

固体理论, Homework 06

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证明算符 $e^{i\phi(x)+i\theta(x)}$ 和 $e^{i\phi(x')+i\theta(x')}$ 当 $x \neq x'$ 时反对易。

Solution: Since

$$e^{i\alpha\phi_R(x)} e^{i\beta\phi_R(y)} = e^{i\beta\phi_R(y)} e^{i\alpha\phi_R(x)} e^{-i\alpha\beta\pi \operatorname{sgn}(x-y)} \quad (1.1)$$

we have

$$\begin{aligned} \{e^{i\phi(x)+i\theta(x)}, e^{i\phi(x')+i\theta(x')}\} &= e^{i\phi(x)+i\theta(x)} e^{i\phi(x')+i\theta(x')} + e^{i\phi(x')+i\theta(x')} e^{i\phi(x)+i\theta(x)} \\ &= e^{i\phi(x)+i\theta(x)} e^{i\phi(x')+i\theta(x')} + e^{i\phi(x)+i\theta(x)} e^{i\phi(x')+i\theta(x')} e^{-i\pi \operatorname{sgn}(x'-x)} \\ &= e^{i\phi(x)+i\theta(x)} e^{i\phi(x')+i\theta(x')} [1 + e^{-i\pi \operatorname{sgn}(x'-x)}] \end{aligned} \quad (1.2)$$

when $x \neq x'$,

$$\begin{aligned} \{e^{i\phi(x)+i\theta(x)}, e^{i\phi(x')+i\theta(x')}\} &= e^{i\phi(x)+i\theta(x)} e^{i\phi(x')+i\theta(x')} [1 + e^{-i\pi(\pm 1)}] \\ &= e^{i\phi(x)+i\theta(x)} e^{i\phi(x')+i\theta(x')} [1 + (-1)] \\ &= 0 \end{aligned} \quad (1.3)$$

2 Umklapp 过程:

考虑一个相互作用, 将两个左侧费米点的费米子散射到右侧费米点, 或者反过来。a) 这样的相互作用对 k_F 有什么要求? b) 写下这个相互作用对应的哈密顿量的二次量子化形式。c) 用玻色化方法将该相互作用项用玻色化之后的标量场表示出来。

Solution:

a) $k_F = \pi/2$

b)

$$H_u = v \int dx [\Psi_R^\dagger \Psi_R^\dagger \Psi_L \Psi_L + \Psi_L \Psi_L \Psi_R^\dagger \Psi_R^\dagger] \quad (2.1)$$

c)