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# Mission impossible: Disproving failed conjectures

**Bachelor Thesis**

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This is to certify that:

(i) the thesis comprises only my original work toward the Bachelor Degree

(ii) due acknowlegement has been made in the text to all other material used

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# Acknowledgments

# Abstract

**Keywords:** Automated Theorem Proving, Model Construction, Effectively propositional logic, Range Restricting Transformations, Theorem Prover E, Bachmair and Ganzinger Model Construction Technique.

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# Chapter 1

# Introduction

**Constructing Models** has a huge importance in many fields specially in debugging tasks. It helps in modelling and highlighting the existence of bugs. And because of this it is used extensively in the field of Software and Hardware verification, also in analyzing and verifying Theorems.

**So a concrete example** to show its importance is the verification of Timsort algorithm, which was explained in details here [1]. Timsort is a hybrid sorting algorithm that was developed in 2002. It combines merge sort and insertion sort. And it was developed in the beginning to be used in python, but later on it was added to java. And today in Open JDK, Sun's JDK, and Android SDK it is the default sorting algorithm since it showed a great performance over real data. That resulted in using it in billions of devices because of the popularity of those platforms. In 2015, a formal verification for Timsort was tried to be done by a team using KeY, a verification tool for java programs that could be found here http://www.key-project.org/. And their analysis showed that the TimSort algorithm was broken and they found a bug and they corrected it, then after that they were able to formally verify the correction of the algorithm. And all the happened by the help of KeY. So it really beneficial to have models.

**Having that importance in mind,** we needed in this project to have an explicit model given to user when running problems on the theorem prover E, specifically for a sub-class of FOL named EPR. And in order to achieve that goal, Transformations have been applied to the problem specification to transform it to a certain form, namely range restricted form. Afterwards, Bachmair and Ganzinger Model Construction Technique is applied to extract the explicit model from the saturated set of the problem specification.

**So a discussion of the work done** will be given in the following order. We will have a background on the topic in chapter 2. Then in chapter 3 we will discuss the methodology followed and the implementation. Then an overview of related work will be

in chapter 4. Afterwards in chapter 5 we will explain the procedures followed to test the accuracy and efficiency of the implemented techniques. Moreover, A discussion of the results will be found in chapter 6. Then a conclusion will be given in chapter 7. And last but not least a discussion for related future work will be in chapter 8.

# Chapter 2

# Background

This chapter is devoted for introducing and familiarizing the reader to the theoretical concepts behind this project, and give the reader a background on the theorem prover E as well. So it will include the following sections:

- A background on FOL in section

- A background on EPR in section

- A background on E in section

- A background on Problem set in section

## 2.1 First order logic (FOL)

FOL, which is also known as first order predicate logic, is an expressive logic that allows us to formulate and encode most of our spoken language sentences in a defined way such that it could be further handled with rules such as simplification, inference rules. That allows automating the reasoning for FOL problems.

In order to have a general background on the topics of FOL, we will devote this section to that mission. So we are going to discuss the following points:

- Summary over FOL and Example on using it

- Decidability of FOL

- Herbrand Universe

- Equality

### 2.1.1 FOL overview and Example on it

**Overview on FOL**

Quantifiers, variables, and functions are what signifies FOL over propositional logic. Being able to formulate "some", "all" is what added a lot to the expressiveness of FOL. So a review on the syntax and the structure of FOL if needed will be found in the appendix here B.

**Example on FOL**

A very famous example on FOL is the following:

Listing 2.1: Example on FOL

```
We want to formulate the following three sentences in FOL:

  (1)  All Humans are mortals.
  (2)  Socrates is a human.
  (3)  Therefore, Socrates is mortal.

So in FOL syntax they are:

  (1)  ∀X(Human(X) → Mortal(X))
  (2)  Human(socrates)
  (3)  Mortal(socrates)

Where Human and Mortal are predicates here since they represent
    property over the elements of the domain. While socrates is a
    constant or in other words a function symbol with arity 0.
```

### 2.1.2 Decidability of FOL

The Problem of proving validity, or in other words unsatisfiability, of a formula in First order logic (FOL) had a lot of attention and experiments in the beginning. A lot of trials were made to prove that it is decidable as mentioned in [6]. But those trials were not successful. Later on, in the same year both [34, 7] proved that this problem in general is un-decidable.

Some algorithms were developed such that if the formula is unsatisfiable it will give a refutation, or proof in simpler words, however if it was not unsatisfiable then the algorithm may halt/terminate and give the correct result and may not halt/terminate at all, so this problem is semi-decidable. An example for such procedure is resolution, with some refinements, in which its basic idea was first developed in [25]. So First order logic (FOL) is a semi-decidable logic.

### 2.1.3 Universe of FOL

Herbrand devised a procedure for generating the Universe of set of first order formulas; where the universe is the set of constants and grounded function symbols that represent terms in predicates. The procedure is defined recursively. So in most of the cases the Herbrand Universe of any set of first order formulas is infinite.

### 2.1.4 Equality

The Equality relation adds a lot of expressiveness and it is so intuitive to have it in logic. Adding equality to FOL is very beneficial such that it eases encoding problems to FOL, however it adds a burden for how it should be handled. So FOL with equality adds a distinguished predicate symbol $=$ or $\approx$ that represents equality to the set of predicate symbols. Moreover, it adds to the problem specification some axioms that helps in dealing with such as: reflexivity and symmetry.

In general, Equations are of the form: $t1 \approx t2$, Where each of the t1 and t2 is a term.

## 2.2 Effectively propositional calculus (EPR)

This section is discussing important points related to EPR, so the following points will be covered:

- subsection – will explain what EPR is.

- Example on EPR will be given in subsection –.

- Universe and Decidability of EPR will discussed in subsection –.

- And last but not least in subsection – we will talk about Equality in EPR.

### 2.2.1 What is EPR

Effectively propositional calculus (EPR) or sometimes known as Bernays-Schönfinkel class is a fragment of First order logic (FOL) in which all of its formulas follow the following format:

Listing 2.2: Format of EPR formula

```
                    ∃∗  ∀∗  F
  where  F  is  the  formula.
  Moreover,  F  has  no  proper  functions  symbols  (all  functions
      present  are  nullary  ones  "constants").
```

EPR fragment has a huge number of applications, e.g. Planning and Verifcation.

## 2.2.2   Example on EPR

Example for a formula in EPR:

Listing 2.3: Example for an EPR formula

$$\exists X \exists Y \forall Z \left( P(a, X, Y, Z) \right).$$

Keeping **??** in mind, this will make every skolemized formula that originally was in EPR format have no proper function symbols. After transforming the above equation, it will be:

Listing 2.4: Example for a skolemized EPR formula

$$\forall Z \left( P(a, b, c, Z) \right).$$

## 2.2.3   Decidability and Universe of EPR

Since the domain of EPR formulas contains only constants, and the number of constants in a formula is finite. So Herbrand universe of EPR formulas is finite. From that we could prove that EPR fragment is a decidable fragment in FOL, because of the finite number of ground interpretations it can have.

## 2.2.4   Equality in EPR

As we have mentioned before that EPR formulas will never contain proper function symbols. So terms will be one of two options:

- constant

- variable

And since Equality may be added to FOL as discussed in subsection –, then we should consider the case if the formulas are from the EPR fragment. Equations are in the following form $t1 \approx t2$, where t1 and t2 are terms. And since terms in EPR are only variables and constants. So the ground case of the Equation will contain only of constants. This result is important and we will refer to it later in chapter 3.

## 2.3 Problem Set

TPTP problems are first order problems, that are considered benchmarks to measure the performance of the different automated theorem provers. A description for TPTP could be found in [31], and the problem set could be downloaded from http://www.cs.miami.edu/~tptp/. TPTP problem set consists of different categories of problems, or divisions in other words, such as : PUZ, NLP, GRP. Those categories depends on the problems scientific meaning, e.g., NLP represents Natural language processing problems.

TPTP has its own language; TPTP for problems and TSTP for solutions. TPTP language could be written in two formats. The first of them is FOF, which is first order form. While the second form is CNF, which is clause normal form. Having a unified language for problems and solutions had a great impact on the field of Automated theorem proving (ATP). And this allows the integration of different ATP systems ,i.e., Automated theorem provers, since they have a specific language that could communicate in.

An example for a problem, i.e., GRP004-1 problem, from the TPTP problem set is given below in CNF format:

Listing 2.5: GRP004-1.p problem

```
%----------------------------------------------------------------------------
% File      : GRP004-1 : TPTP v6.1.0. Released v1.0.0.
% Domain    : Group Theory
% Problem   : Left inverse and identity => Right inverse exists
% Version   : [Cha70] axioms : Incomplete.
% English   : In a group with left inverses and left identity every element
%             has a right inverse.

% Refs      : [Luc68] Luckham (1968), Some Tree-paring Strategies for Theore
%           : [Cha70] Chang (1970), The Unit Proof and the Input Proof in Th
%           : [CL73]  Chang & Lee (1973), Symbolic Logic and Mechanical Theo
% Source    : [Cha70]
% Names     : Example 3 [Luc68]
%           : Example 4 [Cha70]
%           : Example 4 [CL73]
%           : EX4 [SPRFN]

% Status    : Unsatisfiable
% Rating    : 0.00 v5.4.0, 0.11 v5.3.0, 0.10 v5.2.0, 0.00 v2.1.0, 0.00 v2
%     .0.0
% Syntax    : Number of clauses     :    5 (   0 non-Horn;   3 unit;   3 RR)
%             Number of atoms       :   11 (   0 equality)
%             Maximal clause size   :    4 (   2 average)
%             Number of predicates  :    1 (   0 propositional; 3-3 arity)
%             Number of functors    :    3 (   2 constant; 0-1 arity)
%             Number of variables   :   15 (   1 singleton)
%             Maximal term depth    :    2 (   1 average)
% SPC       : CNF_UNS_RFO_NEQ_HRN

% Comments  : [Luc68] is actually the right to left version.
%----------------------------------------------------------------------------
cnf(left_inverse,axiom,
    ( product(inverse(X),X,identity) )).
cnf(left_identity,axiom,
    ( product(identity,X,X) )).
cnf(associativity1,axiom,
    ( ~ product(X,Y,U)
    | ~ product(Y,Z,V)
    | ~ product(U,Z,W)
    | product(X,V,W) )).
cnf(associativity2,axiom,
    ( ~ product(X,Y,U)
    | ~ product(Y,Z,V)
    | ~ product(X,V,W)
    | product(U,Z,W) )).
cnf(prove_there_is_a_right_inverse,negated_conjecture,
    ( ~ product(a,X,identity) )).
%----------------------------------------------------------------------------
```

## 2.4 The theorem prover E

### 2.4.1 What is E

E is a fully automatic theorem prover for FOL with equality implemented in ANSI-C that was released in 1998. Moreover, It is a saturation-based theorem prover. The Calculus implemented in E is variant of the superposition calculus proposed in [2].

### 2.4.2 What can E do

Since superposition calculus is a refutation one. So the current state of E, that it could prove the un-satisfiability of set of axioms with the negation of the conjecture(s) by finding the empty clause, or returning the saturated set if the empty clause was not found and no more new clauses can be inferenced/simplified.

The returned saturated set is considered a model in the sense that it satisfies all the formulae in the give specification. However, an explicit model for the specification is not provided. Worth mentioning that it is not guaranteed in E to terminate/halt for EPR problems. This could be seen from the performance of E in EPR problems it does not terminate in more than 50% of them.

### 2.4.3 The Code flow of E

This part is devoted from giving a brief on the main procedure in E. Figure 2.1 is a simplified version of the code flow in E.

**Code Flow of E**

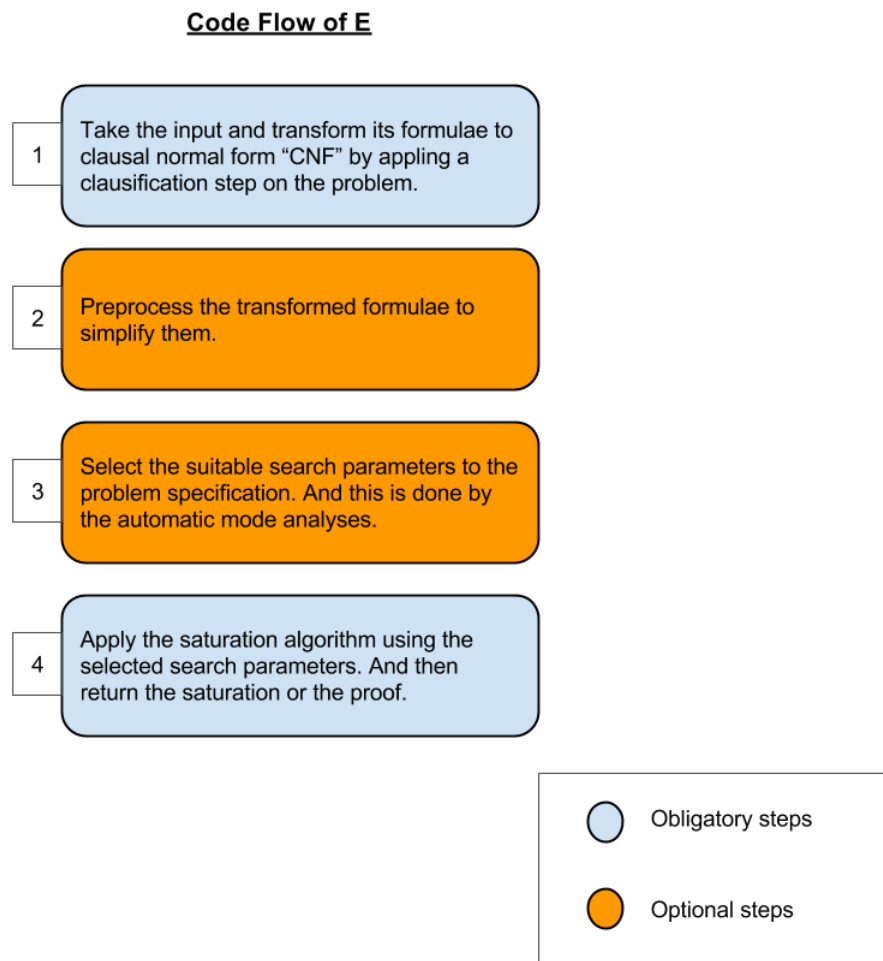| 1 | Take the input and transform its formulae to clausal normal form "CNF" by appling a clausification step on the problem. |
| 2 | Preprocess the transformed formulae to simplify them. |
| 3 | Select the suitable search parameters to the problem specification. And this is done by the automatic mode analyses. |
| 4 | Apply the saturation algorithm using the selected search parameters. And then return the saturation or the proof. |

○ Obligatory steps

● Optional steps

Figure 2.1: E Simplified code flow

### 2.4.4 Latest results

The latest results showed performance approaching 70% over all the CNF and FOF problems of the TPTP problem set and this is according to [27].

# Chapter 3

# Methodology and Implementation

Here in this chapter we will discuss the methods applied to extract an explicit model from an EPR problem specification. For how they are implemented using E's data structures and configurations, and how to run the program, please refer to chapter C in the appendix.

**In order to achieve our goal,** we had to transform the problems' clauses, which are the axioms of the specification and the negated conjectures (if found), to clauses in range restricted form through a simplified form of range restricted transformations. The original range restricted transformations and there simplified version will be discussed in section 3.1.

## 3.1 Transformations

The methods applied in the project to extract the models follows the transformations discussed in [4]. A brief on the original transformations will be given in 3.1.1.

Moreover, as mentioned before that this project is concerned with a sub-class of FOL which is EPR some simplifications to the transformations were made as the removed steps will have no meaning in the context of EPR problems. Those simplifications will be discussed in 3.1.2.

### 3.1.1 Original transformations

**The original procedures** discussed in [4] generally work for all sub-classes of FOL. Those procedures should be applied to a given set of axioms in a specific form called implication form, sometimes it is called a sequent as in here —ref—, which is explained here – , and here – to know how to transform to that form. Moreover, those transformations are guranteed to terminate for any given problem set, which is a gain for us since some of the EPR problems were not terminating in the original configurations of E.

**What are the Transformations**

**The Transformations** are series of procedures, mainly about changing the clauses to certain form named range restricted form. Since the transformations deal with clauses in implication form, then we could define **Clauses in range restricted form** to be clauses in which all the variables that appear in the succeedent must exist in the anticedent as well.

**An example for a range restricted clause** is found below:

Listing 3.1: Range Restricted Clause Example

$$\forall X \forall Y \left( P(X) \wedge Q(Y) \longrightarrow R(X, Y) \right).$$

where the **antecedent** here is $P(X) \wedge Q(Y)$;
whereas the **succeedent** is $R(X, Y)$.

**How do the transformations work**

**Transformations** add a domain predicate to the specification that will help in finding the Model by saying what are the elements of the domain, or the elements of the universe in other words.

Moreover, There are three types of procedures in the transformations:

- **Range restricting transformations**
  are the first two transformations, and they are the most important type of them. All the rest were added to enhance and improve the range restricting ones. They are responsible for transforming the input clauses to the range restricted form and enumerating the universe/domain of the problem in a way or another. Only one of the two transformations should applied, since they perform the same functionality but in different ways. The two transformations are:

  - Classical Range Restricting Transformation (CRR): it enumerates the Herbrand Universe in a naive simple way.
  - Range Restricting Transformation (RR): it was introduced to improve the naive implementation of the CRR. So it only adds elements to the domain only when it is needed.

- **Shifting transformations**
  are the second two transformations. They are optional to be used. They complement one another not replace each other. They were introduced mainly to prevent the non-termination of the transformations and to prevent generating and redundant and unpleasant clauses from the steps in RR as well. And the two shifting transformations are:

  – Basic Shifting Transformation (BS)

  – Partial Flattening Transformation (PF)

- **Blocking Transformation (BL)**
  is the last transformation and it is optional as well. And It was introduced to detect periodicity that may occur because of function terms.

**Their order of application** is:

1. One of the two range restricting CRR or RR

2. The Partial Flattening PF

3. The Basic Shifting BS

4. The Blocking BL

Where the output of a lower number transformation is the input to the higher one. A Flow Chart that summarizes what was explained will be found in Figure 3.1.
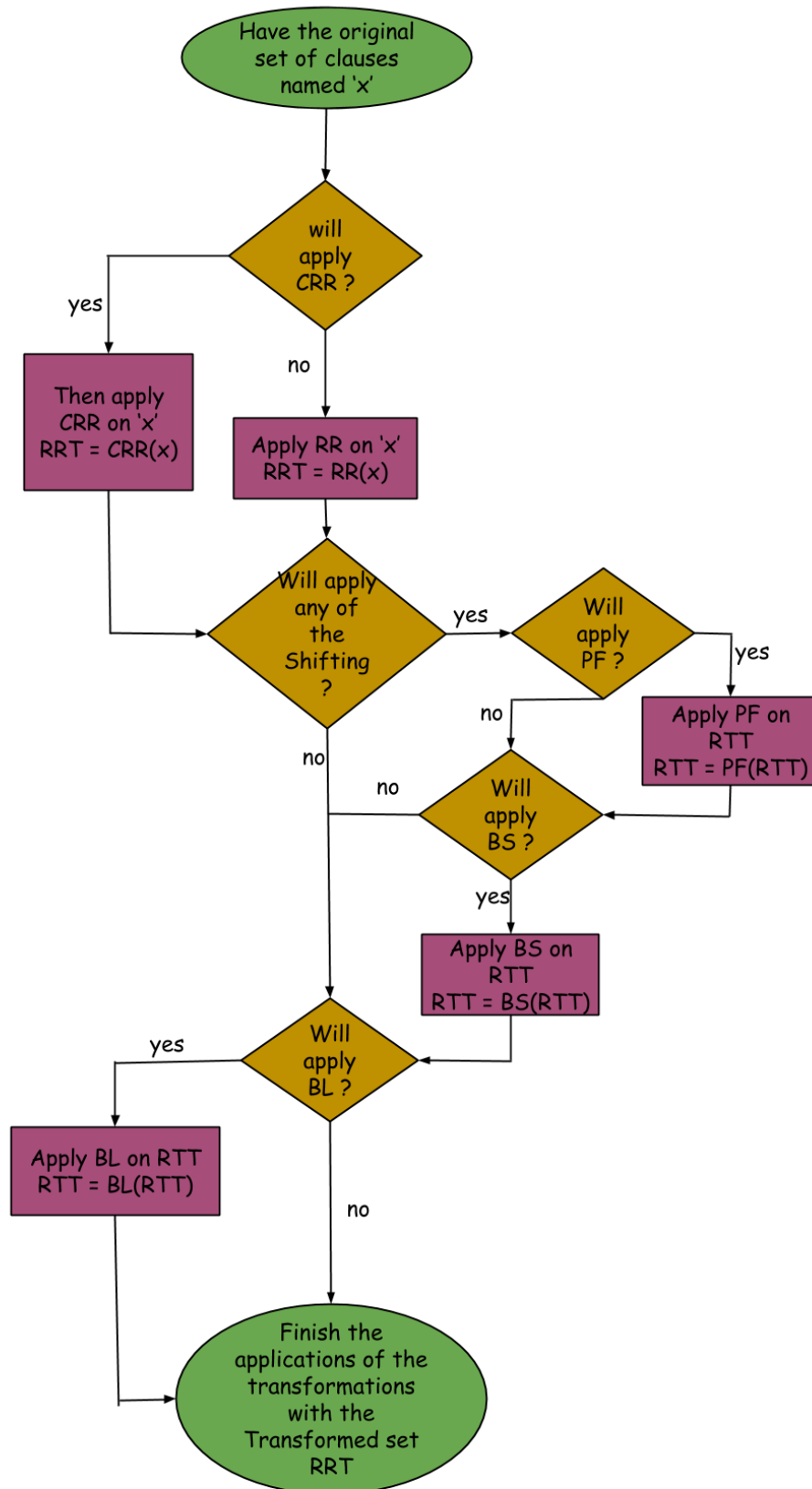
Figure 3.1: Flow of the original transformations

## 3.1.2 Simplified transformations

**This subsection** is concerned about discussing the simplifications that were added to the original transformations, that was explained in subsection 3.1.1. So this subsection will cover the following points:

- The reason for doing such simplifications.

- What are the Simplifications.

- The resulted simplified transformations.

- Examples for the Output of the transformations, or let's name it Intermediate Output.

- Discussion on the Output of the prover.

**The Reason for doing the simplifications**

**As mentioned before** in subsection 3.1.1, the original transformations is generic for all sub-classes of FOL. So naturally it contains many steps that deals with proper function symbols. However, what concerns us in this project is only the EPR sub-class of problems. And by keeping in mind the definition of EPR as mentioned here in 2.2.1, and by knowing that there is no existence of proper function symbols, and that there won't be even after the skolemization step in any EPR problem, So implementing the original transformations, as they were, didn't seem logical as it won't be used in any of the problems, and it would take much more time to implement it practically. So those were the motives for having such simplifications.

**What are the steps of the simplifications**

**The simplifications** made were very simple, and they were applied to all procedures of the original transformations. The steps of the simplifications are:

1. Every step that only deals with proper function symbols is removed since they are not existing in EPR. Ex.: some steps in CRR and RR that loops on all the proper function symbols in the given problem.

2. Any step or procedure that was introduced because of the existence of proper function symbols in problems were removed as well. Ex.: BS, PF, and BL.

**The resulted simplified transformations**

**After applying** the simplifications to the procedures of the transformations, the following steps were the only needed:

- Some steps in CRR.

- Some steps in RR.

For a chart representing the original CRR and what is removed from it, you can view Figure 3.2, while Figure 3.3 for RR. And for a detailed explanation for the steps, you can check [4].
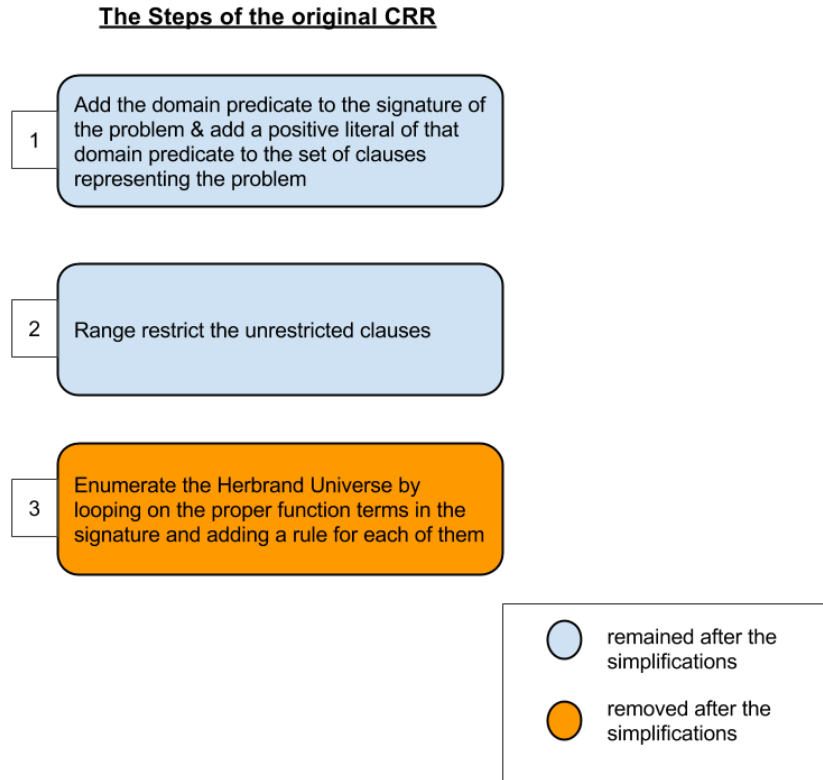
**The Steps of the original CRR**

1 | Add the domain predicate to the signature of the problem & add a positive literal of that domain predicate to the set of clauses representing the problem

2 | Range restrict the unrestricted clauses

3 | Enumerate the Herbrand Universe by looping on the proper function terms in the signature and adding a rule for each of them

○ remained after the simplifications

● removed after the simplifications

Figure 3.2: Original CRR with the kept and removed steps after the simplifications

**The Steps of the original RR**

| | |
|---|---|
| 1 | Add the domain predicate to the signature of the problem & add a positive literal of that domain predicate to the set of clauses representing the problem. |
| 2 | Add the domain elements from the Clause bodies by looping on the predicates in the antecedent of each clause and add their arguments. |
| 3 | Range restrict the unrestricted clauses. |
| 4 | Add a rule to add domain elements of the predicates found in the signature. |
| 5 | Add a rule to add domain elements of the proper function terms found in the signature. |

○ remained after the simplifications
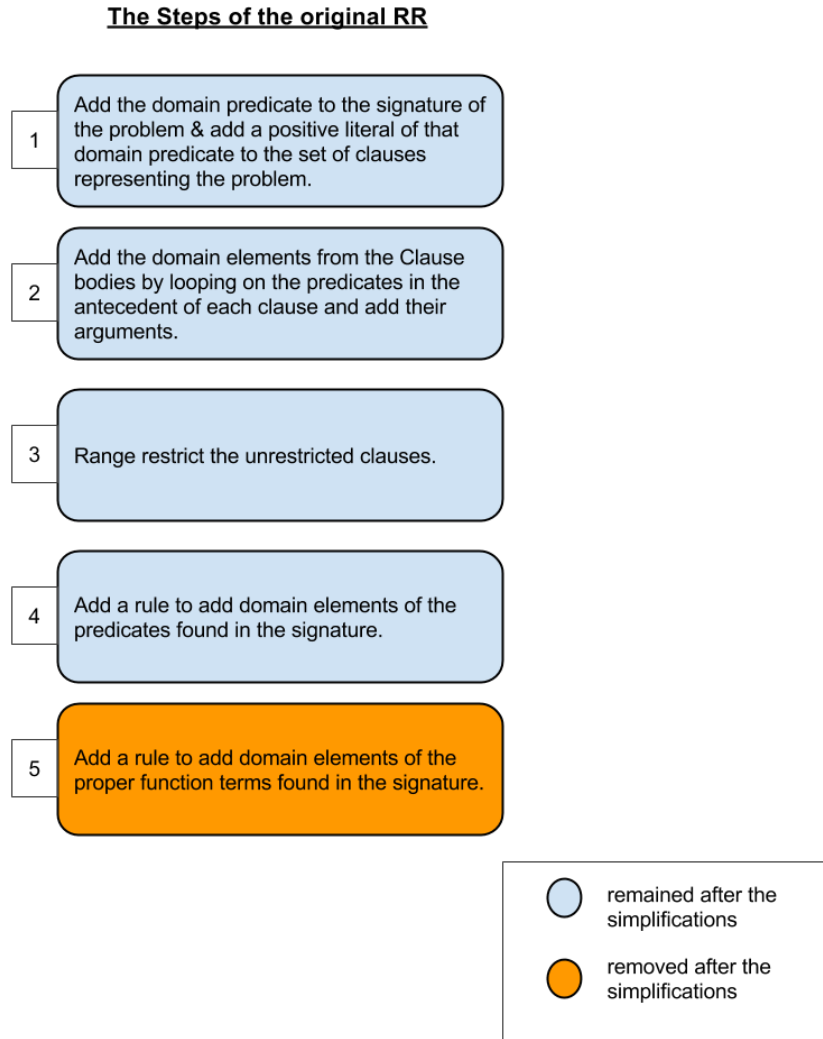
● removed after the simplifications

Figure 3.3: Original RR with the kept and removed steps after the simplifications

**The Intermediate output**

**Here we name** the output of applying the simplified transformations on a set of (counter) satisfiable set of clauses an **Intermediate output**, since the the Output should be the returned explicit (counter) model. So In this part we give an example for a problem, and then show what is the Intermediate output of CRR, and RR.

Listing 3.2: Satisfiable CNF problem Example

```
cnf(agatha,axiom,(~lives(a))).
cnf(butler,axiom,(dies(X)
                |dies(Y)
                |~lives(a)
                |~lives(Y))).
```

Listing 3.3: CRR output Example

```
        cnf(i_0_1,axiom,(~lives(a))).
        cnf(i_0_2,axiom,(dies(X1)|dies(X2)
                        |~lives(a)|~lives(X2)
                        |~dom(X1))).
        cnf(i_0_3,plain,(dom(a))).
```

Listing 3.4: RR output Example

```
        cnf(i_0_1,axiom,(~lives(a))).
        cnf(i_0_2,axiom,(dies(X1)|dies(X2)
                        |~lives(a)|~lives(X2)
                        |~dom(X1))).
        cnf(i_0_3,plain,(dom(a))).
        cnf(i_0_4,plain,(dom(a)|~lives(X1))).
        cnf(i_0_5,plain,(dom(a)|~lives(X2))).
        cnf(i_0_6,plain,(dom(X4)|~lives(X4))).
        cnf(i_0_7,plain,(dom(X5)|~dies(X5))).
```

**Discussion on the Output of the prover**

**After applying the transformations** on the clauses of a given problem, then this Intermediate output should be given as input to the normal proof procedure in E. And the final output for a (counter) satisfiable problem should be the saturated set, from which the explicit model should be extracted easily. However, this wasn't the case while running the problems, and the explicit model wasn't clear in the saturated set.

**So a Model Construction Technique** for saturation-based theorem provers was applied to the saturated set. And that Technique will be discussed in the following section 3.2

## 3.2   Model Construction

A Model Construction Technique had to be applied to the saturated set of clauses as discussed in the part related to the discussion on the output of the prover when the input was the range restricted version of the original set of clauses, and that was explained in subsection 3.1.2. Here in this section we will cover the following points:

- What is the Model Construction Technique used

- How does it work

- Discussion on the Output

**What is the used technique for Model Construction**

**The Model Construction Technique used** is specific for resolution based theorem provers, and it is the ground positive case of Bachmair and Ganzinger Model Construction Technique that was devised here in [18]. This technique originated from the proof of Bachmair and Ganzinger that their resolution based theorem proving technique is complete. That proof will be found in [3].

**How does Bachmair and Ganzinger Model Construction Technique works**

**The chosen and implemented Bachmair and Ganzinger Model Construction Technique** works for a saturated (counter) satisfiable set positive ground set of clauses. That has been generated using an ordered resolution system with simplification.

**This Technique needs a term ordering** that has been lifted to literals then lifted to clauses. That clause ordering should be total on ground clauses, and it should be the same one used in the saturation procedure.

**The algorithm works** as follows:

1. sort the positive ground clauses using the ordering in an ascending order

2. sort the literals in each clause to define the maximal literal

3. for each of the clauses starting from the smaller in terms of the ordering if not already true by the chosen true literals, then add the its maximal literal to the set of the chosen true literals

**A pseudo-code** for the implemented version of the algorithm is given below:

Listing 3.5: Ground Positive Case for Bachmair and Ganzinger Model Construction

```
Input:  clauses_set % saturated set of clauses
        , ordering % ordering used in the saturation
{
        % model is the set of positive literals in the
        % constructed model, at the beginning it is
        % an empty set of literals
        model = {}

        % this sorts the set of clauses ascendingly
        sort_clauses(clauses_set, ordering)
        % it marks the maximal literal in each clause
        mark_maximal_literals(clauses_set, ordering)

        for clause:clause_set do:
        {
          % here it checks whether the current clause
          % is true by the partial model we have or not
          if not is_clause_true_by_model(clause, model) then:
          {
                % if not true yet the it gets the marked
                % maximal literal and adds it to model
                model = model + get_maximal_literal(clause)
          }
          end_if
        }
        end_for
}
Output:  model
```

**An Example** for applying the Bachmair and Ganzinger Model Construction Technique is given below:

Listing 3.6: Example for applying Bachmair and Ganzinger Model Construction Technique

```
Let  the  unordered  saturated  set  of  clauses  be:
{
        R(a,b)|Q(b),
        P(a),
        P(a)|Q(b),
        R(b,b)
}

Let  the  order  of  the  present  clauses:
{
        P(a) < Q(b) < R(a, b) < R(b, b)
}

% the  left  most  literal  in  each  of  the
% ordered  clauses  is  the  maximal
```

| Order | Ordered Clause | Partial Model | Change in Model |
|-------|----------------|---------------|-----------------|
| (1) | P(a) | | P(a) |
| (2) | Q(b) \| P(a) | P(a) | — |
| (3) | R(a, b) \| Q(b) | P(a) | R(a, b) |
| (4) | R(b, b) | P(a), R(a, b) | R(b, b) |

Table 3.1: Table to test captions and labels

```
Then  the  explicit  Model:
{
        P(a),
        R(a,b),
        R(b,b)
}
```

**Discussion on the Output**

**The output of the Model Construction Technique** is the final one. It gives back the positive literals in the constructed explicit model. An Example for the output is given below in 3.7.

Listing 3.7: Example for the returned Model

```
dom( t ) .
bird ( t ) .
fly ( t ) .
```

**Moreover, we augmented** the output with a visual part. It prints out a dot graph representing the positive clauses linked with the positive literal, that belongs to the model, that makes them true or satisfiable in other words. Example of the returned dot graph will be listed in 3.8, and a rendered Figure for the same graph will be in 3.4.

Listing 3.8: Example of returned dot graph

```
digraph model {
 rankdir=LR;
 subgraph cluster_model {
  label="Model";
  0 [shape=ellipse , fillcolor=lightskyblue1 , style=filled , label="
     fly ( t )"]
  1 [shape=ellipse , fillcolor=lightskyblue1 , style=filled , label="
     bird ( t )"]
  2 [shape=ellipse , fillcolor=lightskyblue1 , style=filled , label="
     dom( t )"]
 }
 subgraph cluster_clauses {
  label="Positive Ground Clauses";
  3 [shape=box, fillcolor=lightpink1 , style=filled , label="cnf(
     i_0_3 , plain , ( fly ( t )))."]
  3 -> 0
  4 [shape=box, fillcolor=lightpink1 , style=filled , label="cnf(
     i_0_1 , plain , ( bird ( t )))."]
  4 -> 1
  5 [shape=box, fillcolor=lightpink1 , style=filled , label="cnf(
     i_0_2 , plain , (dom( t )))."]
  5 -> 2
 }
}
```
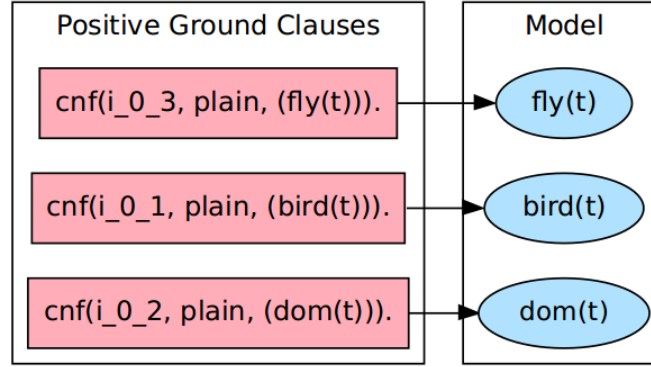
Figure 3.4: Rendered dot graph of Model

## 3.3   Code FLow

**So a small summary** for the flow of the code is given below, and simplified flow chart is in Figure  3.5:

1. The input is an EPR (counter) satisfiable problem.

2. Simplified Range Restricted Transformations got applied on the input.

3. The intermediate output from the transformations act as input for the normal proof procedure.

4. The output from the prover, which is the saturated set of clauses, is the input for the Bachmair and Ganzinger Model Construction Technique.

5. Then the final output is the positive literals of the explicit Model for the given problem.
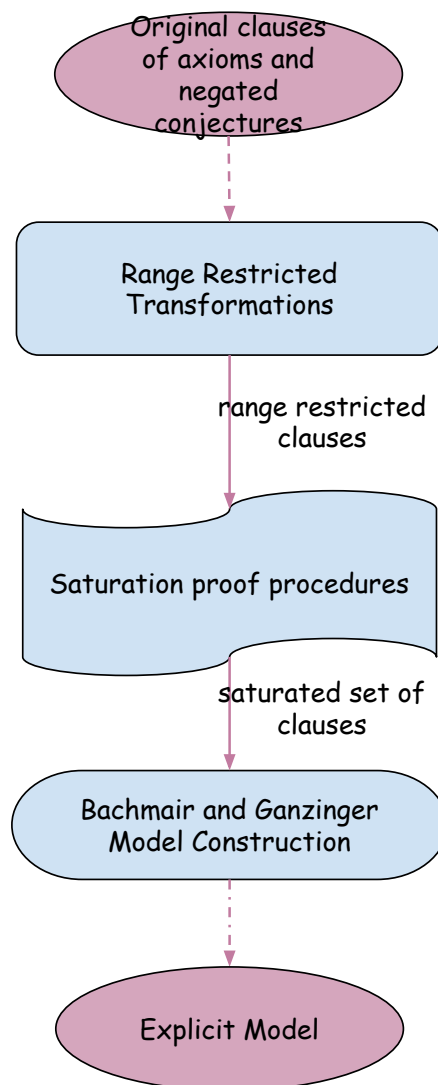
Figure 3.5: Simplified Flow chart for the code flow

# Chapter 4

# Related Work

Automated Theorem Proving has many applications in various fields, e.g., Mathematical Reasoning, Knowledge Representation, Planning and Scheduling. First we mention a defintion for ATP, according to [17], we could define ATP to be "is the use of a computer to prove to prove non-numerical results, i.e. determine their truth (validiy)".

The main process in automated theorem proving is proving the validity of a conjecture given a set of axioms, or if we are talking from the context of refutation-based theorem proving then the main process is proving the unsatisfiability of a given specification. And a lot of work and research was done to develop and enhance different calculi for that process.

The dual task for proving, or process in other words, is disproving, where a (counter) model is returned from the disproving procedure. Despite of the importance of the disproving process, less work ,compared to the proving process, were devoted for its development.

In this chapter we will discuss the related work to the topic and bit of literature review over it. So it divied into the following sections:

- Different Theorem provers

- Model Construction Techniques

- CASC competition

- Reasoning in EPR

- More on Range Restricting Transformations

## 4.1   Different theorem provers

A lot of classifications can be done to automated theorem provers. One may target the different calculi applied by each of them such as Model Evolution Calculus as applied in Darwin for example or Instantiation-based theorem provers as in iProver or Resolution based as implemented in E.

Other classification may target the output of each, where a distinction between the goal of each prover is considered. E.g., E is a refutation-based theorem prover that proves unsatisfiability, or in simpler words it proves the validity of a conjecture. However, The main goal for iProver is to give a (counter) model for unvalid conjectures. It is notable to say that some automated theorem provers give both output according to the given problem.

Another Taxonomy for automated theorem provers relies on whether they are fully automated or interactive. Interactive automated theorem provers needs hints from human beings, however in the case of fully automated theorem proving no assistance is needed from humans. An example for a fully automated one is E, Vampire and Dawrin. On the other hand, IMPS [9] is an interactive example for a proof system.

## 4.2   Model Construction Techniques

Model Construction, or building, can be done using different approaches that depends on the type of logic it relates to. E.g., A very famous procedure for propositional logic is DPLL algorithm, which stands for Davis-Putnam-Logemann-LoveLand algorithm, it is mainly used to decide the satisfiability of a logical formula. In propositional logic, this problem can be reduced to the SAT problem, boolean satisfiability problem. Model evolution calculus is another example that its idea is based on DPLL, however it lifts DPLL in order to be applied for first order logic problems.

Another technique that is applied is Instantiation-based techniques, where it applies instantiation on the active clauses and then give the grounded set to a SAT solver, if the SAT solver detected the un-satisfiability of the clause set then it terminates, however if not then it generates new clauses using inference rules then repeats the whole process until saturation is reached.

Another set of tool that helps in the process of Model Generation, are Model finders. The functionality of those type of tools is mainly giving back a model for a satisfiable set of clauses with specifying the number of domain elements that you need to find for. Those tools is used for example in some Model evolution based theorem provers such as FM-Darwin.

There are different styles of model finders such as MACE-style and SEM-style model finders. MACE-style model finders work by transforming the first order problem to an equivalent propositional one considering the size of the domain while doing the transformation. And then use a SAT-solver on the transformed set. On the other hand,

SEM-style model finders depends on searching and backtracking. An example for a MACE-style model finder is Paradox. However, Finder is an example for a SEM-style model finder. [8]

## 4.3 CASC competition

Nowadays, Automated theorem provers have a yearly competition named CASC competition [21, 29]. It is held on CADE or IJCAR conferences. In CASC different automated theorem provers compete to prove its efficiency over other theorem provers. The CASC competition's problems are chosen from the TPTP problem set. There are various division in the Competition, e.g. FOF, which is first order form non-propositional theorems, another one is SAT, which is clause normal form really non-propositional non theorems. The criteria for judgement is average time needed to solve problems, the number of solved problems, the numbers of problems solved + a solution output . Each division has its own contestants, and its own winners.

An Important division in the CASC competition, that is so related to that topic, is the EPR division, which represents effectively propositional clause normal form theorems and non-theorems. Moreover, EPR division has two sub-divisions, and they are:

- EPT: which have the theorems related problems, or in other words the unsatisfiable set of problems.

- EPS: which have the non-theorems, that is the satisfiable problems.

The latest CASC competition, that was CASC-J7, was held during the 7th International Joint Conference on Automated Reasoning (IJCAR) on 20th July 2014 http://cs.nyu.edu/ijcar2014/. The latest results for the EPR division is summarized in the following Figure 4.1:
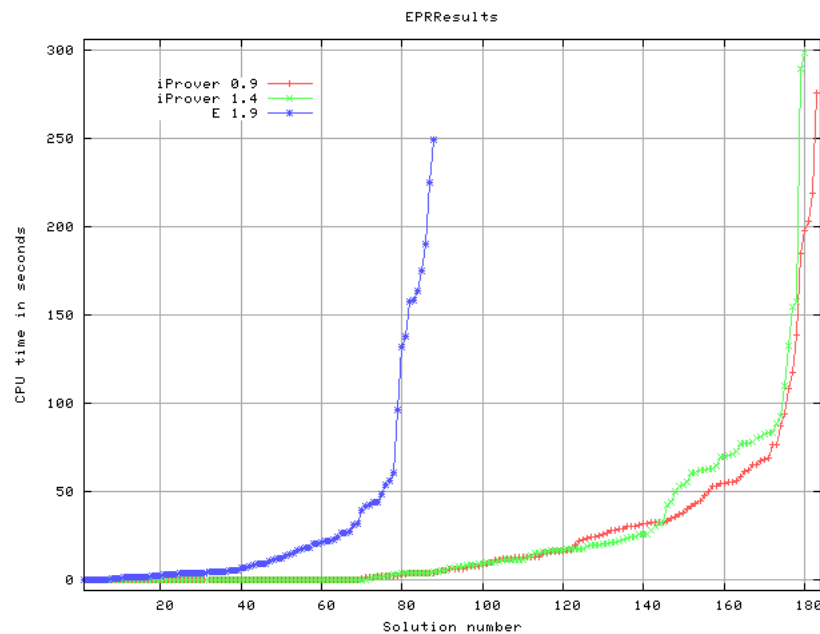
Figure 4.1: EPR division results taken from http://www.cs.miami.edu/~tptp/CASC/
J7/WWWFiles/ResultsPlots.html

Moreover, latest results for the sub-divisions of EPR is given in Fiqure  4.2; where
sub-figure  4.2b shows the EPS results, while sub-figure  4.2a shows the EPT results.
And Table 4.1 is a summary for the results.
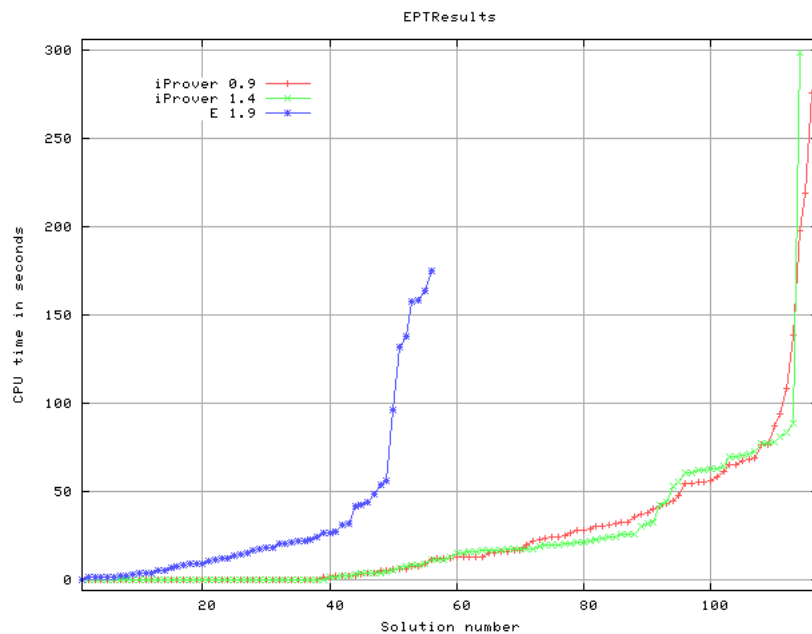
(a) EPT sub-division results
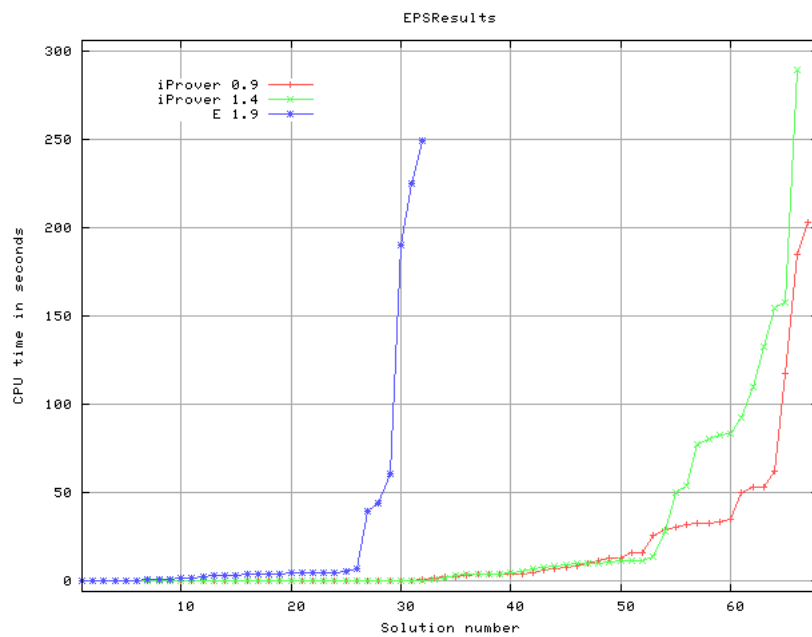


(b) EPS sub-division results

Figure 4.2: EPR sub-divisions results both taken from http://www.cs.miami.edu/~tptp/CASC/J7/WWWFiles/ResultsPlots.html

| criteria | iProver-0.9 | iProver-1.4 | E-1.9 |
|:---:|:---:|:---:|:---:|
| Solved(200) | 183 | 180 | 88 |
| Average-CPU-time | 22.96 | 22.80 | 31.02 |
| Solutions | 62 | 174 | 88 |
| New-Solved(10) | 10 | 9 | 5 |

Table 4.1: Summary for EPR division results

## 4.4   EPR reasoning Techniques

Reasoning in EPR is very interesting, since it is a fragment of FOL that could be transformed to propositional logic with some processing such as grounding techniques. So both reasoning techniques done for FOL and propositional logic are valid for EPR fragment, however for the propositional case further handling will be required.

According to [20], we could classify some of the methods used in reasoning for EPR fragment into two categories:

- Grounding based methods, which are mainly techniques applied to limit the number of generated clauses in the grounding step, afterwards normal methods of reasoning for propositional calculus could be applied as the EPR problem will be reduced to a propositional one. Some of those grounding based methods are mentioned below:

    - Splitting
    - Pure Predicates
    - Linking restrictions
    - Sort inference
    - Incremental Search

- Non-grounding based methods, which are methods using normal inference systems and calculi used for generic FOL.

    - Resolution Calculus, which was first proposed in [25]
    - Model evolution Calculus, which was proposed by [5]
    - Instantiation Calculus, which was proposed in [12]

Resolution is the oldest one of all the three calculi. In the beginning the results mentioned in [13] showed that it is difficult to be able to decide EPR fragment using a resolution procedure. Later on, a resolution procedure was found to be able to decide EPR [10]. However, resolution is inefficient at least practically for EPR and this could viewed from the results of the EPR division in the yearly CASC competition.

Model evolution calculus has the same decidability for FOL as resolution with saturation as proposed here in [3]. So it is refutationally complete and sound. But it has an advantage over resolution, since it is in general a decision procedure for EPR fragment, however this is not the case with most resolution procedures. And practically the automated theorem provers that implement the Model evolution calculus have better results in the CASC competition for EPR division.

Instantiation Calculus has the same decidability for FOL as resolution with saturation and model evolution calculus. And it is decision procedure for EPR fragment like Model evolution calculus. And the automated theorem provers that depends on Instantiation calculus have a very good results in the CASC competition for the EPR division [32, 33], at least the first on this division in the last CASC competition. CASC-J7. is an Instantiation based theorem prover.

Table 4.2 found below summarizes the comparison between the three Non-grounding methods for reasoning in EPR.

| Point of Comparison | Resolution | Model Evolution | Instantiation |
|---|---|---|---|
| Relatively New | No | Yes | Yes |
| Semi-decidable for FOL | Yes | Yes | Yes |
| Decidable for EPR | No, in general | Yes | Yes |
| Practically proved efficiency in EPR division | No | Yes | Yes |
| Example for Prover | E | Darwin | iProver |

Table 4.2: Summary for Non-grounding methods for reasoning in EPR

## 4.5 Range Restricting Transformations

According to [4], The range restricted transformations implemented in this project, that was originally devised in [4], were also added to other theorem provers MSPASS and KRHyper. And the results showed improvement and effectiveness in trying them over the satisfiable set problems in TPTP, specifically Version 3.1.1.

# Chapter 5

# Testing, Validation and Evaluation

Here in this chapter, we explain what was done for validating the output and Evaluating the program. So we divide this chapter into two sections:

- Validating and Evaluating the Simplified Transformations.

- Validating and Evaluating the Implementation of Bachmair and Ganzinger Model Construction Technique.

## 5.1 Simplified Transformations

### 5.1.1 Evaluating Memory Efficiency

The first part of the implementation which is related to applying the transformations to the the clause set introduces no memory leaks to the whole program.

Tools to check this:

1. Script implemented in E for giving a summary on the allocated and de-allocated memory structures, and the results showed that they are equal.

   ```
   # ————————————————————————————————————————
   # Total SizeMalloc()ed memory: 68536168 Bytes (131507 requests)
   # Total SizeFree()ed   memory: 68536168 Bytes (131507 requests)
   # New requests: 214 (197 by SecureMalloc(), 17 by SecureRealloc())
   # Total SecureMalloc()ed memory: 277647 Bytes
   # Returned: 214 (214 by FREE(), 0 by SecureRealloc())
   # SecureRealloc(ptr): 19 (17 Allocs, 0 Frees, 2 Reallocs)
   # ————————————————————————————————————————\\
   ```

2. Tool named 'valgrind' which also showed the same results as the above script.

### 5.1.2 Accuracy of the transformations

In all tested problems, The results of the Transformations were the same as an implemented program called Yarralumla http://users.cecs.anu.edu.au/~baumgart/systems/yarralumla/, implemented by Peter Baumgartner, one of the authors of [4]. So we believe it is correct.

## 5.2 Bachmair and Ganzinger Model Construction Technique

### 5.2.1 Evaluating Memory Efficiency

The same tools mentioned for The Transformations in 5.1.1 part were used. And the same results were achieved for the Implemented Model Construction Technique.

### 5.2.2 Accuracy of Implementation

To Be added later

# Chapter 6

# Results

Results

# Chapter 7

# Conclusion

Conclusion

# Chapter 8

# Future Work

This project has a fertile environment were many aspects could be added and extended in a simple way. And those points will be discussed in the sections of this chapter in details. Moreover, those points could be viewed in two different categories, the first of them is implementation view and it will be discussed further in section 8.1, while the other is a testing and evaluational view and this will be presented in section 8.2.

## 8.1 Implementational Future Work

This section is devoted to discuss the related enhancements and implementations that could be added in E to achieve the goal of model construction. Most of those points will also help in evaluating the implemented technique in a way or another.
Some Points include modifications for the implemented part such as sub-section 8.1.1, while others will need a whole new implementation as in sub-section 8.1.4.

### 8.1.1 Extension for Transformations

As discussed before the original transformations mentioned were simplified because it was only intended for EPR sub-class in FOL. So a great extension that could be done is to extend the transformations to all sub-classes of FOL instead of only EPR by adding the simplified steps in the implemented crr and rr procedures on one hand, and by adding the shifting and blocking transformations on the other hand.

### 8.1.2 Adding Splitting Techniques

Another addition for that project that could be added is implementing splitting techniques. Since it was one of the limitations that did not make the implemented Transformations work on their own and needed further handling by augmenting it with the Bachmair and Ganzinger Model construction Technique.

So this could be done by adding a suitable splitting technique with backtracking, and in this case the Bachmair and Ganzinger Model Construction Technique won't be enabled, however another part for further handling would be needed to extract the explicit model from the saturated splitted set of clauses.

### 8.1.3  Implement Other Model Construction Techniques

Augmenting E with other Model Construction Techniques would add a value for it. As well as, it will make us have a good evaluation on the implemented Bachmair and Ganzinger Model Construction Technique since we will have a meaningful comparison on the performance and the effect of each of the different techniques.

### 8.1.4  More on Bachmair and Ganzinger Model Construction Technique

Adding the General case for Bachmair and Ganzinger Model Construction Technique that deals with the non ground case for the saturated set of clauses, as explained here in [18], may have a good output since the transformations will not be used in this case and it will act directly on the saturated set without having them acting. And this will be a good research point to compare the effect of the transformations as a Model Construction Technique.

## 8.2    Testing and Evaluational Future Work

Testing is very important to be able to evaluate any project. So the coming points are of a great importance to have a fair judgement on the implemented techniques and to discover the limitations of applying them in saturation-based theorem provers.

### 8.2.1  Testing on a Server

Testing medium sized and large sized problems on a large server would be of a great importance. Since only a personal computer of 4 GB RAM were used in the testing so only small sized problems were able to run on it without crashing. And the results of these problems is important to have a full overview on the performance and the impact of the project specially on those problems in the EPR set that were not terminating in the original configurations. Only after that we could have a fair evaluation on the project.

### 8.2.2  More Testing on the Transformations

More Testing on the Transformations is needed with consideration of the prover itself to know why the transformations alone did not perform what it was supposed to do. Then after knowing the reason that could be enhanced accordingly.

### 8.2.3 More Testing on the Bachmair and Ganzinger Model Construction Technique

Also More Testing on the implemented Bachmair and Ganzinger Model Construction Technique is of major importance at least to know the limitations of applying it in saturation-based theorem provers.

# Appendix

# Appendix A

# Lists

**FOL**          First order logic

**ATP**          Automated theorem proving

**EPR**          Effectively propositional calculus

**CRR**          Classical Range Restricting Transformation

**RR**          Range Restricting Transformation

**BS**          Basic Shifting Transformation

**PF**          Partial Flattening Transformation

**BL**          Blocking Transformation

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# Appendix B

# Syntax of FOL

**Syntax of FOL**

The syntax of FOL consists of:

- Predicates, which is a mapping for properties in a language. Moreover, the symbols used for representing predicates are finite and specific for each problem.

- Terms, which consists of functions and variables as shown below:

  - Functions, which itself can be divided according to the arity of the function symbol as follows:
    * if (arity == 0), then it is considered a constant
    * if (arity > 0), then it is a proper function symbol
  - Variables, and they are infinite list of symbols

- Special symbols

  - $\bot$ which represents false
  - $\top$ which represents true

Atoms which are the basic building blocks of a formula, follow the following format:

$$p(t1, ..., tk)$$

where p is a predicate symbols, any ti is a term, and k is the arity of the predicate symbol p. A Literal is an atom or a negated atom. A formula consists of only one atom is called an Atomic formula.

Compound formulas are formed by:

- Connectives, and they are divided into:

- $-$ $\rightarrow$ : implication
  - $-$ $\neg$ or $\sim$ : negation
  - $-$ $\vee$ or $|$ : disjunction
  - $-$ $\wedge$ or $\&$ : conjunction
  - $-$ $\equiv$ : equivalence

- Quantifiers, and they are the following two:

  - $-$ $\forall$ which is the universal quantifier
  - $-$ $\exists$ which is the existential quantifier

A Clause is a disjunction of Literals. A ground clause is a clause having no variables. A positive clause is a clause who has no negated atoms. While a negative clause is a clause who contains only negated atoms. A mixed clause is a clause who consists of both atoms and negated atoms. A unit clause is a clause containing one Literal.

# Appendix C

# Implementation Details

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