Harrison-Beard-ProbSet5

July 24, 2018

```
In [3]: import numpy as np
        from numpy import linalg as la

        import pandas as pd
        from matplotlib import pyplot as plt

        import scipy
        from scipy.stats import norm
        from scipy.stats import chi2
        import scipy.optimize as opt
        import scipy.integrate as integrate
        from scipy import stats as sts

        import math
```

Thank you to Natasha, Matt, and Tim.

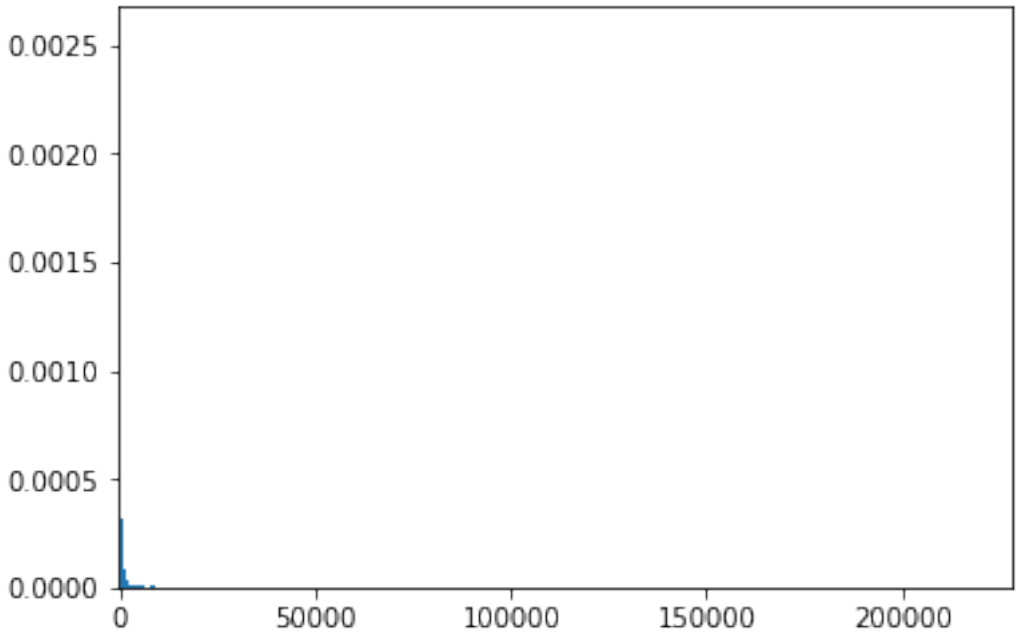
Problem 1.

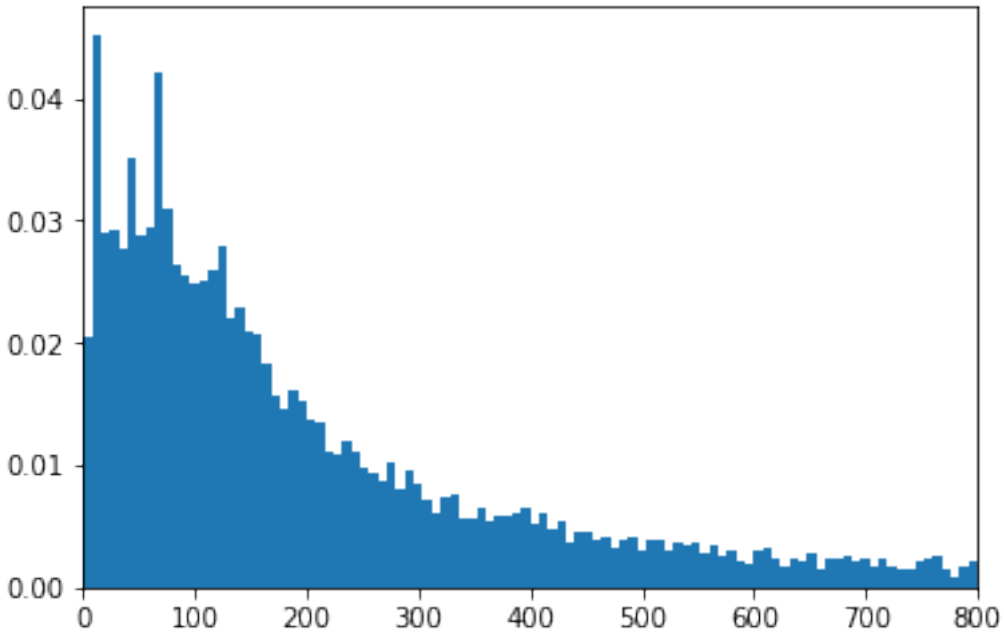
0.1 Part (a).

```
In [9]: def p1a(ret=False):

            claims = np.loadtxt("clms.txt")

            w = 1/len(claims) * np.ones_like(claims)

            claims_t = claims[claims<800]
            w_t = 1/len(claims_t) * np.ones_like(claims_t)

            if ret: return claims, claims_t, w_t



            plt.hist(claims, density = True, bins = 1000, weights = w)
            plt.xlim(-500, claims.max())
            plt.show()
```

```
    plt.hist(claims_t, weights = w_t, bins = 100)
    plt.xlim(0,800)
    plt.show()
```

In [10]: p1a()



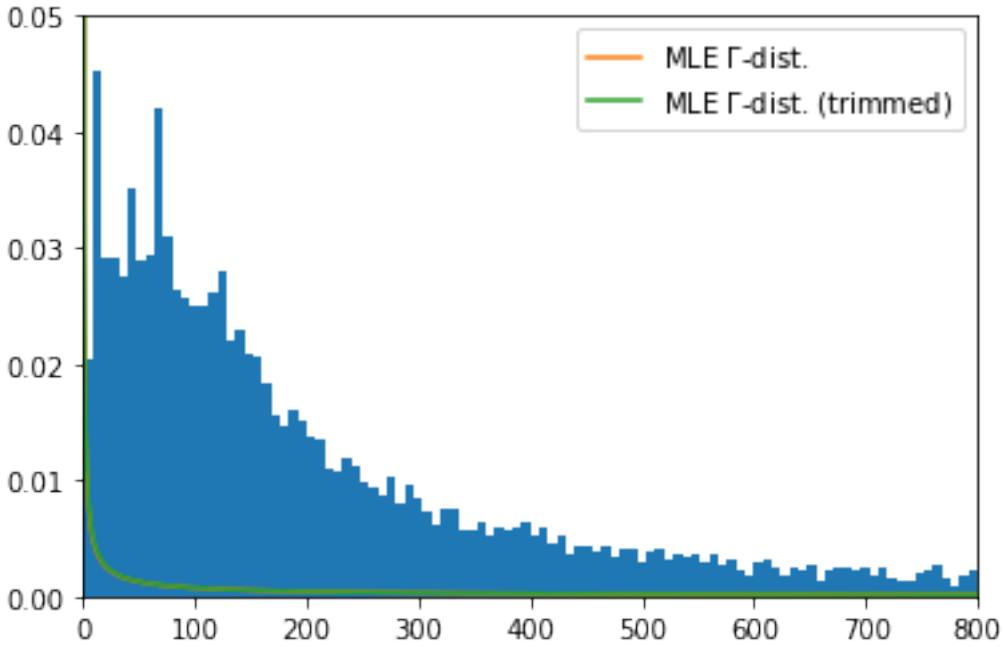


0.2 Part (b).

```
In [11]: def p1b1(x,,):
             return (

                 1 / (** * math.gamma()) * x**(-1) * np.exp(-1*(x/))

             )

In [12]: def p1b2(args, x):

             , = args

             return -1*np.sum(np.log(p1b1(x,,)))


In [13]: def p1b3(ret=False):

             claims, claims_t, w_t = p1a(ret = True)

             _0 = np.var(claims) / np.mean(claims)
             _0 = np.mean(claims) / _0
             params_0 = [_0, _0]

             bounds = ((1e-5,None),(1e-5,None))
             optimized = opt.minimize(p1b2, params_0, args=claims, bounds=bounds)
```

```
    optimized_t = opt.minimize(p1b2,params_0,args=claims_t,bounds=bounds)

    , = optimized.x

    e = optimized.fun

    if ret: return e

    _t, _t = optimized_t.x

    X = np.linspace(1e-2, 800, 1000)

    grid = p1b1(X, , )
    grid_t = p1b1(X, _t, _t)

    plt.hist(claims_t, weights=w_t, bins=100)
    plt.plot(X,grid,label="MLE $\Gamma$-dist.")
    plt.plot(X,grid_t,label="MLE $\Gamma$-dist. (trimmed)")

    plt.xlim(0,800)
    plt.ylim(0,5e-2)

    plt.legend()
    plt.show()

    print(
    "   :  ",,"\n",
    "  :  ",,"\n\n",
    "  _t:",_t,"\n",
    "  _t:",_t
    )
```

In [14]:
```
p1b3()
```



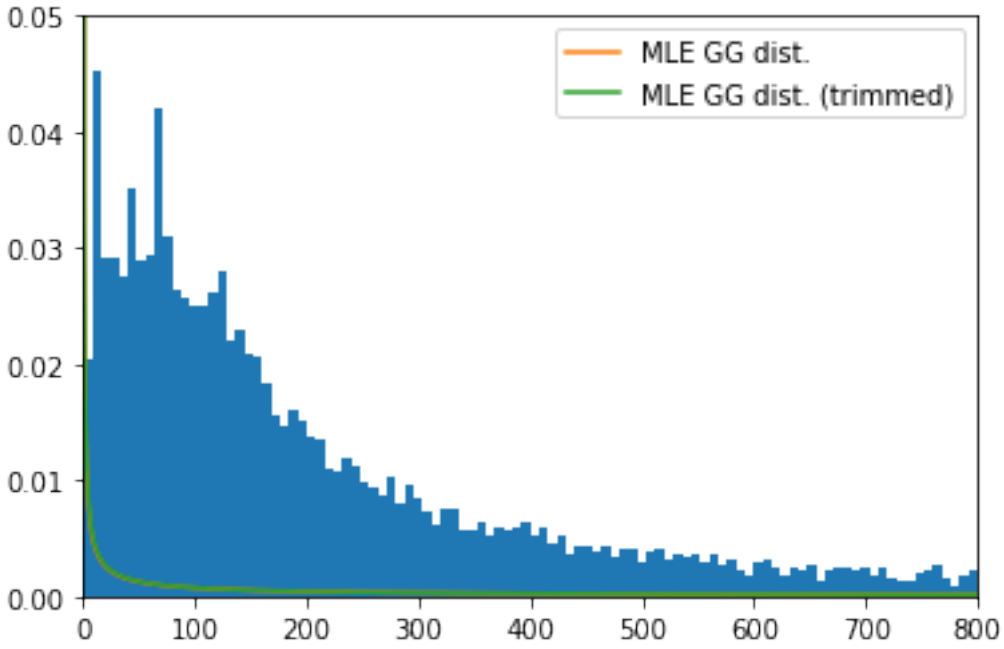

```
:   0.22175531882397229
:   21911.06469927622

_t: 0.20348864099178415
_t: 21911.064699783838
```

0.3 Part (c).

```
In [15]: def p1c1(x,,,m):
             return (

                 m/(** * math.gamma(/m)) * x**(-1)*np.exp(-1*(x/)**m)

             )

In [16]: def p1c2(args,x):
             ,,m = args
             return -1*np.sum(np.log(p1c1(x,,,m)))

In [17]: def p1c3(ret=False):
             claims, claims_t, w_t = p1a(ret = True)

             _0 = np.var(claims) / np.mean(claims)
             _0 = np.mean(claims) / _0
             m_0 = 1
```

```
 params_0 = [_0, _0, m_0]

b = (1e-5,None)
bounds = (b,b,b)

optimized = opt.minimize(p1c2, params_0, args=claims, bounds=bounds)
optimized_t = opt.minimize(p1c2, params_0, args= claims_t,
                           bounds = bounds)

,,m = optimized.x

e = optimized.fun

if ret: return e

_t,_t,m_t = optimized_t.x

X = np.linspace(1e-2,800,1000)

grid = p1c1(X,,,m)
grid_t = p1c1(X,_t,_t,m_t)

plt.hist(claims_t,weights = w_t, bins=100)
plt.plot(X,grid,label="MLE GG dist.")
plt.plot(X,grid_t,label = "MLE GG dist. (trimmed)")

plt.xlim(0,800)
plt.ylim(0,5e-2)

plt.legend()
plt.show()

print(
"    :  ",,"\n",
"  :  ",,"\n",
"  m:  ",m,"\n\n",
"  _t:",_t,"\n",
"  _t:",_t,"\n",
"  m_t:",m_t
)
```

In [18]:
```
p1c3()
```




```
:   0.2217538681829344
:   21911.064372887282
m:   0.9976836233858909

_t: 0.2034851449617037
_t: 21911.064638789405
m_t: 0.8868958758893984
```

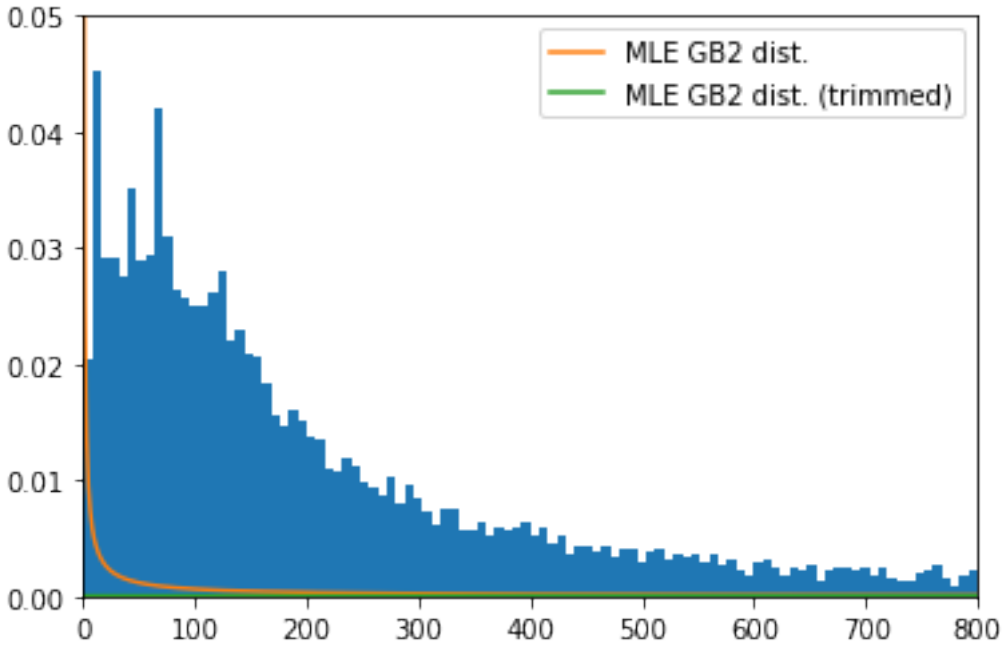
0.4 Part (d).

```
In [19]: def p1d1(x,,,p,q):
             return(

                 *x**(*p-1)/(**(*p)*scipy.special.beta(p,q)*(1+(x/)**)**(p+q))
             )

In [20]: def p1d2(args,x):
             ,,p,q = args

             return -1 * np.sum(np.log(p1d1(x,,,p,q)))

In [21]: def p1d3(ret=False):

             claims, claims_t, w_t = p1a(ret = True)
```

```
    _0 = np.var(claims)/np.mean(claims)
    _0 = np.mean(claims) / _0
    p_0, q_0 = 1,1

    params_0 = [_0,_0,p_0,q_0]

    b = (1e-5,None)
    bounds = (b,b,b,b)

    optimized = opt.minimize(p1d2,params_0, args = claims, bounds = bounds)
    optimized_t = opt.minimize(p1d2, params_0, args=claims_t, bounds=bounds)

    ,,p,q = optimized.x

    e = p1d2([,,p,q],claims)

    if ret: return e

    _t,_t,p_t,q_t = optimized_t.x

    X = np.linspace(1e-2,800,1000)

    grid = p1b1(X,,)
    grid_t = p1b1(X,_t,_t)

    plt.hist(claims_t,weights = w_t, bins=100)
    plt.plot(X,grid,label = "MLE GB2 dist.")
    plt.plot(X,grid_t,label = "MLE GB2 dist. (trimmed)")

    plt.xlim(0,800)
    plt.ylim(0,5e-2)

    plt.legend()
    plt.show()

    print(
    "   :  ",,"\n",
    "  :  ",,"\n",
    "  p:  ",p,"\n",
    "  q:  ",q,"\n\n"
    "  _t:",_t,"\n",
    "  _t:",_t,"\n",
    "  p_t:",p_t,"\n",
    "  q_t:",q_t,"\n\n",
    "  e:  ",e
    )
```

In [22]:
```
p1d3()
```

```
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: overflow encount
  after removing the cwd from sys.path.
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: invalid value en
  after removing the cwd from sys.path.
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: overflow encount
  after removing the cwd from sys.path.
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: invalid value en
  after removing the cwd from sys.path.
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: divide by zero e
  after removing the cwd from sys.path.
```




```
   :    0.1076732352007127
   :    21911.069331861727
  p:    61.5648216721421
  q:    102.69247959910427

 _t: 0.9787247608992424
  _t: 21911.05377033113
  p_t: 1.1884189025004108
  q_t: 120.13896050042712

  e:    74861.6169378541
```

0.5 Part (e).

```
In [23]: def p1e():

             e_ga = p1b3(ret = True)
             e_gg = p1c3(ret = True)
             e_gb2 = p1d3(ret = True)

             lr_ga = 2*(np.log(e_gb2)-np.log(e_ga))
             lr_gg = 2*(np.log(e_gb2)-np.log(e_gg))

             p_ga = chi2.cdf(lr_ga, df = 4)
             p_gg = chi2.cdf(lr_gg, df = 4)

             print(
                 "   ^2_GG:",p_gg,"\n",
                  "  ^2_GA:",p_ga
             )
```

```
In [24]: p1e()
```

```
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: overflow encount
  after removing the cwd from sys.path.
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: invalid value en
  after removing the cwd from sys.path.
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: overflow encount
  after removing the cwd from sys.path.
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: invalid value en
  after removing the cwd from sys.path.
/anaconda3/lib/python3.6/site-packages/ipykernel_launcher.py:4: RuntimeWarning: divide by zero e
  after removing the cwd from sys.path.


   ^2_GG: 0.0
   ^2_GA: 0.0
```

Problem 2.

```
In [25]: # Globals
         SERIES = pd.read_csv("MacroSeries.txt",header=None)
         SERIES.columns = ["c","k","w","r","y"]
```

0.6 Part (a).

```
In [26]: def p2a1(w,k,):
             return np.log(w/((1-)*k**))
```

```
In [27]: def p2a2(w,k,,,,):
             z = p2a1(w,k,)
             val = 0
             for i in range(len(z)):
                 if i == 0:
                     pdf = norm.pdf(,loc=,scale=)
                 else:
                     pdf = norm.pdf(z[i],loc=(*z[i-1]+(1-)*),scale=)
                 val+=np.log(pdf)
             return val

In [28]: def p2a3(params,*args):
             ,,,=params
             w,k=args
             return -1*p2a2(w,k,,,,)

In [43]: def p2a4():
             params_0 = np.array([.4,.4,.4,.8])
             mle_args = (SERIES.w, SERIES.k)

             optimized = opt.minimize(p2a3,params_0,args=mle_args,
                  method="L-BFGS-B",bounds=((1e-2,.99),(1e-2,None),(1e-2,None),
                                            (-.99,.99)))

             _mle,_mle,_mle,_mle = optimized.x
             loglik_max = -1*optimized.fun

             vcv_ih = optimized.hess_inv.todense()

             print(
                 "   _mle:",_mle,"\n",
                  "  _mle:",_mle,"\n",
                  "  _mle:",_mle,"\n",
                  "  _mle:",_mle,"\n",
                  "  log likelihood max:",loglik_max,"\n",
                  "  inverse hessian of vcv:\n",vcv_ih
             )



In [44]: p2a4()

   _mle: 0.5534077576357248
   _mle: 8.106499434665157
   _mle: 0.09278026792905704
   _mle: 0.8246485429070475
   log likelihood max: 95.85827569290834
   inverse hessian of vcv:
 [[ 2.94783896e+00 -3.91105055e+01 -2.20282338e-01 -8.61238189e-01]
```

```
 [-3.91105055e+01  5.18902999e+02  2.92239258e+00  1.14268028e+01]
 [-2.20282338e-01  2.92239258e+00  1.66392463e-02  6.39042748e-02]
 [-8.61238189e-01  1.14268028e+01  6.39042748e-02  2.56555043e-01]]
```

0.7 Part (b).

```
In [45]: def p2b1(r,k,):
             return np.log(r/(*k**(-1)))

In [46]: def p2b2(r,k,,,,):
             z = p2b1(r,k,)
             val = 0
             for i in range(len(z)):
                 if i ==0 :
                     pdf = norm.pdf(,loc=,scale=)
                 else:
                     pdf = norm.pdf(z[i],loc=(*z[i-1]+(1-)*),scale=)
                 val += np.log(pdf)
             return val


In [47]: def p2b3(params, *args):
             ,,,=params
             r,k=args
             return -1*p2b2(r,k,,,,)


In [48]: def p2b4():
             params_0 = np.array([.4,.4,.4,.8])
             mle_args = (SERIES.r, SERIES.k)
             optimized = opt.minimize(p2b3,params_0,args=mle_args,
                  method= "L-BFGS-B",bounds=((1e-2,.99),(1e-2,None),(1e-2,None),
                                             (-.99,.99)))
             _mle,_mle,_mle,_mle=optimized.x
             loglik_max = -optimized.fun

             vcv_ih = -1*optimized.hess_inv.todense()

             print(
                 "   _mle:",_mle,"\n",
                  "  _mle:",_mle,"\n",
                  "  _mle:",_mle,"\n",
                  "  _mle:",_mle,"\n",
                  "  log likelihood max:",loglik_max,"\n",
                  "  inverse hessian of vcv:\n",vcv_ih
             )
```

```
In [49]: p2b4()
```

```
   _mle: 0.553376197472035
   _mle: 7.56982093560222
   _mle: 0.09278048277785769
   _mle: 0.8246601896600757
   log likelihood max: 95.85827566348482
   inverse hessian of vcv:
 [[-1.37999828e+02  2.40074534e+03  5.32604011e+00  4.43546900e+01]
 [ 2.40074534e+03 -4.17656334e+04 -9.26368535e+01 -7.72261481e+02]
 [ 5.32604011e+00 -9.26368535e+01 -2.06326662e-01 -1.68846120e+00]
 [ 4.43546900e+01 -7.72261481e+02 -1.68846120e+00 -1.50402824e+01]]
```

0.8 Part (c).

```
In [50]: def p2c1():
             params_0=np.array([.4,.4,.4,.8])
             mle_args = (SERIES.w,SERIES.k)
             optimized = opt.minimize(p2a3,params_0,args=mle_args,
          method="L-BFGS-B",bounds=((1e-2,.99),(1e-2,None),(1e-2,None),(-.99,.99)))
             return optimized.x


In [51]: def p2c2(r,k,z_m1,params):
             ,,,=params
             return 1-norm.cdf(p2b1(r,k,),(*z_m1+(1-)*),)

In [52]: def p2c3():
             print("Probability that the interest rate is greater than one:",
                   p2c2(1,7_500_000,10,p2c1()))

In [53]: p2c3()
```

```
Probability that the interest rate is greater than one: 1.0
```

Problem 3.

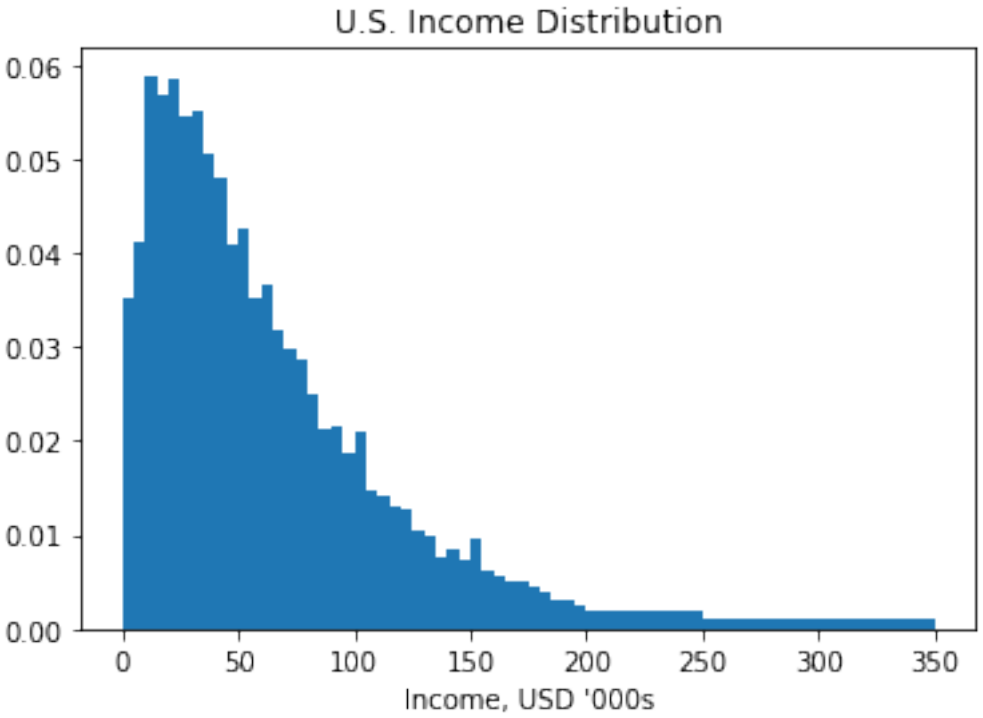
```
In [54]: # Globals
         INCOMES = pd.read_table("usincmoms.txt",header=None)
         INCOMES.columns = ["Percent","Midpoint"]
         INCOMES.Midpoint = INCOMES.Midpoint/1_000
```

0.9 Part (a).

```
In [55]: def p3a(ret=False):
             w=INCOMES.Percent.copy()
```

```
    w[41],w[40]=w[41]/20,w[40]/10
    bins=np.append(np.linspace(0,200,41),[250,350])

    if ret: return bins,w

    plt.hist(INCOMES.Midpoint, bins=bins, weights=w)

    plt.xlabel("Income, USD '000s")
    plt.title("U.S. Income Distribution")
    plt.show()
```

```
In [56]: p3a()
```



0.10 Part (b).

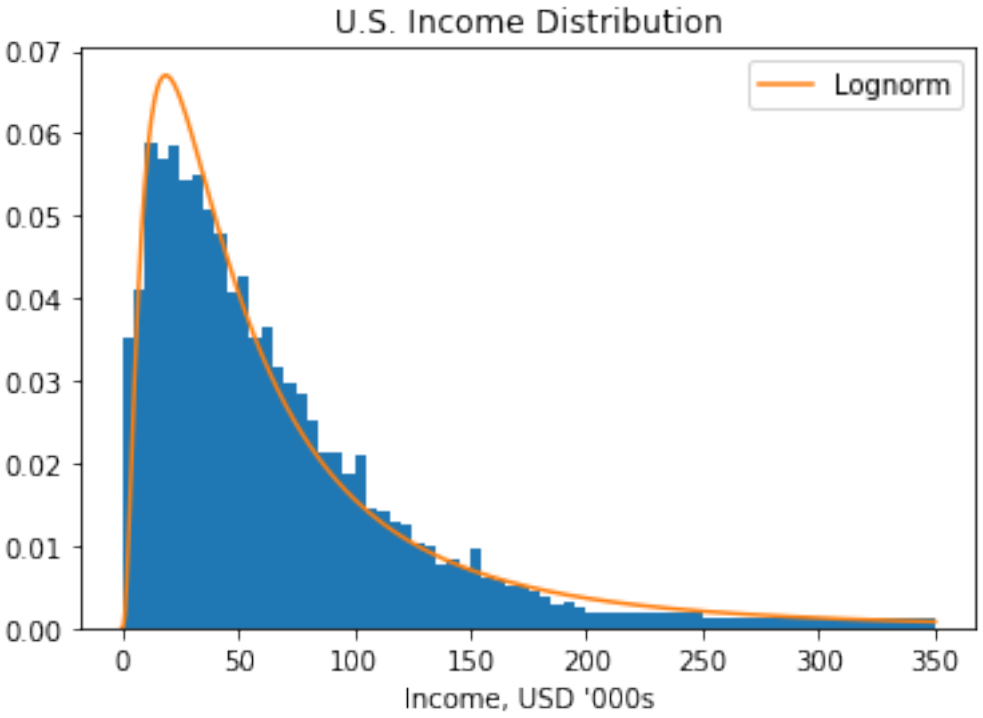
```
In [89]: def p3b1(x,,):
             return (

             (1/(x**np.sqrt(2*np.pi)))*np.exp(-1*((np.log(x)-)**2/(2***2)))

             )
```

```
In [168]: def p3b2(,):
              bins,w=p3a(ret=True)
```

```
                moments = np.zeros(len(bins)-1)

                for i in range(len(bins)-1):
                    moments [i] = (integrate.quad(
                    lambda x: p3b1(x,,),bins[i],bins[i+1])[0])
                return moments


In [91]: def p3b3(dat,,,mode=False):
             modat = np.array(dat.Percent)
             model = p3b2(,)
             if mode: return model-modat
             else: return (model-modat)/modat


In [92]: def p3b4(params,args):
             ,=params
             dat,W=args
             e=p3b3(dat,,)
             return e.T@W@e

In [154]: def p3b5(ret=False):

              bins,w = p3a(ret=True)

              _0 = np.log(np.sum(INCOMES.Percent*INCOMES.Midpoint))
              _0 = 3
              params_0=np.array([_0,_0])

              W=np.diag(INCOMES.Percent)
              gmm_args = np.array([INCOMES,W])

              optimized = opt.minimize(p3b4,params_0,args=gmm_args,method="L-BFGS-B",bounds=((No

              _gmm, _gmm = optimized.x

              X=np.linspace(1e-9,350,500)

              if ret: return optimized.fun,_gmm,_gmm

              plt.hist(INCOMES.Midpoint,bins=bins,weights=w)

              plt.xlabel("Income, USD '000s")
              plt.title("U.S. Income Distribution")

              plt.plot(X,5*p3b1(X,_gmm,_gmm),label="Lognorm")
```

```
            plt.legend()
            plt.show()
```

```
In [155]: p3b5()
```



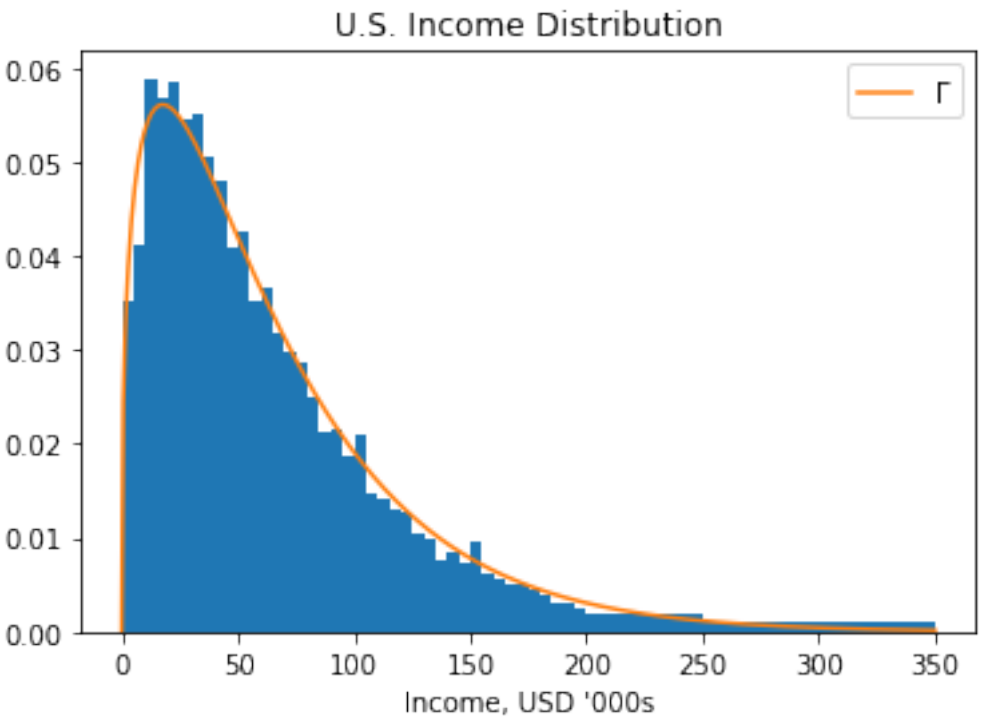
0.11 Part (c).

```
In [103]: def p3c1(x,,):
            return(

            1/(***(math.gamma()))*x**(-1)*np.exp(-(x/))

            )
```

```
In [167]: def p3c2(,):
            bins,w=p3a(ret=True)

            moments = np.zeros(len(bins)-1)
            for i in range(len(bins)-1):
                moments[i] = integrate.quad(lambda x: p3c1(x,,),bins[i],bins[i+1])[0]
            return moments
```

```
In [105]: def p3c3(dat,,,mode=False):
            modat = np.array(INCOMES.Percent)
```

```
                model = p3c2(,)
                if mode: return model - modat
                else: return (model-modat) / modat

In [106]: def p3c4(params, args):
                ,=params
                dat,W=args

                e=p3c3(dat,,,mode=False)

                return e.T@W@e

In [153]: def p3c5(ret=False):

                bins,w=p3a(ret=True)

                _0,_0=5,25
                params_0 = np.array([_0,_0])

                W=np.diag(INCOMES.Percent)
                gmm_args = np.array([INCOMES,W])

                optimized = opt.minimize(p3c4,params_0,method="L-BFGS-B",
                     args=gmm_args,bounds=((1e-10,None),(1e-10,None)))

                _gmm,_gmm = optimized.x

                X = np.linspace(1e-9, 350, 500)

                if ret: return optimized.fun, _gmm, _gmm

                plt.hist(INCOMES.Midpoint, bins= bins, weights=w)

                plt.xlabel("Income, USD '000s")
                plt.title("U.S. Income Distribution")

                plt.plot(X,5*p3c1(X,_gmm,_gmm),label="$\Gamma$")

                plt.legend()
                plt.show()

In [156]: p3c5()
```

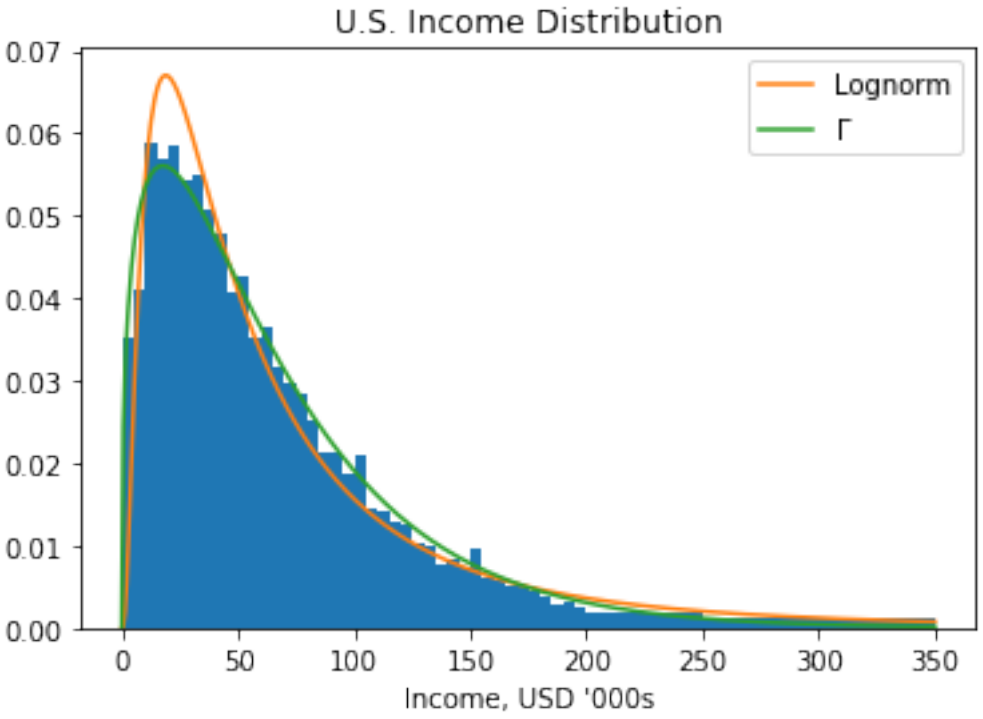


0.12 Part (d).

```
In [164]: def p3d():

              ge, _gmm, _gmm = p3c5(ret=True)
              lne, _gmm, _gmm = p3b5(ret=True)

              bins,w=p3a(ret=True)

              X = np.linspace(1e-9,350,500)

              plt.hist(INCOMES.Midpoint,bins=bins,weights=w)

              plt.xlabel("Income, USD '000s")
              plt.title("U.S. Income Distribution")

              plt.plot(X,5*p3b1(X,_gmm,_gmm),label="Lognorm")
              plt.plot(X,5*p3c1(X,_gmm,_gmm),label="$\Gamma$")

              plt.legend()
              plt.show()

              print( "Gamma and Lognorm errors:", ge, lne)
```

```
    return ge, lne
```

```
In [166]: p3d()
```




```
Gamma and Lognorm errors: 0.014084919474147941 0.03551648133439868
```

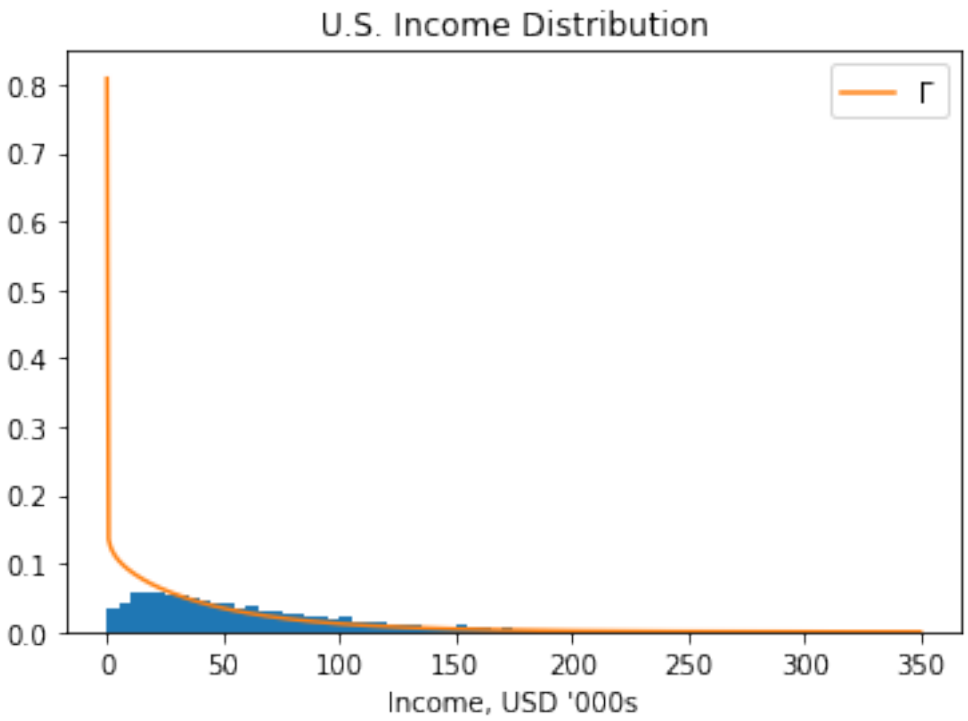
```
Out[166]: (0.014084919474147941, 0.03551648133439868)
```

0.13 Part (e).

```
In [174]: def p3e1(,):

              bins,w=p3a(ret=True)

              moments = np.zeros(len(bins)-1)
              for i in range(len(bins)-1):
                  moments[i] = integrate.quad(lambda x: p3c1(x,,),bins[i],bins[i+1])[0]
              return moments
```

```
In [175]: def p3e2(dat,,,mode=False):
              modat = np.array(INCOMES.Percent)
              model = p3e1(,)
              if mode: return model - modat
              else: return (model-modat)/modat
```

```
In [176]: def p3e3(params, args):
              ,=params
              dat,W=args
              e=p3e2(dat,,,mode=False)
              return e.T@W@e

In [195]: def p3e4():

              bins,w=p3a(ret=True)

              _0,_0 = 1.1, 50

              params_0 = np.array([_0,_0])

              ge, _gmm_0, _gmm_0 = p3c5(ret=True)
              e=p3e2(INCOMES,_gmm_0,_gmm_0)

              vcv=np.outer(e.T,e)
              W=la.pinv(vcv)

              gmm_args = np.array([INCOMES,W])

              optimized = opt.minimize(p3e3,params_0,args=gmm_args, bounds=((1e-10,None),(1e-10,

              _gmm,_gmm = optimized.x

              X=np.linspace(1e-9,350,500)

              plt.hist(INCOMES.Midpoint, bins=bins, weights=w)

              plt.xlabel("Income, USD '000s")
              plt.title("U.S. Income Distribution")

              plt.plot(X,5*p3c1(X,_gmm,_gmm),label="$\Gamma$")

              plt.legend()
              plt.show()

              return optimized.fun

In [196]: p3e4()
```



Out[196]: 2.2526548410291274e-17

Something seems to have gone wrong here. Not sure what.

Problem 4.

```
In [207]: # Globals
          SERIES = pd.read_csv("MacroSeries.txt",header=None)
          SERIES.columns = ["c","k","w","r","y"]

In [208]: def p41(w,k,):
              return(

              np.log(w/((1-)*k**))

              )

In [209]: def p42(z_p1,z,,):
              return(

              z_p1 -  * z - (1-) *

              )
```

```
In [210]: def p43(z_p1,z,,):
              return(

              (z_p1 -  * z - (1-) *  ) * z

              )

In [227]: def p44(z_p1,,,k_p1,c,c_p1):
              return(

              **np.exp(z_p1) * k_p1 ** (-1) * (c/c_p1) - 1

              )

In [228]: def p45(z_p1,,,k_p1,c,c_p1,w):
              return(

              (**np.exp(z_p1)*k_p1**(-1)*(c/c_p1)-1)*w

              )

In [229]: def p46(params, dat):

              ,,, = params
              z =       p41(dat.w,dat.k,)[1:]
              z_p1 =    p41(dat.w,dat.k,)[:-1]
              k =       dat.k[1:]
              k_p1 =    dat.k[:-1]
              w =       dat.w[1:]
              c =       dat.c[1:]
              c_p1 =    dat.c[:-1]

              return (np.array([np.mean(p42(z_p1,z,,)),
                                np.mean(p43(z_p1,z,,)),
                                np.mean(p44(z_p1,,,k_p1,c,c_p1)),
                                np.mean(p45(z_p1,,,k_p1,c,c_p1,w))
                               ]))

In [230]: def p47(params,dat,mode=True):

              model = np.zeros(4)
              modat = p46(params,dat)

              if mode: return model-modat
              else: return (model-modat)/modat

In [231]: def p48(params, args):

              dat,W = args
```

```
            e = p47(params,dat)

            return e.T@W@e
```

In [234]:
```
def p49():

    ,,,  = .4,.9,15,.9 # guesses
    params_0 = (,,,)

    W=np.eye(4)

    args=[SERIES,W]

    optimized=opt.minimize(p48,params_0,args=args,method="L-BFGS-B",
                           bounds=((1e-10,1),(1e-10,1),(1e-10,None),(-1,1)))

    print(p46(optimized.x,SERIES),"\n")

    _1,_1,_1,_1=optimized.x
    e=optimized.fun

    print(
        "   :",_1,"\n",
         "  :",_1,"\n",
         "  :",_1,"\n",
         "  :",_1,"\n",
         "  e:",e
    )
```

In [235]:
```
p49()
```

```
[ 0.35975077  3.52260775 -0.00609967  1.94523833]

   : 0.4320475665588142
   : 0.9357322335853419
   : 0.9590620852936956
   : 1.0
   e: 16.32217535688303
```