Notes of RPA

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January 23, 2020

$$E^C = -\frac{1}{2}\operatorname{Im}\int_0^1 \mathrm{d}\alpha \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \int \mathrm{d}x_1\mathrm{d}x_2 \frac{\chi_\alpha(\omega, x_1, x_2) - \chi_0(\omega, x_1, x_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} \tag{0.1}$$

$$\chi_\alpha^{\mathrm{RPA}}(\omega, x_1, x_2) = \chi_0 + \int \mathrm{d}x_1'\mathrm{d}x_2' \chi_0(\omega, x_1, x_1') \frac{\alpha}{|\mathbf{r}_1' - \mathbf{r}_2'|} \chi_\alpha^{\mathrm{RPA}}(\omega, x_2', x_2) \tag{0.2}$$

$$\chi_\alpha^{\mathrm{RPA}}(\omega, x_1, x_2) = -\sum_n \left(\frac{\rho_{\alpha n}(x_1)\rho_{\alpha n}(x_2)}{\Omega_{\alpha n} - \omega - \mathrm{i}\,\eta} + \frac{\rho_{\alpha n}(x_1)\rho_{\alpha n}(x_2)}{\Omega_{\alpha n} + \omega + \mathrm{i}\,\eta} \right) \tag{0.3}$$

$$\begin{aligned} E^{C,\mathrm{RPA}} = \frac{1}{2}\operatorname{Im}\int_0^1 \mathrm{d}\alpha \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \int \mathrm{d}x_1\mathrm{d}x_2 \\ \sum_n \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|} \left(\frac{\rho_{\alpha n}(x_1)\rho_{\alpha n}(x_2)}{\Omega_{\alpha n} - \omega - \mathrm{i}\,\eta} + \frac{\rho_{\alpha n}(x_1)\rho_{\alpha n}(x_2)}{\Omega_{\alpha n} + \omega + \mathrm{i}\,\eta} - \frac{\rho_{0n}(x_1)\rho_{0n}(x_2)}{\Omega_{0n} - \omega - \mathrm{i}\,\eta} - \frac{\rho_{0n}(x_1)\rho_{0n}(x_2)}{\Omega_{0n} + \omega + \mathrm{i}\,\eta} \right) \end{aligned} \tag{0.4}$$

$\because$

$$\operatorname{Im}\frac{1}{a + \mathrm{i}\,\eta} = -\pi\delta(a) \quad \Rightarrow \tag{0.5}$$

$$\operatorname{Im}\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{1}{\Omega_{\alpha n} - \omega - \mathrm{i}\,\eta} = -\operatorname{Im}\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{1}{\omega - \Omega_{\alpha n} + \mathrm{i}\,\eta} = \int_0^\infty \mathrm{d}\omega\delta(\omega - \Omega_{\alpha n}) \tag{0.6}$$

$$\operatorname{Im}\int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{1}{\Omega_{\alpha n} + \omega + \mathrm{i}\,\eta} = -\int_0^\infty \mathrm{d}\omega\delta(\omega + \Omega_{\alpha n}) \tag{0.7}$$

$\therefore$

$$E^{C,\mathrm{RPA}} = \frac{1}{2}\int_0^1 \mathrm{d}\alpha \int \mathrm{d}x_1\mathrm{d}x_2 \sum_n \frac{\rho_{\alpha n}(x_1)\rho_{\alpha n}(x_2) - \rho_{0n}(x_1)\rho_{0n}(x_2)}{|\mathbf{r}_1 - \mathbf{r}_2|} \tag{0.8}$$