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Efficient unmanned aerial vehicle formation rendezvous trajectory planning using Dubins path and sequential convex programming

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ABSTRACT

Trajectory planning of formation rendezvous of multiple unmanned aerial vehicles (UAVs) is formulated as a mixed-integer optimal control problem, and an efficient hierarchical planning approach based on the Dubins path and sequential convex programming is proposed. The proposed method includes the assignment of rendezvous points (high level) and generation of cooperative trajectories (low level). At the high level, the assignment of rendezvous points to UAVs is optimized to minimize the total length of Dubins-path-based approximate trajectories. The assignment results determine the geometric relations between the UAVs' goals, which are used as equality constraints for generating trajectories. At the low level, trajectory generation is treated as a non-convex optimal control problem, which is transformed to a non-convex parameter optimization and then solved via sequentially performing convex optimization. Numerical experiments demonstrate that the proposed method can generate feasible trajectories and can outperform a typical nonlinear programming method in terms of efficiency.

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1. Introduction

Owing to their ability to enhance task efficiency and enrich mission functionality through multiple-vehicle cooperation (Shima and Rasmussen 2009), unmanned aerial vehicle (UAV) formations are attracting significant interest in a wide range of applications, *e.g.* imaging and mapping, forest fire monitoring, border patrols, search and rescue, precision agriculture and light shows. Before multiple vehicles can cooperatively execute missions in formation, they need to achieve specified formation conditions, including desired formation shapes and required arrival time (Jouffroy *et al.* 2016).

There are two categories of approach by which to achieve formation rendezvous: reactive methods and planning methods (Siegwart, Nourbakhsh, and Scaramuzza 2011). Reactive methods (Harl 2008; Lin and Liu 2015; Zhang *et al.* 2015) design online cooperative controllers to generate guidance commands for forming desired shapes. Planning methods (McLain and Beard 2000; Lee and Kim 2007; Jouffroy *et al.* 2016) generate rendezvous trajectories in advance, considering the feasibility and optimality of entire trajectories, and then UAVs track the planned trajectories to achieve formation rendezvous. Although reactive controllers adapt well to the dynamic environment and scale well to the number of UAVs in theory, they rely on constant communication among UAVs and can

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hardly consider the overall performance of trajectories. Thus, current multi-UAV cooperative applications mainly employ planning methods, and this work also focuses on planning-based formation rendezvous methods.

In traditional planning methods for rendezvous, the outputs are paths (*i.e.* geometric curves) without time information. One method is to generate rendezvous paths with identical lengths under the assumption that all the UAVs fly at an identical velocity (McLain and Beard 2000; McLain *et al.* 2001). Another way is to generate paths with similar lengths and then adjust the velocity of the UAVs to achieve simultaneous arrival (Chandler, Rasmussen, and Pachter 2000; Lee and Kim 2007). Nevertheless, path-planning-based formation rendezvous may result in unacceptable errors between the planned path and the actual flight path by using path-following controllers, especially for the flight time. An alternative method is to conduct trajectory planning subject to rendezvous time constraints directly, since the trajectories include the flight-time information and trajectory tracking can provide better tracking performance than path following (Aguiar *et al.* 2005; Aguiar and Hespanha 2007; Aguiar, Hespanha, and Kokotović 2008).

Typical trajectory planning methods for UAVs include nonlinear programming (Chen *et al.* 2013; Kim and Kim 2014), mixed-integer programming (Kuwata and How 2011), rapidly exploring random tree (Alves Neto, Macharet, and Campos 2010) and heuristic-based intelligent algorithms (Avigad, Eisenstadt, and Cohen 2011). However, these methods scale poorly to the number of UAVs as the number of design variables and constraints grows. Convex programming has seen increasing applications in trajectory planning because of its appealing efficiency even for high-dimensional problems with thousands of variables and constraints (Boyd and Vandenberghe 2004). Application examples include planetary landing (Acikmese and Ploen 2007), distributed spacecraft coordination (Tillerson, Inalhan, and How 2002) and formation reconfiguration (Acikmese *et al.* 2006). Furthermore, sequential convex programming (SCP) combining convex programming with successive convexification techniques was investigated for trajectory planning problems with non-convex constraints, *e.g.* spacecraft rendezvous and proximity operations (Lu and Liu 2013), spacecraft swarm guidance (Morgan, Chung, and Hadaegh 2012, 2014) and entry flight (Liu, Shen, and Lu 2015). However, few studies have been conducted on SCP-based UAV trajectory planning. Augugliaro, Schoellig, and D'Andrea (2012) first applied SCP to generate collision-free trajectories for a fleet of quadrocopters with linear dynamics. Chen, Cutler, and How (2015) proposed a decoupled incremental SCP method to increase the probability of finding feasible trajectories of multiple UAVs and achieve improvement in computational tractability. Both of these SCP-based planning methods were developed for quadrotor UAVs with linear dynamics. Based on the previous work, this article extends SCP to rendezvous trajectory planning of fixed-wing UAVs with nonlinear dynamics in a three-dimensional (3D) space.

Most studies on multi-UAV trajectory planning assume that the goal position of each UAV is predefined. In a general formation rendezvous problem, the desired formation shape is given, but the formation location and the relative positions of UAVs are both unknown. In this work, the desired formation location is limited in a rendezvous zone instead of being given a specific coordinate. In such a scenario, the formation rendezvous problem involves two parts: target assignment and trajectory generation. Because the exact assignment costs depend on UAV trajectories, the two parts must be solved simultaneously to obtain the optimal solution. In this way, evaluating the costs of a candidate assignment requires computation of the corresponding cooperative trajectories, which leads to intensive computational burden. To alleviate the computational complexity, approximate trajectories are commonly used to compute the costs in the assignment problem. For example, a straight-line trajectory was employed by Turpin, Michael, and Kumar (2014) for linear systems to compute the assignment costs. The approximate trajectories for assignment were generated by solving convex problems without collision constraints (Morgan *et al.* 2016). In this article, a Dubins path is employed as the approximate trajectory, and a hierarchical planning method based on the Dubins path and SCP is proposed to solve the rendezvous assignment and trajectory planning problem.

The rest of the article is organized as follows. Section 2 presents the formulation of the considered trajectory planning problem for multi-UAV formation rendezvous. In Section 3, a hierarchical planning framework based on the Dubins path and SCP is proposed. In Section 4, the Dubins-path-based assignment method of rendezvous points is developed. Section 5 presents the techniques of parameterization and convexification and the detailed procedures of SCP. In Section 6, the proposed method is demonstrated on multiple numerical examples. Finally, the article is concluded in Section 7.

2. Problem formulation

In this section, the rendezvous trajectory planning problem of fixed-wing UAV formation is presented. The problem is formulated as a mixed-integer optimal control problem, which is an extended optimal control problem with integer decision variables.

2.1. Formation rendezvous problem of multiple UAVs

In this article, formation rendezvous means guiding multiple fixed-wing UAVs to form a specific formation shape in the rendezvous zone from their initial conditions. Figure 1 depicts a two-dimensional (2D) scenario of multi-UAV formation rendezvous. The five UAVs need to form a V-shaped formation in a given circular rendezvous zone. According to the desired formation shape, the relative offsets between rendezvous points are determined, but the desired formation location and the relative positions of UAVs need to be optimized. Thus, this multi-UAV application requires solving the assignment problem of rendezvous points to decide the relative offsets among the UAVs' goals, and the multi-UAV trajectory generation problem to simultaneously arrive at the goals without collision.

The rendezvous constraints of UAV formation involve the desired formation shape and the specified rendezvous zone.

According to the desired formation shape, as shown in Figure 2, the relative geometric relations between the rendezvous points can be determined. To help the UAVs maintain their formation flight after rendezvous, the flight states, including speed, heading angle and flight path angle at rendezvous points, are also required to be identical. Thus, the UAV states at rendezvous points are subject to a series of linear equality constraints.

The number of candidate rendezvous points is the same as number of UAVs, denoted as N. The rendezvous points are numbered based on the desired formation shape. Denote the desired states at rendezvous point j as $\boldsymbol{g}_j, j \in \{1, 2, \ldots, N\}$. Without loss of generality, the first rendezvous point (*i.e.* $j = 1$) is selected as the reference point. Then, the constraints are expressed by the state offsets $\Delta \boldsymbol{s}$ between the rendezvous points and the reference point:

$$\boldsymbol{g}_j - \boldsymbol{g}_1 = \Delta \boldsymbol{s}_j, j = \{1, 2, \ldots, N\} \tag{1}$$

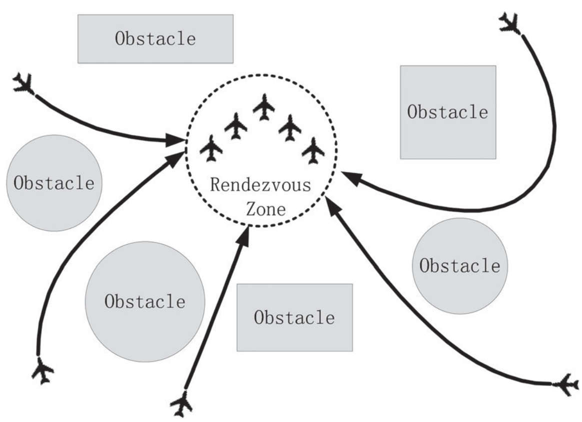

Figure 1. Illustration of a scenario of formation rendezvous of unmanned aerial vehicles.

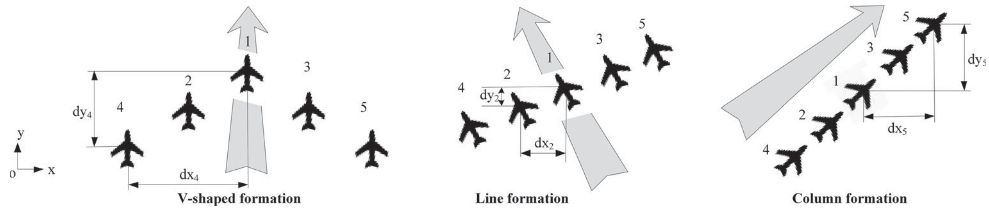

Figure 2. Typical formation shapes of unmanned aerial vehicles.

The rendezvous points must be located in the specified rendezvous zone. In this article, the rendezvous zone is considered as a cylinder with centre $[x_T, y_T]$ and radius r_T. Then, the rendezvous zone constraints can be described as

$$||\boldsymbol{g}_j - (x_T, y_T)||_2 \leq r_T, j = \{1, 2, \ldots, N\} \tag{2}$$

To describe the corresponding relationships between rendezvous points and UAVs, the assignment matrix $\boldsymbol{\psi}$ is defined, as given in Equation (3). If rendezvous point j is assigned to UAV i, $\boldsymbol{\psi}_{ij} = 1$; otherwise, $\boldsymbol{\psi}_{ij} = 0$.

$$\boldsymbol{\psi} = \begin{pmatrix} \boldsymbol{\psi}_{11} & \boldsymbol{\psi}_{12} & \cdots & \boldsymbol{\psi}_{1N} \\ \boldsymbol{\psi}_{21} & \boldsymbol{\psi}_{22} & \cdots & \boldsymbol{\psi}_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\psi}_{N1} & \boldsymbol{\psi}_{N2} & \cdots & \boldsymbol{\psi}_{NN} \end{pmatrix} \tag{3}$$

Each rendezvous point must be assigned to a UAV and each UAV must select a rendezvous point, which is expressed as the equality constraint in Equation (4):

$$\boldsymbol{\psi} \cdot \boldsymbol{\psi}^T = \boldsymbol{I}_N \tag{4}$$

where $\boldsymbol{I}_N$ is an $N \times N$ identity matrix. Using the assignment matrix, the relations between states at rendezvous points and terminal states $\boldsymbol{s}(t_f)$ of UAVs are expressed as

$$\begin{pmatrix} \boldsymbol{s}_1^T(t_f) \\ \boldsymbol{s}_2^T(t_f) \\ \vdots \\ \boldsymbol{s}_N^T(t_f) \end{pmatrix} = \boldsymbol{\psi} \cdot \begin{pmatrix} \boldsymbol{g}_1^T \\ \boldsymbol{g}_2^T \\ \vdots \\ \boldsymbol{g}_N^T \end{pmatrix} \tag{5}$$

Note that the location of target formation is not unique under the constraints of the desired formation shape and specified rendezvous zone. For example, three feasible solutions to a V-shaped formation in a cylinder region are provided in Figure 3. Thus, the assignment result of rendezvous points does not decide the terminal state values of UAVs, but determines the relative offsets between terminal states of different UAVs.

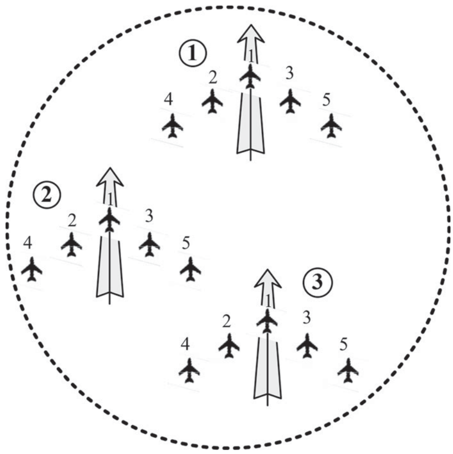

Figure 3. Three feasible rendezvous results of unmanned aerial vehicle in the rendezvous zone.

2.2. Mixed-integer optimal control formulation

Assume that there are N fixed-wing UAVs in a 3D Euclidean space and denote the index of a UAV as $i \in \{1, 2, \ldots, N\}$. The 3D point-mass motion equations for UAVs are shown as follows:

$$\begin{cases} dx_i/dt = V_i \cos\gamma_i \cos\chi_i \\ dy_i/dt = V_i \cos\gamma_i \sin\chi_i \\ dh_i/dt = V_i \sin\gamma_i \\ dV_i/dt = u_{1,i} \\ V_i \cdot \cos\gamma \cdot d\chi_i/dt = u_{2,i} \\ V_i \cdot d\gamma_i/dt = u_{3,i} \end{cases} \tag{6}$$

where t is the time, (x_i, y_i) is the horizontal position, h_i is the altitude, V_i is the velocity, χ_i is the heading angle, and γ_i is the flight-path angle. The trajectory states $\boldsymbol{s_i}$ of UAV i are expressed as $\boldsymbol{s_i} = (x_i, y_i, h_i, V_i, \chi_i, \gamma_i)^T$. The acceleration components $(u_{1,i}, u_{2,i}, u_{3,i})$ in the path coordinate system are the control variable $\boldsymbol{u_i}$ of the UAV. $u_{1,i}$ is the tangential acceleration; $u_{2,i}$ and $u_{3,i}$ are normal acceleration. The above dynamics model of UAVs is a nonlinear system and is rewritten in the following matrix form for simplification:

$$d\boldsymbol{s}_i/dt = f(\boldsymbol{s}_i, \boldsymbol{u}_i) \tag{7}$$

The trajectory states of each UAV at the initial time $t_0 = 0$ are assumed to be given, as described in Equation (8), where $\boldsymbol{s}_{i,0}$ denotes the given initial states of UAV i.

$$\boldsymbol{s}_i(t_0) = \boldsymbol{s}_{i,0} \tag{8}$$

Considering the mission space and the flight performance, the UAV positions, altitudes and velocities are restricted by the following boundary constraints:

$$\begin{gathered} x_{\min} \le x_i(t) \le x_{\max}, y_{\min} \le y_i(t) \le y_{\max} \\ h_{\min} \le h_i(t) \le h_{\max}, V_{\min} \le V_i(t) \le V_{\max} \end{gathered} \tag{9}$$

The magnitude of the control for each UAV is limited by the minimum and maximum allowable acceleration in the corresponding direction, which is described as

$$\boldsymbol{u}_{i,\min} \le \boldsymbol{u}_i(t) \le \boldsymbol{u}_{i,\max} \tag{10}$$

The obstacles are assumed to be cylinders and obstacle avoidance constraints are expressed as Equation (11), where (x_c, y_c) is the centre and r_c is the radius of the obstacle.

$$||(x_i(t), y_i(t)) - (x_c, y_c)||_2 \geq r_c \tag{11}$$

To avoid collision between vehicles, any two UAVs should be separated by a safe distance R, as defined below:

$$||(x_i(t), y_i(t), h_i(t)) - (x_j(t), y_j(t), h_j(t))||_2 \geq R, i \neq j \tag{12}$$

Considering the aforementioned constraints, multi-UAV formation rendezvous trajectory planning can be formulated as the mixed-integer optimal control problem in Equation (13), where the objective is to minimize the $L2$ norm of the control input, and the variables include the integer assignment matrix, state trajectories and control trajectories:

$$\begin{aligned} \text{Problem P1}: \quad \boldsymbol{\psi}^*, s^*(t), u^*(t) &= \underset{\boldsymbol{\psi}, s(t), u(t)}{\arg\min} \sum_{i=1}^{N} \int_0^{t_f} ||\boldsymbol{u}_i(t)||_2 \, \mathrm{d}t \\ &\text{subject to Equations (1)–(5) and (7)–(12)} \end{aligned} \tag{13}$$

3. Hierarchical planning framework

Multi-UAV formation rendezvous was cast as a mixed-integer optimal control problem in Section 2. There are two methods for general optimal control problems: direct methods and indirect methods. For indirect methods, the trajectories are obtained by solving the necessary optimality condition, which is a two-point boundary value problem (TPBVP). However, it is usually hard to solve TPBVPs in real-world applications and to derive the optimality condition for the problem with path constraints and integer variables. On the other hand, direct methods employ the collocation approach to convert the optimal control problem into a nonlinear programming problem, which can be solved effectively by state-of-the-art optimization algorithms. Thus, direct methods are much more widely used than indirect methods. But for mixed-integer optimal control, the resulting problem by collocation is a mixed-integer nonlinear programming problem, which is non-deterministic polynomial time hard to solve (Bonami *et al.* 2013). Besides, the dimensions of the problem are greatly increased when considering cooperation among multiple UAVs. Thus, it is computationally intractable to directly solve P1 to obtain the cooperative trajectories for multi-UAV formation rendezvous.

To decrease the computational complexity, the idea of hierarchical planning is introduced to solve the rendezvous trajectory planning problem. In the hierarchical planning framework, the problem is decomposed into two parts: assignment of rendezvous points (high level) and generation of cooperative trajectories (low level), as shown in Figure 4. The high-level part determines the corresponding relationships between UAVs and rendezvous points by solving an integer programming problem. The low-level part generates the rendezvous trajectories based on the assignment results by solving an ordinary optimal control problem without integer variables. The two parts need to be iteratively invoked to acquire the optimal solution, as shown in Figure 4. Since feasible suboptimal results are generally satisfied in real applications, few iterations are required unless any constraints are violated.

The assignment problem of rendezvous points for multi-UAV formation is described as P2 in Equation (14), which is a classical integer programming problem. In this problem, the approximate trajectory $\tilde{L}$ is employed to estimate the cost function $C(\boldsymbol{\psi}, \tilde{L})$, since the exact trajectories cannot be acquired before the low-level planning is completed. The detailed methods used to generate approximate trajectories and compute assignment costs are presented in Section 4.

$$\begin{aligned} \text{Problem P2}: \boldsymbol{\psi}^* = \underset{\boldsymbol{\psi}}{\arg\min}\, C(\boldsymbol{\psi}, \tilde{L}_i) \\ \text{subject to Equations (3) and (4)} \end{aligned} \tag{14}$$

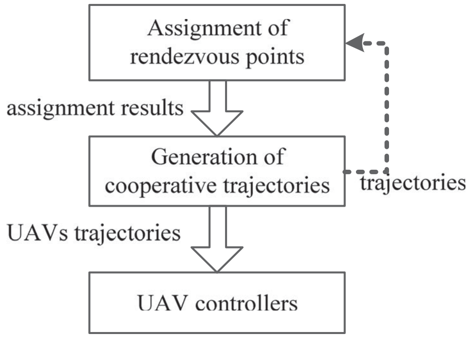

Figure 4. Hierarchical planning framework for formation rendezvous of unmanned aerial vehicles (UAVs).

The problem of cooperative trajectories generation is expressed as P3 in Equation (15). In P3, the assignment matrix is not a variable but a known parameter. The assignment matrix, together with the desired formation shape, determines the relations between the UAVs' goals, which are treated as the terminal constraints in the low-level planning. From P3, the low-level planning is an optimal control problem subject to nonlinear state constraints and non-convex path constraints.

$$\text{Problem P3: } s^*(t), u^*(t) = \underset{s(t),u(t)}{\arg\min} \sum_{i=1}^{N} \int_0^{t_f} ||\boldsymbol{u}_i(t)||_2 \, \mathrm{d}t$$
$$\text{subject to Equations (1), (2), (5) and (7)–(12)} \tag{15}$$

4. Assignment of rendezvous points

This section presents the method of assigning rendezvous points to UAVs based on the Dubins path. The assignment of rendezvous points is implemented on the high-level part of the hierarchical framework to provide corresponding relationships between UAVs and rendezvous points, which are used as inputs to the low-level planning problem.

In this work, the high-level planning problem is described as P4 in Equation (16). The objective of assignment is to minimize the total length of UAVs from their initial states to the assigned rendezvous points.

$$\text{Problem P4: } \boldsymbol{\psi}^* = \underset{\boldsymbol{\psi}}{\operatorname{argmin}} \sum_{i=1}^{N} \sum_{j=1}^{N} (\boldsymbol{\psi}_{ij} \cdot \tilde{l}_{ij})$$
$$\text{subject to Equations (3) and (4)} \tag{16}$$

where $\tilde{l}_{ij}$ denotes the length of the approximate trajectory from the initial state of UAV i to the rendezvous point j. The trajectory length from each UAV's initial location to each rendezvous point is computed in advance; thus, $\tilde{l}_{ij}$ is a known parameter in P4.

To compute the trajectory length approximately, a Dubins path is applied to consider the dynamics performance of fixed-wing UAVs. The Dubins path is the shortest path that satisfies the minimum turning radius between two points along specific orientations. Dubins (1957) proved that the shortest path must be one of the following six candidate paths: RSR, RSL, RLR, LSR, LSL and LRL, as shown in Figure 5. Thus, the Dubins path can be obtained by selecting the shortest one from the six possible paths.

Since the exact positions of the rendezvous points are also unknown, the trajectory is approximately generated by taking the centre of the rendezvous zone as the reference point of the formation.

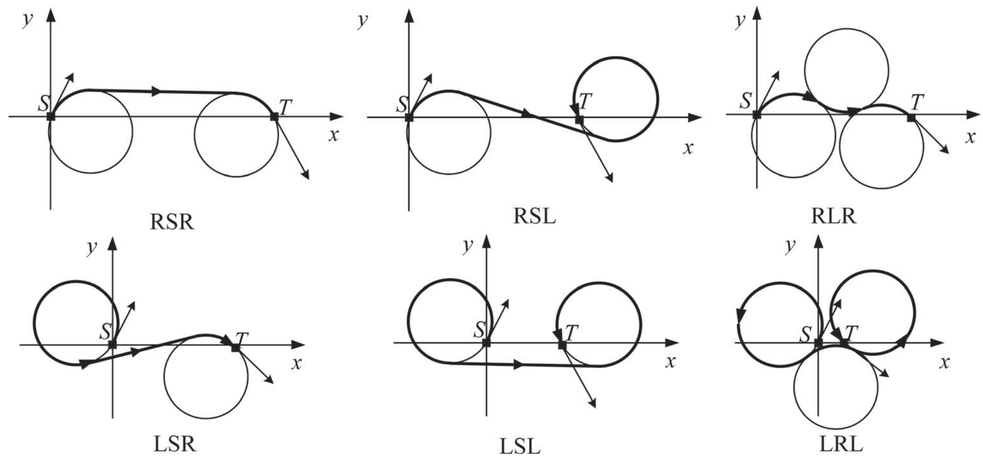

Figure 5. Illustration of six possible candidates for the Dubins path.

This approximation is effective when the radius of the rendezvous zone is relatively small compared with the distance between the initial positions and the rendezvous centre. Thus, the trajectory used at the high level is a 2D approximate trajectory, where the reference point for rendezvous is approximated and collision avoidance is not considered.

The assignment of rendezvous points, problem P4, is a typical combinatorial optimization problem. Many integer programming methods have been developed to solve such problems, such as the branch-and-bound method (Le Thi, Nguyen, and Pham Dinh 2012), genetic algorithm (Eun and Bang 2009) and particle swarm optimization (Sujit, George, and Beard 2008). These algorithms provide good performance on complex problems with high-dimensional variables or nonlinear constraints. However, the assignment problem of rendezvous points in this work does not need to use these advanced methods. Because each UAV selects only one rendezvous point and each rendezvous point is assigned to only one UAV, there are $N!$ possible solutions for an assignment problem. When the number of UAVs is not large, the costs of each possible solution can be calculated directly, since evaluating one candidate solution needs only $N - 1$ addition operations.

To estimate the effectiveness of the enumeration method for problem P4, Monte Carlo simulation is conducted. The test environment is the same as that described later, in Section 6. For different scale problems, 100 runs of each case are conducted to evaluate the computation time of acquiring the

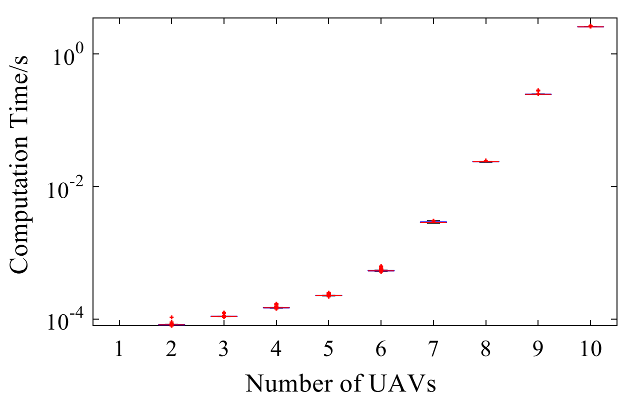

Figure 6. Boxplot of computation time for different numbers of unmanned aerial vehicles (UAVs).

assignment results. Figure 6 provides the boxplots of the statistical results of the computation time. Note that Figure 6 is plotted in a log-linear scale. When the formation consists of no more than eight UAVs, the computation time is less than 0.025 s by the enumeration method. It takes about 0.25 s and 2.7 s to assign rendezvous points for nine UAVs and 10 UAVs, respectively.

The scale of UAV formation is usually small ($N < 10$), in contrast to the concept of UAV swarms, which may contain tens of UAVs. Thus, the enumeration method is applicable to the considered assignment problem for formation rendezvous. However, it should be noted that the enumeration method is inefficient for general combinatorial optimization problems. For large-scale ($N > 10$) assignment problems or other computational intractable problems, heuristic algorithms (*e.g.* integer evolutionary algorithms) can be adopted to replace the enumeration method.

5. Generation of cooperative trajectories

The generation of multi-UAV cooperative trajectories at the low level is a non-convex optimal control problem. In order to apply sequential convex programming to obtain the cooperative trajectories, the non-convex optimal control problem is first transformed to a parameter optimization via discretization of the states and controls. Then, the convex subproblems are constructed by replacing the non-convex constraints with convex approximations around the nominal trajectories. Finally, the locally optimal solution of original non-convex optimization can be obtained by sequentially solving a series of convex optimization subproblems.

5.1. Parameterization of optimal control problem

The direct collocation method is employed to convert the optimal control problem into a parameter optimization problem. In detail, the flight time is uniformly divided into K intervals in terms of the time step h, and then the trajectory of each UAV is approximated with $K+1$ discrete points. The discrete states and controls at the time $t_k = t_0 + k \cdot h$, $k = 0,1, \ldots ,K$ are denoted as $\boldsymbol{s}_i(k)$ and $\boldsymbol{u}_{\mathrm{i}}(k)$, respectively.

Based on the trapezoidal rule, the dynamics constraints described by differential equations in Equation (7) can be transcribed as the following algebraic equations, which are nonlinear equality constraints about discrete states and controls:

$$\boldsymbol{s}_i(k) = \boldsymbol{s}_i(k-1) + \frac{h}{2} \cdot [f(\boldsymbol{s}_i(k), \boldsymbol{u}_i(k)) + f(\boldsymbol{s}_i(k-1), \boldsymbol{u}_i(k-1))], \quad k = 1, 2, \ldots, K \tag{17}$$

The initial conditions, admissible controls and state bounds at discrete points are expressed using discrete states and controls as in Equations (18), (19) and (20), respectively:

$$\boldsymbol{s}_i(0) = \boldsymbol{s}_{i,0} \tag{18}$$

$$\boldsymbol{u}_{i,\min} \le \boldsymbol{u}_i(k) \le \boldsymbol{u}_{i,\ \max} \tag{19}$$

$$\begin{aligned} &x_{\min} \le x(k) \le x_{\max}, y_{\min} \le y(k) \le y_{\max} \\ &h_{\min} \le h(k) \le h_{\max}, V_{\min} \le V(k) \le V_{\max} \end{aligned} \tag{20}$$

By imposing the constraints on discrete points, obstacle avoidance is described in Equation (21) to ensure that each discrete point is located outside the obstacles; collision avoidance is formulated in Equation (22) to guarantee that the distance between any two vehicles at each discrete time is larger than the safe distance:

$$||(x_i(k), y_i(k)) - (x_c, y_c)||_2 \geq r_c \tag{21}$$

$$||(x_i(k), y_i(k), h_i(k)) - (x_j(k), y_j(k), h_j(k))||_2 \geq R, i \neq j \tag{22}$$

From the high-level planning, the assignment results of rendezvous points are acquired, then the relative offsets among UAVs' goals are determined. Thus, formation constraints can be described as a set of linear equality constraints on the terminal states, as shown in Equation (23):

$$\boldsymbol{s}_i^T(K) - \boldsymbol{s}_1^T(K) = \Delta \boldsymbol{s}'_i, i = \{1, 2, \ldots, N\} \tag{23}$$

where $\Delta \boldsymbol{s}'_i$ denotes the offsets of terminal states between UAV i and UAV 1, and it is acquired according to the assignment result and the desired formation shape.

The rendezvous zone limits the location of each UAV goal, which is described as

$$||(x_i(K), y_i(K)) - (x_T, y_T)||_2 \leq r_T, i = \{1, 2, \ldots, N\} \tag{24}$$

The following non-convex parameter optimization problem (P5) can be derived from the non-convex optimal control problem, after the objective of P3 has also been discretized. The design variables of P5 include states and controls of each UAV at each discrete time. Thus, the dimensions of the design variables are $9N(K+1)$.

$$\text{Problem P5: min} \sum_{i=1}^{N} \sum_{k=0}^{K} ||u_i(k)||_2 \cdot h$$

$$\text{subject to Equations(17) –(24)} \tag{25}$$

Now that trajectory generation of multiple UAVs has been transformed as a non-convex optimization problem, SCP can be applied to search for the locally optimal trajectories for multi-UAV formation rendezvous.

5.2. Formulation of convex optimization subproblems

The method used to construct convex optimization subproblems is presented in this subsection. From the non-convex optimization formulation in Equation (25), the objective function and constraints of (18), (19), (20), (23) and (24) are already in convex form. Thus, the nonlinear dynamics, obstacle avoidance and collision avoidance constraints need to be convexified. The convexification is achieved by replacing the non-convex constraints with convex approximations around nominal trajectories, which are the solutions to the previous subproblem.

To be compatible with the convex optimization, the dynamics are linearized around nominal trajectory states and controls $\{\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i\}$, as shown in Equation (26):

$$d\boldsymbol{s}_i/dt = f(\boldsymbol{s}_i, \boldsymbol{u}_i) = \boldsymbol{A}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) \cdot \boldsymbol{s}_i + \boldsymbol{B}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) \cdot \boldsymbol{u}_i + \boldsymbol{b}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) \tag{26}$$

where

$$\boldsymbol{A}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) = \frac{\partial f}{\partial \boldsymbol{s}_i}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i), \boldsymbol{B}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) = \frac{\partial f}{\partial \boldsymbol{u}_i}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) \tag{27}$$

$$\boldsymbol{b}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) = f(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) - \boldsymbol{A}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) \cdot \bar{\boldsymbol{s}} - \boldsymbol{B}(\bar{\boldsymbol{s}}_i, \bar{\boldsymbol{u}}_i) \cdot \bar{\boldsymbol{u}}_i \tag{28}$$

Then, the discretized dynamics constraints (17) can be approximated by the following linear equality constraints with respect to states and controls:

$$\begin{aligned} \boldsymbol{s}_i(k) = \boldsymbol{s}_i(k-1) + h/2 \cdot [\bar{\boldsymbol{A}}_i(k) \cdot \boldsymbol{s}_i(k) + \bar{\boldsymbol{B}}_i(k) \cdot \boldsymbol{u}_i(k) + \bar{\boldsymbol{b}}_i(k)] + \\ h/2 \cdot [\bar{\boldsymbol{A}}_i(k-1) \cdot \boldsymbol{s}_i(k-1) + \bar{\boldsymbol{B}}_i(k-1) \cdot \boldsymbol{u}_i(k-1) + \bar{\boldsymbol{b}}_i(k-1)] \end{aligned} \tag{29}$$

where $\bar{\boldsymbol{A}}_i(k)$, $\bar{\boldsymbol{B}}_i(k)$ and $\bar{\boldsymbol{b}}_i(k)$ are determined by the nominal trajectories.

The avoidance constraints of cylinder obstacles in Equation (21) are concave functions. To acquire the convex approximations of obstacle avoidance, they are approximated by the affine forms, as follows:

$$\frac{\boldsymbol{\Delta}}{||\boldsymbol{\Delta}||_2} \cdot [(x_i(k), y_i(k)) - (\bar{x}_i(k), \bar{y}_i(k))]^T \geq r_c - ||\boldsymbol{\Delta}||_2 \tag{30}$$

where $\boldsymbol{\Delta} = (\bar{x}_i(k), \bar{y}_i(k)) - (x_c, y_c)$ and $(\bar{x}_i(k), \bar{y}_i(k))$ is the nominal horizontal position of UAV i at time k. The origin safe zone expressed by Equation (21) includes all the space outside the obstacle, while the convexified collision-free zone is a half space, which is tangential to the obstacle zone and perpendicular to the line connecting the nominal position and the obstacle centre (Morgan, Chung, and Hadaegh 2012).

Similarly, the inter-UAV collision avoidance constraints are convexified by linear approximations, as follows:

$$\frac{(\bar{\boldsymbol{p}}_i(k) - \bar{\boldsymbol{p}}_j(k)) \cdot (\boldsymbol{p}_i(k) - \boldsymbol{p}_j(k))^T}{||\bar{\boldsymbol{p}}_i(k) - \bar{\boldsymbol{p}}_j(k)||} \geq R, \quad i \neq j \tag{31}$$

where $\bar{p}_i(k) = (\bar{x}_i(k), \bar{y}_i(k), \bar{h}_i(k))$ is the nominal position of UAV i at time k. The convexified constraints require that any two UAVs cannot simultaneously be located in a band whose width is equal to the given safe distance R (Chen, Cutler, and How 2015).

According to the aforementioned approximate convex constraints around nominal trajectories, the convex optimization subproblem can be constructed as follows:

$$\text{Problem P6: min} \quad \sum_{i=1}^{N} \sum_{k=0}^{K} ||u_i(k)||_2 \cdot h$$

$$\text{subject to Equations (18) –(20), (23), (24) and(29) –(31)} \tag{32}$$

5.3. Solution by sequential convex programming

Sequential convex programming is a local optimization method for solving non-convex optimization using convex optimization. By sequentially solving the convex subproblem P6, the trajectories of formation rendezvous can be obtained.

The procedure of the SCP-based trajectory generation method for multi-UAV formation is shown in Algorithm 1. To alleviate the sensitivity to the initial guess, the iteration process of SCP is divided into two phases. In the first phase (lines 1–8), the optimization is conducted without the constraints of obstacle avoidance and inter-UAV collision avoidance. Then, the complete subproblem is iteratively solved to acquire the final cooperative trajectories in the second phase (lines 9–17). The trajectories generated in the first phase are employed as the nominal trajectories at the first iteration of the second phase.

To construct the subproblem at each iteration of SCP, the nominal trajectories are needed. For the first iteration, the nominal trajectories are initialized based on the Dubins path. The horizontal positions are set according to the Dubins path and the altitudes are acquired by linear interpolation between the initial altitude and final altitude. In the following iterations of SCP, the solutions in the previous iteration are used as the nominal trajectories to construct a new subproblem. According to previous studies on SCP-based non-convex optimization (Liu and Lu 2014), once the convex constraints and concave constraints are satisfied in an iteration of SCP, they will be satisfied in later iterations under mild conditions. Nevertheless, the origin dynamics constraints of UAVs are neither convex nor concave. Thus, the linearized dynamics constraints are taken as penalty terms in the optimization subproblems to decrease the probability that the subproblem is unfeasible.

To provide accurate approximations of linearized dynamics and help the SCP method to converge, the trust region is defined for the trajectory states in the optimization subproblem, described

Algorithm 1: SCP-based low-level trajectory generation method

Require: initial $\mathbf{s}_i^0$, $\mathbf{u}_i^0 \boldsymbol{L}^0$ and ϵ; $q = 0, \boldsymbol{L} = \boldsymbol{L}^0$
1 **while**$\{\mathbf{s}_i^q, \mathbf{u}_i^q\}$ does not satisfy (18)–(20), (23), (24), (29) and (34)
2 $q = q + 1$;
3 $\bar{\mathbf{s}}_i = \mathbf{s}_i^{q-1}$; $\bar{\mathbf{u}}_i = \mathbf{u}_i^{q-1}$;
4 $\{\mathbf{s}_i^q, \mathbf{u}_i^q\} \leftarrow$ the solution to P6 without (30), (31) but with (33)
5 **if**$\{\mathbf{s}_i^q, \mathbf{u}_i^q\}$ satisfies (18)–(20), (23), (24) and (29)
6 $\boldsymbol{L} = \boldsymbol{L}/2$;
7 **end if**
8 end while
9 $\boldsymbol{L} = \boldsymbol{L}^0$
10 **while**$\{\mathbf{s}_i^q, \mathbf{u}_i^q\}$ does not satisfy (18)–(20), (23), (24), (29)–(31) and (34)
11 $q = q + 1$;
12 $\bar{\mathbf{s}}_i = \mathbf{s}_i^{q-1}$; $\bar{\mathbf{u}}_i = \mathbf{u}_i^{q-1}$;
13 $\{\mathbf{s}_i^q, \mathbf{u}_i^q\} \leftarrow$ the solution to *P*6 with (33)
14 **if**$\{\mathbf{s}_i^q, \mathbf{u}_i^q\}$ satisfies (18)–(20), (23), (24) and (29)–(31)
15 $\boldsymbol{L} = \boldsymbol{L}/2$;
16 **end if**
17 **end while**
return$\{\mathbf{s}_i^q, \mathbf{u}_i^q\}$

as follows:

$$|\boldsymbol{s}_i(k) - \bar{\boldsymbol{s}}_i(k)| \leq \mathbf{L} \tag{33}$$

where $\boldsymbol{L}$ is the size of the trust region. Note that the preceding inequality equation is componentwise for each state variable. The trust region is gradually constricted to help the SCP method to converge, as shown in lines 5–7 and 14–16 of Algorithm 1.

The convergence criteria of the SCP method are that all the trajectory constraints are satisfied and the states obtained from the subproblem are consistent with the nominal states within the permitted errors, as shown in Equation (34):

$$|\boldsymbol{s}_i - \bar{\boldsymbol{s}}_i| \leq \varepsilon \tag{34}$$

6. Numerical experiments

In this section, the effectiveness and efficiency of the proposed method are demonstrated on three cases of UAV formation rendezvous. The simulations are implemented in a MATLAB® environment on a computer equipped with an Intel Core i5-2310 at 2.90 GHz. In the high-level planning, the assignment method based on the Dubins path given in Section 4 is employed. In the low-level planning, the trajectory-generating results obtained by SCP are compared with those obtained by General Purpose OPtimal Control Software-II (GPOPS-II) (Patterson and Rao 2014). GPOPS-II is a widely used toolbox for optimal control problems, which employs a typical nonlinear programming algorithm and Radau pseudospectral method. Convex optimization subproblems in the SCP-based method are solved using SeDuMi (Sturm 1999).

6.1. Simulation scenarios

Three formation shapes for rendezvous are considered: V-shaped formation, line formation and column formation. As mentioned in Section 2, the absolute locations of rendezvous points are not given, but the desired formation shape is known and can be expressed by the geometric relations between rendezvous points. Taking rendezvous point 1 as the reference point, the horizontal offsets between the rendezvous points and the reference point for different formation shapes are listed in Table 1. The

Table 1. Information on desired formation shapes.

Rendezvous point no.	Horizontal offsets for V formation (m)	Horizontal offsets for line formation (m)	Horizontal offsets for column formation (m)
Point 1	(0, 0)	(0, 0)	(0, 0)
Point 2	(−90, −60)	(−130, 0)	(0, −130)
Point 3	(+90, −60)	(+130, 0)	(0, +130)
Point 4	(−180, −120)	(−260, 0)	(0, −260)
Point 5	(+180,−120)	(+260, 0)	(0, +260)
Point 6	(−270, −180)	(−390, 0)	(0, −390)
Point 7	(+270, −180)	(+390, 0)	(0, +390)

Table 2. Initial states and final states of unmanned aerial vehicles (UAVs).

UAV no.	Initial states (x, y, h, V, χ, γ)	Final states (x, y, h, V, χ, γ)
UAV 1	(2500 m, 1200 m, 600 m, 30 m/s, 0, 0)	(–, –, 500 m, 30 m/s, 0, 0)
UAV 2	(2000 m, 2000 m, 600 m, 30 m/s, $-\pi/4$, 0)	(–, –, 500 m, 30 m/s, 0, 0)
UAV 3	(5500 m, 3500 m, 400 m, 30 m/s, $3\pi/2$, 0)	(–, –, 500 m, 30 m/s, 0, 0)
UAV 4	(2500 m, 2800 m, 450 m, 30 m/s, $\pi/2$, 0)	(–, –, 500 m, 30 m/s, 0, 0)
UAV 5	(5000 m, 1500 m, 450 m, 30 m/s, 0, 0)	(–, –, 500 m, 30 m/s, 0, 0)
UAV 6	(3000 m, 1500 m, 650 m, 30 m/s, π, 0)	(–, –, 500 m, 30 m/s, 0, 0)
UAV 7	(5800 m, 2200 m, 400 m, 30 m/s, $3\pi/2$, 0)	(–, –, 500 m, 30 m/s, 0, 0)

Table 3. Information on obstacles.

Obstacle no.	Centre (x_c, y_c)	Radius (r_c)
Obstacle 1	(3000 m, 2700 m)	200 m
Obstacle 2	(3000 m, 2000 m)	250 m
Obstacle 3	(3500 m, 1500 m)	300 m
Obstacle 4	(4500 m, 1500 m)	300 m
Obstacle 5	(5000 m, 2700 m)	200 m
Obstacle 6	(5000 m, 2000 m)	250 m

indices of rendezvous points for each formation shape can be found in Figures 2 and 8. The centre of the rendezvous zone is (4000 m, 2500 m) and the radius of the zone is 400 m.

For the different desired shapes of formation rendezvous, the same initial conditions of UAVs are used, as listed in Table 2. The desired final states of the different UAVs are identical, except for the horizontal positions, as given in Table 2. The rendezvous time of the UAV formation is set as 80 s.

In the mission space, there are six cylinder obstacles, whose centres and radii are given in Table 3. The minimum safe distance between UAVs is set as 100 m.

In all of the cases, the flight performances of UAVs are identical. The bounds on the position, altitude, velocity, flight path angle and controls are defined as follows:

$$\begin{aligned}
&x_{i,\min} = -\infty, x_{i,\max} = +\infty, y_{i,\min} = -\infty, y_{i,\max} = +\infty, h_{i,\min} = 200\ \text{m}, h_{i,\max} = 800\ \text{m} \\
&V_{i,\min} = 20\ \text{m/s}, V_{i,\max} = 40\text{m/s}, \gamma_{i,\min} = -\pi/6, \gamma_{i,\max} = \pi/6 \\
&\boldsymbol{u}_{i,\min} = (-5, -5, -10)\text{m/s}^2, \boldsymbol{u}_{i,\max} = (10, 5, 5)\ \text{m/ s}^2
\end{aligned} \tag{35}$$

6.2. Simulation results

To solve the preceding trajectory planning problems of formation rendezvous, the proposed hierarchical method based on SCP and the Dubins path is employed. In the SCP-based low-level planning, $K = 40$ is used to uniformly divide the flight time, and the initial size of trust region $\boldsymbol{L}^0$ is set as (2000 m, 2000 m, 200 m, 20 m/s, 2π, π). To demonstrate the merits of the proposed method, GPOPS-II with default tuning parameters is used for comparison.

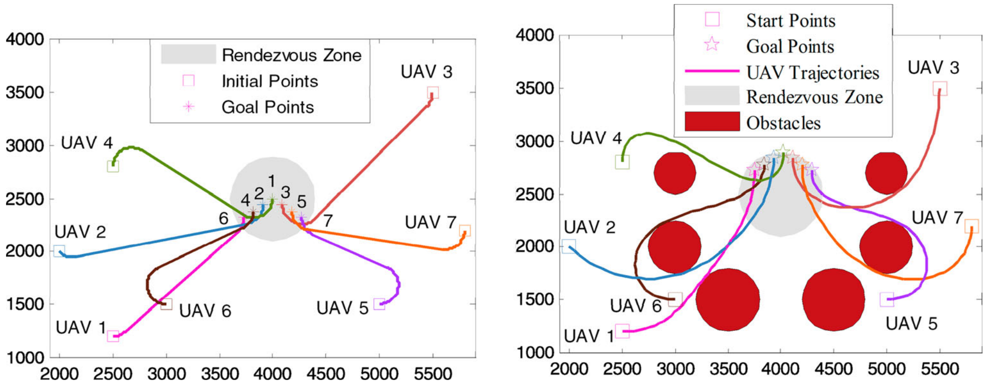

Figure 7. Assignment results (left) and planned trajectories (right) for a V-shaped formation of unmanned aerial vehicles (UAVs).

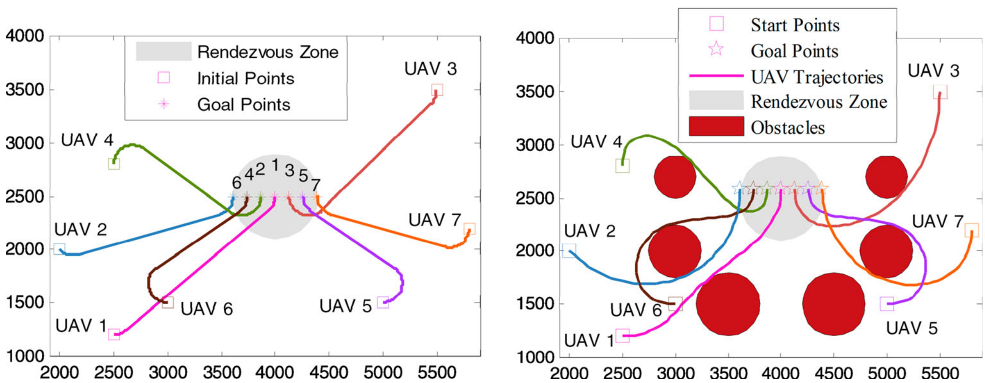

Figure 8. Assignment results (left) and planned trajectories (right) for a line formation of unmanned aerial vehicles (UAVs).

Figures 7–9 show the assignment results of rendezvous points to seven UAVs and planned trajectories for different desired formation shapes. In the high-level planning, the corresponding relationships between UAVs and rendezvous points are acquired, as shown by the labelled indices of UAVs and points. The Dubins paths from the initial states to the rendezvous states of the UAVs are demonstrated and they are employed as the initial guesses of the UAVs' trajectories in the low-level planning. Since all the possible solutions are estimated, the optimality of the assignment results can be guaranteed. Besides, the computation time of the assignment (including generation of Dubins paths) for all three cases is about 0.002 s. Thus, the high-level planning method can efficiently produce an approximate optimal solution to the assignment problem of rendezvous points.

In the low-level planning, from the trajectory results shown in Figures 7–9, the terminal positions of UAVs locate in the specified rendezvous zone and satisfy the requirements of the formation shape for each case. The planned trajectories are smooth and collision free. The UAVs that are close to the rendezvous points detour to achieve simultaneous arrival with other UAVs. Thus, the low-level planning can generate feasible trajectories for UAV formation rendezvous.

To compare the optimality and efficiency of SCP and GPOPS-II, trajectory planning tests for different number of UAVs are implemented. The statistical results of the objective function and computation time are given in Figures 10–12.

From the comparison results of optimized objectives shown in Figures 10–12, the optimality of results by SCP and GPOPS-II are comparable for single-UAV trajectory planning. For multiple-UAV trajectory planning, SCP can provide better optimality than GPOPS-II for the three cases. For the formation with seven UAVs, the objective values obtained by SCP are about 12%, 14% and 18% less

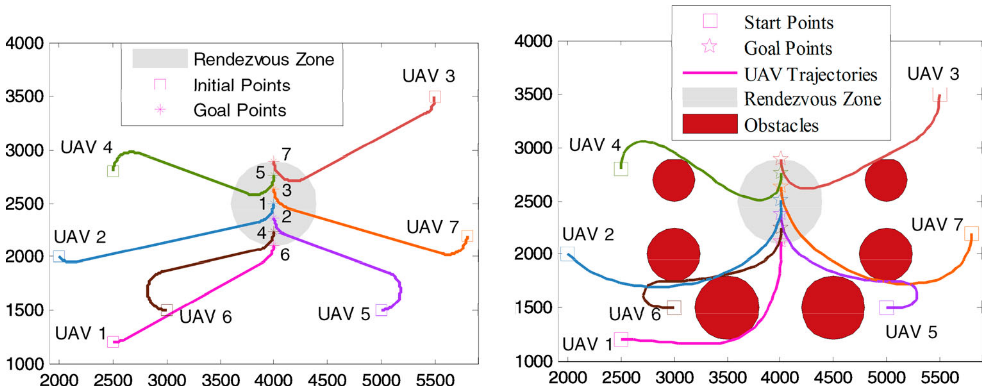

Figure 9. Assignment results (left) and planned trajectories (right) for a column formation of unmanned aerial vehicles (UAVs).

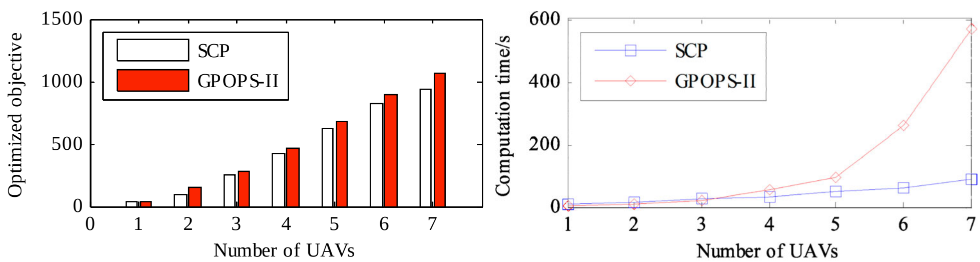

Figure 10. Comparison results of sequential convex programming (SCP) and GPOPS-II for a V-shaped formation of unmanned aerial vehicles (UAVs).

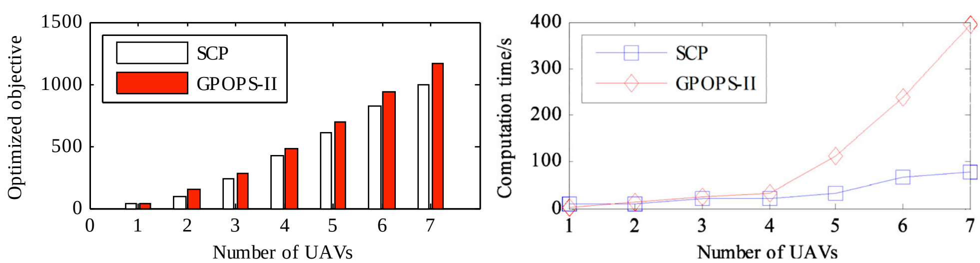

Figure 11. Comparison results of sequential convex programming (SCP) and GPOPS-II for line formation of unmanned aerial vehicles (UAVs).

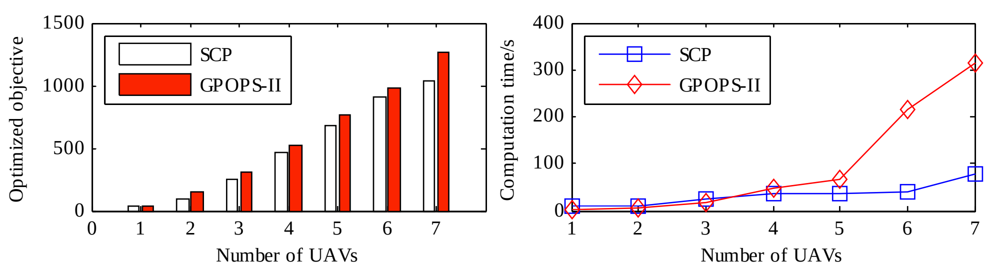

Figure 12. Comparison results of sequential convex programming (SCP) and GPOPS-II for column formation of unmanned aerial vehicles (UAVs).

than those obtained by GPOPS-II for the V-shaped formation, line formation and column formation, respectively.

From the computation times of SCP and GPOPS-II given in Figures 10–12, when the number of UAVs is no more than three, the computation time of SCP is close to that of GPOPS-II. But for trajectory planning of more than three UAVs, SCP shows higher efficiency than GPOPS-II. The efficiency advantage of SCP over GPOPS-II increases with the number of UAVs. For the formation with seven UAVs, the average computation time of SCP for the three cases is 88.5 s, while the average computation time of GPOPS-II is 426.5 s. In addition, GPOPS-II employs the Dubins-path-based initialization technique, which can improve the efficiency of the method. If ordinary linear interpolation is used to generate the initial trajectories, GPOPS-II takes more time than SCP, even for the single-UAV trajectory planning problem.

The results of comparison studies demonstrate that the proposed method outperforms GPOPS-II in terms of optimality and efficiency for trajectory planning of multi-UAV formation rendezvous.

7. Conclusions

Trajectory planning of fixed-wing UAV formation rendezvous is formulated as a mixed-integer optimal control problem. A hierarchical planning method based on the Dubins path and SCP is proposed to decompose the challenging problem as a classical assignment problem and an ordinary optimal control problem. The effectiveness and efficiency of the proposed method are demonstrated on multiple formation rendezvous cases. The comparison results show that the proposed method can provide better optimality and efficiency than typical nonlinear programming methods for multiple UAVs.

In future work, the decentralized planning method based on the Dubins path and SCP will be developed for large-scale UAV swarms. In addition, the SCP method will be enhanced with proper constraints handling techniques and global convergence techniques to further improve efficiency and convergence.

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