



# An *Ab Initio* Discussion on Anomalous Nuclear Magnetic Moment

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# Overview

Introduction

*Ab Initio* Calculation  
Proton/Neutron  
Multi-baryon nucleus





In Chapter 14 of our textbook, a brief introduction of NMR theory is given.

Equation 14.8 gives a naïve magnetic moment

$$\boldsymbol{\mu} = \mu_N \mathbf{I} = \frac{q}{2m_N} \mathbf{I} \quad (1.1)$$

and Equation 14.9 modifies that by introducing  $g$ -factor

$$\boldsymbol{\mu} = g\mu_N \mathbf{I} \quad (1.2)$$

or

$$\gamma = g\mu_N \quad (1.3)$$

However, why should we introduce  $g$ -factor, and can it be explained physically or calculated *ab initio*?





Nuclear Magnetic Moment, or  $g$ -factor, can be measured by experiment, and won't vary in different chemical environments.

However, **why** the  $g$ -factors look "**anomalous**"?

	$^1\text{H}$	$^2\text{H}$	$^7\text{Li}$	$^{19}\text{F}$
$g$	5.58	0.86	2.17	5.25
$\mu(\mu_N)$	2.79	0.86	3.25	2.63





# Calculation of Proton/Neutron Magnetic Moment[2]

The magnetic moment of the proton differs from that expected for a point-like Dirac fermion.

Since quarks are fundamental Dirac fermions, the operators for the total magnetic moment and z-component of the magnetic moment are

$$\boldsymbol{\mu} = \frac{Q}{m} \mathbf{S} \qquad \mu_z = \frac{Q}{m} S_z \quad (2.1)$$

thus

$$\mu_{u,z} = \frac{2}{3} \frac{1}{m_u} \frac{1}{2} = \frac{1}{3m_u} = \frac{2m_p}{3m_u} \mu_N \quad (2.2)$$

$$\mu_{d,z} = -\frac{1}{3} \frac{1}{m_d} \frac{1}{2} = -\frac{1}{6m_d} = -\frac{m_p}{3m_d} \mu_N \quad (2.3)$$

where  $\mu_N = \frac{1}{2m_p}$





The proton wavefunction is

$$|p \uparrow\rangle = \frac{1}{\sqrt{6}}(2u \uparrow u \uparrow d \downarrow - u \uparrow u \downarrow d \uparrow - u \downarrow u \uparrow d \uparrow) \quad (2.4)$$

thus

$$\begin{aligned} \mu_p &= \frac{1}{6} \langle p \uparrow | \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)} | p \uparrow \rangle \\ &= \frac{4}{6} \langle u \uparrow u \uparrow d \downarrow | \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)} | u \uparrow u \uparrow d \downarrow \rangle \\ &\quad + \frac{1}{6} \langle u \uparrow u \downarrow d \uparrow | \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)} | u \uparrow u \downarrow d \uparrow \rangle \\ &\quad + \frac{1}{6} \langle u \downarrow u \uparrow d \uparrow | \hat{\mu}_z^{(1)} + \hat{\mu}_z^{(2)} + \hat{\mu}_z^{(3)} | u \downarrow u \uparrow d \uparrow \rangle \\ &= \frac{2}{3}(\mu_u + \mu_u - \mu_d) + \frac{1}{6}(\mu_u - \mu_u + \mu_d) + \frac{1}{6}(-\mu_u + \mu_u + \mu_d) \\ &= \frac{4}{3}\mu_u - \frac{1}{3}\mu_d \end{aligned} \quad (2.5)$$

thus

$$\mu_p = \frac{4}{3} \frac{2m_p}{3m_u} \mu_N - \frac{1}{3} \left( -\frac{m_p}{3m_d} \mu_N \right) = \left( \frac{8}{9} \frac{m_p}{m_u} + \frac{1}{9} \frac{m_p}{m_d} \right) \mu_N \quad (2.6)$$





Similarly, for neutron

$$\begin{aligned}\mu_n &= \frac{4}{3}\mu_d - \frac{1}{3}\mu_u \\ &= -\left(\frac{4}{9}\frac{m_p}{m_d} + \frac{2}{9}\frac{m_p}{m_u}\right)\mu_N\end{aligned}\quad (2.7)$$

Sadly, we cannot measure mass of quarks directly, due to "color confinement".

More sadly, mass of quarks fitted from experiments varies with cases.

By naïve estimation, we take  $m_u \approx m_d \approx \frac{1}{3}m_p \approx \frac{1}{3}m_n$ , thus

$$\mu_p = 3\mu_N \qquad \mu_n = -2\mu_N \quad (2.8)$$

compared with experimental values

$$\mu_p = 2.79\mu_N \qquad \mu_n = -1.91\mu_N \quad (2.9)$$

The ratio between them is more accurate

$$\left(\frac{\mu_p}{\mu_n}\right)^{(th)} = -1.5 \qquad \left(\frac{\mu_p}{\mu_n}\right)^{(exp)} = -1.46 \quad (2.10)$$



# What about Multi-baryon nucleus?

classification of nucleus by number of protons and neutrons:

- odd-odd
- odd-even
- even-even

Discussion based on Shell Model[1]

	spin	magnetic moment
even-even	0	0
odd-even	by last nucleon	by last nucleon
odd-odd	by coupling of last 2 nucleons	by coupling of last 2 nucleons







## References



Xiting Lu. *Atomic Nuclear Physics*. 2000.



Mark Thomson. *Modern Particle Physics*. Cambridge University Press, 2013.





**Thank You**

