Code for Exercise 4.

For this problem, I am using the dynamic programming template created by Jason DeBacker.

- Let our control be
$$\{\text{work}, \text{don't work}\} \to \text{binary}(0, 1 \text{ choice}).$$
Call this control z.
- The state is w_t and b, since we know both at the time of the decision.
- The transition equation is $w' = \mathrm{E}_0\left[\sum_{t=0}^{\infty} \beta^t e^{\mathcal{N}(\mu,\sigma)}\right]$ if $z = 0$; otherwise, $w' = \mathrm{E}_0\left[\sum_{t=0}^{\infty} \beta^t b\right]$ if $z = 1$.

First let's import the relevant modules:

```
import numpy as np
import matplotlib.pyplot as plt
%matplotlib inline
```

Here, we set our parameters equal to those specified in the problem:

```
β = 0.96 # rate of time preference
b = 0.05 # unemployment benefits
μ = 0.0 # mean of log wages
σ = 0.15 # SD of wage draws
```

We now set up our state space grid:

```
'''
------------------------------------------------------------------------
Create Grid for State Space
------------------------------------------------------------------------
lb_w      = scalar, lower bound of grid
ub_w      = scalar, upper bound of grid
size_w    = integer, number of grid points in state space
w_grid    = vector, size_w x 1 vector of grid points
------------------------------------------------------------------------
'''
lb_w = .5
ub_w = 10
size_w = 200  # Number of grid points
w_grid = np.linspace(lb_w, ub_w, size_w)

'''
------------------------------------------------------------------------
Create grid of current utility values
------------------------------------------------------------------------
Y        = matrix, current income
W        = wage, stochastically determined
------------------------------------------------------------------------
'''
```

```python
Y = np.zeros((size_w, size_w))
W = []
for i in range(size_w):
    W.append(np.exp(np.random.normal(μ,σ)))
W=np.array(W)
for i in range(size_w): # loop over w
    for j in range(size_w): # loop over w'
        Y[i,j] = np.max((W[i]/(1-β), b + β*w_grid[j]))

# replace 0 and negative consumption with a tiny value
# This is a way to impose non-negativity on cons
Y[Y<=0] = 1e-15

'''
------------------------------------------------------------------------
Value Function Iteration
------------------------------------------------------------------------
VFtol     = scalar, tolerance required for value function to converge
VFdist    = scalar, distance between last two value functions
VFmaxiter = integer, maximum number of iterations for value function
V         = vector, the value functions at each iteration
Vmat      = matrix, the value for each possible combination of k and k'
Vstore    = matrix, stores V at each iteration
VFiter    = integer, current iteration number
TV        = vector, the value function after applying the Bellman operator
PF        = vector, indicies of choices of k' for all k
VF        = vector, the "true" value function
------------------------------------------------------------------------
'''
VFtol = 1e-8
VFdist = 5.0
VFmaxiter = 3000
V = np.zeros(size_w)
Vmat = np.zeros((size_w, size_w))
Vstore = np.zeros((size_w, VFmaxiter))
VFiter = 1
while VFdist > VFtol and VFiter < VFmaxiter:
    for i in range(size_w): # loop over w
        for j in range(size_w): # loop over w'
            Vmat[i, j] = Y[i, j] + β * V[j]

    Vstore[:, VFiter] = V.reshape(size_w,) # store value function at each iteration for
    TV = Vmat.max(1) # apply max operator to Vmat (to get V(w))
    PF = np.argmax(Vmat, axis=1)
    VFdist = (np.absolute(V - TV)).max()  # check distance
    V = TV
    VFiter += 1
```

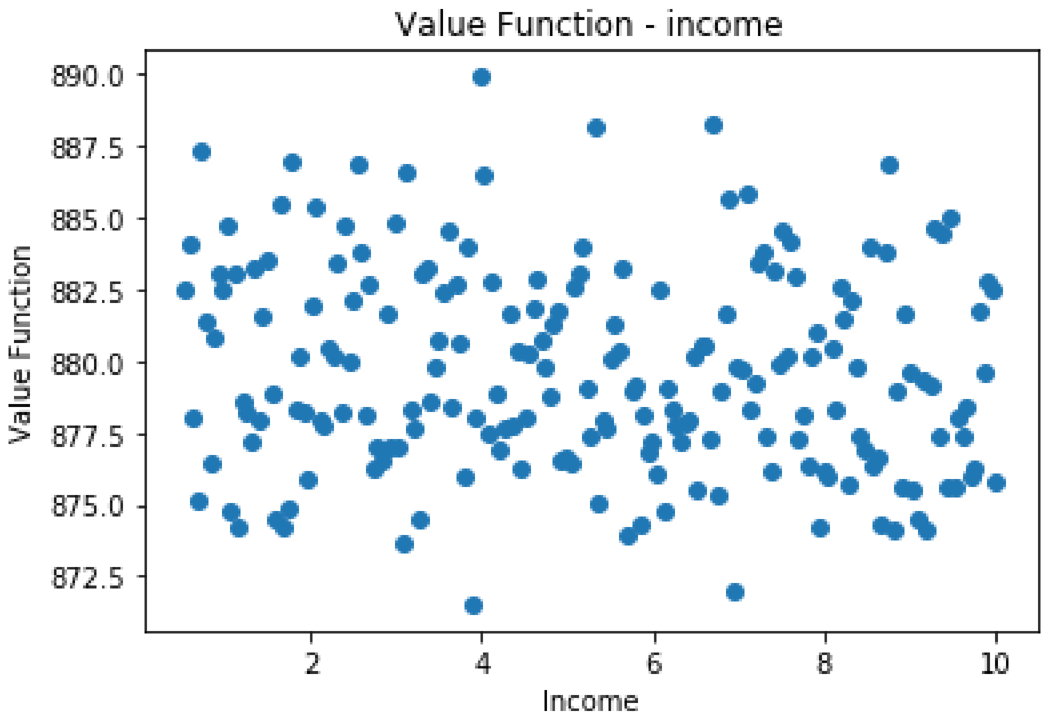
```python
if VFiter < VFmaxiter:
    print('Value function converged after this many iterations:', VFiter)
else:
    print('Value function did not converge')


VF = V # solution to the functional equation
```

```
Value function converged after this many iterations: 541
```

```python
'''
------------------------------------------------------------------------
Find consumption and savings policy functions
------------------------------------------------------------------------
optW  = vector, the optimal choice of w' for each w
optC  = vector, the optimal choice of y' for each y
------------------------------------------------------------------------
'''
optW = w_grid[PF]
optY = optW
```

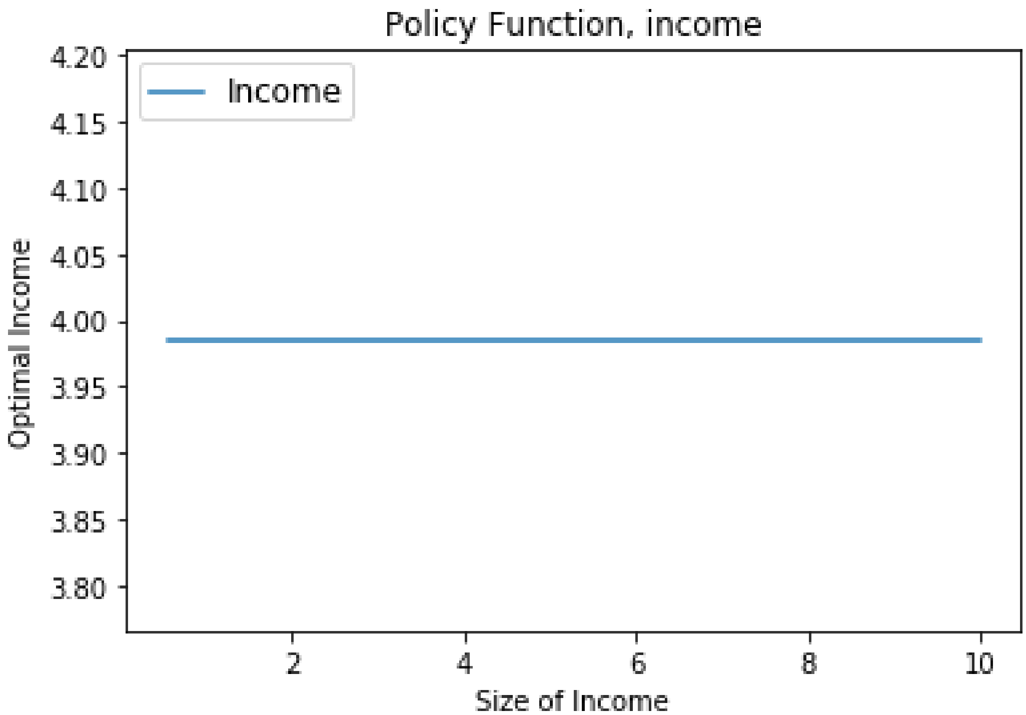
Plotting the **value function**, we have:

```python
# Plot value function
plt.figure()
# plt.plot(wvec, VF)
plt.scatter(w_grid[1:], VF[1:])
plt.xlabel('Income')
plt.ylabel('Value Function')
plt.title('Value Function - income')
plt.show()
```



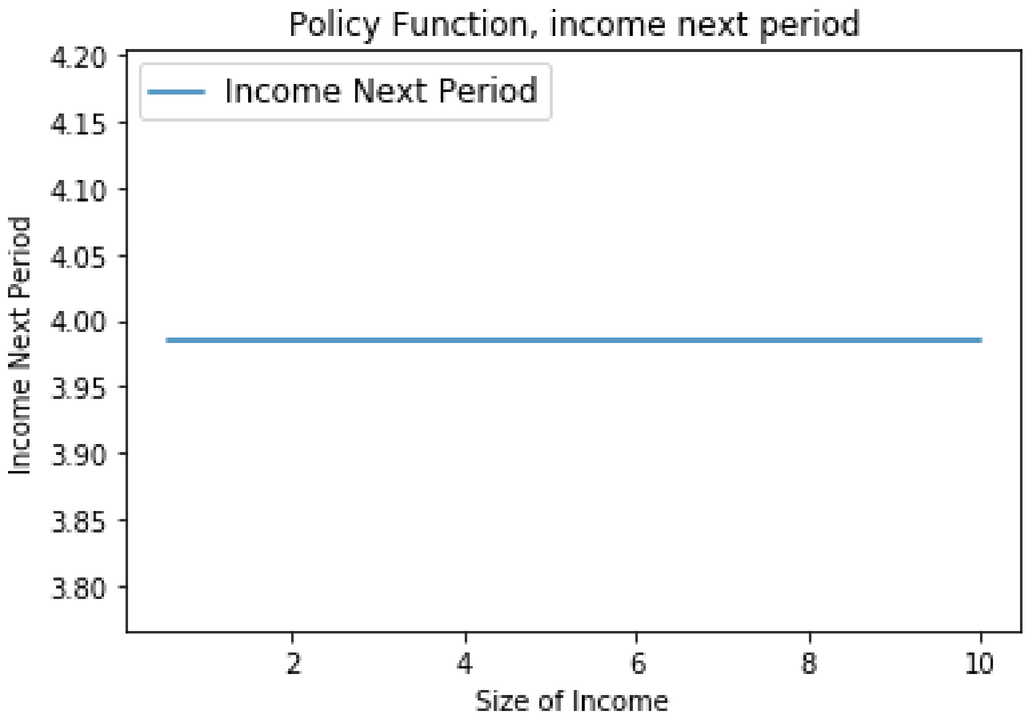

```
#Plot optimal consumption rule as a function of capital size
plt.figure()
fig, ax = plt.subplots()
ax.plot(w_grid[1:], optY[1:], label='Income')
# Now add the legend with some customizations.
legend = ax.legend(loc='upper left', shadow=False)
# Set the fontsize
for label in legend.get_texts():
    label.set_fontsize('large')
for label in legend.get_lines():
    label.set_linewidth(1.5)  # the legend line width
plt.xlabel('Size of Income')
plt.ylabel('Optimal Income')
plt.title('Policy Function, income')
plt.show()
```

```
<Figure size 432x288 with 0 Axes>
```




```python
#Plot optimal consumption rule as a function of capital size
 plt.figure()
 fig, ax = plt.subplots()
 ax.plot(w_grid[1:], optW[1:], label='Income Next Period')
 # Now add the legend with some customizations.
 legend = ax.legend(loc='upper left', shadow=False)
 # Set the fontsize
 for label in legend.get_texts():
     label.set_fontsize('large')
 for label in legend.get_lines():
     label.set_linewidth(1.5)  # the legend line width
 plt.xlabel('Size of Income')
 plt.ylabel('Income Next Period')
 plt.title('Policy Function, income next period')
 plt.show()
```

```
<Figure size 432x288 with 0 Axes>
```


Policy Function, income next period
Income Next Period
4.20
4.15
4.10
4.05
4.00
3.95
3.90
3.85
3.80
Income Next Period
2
4
6
8
10
Size of Income