. This case did not work. We are forced to not short-sell real estate, which has a negative mean return throughout 2007 – 2011. Therefore, this case somewhat breaks the model, as seen on the graph below.



Moving on to case 2, which allowed short selling, and set caps on each stock at -100% and 100% of the portfolio. This one was chosen to see how much the model would maximize short-selling real estate, and what the model would choose to maximize. Interestingly, despite silver having a higher mean return throughout this time period, the model choose to maximize gold, holding 100% of the portfolio in gold at the tangency portfolio. Even more interesting is that the model chose to short silver, which does not quite make sense.

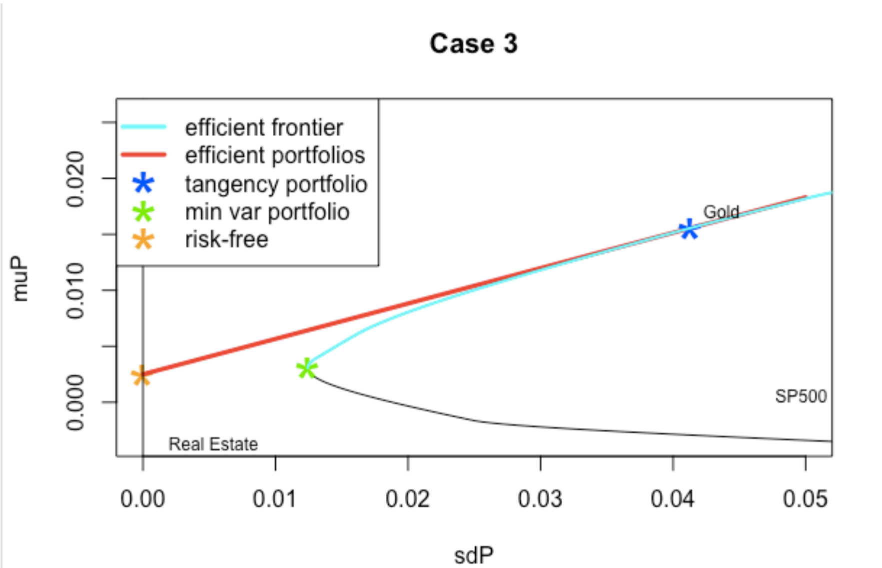
```
> final_results2
     S.P.500.Returns Gold.Returns Silver.Returns Real.Estate.House.Price.Index.Returns
[1,]            0.06         1.70           1.77                                 -0.36
[2,]            0.47         1.00          -0.16                                 -0.50
[3,]            0.02        -0.01           0.00                                  0.99
     Crude.Oil.Returns
[1,]              1.35
[2,]              0.18
[3,]              0.00
```

The model graph looked somewhat more normal than that for case 1, especially when remembering it allows for short selling. However, perhaps adding stricter constraints to the model would lead to a more sensical answer.



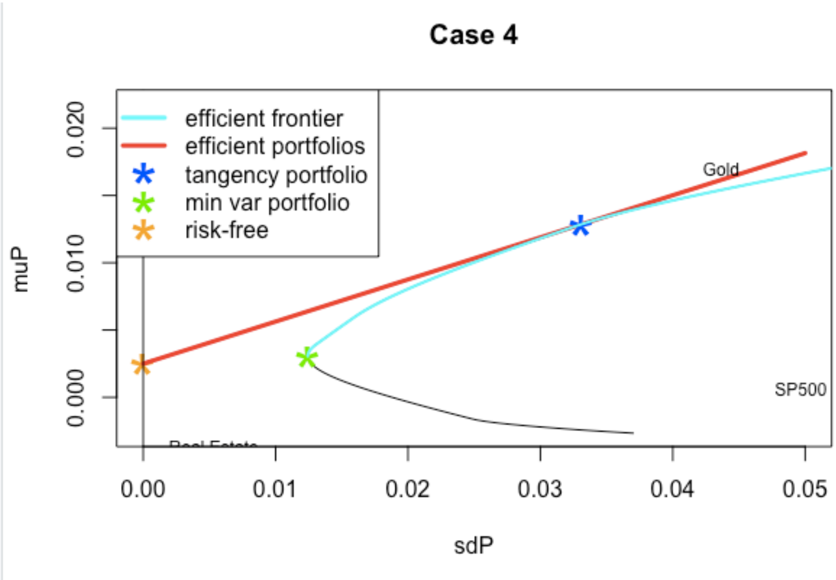
For Case 3, we capped an asset weight at .5, and set a minimum weight of -.5. Following these constraints, the model decided to maximize both gold and S&P500, while curiously not minimizing real estate. Still, the resulting graph was trending towards looking like what the graphs we found in class, so we decided to try one more test case.

```
> final_results3
     S.P.500.Returns Gold.Returns Silver.Returns Real.Estate.House.Price.Index.Returns
[1,]            0.06         1.70           1.77                                 -0.36
[2,]            0.50         0.50           0.16                                 -0.35
[3,]            0.21         0.33          -0.09                                  0.50
     Crude.Oil.Returns
[1,]              1.35
[2,]              0.19
[3,]              0.05
```



Case 4 further constrained to optimal portfolio to have the same weight constraints ($-.15<w_j<.5$) as we used in class. The resulting model tried to short real estate up to the allowed maximum constraint, while reducing the percentage of the portfolio in S&P500, Silver, and Oil. This can therefore not be the optimal portfolio, as Case 3 did not have real estate at the minimum constraint. This therefore suggests that -.35 is the optimal portfolio weight for real estate, and that Case 3 is a better model to use for later applications (i.e. VaR) than the other models.

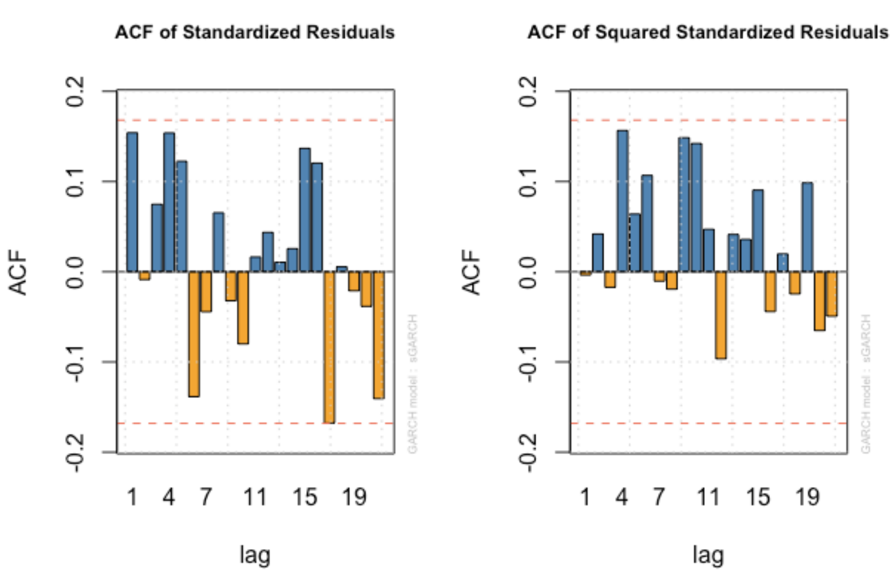
```
> final_results4
     S.P.500.Returns Gold.Returns Silver.Returns Real.Estate.House.Price.Index.Returns
[1,]            0.06         1.70           1.77                                 -0.36
[2,]            0.41         0.50           0.08                                 -0.15
[3,]            0.21         0.33          -0.09                                  0.50
     Crude.Oil.Returns
[1,]              1.35
[2,]              0.16
[3,]              0.05
```



5. GARCH Models:

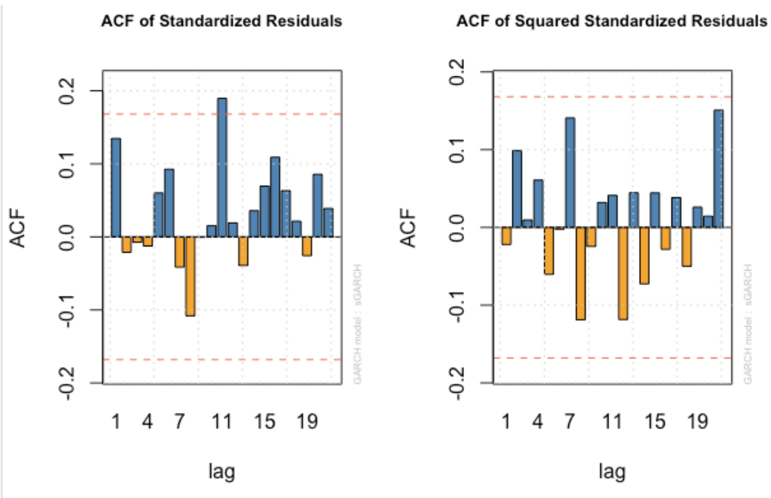
We decided to use GARCH modeling to see how volatile each of our assets in our portfolio was. The time period we used for this analysis, 2007 – 2017 (an extension of our general dataset), saw many changes to these assets. SP500 returns should have high volatility throughout 2007 – 2009. Oil should also see high volatility throughout the mid-2010s, as fracking for natural gas became a popular energy extraction method.

Going through the models for each asset one by one, we start with SP500 returns. Here, we saw that the initial test of an ARCH(1) model fit the data, with the ACF close to zero on both graphs, and both graphs showing squared residuals that are within the test bounds (the red dotted lines on each graph). Therefore, we have a model that accurately captures the change in variance over time for the S&P 500 returns.

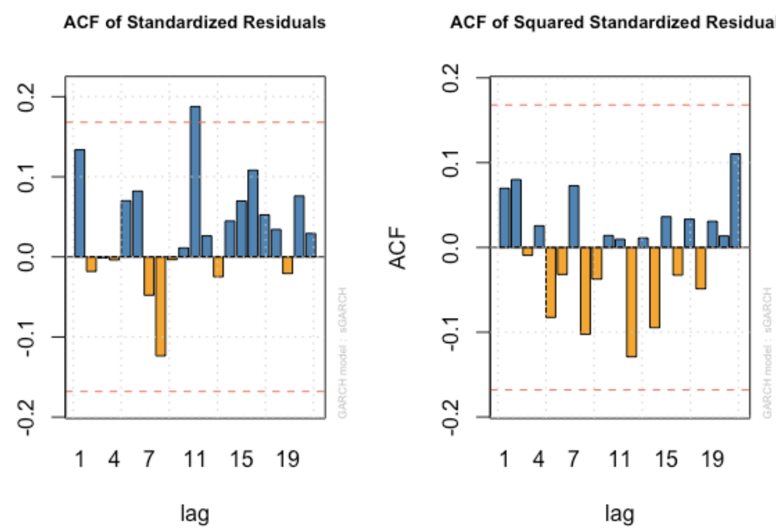


However, when fitting an ARCH(1) model to our gold data, we did not get lucky and get an initial fit, due to the ACF of the Standardized Residuals violating the test bounds. We therefore tried a GARCH(1,1) model which did improve the ACF of the Squared Standardized Residuals.

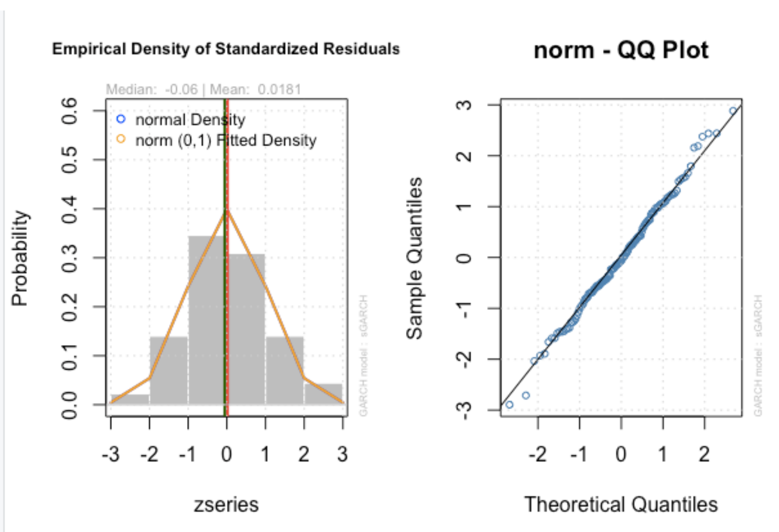
ARCH(1) Model



GARCH(1,1) Model



When further examining gold, we found that the right tail was slightly heavier than the left tail, which could either indicate slight right skewness or just be a product of random variation.



Examining final model fit, we find the following results:

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic p-value
Lag[1]                      2.485  0.1150
Lag[2*(p+q)+(p+q)-1][2]     2.508  0.1909
Lag[4*(p+q)+(p+q)-1][5]     2.663  0.4720
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                     0.6763  0.4109
Lag[2*(p+q)+(p+q)-1][5]    1.6346  0.7068
Lag[4*(p+q)+(p+q)-1][9]    2.7750  0.7955
d.o.f=2
```

The relatively high p-values on the standardized squared residuals shows that the choice of a GARCH(1,1) model was appropriate for this asset. For the next few assets, I will summarize the findings, showing a particularly interesting graph or result.

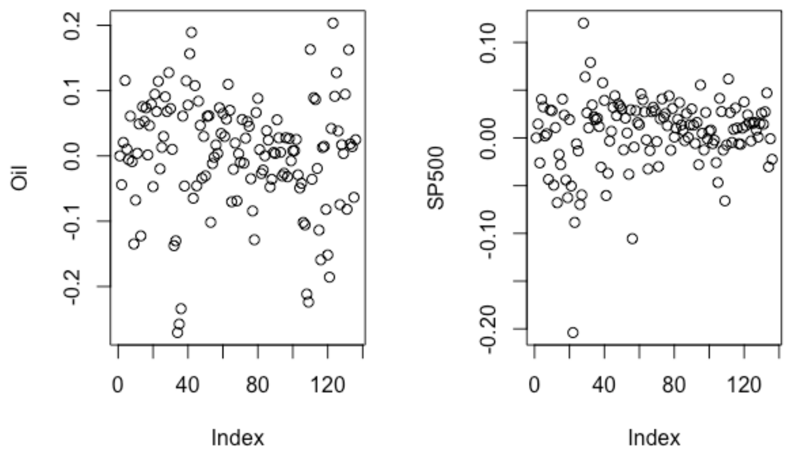
For silver, we found that ARCH(1) model, while containing 1 residual outside the bound, still fit better than the GARCH(1,1) model. However, while the ARCH(1) model was better, it was by no means particularly good, with lower p-values than we had seen for the models for gold and for the S&P500.

```
Weighted Ljung-Box Test on Standardized Residuals
------------------------------------
                        statistic  p-value
Lag[1]                      7.001 0.008144
Lag[2*(p+q)+(p+q)-1][2]     7.554 0.008492
Lag[4*(p+q)+(p+q)-1][5]     8.216 0.026042
d.o.f=0
H0 : No serial correlation

Weighted Ljung-Box Test on Standardized Squared Residuals
------------------------------------
                        statistic p-value
Lag[1]                      0.336  0.5621
Lag[2*(p+q)+(p+q)-1][2]     1.009  0.4954
Lag[4*(p+q)+(p+q)-1][5]     1.957  0.6286
d.o.f=1
```

For real estate, we found that real estate volatility was very similar to gold, with both having a slightly better GARCH(1,1) model than ARCH(1) model, and both having a slightly heavier right tail than left.

Finally, for oil, we found that oil was remarkably volatile in our time period, with the following being a graph of monthly volatility clustering (with SP500 returns next to it to show the high level of volatility):



Given this high level of volatility, it should not be surprising that the ACF of the standard and squared residuals were the highest of any potential asset, and that oil did not fit well in either the ARCH or GARCH models.

6. Theory - Risk Management Perspective:

In finance, there are many risks that can occur that could jeopardize future expected returns on your portfolio, such as credit risk, liquidity risk, and operational risk. However, for this part of the project, we will focus on a well-documented and researched risk: Market risk, also known as systematic risk. Market risk is an umbrella term to describe the type of risk that occurs due to the movement of market prices, and has subcategories such as equity risk, currency risk, and commodity risk. This movement of market prices occurs when the entire market or industry is affected by a certain major factor, and this factor can create a situation that can be difficult to overcome. An example of market risk is the Great Recession, where the collapse of the subprime loan industry led to a massive liquidity and credit crunch that then spread to the entire US economy, with major victims including Bear Stearns and Lehman Brothers. This kind of risk shows how difficult it is to protect your portfolio, no matter what method you may use, as the economy is so interconnected it can be hard to fully weather the storm of a recession.

Market risk is a major factor in the uncertainty of the performance of a portfolio. A major risk metric used across the industry is variance. In general, variance is useful for portfolios where the returns of the portfolio have a multivariate normal distribution. The mean of this type of distribution in a portfolio is $w\boldsymbol{\mu}$, where the variance of the portfolio is $w^T\boldsymbol{\Omega}w$, where w is the weights of the n assets of a portfolio, $\boldsymbol{\mu}$ is the expected returns for each asset, and $\boldsymbol{\Omega}$ is the covariance matrix of the portfolio.

While, theoretically, variance is a great risk metric for portfolios, not all portfolios can be described using a multivariate normal distribution. Therefore, to create a more "catch-all" risk metric, VaR, or Value at Risk, was developed in the mid 1990s by many financial portfolio managers. VaR focuses on the amount a portfolio could lose, based on the alpha probability chosen by the portfolio manager, given normal market circumstances in a set time period. It is a useful metric due to its' ability to be used in various different markets with inherently different risks. For our purposes, VaR is extremely useful, as it allows us to compare alternative investment opportunities, such as commodities, to the more traditional securities in our portfolio.

VaR became prevalent during the 1990s due to the rise of portfolio managers attempting to outperform a benchmark like the S&P 500. With mutual funds and hedge funds becoming more popular, VaR, and VaR-adjacent methods, became popular in Wall Street. These methods included Expected Shortfall (ES), which is the expected loss given that the loss exceeds the VaR at alpha, and Entropic Value at Risk (EVaR), which is the upper bound of VaR and cVar obtained via the Chernoff Inequality. Going back to ES, reducing expected loss is key to minimizing the losses a portfolio may suffer during an economic downturn.

While VaR is a popular tool in finance, it does have its' critics. VaR was blamed for leading many companies into over-leveraging themselves heading into the Great Recession, with many leveraging themselves up to and over 30-1. In an article for the Global Association of Risk Professionals, David Einhorn (Cornell '91) argued that companies using VaR, through focusing on 95 or 99% risk thresholds, ignored the catastrophic 1% occurrences that would fell many old and revered Wall Street Institutions. As an example, he critiqued Bear Sterns' usage of VaR that would lead to them leveraging $395 billion worth of assets with $10.5 billion in assets. While this is certainly a valid complaint, VaR seems to suffer from a common problem across this country: An inability by the average American to accurately interpret probabilities and their outcomes. Therefore, while VaR might provide sound models, those without sufficient training or understanding of VaR may not understand exactly what risk they take by ambitiously following their VaR based models.

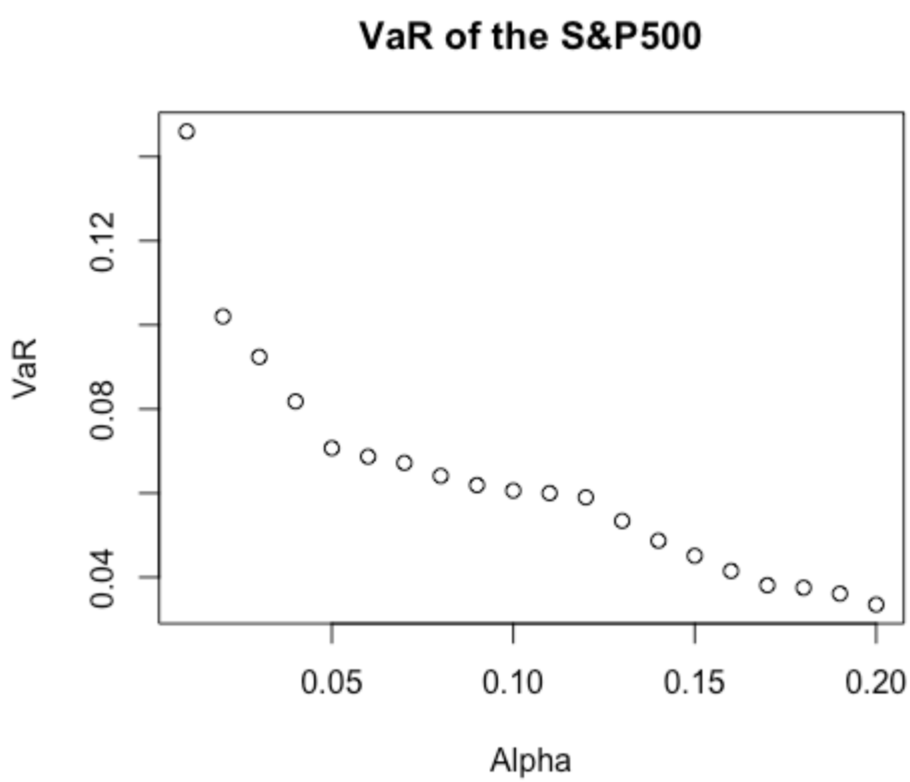
It is important to evaluate the returns of the portfolio we are researching at a proper risk horizon. When dealing with portfolios, the risk horizon, or the time period in which the risk is being forecasted, tends to be longer since models such as the multivariate model tend to model data across a month or further out. A lot of this is based on your asset liquidity, and the external constraints that are involved in the position you're in for your portfolio.

7. Risk Management – Analysis:

Since VaR is an important risk metric, we are going to apply them to the different portfolios that we developed for the Great Recession period. To begin, we used nonparametric estimation for our return distribution because we did not want to have bias when it comes to wrongfully selecting a type of distribution. We applied nonparametric estimation for the SP500 returns to assess the type of risk performance the index went under in a bear market.

When doing this initial calculation, we started out with an alpha of 0.02 and we were able to receive a VaR and ES of 10.2% and 15.5%, respectively. This represents a risk horizon of one month since our data deals with monthly returns. These are pretty large risks for the S&P 500 portfolio, as the scale of these kind of risks would almost certainly cause investors to lose a lot of confidence in the performance of the stock market. If we zoom in, and focus more specifically on the official endpoints of the Great Recession, December 2007 to June 2009, we calculated a VaR and ES of 6.62% and 6.97%, which is smaller than the previous values due to the smaller length of monthly returns.

Going back to the returns of the S&P 500 from 2007 to 2011, we plotted the VaR of the S&P 500 based on alpha. As expected, we saw that the VaR of the S&P 500 decreases at a decreasing rate when alpha increases. The average VaR for alphas between 0.01 and 0.20 is 6.34%.



Next, we examined a sample portfolio that would include the same elements as before, namely S&P 500, gold, silver, real estate, and oil. We started out by putting 50% of our portfolio into the S&P 500, and 12.5% into each of the other 4 alternative investments. However, with the portfolio constructed in this fashion, we have shifted away from a nonparametric estimation to a model-based estimation. This occurs since we have shifted from using a single benchmark, S&P 500 returns, to a portfolio with multiple asset that have nonparametric estimates which are not well-suited for this many asset returns.

With this settled, we started with a multivariate normal distribution for returns for each of the assets in our portfolio. When calculating the VaR and the ES for our new distribution over these new assets, we again used an alpha of 0.02. This ended up finding a VaR and an ES of 5.22% and 6.26% respectively. These VaR and ES numbers are much smaller than the VaR and ES that we received when only assessing the S&P 500. We also adjusted the risk horizon of our risk model, and looked at a 5-year forecast to see how much our results differed. Our results for this scenario are 10.06% and 12.38% for the VaR and the ES respectively. The VaR and ES approximately doubles when the risk horizon is extended to 5 years with our normal distribution.

Now we looked at the optimal weight portfolio that we analyzed earlier in the paper and try to apply it to our risk models. The weight tangency portfolio that we calculated had weights of 0.50, 0.50, 0.16, -0.35, 0.19 for the S&P 500, gold, silver, real estate, and oil, respectively. Looking at the VaR and the ES, we got 6.88% and 8.39% with an alpha of 0.02. This is a little surprising because these results were higher than the portfolio weights of 0.50, 0.125, 0.125, 0.125, and 0.125 for the assets. This is most likely due to the shorting of real estate, which we were unable to effectively model using VaR. Looking back at the optimal portfolio, real estate suffered negative mean returns throughout this time period, so the optimal portfolio would try to short it.

This risk model showed that other commodities, such as gold, silver, and oil, served as good alternative investments than real estate during the Great Recession. This probably due to the result of the housing market crash that was the precursor to the Great Recession starting. There would still need to be more information about whether this movement of real estate prices are common whenever the stock market declines dramatically. Still, the effectiveness of commodities as viable, and sometimes optimal, investments during the Great Recession shows the possibility of that they may be worthwhile adds to a portfolio.

8. Summary of Results and Conclusion:

This project found some interesting conclusions. We found that including commodities, particularly gold and oil, as potential options in the portfolio was beneficial, as gold, silver, and oil all had higher mean returns between 2007 – 2011 than the S&P 500. While some commodities are useful, the particular optimal portfolios we calculated would not be useful today. The two extremes here would be oil and real estate. Oil has suffered a massive downturn in value since 2011, with its' current price roughly half of what it was back then. This is due to a combination of factors, namely a steady oil production from OPEC countries while many oil consumers have reduced their intake, and the advent of fracking that has increased available oil supplies, especially in the United States and Canada. For the optimal portfolio, real estate prices were crashing due to subprime loans, and all variants of the optimal portfolio either shunned or shorted real estate as much as they could. Now, real estate values have been increasing consistently from 2012 through 2020, and a new optimal portfolio would almost certainly not short it.

There is one asset that has shown an interesting valuation pattern: Gold. Gold's value peaked during the recession, probably because it is a hedge against inflation during a chaotic time, and had rallied tremendously during the COVID-19 downturn in the economy. Gold futures increased from $1,479/Troy ounce in December 2019 to $1,968/Troy ounce in August, which corresponds to the end of the COVID-19 stock market-related crash. During more fortunate times, gold does not seem to be a good investment. However, when suffering through a bear market, it seems like the only thing that glitters is gold.

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