Lecture 8 玻色化 · 一维系统.

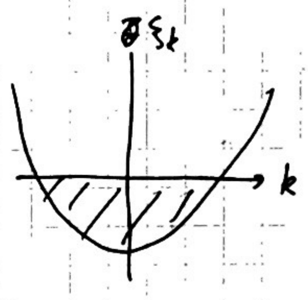
一维相互作用电子气系统：非费米液体. Luttinger - Tomonaga Liquid

自由费米子 ⇒ 玻色化 ⇒ 相互作用在玻色化变换后的形式 ⇒ 非费米液体行为.

1) 一维费米面的低能有效模型.

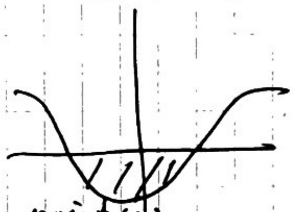
* 一维自由费米子色散.

自由空间: $\epsilon_k = \frac{k^2}{2m}$. $\xi_k = \frac{k^2}{2m} - \mu$

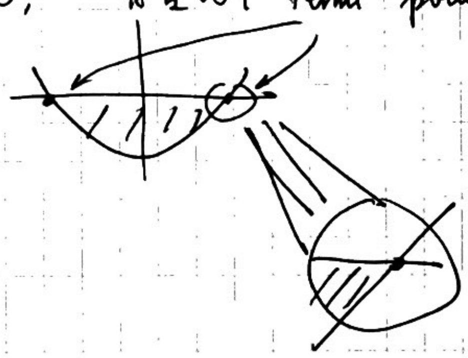


晶格: 例如紧束缚模型

$\xi_k = -t\cos k - \mu$.



一般地, 存在两个 Fermi point(s).



线性化: Fermi point 附近近似线性色散.

$\xi_k = \epsilon_k - \mu \approx \pm v_F (k \mp k_F)$.

(假设两个 Fermi points 分别在 $\pm k_F$)

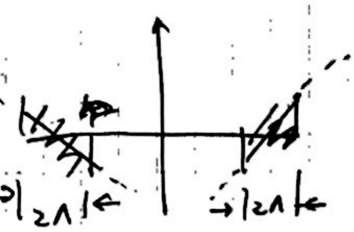
* $\pm k_F$ 附近, 长波极限下的费米子场算符.

$$\psi_{n\sigma} \approx e^{ik_F na}\psi_{\sigma+}(na) + e^{-ik_F na}\psi_{\sigma-}(na) + \text{高能自由度}.$$

$$\hat{\psi}_{n\sigma} = \sum_k e^{ikna}\hat{\psi}_{k\sigma}$$

$$\approx \sum_{k\approx k_F} e^{ikna}\psi_{k\sigma} + \sum_{k\approx -k_F} e^{ikna}\psi_{k\sigma} + \cdots$$

$$= \sum_{q<\Lambda} e^{i(k_F+q)na}\psi_{k_F+q,\sigma} + e^{i(-k_F+q)na}\psi_{-k_F+q,\sigma} + \cdots$$



$$= e^{ik_F na}\psi_{\sigma+}(na) + e^{-ik_F na}\psi_{\sigma-}(na) + \cdots$$

w/ $\psi_{\sigma\pm}(na) = \sum_{q<\Lambda} e^{iqna}\psi_{\pm k_F+q,\sigma} = \sum_{q<\Lambda} e^{iqna}\psi_{\sigma\pm}(q)$

$$\psi_{\sigma\pm}(q) = \psi_{\pm k_F+q,\sigma}.$$

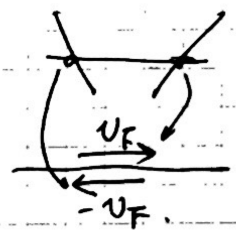
$$H_{eff} = \sum_{q,\sigma,a=\pm}\epsilon_{qa}\psi^\dagger_{\sigma a}(q)\psi_{\sigma a}(q), \qquad \epsilon_{qa} = \pm v_F q.$$

$H_{eff} = \sum_{q a \sigma} a \cdot v_F\, q\, \psi^\dagger_{\sigma a}(q)\, \psi_{\sigma a}(q)$.

Fourier 变换: $q \longrightarrow -i\partial_x$.

$\therefore\ H_{eff} = -v_F \int dx \sum_{a\sigma} \psi^\dagger_{\sigma a}(x)\, a\cdot i\partial_x\, \psi_{\sigma a}$.

$= -v_F \int dx \sum_{\sigma} \psi^\dagger_{\sigma +} i\partial_x \psi_{\sigma +} - v_F \int dx \sum_{\sigma} \psi^\dagger_{\sigma -} (-i\partial_x) \psi_{\sigma -}$



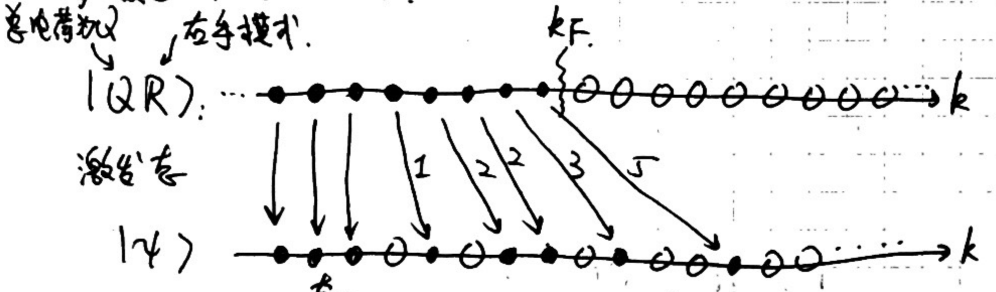
下面，我们以 $a=+$ 为例. 忽略 $\sigma=\uparrow\downarrow$ 自由度.

$+/-$: R/L 向右/向左运动的手征费米子.

正常一维体系: 净手性为零. 向左/向右相"抵消".

$H_R = -v_F \int dx\, \psi^\dagger i\partial_x \psi$.

2) 玻色化: 物理图象.



$\Delta Q=0$的激发可以用对应动量的增加 $\Delta k = n$ 来标记. 粒子/空穴对激发: $\psi_R^\dagger(k+\Delta k)\psi_R(k)$.

如上图: $1\times \Delta k = 1$; $2\times\Delta k = 2$; $3\times\Delta k=3$; $5\times\Delta k = 5$

$|\psi\rangle = b^\dagger_{R1} (b^\dagger_{R2})^2 b^\dagger_{R3} b^\dagger_{R5} |QR\rangle$.

b_{Rn}: 动量 $=\frac{2n\pi}{L}$; 能量 $= \frac{2n\pi}{L} v_F \propto n$

$\epsilon_q = v_F q \propto q$

$\Rightarrow$ 无相互作用玻色子.

$H = H_0 + \frac{2\pi v_F}{L} \sum_n n\, b^\dagger_{Rn} b_{Rn}$.

3) 玻色化, 数学推导.

考虑粒子/空穴对: $\rho_R(p) = \frac{1}{\sqrt{L}} \sum_k \psi^\dagger_R(k+p)\psi_R(k)$.

$= \frac{1}{\sqrt{L}} \int dx\, e^{ipx} \rho_R(x)$.

$\langle \rho_R(x) \rangle_{GS} \neq 0$: 背景电荷密度

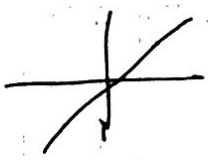
"但对于 R 模式, $\langle\rho_R(x)\rangle$ 却无法定义:

费米海为"无底洞"

$\langle\rho_R(x)\rangle$ 依赖于 cut-off. Λ

故定义 Normal ordering: $:A: \equiv A - \langle A\rangle_{GS}$.

令 $\psi_R(k) = \begin{cases} c_k, & k>0 \\ \tilde{c}_k^+, & k<0. \end{cases}$



$$:\psi_R^+(k)\psi_R(k): = \begin{cases} c_k^+ c_k, & k>0 \\ -\tilde{c}_k^+\tilde{c}_k, & k<0 \end{cases}$$

$: :$ 把 $\tilde{c}^+$ 或 c^+ 放到左边; $\tilde{c}$ 或 c 放到右边.

定义 $\rho_R(x) = :\psi_R^+(x)\psi_R(x):$

$$\rho_R(p) = \sum_k :\psi_R^+(k-p)\psi_R(k): \; = \cancel{\sum_k :\psi_R^+(k)\ldots}$$

4) 计算 $[\rho_R(x), \rho_R(x')]$:

首先, 计算

$$[\rho_R(-p), \rho_R(p')] = \frac{1}{L}\Big[\sum_k :\psi_R^+(k+p)\psi_R(k):, \sum_{k'} :\psi_R^+(k'-p')\psi_R(k'):\Big]$$

这里可以去掉 $: :$ 因为 $:A:$ 与 A 的区别为 c-数, 与算符对易.

$$= \frac{1}{L}\sum_{kk'}\big(\psi_R^+(k+p)\psi_R(k)\psi_R^+(k'-p')\psi_R(k') + \psi_R^+(k+p)\psi_R^+(k'-p')\psi_R(k)\psi_R(k')$$
$$- \psi_R^+(k'-p')\psi_R^+(k+p)\psi_R(k')\psi_R(k) + \psi_R^+(k'-p')\psi_R(k')\psi_R^+(k+p)\psi_R(k)\big)$$

$$= \frac{1}{L}\sum_{kk'}\big(\psi_R^+(k+p)\{\psi_R(k), \psi_R^+(k'-p')\}\psi_R(k')$$
$$- \psi_R^+(k'-p')\{\psi_R^+(k+p), \psi_R(k')\}\psi_R(k)\big)$$

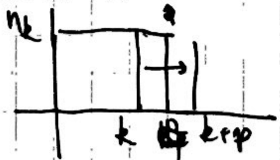
$$= \frac{1}{L}\sum_{kk'}\big(\delta_{k,k'-p'}\psi_R^+(k+p)\psi_R(k') - \delta_{k+p,k'}\psi_R^+(k'-p')\psi_R(k)\big).$$

$$= \frac{1}{L}\sum_k\big(\psi_R^+(k+p)\psi_R(k+p') - \psi_R^+(k+p-p')\psi_R(k)\big).$$

$$= \frac{1}{L}\sum_k\big(\underbrace{:\psi_R^+(k+p)\psi_R(k+p'): - :\psi_R^+(k+p-p')\psi_R(k):}_{0 \text{ b/c } k\to k+p'}\big)$$

$$+ \frac{1}{L}\sum_k\big(\langle\psi_R^+(k+p)\psi_R(k+p')\rangle - \langle\psi_R^+(k+p-p')\psi_R(k)\rangle\big)$$

$$= \frac{1}{L}\delta_{pp'}\sum_k\big(\langle\psi_R^+(k+p)\psi_R(k+p)\rangle - \langle\psi_R^+(k)\psi_R(k)\rangle\big)$$



$n_{k+p} - n_k = -1$, if $k < 0 < k+p$, or $-p<k<0$.

$$= \delta_{pp'}(-1)\int_{-p}^{0}\frac{dk}{2\pi}$$

$$= -\frac{p}{2\pi}\delta_{pp'}.$$

$$\boxed{\begin{aligned} \sqrt{\tfrac{2\pi}{p}}\,\rho_R(p) &= b_{Rp} \\ \sqrt{\tfrac{2\pi}{p}}\,\rho_R(-p) &= b_{Rp}^+ \\ [b_{Rp}, b_{Rp'}^+] &= \delta_{pp'} \end{aligned}}$$

Fourier 变换: $p \to -i\partial_x$

$$\Rightarrow [\rho_R(x), \rho_R(x')] = -\frac{i}{2\pi}\partial_x\delta(x-x').$$

证明: $[\rho_R(x), \rho_R(x')] = \frac{1}{L} \sum_{pp'} [\rho_R(p), \rho_R(p')] \, e^{-ipx+ip'x'}$

$= \frac{1}{L} \sum_{pp'} \left(-\frac{p}{2\pi}\right) \delta_{pp'} \, e^{-ipx+ip'x'}$

$= \frac{1}{L} \sum_{p} \left(-\frac{p}{2\pi}\right) e^{ip(x'-x)}$

$= -\frac{1}{2\pi} \int \frac{dp}{2\pi} \, i\partial_x e^{-ip(x-x')}$ $\quad \left[p e^{-ip(x-x')} = i\partial_x e^{-ip(x-x')} \right]$

$= -\frac{i}{2\pi} \partial_x \int \frac{dp}{2\pi} e^{-ip(x-x')}$

$= -\frac{i}{2\pi} \partial_x \delta(x-x')_{R}$.

5) 玻色化场:

引入 ϕ, s.t. $\rho_R(x) = \frac{1}{2\pi} \partial_x \phi_R(x)$.

$-i\partial_x = p$:

$\rho_R(p) = \frac{1}{2\pi} (ip) \phi_R(p)$

$\phi_R(p) = \frac{2\pi}{ip} \rho_R(p)$.

严格的定义: $\phi_R(x) = \frac{1}{\sqrt{L}} \sum_p \phi_R(p) e^{ipx}$

$= \frac{2\pi}{\sqrt{L}} \sum_p \underbrace{e^{-\alpha|p|/2}}_{\text{soft cut-off}} \frac{1}{ip} e^{ipx} \rho_R(p)$

$[\phi_R(-p), \rho_R(p')] = \frac{2\pi}{-ip} [\rho_R(-p), \rho_R(p')]$

$= \frac{2\pi}{-ip} \left(-\frac{p}{2\pi}\right) \delta_{pp'} = -i\delta_{pp'}$

因此 ϕ_R, $\underbrace{\rho_R}_{= \frac{1}{2\pi}\partial_x \phi_R}$ 基本上互为共轭变量.

定义: $\begin{cases} \phi_R(x) = \phi(x) + \theta(x) \\ \phi_L(x) = \phi(x) - \theta(x). \end{cases}$ or $\begin{cases} \phi(x) = \frac{1}{2}[\phi_R(x) + \phi_L(x)] \\ \theta(x) = \frac{1}{2}[\phi_R(x) - \phi_L(x)]. \end{cases}$

Claim. $[\phi(x'), \partial_x \theta(x)] = -i\pi \delta(x'-x)$.

证明: $\phi(x) = \frac{\pi}{\sqrt{L}} \sum_p e^{-\alpha|p|/2} \frac{1}{ip} e^{ipx} [\rho_R(p) + \rho_L(p)]$.

$\partial_x \theta(x) = \partial_x \frac{\pi}{\sqrt{L}} \sum_p e^{-\alpha|p|/2} \frac{1}{ip} e^{ipx} [\rho_R(p) - \rho_L(p)]$.

$= \frac{\pi}{\sqrt{L}} \sum_p e^{-\alpha|p|/2} e^{ipx} [\rho_R(p) - \rho_L(p)]$.

$$\therefore\ [\phi(x'), \partial_x\theta(x)] = \frac{\pi^2}{L}\sum_{pp'} e^{-\alpha|p|/2} e^{-\alpha|p'|/2} \frac{1}{ip'} e^{ip'x'} e^{ip x} [\rho_R(p') + \rho_L(p'), \rho_R(p) - \rho_L(p)].$$

$$= \frac{\pi^2}{L}\sum_{pp'} e^{-\alpha(|p|+|p'|)/2} \frac{1}{ip'} e^{ip'x' + ipx}\, 2\cdot\frac{p}{2\pi}\delta_{p',-p}$$

$$= \frac{\pi^2}{L}\sum_{p} e^{-\alpha|p|} \frac{1}{-ip} e^{ip(x-x')} \frac{-p}{\pi}$$

这里 $[\rho_L(p'), \rho_L(p)] = -\frac{2\pi}{p'}\delta_{p',-p}$.

$$= -i\pi \frac{1}{L}\sum_p e^{-\alpha|p|} e^{ip(x-x')}$$

$$= -i\pi\delta(x-x').$$

Recall, $[\hat{P}(x), \hat{Q}(x')] = -i\delta(x-x')$

故 $\frac{1}{\pi}\partial_x\theta$ 可视为 ϕ 的正则动量.

6) Hamiltonians ~~& Lagrangian~~:

Recall $H = E_0 + v_F \sum_{q>0} q\, b^{\dagger}_{Rq} b_{Rq} + v_F\sum_{q>0}(-q)\, b^{\dagger}_{Lq} b_{Lq}$.

w/ $b_{Rq} = \sqrt{\frac{2\pi}{q}}\rho_R(q)$. $b_{L|q|} = \sqrt{\frac{2\pi}{|q|}}\rho_L(-|q|)$.

$$\therefore\ H = E_0 + 2\pi v_F\sum_{q>0} \rho_R(-q)\rho_R(q) + \rho_L(q)\rho_L(-q).$$

$$= E_0' + \pi v_F\sum_q \rho_R(-q)\rho_R(q) + \rho_L(-q)\rho_L(q). \qquad ①$$

$$\int dx(\nabla\phi)^2 = \frac{\pi^2}{L}\int dx \sum_{pp'} e^{-\alpha\frac{|p|+|p'|}{2}} e^{ipx+ip'x}[\rho_R(p)+\rho_L(p)]\cdot[\rho_R(p')+\rho_L(p')]$$

$$= \pi^2\sum_{pp'}\delta_{p+p',0}[\rho_R(p)+\rho_L(p)][\rho_R(p')+\rho_L(p')]$$

$$= \pi^2\sum_p[\rho_R(-p)+\rho_L(-p)][\rho_R(p)+\rho_L(p)].$$

类似地, $\int dx(\nabla\theta)^2 = \pi^2\sum_p[\rho_R(-p)-\rho_L(-p)][\rho_R(p)-\rho_L(p)]$.

$$\therefore\ \int dx[(\nabla\phi)^2 + (\nabla\theta)^2] = 2\pi^2\sum_p\left(\rho_R(-p)\rho_R(p) + \rho_L(-p)\rho_L(p)\right). \qquad ②$$

由比较 ① 与 ②,

$$H = E_0' + \frac{v_F}{2\pi}\int dx[(\nabla\phi)^2 + (\nabla\theta)^2].$$

~~7) Lagrangian~~

引入 $K=1$: $H = \frac{v_F}{2\pi}\int dx\left[\frac{1}{K}(\nabla\phi)^2 + K(\nabla\theta)^2\right]$.

K: Luttinger parameter. 自由费米子 $K=1$.

7) Lagrangians.

$-\frac{1}{\pi}\nabla\theta$ 为 ϕ 的正则动量.

$$L=\int dx \frac{1}{\pi}\nabla\theta\,\partial_t\phi - H$$

$$\mathcal{L}=\frac{1}{\pi}\nabla\theta\,\partial_t\phi-\frac{v_F}{2\pi K}(\nabla\phi)^2-\frac{v_F}{2\pi}K(\nabla\theta)^2.$$

虚时： $t\mapsto \tau=it$.

$$Z=\int D\theta D\phi\, e^{i\int dt\,dx\,\mathcal{L}}=\int D\theta D\phi\, e^{i\int dt\,dx\left[\frac{1}{\pi}\nabla\theta\,\partial_t\phi-\frac{v_F}{2\pi K}(\nabla\phi)^2-\frac{v_F}{2\pi}K(\nabla\theta)^2\right]}$$

$$\to\quad Z=\int D\theta D\phi\, e^{-\int d\tau dx\left[-\frac{i}{\pi}\nabla\theta\,\partial_\tau\phi+\frac{v_F}{2\pi K}(\nabla\phi)^2-\frac{v_F}{2\pi}K(\nabla\theta)^2\right]}$$

∴ 虚时下，$\mathcal{L}=-\frac{i}{\pi}\nabla\theta\,\partial_\tau\phi+\frac{v_F}{2\pi K}(\nabla\phi)^2+\frac{v_F}{2\pi}K(\nabla\theta)^2.$

*高斯积分：配方

$$\mathcal{L}=\frac{v_F}{2\pi K}(\nabla\phi)^2+\frac{v_FK}{2\pi}\left[\nabla\theta+\frac{2\pi}{v_FK}\left(-\frac{i}{2\pi}\right)\partial_\tau\phi\right]^2+\frac{1}{2\pi v_FK}(\partial_\tau\phi)^2.$$

因此我们可以利用高斯积分"积掉"θ：

$$Z=\int D\theta D\phi\, e^{-\int d\tau dx\left[\frac{v_F}{2\pi K}(\nabla\phi)^2+\frac{v_FK}{2\pi}\left[\nabla\theta-\frac{i}{v_FK}\partial_\tau\phi\right]^2+\frac{1}{2\pi v_FK}(\partial_\tau\phi)^2\right]}$$

$$=\int D\phi\, e^{-\int d\tau dx\,\frac{1}{2\pi K v_F}\left[v_F^2(\nabla\phi)^2+(\partial_\tau\phi)^2\right]}.$$

类似地，$\mathcal{L}=-\frac{i}{\pi}\nabla\theta\,\partial_\tau\phi+\frac{v_F}{2\pi K}(\nabla\phi)^2+\frac{v_F}{2\pi}K(\nabla\theta)^2$

$$-\frac{i}{\pi}\nabla\theta\,\partial_\tau\phi=\frac{i}{\pi}\theta\,\nabla\partial_\tau\phi+\text{全微分项}$$

$$=-\frac{i}{\pi}\partial_\tau\theta\,\nabla\phi+\cdots.$$

反过来积掉 ϕ，$\Rightarrow$ $\mathcal{L}=\frac{K}{2\pi v_F}\left[(\partial_\tau\theta)^2+v_F^2(\partial_x\theta)^2\right].$

ϕ 与 θ：一对"对偶变量"，$K\leftrightarrow K^{-1}$.

8) 相互作用一维电子气.

$H = H_0 + H_{int}$

$H_0 \Rightarrow$ $\cancel{H_0}$

$$H_0 = \frac{v_F}{2\pi}\int dx\left[(\nabla\phi)^2 + (\nabla\theta)^2\right].$$

Ex: $H_{int} = \frac{U}{2}\int dx\ [\rho_R(x) + \rho_L(x)][\rho_R(x) + \rho_L(x)]$.

— 排斥相互作用.

$\rho_R(x) = \frac{1}{2\pi}\nabla\phi_R(x)$; $\rho_L = \frac{1}{2\pi}\nabla\phi_L$.

$\rho_R + \rho_L = \frac{1}{2\pi}\nabla(\phi_R + \phi_L) = \frac{1}{\pi}\nabla\phi$

$\therefore\ H_{int} = \frac{U}{2\pi^2}\int dx\,(\nabla\phi)^2$.

$H \Rightarrow\ H = \int dx\ \left[\left(\frac{v_F}{2\pi} + \frac{U}{\pi^2}\right)(\nabla\phi)^2 + \frac{v_F}{2\pi}(\nabla\theta)^2\right]$.

$(\nabla\phi)^2$ 与 $(\nabla\theta)^2$ 系数不再相同.

$v_F \to v_F' = v_F\left(1 + \frac{U v_F}{\pi}\right)^{\frac{1}{2}}$.

$K = 1 \to K' = \left(1 + \frac{U v_F}{\pi}\right)^{-\frac{1}{2}}$

$$H = \frac{v_F'}{2\pi}\int dx\left[\frac{1}{K}(\nabla\phi)^2 + K(\nabla\theta)^2\right]$$

相互作用 $\Rightarrow K \neq 1$.

9) 费米子格林函数: $K \neq 1 \Rightarrow$ 非费米液体.

玻色化表象下的费米子算符.

Short story: $[\rho_R(x), e^{i\phi_R(x')}] = -\delta(x-x')\,e^{i\phi_R(x')}$.

对比 $[\psi^\dagger\psi(x), \psi(x')] = -\delta(x-x')\,\psi(x') \Rightarrow \psi_R(x) \sim e^{i\phi_R(x)}$.

证明: $[\phi_R(-p), \rho_R(p')] = -i\delta_{pp'}$

$\Rightarrow [\rho_R(x), \phi_R(x')] = i\delta(x-x')$.

$[\rho_R(x), e^{i\phi_R(x')}]$

$= [\rho_R(x), \sum_{n=0}^{\infty}\frac{i^n}{n!}\phi_R^n(x')]$

$= \sum_{n=1}^{\infty}\frac{i^n}{n!}[\rho_R(x), \phi_R^n(x')]$ (因 $[\rho_R(x), \phi_R^0(x')] = 0$.) ($\phi_R^0 = 1$)

$= \sum_{n=1}^{\infty}\frac{i^n}{n!}\, n\,[\rho_R(x), \phi_R(x')]\,\phi_R^{n-1}(x')$

$= \sum_{n=1}^{\infty}\frac{i^n}{(n-1)!}\, i\,\delta(x-x')\,\phi_R^{n-1}(x')$

$= -\delta(x-x')\sum_{n=1}^{\infty}\frac{i^{n-1}}{(n-1)!}\phi_R^{n-1}(x') = -\delta(x-x')\,e^{i\phi_R(x')}$ □

$\therefore\ \psi_R \sim e^{i\phi_R} = e^{i(\theta+\phi)}$

Long story: see [Sachdev] II B 或者教材 §10.2

$$\langle \psi_R^\dagger(x)\,\psi_R(0)\rangle = \langle e^{-i\theta(x)-i\phi(x)}\, e^{i\theta(y)+i\phi(y)}\rangle$$

Thm. 高斯变量: $\langle e^{A}\rangle = e^{\frac12\langle A^2\rangle}$ (Wick定理).

$$\langle \psi_R^\dagger(x)\,\psi_R(0)\rangle = \exp\left\{-\frac12\left\langle[\theta(x)+\phi(x)-\theta(y)-\phi(y)]^2\right\rangle\right\}$$

$$= \exp\left\{-\frac12\left\langle[\theta(x)-\theta(y)]^2\right\rangle\right\}\exp\left\{-\frac12\left\langle[\phi(x)-\phi(y)]^2\right\rangle\right\}$$

$$\exp\left\{\frac12\langle\theta(x)\phi(y)\rangle + \frac12\langle\theta(y)\phi(x)\rangle\right\}.$$

计算 $\langle\theta(x)\theta(y)\rangle$: 可用 $\mathcal{L} = \frac{K}{2\pi}[(\partial_\tau\theta)^2+(\partial_x\theta)^2]$

$$S = \int dx\,d\tau\,\frac{K}{2\pi}[(\partial_\tau\theta)^2+(\partial_x\theta)^2] = \sum_{\omega_n,k}\frac{K}{2\pi}\theta(-k,-\omega_n)(\omega_n^2+k^2)\theta(k,\omega_n)$$

$$\therefore \langle\theta^*(k,\omega_n)\theta(k,\omega_n)\rangle = \frac{\pi}{K(\omega_n^2+k^2)}.$$

$$\langle\theta(x)\theta(y)\rangle = \int\frac{dk}{2\pi}\frac{d\omega}{2\pi}\frac{\pi}{K(\omega^2+k^2)}e^{ik(x-y)}.$$

$$\frac12\langle[\theta(x)-\theta(0)]^2\rangle = \frac{\pi}{K}\int\frac{dk\,d\omega}{(2\pi)^2}\frac{1-e^{ikx}}{\omega^2+k^2}$$

$$= \frac{\pi}{K}\int\frac{k\,dk\,d\theta}{4\pi^2}\frac{1-e^{ikx\cos\theta}}{k^2}$$

$$\sim \frac{K}{2}\ln|x| \quad \text{当 } x\gg 1.$$

类似地, $\frac12\langle[\phi(x)-\phi(0)]^2\rangle \sim \frac{1}{2K}\ln|x|$. 用 $\mathcal{L} = \frac{1}{2\pi K}[(\partial_\tau\phi)^2+(\partial_x\phi)^2]$.

计算 $\langle\theta(x)\phi(y)\rangle$: 需用到 θ 和 ϕ:

$$\mathcal{L} = -\frac{i}{\pi}\nabla\theta\,\partial_\tau\varphi + \frac{1}{2\pi K}(\nabla\phi)^2 + \frac{K}{2\pi}(\nabla\theta)^2.$$

对称性: $\theta\to-\theta$; $\tau\to-\tau$ $\mathcal{L}$不变

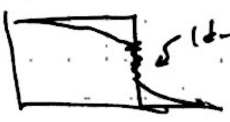
$$\therefore \langle\theta(x,0)\phi(y,0)\rangle = \langle-\theta(x,0)\phi(y,0)\rangle = 0.$$

$$\therefore \langle\psi_R^\dagger(x)\psi_R(0)\rangle \sim \exp\left(-\frac{K}{2}\ln|x| - \frac{1}{2K}\ln|x|\right) \sim |x|^{-\frac12(K+\frac1K)}.$$

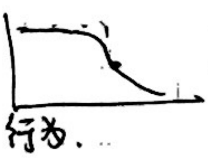
($K=1$: $\langle\psi_R^\dagger(x)\psi_R(0)\rangle\sim|x|^{-1}$.)

对应 $n_k = \langle\psi_k^\dagger(k)\psi_k(k)\rangle_0 \sim |k|^{\frac12(K+\frac1K)-1}$ (Fourier变换+更细分析).

自由费米子: $k\to k_F$ ($k\to 0$). $n_k\sim(k-k_F)^0$



相互作用: $n_k\sim(k-k_F)^{\frac12(K+\frac1K)-1}$



没有跳变 ⇒ 非费米液体行为.