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29 Bose-Einstein and Fermi-Dirac Distributions

29.1

29.2 Wave Function of Identical Particles

29.3 The Statistics of Identical Particles

suppose energy of each particle is E

$$\mathcal{Z} = \sum_N \sum_\alpha \mathrm{e}^{\beta(\mu N - E_\alpha)} = \sum_N \mathrm{e}^{N\beta(\mu - E)} \tag{29.3.1}$$

thus

$$\langle n \rangle = \frac{\sum_N N\,\mathrm{e}^{N\beta(\mu-E)}}{\sum_N \mathrm{e}^{N\beta(\mu-E)}} = -\frac{1}{\beta \mathcal{Z}}\frac{\partial \mathcal{Z}}{\partial E} = -\frac{1}{\beta}\frac{\partial \ln \mathcal{Z}}{\partial E} \tag{29.3.2}$$

for fermions

$$\mathcal{Z} = \sum_{N=0}^{1} \mathrm{e}^{N\beta(\mu-E)} = 1 + \mathrm{e}^{\beta(\mu-E)} \tag{29.3.3}$$

Fermi-Dirac distribution fxn

$$f_D(E) \equiv \langle n \rangle = \frac{\mathrm{e}^{\beta(\mu-E)}}{1+\mathrm{e}^{\beta(\mu-E)}} = \frac{1}{\mathrm{e}^{\beta(E-\mu)+1}} \tag{29.3.4}$$

for bosons

$$\mathcal{Z} = \sum_{N=0}^{\infty} \mathrm{e}^{N\beta(\mu-E)} = \frac{1}{1-\mathrm{e}^{\beta(\mu-E)}} \tag{29.3.5}$$

Bose-Einstein distribution fxn

$$f_B(E) \equiv \langle n \rangle = \frac{\mathrm{e}^{\beta(\mu-E)}}{1-\mathrm{e}^{\beta(\mu-E)}} = \frac{1}{\mathrm{e}^{\beta(E-\mu)}-1} \tag{29.3.6}$$

30 Quantum Gases and Condensates

30.1 non-interacting q fluid

$$\mathcal{Z} = \prod_k \mathcal{Z}_k^{2S+1} \tag{30.1.1}$$

where $\mathcal{Z}_k$ refers to (29.3.3)(29.3.5)
i.e.

$$\mathcal{Z}_k = (1 \pm \mathrm{e}^{(\mu - E)})^{\pm 1} \tag{30.1.2}$$

grand potential

$$\begin{aligned}
\Phi &= -\frac{1}{\beta}\ln\mathcal{Z} \\
&= \mp\frac{2S+1}{\beta}\sum_k \ln\bigl(1 \pm \mathrm{e}^{(\mu-E)}\bigr) \\
&= \mp\frac{1}{\beta}\int_0^\infty \ln\bigl(1 \pm \mathrm{e}^{(\mu-E)}\bigr) g(E)\mathrm{d}E
\end{aligned} \tag{30.1.3}$$

$$g(k)\mathrm{d}k = \frac{4\pi k^2 \mathrm{d}k}{(2\pi/L)^3}(2S+1) = \frac{(2S+1)Vk^2\mathrm{d}k}{2\pi^2} \tag{30.1.4}$$

$$g(E)\mathrm{d}E = \frac{(2S+1)VE^{1/2}\mathrm{d}E}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2} \tag{30.1.5}$$

thus

$$\begin{aligned}
\Phi &= \mp\frac{(2S+1)V}{4\pi^2\beta}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \ln\bigl(1 \pm \mathrm{e}^{\beta(\mu-E)}\bigr)E^{1/2}\mathrm{d}E \\
&= \mp\frac{(2S+1)V}{4\pi^2\beta}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \ln\bigl(1 \pm \mathrm{e}^{\beta(\mu-E)}\bigr)\frac{2}{3}\,\mathrm{d}\bigl(E^{3/2}\bigr) \\
&= \mp\frac{2}{3}\frac{(2S+1)V}{4\pi^2\beta}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left\{\Bigl[\ln\bigl(1 \pm \mathrm{e}^{\beta(\mu-E)}\bigr)E^{3/2}\Bigr]\Big|_0^\infty - \int_0^\infty E^{3/2}\frac{\mp\beta\,\mathrm{e}^{\beta(\mu-E)}}{1 \pm \mathrm{e}^{\beta(\mu-E)}}\mathrm{d}E\right\} \\
&= \mp\frac{2}{3}\frac{(2S+1)V}{4\pi^2\beta}\left(\frac{2m}{\hbar^2}\right)^{3/2}\left\{0 \pm \beta\int_0^\infty E^{3/2}\frac{1}{\mathrm{e}^{\beta(E-\mu)} \pm 1}\mathrm{d}E\right\} \\
&= \frac{2}{3}\frac{(2S+1)V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\int_0^\infty \frac{E^{3/2}}{\mathrm{e}^{\beta(E-\mu)} \pm 1}\mathrm{d}E
\end{aligned} \tag{30.1.6}$$