Notes of Particle Physics, LIU Zuowei

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Contents

9 Symmetries and the Quark Model

Standard Model can be written in group theory
EM – U(1)
weak –
strong – SU(3)
EM, weak – SU(2)

9.1 Symm. in QM

$$\psi \to \psi' = \hat{U}\psi \tag{9.1}$$

$$\langle \psi \mid \psi \rangle = \langle \psi' \mid \psi' \rangle = \left\langle \psi \middle| \hat{U}^\dagger \hat{U} \middle| \psi \right\rangle \tag{9.2}$$

thus $\hat{U}$ must be unitary

$$\hat{U}^\dagger \hat{U} = I \tag{9.3}$$

...

$$[\widehat{\mathrm{H}}, \hat{U}] = 0 \tag{9.4}$$

A finite continuous symm operation can be built up from a series of infinitesimal transformations like

$$\widehat{\mathrm{U}} = I + \mathrm{i}\,\epsilon\,\widehat{\mathrm{G}} \tag{9.5}$$

$\widehat{\mathrm{G}}$: generator of the transformation 产生算符

$$\hat{U}^\dagger \hat{U} = I \;\Rightarrow\; \widehat{\mathrm{G}}^\dagger = \widehat{\mathrm{G}} \text{ (Hermitian)} \tag{9.6}$$

$$[\widehat{\mathrm{H}}, \widehat{\mathrm{G}}] = 0 \tag{9.7}$$

thus

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \widehat{\mathrm{G}} \right\rangle = \mathrm{i}\left\langle [\widehat{\mathrm{H}}, \widehat{\mathrm{G}}] \right\rangle = 0 \text{ (conserved)} \tag{9.8}$$

Noether's Theorem: Symmetry $\Rightarrow$ Conserved quantity
E.g.: translational symm. – momentum conserved

9.1.1 Finite Transformation

$$\widehat{\mathrm{U}}(\boldsymbol{\alpha}) = \mathrm{e}^{\mathrm{i}\,\boldsymbol{\alpha}\cdot\widehat{\mathrm{G}}} \tag{9.9}$$

9.2 Flavor Symmetry

Heisenberg: proton and neutron are 2 states of nucleon.
In isospin space

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{9.10}$$

total isospin $I = 1/2$
3rd component of isospin $I_3 = \pm 1/2$

9.2.1 Flavor Symm. of the Strong Interaction

flavor space

$$\mathrm{u} = \begin{pmatrix}1\\0\end{pmatrix} \quad \mathrm{d} = \begin{pmatrix}0\\1\end{pmatrix} \tag{9.11}$$

$$\begin{pmatrix}\mathrm{u}'\\ \mathrm{d}'\end{pmatrix} = \widehat{\mathrm{U}} \begin{pmatrix}\mathrm{u}\\ \mathrm{d}\end{pmatrix} \tag{9.12}$$

$$\widehat{\mathrm{U}} = \begin{pmatrix}U_1 & U_2\\ U_3 & U_4\end{pmatrix} \tag{9.13}$$

4 complex numbers – 8 real parameters
$\widehat{\mathrm{U}}^\dagger \widehat{\mathrm{U}} = I$ – 4 constraints
thus, 4 real parameters needed.

$$\widehat{\mathrm{U}} = \mathrm{e}^{\mathrm{i}\,\alpha_i\,\widehat{\mathrm{G}}_i} \quad i = 1,2,3,4 \tag{9.14}$$

one of these

$$\widehat{\mathrm{U}} = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \mathrm{e}^{\mathrm{i}\,\phi} \quad U(1) \text{ symm} \tag{9.15}$$

the remaining three – $SU(2)$ (special unitary, traceless)
a suitable choice: Pauli matrices

$$\sigma_1 = ... \tag{9.16}$$

def: isospin

$$\widehat{\mathbf{T}} = \frac{1}{2}\boldsymbol{\sigma} \tag{9.17}$$

9.2.2 Isospin Algebra

spin: $|l, m\rangle$
isospin: $|I, I_3\rangle$

$$\mathrm{u} = \begin{pmatrix}1\\0\end{pmatrix} = \left|\frac{1}{2}, \frac{1}{2}\right\rangle \quad \mathrm{d} = \begin{pmatrix}0\\1\end{pmatrix} = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle \tag{9.18}$$

9.3 Combining Quarks into Baryons

$2 \otimes 2 = 3 \oplus 1$
singlet

$$|0,0\rangle = \frac{1}{\sqrt{2}}(\mathrm{ud} - \mathrm{du}) \tag{9.19}$$

triplet

$$\begin{cases} |1,1\rangle = \mathrm{uu} \\ |1,0\rangle = \frac{1}{\sqrt{2}}(\mathrm{ud} + \mathrm{du}) \\ |1,-1\rangle = \mathrm{dd} \end{cases} \tag{9.20}$$

$2 \otimes 2 \otimes 2 = (3 \oplus 1) \otimes 2 = 4 \oplus 2 \oplus 2$
(using spin multiplicity)
actually $4 \oplus 2_S \oplus 2_A$, symm and anti-symm
$3 \otimes 2$

$1-\frac{1}{2} \leq I \leq 1+\frac{1}{2} \;\Rightarrow\; I=\frac{1}{2},\frac{3}{2}$

$$2I+1 \;\Rightarrow\; 4\oplus 2 \tag{9.21}$$

$$\begin{cases} |3/2,3/2\rangle = \\ |3/2,1/2\rangle = \\ |3/2,-1/2\rangle = \\ |3/2,-3/2\rangle = \end{cases} \tag{9.22}$$

$$\begin{cases} |1/2,1/2\rangle_S = \\ |1/2,-1/2\rangle_S = \end{cases} \tag{9.23}$$

$1 \otimes 2$

$$\begin{cases} |1/2,1/2\rangle_A = \\ |1/2,-1/2\rangle_A = \end{cases} \tag{9.24}$$

9.4 Ground State Baryon Wfn.

For proton, $p=$ uud, or neutron

$$\Psi = \eta_{\text{space}}\chi_{\text{spin}}\phi_{\text{flavor}}\xi_{\text{color}} \tag{9.25}$$

spin – 8, flavor – 8, color – 27
But QCD states:
ξ – anti-symm
η – $(-1)^l$, for ground state, symm
And Ψ is anti-symm
So, $\chi\phi$ is symm

$$\Psi = \frac{1}{\sqrt{2}}(\chi_S\phi_S + \phi_A\chi_A) \tag{9.26}$$

9.5 Isospin Representation of Anti-Quarks

A general SU(2) transformation $q \to q' = Uq$

$$\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix} \to \begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix} = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \end{pmatrix} \tag{9.27}$$

with $aa^*+bb^*=1$
take complex conjugation

$$\begin{pmatrix} \bar{\mathrm{u}} \\ \bar{\mathrm{d}} \end{pmatrix} = \begin{pmatrix} a^* & b^* \\ -b & a \end{pmatrix}\begin{pmatrix} \bar{\mathrm{u}} \\ \bar{\mathrm{d}} \end{pmatrix} \tag{9.28}$$

9.6 SU(3) Flavor Symmetry

$$\begin{pmatrix} \mathrm{u}' \\ \mathrm{d}' \\ \mathrm{s}' \end{pmatrix} = \widehat{\mathrm{U}}\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \\ \mathrm{s} \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ ... & & \\ ... & & \end{pmatrix}\begin{pmatrix} \mathrm{u} \\ \mathrm{d} \\ \mathrm{s} \end{pmatrix} \tag{9.29}$$

$$\widehat{\mathrm{U}} = \mathrm{e}^{\mathrm{i}\,\boldsymbol{\alpha}\cdot\widehat{\mathbf{T}}} \tag{9.30}$$

$$\widehat{\mathbf{T}}_i = \frac{1}{2}\boldsymbol{\lambda}_i \quad i=1,2,...,8 \tag{9.31}$$

$$\mathrm{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \mathrm{d} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad \mathrm{s} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{9.32}$$

expand SU(2) of u, d to SU(3)

$$\lambda_1 = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = ... \quad \lambda_3 = ... \tag{9.33}$$

expand SU(2) of u, s and d, s to SU(3)

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = ... \quad \lambda_X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{9.34}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \lambda_7 = ... \quad \lambda_Y = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{9.35}$$

However $\lambda_3, \lambda_X, \lambda_Y$ are not linear independent, thus we take

$$\lambda_8 = \frac{1}{\sqrt{3}}\lambda_X + \frac{1}{\sqrt{3}}\lambda_Y = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \tag{9.36}$$

those eight aka Gell-Mann matrices

9.6.1 SU(3) Flavor States

$$\widehat{\mathbf{T}}^2 = \frac{1}{4}\sum_{i=1}^{8} \lambda_i^2 = \tag{9.37}$$

observables: 3rd component of isospin and flavor hypercharge

$$\widehat{\mathbf{T}}_3 = \frac{1}{2}\lambda_3 \quad \hat{Y} = \frac{1}{\sqrt{3}}\lambda_8 \tag{9.38}$$

$$I_3 = n_u - n_d \quad Y = \frac{1}{3}(n_u + n_d - 2n_s) \tag{9.39}$$

states

$$\mathrm{u} = (1/2, 1/3) \quad \bar{\mathrm{u}} = (-1/2, -1/3) \tag{9.40}$$

$$\mathrm{d} = (-1/2, 1/3) \quad \bar{\mathrm{d}} = (1/2, -1/3) \tag{9.41}$$

$$\mathrm{s} = (0, -2/3) \quad \bar{\mathrm{s}} = (0, 2/3) \tag{9.42}$$

Meson ($q\bar{q}$):

$$\mathrm{u}\bar{\mathrm{u}} = (0,0) \quad \mathrm{u}\bar{\mathrm{d}} = (1,0)\mathrm{u}\bar{\mathrm{s}} = (1/2, 1) \tag{9.43}$$

$$\mathrm{d}\bar{\mathrm{u}} = ... \tag{9.44}$$

$$\mathrm{s}\,\bar{\mathrm{u}} = ... \tag{9.45}$$

9.6.2 The Light Mesons

Flavor symm is approximate.

10 Quantum Chromodynamics

10.1 The Local Gauge Principle

$$\psi(x) \to \psi'(x) = \widehat{\mathrm{U}}(x)\psi(x) = \mathrm{e}^{\mathrm{i}\,q\chi(x)}\,\psi(x) \tag{10.1}$$

$\chi(x)$ is dependent on x. It's local U(1) transformation.

$$\mathrm{i}\,\gamma^\mu\partial_\mu[\mathrm{e}^{\mathrm{i}\,q\chi(x)}\,\psi] = m[\mathrm{e}^{\mathrm{i}\,q\chi(x)}\,\psi] \tag{10.2}$$

$$\mathrm{i}\,\gamma^\mu\,\mathrm{e}^{\mathrm{i}\,q\chi(x)}[\partial_\mu\psi + \mathrm{i}\,q(\partial_\mu\chi)\psi] = \mathrm{e}^{\mathrm{i}\,q\chi(x)}\,m\psi \tag{10.3}$$

$$\mathrm{i}\,\gamma^\mu[\partial_\mu + \mathrm{i}\,q(\partial_\mu\chi)]\psi = m\psi \tag{10.4}$$

def: $D_\mu = \partial_\mu + \mathrm{i}\,qA_\mu$, gauge invariant derivative operator

$$\mathrm{i}\,\gamma^\mu[\partial_\mu + \mathrm{i}\,qA_\mu]\psi = m\psi \tag{10.5}$$

$$A_\mu \to A'_\mu = A_\mu - \partial_\mu\chi \tag{10.6}$$

10.1.1 From QED to QCD

	QED	QCD
symm	U(1)	SU(3)
gauge boson	photon A_μ	gluon $G^a_\mu \quad a = 1, 2, ..., 8$
generator	q	$T^a = \lambda^a/2$
charge	electric chg. $+/-$	color chg. $r/g/b$

elec/color neutral particles do not interact with photon/gluon. But photon has no elec chg, while gluon has color chg.
SU(3) local transf.

$$\psi(x) \to \psi'(x) = \mathrm{e}^{\mathrm{i}\,g_S\boldsymbol{\alpha}(x)\cdot\widehat{\mathbf{T}}}\,\psi(x) \tag{10.7}$$

Dirac Eq. becomes

$$\mathrm{i}\,\gamma^\mu[\partial_\mu + \mathrm{i}\,g_S(\partial_\mu\boldsymbol{\alpha})\cdot\widehat{\mathbf{T}}]\psi = m\psi \tag{10.8}$$

$$\mathrm{i}\,\gamma^\mu[\partial_\mu + \mathrm{i}\,g_S G^a_\mu T^a]\psi = m\psi \tag{10.9}$$

$$G^k_\mu \to (G^k_\mu)' = G^k_\mu - \partial_\mu\alpha_k - g_S f_{ijk}\alpha_i G^j_\mu \tag{10.10}$$

f_{ijk}: structure constant, def by

$$[\lambda_i, \lambda_j] = 2\,\mathrm{i}\,f_{ijk}\lambda_k \tag{10.11}$$

U(1) – Abelian
SU(3) – Non-Abelian, or Yang-Mils

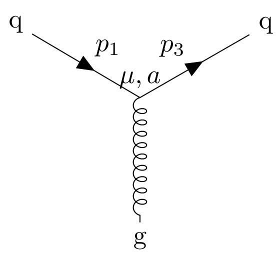
10.2 Color and QCD

10.2.1 Quark-gluon Vertex

$$(-\mathrm{i}\, g_S \gamma^\mu)\left(\frac{1}{2}\lambda^a\right) \quad \text{\# (spinor index)(color index)} \tag{10.12}$$

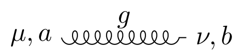
$$u(p) \to c_i u(p) \tag{10.13}$$

where $c_1, c_2, c_3 = r, g, b$



$$j_q^\mu = \bar{u}(p_3) c_j^\dagger \left(-\mathrm{i}\, g_S \gamma^\mu \frac{1}{2}\lambda^a\right) c_i u(p_1) \tag{10.14}$$

gluon propagator



$$-\mathrm{i}\,\frac{g_{\mu\nu}}{q^2}\delta^{ab} \tag{10.15}$$

10.3 Gluons

gluon – octet of colored states

$$r\bar{g}, g\bar{r}, r\bar{b}, b\bar{r}, g\bar{b}, b\bar{g}, \frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g}), \frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b}) \tag{10.16}$$

10.4 Color Confinement

gluon field

$$V(\mathbf{r}) \sim \kappa r \tag{10.17}$$

colored objects are always confined to color singlet states.

10.4.1 Color Confinement and Hadronic States

$3 \otimes \bar{3} = 8 \oplus 1$, the 8 are gluon states and the 1 is meson state (q q).
$3 \otimes 3 = 6 \oplus 3$, and hadrons must be color singlet, thus qq meson does not exist.

For baryon (q q q)

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1 \tag{10.18}$$

while q q q, q q q do not exist.
Other: antibaryon (q q q), pentaquark (q q q q q).

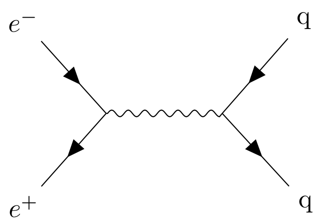
10.4.2 Hadronization and Jets

...

10.5 Running of α_S and Asymptotic Freedom(渐进自由)

... photo

10.6 QCD in e-p Annihilation



$$R = \frac{\sigma(\mathrm{e}^+\,\mathrm{e}^- \to \mathrm{hadrons})}{\sigma(\mathrm{e}^+\,\mathrm{e}^- \to \mu^+\,\mu^-)} = 3 \sum_{\text{flavors}} Q_q^2 \tag{10.19}$$

agree w/ data by error 10%(圈图修正？)

10.7

10.8

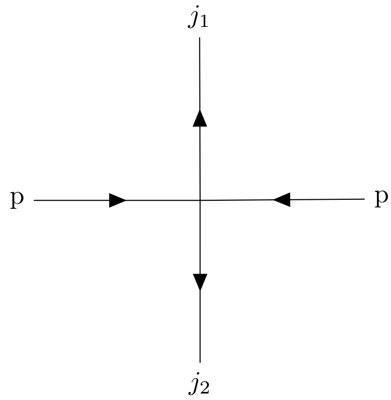
10.9 Hadron-hadron Collisions

elec collision – 10 GeV
hadron coll. – 13 TeV

10.9.1 Kinematics

Variables
e^-p elastic: θ_e
e^-p inelastic: θ_e, E_e
pp: x_1, x_2, Q^2

$p\bar{p} \to 2$ jets $+ X$
$\{x_1, x_2, Q^2\} \to \{\theta_1, \theta_2, P_T\}$



$$P_T = \sqrt{p_x^2 + p_y^2} \tag{10.20}$$

net longitudinal momentum

$$(x_1 - x_2)E_p \tag{10.21}$$

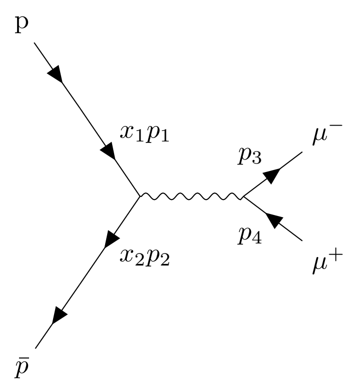
where E_p is the energy og the proton.
def rapidity

$$y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} \tag{10.22}$$

high-energy jets: $p_z =\approx E\cos\theta$

$$y \approx \frac{1}{2} \ln(...) = -\ln\tan\frac{\theta}{2} \equiv \eta \tag{10.23}$$

10.9.2 The Drell-Yan process

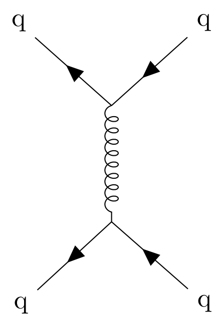


$$\mathrm{d}^2\sigma = ... \tag{10.24}$$

$$\mathrm{d}\sigma\, yM = \frac{8\pi\alpha^2}{9Ms} f\left(\frac{M}{\sqrt{s}}\,\mathrm{e}^{y}, \frac{M}{\sqrt{s}}\,\mathrm{e}^{-y}\right) \tag{10.25}$$

10.9.3 Jet Production at the LHC

parton level diagram



$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}Q^2} = \frac{4\pi\alpha_S^2}{9Q^4}\left[1 + \left(1 - \frac{Q^2}{\hat{s}}\right)^2\right] \tag{10.26}$$

hat means parton level

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}Q^2} g(x_1, x_2)\mathrm{d}x_1\mathrm{d}x_2 \tag{10.27}$$

homework Jun 5
10.6 10.7

11 The Weak Interaction

11.1

QED/QCD	weak
massless gauge boson	massive gauge boson
vector intrxn	vector & axial vector

11.2 Parity

parity transf.

$$\widehat{\mathbf{P}}\,\psi(\mathbf{x}, t) = \psi(-\mathbf{x}, t) \tag{11.1}$$

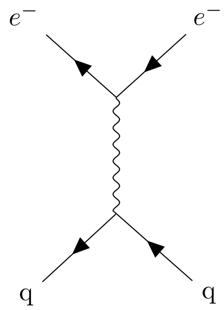
must be unitary and Hermitian.
and $P = \pm 1$.

11.2.1 Intrinsic Parity

(4.9) shows thr parity op. for Dirac spinors is γ^0.
For spin-half particles, $P = +1$; for anti-particles, $P = -1$.
For vector bosons responsible for the EM, strong, weak forces

$$P(\gamma) = P(\mathrm{g}) = P(\mathrm{W}^{\pm}) = P(\mathrm{Z}) = -1 \tag{11.2}$$

11.2.2 Parity Conservation in QED



$$-\,\mathrm{i}\,\mathcal{M} = \bar{u}(p_3)[\mathrm{i}\,eQ\gamma^\mu]u(p_1)\frac{-\,\mathrm{i}\,g_{\mu\nu}}{q^2}\bar{u}(p_4)[\mathrm{i}\,eQ_q\gamma_\nu]u(p_2) \tag{11.3}$$

$$\mathcal{M} = \frac{Q_q e^2}{q^2} j_e \cdot j_q \tag{11.4}$$

$$j_e^\mu = ... \tag{11.5}$$

Parity transf.

$$\widehat{\mathbf{P}}\,u = \gamma^0 u \tag{11.6}$$

$$\widehat{\mathbf{P}}\,\bar{u} = \widehat{\mathbf{P}}\,u^\dagger\gamma^0 = ... = \bar{u}\gamma^0??? \tag{11.7}$$

thus

$$\widehat{\mathbf{P}}\,j_e^0 = \bar{u}\gamma^0\gamma^0\gamma^0 u = j_e^0 \tag{11.8}$$

$$\widehat{\mathbf{P}}\,j_e^k = ... = -j_e^k \tag{11.9}$$

$$\widehat{\mathbf{P}}\,j_e \cdot j_q = \widehat{\mathbf{P}}(j_e^0 j_q^0 - j_e^k j_q^k) = j_e \cdot j_q \tag{11.10}$$

thus, parity is conserved in QED.
And also in QCD.
e.g.

$$\rho^0(1^-) \longrightarrow \pi^+(0^-) + \pi^-(0^-) \tag{11.11}$$

where 1^- denotes J^P.
ang momentum conservation $1 = 0 + 0 + \ell \Rightarrow \ell = 1$
parity consv. $(-1) = (-1)(-1)(-1)^\ell$ ✓

$$\eta^0(0^-) \longrightarrow \pi^+(0^-) + \pi^-(0^-) \tag{11.12}$$

where 1^- denotes J^P.
ang momentum conservation $0 = 0 + 0 + \ell \Rightarrow \ell = 0$
parity consv. $(-1) = (-1)(-1)(-1)^\ell$ ×

	e.g.	P
scalar		+1
vector	$\mathbf{x}, \mathbf{p}$	-1
pseudoscalar	$h = \mathbf{S} \cdot \mathbf{p}$	-1
axial vector	$\mathbf{L}, \mathbf{S}, \mathbf{B}, \boldsymbol{\mu}$	+1

11.2.3 Parity Violation in Nuclear β-decay

11.3 V-A Structure of the Weak Interaction

$$j^\mu \propto \bar{u}(g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) u = g_V j_V^\mu + g_A j_A^\mu \tag{11.13}$$

... parity violation $\propto \dfrac{g_V g_A}{g_V^2 + g_A^2}$
Experiment shows maximal parity violation: $|g_V| = |g_A|$
and it looks like $V - A$(V minus A)

$$j^\mu = g\bar{\psi}\gamma^\mu(1 - \gamma^5)\psi \tag{11.14}$$

Feynman rule

$$... \tag{11.15}$$

11.4 Chiral Structure of the Weak Interaction

$$P_R = ... \tag{11.16}$$

$$\bar{\psi}_{1L}\gamma^\mu\psi_{2R} = 0 = \bar{\psi}_{1R}\gamma^\mu\psi_{2L} \tag{11.17}$$

Only L particles participate in charged weak current.
... R anti-particles ...

$E >> m \;\; \Rightarrow$ chiral states $\approx$ helicity states

11.5 The W-boson Propagator

propagator of photon

$$\frac{-\,\mathrm{i}\, g_{\mu\nu}}{q^2} \tag{11.18}$$

of W-boson

$$\frac{-\,\mathrm{i}}{q^2 - m_W^2}\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right) \tag{11.19}$$

11.5.1 Fermi Theory

for low energy

$$\frac{\mathrm{i}\, g_{\mu\nu}}{m_W^2} \tag{11.20}$$

11.6 Helicity in Pion Decay

$\pi^\pm$, $J^P = 0^-$, formed from $\mathrm{u\bar{d}}/\mathrm{d\bar{u}}$.
decay:

$$\pi^- \to e^-\bar{\nu}_e \tag{11.21}$$

$$\pi^- \to \mu^-\bar{\nu}_\mu \tag{11.22}$$

$$\tag{11.23}$$