An *Ab Initio* Discussion on Anomalous Nuclear Magnetic Moment

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November 10, 2019

Overview

In Chapter 14 of our textbook, a brief introduction of NMR theory is given.
Equation 14.8 gives a naïve magnetic moment

$$\boldsymbol{\mu} = \mu_N \mathbf{I} = \frac{q}{2m_N}\mathbf{I} \tag{1.1}$$

and Equation 14.9 modifies that by introducing g-factor

$$\boldsymbol{\mu} = g\mu_N \mathbf{I} \tag{1.2}$$

or

$$\gamma = g\mu_N \tag{1.3}$$

However, why should we introduce g-factor, and can it be explained physically or calculated *ab initio*?

Nuclear Magnetic Moment, or g-factor, can be measured by experiment, and won't vary in different chemical environments.

However, **why** the g-factors look **"anomalous"**?

	$^{1}\mathrm{H}$	$^{2}\mathrm{H}$	$^{7}\mathrm{Li}$	$^{19}\mathrm{F}$
g	5.58	0.86	2.17	5.25
μ(a.u.)	2.79	0.86	3.25	2.63

Calculation of Proton/Neutron Magnetic Moment[1]

The magnetic moment of the proton differs from that expected for a point-like Dirac fermion.
Since quarks are fundamental Dirac fermions, the operators for the total magnetic moment and z-component of the magnetic moment are

$$\boldsymbol{\mu} = \frac{Q}{m}\mathbf{S} \qquad \boldsymbol{\mu}_z = \frac{Q}{m}\mathbf{S}_z \tag{2.1}$$

thus

$$\mu_{u,z} = \frac{2}{3}\frac{1}{m_u}\frac{1}{2} = \frac{1}{3m_u} = \frac{2m_p}{3m_u}\mu_N \tag{2.2}$$

$$\mu_{d,z} = -\frac{1}{3}\frac{1}{m_d}\frac{1}{2} = -\frac{1}{6m_d} = -\frac{m_p}{3m_d}\mu_N \tag{2.3}$$

where $\mu_N = \dfrac{1}{2m_p}$

The proton wavefunction is

$$|\mathrm{p}\uparrow\rangle = \frac{1}{\sqrt{6}}(2\,\mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow - \mathrm{u}\uparrow\mathrm{u}\downarrow\mathrm{d}\uparrow - \mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow) \tag{2.4}$$

thus

$$\begin{aligned}
\mu_p &= \frac{1}{6}\left\langle \mathrm{p}\uparrow \middle| \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)} \middle| \mathrm{p}\uparrow \right\rangle \\
&= \frac{4}{6}\left\langle \mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow \middle| \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)} \middle| \mathrm{u}\uparrow\mathrm{u}\uparrow\mathrm{d}\downarrow \right\rangle \\
&\quad + \frac{1}{6}\left\langle \mathrm{u}\uparrow\mathrm{u}\downarrow\mathrm{d}\uparrow \middle| \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)} \middle| \mathrm{u}\uparrow\mathrm{u}\downarrow\mathrm{d}\uparrow \right\rangle \\
&\quad + \frac{1}{6}\left\langle \mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow \middle| \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)} \middle| \mathrm{u}\downarrow\mathrm{u}\uparrow\mathrm{d}\uparrow \right\rangle \\
&= \frac{2}{3}(\mu_{\mathrm{u}} + \mu_{\mathrm{u}} - \mu_{\mathrm{d}}) + \frac{1}{6}(\mu_{\mathrm{u}} - \mu_{\mathrm{u}} + \mu_{\mathrm{d}}) + \frac{1}{6}(-\mu_{\mathrm{u}} + \mu_{\mathrm{u}} + \mu_{\mathrm{d}}) \\
&= \frac{4}{3}\mu_{\mathrm{u}} - \frac{1}{3}\mu_{\mathrm{d}}
\end{aligned} \tag{2.5}$$

thus

$$\mu_{\mathrm{p}} = \frac{4}{3}\frac{2m_p}{3m_u}\mu_N - \frac{1}{3}\left(-\frac{m_p}{3m_d}\mu_N\right) = \left(\frac{8}{9}\frac{m_p}{m_u} + \frac{1}{9}\frac{m_p}{m_d}\right)\mu_N \tag{2.6}$$

Similarly, for neutron

$$\begin{aligned}\mu_{\rm n} &= \frac{4}{3}\mu_{\rm d} - \frac{1}{3}\mu_{\rm u} \\ &= -\left(\frac{4}{9}\frac{m_p}{m_d} + \frac{2}{9}\frac{m_p}{m_u}\right)\mu_N \end{aligned} \tag{2.7}$$

Sadly, we cannot measure mass of quarks directly, due to "color confinement".
More sadly, mass of quarks fitted from experiments varies with cases.
By naïve estimation, we take $m_{\rm u} \approx m_{\rm d} \approx \frac{1}{3}m_{\rm p} \approx \frac{1}{3}m_{\rm n}$, thus

$$\mu_{\rm p} = 3\mu_N \qquad \mu_{\rm n} = -2\mu_N \tag{2.8}$$

compared with experimental values

$$\mu_{\rm p} = 2.79\mu_N \qquad \mu_{\rm n} = -1.91\mu_N \tag{2.9}$$

The ratio between them is more accurate

$$\left(\frac{\mu_{\rm p}}{\mu_{\rm n}}\right)^{(th)} = -1.5 \qquad \left(\frac{\mu_{\rm p}}{\mu_{\rm n}}\right)^{(exp)} = -1.46 \tag{2.10}$$

What about Multi-baryon nucleus?

classification of nucleus by number of protons and neutrons:

- odd-odd
- odd-even
- even-even

Discussion based on Shell Model

	spin	magnetic moment
even-even	0	0
odd-even	by last nucleon	by last nucleon
odd-odd	by coupling of last 2 nucleons	by coupling of last 2 nucleons

References

Mark Thomson. Cambridge Unversity Press, 2013.