4. $\bar{X}y$ - expresses disease
$\bar{X}X$ - carrier, doesnt express disease

$p(c) = \frac{1}{2}$ $\quad$ $p(c \mid 1 h.s) = \frac{1}{3}$

ⓐ $p(c \mid n\,h.s) = \dfrac{p(n\,h.s)\,p(c)}{p(n\,h.s)}$

$$= \frac{p(n\,h.s \mid c)\,p(c)}{p(n\,h.s \mid c)\,p(c) + p(n\,h.s \mid \bar{c})\,p(\bar{c})}$$

$$= \frac{(1/2)^n \cdot (1/2)}{(1/2)^n \cdot (1/2) + 1^n \cdot (1/2)}$$

$$= \frac{(1/4)^n}{(1/4)^n + 1/2}$$

ⓑ $\dfrac{(1/2)^7 \cdot (1/2)}{1/2^7 \cdot 1/2 + 1 \cdot (1/2)} = .0078 = .78\%$

$\dfrac{(1/2)^6 \cdot 1/2}{(1/2)^6 \cdot (1/2) + 1 \cdot (1/2)} = .0156 = 1.56\%$

$\boxed{\text{7 healthy sons}}$ → probability of being a carrier will fall below 1%.

5. $p(f|s) = .9$ $\qquad p(s) = 10^{-3}$

$p(f|\bar{s}) = .2$ – false pos rate

(a) $p(s|f) = \dfrac{p(f|s)\,p(s)}{p(f|s)\,p(s) + p(f|\bar{s})\,p(\bar{s})}$

$= \dfrac{(.9)(.001)}{(.9)(.001) + (.2)(.999)}$

$= \boxed{.0045}$

(b) i) $p(s|10f)$ → dependent on person, not the # of trials so $p(s|10f)$ is the same as 5a

$p(s|10f) = \dfrac{(.9)(.001)}{(.9)(.001) + (.2)(.999)}$

$= \boxed{.0045}$

ii) $p(s|10f) = \dfrac{(.9^{10})(.001)}{(.9)^{10}(.001) + (.2^{10})(.999)}$

$= \boxed{.9997}$

(c) $p(s|8f + 2\bar{f}) = \dfrac{(.9^{8})(.001)}{(.9^{8})(.001) + (.1^{2})(.999)}$

$= \boxed{.7245}$

(d) $p(s|f) = \dfrac{(1)(.001)}{(1)(.001) + (.2)(.999)}$

$= \boxed{.00498}$

6 ⓐ $p(\text{"exciting"} \mid s) = 50/500 = .1$

$p(\text{"exciting"}) \bar{s} = 35/200 = .175$

$p(s) = 1/2$

$$p(s \mid \text{"exciting"}) = \frac{p(\text{"exciting"} \mid s)\, p(s)}{p(\text{"exciting"} \mid s) p(s) + p(\text{"exciting"} \mid \bar{s}) p(\bar{s})}$$

$$= \frac{(.1)(.5)}{(.1)(.5) + (.175)(.5)}$$

$$= .364$$

$\boxed{\text{Not rejected @ } p(s) = 1/2}$

$p(s) = 5/7$

$$p(s \mid \text{"exciting"}) = \frac{(.01)(.7143)}{(.01)(.7143) + (.175)(.2857)}$$

$$= .5883$$

$\boxed{\text{Not rejected @ } p(s) = 5/7}$

ⓑ $p(\text{"stock"} \mid s) = 400/2000 = .2$ $\quad p(\text{"stock"} \mid \bar{s}) = .06$

$p(s \mid \text{"undervalued"}) = 200/2000 = .1$ $\quad p(\text{"undervalued"} \mid \bar{s}) = .02$

i) $$p(s \mid \text{"undervalued"}) = \frac{(.01)(.05)}{(.01)(.05) + (.025)(.5)}$$

$$= .8$$

$\boxed{\text{Not rejected with just "undervalued"}}$

ii) $p(s|\text{"stock"}) = \dfrac{(.2)(.5)}{(.2)(.5)+(.06)(.5)}$

$= .7692$

$\boxed{\text{not rejected with just "stock"}}$

iii) $p(s|\text{"undervalued"}) = \dfrac{(.1)(.75)}{(.1)(.75)+(.025)+(.25)}$

$= .923$

$\boxed{\text{Rejected for undervalued}}$

$p(s|\text{"stock"}) = \dfrac{(.2)(.75)}{(.2)(.75)+(.06)(.025)}$

$= .909$

$\boxed{\text{Rejected for stock}}$

iv) $p(s|\text{under, stock}) = \dfrac{p(\text{under}|s)\,p(\text{stock}|s)\,p(s)}{p(\text{under}|s)\,p(\text{stock}|s)\,p(s)+p(\text{under}|\bar{s})\,p(\text{stock}|\bar{s})\,p(\bar{s})}$

$= \dfrac{(.1)(.2)(.025)}{(.1)(.2)(.025)+(.025)(.06)(.75)}$

$= .816$

$\boxed{\text{Not rejected}}$

7. (a) $v[x] = Np(1-p)$

$N = 20 \qquad p = 1/6 \qquad q = 5/6$

$v[x] = (20)(1/6)(5/6)$

$= \boxed{\frac{25}{9} \text{ or } 2.7\overline{7}}$

(b) i) $E[x] = 1/2(6) + 1/10(1) + 1/10(2) + 1/10(3) + 1/10(4) + 1/10(5)$

$= 4.5 \times 5$

$= \boxed{22.5}$

ii) $v[x] = E[x^2] - E[x]^2$

$E[x^2] = 1/2(6^2) + 1/10(1^2) + 1/10(2^2) + 1/10(3^2) + 1/10(4^2) + 1/10(5^2)$

$= 23.5$

$v[x] = 23.5 - 4.5^2$

$= 3.25 \cdot 5 = \boxed{16.25}$

iii) $\sigma[x] = \sqrt{v[x]}$

$\sigma[x] = \sqrt{16.25}$

$= \boxed{4.0311}$