AMS 562 Homework 3

Due: Monday, 10/22, 11:59 pm

Background:

Nonlinear root finding builds the foundation for optimization problems. Given any function $f : \mathbb{R} \to \mathbb{R}$, which is continuous and smooth, we want to find a(all) root(s) of it, say x, so that $f(x) = 0$. One of the most basic algorithms for the root finding problems is *interval bisection method*, which is built on top of the *intermediate value theorem*, i.e. a root $x^* \in [a, b]$ exits for a continuous and smooth function f if $f(a)f(b) \leq 0$. The following shows the *pseudo code* of the interval bisection method.

Algorithm: Interval Bisection
Inputs: function f, interval $[a, b]$
Output: x^*, s.t. $f(x^*) = 0$

while $(b - a) > \epsilon$**, do**
    $m = a + \frac{b-a}{2}$
    if $f(m)$ and $f(a)$ have the same sign**, then**
        $a = m$
    else
        $b = m$
    end if
end while
$x^* = m$

Your task is:

Write a program that uses the interval bisection method to find **a single** root of $\sin(x)$ and $x^2 - 2$. In addition, report the cost of iterations regarding the loop statement. Your program should take **three command line** input arguments, i.e. `./my_prog <func-choice> <a> <b> <maxit>`, where `a` and `b` define the interval, and `maxit` is maximal iteration counts allowed. Notice that the implementation of the bisection method must go **after** the `main` function.

Hints:

1. Regarding `func-choice`, you can use either string or integers, but you must let the user know what to input.

2. ϵ is a numerical tolerance, which should be small, say 1×10^{-6}.