WIKIPEDIA

Nuclear magnetic moment

The **nuclear magnetic moment** is the magnetic moment of an atomic nucleus and arises from the spin of the protons and neutrons. It is mainly a magnetic dipole moment; the quadrupole moment does cause some small shifts in the hyperfine structure as well. All nuclei that have nonzero spin also possess a nonzero magnetic moment and vice versa, although the connection between the two quantities is not straightforward or easy to calculate.

The nuclear magnetic moment varies from isotope to isotope of an element. For a nucleus of which the numbers of protons and of neutrons are *both even* in its ground state (i.e. lowest energy state), the nuclear spin and magnetic moment are both always zero. In cases with odd numbers of either or both protons and neutrons, the nucleus often has nonzero spin and magnetic moment. The nuclear magnetic moment is not sum of nucleon magnetic moments, this property being assigned to the tensorial character of the nuclear force, such as in the case of the most simple nucleus where both proton and neutron appear, namely deuterium nucleus, deuteron.

Contents

Measurement methods

The methods for measuring nuclear magnetic moments can be divided into two broad groups in regard to the interaction with internal or external applied fields.[1] Generally the methods based on external fields are more accurate.

Shell model

According to the shell model, protons or neutrons tend to form pairs of opposite total angular momentum. Therefore, the magnetic moment of a nucleus with even numbers of each protons and neutrons is zero, while that of a nucleus with an odd number of protons and even number of neutrons (or vice versa) will have to be that of the remaining unpaired nucleon. For a nucleus with odd numbers of each protons and neutrons, the total magnetic moment will be some combination of the magnetic moments of both of the "last", unpaired proton and neutron.

The magnetic moment is calculated through j, l and s of the unpaired nucleon, but nuclei are not in states of well defined l and s. Furthermore, for odd–odd nuclei, there are two unpaired nucleons to be considered, as in deuterium. There is consequently a value for the nuclear magnetic moment associated with each possible l and s state combination, and the actual state of the nucleus is a superposition of these. Thus the real (measured) nuclear magnetic moment is between the values associated with the "pure" states, though it may be close to one or the other (as in deuterium).

g-factors

The values of $g^{(l)}$ and $g^{(s)}$ are known as the *g*-factors of the nucleons.

The measured values of $g^{(l)}$ for the neutron and the proton are according to their electric charge. Thus, in units of nuclear magneton, $g^{(l)} = 0$ for the neutron and $g^{(l)} = 1$ for the proton.

The measured values of $g^{(s)}$ for the neutron and the proton are twice their magnetic moment (either the neutron magnetic moment or the proton magnetic moment). In nuclear magneton units, $g^{(s)} = -3.8263$ for the neutron and $g^{(s)} = 5.5858$ for the proton.

Gyromagnetic ratio

The gyromagnetic ratio, expressed in Larmor precession frequency $f = \frac{\gamma}{2\pi}B$, is of great relevance to nuclear magnetic resonance analysis. Some isotopes in the human body have unpaired protons or neutrons (or both, as the magnetic moments of a proton and neutron do not cancel perfectly) [2][3][4] Note that in the table below, the measured magnetic dipole moments, expressed in a ratio to the nuclear magneton, may be divided by the half-integral nuclear spin to calculate dimensionless g-factors. These g-factors may be multiplied by 7.622 593 285(47) MHz/T,[5] which is the nuclear magneton divided by Planck's constant, to yield Larmor frequencies in MHz/T. If divided instead by the reduced Planck constant, which is 2π less, a gyromagnetic ratio expressed in radians is obtained, which is greater by a factor of 2π.

The quantized difference between energy levels corresponding to different orientations of the nuclear spin $\Delta E = \gamma\hbar B$. The ratio of nuclei in the lower energy state, with spin aligned to the external magnetic field, is determined by the Boltzmann distribution.[6] Thus, multiplying the dimensionless g-factor by the nuclear magneton ($3.152\,451\,2550(15) \times 10^{-8}$ eV·T^{-1}) and the applied magnetic field, and dividing by Boltzmann's constant ($8.617\,3303(50) \times 10^{-5}$ eV·K^{-1}) and the Kelvin temperature.

Mass	Element	Magnetic dipole moment[7][8] (nuclear magneton units)	Nuclear spin number[7]	g-factor[9]	Larmor frequency (MHz/tesla)	Gyromagnetic ratio[10] (rad s^{-1} μT^{-1}) (free atom)	Isotopic abundance	NMR Sensitivity[2] (relative to ^{1}H)
Formula		μ_Z/μ_N (measured)[9]	I	$g = \mu/I$[8]	$\nu/B = g\mu_N/h$	$\omega/B = \gamma = g\mu_N/\hbar$		
1	H	2.79284734(3)	1/2	5.58569468	42.6	267.522208	99.98%	1
2	H	0.857438228(9)	1	0.857438228	6.5	41.0662919	0.02%	
7	Li	3.256427(2)	3/2	2.1709750	16.5	103.97704	92.6%	
13	C	0.7024118(14)	1/2	1.404824	10.7	67.28286	1.11%	0.016
14	N	0.40376100(6)	1	0.40376100	3.1	19.337798	99.63%	0.001
19	F	2.628868(8)	1/2	5.253736	40.4	251.6233	100.00%	0.83
23	Na	2.217522(2)	3/2	1.4784371	11.3	70.808516	100.00%	0.093
31	P	1.13160(3)	1/2		17.2	108.394	100.00%	0.066
39	K	0.39147(3)	3/2	0.2610049	2.0	12.500612	93.1%	

Calculating the magnetic moment

In the shell model, the magnetic moment of a nucleon of total angular momentum j, orbital angular momentum l and spin s, is given by

$$\mu = \langle (l,s),j,m_j=j|\mu_z|(l,s),j,m_j=j\rangle\,.$$

Projecting with the total angular momentum **j** gives

$$\begin{aligned}\mu &= \left\langle (l,s),j,m_j=j\left|\vec{\mu}\cdot\vec{j}\right|(l,s),j,m_j=j\right\rangle \frac{\langle (l,s)j,m_j=j\,|j_z|\,(l,s)j,m_j=j\rangle}{\left\langle (l,s)j,m_j=j\left|\vec{j}\cdot\vec{j}\right|(l,s)j,m_j=j\right\rangle}\\ &= \frac{1}{j+1}\left\langle (l,s),j,m_j=j\left|\vec{\mu}\cdot\vec{j}\right|(l,s),j,m_j=j\right\rangle\end{aligned}$$

$\vec{\mu}$ has contributions both from the orbital angular momentum and the spin, with different coefficients $g^{(l)}$ and $g^{(s)}$:

$$\vec{\mu} = g^{(l)}\vec{l} + g^{(s)}\vec{s}$$

by substituting this back to the formula above and rewriting

$$\begin{aligned}\vec{l}\cdot\vec{j} &= \frac{1}{2}\left(\vec{j}\cdot\vec{j}+\vec{l}\cdot\vec{l}-\vec{s}\cdot\vec{s}\right)\\ \vec{s}\cdot\vec{j} &= \frac{1}{2}\left(\vec{j}\cdot\vec{j}-\vec{l}\cdot\vec{l}+\vec{s}\cdot\vec{s}\right)\\ \mu &= \frac{1}{j+1}\left\langle (l,s),j,m_j=j\left|g^{(l)}\frac{1}{2}\left(\vec{j}\cdot\vec{j}+\vec{l}\cdot\vec{l}-\vec{s}\cdot\vec{s}\right)+g^{(s)}\frac{1}{2}\left(\vec{j}\cdot\vec{j}-\vec{l}\cdot\vec{l}+\vec{s}\cdot\vec{s}\right)\right|(l,s),j,m_j=j\right\rangle\\ &= \frac{1}{j+1}\left(g^{(l)}\frac{1}{2}\left[j(j+1)+l(l+1)-s(s+1)\right]+g^{(s)}\frac{1}{2}\left[j(j+1)-l(l+1)+s(s+1)\right]\right)\end{aligned}$$

For a single nucleon $s = 1/2$. For $j = l + 1/2$ we get

$$\mu_j = g^{(l)}l + \frac{1}{2}g^{(s)}$$

and for $j = l - 1/2$

$$\mu_j = \frac{j}{j+1}\left(g^{(l)}(l+1) - \frac{1}{2}g^{(s)}\right)$$

See also

- Deuterium magnetic moment
- Electron magnetic moment
- Gyromagnetic ratio
- Magnetic moment
- Neutron magnetic moment
- Nuclear magneton
- Proton spin crisis

References

1. Blyn Stoyle, *Magnetic moments* , p 6
2. R. Edward Hendrick (2007-12-14). "Fundamentals of Magnetic Resonance Imaging" (https://books.google.com/books?id=GCIrzKiSKqEC&pg=PA10). Springer. p. 10.
3. K. Kirk Shung; Michael Smith; Benjamin M.W. Tsui (2012-12-02). "Principles of Medical Imaging" (https://books.google.com/books?id=fUH_HrIHh30C&pg=PA216). Academic Press. p. 216.
4. Manorama Berry; et al., eds. (2006). "Diagnostic Radiology : Neuroradiology : Head and Neck Imaging" (https://books.google.com/books?id=bR6YTH1fycUC&pg=PA74). Jaypee Brothers.
5. "nuclear magneton in MHz/T: μ_N/h" (http://physics.nist.gov/cgi-bin/cuu/Results?search_for=nuclear+magneton). NIST (citing CODATA recommended values). 2014.
6. "Nuclear magnetic resonance spectroscopy" (http://teaching.shu.ac.uk/hwb/chemistry/tutorials/molspec/nmr1.htm). Sheffield Hallam University.
7. Gladys H. Fuller (1975). "Nuclear spins and moments" (https://srd.nist.gov/JPCRD/jpcrd85.pdf) (PDF). *J Phys Chem Ref Data*. **5** (4). Magnetic dipole moments are given with a diamagnetic correction applied; the correction values are detailed in this source.
8. NJ Stone (February 2014). "Table of nuclear magnetic dipole and electric quadrupole moments" (https://www-nds.iaea.org/publications/indc/indc-nds-0658.pdf) (PDF). IAEA. For some nuclei multiple magnetic dipole values were given based on different methods and publications. For brevity only the first of each in the table is shown here.
9. "Almanac 2011" (https://lsa.umich.edu/content/dam/chem-assets/chem-docs/BrukerAlmanac2011.pdf) (PDF). Bruker. 2011.
10. From Bruker's Almanac, PDF page 118 (numbers here have been multiplied by 10 to account for different units)

Bibliography

- Nersesov, E.A. (1990). *Fundamentals of atomic and nuclear physics*. Moscow: Mir Publishers. ISBN 5-06-001249-2.
- Sergei Vonsovsky (1975). *Magnetism of Elementary Particles*. Mir Publishers.
- Hans Kopfermann *Kernmomente* and *Nuclear Momenta (Akademische Verl., 1940, 1956, and Academic Press, 1958)*

External links

- **Nuclear Structure and Decay Data - IAEA (http://www-nds.iaea.org/queryensdf)** with query on Magnetic Moments
- **magneticmoments.info/wp (http://magneticmoments.info/wp)** A blog with all recent publications on electromagnetic moments in nuclei
- **[1] (http://faculty.missouri.edu/~glaserr/8160f09/STONE_Tables.pdf)** Table of nuclear magnetic dipole and electric quadrupole moments, N.J. Stone
- RevModPhys Blyn Stoyle (http://journals.aps.org/rmp/abstract/10.1103/RevModPhys.28.75)