

电路IB

第十章 二端口网络

参数矩阵

开路参数矩阵Z

短路参数矩阵Y

传输参数A

混合参数H

$$\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix}; \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \end{bmatrix} \quad \begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix} \quad \begin{bmatrix} \dot{U}_1 \\ \dot{I}_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{U}_2 \end{bmatrix}$$

互易

$$z_{12} = z_{21}$$

$$Y_{12} = Y_{21}$$

$$A_{11}A_{22} - A_{21}A_{12} = 1$$

$$H_{12} = -H_{21}$$

互易+对称

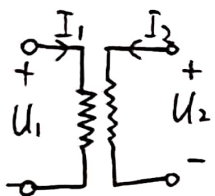
$$+ z_{11} = z_{22}$$

$$+ Y_{11} = Y_{22}$$

$$+ A_{11} = A_{22}$$

$$+ H_{11}H_{22} - H_{12}H_{21} = 1$$

理想变压器



$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} n & 0 \\ 0 & 1/n \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ -\dot{I}_2 \end{bmatrix}$$

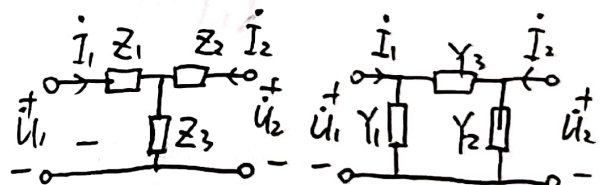
互易条件

二端口等效电路

- 独立无源二端口网络, 可以用最简二端口网络等效代替
- 一般二端口有四个独立参数, 互易二端口可由三阻抗或导纳元件组成。

形成T型网络, Π型网络

$$Z = \begin{bmatrix} z_1 + z_3 & z_3 \\ z_3 & z_2 + z_3 \end{bmatrix}, Y = \begin{bmatrix} Y_1 + Y_3 & -Y_3 \\ -Y_3 & Y_2 + Y_3 \end{bmatrix}$$



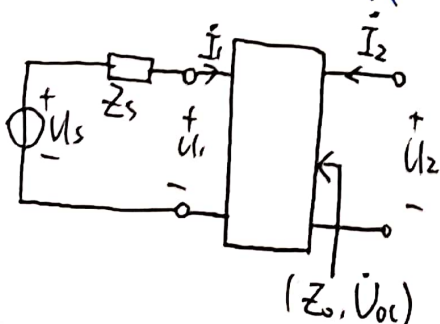
非互易网络等效(伏安控制源)

$$\dot{U}_1 = z_{11}\dot{I}_1 + z_{12}\dot{I}_2$$

外加电流控制的电压源

$$\dot{U}_2 = z_{12}\dot{I}_1 + z_{22}\dot{I}_2 + (z_{21} - z_{12})\dot{I}_1$$

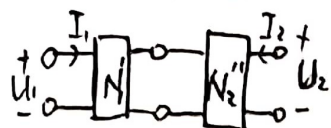
二端口网络与电源负载联接(含源阻抗戴维南电路)



$$\dot{U}_{oc} = \frac{\dot{U}_s}{A_{21}Z_s + A_{11}}$$

$$Z_o = \left. \frac{\dot{U}_2}{\dot{I}_2} \right|_{U_s=0} = \frac{A_{22}Z_s + A_{12}}{A_{21}Z_s + A_{11}}$$

二端口互联 (级联)

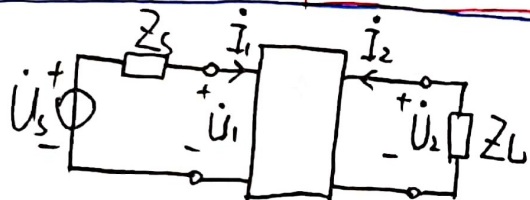


$$A = A' \cdot A''$$

$$\left. \begin{array}{l} \text{串} \\ \text{并} \end{array} \right\} \begin{array}{l} Z = Z' + Z'' \\ Y = Y' + Y'' \end{array}$$

输入/输出阻抗

$$\text{输入 } Z_i = \frac{U_1}{I_1} = \frac{A_{11}Z_L + A_{12}}{A_{21}Z_L + A_{22}} \quad \text{输出 } Z_o = \frac{U_2}{I_2} = \frac{A_{22}Z_S + A_{12}}{A_{21}Z_S + A_{11}}$$



特性阻抗 二端口与负载、电源匹配连接

$$\text{要求 } Z_o = Z_L \quad Z_S = Z_i \Rightarrow Z_{C1} = Z_i = Z_S = \sqrt{\frac{A_{11}A_{22}}{A_{21}A_{12}}} \quad \text{输入端口特性阻抗}$$

$$Z_{C2} = Z_o = Z_L = \sqrt{\frac{A_{22}A_{12}}{A_{11}A_{21}}} \quad \text{输出端口特性阻抗}$$

$$\text{二端口对称} \Rightarrow A_{11} = A_{22} \Rightarrow Z_{C1} = Z_{C2} = \sqrt{\frac{A_{12}}{A_{21}}} \Rightarrow Z_L = Z_C = Z_0 = \sqrt{\frac{A_{12}}{A_{21}}}$$

$$U_1 = (A_{11} + \sqrt{A_{12}A_{21}}) U_2 \quad I_1 = (A_{11} + \sqrt{A_{12}A_{21}}) (-I_2)$$

$$\text{传输系数 } \Gamma = \ln(A_{11} + \sqrt{A_{12}A_{21}}) = \alpha + \beta j$$

$$\alpha = \ln\left(\frac{U_1}{U_2}\right) = \ln\left(\frac{I_1}{I_2}\right) \text{ 衰减系数, } \beta = \psi_{U1} - \psi_{U2} = \psi_{I1} - \psi_{I2} \text{ 相位系数}$$

图论与网络方程

基本割集, \rightarrow 单树支割集 决定了独立的KCL方程 连支电流为一组独立变量

基本回路 \rightarrow 单连支回路. 决定独立KVL方程 全部支路电压中, 树支电压为一组独立变量

关联矩阵A,

(表示节点, 支路联系关系), n 节点 b 支路, 线图, 行对应 $(n-1)$ 节点, 列对应支路,

$$a_{ij} = \begin{cases} 1 & \text{支路 } j \text{ 从节点 } i \text{ 发出} \\ -1 & \\ 0 & \end{cases} \quad \text{大小为 } (n-1) \times b$$

以 n 节点参考, 支路电压 $U = [U_1, U_2, \dots, U_b]^T$, $U_n = [U_{n1}, \dots, U_{n(n-1)}]^T$

$$\text{KCL: } AI = 0 \quad \text{KVL: } A^T U_n = U$$

基本回路矩阵B,

(表示基本回路, 支路包含关系) 行基本回路, 列对应支路

$$b_{ij} = \begin{cases} 1 & \text{包含, 方向相同} \\ -1 & \\ 0 & \end{cases} \quad \text{大小 } b_l \times b$$

$$\text{KCL: } B^T I_l = I \quad \text{KVL: } BU = 0$$

$$I_t = B_t^T I_l \quad U_l = -B_t U_t$$

基本割集矩阵C

(表示基本割集与支路的包含关系) 行基本割集, 列对应支路.

$$c_{ij} = \begin{cases} 1 & \text{基本割集 } i \text{ 包含 } j, \text{ 方向相同} \\ -1 & \\ 0 & \end{cases}$$

$$\text{KCL: } CI = 0, \quad \text{KVL: } C^T U_T = U \quad \text{—— 树支电压列向量.}$$

$$I_t = -C_l I_l \quad U_l = C_l^T U_t$$

$$AI = AB^T I_l = 0 \quad BU = BC^T U_t = 0$$

$$\Rightarrow AB^T = 0 \quad BA^T = 0 \quad / \quad BC^T = 0, \quad CB^T = 0$$

$$CI = CB^T I_l = 0 \Rightarrow B_t = -C_l^T$$

网络矩阵关系.

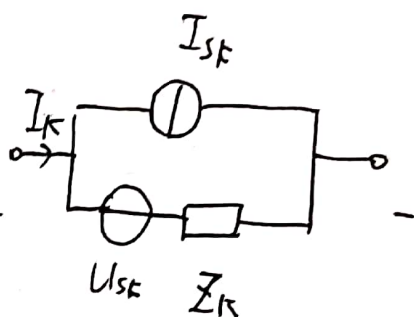
$$AB^T = 0 \quad / \quad A^T B = 0, \quad BC^T = 0, \quad CB^T = 0$$

$$BC \text{ 若出现单位阵, } BC^T = 0 \quad B_t = -C_l^T$$

广义支路及其方程矩阵形式.

$$U = ZI - ZI_s + U_s$$

$$\begin{bmatrix} U_l \\ U_b \end{bmatrix} = \begin{bmatrix} Z_l \\ Z_b \end{bmatrix} \begin{bmatrix} I_l \\ I_b \end{bmatrix} - \begin{bmatrix} Z_l \\ Z_b \end{bmatrix} \begin{bmatrix} I_{s1} \\ I_{sb} \end{bmatrix} + \begin{bmatrix} U_{s1} \\ U_{sb} \end{bmatrix}$$



$Z_k \neq 0$ 时, 存在逆矩阵 Z^{-1} , $Y = Z^{-1}$

$$I = YU - YU_s + I_s$$

电路方程矩阵形式

$$\text{节点电压方程} \quad AI = 0 \Rightarrow AYAT^T U_n = AYU_s - AI_s$$

$$\text{回路电流方程} \quad BU = 0 \Rightarrow BZB^T I_l = BZI_s - BU_s$$

广义割集分析法 待求 U_t .

广义节点分析法

$$CYC^T U_t = CYU_s - CI_s$$

$$Y_t$$

状态方程

$$\dot{X} = AX + BV$$

$$X = [U_c, \dots, i_{L1}, \dots]^T$$

列写单电容支路节点电压方程, 单电感支路回路KVL方程

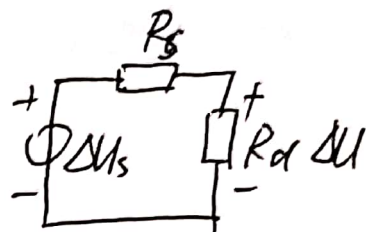
非线性电阻电路

流控、压控型非线性电阻 (自变量) + 单调型

分段线性分析法 / 折线法

小信号分析

直流确定工作点, 动态电导 \rightarrow 动态电阻 \rightarrow 小信号叠加



第七章 均匀传输线

一次参数 R_0, L_0, G_0, C_0

$$\text{均匀线方程} \begin{cases} -\frac{\partial u}{\partial x} = R_0 i + L_0 \frac{\partial i}{\partial t} \\ -\frac{\partial i}{\partial x} = G_0 u + C_0 \frac{\partial u}{\partial t} \end{cases}$$

$$\text{传播常数 } \gamma = \alpha + j\beta = \sqrt{(R_0 + j\omega L_0)(G_0 + j\omega C_0)}$$

$$\text{特性/波阻抗 } Z_c = \frac{Z_0}{\gamma} = \sqrt{\frac{R_0 + j\omega L_0}{G_0 + j\omega C_0}}$$

$$\begin{bmatrix} \dot{U}(x) \\ \dot{I}(x) \end{bmatrix} = \begin{bmatrix} \cosh \gamma x & -Z_c \sinh \gamma x \\ -\frac{1}{Z_c} \sinh \gamma x & \cosh \gamma x \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{U}(x) \\ \dot{I}(x) \end{bmatrix} = \begin{bmatrix} \cosh \gamma x' & Z_c \sinh \gamma x' \\ \frac{1}{Z_c} \sinh \gamma x' & \cosh \gamma x' \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ \dot{I}_2 \end{bmatrix}$$

$$\text{输入特性阻抗 } Z_i = \frac{\dot{U}_1}{\dot{I}_1} = Z_c \frac{Z_L \cosh \gamma l + Z_c \sinh \gamma l}{Z_c \sinh \gamma l + Z_L \cosh \gamma l}$$

$$Z_{i0} = \frac{Z_c}{\tanh \gamma l} \quad Z_{is} = Z_c \tanh \gamma l$$

均匀线上行波

$$\text{相速 } v_p = \frac{dx}{dt} = \frac{\omega}{\beta} = \lambda f$$

$$\text{瞬时值 } v^+(x,t) = \sqrt{2} U' e^{-\alpha t} \cos(\omega t - \beta x + \psi')$$

$$\lambda = v_p \cdot T = \frac{\omega}{\beta} \cdot \frac{2\pi}{\omega} = \frac{2\pi}{\beta}$$

$$\text{终端反射系数 } N_2 = \frac{Z_L - Z_C}{Z_L + Z_C}$$

$Z_L = Z_C$ 时 $N_2 = 0$, 负载与传输线匹配

$Z_L \rightarrow \infty$ $N_2 = 1$ 全反射

$$\text{始端反射系数 } N_1 = \frac{R_1 - Z_C}{R_1 + Z_C}$$

R_1 始端电阻

自然功率

终端匹配时, $Z_L = Z_C$, 线路任一点电压电流之比均为 Z_C .

$$\text{终端负载吸收功率称自然功率 } P_2 = U_2 I_2 \cos \varphi_c = U_2 I_2 \cos \varphi_c = U_1 I_1 e^{-2\alpha L} \cos \varphi_c$$

$$\text{输入功率 } P_1 = U_1 I_1 \cos \varphi_c \quad \eta = \frac{P_2}{P_1} = e^{-2\alpha L}$$

无损线, 驻波

$R_0 = G_0 = 0$ 线路无损, 称无损耗线.

$$Y = \sqrt{j\omega L_0 \cdot j\omega C_0} = j\omega \sqrt{L_0 C_0} = j\beta = j \frac{2\pi}{\lambda} \Rightarrow \alpha = 0, \beta = \omega \sqrt{L_0 C_0}$$

无损线行波传输不衰减, $Z_C = \sqrt{\frac{L_0}{C_0}}$

$$\cosh \gamma x = \cos \beta x \quad \sinh \gamma x = j \sin \beta x$$

$$\begin{bmatrix} \dot{U} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} \cos \beta x & -j Z_C \sin \beta x \\ -j \frac{1}{Z_C} \sin \beta x & \cos \beta x \end{bmatrix} \begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix}$$

$$\begin{bmatrix} \dot{U} \\ \dot{I} \end{bmatrix} = \begin{bmatrix} \cos \beta x & j Z_C \sin \beta x \\ j \frac{1}{Z_C} \sin \beta x & \cos \beta x \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ \dot{I}_2 \end{bmatrix}$$

$$\text{输入阻抗 } Z_i(x') = Z_C \frac{Z_L \cos \beta x' + j Z_C \sin \beta x'}{j Z_C \sin \beta x' + Z_C \cos \beta x'}$$

$$x' = \frac{\lambda}{4} \quad Z_i = \frac{Z_C^2}{Z_L} \quad \text{阻抗变换}$$

驻波.

形成条件: 终端开路, 短路, 接纯电抗负载.

① 行波沿线不衰减, $\alpha=0$, 无损线 ② $|N_2|=1$.

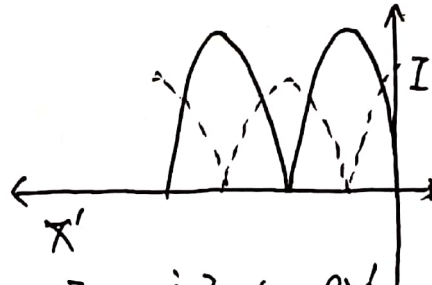
终端开路.



$$Z_i(x') = -jZ_c \cot \beta x'$$

$l < \frac{\lambda}{4}$ 且终端开路, 相当电容.

短路



$$Z_i = jZ_c \tan \beta x'$$

$l < \frac{\lambda}{4}$, 终端短路, 相当电感

纯电抗负载.

$$N_2 = \frac{jX - Z_c}{jX + Z_c} \quad |N_2| = 1$$

电感可用短路无损线等效, 电容可用开路无损线等效.

集中参数等效.

$$\begin{bmatrix} \dot{U}_1 \\ \dot{I}_1 \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & Z_c \sinh \gamma l \\ \sinh \gamma l / Z_c & \cosh \gamma l \end{bmatrix} \begin{bmatrix} \dot{U}_2 \\ \dot{I}_2 \end{bmatrix} \text{ 转为 } Z, Y \text{ 参数进行, T, } \pi \text{ 等效}$$

无损均匀线通解

电流 电压

↓
磁场 电场

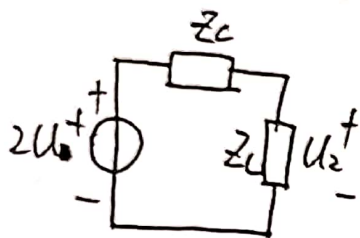
终端反射

$$\Gamma_2 = \frac{Z_L - Z_c}{Z_L + Z_c}$$

$$\Gamma_1 = \frac{Z_1 - Z_c}{Z_1 + Z_c}$$

多次反射, 电压叠加, 电流正向叠加, 反向削减.

假想源法则 一般负载, 确定终端电压, 电流,



$$U_2 = 2U_1^+ - Z_c I_2$$

磁路

$$\Phi = \oint_S B ds \quad H = \frac{B}{\mu} \quad \mu - \text{磁导率}$$

闭合面磁通 $\sum \Phi_k = 0$.

磁位差定律 $\sum U_{mj} = \sum N_k I_k = \sum F_{mk}$. 磁通势 $\sum I$.

$$\int_a^b H \cdot dl, \text{磁位差}$$

磁阻

$$U_m = Hl \Rightarrow R_m = \frac{U_m}{\Phi} = \frac{l}{\mu S} = \frac{1}{\lambda} \quad (\text{单位: } H^{-1})$$

$$\Phi = BS$$

$$\text{气隙截面修正公式 } S = (a + \delta)(b + \delta)$$

恒定磁通磁路计算

1) 分段 (2) 计算各段截面积及长度 (气隙)

(3) 计算磁感应强度 (4) $B \rightarrow H$ 气隙可以 $H = \frac{B_0}{\mu_0}$

(5) 计算磁位差 $\sum U_m = \sum Hl + H_0 \delta$ (6) 磁通势计算

正弦电压作用下铁心磁路磁通

$$\dot{\Phi}_m = -j \frac{\dot{U}}{4.44fN}$$

$$\dot{U} = j4.44fN\dot{\Phi}_m$$

功率损耗 $P_{ho} = \text{系数} \cdot f B_m^n$ $B_m < 1T, n \text{取} 1.6, B_m > 1T, n \text{取} 2$

涡流损耗 $P_{eo} = \frac{\pi^2}{6} f^2 B_m^2 d^2 \gamma$
电导率

单位体积

B 为 $0.75T, 1T, 1.5T, f=50Hz$ 时比损耗

记为 $P_{7.5/50}, P_{1.0/50}, P_{1.5/50}$

$$P_{Feo} = P_{1.0/50} B_m^n \left(\frac{f}{50Hz} \right)^{1.3} \quad B_m < 1T, n \text{取} 1.6, B_m > 1T, n \text{取} 2$$

每千克

正弦磁通磁路计算

$$Z_m = \frac{NI}{\dot{\Phi}_m / \sqrt{2}} = R_m + jX_m \quad \text{磁阻, (磁抗)} \rightarrow \text{耗能}$$

无功分量 I_q

已知 $\dot{\Phi}_m \xrightarrow{B_m} B_m \xrightarrow{\text{磁化曲线}} H_m \xrightarrow{NI_m = H_m l} I_m$

励磁电流

$$I = \sqrt{I_p^2 + I_q^2} \quad \theta = 90^\circ - \varphi = \arctan \frac{I_p}{I_q}$$

若未饱和 $I_q = \frac{I_m}{\sqrt{2}}$, 饱和 $I_q = \frac{I_m}{\sqrt{2} \lambda}$

$$\text{不计电阻和漏磁} L = \frac{N^2}{R_m \lambda}$$

有功分量 I_p

$$\text{得铁耗} I_p = \frac{P_{Fe}}{U} = \frac{P_{Fe}}{4.44fN\dot{\Phi}_m}$$