固体理论, Homework 08

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1 教材 13.1

Consider a linear chain with a single defect placed at the mth site. Let the site amplitude be c_n and W the strength of the defect in a linear chain described by the following evolution equation:

$$Ec_n = \epsilon c_n + V(c_{n+1} + c_{n-1}) + W\delta_{nm}c_m. \tag{1.1}$$

Use the method of defects to show that a single bound state forms outside the continuous band for $W \neq 0$. To implement this method, use the following procedure.
(1) Multiply the eigenvalue equation by e^{ikn} and sum over n.
(2) Then multiply the resultant equation by e^{-ikm} and integrate to obtain as a condition for the location of the bound state

$$1 = W\langle G(E)\rangle \tag{1.2}$$

where

$$\langle G(E)\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\mathrm{d}k}{E-\epsilon(k)} \tag{1.3}$$

with $\epsilon(k) = \epsilon + 2V\cos k$. Show that if $W > 0$, the bound state lies above the band, that is, at an energy $E > \epsilon + 2V$, whereas for $W < 0$, the bound state lies below the band. Repeat the same calculation for $d = 2$.

Solution:

$$\sum_n (E-\epsilon)c_n\,\mathrm{e}^{ikn} = \sum_n V(c_{n+1}+c_{n-1})\,\mathrm{e}^{ikn} + \sum_n W\delta_{nm}c_m\,\mathrm{e}^{ikn} \tag{1.4}$$

$$\sum_n (E-\epsilon)c_n\,\mathrm{e}^{ikn} = \sum_n V(c_{n+1}+c_{n-1})\,\mathrm{e}^{ikn} + Wc_m\,\mathrm{e}^{ikm} \tag{1.5}$$

$$\sum_n (E-\epsilon)c_n\,\mathrm{e}^{ikn}\,\mathrm{e}^{-ikm} = \sum_n V(c_{n+1}+c_{n-1})\,\mathrm{e}^{ikn}\,\mathrm{e}^{-ikm} + Wc_m\,\mathrm{e}^{ikm}\,\mathrm{e}^{-ikm} \tag{1.6}$$

$$\sum_n (E-\epsilon)c_n\,\mathrm{e}^{ik(n-m)} = \sum_n V(c_{n+1}+c_{n-1})\,\mathrm{e}^{ik(n-m)} + Wc_m \tag{1.7}$$

$$\int_{-\pi}^{\pi}\mathrm{d}k\frac{1}{W} = \int_{-\pi}^{\pi}\mathrm{d}k\frac{1}{(E-\epsilon)-(V\,\mathrm{e}^{ik}+V\,\mathrm{e}^{-ik})} \tag{1.8}$$

$$\frac{1}{W} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}k\frac{1}{E-\epsilon-2V\cos k} \tag{1.9}$$

thus

$$\langle G(E)\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi}\mathrm{d}k\frac{1}{E-\epsilon(k)} \tag{1.10}$$

where $\epsilon(k) = \epsilon + 2V\cos k$

2 教材 13.5

Assume that in the Ohmic regime, the general form for the β-function is

$$\beta = d - 2 - \frac{A_d}{g} \tag{2.1}$$

Integrate this quantity in the interval $[L_0, L]$ and obtain the explicit length dependence for the conductance for $d = 1$ and $d = 3$, respectively. Show that $s = 1$, provided that $\epsilon = d - 2 \ll 1$.

Solution: Since

$$\beta = \frac{\mathrm{d}\ln g(L)}{\mathrm{d}\ln L} \tag{2.2}$$

we have

$$\beta \mathrm{d}\ln L = \mathrm{d}\ln g \tag{2.3}$$

$$\left(d - 2 - \frac{A_d}{g}\right)\frac{1}{L}\mathrm{d}L = \frac{1}{g}\mathrm{d}g \tag{2.4}$$

$$\left(d - 2 - \frac{A_d}{g}\right)\frac{g}{L} = \frac{\mathrm{d}g}{\mathrm{d}L} \tag{2.5}$$

solve it, we get

$$g(L) = \frac{A_g}{d-2} + cL^{d-2} \tag{2.6}$$

thus

$$\frac{g(L) - g(0)}{g(L_0) - g(0)} = \left(\frac{L}{L_0}\right)^{d-2} \tag{2.7}$$

where $g(0) = \dfrac{A_g}{d-2} = g_c$.
Here we get

$$\nu = \frac{1}{d-2} \tag{2.8}$$

thus

$$s = (d-2)\nu = 1 \tag{2.9}$$