绪论

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成绩：

1. 平时20%
2. 作业30%
3. 考试50%

书：

- Mark Thomson "Mordern Particle Physics"
- David Griffiths "Introduction to Elementary Particles"
- Francis Halzen & Alan D, Martin "Quarks & leptons"

基本粒子以及它们之间的相互作用
  电子(e)+原子核(p,n)+ν_0
  重力，电磁力，强相互作用，弱相互作用
质子中子由quark组成
  p=u+u+d
  n=d+d+u
  $e^-/\nu_e/u/d$ <-1st generation
  $\mu^-/\nu_\mu/c/s$ <- 2ed generation
  $\tau^-/\mu_\tau/t/b$ <- 3rd
↑spin-1/2
  Q:-1,0,2/3,-1/3
  最重的粒子： top quark $m_t = 170GeV$, 质子,中子~1GeV
spin-1:

1. EM->photon
2. strong->gloun
3. weak->$W^\pm/Z^0$

Higgs: spin-0,m_H=125GeV
with m_Z=91GeV,m_W=80GeV called EW scale

Standard Model
  Effective theory
相互作用
  universal 3-point vertex
  费曼图
  EM:$\alpha = 1/137$
  strong:$\alpha_s \sim 1$
  weak:$\alpha_w \sim 1/30$
Feynman diagrams
Feynman rules
  $\sigma(e^-e^- \to e^-e^-)$
  微扰计算<=>$\alpha = 1/137$

Natural Units

S.I.: kg,m,s,...

$m_e = 9.1 * 10^-31kg$

$barn = 10^{-28}m^2$

[kg, m, s]->[GeV,\hbar,c]

$GeV = 10^9eV = 1.6 * 10^{-10}J$

$\hbar = 1.055 * 10^{-34}Js$

$c = 3 * 10^8m/s$

set $\hbar = c = 1$

E$^{2=p}$2+m^2

E,p,m->GeV

$T, L- > GeV^{-1}$

$\sigma- > GeV^{-2}$

e.g.$< r^2 >^{1/2} = 4.1GeV^{-1}$

$[L] = [E]^{-1}[c]^m[\hbar]^n$

量纲分析

$[RHS] = [E]^{-1}[L/T]^m[ET]^n$

$1 = m; 0 = -1 + n; 0 = -m + n;$

$m = 1, n = 1$

$< r^2 >^{1/2} = 4.1GeV^{-1}c\hbar = 0.8fm$

Special relativity

contravariant 4-vector

$X^\mu = (t, x, y, z)$

$\mu = 0, 1, 2, 3$

$X^{\mu'} = (t', x', y', z')$

$X^{\mu'} = \Lambda^\mu_\nu X^\nu$

$t^2 - x^2 - y^2 - z^2 = t'^2 - x'^2 - y'^2 - z'^2$

covariant 4-vector

$X_\mu = (t, -x, -y, -z)$

$X^\mu X_\mu = L.I.$(Lorantz Invariant)

$X^\mu X_\mu = X'^\mu X'_\mu$ (in Eistein summation convention)

contaction on Lorentz index

$A^\mu B_\mu = L.I.$(for 4-vector)

$p^\mu = (E, p)$

$p^\mu p_\mu = m^2$

$E^2 - p^2 = m^2$

$\partial_\mu = \frac{\partial}{\partial X^\mu} = (\partial_t, \partial_x, \partial_y, \partial_z)$

$\partial^\mu(\partial_t, -\partial_x, -\partial_y, -\partial_z)$

$p^\mu = i\partial^mus$

$\partial^\mu\partial_\mu = \Box = \partial_t^2 - \nabla^2$

Mandelstam variables

1+2->3+4

$p_1^\mu, p_2^\mu \to p_3^\mu, p_4^\mu$

$p_1 + p_2 = p_3 + p_4$

$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$

$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$

$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$
[img]

散射截面
$10^{-12} GeV^{-2} = 10^{-12} GeV^{-2} (c\hbar)^2$
$= (\text{带入}) = 4 * 10^{-40} cm^2 = 4 * 10^{-16} barn = 0.4 fb$

Lorentz invariant quantity
4-vector quantity
X^μ, X_μ
metric tensor
$g_{\mu\nu}$
$A \cdot B = g_{\mu\nu} A^\mu B^\mu$

Fermi's Golden Rule
$H_0 \phi_k = E_k \phi_k$
$< \phi_i | \phi_j >= \delta_{ij}$
Schrodinger Eq. $i d\psi/dt = [H_0 + H']\psi$
$\phi(x,t) = \sum_k c_k(t) \phi_k(x) e^{-iE_k t}$
$LHS = i \sum_k (c_k'(t) \phi_k(x) e^{-iE_k t} + c_k(t) \phi_k(x)(-iE_k) e^{-iE_k t})$
$RHS = \sum_k (c_k(t) E_k \phi_k(x) e^{-iE_k t} + c_k(t) H' \phi_k(x) e^{-iE_k t})$
$i \sum_k (c_k'(t) \phi_k(x) e^{-iE_k t}) = \sum_k (c_k(t) H' \phi_k(x) e^{-iE_k t})$
Assumption:
$t = 0, |i >= \phi_i => c_k(0) = \delta_{ik}$
$H' \approx const. \geq 1$
$c_i(t) \approx 1, c_{k \neq i} \approx 0$
Then:
$i \sum_k (c_k'(t) \phi_k(x) e^{-iE_k t}) \approx H' \phi_i e^{-iE_i t}$
<f|\times:
$i \sum_k (c_k'(t) \delta_{fk} e^{-iE_k t}) =< f|H'|i > e^{-iE_i t}$
$i c_f'(t) e^{-iE_f t} =< f|H'|i > e^{-iE_i t}$
$dc_f(t)/dt = -i < f|H'|i > e^{i(E_f - E_i)t}$
Transition matrix element
$T_{fi} =< f|H'|i >= \int_v d^3x \phi_f^*(x) H' \phi_i(x)$
@t=T:
$c_f(T) = \int_0^T dc_f(t)/dt dt$
$= \int_0^T dt - iT_{fi} e^{i(E_f - E_i)t}$
$= -iT_{fi} \int_0^T e^{i(E_f - E_i)t} dt$
Probility:
$P_{fi} = |c_f(T)|^2 = T_{fi} T_{fi}^* \int_0^T \int_0^T e^{i(E_f - E_i)t} e^{-i(E_f - E_i)t'} dt dt'$
Transition rate
$d\Gamma_{fi} = P_{fi}/T = |T_{fi}|^2 / T \int_{-T/2}^{t/2} \int_{-T/2}^{t/2} e^{i(E_f - E_i)t} e^{-i(E_f - E_i)t'} dt dt'$
$= |T_{fi}|^2 \text{sinc}^2(x) T, x = (E_f - E_i)T/2$
$\int_\infty^\infty sinc^2 x dx = \pi, when |x| < 3, I$(the double int) is significant
in N.I.$x = (E_f - E_i)T/2\hbar, \Delta E \leq 4\hbar/T \approx 10^{-15} eV$
$E_f \approx E_i$ <-energy is conserved
delta function
$\int_{-\infty}^\infty e^{i(k-k_0)x} dx = 2\pi\delta(k - k_0)$

$d\Gamma_{fi}$

$= |T_{fi}|^2 \lim_{t\to\infty} 1/T \int_{-T/2}^{T/2}\int_{-T/2}^{T/2} e^{i(E_f-E_i)t}e^{-i(E_f-E_i)t'}dtdt'$

$= |T_{fi}|^2 \lim_{t\to\infty} 1/T \int_{-T/2}^{T/2} e^{i(E_f-E_i)t}\delta(E_f-E_i)dt$

dn final state in $E_f->E_f+dE_f$

Total transition rate

$\Gamma_{fi} = \int dE_f dn/dE_f d\Gamma_{fi}$

$= 2\pi \int |T_{fi}|^2 dn/dE_f \delta(E_f-E_i) \lim t\to\infty (1/t\int_{-T/2}^{T/2} dt) dE_f$

$= 2\pi \int |T_{fi}|^2 dn/dE_f \delta(E_f-E_i)dE_f$

$= 2\pi |T_{fi}|^2 dn/dE_f|_{E_i}$

$= 2\pi |T_{fi}|^2 \rho(E_i)$

ρ:desity of states

a->1+2

1st-order:

$T_{fi} =< \psi_1\psi_2|H'|\psi_a >$

Born Approx:

Plane wave in a box with length a

$A^2 = 1/a^3, \phi = Ae^{ipx-Et}$

周期性边界条件$(p_x,p_y,p_z) = 2\pi/a(n_x,n_y,n_z)$

$d^3p = (2\pi/a)^3 d^3n = d^3n(2\pi)^3/V$

$dn = d^3n = V/(2\pi)^3 d^3p = 4\pi V/(2\pi)^3 p^2 dp$

$dn/dE = dn/dp|dp/dE| = \frac{4\pi p^2}{(2\pi)^3} V|dp/dE|$

set V=1 as it will vanish

$dn = d^3p/(2\pi)^3$

N,final state particles:

$dn = \Pi_{i=1}^{N-1} d^3p_i/(2\pi)^3$

(phase space)

$dn = \Pi_{i=1}^{N} d^3p_i/(2\pi)^3 \delta^3(p_a - \sum_{i=1}^N p_i)(2\pi)^3$

$\int_{V=1} d^3x|\psi'|^2 = 2E \leftarrow 2E$ particles in unit volume

$\psi = \sqrt{2E}\psi$

$\Gamma_{fi} = 2\pi \int |T_{fi}|^2 \delta E_a - E_1 - E_2 dn$

$= 2\pi \int \frac{|\sqrt{2E_A 2E_1 2E_2} T_{fi}|^2}{2E_A 2E_1 2E_2} \delta E_a - E_1 - E_2 \Pi_{i=1}^N d^3p_i/(2\pi)^3 \delta^3(p_a - \sum_{i=1}^N p_i)2\pi)^3$

$= (2\pi)^4/(2E_a)\int |<\psi_1'\psi_2'|H'|\psi_a'>|^2 \delta^4 p_a - p_1 - p_2 d^3p_1/(2E_1(2\pi^3))d^3p_2/(2E_2(2\pi)^3)$

everthing in the integral in Lorentz invariant, but E_a is not L.I. soΓ_{fi} is not L.I.

$\Gamma_j = \partial$ decay rates

N initial particles

$\delta N = -N\Gamma_1\delta t - N\Gamma_2\delta t...$

$= -N(\sum_j \Gamma_j)\delta t = -N\Gamma\delta t$

Γ:total decay rate

$N(t) = N(0)e^{-\Gamma t} = N(0)e^{-t/\tau}$

$\tau = 1/\Gamma$

BR=branching ratio

$BR(j) = \Gamma(j)/\Gamma$

a->1+2
$\Gamma_{fi} = 1/(2E_a)\int |M_{fi}|^2 (2\pi)^4 \delta^4(p_a - p_1 - p_2) d^3p_1/((2\pi)^3 2E_1) d^3p_2/((2\pi)^3 2E_2)$
τ $1/\Gamma$ E_a
静止参考系寿命最短~τ
$\int_{-\infty}^{\infty} \delta(f(x))g(x)dx = \sum_{i=1}^{N} g(x)/|f'(x_i)| where f(x_i) = 0$
Rest frame of a

- $p_a = (m_a, 0)$
- $\int \Delta(p_1, p_2)\delta^3(0 - p_1 - p_2)d^3p_2 = \delta(p_1, -p_1)$
- $\Gamma = 1/(2m_a)\int |M|^2 (2\pi)^4 \delta(m_a - E_1 - E_2)/((2\pi)^6 4E_1E_2)d^3p_1$
- $g(p) = |M(p)|^2 (2\pi)^4/((2\pi)^6 4E_1(p)E_2(p))p^2$
- $f(p) = m_a - \sqrt{p^2 + m_1^2} - \sqrt{p^2 + m_2^2}$
  - $f(p) = 0 : p^* = 1/\sqrt{2m_a}\sqrt{(m_a^2 - (m_1 + m_2)^2)(m_a^2 - (m_1 - m_2)^2)}$
  - $note : p^* = |p_1^{CM}|$
- $\Gamma = 1/(2m_a)\int |M(p^*)|^2/(2\pi)^2 \frac{(p^*)^2}{4E_1(p^*)E_2(p^*)}/|p^*/E_1(p^*) + p^*/E_2(p^*)|d\Omega$
  - $= \frac{1}{32\pi^2 m_a^2} p^* \int |M|^2 d\Omega$

Interaction cross section

Interaction rate

- per target paricle:$\Gamma = nv\sigma = \sigma\phi_a$
  - $\phi_a(flux) = dN_a/dAdt = ndAvdt/dAdt$
  - $\delta N_b = n_b A v \delta t$
  - $\delta p = \delta N_b \sigma/A = n_b \sigma v \delta t$
  - $\Gamma = n_b \sigma v$
- total interaction rate
  - $\Gamma(n_a V) = n_a n_b v V \sigma$
- crose section $\sigma = \frac{\text{total interaction rate}}{\text{flux}\times\#\text{ of target particles}}$

a+b->1+2
$rate = n_a n_b V(v_a + v_b)\sigma$
$NR : n_a = n_b = 1/V(setV = 1)$
$\Gamma = \sigma(v_a + v_b)$
$\sigma = Gamma_{fi}/(v_a + v_b)$
$\Gamma_{fi} = 1/(2E_aE_b)\int |M_{fi}|^2 (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2)d^3p_1/((2\pi)^3 2E_1)d^3p_2/((2\pi)^3 2E_2)$
$\sigma = 1/(2E_aE_b(v_a + v_b))\int |M_{fi}|^2 (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2)d^3p_1/((2\pi)^3 2E_1)d^3p_2/((2\pi)^3 2E_2)$

- $Flux = (2E_aE_b(v_a + v_b)) = 4\sqrt{(p_ap_b)^2 - m_a^2m_b^2} is L.I.$

$\sigma = 1/(64\pi^2 s)|p_f^*|/|p_i^*|\int |M|^2 d\Omega$
where $s = (p_1 + p_2)^2$

differential xsec
$\sigma = \int d\sigma/d\Omega d\Omega$
$d\sigma/d\Omega = 1/(64\pi^2 s)|p_f^*|/|p_i^*||M|^2$

e+p=e+p(1+2=3+4)
In lab:p_2=0；1,3间夹角与CoM不同
CoM:1,3间夹角\theta
$t = (p_1 - p_3)^2 L.I.$
CoM:
$t = p_1^2 + p_3^2 - 2p_1p_3 = m_1^2 + m_3^2 - 2(E_1E_3 - P_1 \cdot P_3)$
$= m_1^2 + m_3^2 - 2E_1E_3 + 2|P_1||P_3|\cos\theta$
$dt = 2|P_1||P_3|d(\cos\theta)$
$d\Omega = d\cos\theta d\phi = \frac{dtd\phi}{2|P_1||P_3|}$
$d\sigma/d\Omega = 2|P_1||P_3|d\sigma/(dtd\phi) = 1/(64\pi^2 s)|p_f^*|/|p_i^*||M|^2$
$d\sigma/dt = \frac{1}{64\pi s}\frac{1}{|P_i^*|}|M|^2$ this is L.I.
$|p_i^*|^2 = 1/(4s)(s-(m_1+m_2)^2)(s-(m_1-m_2)^2)$
Lab:
$m_e << m_p << E$
$p_1 \approx (E_1, 0, 0, E_1)$
$p_2 \approx (m_p, 0, 0, 0)$
$p_3 \approx (E_3, 0, E_3\sin\theta, E_3\cos\theta)$
$p_4 \approx (E_4, P_4)$
$|p_i^*|^2 = (s - m_p^2)^2/4s$
$s = (p_1 + p_2)^2 = 2E_1m_p + m_p^2$
$|p_i^*|^2 = E_1^2m_p^2/s$
$t = -2E_1E_3(1 - \cos\theta)$
notice E_3 is relavent with \theta in lab
$t = (p_2 - p_4)^2 = 2m_p(m_p - E_4) = 2m_p(E_3 - E_1)$
$E_3 = m_pE_1/(m_p + E_1(1 - \cos\theta))$
$dt = 2m_pdE_3$
$d\sigma/dt = \sigma/2m_pdE_3 = d\sigma/d\cos\theta|d\cos\theta/dE_3|1/(2m_p)$
$(d\sigma/d\Omega)_{lab} = 1/(64\pi^2)(E_3/(E_1m_p))^2|M_{fi}|^2$

4 Dirac Equation

Schodinger Eq.(NR)
$E = p^2/2m, p = \nabla/i$
Klein-Gordon Eq.
$E^2 = p^2 + m^2$
$-\partial_t^2 = -\nabla^2 + m^2$
$\partial_t^2 - \nabla^2 + m^2\psi = 0$
Negetive energy
Dirac
$E\psi = (\alpha \cdot p + \beta m)\psi$
$i\partial_t\psi = (\alpha \cdot \nabla/i + \beta m)\psi$
$E^2\psi = (i\partial_t)(i\partial_t)\psi$
$= (i\partial_t)(-i\alpha \cdot \nabla + \beta m)\psi$
$= (-i\alpha \cdot \nabla + \beta m)(i\partial_t)\psi$
$= (-i\alpha \cdot \nabla + \beta m)(-i\alpha \cdot \nabla + \beta m)\psi$
$= (-i\alpha_a\partial_a + \beta m)(-i\alpha_b\partial_b + \beta m)\psi$

$= (-\alpha_a \partial_a \alpha_b \partial_b - i\alpha_a \partial_a \beta m - i\beta m \alpha_b \partial_b + \beta^2 m^2)\psi$
$= (-\alpha_a \alpha_b \partial_a \partial_b - im\partial_a(\alpha_a \beta + \beta\alpha_a) + \beta^2 m^2)\psi$
$(p^2 + m^2)\psi = (-\partial_a^2 + m^2)\psi$
$E^2 = p^2 + m^2$
$\beta^2 = 1, \beta^\dagger = \beta$
$\alpha_a \beta + \beta\alpha_a = 0$
$\alpha_a^2 = 1, \alpha_a^\dagger = \alpha_a$
$\alpha_a \alpha_b + \alpha_b \alpha_a = 0, a \neq b$
last come from:
$\alpha_a \alpha_b = \delta_{ab}$
$(\alpha_a \alpha_b \partial_a \partial_b + \alpha_a \alpha_b \partial_a \partial_b)/2$
$= 1/2(\alpha_a \alpha_b + \alpha_b \alpha_a)\partial_a \partial_b$
1.$\mathrm{tr}(\alpha_a) = \mathrm{tr}(\beta) = 0$
Ex.$\mathrm{tr}(\alpha_a) = \mathrm{tr}(\alpha_a \beta^2) = \mathrm{tr}(\beta\alpha_a \beta) = -\mathrm{tr}(\alpha_a \beta^2) = -\mathrm{tr}(\alpha_a)$
2.Eigenvalue$= \pm 1$
Ex.$\alpha_a x = \lambda x, \alpha_a \alpha_a x = \alpha_a \lambda x,$
$\alpha_a^2 = \lambda^2 x, \lambda^2 = 1$
3.$\alpha_a \& \beta$ are even dimension
4.$\alpha_a^\dagger = \alpha_a, \beta^\dagger = \beta$
$\alpha \& \beta$ 4*4 matrix
$H_D = \alpha \cdot p + \beta m$

Dirac spinor:
$\psi = (\psi_1, \psi_2, \psi_3, \psi_4)$

abstract algebra
explicit representation

Dirac-Pauli representation:
$\beta = ((1,0),(0,-1)), \alpha_i = ((0,\sigma_i),(\sigma_i,0))$

Probability desity
①$i\partial_t \psi = (\alpha \cdot p + \beta m)\psi = (\alpha_a \partial_a / i + \beta m)\psi$
②$-i\partial_t \psi^\dagger = \psi^\dagger(\alpha^\dagger \cdot p^\dagger + \beta^\dagger m) = (i\partial_a \psi^\dagger \alpha_a + \psi^\dagger \beta m)$
$\psi^\dagger$① $-$ ②ψ :
$i\psi^\dagger \partial_t \psi + (\partial_t \psi^\dagger)\psi = 1/i\psi^\dagger \alpha_a \partial_a \psi + 1/i(\partial_a \psi^\dagger)\alpha_a \psi$
$i\partial_t(\psi^\dagger \psi) = 1/i\partial_a(\psi^\dagger \alpha_a \psi)$
$\partial_t(\psi^\dagger \psi) + \partial_a(\psi^\dagger \alpha_a \psi) = 0$
$\partial_t \rho + \nabla \cdot J = 0$
$\rho = \psi^\dagger \psi \geq 0$

$H = p^2/2m, L = r \times p$
$[H, L] = [p_a^2/2m, e_i \epsilon_{ijk} r_j p_k]$
$= \epsilon_{ijk} p_k e_i / 2m[p_a^2, r_j]$
$[p_a^2, r_j] = p_a[p_a, r_j] + [p_a, r_j]p_a$
$= p_a \delta_{aj}/i + \delta_{aj}/ip_a$
$= 2p_j/i$
$[H, L] = \epsilon_{ijk} p_k e_i / 2m 2p_j / i$
$= 0$
$[L_a, L_b] = i\epsilon_{abc} L_c$

For H_D?(Not)

$[H_D, L] = [\alpha \cdot p + \beta m, r \times p]$
$= [\alpha_a \cdot p_a, e_i \epsilon_{ijk} r_j p_k]$
$= \alpha_a e_i \epsilon_{ijk} p_k [p_a, r_j]$
$= \alpha_a e_i \epsilon_{ijk} p_k \delta_{aj}/i$
$= 1/i \alpha_a e_i \epsilon_{ijk} \alpha_j p_k$
$= \alpha \times p/i \neq 0$

Construct another operator

$S = 1/2\Sigma = 1/2((\sigma, 0), (0, \sigma))$
since pauli:
$[\sigma_a, \sigma_b] = 2i\epsilon_{abc}\sigma_c$
$[H_D, S] = [\alpha \cdot p + \beta m, 1/2\Sigma]$
$= 1/2 p_a e_b [\alpha_a, \Sigma_b] + m/2 e_b [\beta, \Sigma_b]$
$[\alpha_a, \Sigma_b] = \alpha_a \Sigma_b - \Sigma_b \alpha_a$
(in DP Rep.)$= ((0, \sigma_a), (\sigma_a, 0))((\sigma_b, 0), (0, \sigma_b)) - ((\sigma_b, 0), (0, \sigma_b))((0, \sigma_a), (\sigma_a, 0))$
$= ((0, [\sigma_a, \sigma_b]), ([\sigma_a, \sigma_b], 0))$
$= 2i\epsilon_{abc}\alpha_c$
$[\beta, \Sigma_b] = 0$
$[H_D, S] = ip_a e_b \epsilon_{abc} \alpha_c = i\alpha \times p$
$[S_a, S_b] = i\epsilon_{abc} S_c$

$[H_D, L + S] = 0$
$J = L + S$

$S^2 = 1/4\Sigma^2 = 3/4I$
$S^2|sm >= s(s+1)|sm >$
$s(s+1) = 3/4$
$s = 1/2$

Magnatic moment

$E = -\mu \cdot B$
$E\psi = (\alpha \cdot p + \beta m)\psi$
minial substitution $A^\mu = (\phi, A)$
$E- > E - q\phi$
$p- > p - qA$
$p^\mu = p^\mu - qA^\mu$
$(E - q\phi)\psi = (\alpha \cdot (p - qA) + \beta m)\psi$
spin-1/2 in EM
$((-m + E - q\phi, -\sigma \cdot (p - qA)), (-\sigma \cdot (p - qA), m + E - q\phi))((\psi_a), (\psi_b)) = 0$
i.e.
$(E - q\phi - m)\psi_A = \sigma \cdot (p - qA)\psi_B$
$(E - q\phi + m)\psi_B = \sigma \cdot (p - qA)\psi_A$
$NR. E\, m >> q\phi$
$2m\psi_B = \sigma \cdot (p - qA)\psi_A$
$\psi_B = \sigma \cdot (p - qA)/2m\psi_A$
$(E - q\phi - m)\psi_A = \sigma \cdot (p - qA)/2m\sigma \cdot (p - qA)\psi_A$

$(\sigma\cdot a)(\sigma\cdot b)=\sigma_i a_i \sigma_j b_j$
$\quad = a_i b_j(\delta_{ij}+i\epsilon_{ijk}\sigma_k)$
$\quad = a\cdot b+i\sigma\cdot(a\times b)$

$(E-q\phi-m)\psi_A=1/2m((p-qA)^2+i\sigma\cdot((p-qA)\times(p-qA)))\psi_A$
where
$\quad (p-qA)\times(p-qA))=-qp\times A-qA\times p$
$(E-q\phi-m)\psi_A=1/2m((p-qA)^2-q\sigma\cdot(\nabla\times A+A\times\nabla))\psi_A$
where
$\quad (\nabla\times A+A\times\nabla)\psi_A=(\nabla\times A)\psi_A=B\psi_A$
$E\psi_A=(m+1/2m(p-qA)^2+q\phi-q/2m\sigma\cdot B)\psi_A$
Magnetic moment
$\quad \mu=\frac{q}{2m}\sigma=\frac{q}{m}S$
$\mu_L=q/2mL$

$\mu=g\frac{q}{2m}S,\text{Dirac Eq}:g=2$

1948:
Schwinger:
$a=(g-2)/2=\alpha/2\pi$

$(i\partial_t+i\alpha\cdot\nabla-\beta m)\psi=0$
$(i\beta\partial_t+i\beta\alpha\cdot\nabla-m)\psi=0$
$\gamma=\beta\alpha$
$\gamma^\mu=(\beta,\beta\alpha)$
$\partial_\mu=(\partial_t,\nabla)=\partial/\partial x^\mu$
note it is not 4-vector
$(i\gamma_0\partial_t+i\gamma\cdot\nabla-m)\psi=0$
$(i\gamma^\mu\partial_\mu-m)\psi=0$

Solutions to the Dirac equation
plane wave solution:
$\quad \psi(x,t)=u(E,p)e^{i(p\cdot x-Et)}=u(p^\mu)e^{-ip\cdot x}$
u is also a spinor
$p_\mu=i(\partial_t-\nabla)=i\partial_\mu$
$\quad \partial_\mu\psi=u(p^\mu)\partial_\mu e^{-ip\cdot x}$
$\qquad =u(p^\mu)e^{-ip\cdot x}\partial_\mu(-ip_\mu x^\mu)$
$\qquad =u(p^\mu)e^{-ip\cdot x}(-ip_\mu)$
$(\gamma^\mu p_\mu-m)u(p^\mu)e^{-ip\cdot x}=0$
$(\gamma^\mu p_\mu-m)u(p^\mu)=0$可看作u-spinor的Dirac-Eq

静止$p=0$
$(\gamma^0E-m)u=0$
Dirac-Pauli rep:
$E\,\mathrm{diag}(1,1,-1,-1)\psi=m\psi$
$u_1=N(1,0,0,0),E=m,S=1/2$
$u_2=N(0,1,0,0),E=m,S=-1/2$
$u_3=N(0,0,1,0),E=-m,S=1/2$
$u_4=N(0,0,0,1),E=-m,S=-1/2$

$S = 1/2\Sigma = 1/2((\sigma, 0), (0, \sigma))$
$S_z = 1/2\mathrm{diag}(1, -1, 1, -1)$

$p \neq 0$
$((E - m, \sigma \cdot p), (\sigma \cdot p, -E - m))((u_A), (u_B)) = 0$
$(E - m)u_A = \sigma \cdot pu_B$
$\sigma \cdot pu_A = (E + m)u_B$
2 choices for $u_A : u_A = (1, 0); u_A = (0, 1)$
$u_B = \sigma \cdot p/(E + M)u_A = 1/(E + m)((p_z, p_x - ip_y), (p_x + ip_y, -pz))u_A$
$u_1 = N1(1, 0, p_z/(E + m), (p_x + ip_y)/(E + m))$
$u_2 = N2(0, 1, (p_x - ip_y)/(E + m), -p_z/(E + m))$
same, choose $u_B = (1, 0); u_B = (0, 1)$
$u_3 = N3(p_z/(E - m), (p_x + ip_y)/(E - m), 1, 0)$
$u_4 = N4((p_x - ip_y)/(E - m), -p_z/(E - m), 0, 1)$

Dirac Sea

Feynman-Stuekelberg
$E < 0:$
particle propagate backward in time:$E < 0, p$
<=>anti-particle propagate forward in time:$E_{ap} = -E > 0, p_{ap} = -p$

$\psi = u(p)e^{-iEt + ip \cdot x}$
$\psi = v(p)e^{iEt - ip \cdot x}$
$(\gamma^\mu p_\mu + m)v = 0$
$v_1 = N1((p_x - ip_y)/(E + m), -p_z/(E + m), 0, 1), E = m, S^v = 1/2$
$v_2 = N2(p_z/(E + m), (p_x + ip_y)/(E + m), 1, 0), E = m, S^v = -1/2$
$v_3 = N3(1, 0, p_z/(E - m), (p_x + ip_y)/(E - m)), E = -m, S^v = 1/2$
$v_4 = N4(0, 1, (p_x - ip_y)/(E - m), -p_z/(E - m)), E = -m, S^v = -1/2$

一般选择两组正能解1,2
$N = \sqrt(E + m)$
$\rho = \psi^2 = 2E$

$E = i\partial_t$
$E\psi_u = i\partial_t(ue^{-ipx}) = E\psi$
$E\psi_v = i\partial_t(ve^{ipx}) = -E\psi$
$E^v = -i\partial_t$
$p^v = i\nabla$
$[H_D, L + S] = 0, L^v = r \times p^v$
$S^v = -S$
$u_1, u_2, v_1, v_2$are not eigenstates of S_z
onlt if $p = p_z e_z$,they are S_z,eigenstates

Helicity(本质是自旋) not LI
$h = S \cdot p/|p|$
$[H_D, S_z] \neq 0$
$[H_D, h] = 0$

RH:h=1/2;LH:h=-1/2
$hu = \lambda u$
$1/2p\ \mathrm{diag}(\sigma \cdot p, \sigma \cdot p)u = \lambda u$
$\sigma \cdot p\sigma \cdot pu_A = (2p\lambda)^2 u_A$
$p^2 = (2p\lambda)^2$
$\lambda = \pm 1/2$
meanwhile
$(\gamma^\mu p_\mu - m)u = 0$
$\sigma \cdot pu_A = (E+m)u_B$
$\sigma \cdot pu_A = 2p\lambda u_A$
$p = (psc, pss, pc)$球坐标
$u_A = (a, b)$
$u_\uparrow = (\cos\theta/2, \sin\theta/2e^{i\phi}, p/(E+m)\cos\theta/2, p/(E+m)\sin\theta/2e^{i\phi})$

Charge conjugation
$p_\mu - > p_\mu - qA_\mu$
$\gamma^\mu(\partial_\mu + ieQ_e A_\mu)\psi + im\psi = 0$
$e = |e|, Q_e = -1$
$(-i\gamma^2)(\gamma^{\mu *}(\partial_\mu - ieQ_e A_\mu)\psi^* - im\psi^*) = 0$
$\gamma^2\gamma^{\mu *} = -\gamma^\mu\gamma^2$
$\gamma^\mu(\partial_\mu - ieQ_e A_\mu)\psi' + im\psi' = 0$
$\psi' = i\gamma^2\psi^*$
$\gamma^\mu(\partial_\mu + ie(-Q_e)A_\mu)\psi' + im\psi' = 0$
so its a same mass opposite charge:$\psi' = i\gamma^2\psi^*$
i.e.
$i\gamma^2 u_1^* = v_1$
intrinsic parity:P
$t' = t$
$x' = -x$
$P^2 = 1$
$(i\gamma^0\partial_t + i\gamma^1\partial_i - m)\psi = 0$
$\psi'(x', t) = P\psi(x, t)$
$(i\gamma^0\partial_t' + i\gamma^1\partial_i' - m)\psi' = 0$
$\psi = P\psi'$
$(i\gamma^0\partial_t + i\gamma^1\partial_i - m)P\psi' = 0$
$\gamma^0(i\gamma^0\partial_t' - i\gamma^i\partial_i' - m)p\psi' = 0$
$(i\gamma^0\partial_t' + i\gamma^i\partial_i' - m)(\gamma^0 P\psi') = 0$
$\psi' = \gamma^0 P\psi'$
take $P = \gamma^0$
$\psi' = \gamma^0\psi$

Chapter 5 Interaction by particke exchange

Fermi's golden rule $i- > f$
Transition rate $\Gamma_{fi} 2\pi |T_{ij}|^2 \rho$
Transition matrix element
$T_{fi} =< f|V|i > + \sum_{j\neq i} \frac{<f|V|j><j|V|i>}{E_i - E_j} + \dots$

a+b->c+d

$e^- + p \to e^- + p$

$|i>=|a+b>$

$|j>=|c+x+b> or |a+x+d>$

$|f>=|c+d>$

$T^{ab}_{ij} = (<d|V|b+x><c+x|V|a>)/(E_a+E_b-(E_c+E_x+E_b))$

NE matrix element => L.I. matrix element

$V_{ji} = M_{ji}\Pi_k 1/\sqrt{2E_k}$

$M = g$ <- couplong constant for a scalar interaction e.g. Higgs

$T^{ab}_{ij} = g^2/(E_a - E_c - E_x\sqrt{2E_a 2E_b 2E_c 2E_d (2E_x)^2})z$

for L.I.:

$M^{ab}_{fi} = 1/(2E_x)g^2/(E_a - E_c - E_x)$

$M^{ba}_{fi} = 1/(2E_x)g^2/(E_b - E_d - E_x)$

 with $E_a + E_b = E_c + E_d$

$M_{fi} = M^{ab}_{fi} + M^{ba}_{fi}$

 $= g^2/(2E_x)(1/(E_a - E_c - E_x) + 1/(E_c - E_a - E_x))$

$p_a = p_c + p_x$

$E_x^2 = p_x^2 + m_x^2$ <- on mass shell (or on shell)

 $= (p_a - p_c)^2 + m^2$ (this is valid for both path)

$M_{fi} = g^2/(2E_x)\frac{-2E_x}{E_x^2-(E_a-E_c)^2}$

 $= -g^2/(E_x^2 - (E_a - E_c)^2)$

 $= g^2/((E_a - E_c)^2 - (p_a - p_c)^2 - m^2)$

 $= g^2/(p_a - p_c)^2 - m_x^2$ (四矢量,定义$q = p_a - p_c$, and assign "q" to x)

 $= g^2/(q^2 - m_x^2)$

notice that $q^2 \neq m^2$ so q for x is off-shell for time-ordered diagrams

Feynman diagram

 $q = p_a - p_c$ is a 4-momentum of x

 E & p are conserved

 $q^2 \neq m_x^2$ is off shell "virtual particle"

$1/(q^2 - m_x^2)$<- propagator

QED=quantum eletrodynamics

Dirac eq.+minal substitution

 $\partial\mu \to \partial_\mu + iqA_\mu$

 $(\gamma^\mu\partial_\mu + iq\gamma^\mu A_\mu + im)\psi = 0$

$i\partial_t\psi = H\psi$

$H = m\gamma^0 - i\gamma^0\gamma\cdot\nabla + q\gamma^0\gamma^\mu A_\mu$

$V_D = q\gamma^0\gamma^\mu A_\mu$ <- interaction between photon & fermion

$M(e^-e^- -> e^-e^-)\ \sum_\lambda <4|V_D|2><3|V_D|1>/q^2$

$photon: A_\mu = \epsilon^\lambda_\mu e^{i(p\cdot x - Et)}$

 λ mean LH,RH

$<3|V_D|1>\ \int d^3x u_3^\dagger e^{i\cdots}\ldots$ book P123

 $u_3^\dagger eQ_Q\gamma^0\gamma^\mu\epsilon^\lambda_\mu u_1$

$<4|V_D|2>\ u_4^\dagger eQ_Q\gamma^0\gamma^\mu\epsilon^{\lambda *}_\mu u_2$

$\sum_\lambda \epsilon_m u^\lambda\epsilon^{\lambda *}_\nu = -g_{\mu\nu}$

$M(e^-e^- \to e^-e^-) = -(eQ_e\bar{u}_3\gamma^\mu u_1)g_{\mu\nu}q^2(eQ_e\bar{u}_4\gamma^\nu u_2)$

Feynman Rules for QED
initial state fermion: $u(p)$ ->·
final state fermion: $\bar{u}(p)$ ·->
initial state anti-fermion: $\bar{v}(p)$ <-·
final state anti-fermion: $v(p)$ ·<-
initial state photon: $\epsilon_\mu(p)$ ~·γ
final state photon: $\epsilon_\mu(p)^*$ ·~γ
photon propagateor: $-ig_{\mu\nu}q^2$ ·~·
fermion propagator: $-\frac{i\gamma^\mu p_\mu+m}{q^2-m^2}$ ·--·
QED vertex: $-ieQ\gamma^\mu \mid \sim$

Ex.$e^-e^- \to e^-e^-$
against arrow
$-iM = u\bar{u}_3(-ieQ\gamma^\mu)u_1(-ig_{\mu\nu}q^2)\bar{u}_4(-ieQ\gamma^\nu)u_2$

Ex.$e^+e^- \to \mu^+\mu^-$
  $\mu =$muon=2nd generation charged leptons
  $e^-(p_1), e^+(p_2), \mu^-(p_3), \mu^+(p_4)$
  arrow on line = partical flow/"charge flow"
$-iM = \bar{v}_2(-ieQ\gamma^\mu)u_1(-ig_{\mu\nu}q^2)\bar{u}_3(-ieQ\gamma^\nu)v_4$
  $= ie^2\bar{v}_1\gamma^\mu u_1\bar{u}_3\gamma_\mu u_4/q^2$
  where $q = p_1 + p_2$

6.e^-e^+ annihilation

Leading Order diagram
  tree diagram
  $M \sim e^2 \sim \alpha$
Next Leading Order diagram
  loop diagram
  $M \sim e^4 \sim \alpha^2$
NNLO....
E.G. P129
$M_{tot} = M_{LO} + M_{NLO} + M_{NNLO} + ...$
$|M_{tot}|^2 = |M_{LO}|^2 + |M_{NLO}| + M_{LO}M^*_{NLO} + M^*_{LO}M_{NLO} + ...$
  $\alpha^2 + \alpha^4 + \alpha^3$
$\alpha = 1/137$
$\therefore M_{tot} = M_{LO}$
$-iM = \bar{v}_2(-ieQ\gamma^\mu)u_1(-ig_{\mu\nu}q^2)\bar{u}_3(-ieQ\gamma^\nu)v_4$
  $= ie^2\bar{v}_1\gamma^\mu u_1\bar{u}_3\gamma_\mu u_4/q^2$

Possible helicity states
$e^-_\uparrow e^+_\uparrow, ...$
$|M(e^-_\uparrow e^+_\uparrow -> \mu^+\mu^-)|^2 = \sum_{\mu^+\mu^- \text{spins}} |M(e^-_\uparrow e^+_\uparrow -> \mu^+\mu^)|$
unpolarized(average over initial states)
$\overline{|}M|^2 = 1/4\sum_{e^+e^- \text{spins}} |M(e^-e^+ -> \mu^+\mu^)|^2$

$\sqrt{s} >> m_\mu$
$p_1 = (E, 0, 0, E)$
$p_2 = (E, 0, 0, -E)$
$p_3 = (E, E\sin\theta, 0, E\cos\theta)$
$p_4 = (E, -E\sin\theta, 0, -E\cos\theta)$
$|\bar{M}|^2 = e^4(1+\cos^2\theta)$
$d\sigma/d\Omega = 1/(64\pi^2 s)e^4(1+\cos^2\theta) = \alpha^2/(4s)(1+\cos^2\theta)$
$\sigma = 4\pi\alpha^2/(3s)$
total spin$=\pm 1$ along $z =>$contribute helicity amplitude
initial state $e^- e^+$

$|s, s_z >= |1, 1 > e^-_\uparrow e^+_\downarrow$

$|s, s_z >= |1, -1 >$

final state

$|s, s_n >= |1, 1 >, |1, -1 >$

$M(e^-_\uparrow e^+_\downarrow -> \mu^-_\uparrow \mu^+_\downarrow) \approx_\theta < 1, 1|1, 1 >_z \ \ (1+\cos\theta)$

Chirality(手征)
$\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$
In Pauli-Dirac Rep
$\gamma_5 = ((0, 1), (1, 0))$
$\gamma_5^2 = 1$
$\gamma_5^\dagger = \gamma_5$
$\gamma_5\gamma^\mu = -\gamma^\mu\gamma_5$
chiral states: γ_5 eigenstates

LH,RH chiral states

$\gamma u_R = u_R$
$\gamma u_L = -u_L$
$\gamma v_R = -v_R$
$\gamma v_L = v_L$
antipartical $S^v = -S$
chiral states are identical to massless helicity states

$uR = (c, se, c, se)$

$uL = (-s, ce, s, -ce)$

$vR = (s, -ce, -s, ce)$

$vL = (c, se, c, se)$

high energy limit ~ massless limit

chiral projection operators

$P_R = 1/2(1+\gamma_5)$

$P_L = 1/2(1-\gamma_5)$

$P_R + P_L = 1$

$P_R P_R = P_R; P_L P_L = P_L$

$P_L P_R = 0$

$P_R U_R = U_R,$
$P_R U_L = P_R V_R = 0,$
$P_R V_L = V_L$
similar for P_L
chirality in QED
$\bar{\psi}\gamma^\mu\psi = (\bar{\psi}_L + \bar{\psi}_R)\gamma^\mu(\psi_L + \psi_R)$

consider cross terms:
$\bar{\psi}\gamma^\mu\psi = (P_L\psi)^\dagger\gamma^0\gamma\mu P_R\psi$
$= \psi^\dagger P_L\gamma^0\gamma^\mu P_R\psi$
$= \psi^\dagger\gamma^0\gamma^\mu P_L P_R\psi(Hermit)$
$= 0$
$\bar{v}_{2R}\gamma^m uu_{1L}$
$\bar{v}_{2L}\gamma^m uu_{1R}$

e-p elastic sacttering & deep inelastic sacttering

λ:wavelength of virtual photon
r_p:proton radius
1.$\lambda >> r_p$, elastic, very low E, 1/r potential
2.$\lambda\, r_p$, elastic, form factors: charge & magnetic
3.$\lambda < r_p$, quark, inelastic
4.$\lambda << r_p$

2 physics obserbables:
$\theta \& E_3$
for elastic: $E_3 = E_3(\theta), d\sigma/d\cos\theta$
inelastic: $d\sigma/(d\cos\theta dE_3)$

elastic:
1.Rutherford: NR e-
2.Mott: e- is relativistic, neglect proton recoil
Lab frame:
$d\sigma/d\Omega = 1/(64\pi^2)(1/(m_p + E - 1(1-\cos\theta)))^2|\bar{M}|^2$
$-iM = \bar{u}_3(-ieQ_e\gamma^\mu)u(p_1)(-ig_{\mu\nu}/q^2)\bar{u}_4(-ieQ_p\gamma^\mu)u(p_2)$
$Q_e = -1, Q_p = 1$
$|\bar{M}|^2 = 1/4\sum_{helicity} M^*M$
$M = e^2/q^2\bar{u}_3\gamma^\mu u_1\bar{u}_4\gamma_\nu u_2$
helicity states
$u_{up} = N(c, sei\phi, kc, ksei\phi)$
$u_{down} = N(-s, cei\phi, ks, -kcei\phi)$
$N = \sqrt{E+m_e}, c = \cos\theta/2$
$k = p/(E+m_e) = \beta\gamma/(\gamma+1)$
$NR: K->0; R: K\,1$
k is the same for elastic scattering
initial $e-: (\theta,\phi) = (0,0)$
final $e-: (\theta,\phi) = (\theta,0)$
$M = e^2/q^2 j_e^\mu j_\nu^p$
$j^e_{upup} = (E+m_e)((1+k^2)c, 2ks, 2iks, 2kc)$
$j^e_{downdown} = (E+m_e)((1+k^2)c, 2ks, -2iks, 2kc)$
$j^e_{downup} = (E+m_e)((1-k^2)s, 0, 0, 0)$
$j^e_{updown} = (E+m_e)(-(1-k^2)s, 0, 0, 0)$
$ifk\,1, j^e_{downup} = j^e_{updown} = 0$

helicity is effectively conserved at verticea in high-E limits
initial $p : \theta = 0, \phi = 0, k = 0$
final $p : \theta = \eta, \phi = 0, k\ 0$
$j^p_{upup} = -j^p_{downdown} = 2mp(c_n, 0, 0, 0)$
$j^p_{downup} = j^p_{updown} = 2mp(-s\eta, 0, 0, 0)$
p is NR
$|\bar{M}|^2 = 1/4 \sum_{helicity} M^* M$
$= e^2/q^2(E_e + m_e)^2(2m_p)^2((1 - k_e^2)^2 + 4c^2k^2)$
$= 16e^4 m_e^2 m_p^2/q^4(1 + \beta^2\gamma^2 \cos^2\theta/2)$
$E_e = \gamma m_e$
$q^4 = (p_1 - p_3)^4 = 16\rho^4 \sin^4\theta/2$
$|\bar{M}|^2 = e^4 m_e^2 m_p^2/(p^4 \sin^4\theta/2)(1 + \beta^2\gamma^2 \cos^2\theta/2)$
1.e- is NR, Rutherford, $(1 + \beta^2\gamma^2 \cos^2\theta/2) \propto 1$
$d\sigma/d\Omega = 1/(64\pi^2)\frac{1}{m_p^2}\frac{e^4 m_e^2 m_p^2}{p^4 \sin^4\theta/2}$
$= \frac{\alpha^2}{4} 1/(p^4/m_e^2) 1/\sin^4\theta/2$
$= \frac{\alpha^2}{16} 1/E_k^2 1/\sin^4\theta/2$
p only Q_p contribution
2.e- is R, Mott,$(1 + \beta^2\gamma^2 \cos^2\theta/2)\ \beta^2\gamma^2 \cos^2\theta/2$
$d\sigma/d\Omega = 1/(64\pi^2)\frac{1}{m_p^2}\frac{e^4 m_e^2 m_p^2}{p^4 \sin^4\theta/2}\beta^2\gamma^2 \cos^2\theta/2$
$p_e = \gamma\beta m_e \approx E_1$
$(d\sigma/d\Omega) = \alpha^2/(4E_1^2 \sin^4(\theta/2)) \cos^2(\theta/2)$

Form factor
charge distribution
$Q\rho(r)$
$\int \rho(r) dr^3 = 1$
Potential $V(r) = \int Q\rho(r')/(4\pi|r - r'|)d^3r'$
Born Appro:
initial: $\psi_i = e^{ip_i \cdot r - Et}$
final: $\psi_f = e^{ip_f \cdot r - Et}$
LO matrix: $M =< \psi_f|V|\psi_i >$
$M = \int d^3r e^{-ip_3\cdot r} e^{ip_1 \cdot r} V(r)$
$= \int d^3r e^{iq\cdot r} \int d^3r' \frac{Q\rho(r')}{4\pi|r - r'|}$
$R = r - r', q = p_1 - p_3$
$M = \int d^3R \frac{Q}{4\pi|R|} e^{iq\cdot R} \int d^3r' \rho(r') e^{iq\cdot r'}$
=(pointlike particle matirx)($F(q^2)$ Form factor)
Form factor = Fourier transform of the density distribution
$(d\sigma/d\Omega) = \alpha^2/(4E_1^2 \sin^4(\theta/2)) \cos^2(\theta/2)|F(q^2)|^2$
$q < 1/r_p\ \lambda > r_p$
1.$F(0) = 1$
2.$F(q^2) < 1$
3.$F(Q^2 \to \infty) = 0$

Relativistic e-p scattering
$p_1 = (E_1, 0, 0, E_1)$
$p_2 = (m_p, 0, 0, 0)$
$p_3 = (E_3, 0, E_3 s, E_3, c)$
$p_e = (E_4, p_4)$
$tracetechnique- > |\bar{M}|^2$
$q^2 = (p_1 - p_3)^2 = 2m_e^2 - 2(E_1 E_3 (1-c))$
$= -2(E_1 E_3 (1-c)) < 0$
dim $Q^2 = -q^2$
Relativistic $e^- p$ scattering xsec
$(d\sigma/d\Omega) = \frac{\alpha^2}{(4E_1^2 \sin^4(\theta/2))} \frac{E_3}{E_1} (\cos^2 \theta/2 + Q^2/2m_p^2 \sin^2(\theta/2))$
=(normal)(energy loss)(charge+spin-spin interaction)
when $Q^2 << 2m_p^2, E_3 \ E_1, (d\sigma/d\Omega)_{Mott}$
$E_3 = \frac{E_1 m_p}{m_p + E_1(1-\cos\theta)}$
$Q^2 = 2E_1 E_3 (1-\cos\theta)$
Rosenbluth formula
$(d\sigma/d\Omega) = \frac{\alpha^2}{(4E_1^2 \sin^4(\theta/2))} \frac{E_3}{E_1} (\frac{G_E^2(Q^2)+\tau G_M(Q^2)}{1+\tau} \cos^2 \theta/2 + 2\tau G_M^2(Q^2) \sin^2(\theta/2))$
$\tau = Q^2/(4m_p^2)$
$@lowQ^2$
$G_E(Q^2) \ G_E(q^2) = \int e^{iq\cdot r} \rho(r) d^3 r$
$G_M(Q^2) \ G_M(q^2) = \int e^{iq\cdot r} \mu(r) d^3 r$
$G_E(0) = 1$
$G_M(0) = 2.79$
$\frac{(d\sigma/d\Omega)}{(d\sigma/d\Omega)_{Mott}} = (\frac{G_E^2(Q^2)+\tau G_M(Q^2)}{1+\tau} + 2\tau G_M^2(Q^2) \tan^2(\theta/2))$
1.low $Q^2 : \ G_E^2$
2.hihg $Q^2 : \ (1 + 2\tau \tan^2 \theta/2) G_M^2$
$Q^2 = \frac{2m_p E_1 (1-\cos\theta)}{\ldots}$

Chapter8 Deep Inelastic Scattering(DIS)

reconstruct an invariant mass
$W^2 = p_4^2, p_4$ is sum of particle produced from proton
$W^2 = m_p^2$ for elastic scaterring
need 2 variables which can be selected from L.I.ones:
$W, Q^2 = -q^2, x = Q^2/(2p_2 q), y = (p_2 q)/(p_1 p_2), \nu = p_2 q/m_p$
$Q^2 = -(m_e^2 - 2(E_1 E_3 - p_1 p_3))$
$= 2E_1 E_3 (1-\cos\theta)$
Bjorken X
$X = Q^2/(2p_2 q)$
$W = (p_2 + q)^2$
$2p_2 q = W - m_p^2 + Q^2$
$X = Q^2/(Q^2 + W^2 - m_p^2)$
since there will be heavy produced: $W > m_p$, so $0 < X < 1$
elastic:$X = 1$

$y=(p_2q)/(p_1p_2)$
evalute in proton rest frame
$p_2=(m_p,0,0,0)$
$y=(E_1-E_3)/E_1$
$\nu=E_1-E_3$

kinematics can be described by any 2 of X,Q^2,y,ν except $y\&\nu$

DESY:
$E_1=4.879Gev$
$\theta=10^\circ$
$d^2\sigma/(d\Omega dE_3)\leftrightarrow E$
P200 Figure8.2
$W^2=(p_2+q)^2=m_p^2+2p_2(p_1-p_3)+(p_1-p_3)^2$
$=m_p^2+2m_p(E_1-E_3)+2E_1E_3(1-\cos\theta)$
$=(m_p^2+m_pE_1)-2(m_p+E_1(1-\cos\theta))E_3$
can calculate the pike in the figure, $W=m_p,1.2GeV,2.24,3.85$
resonance->excited states of proton
$FWHM=\Gamma=1/\tau,\tau=$lifetime
Figure 8.3

Rosenbluth formula(elastic)
$(d\sigma/d\Omega)=\frac{\alpha^2}{(4E_1^2\sin^4(\theta/2))}\frac{E_3}{E_1}(\frac{G_E^2(Q^2)+\tau G_M(Q^2)}{1+\tau}\cos^2\theta/2+2\tau G_M^2(Q^2)\sin^2(\theta/2))$
$(d\sigma/d\Omega)=\frac{4\pi\alpha^2}{Q^4}(\frac{G_E^2(Q^2)+\tau G_M(Q^2)}{1+\tau}(1-y-m_p^2y^2/Q^2)+1/2y^2G_M^2)$
$f_2(Q^2)=\frac{G_E^2(Q^2)+\tau G_M(Q^2)}{1+\tau},f_1(Q^2)=G_M^2$

Structure function(inelastic)
$d^2\sigma/(dxdQ^2)=4\pi\alpha^2/Q^4((1-y-m_p^2y^2/Q^2)F(x,Q^2)/x+y^2F_1(x,Q^2))$
$m_p^2y^2/Q^2<<1(DIS)$

SLAC $5GeV<E_1<20GeV$
2 striking features:
1.Bjorken scaling
$F_1(x,Q^2)->F_1(X)$
$F_2(x,Q^2)->F_2(X)$
almost independent of Q^2
->point like contituents with proton
2.Collan-Gross relation
$F_2(X)-2XF_1(X)$
constituents are Dirac particles

Feynman:partons 部分子 ~ quark
旁观者spectator
ep scattering cross section=(parton distribution functions)(eq scatering section)
PDF:parton distribution functions
eq:
$d\sigma/d\Omega=Q_q^2e^4/(8\pi^2s)\frac{1+1/4(1+\cos^2\theta)}{(1-\cos\theta)^2}$
in CoM
$d\sigma/dq=Q_q^2e^4/(8\pi q^4)(1+(1+q^2/s)^2)$

Infinite momentum frame
$E_p >> m_p$
$p_2 = (E_2, 0, 0, E_2)$
$p_q = \xi p_2 = (\xi E_2, 0, 0, \xi E_2)$
where ξ is momentum fraction

8.2,3,6,8

$m_q^2 = (\xi p_2 + q)^2$
  $= m_q^2 + 2\xi p_2 q + q^2$
so $\xi = X$

ep system:
  $s = 2p_1 p_2$
eq system:
  $p_q = xp_2$
  $s_q = Xs$
$X = Q^2/(2p_2 q)$
$y_q = \frac{p_q q}{p_q p_1} = y$
$X_q = 1 \leftarrow$elastic
$s_q = XS$
$y_q = y$
$X_q = 1$

Parton Distribution Functions(PDFs)
$u(x)dx =$# of up quark in x->dx in p
$d(x)dx =$# of down quark in x->dx in p
$d\sigma/dQ^2 = \sum_q (q(x)dx)(\frac{4\pi\alpha^2}{Q^4}(1-y+y^2/2)Q_q^2)$
$d^2\sigma/(dxdQ^2) = \frac{4\pi\alpha^2}{Q^4}(1-y+y^2/2)\sum_q Q_q^2 q(x)$
compare to ep:
$d^2\sigma/(dxdQ^2) = 4\pi\alpha^2/Q^4((1-y-m_p^2 y^2/Q^2)F(x,Q^2)/x + y^2 F_1(x,Q^2))$
quark-parton model predicts
$F_2(x,Q^2)/x = \sum_q Q_q^2 q(x) = 2F_1(x,Q^2)$
1.Bjorken scaling=>$F_1 \& F_2$ have no Q^2-dep
2.Collan-Gross:$F_2 = 2XF_1$

PDFs cannot be computed in QFT, have to be extracted from exp

ep casttering:
  $F_2 = X\sum_q Q_q^2 q(x)$
  $= X(4/9(u(x) + \bar{u}(x)) + 1/9(d(x) + \bar{d}(x)))$
valence quark
p=uud
n=udd
sea quarks

$F_2^{en} = X(4/9(d(x) + \bar{d}(x)) + 1/9(u(x) + \bar{u}(x)))$
iso spin summmetry
$u(x) = u^p(x) = d^n(x)$
$d(x) = d^p(X) = u^n(x)$

$\int_0^1 dxF_2^{ep}(X) = \int_0^1 dxX(4/9(u(x) + \bar{u}(x)) + 1/9(d(x) + \bar{d}(x)))$
$= 4/9f_u + 1/9f_d$
$\int_0^1 dxF_2^{en}(X) = 4/9f_d + 1/9f_u$

SLAS:
$f_u = 0.36$
$f_d = 0.18$
so
1.f_u~$2f_d$
2.$f_u + f_d$~50% ->gluon

$u(x) = u_v + u_s$
take into account sea quarks
$F^{ep} = X(4/9u_v + 1/9d_v + 10/9s)$
$F^{en} = X(4/9d_v + 1/9u_v + 10/9s)$
$R = F^{ep}/F^{en} = \frac{4u_v+d_v+10s}{4d_v+u_v+10s}$
1.X->0,sea quark dominate, R->1
2.X->,valence quark dominate,$R_{naive} = 3/2$, but $E_{exp} - > 4$
because pauli's rule, $X \to 1, d_v$ will disapear

$Q^2\ 2 * 10^4 GeV^2, \lambda\ 10^-18m$, Bjorken still fitts well
有效理论

SM in Group theory
EM:U(1)
weak:SU(2)
strong:SU(3)

Chapter 9 Symmetries & the quark model

Symmetry: invariance under some transformation
wavefunction: ψ
transformation:$\psi' = U\psi$
$< \psi'|\psi' >= 1, = U^\dagger U = 1$
unitary transformation
symmetry:$[H, U] = 0$
Infinitesimal transformation
$U(\epsilon) = 1 + i\epsilon G$
ϵ:infinitesimal parameter
G:generator of the transformation
unitary condition
$1 = (1 - i\epsilon G^\dagger)(1 + i\epsilon G)$
$= 1 + i\epsilon(G - G^\dagger)$
$G = G^\dagger$ (Hermitian)

symmetry->unitary transformation with Hermitian generators
$[H, U] = 0 - > [H, G] = 0$
$d < G > /dt = i < [H, G] >= 0 - >< G >$ conserved
Noether's theorem: symmetry-> conservation

translational invariance
$x' = x + \epsilon$
$\psi(x) - > \psi'(x') = \psi(x+\epsilon) = \psi(x) + \partial\psi/\partial x \epsilon + \ldots$
$\quad = (i + i\epsilon 1/i\partial/\partial x)\psi$
generator $G = 1/i\partial/\partial x.$
$< G >$ is conserved. And it is known as momentum conservation

Finite transformation
$U(\alpha) = \lim_{n\to\infty}(1 + i/n\alpha \cdot G)^n = exp(i\alpha \cdot G)$

isospin(同位旋)
p & n : nucleon
consider p&n as nucleon's two states
in isospin space
$p = (1, 0)$
$n = (0, 1)$
p&n isospin doublet with
I=total isospin=1/2
I_3=3rd comp'=$\pm 1/2$
Isospin $\neq$ spin
both isospin & spin obey SU(2) algebra

Flavor symmetry
flavor symmery between up & down quarks
in flavor space
$u = (1, 0)$
$d = (0, 1)$
$(u', d') = ((U_{11}, U_{12}), (U_{21}, U_{22}))(u, d)$
U:2*2 matrix complex, with 8 real variables
since $U^\dagger U = 1$ have 4 constrains, so there is 4 dof

$U = \exp(i\alpha_a G_a), a = 1, 2, 3, 4$
α_a: real
G_a:linearly indepedent 2*2 matrices(generator)
$G = ((1, 0), (0, 1)) => U = ((1, 0), (0, 1))e^{i\alpha}$
$U(i)$ transform
  SU=special unitary (det U=1)
remove this U(1),the remaining 3 generators=>SU(2)
Pauli matrix (traceless)
$\sigma_1 = ((0, 1), (1, 0))$
$\sigma_2 = ((0, -i), (i, 0))$
$\sigma_3 = ((1, 0), (0, -1))$
$G => T + 1/2\sigma$ (isospin)

orbit angular momentum |l m>
spin:$|s, s_z >= |1/2, \pm 1/2 >$
isospin:$|I, I_3 >$
flavor space:
  $u = (1, 0) = |1/2, 1/2 >$
  $d = (0, 1) = |1/2, -1/2 >$

$T^2\phi(I,I_3) = I(I+1)\phi(I,I_3)$
$T_3\phi(I,I_3) = I_3\phi(I,I_3)$

Isospin ladder operator
$T_\pm = T_1 \pm iT_2$
$T_\pm\phi(I,I_3) = \sqrt{I(I+1) - I_3(I_3 \pm 1)}\phi(I,I_3 \pm 1)$

4 possible combinations of 2 isospin doublets
isospin triplet: I=1
isospin singlet: I=0
2*2=3+1
triplet:(symmetry)
$\phi(1,1) = uu$
$\phi(1,0) = 1/sqrt2(ud + du)$
$\phi(1,-1) = dd$
singlet:(anti-symmetry)
$\phi(0,0) = 1/sqrt2(ud - du)$

Isospin states with 3 quarks
$2 * 2 * 2 = (3+1) * 2 = 4 + 2_S + 2_A$
$I_{tot} = I_1 + I_2$
$I_{tot}^2 = I_1^2 + I_2^2$
$|I_1 - I_3| \leq I \leq I_1 + I_3$

ex:$3 * 2$
$I_1 = 1, I_2 = 1/2$
$I_{tot} \in (1/2, 3/2)$
$2 * 1/2 + 1 = 2$
$2 * 3/2 + 1 = 4$

$\phi(3/2, 3/2) = uuu$
$\phi(3/2, 1/2) = 1/\sqrt{3}(duu + udu + uud)$
$\phi(3/2, 1/2) = 1/\sqrt{3}(udd + dud + ddu)$
$\phi(3/2, -3/2) = ddd$
2_S :
$\phi_S(1/2, 1/2) = 1/\sqrt{6}(udd + dud - 2ddu)$
$\phi_s(1/2, -1/2) == 1/\sqrt{6}(2uud - udu - duu)$
2_A :
$\phi_A(1/2, 1/2) = 1/\sqrt{2}(ud - du)u$
$\phi_A(1/2, -1/2) = 1/\sqrt{2}(ud - du)d$

proton: p=uud
is a bond state of 3 quarks
proton wave function:
$\psi = \eta_{space} X_{spin} \phi_{flavor} \xi_{color}$
~()(8)(8)()
in $\phi(I, I_3)$,replacing u/d with up/down $=> X_{spin}$
$X_s = (1/2, 1/2) = 1/\sqrt{4}$ (2upupdown-updownup-downupup)
QCD:ξ_{color} is antisymmetric
$\eta_{space} is (-1)^l$

ϕ is antisymmetric with exchangeing any two quarks
$X\phi$ is symetric

$\phi = 1/\sqrt{2}(\phi_S x_S + \phi_A X_a)$
  is symmetric w.r.t. any 2 quarks
p&n wavefunction

SU(2):special unitary
4 dof

$q' = (u', d') = ((a, b), (-b^*, a^*))(u, d)$
$aa^* + bb^* = 0$
charge conjugation: $\psi' = C\psi = i\gamma^2\psi^*$
define $\bar{u} = i\gamma^2 u^*$
1.complex conjugation:
$(u'^*, d'^*) = ((a^*, b^*), (-b, a))(u^*, d^*)$
2.times $i\gamma^2$
$(\bar{u}', \bar{d}') = ((a^*, b^*), (-b, a))(\bar{u}, \bar{d})$
define $\bar{q} = (-\bar{d}, \bar{u})$
$(-\bar{d}', \bar{u}') = ((a, b), (-b^*, a^*))(-\bar{d}, \bar{u})$
$\bar{q}' = U\bar{q}$

SU(3) flavor symmetry
$(u', d', s') = U(u, d, s)$
(18-9 constraints($UU^\dagger = 1$) =9 dof)-1=8
SU(3)group-> 8 generators
$U = exp(i\alpha^a T^a)$
generator:$T^a = 1/2\lambda^a$
acton flavor space:
$u = (1, 0, 0), d = (0, 1, 0), s = (0, 0, 1)$
ud
$\lambda_1 = ((0, 1, 0), (1, 0, 0), (0, 0, 0))$
$\lambda_2 = ((0, -i, 0), (i, 0, 0), (0, 0, 0))$
$\lambda_3 = ((1, 0, 0), (0, -1, 0), (0, 0, 0))$
us
$\lambda_4 = ((0, 0, 1), (0, 0, 0), (1, 0, 0))$
$\lambda_5 = ((0, 0, -i), (0, 0, 0), (i, 0, 0))$
$\lambda_x = ((1, 0, 0), (1, 0, 0), (0, 0, -1))$
ds
$\lambda_6 = ((0, 0, 0), (0, 0, 1), (0, 1, 0))$
$\lambda_7 = ((0, 0, 0), (0, 0, -i), (0, i, 0))$
$\lambda_y = ((0, 0, 0), (0, 1, 0), (0, 0, -1))$
$\lambda_x - \lambda_3 = \lambda_y$
$\lambda_8 = (\lambda_x + \lambda_y)/\sqrt{3}$
  $= 1/\sqrt{3}((1, 0, 0), (0, 1, 0), (0, 0, -2))$
Gell-Mann matrices

$SU(2):$
$T^2 = \sum T_i^2$
T_3
$SU(3):$
$T^2 = \sum_i^8 T_i^2$
$\quad = 1/4(\sum_i^8 \lambda_i^2)$
$\quad = 4/3 I_{3*3}$
$T_3 = \lambda_3/2$
$Y = \lambda_8/\sqrt{3}$ →flavor hypercharge
flavor:
$T_3 = 1/2(n_u - n_d)$
$Y = 1/3(n_u + n_d - 2n_s)$

meson=$q\bar{q}$
(I_3, Y)
$u = (1/2, 1/3)\bar{u} = (-1/2, -1/3)$
$d = (-1/2, 1/3)\bar{d} = (1/2, -1/3)$
$s = (0, -2/3)\bar{s} = (0, 2/3)$
$u\bar{u} = (0,0)$
$u\bar{d} = (1,0)$
$u\bar{s} = (1/2, 1)$
$d\bar{s} = (-1, 0)$
$d\bar{d} = (0,0)$
$d\bar{s} = (-1/2, 1)$
$s\bar{s} = (-1/2, -1)$
$s\bar{s} = (1/2, -1)$
$s\bar{s} = (0,0)$

$T_\pm = 1/2(\lambda + 1 \pm \lambda_2)$
$V_\pm = 1/2(\lambda + 4 \pm \lambda_5)$
$U_\pm = 1/2(\lambda + 6 \pm \lambda_7)$

$3 \times \bar{3} = 8 + 1$
$|\psi_s >= 1/\sqrt{3}(u\bar{u} + d\bar{d} + s\bar{s})$

flavor symmetry is approximate

Chapter10 QCD

gauge invarieance
$E\&B \to \phi\&A$
E & B do not change under the following gauge transformation
$\phi' = \phi - \partial f/\partial t$
$A' = A - \nabla f$

gauge symmetry:
  the invariance of physics undera local phase tranf
  $\phi'(x) = U(x)\phi(x) = e^{iqf(x)}\phi(x)$
local: f(x) is a function of x
global: f(x)=constant

Free dirac eq is not gauge invariant
$i\gamma^\mu \partial_\mu \psi = m\psi$
$i\gamma^\mu \partial_\mu e^{iqf(x)}\psi = me^{iqf(x)}\psi$
$i\gamma^\mu (e^{iqf(x)}\partial_\mu(iqf(x))\psi + e^{iqf(x)}\partial_\mu \psi) = me^{iqf(x)}\psi$
$i\gamma^\mu (\partial_\mu + iq\partial_\mu f(x))\psi = m\psi$

take
$i\gamma^\mu (\partial_\mu + iqA_\mu)\psi = m\psi$
D_μ: covariant derivative
$A_\mu = (\phi - A)$
$A'_\mu = A_\mu - \partial_\mu f(x)$
$i\gamma^\mu (\partial_\mu + iq(A - \partial_\mu f(x)))e^{iqf(x)}\psi = me^{iqf(x)}\psi$
$i\gamma^\mu (iqA_\mu + \partial_\mu)\psi = m\psi$ <-gauge invariance

q is charge, invariance

$QED : U(1), A_\mu, Q$
$QCD : SU(3), G^a_\mu, T^a = \lambda^a/2$

SU(3) transformation
$\psi' = exp(ig_s\alpha(x) \cdot T)\psi$
$T^a = \lambda^a/2$
$\alpha^a(x) = 8$ functions
ψ is a 3-component vector

color
red=r
green=g
blue=b
$r = (1, 0, 0), g = (0, 1, 0), b = (0, 0, 1)$

$i\gamma^\mu (\partial_\mu + ig_s G^a_\mu T^a)\psi = m\psi$
invariant under SU(3) gauge transf
$\psi' = exp(ig_s\alpha^a(x)T^a)\psi$
$G^{k'}_\mu = G^k_\mu - \partial_m u\alpha^a(x) - g_s f_{ijk}\alpha_i(x)G^j_\mu$
f_{ijk} =structure constant
$[\lambda_i, \lambda_j] = \alpha_i\lambda_{ijk}\lambda_k$
$g_s T^a \gamma^\mu G^a_\mu \psi$

U(1) Abelian
SU(3) Non-Abelian

QED	QCD
a massless photon	8 massless gluons
U(1)gauge	SU(3)gauge
1 generator	8 generators
eletric charge	3 color charges
电中性的离子不跟光子作用	色中性粒子不跟胶子作用

$-ig_s\gamma^\mu$:spinor index
$1/2\gamma^a$:color index
$U(p) \to C_i u(p)$
(color states)(Dirac spinors)
$C_1 = r, C_2 = g, C_3 = b$
$\bar{U}(p) - > \bar{u}(p) C_i^\dagger$
$j^\mu = \bar{U}(p_3) C_j^\dagger (-ig_s \gamma^\mu \lambda^a/2) C_i U(p_1)$
$= -i/2 g_s (C_j^\dagger \lambda^a C_i)(\bar{u}(p_3)\gamma^\mu u(p_1))$
$= -i/2 g_s \lambda_{ji}^a \bar{u}_3 \gamma^\mu u_1$

Feynman rule: $(-i/2 g_s \lambda_{ji}^a \gamma^\mu)$

psi is both 3-comp vector in color and 4-comp in spinor
QCD vortex:$-g_s\lambda^a/2\gamma^\mu$

gluon propagator
$-ig_{\mu\nu}/q^2\delta_{ab}$

color flow
r+b->b+r
$\lambda_4 = ((0,0,1),(0,0,0),(1,0,0))$
$r\bar{b}$ or $\bar{r}b$
gluon(9?):
$r\bar{g}, g\bar{r}, r\bar{b}, b\bar{r}, g\bar{b}, b\bar{g}, r\bar{r}, g\bar{g}, b\bar{b}$
gluon->acotet
physcial gluon
$1/\sqrt{2}(r\bar{r} - g\bar{g})$
$1/\sqrt{6}(r\bar{r} + g\bar{g} - 2b\bar{b})$
not exist in nature:
$G_9 = 1/\sqrt{3}(r\bar{r} + g\bar{g} + b\bar{b})$

color confinement(色禁闭)
V(r)~kr
k~1Gev/fm
seperate $q\bar{q}$ will produce "2 new $q\bar{q}$"
colored objects are always confined to color single states

proton wavefunction
ψ~color*spin*flavor*space
$3\times 3\times 3 = 10+8+8+1$
$\psi^c(qqq) = 1/\sqrt{6}(rgb - rbg + gbr - grb + brg - bgr)$

meson wavefunction($q\bar{q}$)
$\psi^c(q\bar{q}) = 1/\sqrt{3}(r\bar{r} + g\bar{g} + b\bar{b})$
$3\times\bar{3} = 8+1$

there is no qq bound state
$3\times 3 = 6+3$

hadronization
jet
$e^+e^- -> q\bar{q}$
where q turns into a lot of particles, those in total are called jet & the process is hadronization
1.$e^+e^- -> q\bar{q}$
2.$q \equiv \bar{q}$
3.$o \equiv o + o \equiv o$
4.particles+particles
notice the final should still be non-color in sum, so there will be 3 final states:$r\bar{r}, g\bar{g}, b\bar{b}$

α_s running
asymptotic freedom(渐近自由)
α_s $|q|$:
0.5~1GeV:non-perturbative QCD
0.1~100GeV:pQCD

$\sigma(e^+e^- -> \mu\bar{\mu}) = 4\pi\alpha^2/(3s)$ (QED xsec)
$\sigma(e^+e^- -> q\bar{q}) = 3Q_q^2 4\pi\alpha^2/(3s)$ (3 comes from 3 final states)
$\sigma(e^+e^- -> hadrons) = 4\pi\alpha^2/(3s)3\sum_q Q_q^2$

$R = \sigma(e^+e^- -> hadrons)/\sigma(e^+e^- -> \mu\bar{\mu}) = 3\sum_q Q_q^2$
$\sqrt{s} < 3GeV, R = 3((2/3)^2 + (1/3)^2 + (1/3)^2) = 2$
$\sqrt{s} > 3.1GeV, R = 3((2/3)^2 + (1/3)^2 + (1/3)^2 + (2/3)^2) = 10/3(charm)$
$\sqrt{s} > 9.5GeV, R = 3((2/3)^2 * 2 + (1/3)^2 * 3) = 11/3(botom)$
agree with data within 10%

LHC(pp) $\sqrt{s} = 13 \sim 14$ TeV
Tevatron $\sqrt{s} \sim 2TeV, p\bar{p}$

revive:
1.ep elastic:θ_e
2.ep inelastic: θ_e, E_e
3.pp: 3variables
x_1, x_2, Q^2
$p \to^{x_1} q, p \to^{x_2} \bar{q}$
$q - Q^2 - \bar{q}$
Drell-Yan

pp-> 2 jets+X
X is spectator
Q^2, x_1, x_2
consider j_1, j_2 :
θ_1, θ_2, P_T
P_T: transverse momentum
$P_T = \sqrt{p_x^2 + p_y^2}$,as collision happens in z
$(x_1 - x_2)E_p$:boost

Rapidity
$y = 1/2 \ln \frac{E+p_z}{E-p_z}$
$\Delta y = y_1 - y_2$ is L.I.
$p_z\ E \cos\theta$
$y\ 1/2 \ln \frac{1+\cos\theta}{1-\cos\theta} = -\ln\tan(\theta/2) = \eta$
pseudo-rapidity η

Z~91GeV
$q + \bar{q} - \gamma/Z- > \mu^+\mu^-$
$\sigma(q\bar{q}- > \mu^+\mu^-) = 1/3Q_q^2 4\pi\alpha^2/(3s)$
$d^2\sigma = \sigma(q\bar{q}- > \mu^+\mu^-)u^p(x_1)dx_1\bar{u}^{\bar{p}}(x_2)dx_2$
$\hat{s} = x_1x_2s$
$d^2\sigma/(dx_1dx_2) = 4/9\frac{4\pi\alpha^2}{9x_1x_2s}u(x_1)u(x_2)$
$u(x) = u^p(x) = \bar{u}^{\bar{p}}(x)$
s:hadron
$\hat{s}$:parton
$d^2\sigma/(dx_1dx_2) = \frac{4\pi\alpha^2}{9x_1x_2s}(4/9(u(x_1)u(x_2) + \bar{u}(x_1)\bar{u}(x_2)) + 1/9(d(x_1)d(x_2) + \bar{d}(x_1)\bar{d}(x_2)))$
$= \frac{4\pi\alpha^2}{9x_1x_2s}f(x_1, x_2)$

$x_1, x_2- > y, M$
M=invariant mass of $\mu^+\mu^-$
$y = 1/2\ln(\frac{E_3+E_4+p_{3z}+p_{4z}}{E_3+E_4-p_{3z}-p_{4z}})$
$M^2 = x_1x_2s$
$y = 1/2\ln(x_1/x_2)$
$dydM = \partial(y, M)/\partial(x_1, x_2)dx_1dx_2$
$\quad = |((1/(2X_1), -1/(2X_2)), (\sqrt{x_2s}/2\sqrt{x}_1, \sqrt{x_1s}/2\sqrt{x}_2))|dx_1dx_2$
$\quad = s/(2M)dx_1dx_2$

$d^2\sigma = \frac{4\pi\alpha^2}{9x_1x_2s}f(x_1, x_2)dx_1dx_2$
$\quad = \frac{4\pi\alpha^2}{9M^2}f(e^yM/\sqrt{s}, e^-yM/\sqrt{s})2M/sdydM$
$d^2\sigma/dydM = \frac{8\pi\alpha^2}{9Ms}f(\dots)$

Jet production @ LHC (pp)
parton level diagram
$d\hat{\sigma}/dQ^2 = \frac{4\pi\alpha_s^2}{9Q^4}(1 + (1 - Q^2/\hat{s})^2)$
$Q^2 = -q^2$
$\hat{s} = x_1x_2s$
$d\sigma/dQ^2 = d\hat{\sigma}/dQ^2g(x_1, x_2)d_x1dx_2$

$g(x_1,x_2)=u(x_1)u(x_2)+u(x_1)d(x_2)+d(x_1)u(x_2)+d(x_1)d(x_2)$
$d^3\sigma/(dQ^2dx_1dx_2)->d^3\sigma/(dy_3dy_4dP_T)$

6.5交:10.6,10.7

QED/QCD	**WEAK**
massless gauge boson	massive gauge boson W^{\pm,Z}_0
vectoer interaction \bar\psi\gamma^\mu\psi	Vector and axial vector

Intrinsic parity
parity transfer:x->x',t->t
Dirac Eq.
$(i\mathrm{hadrons}^\mu\partial_\mu-m)\psi=0$
$\psi'(x',t')=P\psi(x,t)$
$P=\gamma^0,engin:p=1,particle;p=-1,antiparticle.$
$PP=1$
$P^\dagger P=1$
$P^\dagger=P$
If P commutes with H
$P\psi=p\psi$
$PP\psi=\psi$
$p^2=1$
$p=\pm1$(since P is herition)
particle:$p(e^-)=p(\nu_e)=p(q)=1$
antiparticle:$p(e^+)=p(\bar\nu_e)=p(\bar q)=-1$
boson:$p(\gamma)=p(g)=-1$

parity conservation in QED
$e^-+q\longrightarrow^\gamma e^-q$
$-iM=\bar u_3(ieQ_e\gamma^\mu)u_1\frac{-ig_{\mu\nu}}{q^2}\bar u_4(ieQ_q\gamma_\nu)u_4$
$M=Qqe^2/q^2j_ej_q$
$j_e^\mu=\bar u_3\gamma^\mu u_1$
$j_q^\mu=\bar u_4\gamma^\mu u_2$

take P:
$u->\gamma^0u$
$\bar u=u^\dagger\gamma^0->(\gamma^0u)^dagger\gamma^0=\bar u\gamma^0$
$J_e^\mu=\bar u_3\gamma^0\gamma^\mu\gamma^0u_1=(\bar u_3\gamma^0u_1,-\bar u_3\gamma^0u_1)$
vector current
$M\ j_ej_q=j_e^0j_q^0-j_e^ij_q^i->j_e^0j_q^0-(-j_e^i)(-j_q^i)=j_ej_q$

If a process is allowed in QED/QCD?
Ex:
J^p:J:total angular momentum,p:parity
$P^0(1^-)->Pi^+(0^-)+Pi^-(0^-)$
J conservation: $1=0+0+l:l=1$
parity:$-1=(-1)(-1)(-1)^l=-1$

$(-1)^l$:parity with orbital
so this is possible to happen

Ex:
$\eta(0^-)->Pi^+(0^-)+Pi^-(0^-)$
$J:0=0+0+l\Rightarrow l=0$
$P:-1=(-1)(-1)(-1)^l=1$
Not allowed in QCD

1.saclar:$p=1$
2.vector:$(X,P),p=-1$
3.axial vector:$p=1$,e.g. $L=r\times p,S,B$
4.$P\cdot P=p^2$ as a scalar,$p=1$
5.pseudo-scalar: vector\cdot axial vector
$h\ S\cdot P$(helicity), $p=-1$

1957 Wu at al
$^{60}Co\rightarrow^{60}Ni^*+e^-+\bar{\nu}_e$
Co in B,μ is paralle to B
e^- is ejected, take P:
B,μ does not change; e^- will be in oppsite direction
is parity is conserved, then there will be same posibility happen in different direction
but we see more e^- in lower hemisphere(oppsite to B)
so parity non-conservation

vector current->parity conservation(QED,QCD)
spin-1 boson:vector current & axial vector current

In weak interaction
$M\sim j_1^\mu j_{2\mu}$
$j_1^\mu=g_V\bar{\psi}\gamma^\mu\psi+g_A\bar{\psi}\gamma^\mu\gamma_5\psi$
$\bar{u}\gamma^\mu\gamma_5u->\bar{u}\gamma^0\gamma^\mu\gamma_5\gamma^0u$
$=(-\bar{u}\gamma^0\gamma_5u,\bar{u}\gamma^i\gamma_5u)$

$j_Vj_V:p=1$
$j_Aj_A->j_A^0j_A^0-j_A^ij_A^i=j_Aj_A$
$p=1$
$j_Vj_A->-j_Vj_A$
$p=-1$

$M\sim j_1^\mu j_{2\mu}$
$=g_V^2j_{1V}j_{2V}+g_A^2j_{1A}j_{2A}[p=1]$
$+g_Vg_A(j_{1V}j_{2A}+j_{2V}j_{1A})[p=-1]$
parity violation~$g_Vg_A/(g_V^2+g_A^2)$
if $g_A<<g_V,PV\ g_A/g_V<<1$

Maximal Parity Violation
$|g_V|=|g_A|$
$j_1=g_V\bar{\psi}\gamma^\mu(1\pm\gamma_5)\psi$
weak:$g_V\bar{\psi}\gamma^\mu(1-\gamma_5)\psi$
weak current: V-A
(only left hand particle works in weak)

Weak charged current
$j_\mu = g_W/\sqrt{2}\bar{u}(p')\gamma^\mu(1-\gamma_5)/2u(p)$
g_w:weak coupling constant
vertex factor
$-ig_W/\sqrt{2}1/2\gamma^\mu(1-\gamma_5)$
Chiral(手征)
$P_R = (1+\gamma_5)/2$
$P_L = (1-\gamma_5)/2$
$\bar{\psi}_1\gamma^\mu\psi_2 = \bar{\psi}_{1L}\gamma^\mu\psi_{2L} + \bar{\psi}_{1R}\gamma^\mu\psi_{2R}$
$\bar{\psi}_{1L}\gamma^\mu\psi_{2r} = \bar{\psi}_{1l}\gamma^\mu\psi_{2R} = 0$
$j^\mu = g_W/\sqrt{2}\bar{u}_1\gamma^\mu P_L u_2$
$= g_W/\sqrt{2}\bar{u}_{1L}\gamma^\mu u_{2L}$
1.Only left-handed chiral particles participate in charged weak current
2.Only right handed chiral antiparticles .....(since $P_L v = v_R$)

In high energy limit E>>m
chiral states ~ helicity states

$e^- - W^\pm - > \nu_e$
$e^+ - W^\pm - > \bar{\nu}_e$
$e^- + \bar{\nu}_e$,Take P can not happen

QED photon propagator
$\gamma : -ig_{\mu\nu}/q^2$

$W^\pm$ propagator
$M_W\ 80GeV$
$m_Z\ 91GeV$
Feynman ($W^\pm$) rules
$-i/(q^2-M_W^2)(g_{\mu\nu} - q\mu q\nu/M_W^2)$
$q << M_W :$
$ig_{\mu\nu}/M_W^2$
$\nu_e + d \to^W e^- + u$
$M\ g_W^2/M_W^2$
$\alpha = e^2/4\pi = 1/137$
$\alpha_W = g_W^2/4\pi = 1/30$
$M_{QED}\ 1/q^2$
$M_W\ 1/M_W^2$
$\sigma_{weak}/\sigma_{QED}\ |M_{weak}/M_{QED}|\ (q/M_W)^4$

Weak Interaction
photon propagator $\gamma : -ig_{\mu\nu}/q^2$

$W : -i/(q^2-M_W^2)(g_{\mu\nu} - q\mu q\nu/M_W^2)$

very low energy: $q \to 0$
$ig_{\mu\nu}/M_W^2$ (point interaction)
$\nu_e + d \to^W e^- + u$
at very low energy the line for W can be seen as a point

1934 Fermi throry
$M = G_F g_{\mu\nu} \bar{\psi}_3 \gamma^\mu \psi_1 \bar{\psi}_4 \gamma^\nu \psi_2$
after 1957
$M = G_F/\sqrt{2} g_{\mu\nu} \bar{\psi}_3 \gamma^\mu (1-\gamma_5) \psi_1 \bar{\psi}_4 \gamma^\nu (1-\gamma_5) \psi_2$
G_F:Fermi constant

Full weak theory
$M_{fi} = -(g_w/\sqrt{2}\bar{\psi}_3 \gamma^\mu (1-\gamma_5)/2\psi_1)((g_{\mu\nu} - q_\mu q_\nu/M_W^2)/(q^2 - M_W^2))(g_w/\sqrt{2}\bar{\psi}_4 \gamma^\nu (1-\gamma_5)/2\psi_2)$
$G_F/\sqrt{2} = (g_w/(2\sqrt{2}))^2/M_W^2 = g_W^2/(8M_W^2)$
$(q^2 << M_W^2)$

muon decay
$\Gamma(\mu^- -> e^- \nu_\mu \bar{\nu}_e) = 1/\tau_\mu = G_F^2 m_\mu^2/192\pi^3$
$m_\mu = 106MeV$
$\tau_\mu = 2.2 * 10^-6s$
thus obtain
$G_F = 1.16638 * 10^-5GeV^{-2}$

$G_F/\sqrt{2} = g_w^2/(8M_W^2)$
$M_W = 80GeV$
$\alpha_W = g_W^2/4\pi = 1/30$

for $|q^2| << M_W^2$
$QED\ 1/q^2$
$WEAK\ \ -1/M_W^2$
Interaction rate
$QED/WEAK = M_W^4/q^4 \rightarrow^{\text{q in MeV}} 10^{20}$
when $q\ M_W, QED/WEAK\ 1$

Helicity in pion decay
charged $\Pi^\pm, J^p = 0^-$ (pseudoscalar)
$m(\Pi^\pm)\ 140MeV$
can only decay via weak interaction
$1.\Pi^- -> e^- \bar{\nu}_e$
$2.\Pi^- -> \mu^- \bar{\nu} \mu$
$3.\Pi^- -> \mu^- \bar{\nu} \mu \gamma$

$\Pi^-: d + \bar{u} - W^- -> e^-(\mu^-) + \bar{\nu}_e(\nu_\mu)$
lepton universality
$\Gamma(\Pi^- -> e^- + \bar{\nu}_e)\ \Gamma(\Pi^- -> \mu^- + \bar{\nu}_\mu)$
e will be expected greater since e in lighter than \mu
Measurements
$\Gamma(\Pi^- -> e^- + \bar{\nu}_e)/\Gamma(\Pi^- -> \mu^- + \bar{\nu}_\mu) = 1.230(4) * 10^{-4}$

$m_\nu << E_\nu\ m_\Pi$
chiral\approx helicity (@ high energy)
$\bar{\nu}_e(\bar{\nu}_\mu)$ is RH helicity
total angular momentum conservation -> e^- is RH helicity ("wrong helicity")
If $m_{e^-} << E_{e^-}$,chiral~helicity, decay not allowed
If chiral $\neq$ helicity decay can happen

RH helicity state in term of chiral states
$u_{\uparrow} = 1/2(1+\kappa)u_R + 1/2(1-\kappa)u_L$
$\kappa = p/(E+m) = 0, NR; 1, R;$
for e is R,for μ is NR

$M\ 1-\kappa = 1 - p_l/(E_l + m_l)$
$m_{\Pi} = E_{\nu} + E_l$
$0 = p_{\nu} + p_l$
$m_{\Pi} = |p_l| + \sqrt{|p_l|^2 + m_l^2}$
$|p_l| = (m_{\Pi}^2 - m_l^2)/2m_{\Pi}$
$E_l = (m_{\Pi}^2 + m_l^2)/2m_{\Pi}$
$M\ 1-\kappa = 2m_l/(m_{\Pi} + m_l)$
$\Gamma(\Pi^- -> e^- + \bar{\nu}_e)/\Gamma(\Pi^- -> \mu^- + \bar{\nu}_{\mu})\ m_e^2/m_{\mu}^2 \sim 1/4 * 10^{-4}$
$e^-\ : \beta = 0.99997$ chiral~helicity
$\mu^-\ : \beta = 0.27 \text{chiral} \neq \text{helicity}$

weak lepton current
$j_l^{\nu} = g_w/\sqrt{2}\bar{u}_3 1/2\gamma^{\nu}(1-\gamma_5)v_4$
we need a 4-vector from pion to couple with final state current
$j_{\Pi}^{\mu} = f_{\Pi} p_{\Pi}^{\mu} g_w/\sqrt{2}/2, f_{\Pi} =$decay constant
$\Pi^- -> l^- \nu_l$
$M_{fi} = g_W^2/(4M_W^2) f_{\Pi} p_{\Pi}^{\mu} \bar{u}_3 \gamma_{\mu}(1-\gamma_5)/2v_4$
$Only p_{\Pi}^0 \neq 0 in \Pi^-$ rest frame $p_{\Pi}^0 = m_{\Pi}$
$\bar{u}_3\gamma_0 = u_3^{\dagger}$
$M = g_W^2/(4M_W^2) f_{\Pi} m_{\Pi} u_3^{\dagger}(1-\gamma_5)/2v_4$
$(1-\gamma_5)/2v_4 = v_{4R} \approx v_{4\uparrow}$
$M = g_W^2/(4M_W^2) f_{\Pi} m_{\Pi} u_3^{\dagger} v_{4\uparrow}$
consider 2 possible helicity state for u_3
$u_{3\uparrow}^{\dagger} v_{4\uparrow} \& u_{3\downarrow}^{\dagger} v_{4\uparrow}$

$u_{\uparrow} = \sqrt{E+m}(c, se^{i\phi}, \kappa c, \kappa s e^{i\phi})$
$u_{\downarrow} = \sqrt{E+m}(-s, ce^{i\phi}, \kappa s, -\kappa c e^{i\phi})$
$v_{\uparrow} = \sqrt{E+m}(\kappa s, -\kappa c e^{i\phi}, -s, ce^{i\phi})$

$l^- = (\theta, \phi) = (0, 0)(p_3)$
$\bar{\nu}_l : (\theta, \phi) = (\pi, \pi)(p_4)$
$u_{\uparrow}(p_3) = \sqrt{E_l + m_l}(1, 0, \kappa, 0)$
$u_{\downarrow}(p_3) = \sqrt{E_l + m_l}(0, 1, 0, -\kappa)$
$v_{\uparrow}(p_4) = \sqrt{E_{\nu}}(1, 0, -1, 0)$
$u_{3\downarrow}^{\dagger} v_{4\uparrow} = 0$ angular momentum violation
$u_{3\uparrow}^{\dagger} v_{4\uparrow} = \sqrt{E_l + m_l}\sqrt{p_l}(1-\kappa)$

$M = (g_w^2/2M_W)^2 f_{\Pi} m_l \sqrt{(}m_{\Pi}^2 - m_l^2)$
$\Gamma = ..\int |M|^2$
$\quad = G_F^2/(8\pi m_{\pi}^3) f_{\Pi} m_l^2 (m_{\Pi}^2 - m_l^2)^2$
$\Gamma(\Pi^- -> e^- + \bar{\nu}_e)/\Gamma(\Pi^- -> \mu^- + \bar{\nu}_{\mu}) = 1.26 * 10^{-4}$
$m_{\Pi} = 140 MeV$
$m_e = 0.51 MeV$
$m_{\mu} = 105.7 MeV$

Lepton universality
$W^- \ \bar{\nu}_l + l^-$ for e, μ, τ
to obtain ν
proton$\rightarrow || \rightarrow \Pi^{\pm} \rightarrow^{+B} \Pi^+ \rightarrow^{\text{(8km tunel)}} \mu^+ \& \nu_\mu \rightarrow || \rightarrow \nu_\mu$

Dirac neutrino
Majorana neutrino
Neutrino flavor
ν_e, ν_μ, ν_τ
neutrino oscillation
sun's energy: 40 minutes to melt 1 cm^3 ice (about 6 km if ice is on the surface of sun)
solar neutrino: $2 * 10^{38} \nu_e / s$
On earth $6 * 10^{10} cm^{-2} / s$
(how many will scatters through the earth)