137A Final, Fall 2018

Write your name and Student ID number on the front page of your blue/green book.

There are TEN problems which have a total score of 150 Points. You are responsible for solving all of them

Show your work, and take care to explain what you are doing. Cross out or erase parts of the problem you wish the grader to ignore.

Useful formulas

$$\int_{-\infty}^{\infty} e^{-ax^2}\, dx = \sqrt{\frac{\pi}{a}} \qquad \int_{0}^{\infty} x^n\, e^{-ax}\, dx = \frac{n!}{a^{n+1}}$$

$$\int_{0}^{\pi} \cos^2(nx) dx = \frac{\pi}{2} \qquad \int_{0}^{\pi} \sin(x)\, \cos(x)\, dx = 0$$

$$\int_{0}^{\pi} \sin(x)\, \cos^2(x)\, dx = \frac{2}{3} \qquad \int_{0}^{\pi} \sin^2(nx) dx = \frac{\pi}{2}$$

One-dimensional Schrödinger equation: $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$.

One-dimensional momentum operator: $\hat{p} = -i\hbar\frac{\partial}{\partial x}$.

Harmonic Oscillator

$$V(x) = \frac{1}{2}m\omega^2 x^2.$$

$$\hat{a}_{\pm} = \frac{1}{\sqrt{2\hbar m\omega}}(\mp i\hat{p} + m\omega\hat{x}).$$

$$[\hat{a}_-, \hat{a}_+] = 1$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}m\omega^2 x^2 = \hbar\omega(\hat{a}_+\hat{a}_- + \frac{1}{2}).$$

$$\hat{a}_+\psi_n^{ho} = \sqrt{n+1}\,\psi_{n+1}^{ho},$$
$$\hat{a}_-\psi_n^{ho} = \sqrt{n}\,\psi_{n-1}^{ho}.$$

$$\text{Energy Eigenstates } \hat{H}\psi_n^{ho} = E_n\psi_n^{ho}, n = 0, 1, 2, ...$$

$$\text{Energy } E_n = \left(n + \frac{1}{2}\right)\hbar\omega.$$

$$\psi_0^{ho}(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar}$$

One-dimensional infinite square well:

$$\text{Energies: } E_n = \frac{\hbar^2 n^2 \pi^2}{2ma^2}, \text{ where } a \text{ is the width of the well, and } n = 1, 2, \ldots$$

One-dimensional expectation value: $\langle \hat{O} \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) \hat{O} \psi(x,t) dx$.

Spin

$$S_\pm = S_x \pm i S_y.$$

$$S_\pm |s, m\rangle = \hbar \sqrt{s(s+1) - m(m \pm 1)}\, |s, m \pm 1\rangle\,.$$

Radial Equation in spherical coordinates:

$$-\frac{\hbar^2}{2m} \frac{d^2 u}{dr^2} + \left[V(r) + \frac{\hbar^2}{2m} \frac{l(l+1)}{r^2} \right] u(r) = E u(r), \text{ where } u(r) = rR(r).$$

Spherical Harmonics:

$$Y_0^0 = \frac{1}{2}\sqrt{\frac{1}{\pi}} \qquad Y_1^{-1} = \frac{1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{-i\varphi}$$

$$Y_1^0 = \frac{1}{2}\sqrt{\frac{3}{\pi}}\,\cos\theta \qquad Y_1^1 = \frac{-1}{2}\sqrt{\frac{3}{2\pi}}\,\sin\theta\, e^{i\varphi}$$

Hydrogen energy:

$$E_{nlm} = -R_E \frac{1}{n^2}, \text{ where } R_E = \frac{me^4}{32\hbar^2\pi^2\epsilon_0^2} \text{ is the Rydberg energy.}$$

Inner product in spherical coordinates:

$$\langle f|g\rangle = \int_0^\infty dr\, r^2 \int_{-1}^{1} d\cos(\theta) \int_0^{2\pi} d\phi\, f^*(r,\theta,\phi) g(r,\theta,\phi)$$