（Due: Dec. 15, 2022）

1. (30’) Consider the following LTI system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \qquad (1)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^p$ is the input, $y \in \mathbb{R}^q$ is the output. Show that all eigenvalues of $A + BK$ can be arbitrarily assigned (provided the complex conjugate eigenvalues are assigned in pairs) by selecting a real constant matrix K if and only if (A, B) is controllable.

2. (20’) Consider the LTI system

$$\begin{aligned} \dot{x} &= \begin{bmatrix} -1 & 1 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u \\ y &= [1 \quad 0 \quad 0] x \end{aligned} \qquad (2)$$

Try to design a state feedback control law to shift the eigenvalues to -1, -2, -3 by transforming the above system to controllable form.

3. (50’) Consider system (1). Let $\mathcal{C}$ and $\overline{\mathcal{C}}$ be its controllability matrix and observability matrix. If $rank(\mathcal{C}) = n_1 < n$ and $rank(\mathcal{C}) = n_2 < n$. Show that system (1) can be equivalently transformed into the following canonical form:

$$\begin{aligned} \begin{bmatrix} \dot{\bar{x}}_{co} \\ \dot{\bar{x}}_{c\bar{o}} \\ \dot{\bar{x}}_{\bar{c}o} \\ \dot{\bar{x}}_{\bar{c}\bar{o}} \end{bmatrix} &= \begin{bmatrix} \bar{A}_{co} & 0 & \bar{A}_{13} & 0 \\ \bar{A}_{21} & \bar{A}_{c\bar{o}} & \bar{A}_{23} & \bar{A}_{24} \\ 0 & 0 & \bar{A}_{\bar{c}o} & 0 \\ 0 & 0 & \bar{A}_{43} & \bar{A}_{\bar{c}\bar{o}} \end{bmatrix} \begin{bmatrix} \bar{x}_{co} \\ \bar{x}_{c\bar{o}} \\ \bar{x}_{\bar{c}o} \\ \bar{x}_{\bar{c}\bar{o}} \end{bmatrix} + \begin{bmatrix} \bar{B}_{co} \\ \bar{B}_{c\bar{o}} \\ 0 \\ 0 \end{bmatrix} u \\ y &= \begin{bmatrix} \bar{C}_{co} & 0 & \bar{C}_{\bar{c}o} & 0 \end{bmatrix} \bar{x} + Du \end{aligned} \qquad (3)$$

Further, the state-space equation (1) is zero-state equivalent to the controllable and observable state-space equation

$$\begin{aligned} \dot{\bar{x}}_{co} &= \bar{A}_{co} \bar{x}_{co} + \bar{B}_{co} u \\ y &= \bar{C}_{co} \bar{x}_{co} + Du \end{aligned} \qquad (4)$$

and has the transfer matrix

$$\hat{G}(s) = \bar{C}_{co} (sI - \bar{A}_{co})^{-1} \bar{B}_{co} + D. \qquad (5)$$