

# Notes of **Advanced Physical Chemistry II**

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## Introduction

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## 12 Group Theory: the Exploitation of Symmetry

### Matrices

$\det(\mathbf{A}) = 0 \Rightarrow \mathbf{A}$  is a singular matrix.

### 12.1 The Exploitation of the Symm of a Mol Can Be Used to Significantly Simplify Numerical Calculations

### 12.2 The Symm of Mols Can Be Described by a Set of Symm Elements

$E$	
$C_n$	Rotation by $360^\circ/n$
$\sigma$	
$i$	
$S_n$	

Table 1: Symmetry elements and operators

### Identity

### Rotation

$\sigma_h$	horizontal
$\sigma_v$	vertical
$\sigma_d$	diagonal (vertical and bisects the angle between $C_2$ axis)

Table 2

### Reflection

### Inversion

### Rotation Reflection

$$\hat{S}_n = \hat{\sigma}_h \times \hat{C}_n \quad (12.1)$$

### 12.2.1 Point Groups of Interest to Chemists

$C_{nv}$	Rotation by $360^\circ/n$
$C_{nh}$	
$D_{nh}$	
$D_{nv}$	
$D_{nd}$	
$T_d$	

Table 3: Symmetry elements and operators

## 12.3 The Symm Operators of a Mol Form a Group

A set of operators form a group if they satisfy:

1. closed under multiplication 乘法封闭
2. associative multiplication 乘法结合律
3. only one identity operator 单位元
4. everyone has only one inverse 逆元

### 12.3.1 Point Group for Some Mols

No Symm Axis

$C_1$  – nothing

$C_s$  –  $\sigma$

$C_i$  –  $i$

$C_n$

$S_n$

$C_{nv}$  –  $C_n$  and  $n\sigma_v$

$C_{nh}$  –  $C_n$  and  $\sigma_h$

$D_n$  –  $C_n$  and  $nC_2 \perp C_n$

e.g. 一点点交错的  $C_3H_6$ ,  $C_2$  在 3 个角平分线处

$D_{nd}$  –  $C_n$ (also  $S_{2n}$ ) and  $nC_2 \perp C_n$  and  $n\sigma_d$

$D_{nh}$  –  $C_n$  and  $nC_2 \perp C_n$  and  $\sigma_h$

$T_d$  主轴是  $S_4$

$O_h$

$I_h$

#### 12.4 Symm Operators Can Be Represented by Matrices

#### 12.5 The $C_{3v}$ Point Group Has a 2-D Irreducible Representation

#### 12.6 The Most Important Summary of the Properties of a Point Group Is Its Character Table

basis

class same characters – in a class.

# of class = # of irred representn.

notations

1.  $A_1, B_1, E:2D, T:3D$
2.  $A_1$ : symm wrt  $C_2/\sigma_v$ ,  $A_2$ : antisymm wrt that.
3.  $A'$ : symm wrt  $\sigma_h$ ,  $A''$ : antisymm wrt that.
4.  $A_g, A_u$ :

#### 12.7 Several Mathematical Relations Involve the Characters of Irreducible Representation

notations

XU G.X.	McQuarrie	
$D^{(\nu)}(R)$		
$\chi^{(\nu)}(R)$	$\chi_j(R)$	
$n_\nu$	$d_j$	dimension of repr matrix
$a_\nu$	$a_j$	
$g$	$h$	

Table 4

order

$$\sum_{\nu} n_{\nu}^2 = g \quad (12.2)$$

character

$$\sum_R D_{il}^{(\nu)} D_{jm}^{*(\mu)} = \frac{g}{n_{\nu}} \delta_{\mu\nu} \delta_{ij} \delta_{lm} \quad (12.3)$$

$$\sum_R \chi^{(\nu)}(R) \chi^{*(\mu)}(R) = g \delta_{\mu\nu} \quad (12.4)$$

$$\sum_R \chi^{(\nu)}(R) = 0 \quad (\nu \neq A_1) \quad (12.5)$$

reduce a given reducible repr  $\Gamma$

Suppose

$$\chi(R) = \sum_{\nu} a_{\nu} \chi^{(\nu)}(R) \quad (12.6)$$

thus

$$a_{\nu} = \frac{1}{g} \sum_R \chi(R) \chi^{(\nu)}(R) \quad (12.7)$$

12.8 Use Symm Arguments to Predict Which Elements in a Secular Det Equals 0

12.9 Generating Operators Are Used to Find LCAOs That Are Bases for IrRepr

$$\hat{\mathbf{P}}_j = \frac{d_j}{h} \sum_{\hat{\mathbf{R}}} \chi_j(\hat{\mathbf{R}}) \hat{\mathbf{R}} \quad (12.8)$$

## 13 Molecular Spectroscopy

### 13.1

	micro	far IR	IR	visible & UV
$f/\text{Hz}$				
$\lambda/\text{m}$				
$\bar{\nu}/\text{cm}^{-1}$				
$E/\text{J mol}^{-1}$				
process				

Table 5

13.2 Rotational Transitions Accompany Vibrational Transitions