# Advanced Physical Chemistry II

## HW Part I

王石嵘 161240065

January 7, 2020

### 25 The Kinetic Theory of Gases

2,3,17,26,27,35,37,42

25-2

$$u_{\rm rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3RT}{28.02 \times 10^{-3}}}$$
 (25.1)

thus

$$u_{\rm rms}(200\,{\rm K}) = 421.95\,{\rm m/s}$$
 (25.2)

$$u_{\rm rms}(300\,{\rm K}) = 516.78\,{\rm m/s}$$
 (25.3)

$$u_{\rm rms}(500\,{\rm K}) = 667.16\,{\rm m/s}$$
 (25.4)

$$u_{\rm rms}(1000\,\rm K) = 943.50\,\rm m/s$$
 (25.5)

25-3 Since

$$u_{\rm rms} = \sqrt{\frac{3RT}{M}} \tag{25.6}$$

The RMS speed is increased by  $\sqrt{2}$ .

25-17 Since

$$f(u_x) = \sqrt{\frac{m}{2\pi k_{\rm B}T}} e^{-mu_x^2/2k_{\rm B}T}$$
 (25.7)

when  $u_x > 0$ 

$$\langle u_x \rangle = \int_0^\infty u_x f(u_x) du_x = \sqrt{\frac{m}{2\pi k_B T}} \int_0^\infty u_x e^{-mu_x^2/2k_B T} du_x$$

$$= \sqrt{\frac{m}{2\pi k_B T}} \left( -\frac{k_B T}{m} \right) (0 - 1)$$

$$= \sqrt{\frac{k_B T}{2\pi m}}$$
(25.8)

25-26 Since

$$F(\varepsilon) = \frac{2\pi}{(\pi k_{\rm B} T)^{3/2}} \varepsilon^{1/2} \,\mathrm{e}^{-\varepsilon/k_{\rm B} T} \tag{25.9}$$

Let  $\frac{\mathrm{d}F}{\mathrm{d}\varepsilon} = 0$ , we have

$$\frac{1}{2}\varepsilon^{-1/2} e^{-\varepsilon/k_{\rm B}T} + \varepsilon^{1/2} \left(-\frac{1}{k_{\rm B}T}\right) e^{-\varepsilon/k_{\rm B}T} = 0$$
 (25.10)

$$\varepsilon = \frac{k_{\rm B}T}{2} \tag{25.11}$$

25-27

$$\langle \varepsilon \rangle = \int_0^\infty \varepsilon F(\varepsilon) d\varepsilon$$

$$= \int_0^\infty \frac{2\pi}{(\pi k_{\rm B} T)^{3/2}} \varepsilon^{3/2} e^{-\varepsilon/k_{\rm B} T} d\varepsilon$$

$$= \frac{2\pi}{(\pi k_{\rm B} T)^{3/2}} \frac{3}{4} (k_{\rm B} T)^{5/2} \sqrt{\pi}$$

$$= \frac{3}{2} k_{\rm B} T \tag{25.12}$$

$$\langle \varepsilon^2 \rangle = \int_0^\infty \varepsilon^2 F(\varepsilon) d\varepsilon$$

$$= \int_0^\infty \frac{2\pi}{(\pi k_B T)^{3/2}} \varepsilon^{5/2} e^{-\varepsilon/k_B T} d\varepsilon$$

$$= \frac{2\pi}{(\pi k_B T)^{3/2}} \frac{15}{8} (k_B T)^{7/2} \sqrt{\pi}$$

$$= \frac{15}{4} (k_B T)^2$$
(25.13)

$$\sigma_{\varepsilon}^{2} = \langle \varepsilon^{2} \rangle - \langle \varepsilon \rangle^{2} = \frac{3}{2} (k_{\rm B} T)^{2}$$
(25.14)

thus

$$\frac{\sigma_{\varepsilon}}{\langle \varepsilon \rangle} = \sqrt{\frac{3}{2}} / \frac{3}{2} = \sqrt{\frac{2}{3}} \tag{25.15}$$

which means the fluctuations in  $\varepsilon$  are large with respect to  $\varepsilon$ .

25-35

$$z_{A} = \rho \sigma \sqrt{2} \sqrt{\frac{8RT}{\pi M}} = \frac{PN_{A}}{RT} \sigma \cdot 4 \sqrt{\frac{RT}{\pi M}}$$
$$= \frac{4\sigma N_{A}}{\sqrt{\pi MRT}} P \tag{25.16}$$

where  $\sigma = 0.230 \times 10^{-18} \,\mathrm{m}^2$ 

(a) 
$$z_A = \frac{4 \times 0.230 \times 10^{-18} \times 6.022 \times 10^{23}}{\sqrt{\pi \times 2.016 \times 10^{-3} \times 8.3145 \times 298.15}} \times 133.32 = 1.86 \times 10^7 \,\text{Hz}$$
 (25.17)

(b) 
$$z_A = \frac{4 \times 0.230 \times 10^{-18} \times 6.022 \times 10^{23}}{\sqrt{\pi \times 2.016 \times 10^{-3} \times 8.3145 \times 298.15}} \times 1 \times 10^5 = 1.40 \times 10^{10} \,\text{Hz}$$
 (25.18)

25-37 The probability that an  $O_2$  molecule will travel distance d without a collision is

$$P(d) = 1 - \int_0^d p(x) dx = 1 - \int_0^d \frac{1}{l} e^{-x/l} dx$$

$$= 1 - \frac{1}{l} (-l e^{-x/l}) \Big|_0^d$$

$$= 1 + (e^{-d/l} - 1)$$

$$= e^{-d/l}$$
(25.19)

Since the MFP

$$l = \frac{1}{\sqrt{2}\rho\sigma} = \frac{k_{\rm B}T}{\sqrt{2}\sigma P}$$

$$= \frac{1.38 \times 10^{-23} \times 298.15}{\sqrt{2} \times 0.410 \times 10^{-18} \times 1 \times 10^{5}}$$
$$= 7.10 \times 10^{-8} \text{ m} = 7.10 \times 10^{-5} \text{ mm}$$
(25.20)

we get

(a) 
$$P(1.00 \times 10^{-5} \,\text{mm}) = e^{-1.00 \times 10^{-5} / 7.10 \times 10^{-5}} = 0.869$$
 (25.21)

(b) 
$$P(1.00 \times 10^{-3} \text{ mm}) = e^{-1.00 \times 10^{-3} / 7.10 \times 10^{-5}} = 7.63 \times 10^{-7}$$
 (25.22)

(c) 
$$P(1.00 \,\mathrm{mm}) = \mathrm{e}^{-1.00/7.10 \times 10^{-5}} = 4.20 \times 10^{-6118} \tag{25.23}$$

25-42 Since

$$l = \frac{1}{\sqrt{2}\rho\sigma} = \frac{k_{\rm B}T}{\sqrt{2}\sigma P} \tag{25.24}$$

we have

$$P(l) = \frac{k_{\rm B}T}{\sqrt{2}\sigma l} \tag{25.25}$$

where  $\sigma = 0.230 \times 10^{-18} \, \mathrm{m^2}, \, T = 293.15 \, \mathrm{K}.$  thus

$$P(100 \,\mu\mathrm{m}) = 124 \,\mathrm{Pa}$$
  
 $P(1.00 \,\mathrm{mm}) = 12.4 \,\mathrm{Pa}$   
 $P(1.00 \,\mathrm{m}) = 0.0124 \,\mathrm{Pa}$  (25.26)

#### 26 Chemical Kinetics I: Rate Laws

26-47

$$\Delta^{\ddagger}G^{\circ} = \Delta^{\ddagger}H^{\circ} - T\Delta^{\ddagger}S^{\circ}$$

$$= 31.38 - 325 \times 16.74 \times 10^{-3}$$

$$= 25.94 \,\text{kJ/mol}$$
(26.1)

#### 28 The Rate of a Bimolecular Gas-Phase Reaction

1,4,6,10

Additional Problems

1. 对于单分子气相反应,活化熵变往往可忽略不计,试计算按室温 (200 K) 附近活化焓分别为  $60,\!80,\!100 \mathrm{kJ} \cdot \mathrm{mol}^{-1}$  时之反应比速及  $t_{1/2}$ 。

2