Linear System Theory: HW1

2022/11/03

（Due: Nov. 10, 2022）

1. (5’+10’) Consider the following system

$$\ddot{y}+0.1\dot{y}+y^5=6\sin t \qquad (1)$$

(1) Let $x_1=y, x_2=\dot{y}$. Put system (1) into state space model.

(2) Using Matlab to plot the trajectories of y under two initial conditions, i.e., $[x_1(0),x_2(0)]^T=[2,3]^T$ and $[x_1(0),x_2(0)]^T=[2.01,3.01]^T$. Is there significant difference when $0\le t<10s$? How about when $t\ge 40s$?

2. (5’+5’+5’)

Consider the electrical network shown in Fig. 2. Find its state space model when the state are chosen to be $x=[v_c,i_L]^T$ and $x=[v_c,v_o]^T$, respectively. What is the relation between these two models?

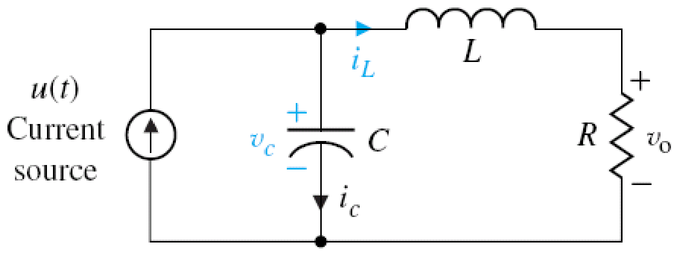

Fig. 2 An electrical network

3. (20’) (C.T.Chen 4th, Pro. 3.29)

Let the eigenvalues of A be distinct and let $q_i\in\mathbb{R}^n$ be a right eigenvector of A associated with the eigenvalue λ_i. Define $Q=[q_1,q_2,\cdots,q_n]$ and define $P=Q^{-1}=[p_1^T,p_2^T,\cdots,p_n^T]^T$ where p_i is the i-th row of P. Show that p_i is a left eigenvector of A associated with λ_i.

4. (40’) (C.T.Chen 4th, Theorem 3.1)

Given a matrix $A\in\mathbb{R}^{m\times n}$ and vector $y\in\mathbb{R}^m$.

(1). Show that a solution x exists in $Ax=y$ if and only if y lies in the range space of A or, equivalently $rank(A)=rank([A \quad y])$ where $[A \quad y]$ is an augmented matrix.

(2). Show that a solution x exists in $Ax=y$ for every y if and only if A have a rank m (full row rank).

5. (10’) (C.T.Chen 1984, Problem 2-13)

Show that similar matrices have the same characteristic polynomials, and consequently the same set of eigenvalues. (Hint: *det(AB)=det(A)det(B)*)