Notes of Advanced Physical Chemistry II

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January 7, 2020

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Introduction

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12 Group Theory: the Exploitation of Symmetry

Matrices

 $det(\mathbf{A}) = 0 \implies \mathbf{A}$ is a singular matrix.

- 12.1 The Exploitation of the Symm of a Mol Can Be Used to Significantly Simplify Numerical Calculations
- 12.2 The Symm of Mols Can Be Described by a Set of Symm Elements

E	
C_n	Rotation by $360^{\circ}/n$
σ	
i	
S_n	

Table 1: Symmetry elements and operators

Identity

Rotation

σ_h	horizontal
σ_v	vertical
σ_d	diagonal (vertical and bisects the angle between C_2 axis)

Table 2

Reflection

Inversion

Rotation Reflection

$$\hat{S}_n = \hat{\sigma}_h \times \hat{C}_n \tag{12.1}$$

12.2.1 Point Groups of Interest to Chemists

C_{nv}	
C_{nh}	Rotation by $360^{\circ}/n$
D_{nh}	
D_{nv}	
D_{nd}	
T_d	

Table 3: Symmetry elements and operators

The Symm Operators of a Mol Form a Group

A set of operators form a group if they satisfy:

- 1. closed under multiplication 乘法封闭
- 2. associative multiplication 乘法结合律
- 3. only one identity operator 单位元
- 4. everyone has only one inverse 逆元

12.3.1 Point Group for Some Mols

No Symm Axis

 C_1 – nothing C_s – σ

 $C_i - i$

 C_n

 S_n

 C_{nv} – C_n and $n\sigma_v$

 $C_{nh} - C_n$ and σ_h

 $D_n - C_n$ and $nC_2 \perp C_n$ e.g. 一点点交错的 C_3H_6, C_2 在 3 个角平分线处

 $D_{nd} - C_n(\text{also } S_{2n}) \text{ and } nC_2 \perp C_n \text{ and } n\sigma_d$

 D_{nh} – C_n and $nC_2 \perp C_n$ and σ_h

 T_d 主轴是 S_4

 O_h

 I_h

- 12.4 Symm Operators Can Be Represented by Matrices
- 12.5 The C_{3v} Point Group Has a 2-D Irreducible Representation
- 12.6 The Most Important Summary of the Properties of a Point Group Is Its Character Table

basis

class same characters - in a class.
of class = # of irred represtn.

notations

- 1. A:, B:, E:2D, T:3D
- 2. A_1 : symm wrt C_2/σ_v , A_2 : antisymm wrt that.
- 3. A': symm wrt σ_h , A'': antisymm wrt that.
- 4. A_g :, A_u :

12.7 Several Mathematical Relations Involve the Characters of Irreducible Representation

notations

XU G.X.	McQuarrie	
$D^{(\nu)}(R)$		
$\chi^{(\nu)}(R)$	$\chi_j(R)$	
$n_{ u}$	d_{j}	dimension of repr matrix
$a_{ u}$	a_{j}	
$\underline{}$	h	

Table 4

order

$$\sum_{\nu} n_{\nu}^2 = g \tag{12.2}$$

character

$$\sum_{R} D_{il}^{(\nu)} D_{jm}^{*(\mu)} = \frac{g}{n_{\nu}} \delta_{\mu\nu} \delta_{ij} \delta_{lm}$$

$$\tag{12.3}$$

$$\sum_{R} \chi^{(\nu)}(R) \chi^{*(\mu)}(R) = g \delta_{\mu\nu}$$
 (12.4)

$$\sum_{R} \chi^{(\nu)}(R) = 0 \quad (\nu \neq A_1)$$
(12.5)

reduce a given reducible repr Γ $\operatorname{Suppose}$

$$\chi(R) = \sum_{\nu} a_{\nu} \chi^{(\nu)}(R)$$
 (12.6)

thus

$$a_{\nu} = \frac{1}{g} \sum_{R} \chi(R) \chi^{(\nu)}(R)$$
 (12.7)

- 12.8 Use Symm Arguments to Predict Which Elements in a Secular Det Equals 0
- 12.9 Generating Operators Are Used to Find LCAOs That Are Bases for IrRepr

$$\widehat{\mathbf{P}}_{j} = \frac{d_{j}}{h} \sum_{\widehat{\mathbf{R}}} \chi_{j}(\widehat{\mathbf{R}}) \widehat{\mathbf{R}}$$
(12.8)

13 Molecular Spectroscopy

13.1

	micro	far IR	IR	visible & UV
f/Hz				
$\lambda/\mathrm{m} \ ar{ u}/\mathrm{cm}^{-1}$				
$\bar{ u}/\mathrm{cm}^{-1}$				
$E/\mathrm{J}\mathrm{mol}^{-1}$				
process				

Table 5

13.2 Rotational Transitions Accompany Vibrational Transitions

Vib & rot energy

$$\tilde{E} = G(\nu) + F(J) \tag{13.1}$$

$$= (v+1/2)\tilde{\nu} + \tilde{B}J(J+1) \tag{13.2}$$

selex rule:

$$\Delta v = \pm 1 \quad \Delta J = \pm 1 \tag{13.3}$$

P - left -
$$\Delta J = -1$$
 - wide
R - right - $\Delta J = +1$ - narrow

13.3

$$\tilde{B}_v = \tilde{B}_e - \tilde{\alpha}_e(v + 1/2) \tag{13.4}$$

$$\tilde{B}_0 > \tilde{B}_1 > \cdots \tag{13.5}$$

which makes P-branches wider.

13.4

$$F(J) = \tilde{B}J(J+1) - \tilde{D}J^2(J+1)^2 \tag{13.6}$$

 \tilde{D} :

13.5 Overtones Are Observed in Vibrational Spectra

$$G(v) = \tilde{\nu}_e \left(v + \frac{1}{2} \right) - \tilde{x}_e \tilde{\nu}_e \left(v + \frac{1}{2} \right)^2$$
(13.7)

 \tilde{x}_e : anharmonicity cons.

$$\tilde{v}_{obs} = \tilde{\nu}_e v - \tilde{x}_e \tilde{\nu}_e v(v+1) \tag{13.8}$$

v = 1, 2, ...

13.6 Electronic Spectra Contains Electronic, Vibrational and Rotational Info

$$\tilde{E} = n\tilde{u}_{el} + \tilde{\nu}_e(v + 1/2) - \tilde{x}_e\tilde{\nu}_e(v + 1/2)^2 + \dots (rot)$$
(13.9)

vibronic transitions:

 $0 \rightarrow 0$

$$\tilde{v}_{0,0} = \tilde{T}_e + \frac{1}{2} (\tilde{\nu}'_e - \tilde{\nu}''_e) - \frac{1}{4} (\tilde{x}'_e \tilde{\nu}' - \tilde{x}''_e \tilde{\nu}'')$$
(13.10)

- 13.7 Franck-Condon Principle Predicts the Relative Intensities of Vibronic Transitions
- 13.8 The Rotational Spectrum of a Polyatomic Mols Depends Upon the Principal Moments of Inertia of the Mol

$$\begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix} \xrightarrow{\text{diagnalization}} \begin{pmatrix} I_A & & \\ & I_B & \\ & & I_C \end{pmatrix}$$

$$(13.11)$$

top	requisition
sph top	$2C_n, n \geq 3$
prolate symm top	
oblate symm top	
asymm	
	sph top prolate symm top oblate symm top

Table 6

- 13.9 The Vibrations of Polyatomic Mols Are Represented by Normal Coordinates
- 13.10 Normal Coordinates Belong to Irreducible Representations of Mol Point Groups Contribution to $\chi(R)$ per unmoved atom

$\hat{\mathbf{R}}$ contribution per unmoved atom

Table 7

Now we get Γ_{3N} .

Subtract the irreducible representations corresponding to translational (x, y, z) and rotational (R_x, R_y, R_z) degrees of freedom, we get Γ_{vib} .

13.11 Selection Rules Are Derived from TD Perturbation Theory

Consider a mol interacting w/ EM radiation. The EM field

$$\mathbf{E} = \mathbf{E}_0 \cos 2\pi \nu t \tag{13.12}$$

$$\widehat{H}^{(1)} = -\mu \cdot \mathbf{E} = -\mu \mathbf{E}_0 \left(e^{i 2\pi \nu t} + e^{-i 2\pi \nu t} \right) / 2$$
(13.13)

$$\Psi(t) = a_1(t)\Psi_1(t) + a_2(t)\Psi_2(t)$$
(13.14)

$$a_1(t) \widehat{\mathbf{H}}^{(1)} \Psi_1 + a_2(t) \widehat{\mathbf{H}}^{(1)} \Psi_2 = i \hbar \left(\Psi_1 \frac{\mathrm{d}a_1}{\mathrm{d}t} + \Psi_2 \frac{\mathrm{d}a_2}{\mathrm{d}t} \right)$$
 (13.15)

$$a_1(t) \left\langle \psi_2 \left| \widehat{\mathbf{H}}^{(1)} \right| \Psi_1 \right\rangle + a_2(t) \left\langle \psi_2 \left| \widehat{\mathbf{H}}^{(1)} \right| \Psi_2 \right\rangle = \mathrm{i} \, \hbar \left(0 + \frac{\mathrm{d} a_2}{\mathrm{d} t} \, \mathrm{e}^{-\mathrm{i} \, E t / \hbar} \right)$$
(13.16)

. . .

$$i\hbar \frac{\mathrm{d}a_2}{\mathrm{d}t} = e^{-i(E_1 - E_2)t/\hbar} \left\langle \psi_2 \left| \widehat{\mathbf{H}}^{(1)} \right| \psi_1 \right\rangle$$
 (13.17)

$$\frac{\mathrm{d}a_2}{\mathrm{d}t} \approx \dots \tag{13.18}$$

13.12 The Selection Rule in the Rigid-Rotator Approx Is $\Delta J=\pm 1$

$$\langle J', M' | \mu_z | J, M \rangle = \int_0^{2\pi} d\phi \int_0^{\pi} Y_{J'}^{M'*} \mu_z Y_J^M \sin\theta d\theta$$
$$= \dots$$
(13.19)

- 13.13 The Harmonic-Oscillator Selection Rule Is $\Delta \nu = \pm 1$
- 14 Nuclear Magnetic Resonance Spectroscopy
- 14.1 Nuclei Have Intrinsic Spin Angular Momenta
- 14.2 Magnetic Moments Interact with Magnetic Fields
- 14.3
- 14.4 The Magnetic Field Acting upon Nuclei in Mols Is Shielded
- 14.5 Chemical Shifts Depend upon the Chemical Environment of the Nucleus
- 14.6 Spin-Spin Coupling

$$\hat{\mathbf{H}} = -\gamma B_0 (1 - \sigma_1) \hat{\mathbf{I}}_{z_1} - \gamma B_0 (1 - \sigma_2) \hat{\mathbf{I}}_{z_2} + 2\pi J_{12} \hat{\mathbf{I}}_1 \cdot \hat{\mathbf{I}}_2$$
(14.1)

$$\widehat{\mathbf{H}}^{(0)} = -\gamma B_0 (1 - \sigma_1) \hat{\mathbf{I}}_{z1} - \gamma B_0 (1 - \sigma_2) \hat{\mathbf{I}}_{z2} \quad \widehat{\mathbf{H}}^{(1)} 2\pi J_{12} \hat{\mathbf{I}}_1 \cdot \hat{\mathbf{I}}_2$$
(14.2)

$$\psi_1^{(0)} = \alpha(1)\alpha(2)$$
 $\psi_2^{(0)} = \beta(1)\alpha(2)$ (14.3)

$$\psi_3^{(0)} = \alpha(1)\beta(2) \qquad \qquad \psi_4^{(0)} = \beta(1)\beta(2) \tag{14.4}$$

$$E_1^{(0)} = -\gamma B_0 \left(1 - \frac{\sigma_1 + \sigma_2}{2} \right) \qquad \qquad E_2^{(0)} = -\gamma B_0 (\sigma_1 - \sigma_2) \tag{14.5}$$

$$E_3^{(0)} = \gamma B_0(\sigma_1 - \sigma_2) \qquad \qquad E_4^{(0)} = \gamma B_0 \left(1 - \frac{\sigma_1 + \sigma_2}{2} \right) \tag{14.6}$$

perturbed to 1st order

$$E_1 = -\gamma B_0 \left(1 - \frac{\sigma_1 + \sigma_2}{2} \right) + \frac{2\pi J_{12}}{4} \tag{14.7}$$

$$E_2 = -\gamma B_0(\sigma_1 - \sigma_2) - \frac{2\pi J_{12}}{4} \tag{14.8}$$

$$E_3 = \gamma B_0(\sigma_1 - \sigma_2) - \frac{2\pi J_{12}}{4} \tag{14.9}$$

$$E_4 = \gamma B_0 \left(1 - \frac{\sigma_1 + \sigma_2}{2} \right) + \frac{2\pi J_{12}}{4} \tag{14.10}$$

Since

$$\nu_0 = \frac{\gamma B_0}{2\pi} \tag{14.11}$$

$$\nu_{1\to 2} = \nu_0 (1 - \sigma_1) - \frac{J_{12}}{2} \tag{14.12}$$

$$\nu_{1\to 3} = \nu_0 (1 - \sigma_2) - \frac{J_{12}}{2} \tag{14.13}$$

$$\nu_{2 \to 4} = \nu_0 (1 - \sigma_2) + \frac{J_{12}}{2} \tag{14.14}$$

$$\nu_{3\to 4} = \nu_0(1-\sigma_1) + \frac{J_{12}}{2} \tag{14.15}$$

14.7 Spin-Spin Coupling Between Chemically Equivalent Protons

14.8 The n+1 Rule

14.9 2nd-Order Spectra

Only for the case in which

$$J << \nu_0 |\sigma_1 - \sigma_2| \tag{14.16}$$

the n+1 spectra.

Lasers, Laser Spectroscopy and Photochemistry 15

15.1

The Dynamics of Spectroscopic Transitions between the Electronic States 15.2 absorption

$$-\frac{\mathrm{d}N_1(t)}{\mathrm{d}t} = B_{12}\rho_{\nu}(\nu_{12})N_1(t) \tag{15.1}$$

emission

- 15.3 Population Inversion, 2-Level System
- Population Inversion, 3-Level System
- 15.5 What is Inside a Laser?
- 15.6 He-Ne Laser
- High-Resolution Laser Spectroscopy

Hyperfine structure

15.8 The Dynamics of Photochemistry Process

def: quantum yield

$$\Phi = \frac{\text{# mols undergoing reaction}}{\text{# photons absorbed}}$$
 (15.2)

The Properties of Gases 16

17 The Boltzmann Factor and Partition Functions

17.1

17.2 Partition Function

$$Q = \tag{17.1}$$

$$Q =$$

$$p_j = \frac{e^{-E_j \beta}}{Q}$$

$$(17.1)$$

- 17.3 Thermodynamic Quantities
- 17.3.1 Energy
- 17.3.2 Work and Heat
- 17.4 Pressure
- 17.4.1 Heat Capacity
- 17.4.2 Entropy

17.5

17.6 The Partition Function of a System of Independent, Distinguishable Mols

$$Q(N, V, T) = \sum_{i,j,k,\dots} e^{-\beta(\varepsilon_i + \varepsilon_j + \varepsilon_k)} = \sum_i e^{-\beta\varepsilon_i} \sum_j e^{-\beta\varepsilon_j} \dots$$
 (17.3)

The Partition Function of a System of Independent, Indistinguishable Mols

$$Q(N, V, T) = \frac{q(V, T)^N}{N!}$$
(17.4)

17.8

$$\langle \varepsilon \rangle = \sum_{j} \frac{\varepsilon_{j} e^{-\beta \varepsilon_{j}}}{q} \tag{17.5}$$

$$\varepsilon = \varepsilon_i^{trans} + \varepsilon_i^{rot} + \varepsilon_k^{vib} + \varepsilon_l^{elec} \tag{17.6}$$

$$\varepsilon = \varepsilon_i^{trans} + \varepsilon_j^{rot} + \varepsilon_k^{vib} + \varepsilon_l^{elec}$$

$$\pi_{ijkl} = \frac{e^{-\beta \varepsilon_i^{trans}} e^{-\beta \varepsilon_j^{rot}} e^{-\beta \varepsilon_k^{vib}} e^{-\beta \varepsilon_l^{elec}}}{q_{trans} q_{rot} q_{vib} q_{elec}}$$
(17.6)

- Partition Functions and Ideal Gases 18
- The Translational PF of Monatomic Ideal Gas

$$q_{\rm trans} = \left(\frac{2\pi m k_{\rm B} T}{h^2}\right)^{3/2} T \tag{18.1}$$

- 18.2 The Electronic PF
- 18.3
- The Rotational PF

$$q_{\text{rot}} = \sum_{J=0} (2J+1) e^{-J(J+1)\Theta_{\text{rot}}/T}$$
 (18.2)

where

$$\Theta_{rot} = \frac{\hbar^2}{2Ik_{\rm B}} \tag{18.3}$$

At low $\Theta_{\rm rot}$

$$q_{\rm rot} = \dots = \frac{T}{\Theta_{\rm rot}} \tag{18.4}$$

Symmetry Number

$$q_{\rm rot} = \frac{T}{\sigma \Theta_{\rm rot}} \tag{18.5}$$

Spherical top

$$q_{\rm rot} = \frac{\sqrt{\pi}}{\sigma} \left(\frac{T}{\Theta_{\rm rot}}\right)^{3/2} \tag{18.6}$$

25 The Kinetic Theory of Gases

25.1

25.2 Speed Distribution

$$f(u_x) = \sqrt{\frac{m}{2\pi k_{\rm B}T}} e^{-mu_x^2/2k_{\rm B}T}$$
 (25.1)

$$\langle u_x^2 \rangle = \frac{k_{\rm B}T}{m} = \frac{RT}{M} \tag{25.2}$$

25.3 Maxwell Distribution

$$F(u) = 4\pi u^2 \left(\frac{m}{2\pi k_{\rm B}T}\right)^{3/2} e^{-mu^2/2k_{\rm B}T}$$
(25.3)

$$\langle u \rangle = \sqrt{\frac{8k_{\rm B}T}{\pi m}} \tag{25.4}$$

$$\langle u^2 \rangle = \frac{3k_{\rm B}T}{m} \tag{25.5}$$

$$u_{mp} = \sqrt{\frac{2k_{\rm B}T}{m}} \tag{25.6}$$

$$F(\varepsilon) = \frac{2\pi}{(\pi k_{\rm B} T)^{3/2}} \varepsilon^{1/2} e^{-\varepsilon/k_{\rm B} T}$$
 (25.7)

$$\langle \varepsilon \rangle = \frac{3}{2} k_{\rm B} T \tag{25.8}$$

25.4 The Frequency of Collisions with a Wall

$$dz = \frac{1}{A} \frac{dN}{dt} \tag{25.9}$$

freq per area

$$z = \frac{\rho}{4} \left\langle u \right\rangle \tag{25.10}$$

25.5 Inter-collision and MFP

$$z_{A} = \rho \sigma \left\langle u_{r} \right\rangle = \rho \sigma \sqrt{2} \left\langle u \right\rangle \tag{25.11}$$

$$l = \frac{u}{z_A} = \frac{1}{\sqrt{2\rho\sigma}} \tag{25.12}$$

$$p(x)\mathrm{d}x = \frac{1}{l} e^{-x/l} \,\mathrm{d}x \tag{25.13}$$

26 Chemical Kinetics I: Rate Laws

26.8 Reaction Rate Constants

$$k(T) = \frac{k_{\rm B}T}{hc^{\circ}} \,\mathrm{e}^{-\Delta^{\ddagger}G^{\circ}/RT} \tag{26.1}$$