imaginally stable & A_c 看 A 特征值

BIBO 看 传函 极点配置

Necessity: If the system is stable,

Let $P = \int_0^\infty e^{A^T t} Q e^{At} dt$ is positive definite

$$A^T P + PA = \int_0^\infty (A^T e^{A^T t} Q e^{At} + e^{A^T t} Q e^{At} \cdot A) dt$$ 证明 $A^T P + PA = -Q$

$$= \int_0^\infty \frac{d}{dt} (e^{A^T t} Q e^{At}) dt = e^{A^T t} Q e^{At} \Big|_{t=0}^{\infty} = -Q$$

Sufficiency: to prove for PD matrix P and Q, then A is stable.

Let λ be an eigenvalue of A, and v is the corresponding eigenvector.

that is $Av = \lambda v$

Take the complex conjugate transpose of $Av = \lambda v$, $v^* A^T = v^* \lambda^*$

$$-v^* Q v = -v^*(-A^T P - PA) v = v^* A^T P v + v^* PAv$$

$$= \lambda^* v^* P v + v^* P \lambda v = (\lambda^* + \lambda) v^* P v = 2Re(\lambda) v^* P v$$

Since P, Q is PD, $Re(\lambda) < 0$ so the system is stable

或者 $V = X^T P X > 0$

$$\dot{V} = (AX)^T P X + X^T P \cdot AX$$

$$= X^T (A^T P + PA) X = X^T (-Q) X < 0$$

能控性矩阵

$$X(t) = e^{At} X_0 + \int_0^t e^{A(t-\tau)} B(\tau) \cdot u(\tau) d\tau$$

$W_c(t) = \int_0^t e^{A\tau} B B^T e^{A^T \tau} d\tau$ 正定 即 能控

取 $u(t) = -B^T e^{A^T(t_1 - t)} W_c^{-1} (e^{At_1} X_0 - X_1)$

$$X(t_1) = e^{At_1} X_0 - \int_0^{t_1} e^{A(t_1-\tau)} B B^T e^{A(t_1-\tau)} d\tau \, W_c^{-1} (e^{At_1} X_0 - X_1)$$

$$= e^{At_1} X_0 - (e^{At_1} X_0 - X_1) = X_1$$

能观 已知 $y(t), u(t) \Longrightarrow x(0)$

定理5.6

对 $\forall Q$. A nonsingular

$A^T P + PA = -Q$ 存在 unique solution $P = \int_0^\infty e^{A^T t} Q^T Q e^{At} dt$

$$\Delta(s) = s^n + \alpha_{n-1} s^{n-1} + \cdots + \alpha_0$$

$$\Delta^*(s) = s^n + \alpha^*_{n-1} s^{n-1} + \cdots + \alpha^*_0$$

$$\bar{k} = [a_0 - a_0^* \quad \cdots \quad a_{n-1} - a_{n-1}^*]$$

$$P = Q_c \cdot H_A = [b \quad Ab \quad \cdots A^{n-1} b] \cdot \begin{bmatrix} \alpha_1 & \alpha_2 & \cdots & \alpha_{n-1} & 1 \\ \alpha_2 & \alpha_3 & \cdots & 1 & \\ \vdots & & ⋰ & & 0 \\ 1 & & & & \end{bmatrix}$$

$$k = \bar{k} P^{-1}$$