Homework 2

Due: Friday, February 4 at noon

Instructions: Submit a pdf of your solutions to the HW 2 assignment on Gradescope. When working on this assignment, you should focus on the following goals:

- Demonstrate that you understand how to write a complete set of equivalence classes modulo m and when two equivalence classes are equal.
- Demonstrate understanding of the definition of a unit/invertible element modulo m.
- Clearly and correctly apply the definition of divisibility in a proof.
- (Problem 4) Use examples to determine if a conjecture is true and then prove or disprove the conjecture.
- Write clear and correct proofs that meet the guidelines linked on the Homework Assignments page in Canvas.

0. If you would like any of these problems to be graded for proficiency with the core skills, list the skill and the corresponding problem.

1. Write out all elements of $\mathbb{Z}_8$ using two different collections of representatives for the equivalence classes. (Note: Your answer will have two different sets; each equivalence class in the first set should be equal to an equivalence class in the second set.)

2. Make a multiplication table for $\mathbb{Z}_{10}$. Use it to identify the units (invertible elements) of $\mathbb{Z}_{10}$. What is $\phi(\mathbb{Z}_{10})$?

3. Find a multiplicative inverse to n modulo $2n-1$. You should justify that your answer is correct, but you do not need to structure your justification as a formal proof.

4. Prove or disprove: For positive integers n and N and any integer x,
$$(x\%N)\%n = x\%n.$$

5. Show that $x^2 - y^2 = 102$ has no solutions in the integers. (Hint: Show that the squares modulo 4 are all congruent to 0 or 1 and use the contrapositive of the reflexivity property of modular congruence.)

6. Suppose that $a = bq + r$, for some integers a, b, q and r. Using only the fact that $\gcd(x, y)$ is the largest common divisor of x and y, prove that $\gcd(a, b) = \gcd(b, r)$. (Hint: $x = y$ if and only if $x \leq y$ and $x \geq y$.)