

Notes of Particle Physics, LIU Zuowei

hebrewsnabla

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Contents

1	Introduction	4
1.1	Elementary Particles	4
1.2	Feynman Diagrams	4
2	Underlying Concepts	5
2.1	Natural Units	5
2.2	Special Relativity	5
2.2.1	The Lorentz Transformation	5
2.2.2	4-Vectors & Lorentz Invariant	5
2.2.3	Mandelstam Variables	6
2.3	Non-relativistic QM	6
2.3.1	Wave Mechanics & Schrödinger Equation	6
2.3.2	Prob. Dens. & Prob. Current	6
2.3.3	TD & Conserved Quantities	6
2.3.4	Commutation relations & Compatible Observables	6
2.3.5	Angular Momentum	6
3	Decay Rates and Cross Sections	6
3.1	Fermi's Golden Rule ★	6
3.2	Phase Space and Wavefn Normalization ★	7
3.2.1	LI Phase Space	7
3.2.2	Fermi G R Revisited	7
3.3	Particle Decays ★	7
3.4	Interaction Cross Sections ★	8
3.4.1	\mathcal{L} .I. Flux	8
3.4.2	Scattering in the CoM frame	8
3.5	Diff. Xsec. ★	9
3.5.1	CoM Frame	9
3.5.2	Lab Frame	9
4	The Dirac Equation	9
4.1	The Klein-Gordon Eq.	9
4.2	The Dirac Eq. ★	9
4.3	Prob. Dens. & Prob. Curr.	10
4.4	Spin ★	10
4.5	Covariant Form of the Dirac Eq.	12

4.5.1	12
4.6 Sol. to the Dirac Eq. ★	12
4.6.1 Particles at Rest	12
4.6.2 General Free-particle Sol.	13
4.7 Antiparticles ★	13
4.7.1	13
4.7.2 the Feynman-Stückelberg Interpretation	13
4.7.3 Normalization	13
4.7.4 Operators for Antiparticles	13
4.7.5 Charge Conjugation	14
4.8 Spin and Helicity States ★	14
4.9 Intrinsic Parity of Dirac Fermions ★	15
5 Interaction by Particle Exchange	15
5.1 1st and 2nd Order Perturbation Theory	15
5.1.1 Time-Ordered Perturbation Theory	15
5.2 Feynman Diagrams and Virtual Particles	15
5.3 Intro to QED	15
5.4 Feynman Rules for QED	15
6 e-p Annihilation	16
6.1 Perturbation Theory	16
6.2 e-p Annihilation	16
6.3 Spin Sums	16
6.4 Helicity Amplitudes	16
6.5 The μ and e Currents	16
6.6 Cross Section	16
6.7	16
6.8 Spin in e-p Anni.	16
6.9 Chirality ★	17
6.9.1 Chiral Projection Operators	17
6.9.2 Chirality in QED	17
7 E-p Elastic Scattering	17
7.1 Probing ...	17
7.2 Rutherford and Mott Scattering ★	17
7.2.1 Rutherford Scattering	18
7.2.2 Mott Scattering	18
7.3 Form Factors ★	19
7.4 Relativistic e-p Elastic Scattering	19
7.5 the Rosenbluth Formula	19
7.5.1 Measuring G	19
7.5.2 Elastic Scattering at high Q^2	20
8 Deep Inelastic Scattering (DIS)	20
8.1 E-p Inelastic Scattering	20
8.1.1 Kinematic Variables	20
8.1.2 IS at Low Q^2	21
8.2 Deep Inelastic Scattering	21
8.2.1 Structure Func.	21

8.3	Elec-quark Scattering	21
8.4	The Quark Model	22
8.4.1	PDF	22
8.4.2	Determination of the PDFs	22
8.4.3	Valence and Sea Quarks	23

1 Introduction

物理楼 318, zuoweiliu@nju.edu.cn
平时 20 作业 30 期末 50

References

- Mark Thomson "Modern Particle Physics"
- David Griffiths "Introduction to Elementary Particles"
- Francis Halzen & Alan D. Martin "Quarks & Leptons"

1.1 Elementary Particles

p – uud
n – ddu
u – upper quark, d – down quark
c – s –
t – top b –

Gen1 – $e^-/\nu_e/u/d$
Gen2 – $\mu^-/\nu_\mu/c/s$
Gen3 – $\tau^-/\nu_\tau/t/b$
all have spin 1/2
charge: u,c,t 2/3; d,s,b -1/3
mass: $m_t \approx 170m_p$

Electromagnetic Interaction, EM – photon
Strong – gluon
Weak – W^\pm/Z^0
Gravity – Higgs boson
 $m_g = 125GeV$, $m_Z = 91GeV$, $m_W = 80GeV$ – EW scale

1.2 Feynman Diagrams

EM $\alpha_{EM} = 1/137$
strong $\alpha_s = 1$
weak $\alpha_w = 1/30$

small α enables perturbative calculation
Feynman rules

2 Underlying Concepts

2.1 Natural Units

$[kg, m, s] \rightarrow [\text{GeV}, \hbar, c]$
 $\hbar = c = 1$ mass: $\text{GeV}/c^2 - \text{GeV}$
length: $\hbar c / \text{GeV} - \text{GeV}^{-1}$
time: $\hbar / \text{GeV} - \text{GeV}^{-1}$

$E, \mathbf{p}, m \rightarrow \text{GeV}$
 $t, l \rightarrow \text{GeV}^{-1}$
 $\sigma \rightarrow \text{GeV}^{-2}$

e.g. $\langle r^2 \rangle^{1/2} = 4.1 \text{ GeV}^{-1} = ?$
 $[L] = [E]^{-1} [\hbar] [c]$
 $\text{GeV}^{-1} = 1.6 \times 10^{-10} \text{ J}^{-1} \cdot 3 \times 10^8 \text{ m s}^{-1} \cdot 1.055 \times 10^{-34} \text{ J s} \approx 0.8 \text{ fm}$

Heaviside-Lorentz Units

$$\epsilon_0 = \mu_0 = 1 \quad (2.1)$$

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \Rightarrow \frac{e^2}{4\pi} = \frac{1}{137} \quad (2.2)$$

2.2 Special Relativity

2.2.1 The Lorentz Transformation

$$\mathbf{X}' = \mathbf{\Lambda} \mathbf{X} \quad (2.3)$$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \quad (2.4)$$

where $\gamma = (1 - \beta^2)^{-1/2}$.

$$\mathbf{X} = \mathbf{\Lambda}^{-1} \mathbf{X}' \quad (2.5)$$

$$\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} \quad (2.6)$$

$$\mathbf{\Lambda} \mathbf{\Lambda}^{-1} = \mathbf{I} \quad (2.7)$$

2.2.2 4-Vectors & Lorentz Invariant

contravariant 4-vector

$$X^\mu = (t, x, y, z) \quad (2.8)$$

when $\mu \rightarrow \mu'$,

$$X^{\mu'} = \Lambda_{\nu}^{\mu'} X^\nu \quad (2.9)$$

$$t^2 - x^2 - y^2 - z^2 = t'^2 - x'^2 - y'^2 - z'^2 \quad (2.10)$$

covariant 4-vector $X_\mu = (t, -x, -y, -z)$
 $X^\mu X_\mu = X^{\mu'} X_{\mu'}$ # Ein sum convention
 thus $X^\mu X_\mu = \mathcal{L}.I.$ # Lorentz invariant

$$X_\mu = G_\mu^\nu X^\nu \quad (2.11)$$

$$G_\mu^\nu \equiv \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (2.12)$$

4-momentum

$$P^\mu = (E, p_x, p_y, p_z) \quad (2.13)$$

$$P^\mu P_\mu = E^2 - \mathbf{p}^2 = m^2 \quad (2.14)$$

4-derivative

$$\begin{pmatrix} \partial_{t'} \\ \partial_{x'} \\ \partial_{y'} \\ \partial_{z'} \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & \gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \gamma\beta & 0 & 0 & \gamma \end{pmatrix} \begin{pmatrix} \partial_t \\ \partial_x \\ \partial_y \\ \partial_z \end{pmatrix} \quad (2.15)$$

i.e.

$$\partial_{\mu'} = \Lambda_\mu^{\nu'} \partial_\nu \quad (2.16)$$

$$\partial^\mu \partial_\mu = \frac{\partial^2}{\partial t^2} - \nabla^2 \equiv \square \quad (2.17)$$

2.2.3 Mandelstam Variables

2.3 Non-relativistic QM

2.3.1 Wave Mechanics & Schrödinger Equation

2.3.2 Prob. Dens. & Prob. Current

2.3.3 TD & Conserved Quantities

2.3.4 Commutation relations & Compatible Observables

2.3.5 Angular Momentum

3 Decay Rates and Cross Sections

3.1 Fermi's Golden Rule ★

Transition rate

$$\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_i) \quad (3.1)$$

3.2 Phase Space and Wavefxn Normalization ★

$\alpha \rightarrow 1 + 2$

1st order:

$$T_{fi} = \langle f | H' | i \rangle = \langle \psi_1^* \psi_2^* | H' | \psi_a \rangle \quad (3.2)$$

Born approx.:

$$\psi(\mathbf{x}, t) = A e^{i(\mathbf{p} \cdot \mathbf{x} - Et)} \quad (3.3)$$

Normalization in a box of side a :

$$\langle \psi | \psi \rangle = 1 \Rightarrow A^2 = 1/a^3 \quad (3.4)$$

PBC gives

$$\mathbf{p} = \mathbf{n} \frac{2\pi}{a} \quad (3.5)$$

thus

$$d^3\mathbf{p} = d^3\mathbf{n} \frac{(2\pi)^3}{V} \quad (3.6)$$

$$dn \equiv d^3\mathbf{n} d^3p \frac{V}{(2\pi)^3} = 4\pi p^2 dp \frac{V}{(2\pi)^3} \quad (3.7)$$

$$\rho(E) = \frac{dn}{dE} = \frac{dn}{dp} \left| \frac{dp}{dE} \right| \quad (3.8)$$

Set $V = 1$, and there are N particles, thus $N - 1$ indep momenta

$$dn = \prod_{i=1}^{N-1} dn_i = \prod_{i=1}^{N-1} \frac{d^3\mathbf{p}_i}{(2\pi)^3} \quad (3.9)$$

$$(3.10)$$

3.2.1 LI Phase Space

$$\Gamma_{fi} = \frac{1}{2E_a} \int |\mathcal{M}_{fi}|^2 [(2\pi)^4 \delta^4(\mathbf{p}_a - \mathbf{p}_1 - \mathbf{p}_2)] \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \quad (3.11)$$

3.2.2 Fermi G R Revisited

$$\Gamma_{fi} = \quad (3.12)$$

In CoM, $E_a \rightarrow m_a$
next subsec.

3.3 Particle Decays ★

静止参考系中

$$p_a = (m_a, 0) \quad (3.13)$$

\therefore

$$\delta^3(\mathbf{p}_a - \mathbf{p}_1 - \mathbf{p}_2) d^3\mathbf{p}_2 = \quad (3.14)$$

$$\Gamma = \frac{1}{2m_a} \int |\mathcal{M}|^2 (2\pi)^4 \delta(m_a - E_1 - E_2) \frac{d^3\mathbf{p}_1}{(2\pi)^6 4E_1 E_2} \quad (3.15)$$

where $\mathbf{p}_2 = -\mathbf{p}_1$. thus $E_2 = \sqrt{m_2^2 + p_1^2}$.
and $d^3\mathbf{p}_1 = p_1^2 dp_1 \sin\theta d\theta d\phi$

$$\Gamma = \frac{1}{8\pi^2 m_a} \int |\mathcal{M}|^2 \delta(m_a - \sqrt{m_1^2 + p_1^2} - \sqrt{m_2^2 + p_1^2}) \frac{p_1^2 dp_1 d\Omega}{4E_1 E_2} \quad (3.16)$$

Since

$$\int_{-\infty}^{\infty} \delta[f(x)]g(x)dx = \sum_i \frac{g(x_i)}{|f'(x_i)|} \quad \text{w/ } f(x_i) = 0 \quad (3.17)$$

thus

$$\Gamma_{fi} = \frac{p^*}{32\pi^2 m_a^2} \int |\mathcal{M}_{fi}|^2 d\Omega \quad (3.18)$$

where $m_a - \sqrt{m_1^2 + p^{*2}} - \sqrt{m_2^2 + p^{*2}} = 0$

3.4 Interaction Cross Sections ★

tot # of target particles

$$\delta N = n_b v \delta t A \quad (3.19)$$

interaction prob.

$$\delta P = \frac{\delta N \sigma}{A} = n_b v \sigma \delta t \quad (3.20)$$

interaction rate per target particle

$$\Gamma_b = \frac{dP}{dt} = n v \sigma = \sigma \phi_a \quad (3.21)$$

$$\phi_a = \frac{dN_a}{dA dt} = \frac{ndA v dt}{dA dt} = n v \quad (3.22)$$

total rate

$$\Gamma_a n_a V = (n_a v)(n_b V) \sigma = \text{flux} \times \# \text{ target} \times \sigma \quad (3.23)$$

3.4.1 \mathcal{L} .I. Flux

$a + b \longrightarrow 1 + 2$

$$\Gamma_{fi} = \frac{v_a + v_b}{V} \sigma \quad (3.24)$$

$$\sigma = \frac{1}{4E_a E_b (v_a + v_b)} \int |\mathcal{M}|^2 (2\pi)^4 \delta^4(p_a + p_b - p_1 - p_2) \frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1} \frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2} \quad (3.25)$$

which is \mathcal{L} .I..

\mathcal{L} .I. flux factor $F = 4E_a E_b (v_a + v_b)$.

3.4.2 Scattering in the CoM frame

photo

$$\sigma = \frac{|\mathbf{p}_f|}{64\pi^2 s |\mathbf{p}_i|} \int |\mathcal{M}|^2 d\Omega \quad (3.26)$$

3.5 Diff. Xsec. ★

$$\sigma = \int \frac{d\sigma}{d\Omega} d\Omega \quad (3.27)$$

3.5.1 CoM Frame

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2} \frac{p_f^*}{p_i^*} |\mathcal{M}_{fi}|^2 \quad (3.28)$$

$$t = (p_i - p_3)^2 \quad \mathcal{L}.I. \quad (3.29)$$

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} \frac{|\mathcal{M}_{fi}|^2}{p_i^{*2}} \quad \mathcal{L}.I. \quad (3.30)$$

3.5.2 Lab Frame

$$e^- + p \longrightarrow e^- + p$$

$$\begin{aligned} p_1 &\approx (E_1, 0, 0, E_1) \\ p_2 &= (m_p, 0, 0, 0) \\ &\dots \end{aligned} \quad (3.31)$$

$$p_i^* \approx \frac{(s - m_p^2)^2}{4s} \quad (3.32)$$

$$\text{where } s = (p_1 + p_2)^2 = m_p^2 + 2E_1 m_p$$

...

4 The Dirac Equation

4.1 The Klein-Gordon Eq.

$$\hat{H}^2 = \hat{\mathbf{p}}^2 + m^2 \quad (4.1)$$

where $\hat{H} = i \frac{\partial}{\partial t}$, thus

$$\frac{\partial^2 \psi}{\partial t^2} = (\nabla^2 - m^2) \psi \quad (4.2)$$

in $\mathcal{L}.I.$ form

$$(\partial^\mu \partial_\mu + m^2) \psi = 0 \quad (4.3)$$

4.2 The Dirac Eq. ★

$$E\psi = (\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta m) \psi \quad (4.4)$$

$$\begin{aligned} -\frac{\partial^2 \psi}{\partial t^2} &= (-i \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} + \beta m)^2 \psi \\ &= \dots \end{aligned} \quad (4.5)$$

must satisfy

$$\begin{aligned}\alpha_x^2 &= \alpha_y^2 = \alpha_z^2 = \beta^2 = I \\ \alpha_i \beta + \beta \alpha_i &= 0 \\ \alpha_i \alpha_j + \alpha_j \alpha_i &= 0\end{aligned}\tag{4.6}$$

$$\text{Tr}(\alpha_i) = \text{Tr}(\beta) = 0\tag{4.7}$$

$$\lambda_{\alpha_i} = \pm 1\tag{4.8}$$

$$\text{len}(\alpha_i) = \text{len}(\beta) = \text{evennumber}\tag{4.9}$$

$$\alpha_i^\dagger = \alpha_i \quad \beta^\dagger = \beta\tag{4.10}$$

Dirac spinor

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}\tag{4.11}$$

There are infinite choices of α and β .

Dirac-Pauli representation:

$$\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \alpha_i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}\tag{4.12}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\tag{4.13}$$

4.3 Prob. Dens. & Prob. Curr.

$$i \frac{\partial \psi}{\partial t} = -i \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} \psi + m \beta \psi\tag{4.14}$$

$$-i \frac{\partial \psi^\dagger}{\partial t} = i \boldsymbol{\nabla} \psi^\dagger \cdot \boldsymbol{\alpha}^\dagger + m \psi^\dagger \beta^\dagger\tag{4.15}$$

$$\psi^\dagger (-i \boldsymbol{\alpha} \cdot \boldsymbol{\nabla} \psi + m \beta \psi) - (i \boldsymbol{\nabla} \psi^\dagger \cdot \boldsymbol{\alpha}^\dagger + m \psi^\dagger \beta^\dagger) \psi = i \psi^\dagger \frac{\partial \psi}{\partial t} + i \frac{\partial \psi^\dagger}{\partial t} \psi\tag{4.16}$$

$$\boldsymbol{\nabla} \cdot (\psi^\dagger \boldsymbol{\alpha} \psi) + \frac{\partial}{\partial t} (\psi^\dagger \psi) = 0\tag{4.17}$$

$$\boldsymbol{\nabla} \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} = 0\tag{4.18}$$

4.4 Spin ★

$$[\hat{H}_D, \hat{\mathbf{L}}] = [\boldsymbol{\alpha} \cdot \hat{\mathbf{p}}, \hat{\mathbf{r}} \times \hat{\mathbf{p}}] = -i \boldsymbol{\alpha} \times \hat{\mathbf{p}} \neq 0\tag{4.19}$$

$$\hat{\mathbf{S}} = \frac{1}{2} \hat{\boldsymbol{\Sigma}} = \frac{1}{2} \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}\tag{4.20}$$

$$[\alpha_i, \Sigma_j] = \begin{pmatrix} 0 & [\sigma_i, \sigma_j] \\ [\sigma_i, \sigma_j] & 0 \end{pmatrix}\tag{4.21}$$

$$[\hat{H}_D, \hat{\boldsymbol{\Sigma}}] = 2i(\boldsymbol{\alpha} \times \hat{\mathbf{p}}) \Rightarrow [\hat{H}_D, \hat{\mathbf{S}}] = i(\boldsymbol{\alpha} \times \hat{\mathbf{p}})\tag{4.22}$$

def

$$\hat{\mathbf{J}} = \hat{\mathbf{L}} + \hat{\mathbf{S}} \quad (4.23)$$

$$[\hat{\mathbf{H}}_D, \hat{\mathbf{J}}] = 0 \quad (4.24)$$

$\hat{\mathbf{J}}$ is a conserved quantity.

$$\hat{\mathbf{S}}^2 = \quad (4.25)$$

EM – magnetic moment ★

$$E = -\boldsymbol{\mu} \cdot \mathbf{B} \quad (4.26)$$

$$E\psi = (\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta m)\psi \quad (4.27)$$

minimal substitution

$$\begin{aligned} E &\rightarrow E - q\phi \\ \hat{\mathbf{p}} &\rightarrow \hat{\mathbf{p}} - q\mathbf{A} \end{aligned} \quad (4.28)$$

where $A = (\phi, \mathbf{A})$

$$(E - q\phi)\psi = [\boldsymbol{\alpha} \cdot (\hat{\mathbf{p}} - q\mathbf{A}) + \beta m]\psi \quad (4.29)$$

$$\begin{aligned} (E - q\phi - m)\psi_A &= \boldsymbol{\sigma} \cdot (\hat{\mathbf{p}} - q\mathbf{A})\psi_B \\ (E - q\phi + m)\psi_B &= \boldsymbol{\sigma} \cdot (\hat{\mathbf{p}} - q\mathbf{A})\psi_A \end{aligned} \quad (4.30)$$

Non-Rela limit: $E \approx m \gg q\phi$

$$2m\psi_B = \boldsymbol{\sigma} \cdot (\hat{\mathbf{p}} - q\mathbf{A})\psi_A \quad \psi_B \ll \psi_A \quad (4.31)$$

$$\begin{aligned} (E - q\phi - m)\psi_A &= \frac{1}{2m}[\boldsymbol{\sigma} \cdot (\hat{\mathbf{p}} - q\mathbf{A})]^2\psi_A \\ &= \frac{1}{2m}[(\hat{\mathbf{p}} - q\mathbf{A})^2 + i\boldsymbol{\sigma} \cdot (\hat{\mathbf{p}} - q\mathbf{A}) \times (\hat{\mathbf{p}} - q\mathbf{A})]\psi_A \end{aligned} \quad (4.32)$$

$$\begin{aligned} &= \frac{1}{2m}[(\hat{\mathbf{p}} - q\mathbf{A})^2 + q\boldsymbol{\sigma} \cdot (\nabla \times \mathbf{A} + \mathbf{A} \times \nabla)]\psi_A \\ &(\nabla \times \mathbf{A} + \mathbf{A} \times \nabla)\psi_A = \mathbf{B}\psi_A \end{aligned} \quad (4.33)$$

$$E\psi_A = \left[m + \frac{1}{2m}(\hat{\mathbf{p}} - q\mathbf{A})^2 + q\phi - \frac{q}{2m}\boldsymbol{\sigma} \cdot \mathbf{B} \right]\psi_A \quad (4.34)$$

From (4.26),

$$\boldsymbol{\mu} = \frac{q}{2m}\boldsymbol{\sigma} = \frac{q}{m}\hat{\mathbf{S}} \quad (4.35)$$

which is spin magnetic moment.

Or,

$$\boldsymbol{\mu} = g\frac{q}{2m}\hat{\mathbf{S}} \quad (4.36)$$

Dirac Eq. explained $g = 2$.

orbital magnetic moment

$$\boldsymbol{\mu} = \frac{q}{2m}\hat{\mathbf{L}} \quad (4.37)$$

In fact, Schwinger gave a correction on g in 1948

$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi} \quad (4.38)$$

4.5 Covariant Form of the Dirac Eq.

$$\beta(i \frac{\partial}{\partial t} + \boldsymbol{\alpha} \cdot \hat{\mathbf{p}} - \beta m)\psi = 0 \quad (4.39)$$

def

$$\gamma^0 = \beta \quad \gamma^1 = \beta\alpha_x \quad \gamma^2 = \beta\alpha_y \quad \gamma^3 = \beta\alpha_z \quad (4.40)$$

and since $\beta^2 = I$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \quad (4.41)$$

where $\partial_\mu = \left(\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$

Note: γ is not usual 4-vector, but (4.41) is Lorentz covariant.

4.5.1

4.6 Sol. to the Dirac Eq. ★

Suppose

$$\psi(\mathbf{r}, t) = u(E, \mathbf{p}) e^{i(\mathbf{p} \cdot \mathbf{r} - Et)} \quad (4.42)$$

where u is a 4-component Dirac spinor

with (4.41)

$$(\beta E - \beta \boldsymbol{\alpha} \cdot \mathbf{p} - m)u(E, \mathbf{p}) e^{i(\mathbf{p} \cdot \mathbf{r} - Et)} = 0 \quad (4.43)$$

$$(\gamma^\mu P_\mu - m)u = 0 \quad (4.44)$$

4.6.1 Particles at Rest

Since $\mathbf{p} = 0$

$$(E\beta - m)u = 0 \quad (4.45)$$

$$E \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} = m \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix} \quad (4.46)$$

the eigenvectors are

$$u_1 = N \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad u_2 = N \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad (4.47)$$

with $E = m$

$$u_3 = N \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad u_4 = \quad (4.48)$$

with $E = -m$

How to explain negative energy?

Feynman-Stueckelberg:

$$e^{-iEt} \equiv e^{iE(-t)} \quad (4.49)$$

E.g. holes in solid state physics. Holes "move" oppositely as the electrons move, like propagating "backward" in time.

or anti-particles, like positron.

4.6.2 General Free-particle Sol.

$$(\gamma^\mu P_\mu - m)u = 0 \quad (4.50)$$

$$\left[\begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} E - \begin{pmatrix} 0 & \boldsymbol{\sigma} \cdot \mathbf{p} \\ -\boldsymbol{\sigma} \cdot \mathbf{p} & 0 \end{pmatrix} - m \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} \right] u = 0 \quad (4.51)$$

$$\begin{pmatrix} (E - m)I & -\boldsymbol{\sigma} \cdot \mathbf{p} \\ \boldsymbol{\sigma} \cdot \mathbf{p} & -(E + m)I \end{pmatrix} \begin{pmatrix} u_A \\ u_B \end{pmatrix} = 0 \quad (4.52)$$

$$u_A = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E - m} u_B u_B = \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E + m} u_A \quad (4.53)$$

$$u_A = \dots = u_A \quad (4.54)$$

thus we can choose $u_A = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

4.7 Antiparticles ★

4.7.1

4.7.2 the Feynman-Stückelberg Interpretation

$$(\gamma^\mu P_\mu + m)v = 0 \quad (4.55)$$

$$\psi = v(p) e^{iEt - i\mathbf{p} \cdot \mathbf{x}} \quad (4.56)$$

...

$$v_1 = N \begin{pmatrix} \dots \\ \dots \\ 0 \\ 1 \end{pmatrix} \quad v_2 = N \begin{pmatrix} \dots \\ \dots \\ 1 \\ 0 \end{pmatrix} \quad (4.57)$$

kick out v_3, v_4, u_3, u_4 whose energy are negative.

4.7.3 Normalization

$$\rho = \psi^\dagger \psi = u_1^\dagger u_1 = |N|^2 \frac{2E}{E + m} \quad (4.58)$$

def

$$\rho = 2E \quad N = \sqrt{E + m} \quad (4.59)$$

4.7.4 Operators for Antiparticles

$$\hat{H}^{(v)} = -i \frac{\partial}{\partial t} \quad \hat{\mathbf{p}}^{(v)} = +i \boldsymbol{\nabla} \quad (4.60)$$

thus

$$\hat{\mathbf{L}}^{(v)} = -\hat{\mathbf{L}}^{(u)} \quad \hat{\mathbf{S}}^{(v)} = -\hat{\mathbf{S}}^{(u)} \quad (4.61)$$

4.7.5 Charge Conjugation

$$E \rightarrow E - q\phi \quad \mathbf{p} \rightarrow \mathbf{p} - q\mathbf{A} \quad (4.62)$$

$$p_\mu \rightarrow p_\mu - qA_\mu \text{ i.e. } i\partial_\mu \rightarrow i\partial_\mu - qA_\mu \quad (4.63)$$

for electron, $q = -e$

$$\gamma^\mu (\partial_\mu - ieA_\mu)\psi + im\psi = 0 \quad (4.64)$$

take complex conj and pre-multiply $-i\gamma^2$

$$-i\gamma^2(\gamma^\mu)^*(\partial_\mu + ieA_\mu)\psi^* - m\gamma^2\psi^* = 0 \quad (4.65)$$

since

$$\gamma^2\gamma^\mu = -\gamma^\mu\gamma^2 \quad (4.66)$$

for $\mu \neq 2$

4.8 Spin and Helicity States ★

$\{u_1, u_2, v_1, v_2\}$ are not $\hat{\mathbf{S}}_z$ eigenstates except $\mathbf{p} = \pm p\hat{\mathbf{k}}$
and $[\hat{H}_D, \hat{\mathbf{S}}_z] \neq 0$, thus we can't find a basis of simultaneous eigenstates.
def: helicity operator

$$\hat{h} \equiv \frac{\hat{\mathbf{S}} \cdot \mathbf{p}}{p} \quad (4.67)$$

$$\hat{h} = \frac{1}{2p} \begin{pmatrix} \boldsymbol{\sigma} \cdot \mathbf{p} & 0 \\ 0 & \boldsymbol{\sigma} \cdot \mathbf{p} \end{pmatrix} \quad (4.68)$$

$$[\hat{h}, \hat{H}_D] = 0 \quad (4.69)$$

suppose $\hat{h}u = \lambda u$

$$\frac{1}{2p} \begin{pmatrix} \boldsymbol{\sigma} \cdot \mathbf{p} & 0 \\ 0 & \boldsymbol{\sigma} \cdot \mathbf{p} \end{pmatrix} u = \lambda u \quad (4.70)$$

$$(\boldsymbol{\sigma} \cdot \mathbf{p})u_A = 2p\lambda u_A \quad (4.71)$$

since $(\boldsymbol{\sigma} \cdot \mathbf{p})^2 = p^2$

$$\lambda = \pm \frac{1}{2} \quad (4.72)$$

according to (4.53)

$$(\boldsymbol{\sigma} \cdot \mathbf{p})u_A = (E + m)u_B \quad (4.73)$$

\therefore

$$u_B = \frac{2\lambda p}{E + m} u_A \quad (4.74)$$

To solve (4.71), let

$$\mathbf{p} = \dots \quad (4.75)$$

$$\hat{h} = \frac{1}{2} \begin{pmatrix} \cos \theta & \sin \theta e^{-i\phi} \\ \sin \theta e^{i\phi} & -\cos \theta \end{pmatrix} \quad (4.76)$$

assume $u_A = \begin{pmatrix} a \\ b \end{pmatrix}$

$$\frac{b}{a} = e^{i\phi} \tan \frac{\theta}{2} \quad (4.77)$$

thus

$$u_{\uparrow} = N \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\phi} \sin \frac{\theta}{2} \\ \dots \\ \dots \end{pmatrix} \quad (4.78)$$

4.9 Intrinsic Parity of Dirac Fermions ★

parity transformation

$$t' = t \quad x' = -x \quad (4.79)$$

parity operator

$$\psi' = \hat{\mathbf{P}} \psi \hat{\mathbf{P}}^2 = I \quad (4.80)$$

$$(4.81)$$

$$\gamma^0 \hat{\mathbf{P}} \propto I \quad (4.82)$$

5 Interaction by Particle Exchange

5.1 1st and 2nd Order Perturbation Theory

5.1.1 Time-Ordered Perturbation Theory

5.2 Feynman Diagrams and Virtual Particles

5.3 Intro to QED

Dirac Eq. + minimal substitution

$$\partial_{\mu} \rightarrow \partial_{\mu} + i q A_{\mu} \quad (5.1)$$

$$i \frac{\partial \psi}{\partial t} + i \gamma^0 \boldsymbol{\gamma} \cdot \boldsymbol{\nabla} \psi - q \gamma^0 \gamma^{\mu} A_{\mu} \psi - m \gamma^0 \psi = 0 \quad (5.2)$$

$$\hat{\mathbf{H}} = (m \gamma^0 - i \gamma^0 \boldsymbol{\gamma} \cdot \boldsymbol{\nabla}) + q \gamma^0 \gamma^{\mu} A_{\mu} \quad (5.3)$$

这一节的例子可以不看。

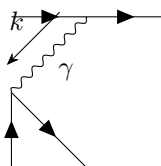
5.4 Feynman Rules for QED

$$e^{-} e^{-} \longrightarrow e^{-} e^{-}$$

$$e^{-} e^{+} \longrightarrow e^{-} e^{+}$$

$$-i \mathcal{M} = \quad (5.4)$$

6 e-p Annihilation



计算 matrix element

6.1 Perturbation Theory

6.2 e-p Annihilation

$$\mathcal{M} = \dots \quad (6.1)$$

6.3 Spin Sums

$$\langle |\mathcal{M}|^2 \rangle = \frac{1}{4} \sum_{\text{spins}} |\mathcal{M}|^2 \quad (6.2)$$

6.4 Helicity Amplitudes

6.5 The μ and e Currents

6.6 Cross Section

$$\mathcal{M}_{RL \rightarrow RL} = e^2(1 + \cos \theta) \quad (6.3)$$

$$\langle |\mathcal{M}|^2 \rangle = e^4(1 + \cos^2 \theta) \quad (6.4)$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4(1 + \cos^2 \theta)}{64\pi^2 s} \quad (6.5)$$

$$\sigma = \frac{4\pi\alpha^2}{3s} \quad (6.6)$$

total spin = $\pm 1 \Rightarrow$ contribution to helicity amplitude. (because photon has spin 1)

6.7

6.8 Spin in e-p Anni.

$$|1, +1\rangle_\theta = \frac{1}{2}(1 - \cos \theta) |1, -1\rangle + \frac{1}{\sqrt{2}} \sin \theta |1, 0\rangle + \frac{1}{2}(1 + \cos \theta) |1, +1\rangle \quad (6.7)$$

thus

$$\mathcal{M}_{RL \rightarrow RL} \propto \langle 1, +1 | 1, +1 \rangle_\theta = \frac{1}{2}(1 + \cos \theta) \quad (6.8)$$

6.9 Chirality ★

def

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix} \quad (6.9)$$

in the limit $E \gg m$

$$\gamma^5 u_\uparrow = +u_\uparrow \quad \gamma^5 u_\downarrow = -u_\downarrow \quad (6.10)$$

def: left- and right-handed chiral states

$$\gamma^5 u_R = +u_R \quad \gamma^5 u_L = -u_L \quad (6.11)$$

$$u_R = N \begin{pmatrix} c \\ s e^{i\phi} \\ c \\ s e^{i\phi} \end{pmatrix} \quad u_L = \quad (6.12)$$

chiral states are identical to massless helicity states.

6.9.1 Chiral Projection Operators

$$\begin{aligned} P_R &= \frac{1}{2}(1 + \gamma^5) \\ P_L &= \frac{1}{2}(1 - \gamma^5) \end{aligned} \quad (6.13)$$

features

$$P_R + P_L = 1 \quad P_R P_R = P_R \quad P_R P_L = 0 \quad (6.14)$$

$$P_R u_R = u_R \quad \dots \quad (6.15)$$

$$P_L u_R = 0 \quad P_L u_L = u_L \quad (6.16)$$

thus any spinor can be decomposed with

$$u = a_R u_R + a_L u_L = P_R u + P_L u \quad (6.17)$$

6.9.2 Chirality in QED

$$\bar{\psi} \gamma^\mu \phi \quad (6.18)$$

7 E-p Elastic Scattering

7.1 Probing ...

7.2 Rutherford and Mott Scattering ★

$$\mathcal{M}_{fi} = \frac{e^2}{q^2} \bar{u}_3 \gamma^\mu u_1 g_{\mu\nu} \bar{u}_4 \gamma^\nu u_2 \quad (7.1)$$

Helicity states

$$u_\uparrow = \dots \quad u_\downarrow = \dots \quad (7.2)$$

where

$$\kappa = \frac{p}{E + m_e} = \frac{\beta_e \gamma_e}{\gamma_e + 1} \quad (7.3)$$

NR: $\kappa \rightarrow 0$, R: $\kappa \approx 1$
init e^- : $(\theta, \phi) = (0, 0)$
fin e^- : $(\theta, \phi) = (\theta, 0)$

$$j_{\uparrow\uparrow}^e = (E + m_e)[(\kappa^2 + 1)c, 2\kappa s, 2i\kappa s, 2\kappa c] \quad (7.4)$$

$\kappa = 1 \Rightarrow j_{\uparrow\downarrow}^e = j_{\downarrow\uparrow}^e = 0$, the same with last week.
Helicity is effectively conserved at vertices in high energy limits.

init p: $\theta = 0, \phi = 0, \kappa = 0$
fin p: $\theta = \eta, \phi = 0, \kappa \approx 0$

$$\begin{aligned} j_{\uparrow\uparrow}^p &= j_{\downarrow\downarrow}^p = 2m_p[c_\eta, 0, 0, 0] \\ j_{\uparrow\downarrow}^p &= j_{\downarrow\uparrow}^p = -2m_p[s_\eta, 0, 0, 0] \end{aligned} \quad (7.5)$$

all NR.

$$\begin{aligned} \langle |\mu|^2 \rangle &= \frac{1}{4} \sum |\mathcal{M}_{fi}^2| = \frac{4m_p^2 m_e^2 e^4 (\gamma_e + 1)^2}{q^4} [(1 - \kappa^2)^2 + 4\kappa^2 c^2] \\ &= \frac{16m_p^2 m_e^2 e^4}{q^4} \left[1 + \beta_e^2 \gamma_e^2 \cos^2 \frac{\theta}{2} \right] \end{aligned} \quad (7.6)$$

$$q^4 = \dots \quad (7.7)$$

- i) e^- is NR, $1 + \beta_e^2 \gamma_e^2 \cos^2 \frac{\theta}{2} \approx 1$ (Rutherford)
- ii) e^- is R, $1 + \beta_e^2 \gamma_e^2 \cos^2 \frac{\theta}{2} \approx \beta_e^2 \gamma_e^2 \cos^2 \frac{\theta}{2}$ (Mott)

7.2.1 Rutherford Scattering

$$\langle |\mu|^2 \rangle = \frac{m_p^2 m_e^2 e^4}{p^4 \sin^4(\theta/2)} \quad (7.8)$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \left(\frac{1}{m_p + E_1 - E_1 \cos \theta} \right) \langle |\mu|^2 \rangle \quad (7.9)$$

Since

$$E_1 \sim m_e \ll m_p \quad (7.10)$$

we have

$$\frac{d\sigma}{d\Omega} = \dots \quad (7.11)$$

7.2.2 Mott Scattering

$E \approx p_e$

$$\langle |\mu|^2 \rangle = \frac{m_p^2 e^4}{E^2 \sin^4(\theta/2)} \cos^2 \frac{\theta}{2} \quad (7.12)$$

$$\frac{d\sigma}{d\Omega} = \dots \quad (7.13)$$

7.3 Form Factors ★

形状因子

charge density $Q\rho(\mathbf{r}')$, where $\int \rho(\mathbf{r}') = 1$

potential

$$V(\mathbf{r}) = \int \frac{Q\rho(\mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' \quad (7.14)$$

In the Born approx.,

$$\psi_i = e^{i(\mathbf{p}_1 \cdot \mathbf{r} - Et)} \quad \psi_f = e^{i(\mathbf{p}_3 \cdot \mathbf{r} - Et)} \quad (7.15)$$

$$\begin{aligned} \mathcal{M}_{fi} &= \langle \psi_f | V(\mathbf{r}) | \psi_i \rangle = \int e^{i\mathbf{q} \cdot \mathbf{r}} \int \frac{Q\rho(\mathbf{r}')}{4\pi|\mathbf{r} - \mathbf{r}'|} d^3\mathbf{r}' d^3\mathbf{r} \\ &= \int e^{i\mathbf{q} \cdot \mathbf{R}} \int \frac{Q}{4\pi|\mathbf{R}|} d^3\mathbf{R} \int \rho(\mathbf{r}') e^{i\mathbf{q} \cdot \mathbf{r}'} d^3\mathbf{r}' \\ &\equiv \mathcal{M}_{fi}^{\text{pt}} F(\mathbf{q}^2) \end{aligned} \quad (7.16)$$

pt: point-like

Form factor

$$F(\mathbf{q}^2) = \int \rho(\mathbf{r}') e^{i\mathbf{q} \cdot \mathbf{r}'} d^3\mathbf{r}' \quad (7.17)$$

i.e. Fourier transform of chg density distribution.

$$F(0) = 1 \quad F(\infty) = 0 \quad (7.18)$$

Note 7.8 留数定理 ★

7.4 Relativistic e-p Elastic Scattering

$$p_1 = (E_1, 0, 0, E_1) \quad (7.19)$$

$$p_2$$

$$q^2 = -2E_1 E_3 (1 - \cos \theta) \quad (7.20)$$

def $Q^2 = -q^2$

$$\begin{aligned} \frac{d\sigma}{d\Omega} &= \frac{1}{64\pi^2} \left(\frac{E_3}{m_p E_1} \right)^2 \langle |\mathcal{M}_{fi}|^2 \rangle \\ &= \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\cos^2 \frac{\theta}{2} + \frac{Q^2}{2m_p^2} \sin^2 \frac{\theta}{2} \right) \end{aligned} \quad (7.21)$$

when $Q^2 \ll m_p^2$ and $E_3 \approx E_1$, reduced to Mott.

7.5 the Rosenbluth Formula

7.5.1 Measuring G

$$\frac{d\sigma}{d\Omega} \bigg/ \left(\frac{d\sigma}{d\Omega} \right)_{\text{Mott}} = \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \quad (7.22)$$

G can be fit by a dipole function

$$G_E^2(Q^2) = \frac{1}{(1 + Q^2/Q_0^2)^2} \quad (7.23)$$

$$G_M^2(Q^2) = 2.79 G_E^2(Q^2) \quad (7.24)$$

charge distr.

$$\rho(r) = \rho_0 e^{-r/a} \quad (7.25)$$

$$a = 0.24 fm$$

7.5.2 Elastic Scattering at high Q^2

homework 0508

chap7 2,3,4,5,8

7.8 Q_0 数据错了

8 Deep Inelastic Scattering (DIS)

8.1 E-p Inelastic Scattering

$e^- + p \rightarrow e^- + X$

invariant mass of the hadronic system X : $W^2 = p_4^2$

elastic: $W^2 = m_p^2$

8.1.1 Kinematic Variables

$$Q^2 = -(p_1 - p_3)^2 = -(2m_e^2 - 2p_1 p_3) = -2m_e^2 + 2E_1 E_3 - 2\mathbf{p}_1 \cdot \mathbf{p}_3 \quad (8.1)$$

inelastic \rightarrow high E, thus

$$Q^2 = 2E_1 E_3 (1 - \cos \theta) = \quad (8.2)$$

def

$$x \equiv \frac{Q^2}{2p_2 q} \quad y \equiv \frac{p_2 q}{p_2 p_1} \quad \nu \equiv \frac{p_2 q}{m_p} \quad (8.3)$$

$$W^2 = (q + p_2)^2 = m_p^2 + q^2 + 2p_2 q \quad (8.4)$$

$$W^2 + Q^2 - m_p^2 = 2p_2 q \quad (8.5)$$

$$x \equiv \frac{Q^2}{Q^2 + W^2 - m_p^2} = \frac{Q^2}{2p_2 q} \quad (8.6)$$

elastic: $x = 1$

inelastic: $0 \leq x \leq 1$, since ...

x expresses the elasticity.

$$y = \frac{p_2 q}{p_2 p_1} = \dots = 1 - \frac{E_3}{E_1} \quad (8.7)$$

y expresses the inelasticity, or fractional energy loss of e^- .

$$\nu = \dots = E_1 - E_3 \quad (8.8)$$

the kinematics of inelastic scattering can be described by any two of the Lorentz-invariant quantities x , Q^2 , y and ν , except $\{y, \nu\}$.

8.1.2 IS at Low Q^2

$$\begin{aligned}
W^2 &= m_p^2 + q^2 + 2p_2q = m_p^2 + (p_1 - p_3)^2 + 2p_2(p_1 - p_3) \\
&= m_p^2 - 2E_1E_3(1 - \cos\theta) + 2m_p(E_1 - E_3) \\
&= [m_p^2 + 2m_pE_1] - [2m_p + 2E_1(1 - \cos\theta)]E_3
\end{aligned} \tag{8.9}$$

...

8.2 Deep Inelastic Scattering

Elastic Rosenbluth formula

$$\frac{d\sigma}{d\Omega} = \dots \tag{8.10}$$

Using def of Q^2 and y

$$\frac{d\sigma}{d\Omega} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{1 + \tau} \left(1 - y - \frac{m_p^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right] \tag{8.11}$$

def: $f_1(Q^2) = \dots, f_2(Q^2) = \dots$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{m_p^2 y^2}{Q^2} \right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2) \right] \tag{8.12}$$

8.2.1 Structure Func.

$$\frac{d\sigma}{dx} Q^2 = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{m_p^2 y^2}{Q^2} \right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \tag{8.13}$$

deep IS: $Q^2 \gg m_p^2 y^2$

$$\frac{d\sigma}{dx} Q^2 = \frac{4\pi\alpha^2}{Q^4} \left[(1 - y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \tag{8.14}$$

SLAC's 2 striking features Bjorken scaling

$$F_1(x, Q^2) \rightarrow F_1(x) \quad F_2(x, Q^2) \rightarrow F_2(x) \tag{8.15}$$

almost independent of Q^2 . point-like constituents within the proton.

Callan-Gross relation

$$F_2(x) = 2xF_1(x) \tag{8.16}$$

8.3 Elec-quark Scattering

[e-p scat. cross sect.] = [pdf(slow)] \times [e-q scat. cross sect.(fast)]

pdf: parton distribution function

e-q scat. in CoM frame

$$\frac{d\sigma}{d\Omega} = \frac{Q_q^2 e^4}{8\pi^2 s} \frac{1 + \frac{1}{4}(1 + \cos\theta)^2}{1 - \cos\theta^2} \tag{8.17}$$

\mathcal{L} .I. form

$$\frac{d\sigma}{dq^2} = \frac{d\sigma}{d\Omega} \left| \frac{d\Omega}{dq^2} \right| = \dots \tag{8.18}$$

8.4 The Quark Model

Infinite Momentum Frame $E_p \gg m_p$
the struck quark

$$p_q = \xi p_2 = (\xi E_2, 0, 0, \xi E_2) \quad (8.19)$$

homework May 15
8.2 8.3 8.6 8.8

$$(\xi p_2 + q)^2 = m_q^2 \quad (\xi p_2)^2 = m_q^2 \quad (8.20)$$

$$\xi = \frac{-q^2}{2p_2 q} = x \quad (8.21)$$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4} \left[1 + \left(1 + \frac{q^2}{s_q} \right)^2 \right] \quad (8.22)$$

where $s_q = (p_1 + xp_2)^2 = xs$
And

$$y_q = \frac{xp_2 q}{xp_2 p_1} = y \quad x_q = 1 \text{ (e-q is elastic)} \quad (8.23)$$

$$\frac{q^2}{s_q} = \frac{-(s_q - m_q^2)x_q y_q}{s_q} = -y \quad (8.24)$$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4} \left[1 + (1 - y)^2 \right] \quad (8.25)$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 Q_q^2}{Q^4} \left[1 - y + \frac{y^2}{2} \right] \quad (8.26)$$

8.4.1 PDF

$$\frac{d\sigma}{dx} Q^2 = \frac{4\pi\alpha^2}{Q^4} \left[(1 - y) + \frac{y^2}{2} \right] \sum_i Q_i^2 q_i(x) \quad (8.27)$$

which predicts Bjorken scaling and CG relation.

8.4.2 Determination of the PDFs

But PDFs cannot be computed in QFT.

$$F_2^{ep} = x \left(\frac{4}{9} u^p(x) + \frac{1}{9} d^p(x) + \frac{4}{9} \bar{u}^p(x) + \frac{1}{9} \bar{d}^p(x) \right) \quad (8.28)$$

valence quark: $p = uud, n = udd$

sea quark: any flavor can be produced via gluon, so anti-quark be taken into the eq above.

$$F_2^{en} = x \left(\frac{4}{9} u^n(x) + \frac{1}{9} d^n(x) + \frac{4}{9} \bar{u}^n(x) + \frac{1}{9} \bar{d}^n(x) \right) \quad (8.29)$$

From symm

$$u^n(x) = d^p(x) \quad (8.30)$$

thus, def

$$F_2^{ep} = x \left(\frac{4}{9} u(x) + \frac{1}{9} d(x) + \frac{4}{9} \bar{u}(x) + \frac{1}{9} \bar{d}(x) \right) \quad (8.31)$$

$$F_2^{en} = x \left(\frac{4}{9} d(x) + \frac{1}{9} u(x) + \frac{4}{9} \bar{d}(x) + \frac{1}{9} \bar{u}(x) \right) \quad (8.32)$$

$$\int_0^1 F_2^{ep}(x) dx = \dots \equiv \frac{4}{9} f_u + \frac{1}{9} f_d \quad (8.33)$$

$$\int_0^1 F_2^{en}(x) dx \equiv \frac{4}{9} f_d + \frac{1}{9} f_u \quad (8.34)$$

By experiment

$$f_u = 0.36 \quad f_d = 0.18 \quad (8.35)$$

8.4.3 Valence and Sea Quarks

$x \rightarrow 0$, $R \rightarrow 1$

$x \rightarrow 1$, $R \rightarrow 1/4$, while $R_{naive} = 2/3$.

why?