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Dual optimization design in formation keeping trajectory of multi UAV

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Abstract: When considering the problem of how to design multi UAV formation keeping optimization model, we combine the kinematic equation of multi UAV, performance index and constrained condition to construct a constrained optimization problem. After some transformation is used in the performance function to decompose the optimization problem, we introduce the Lagrange multiplier vector to construct a Lagrange function of this constrained optimization problem. To obtain the two classes of optimization variables-primal variable and dual variable, we find that all primal variables and dual variables can be formulated to the expressions about one dual variable. Then the gradient projection method from the convex optimization theory is proposed to obtain this dual variable. When this dual variable is solved, all the other optimization variables can also be obtained through simple substitution Finally, the efficiency of the proposed strategy can be confirmed by the simulation results.
Key Words: multi UAV; formation keeping; dual optimization; gradient projection algorithm

1 Introduction

The problem of cooperative control in multi-UAV formation is one of the key technology and research focus. Multi-UAV formation control means that during the mission, how to form and maintain a certain geometry shape to accommodate the platform performance, battlefield environment and tactical tasks. There are two main problems to be solved in multi-UAV formation control. One is the formation constitute, the other is formation keeping. The formation constitute includes how to generate the formation before flight The formation keeping includes the problem of switching in different geometry shapes. Further the main problem of formation keeping is to contract and expand the formation while keeping the geometry shape unchanged.

Because the single route planning is a simple case of the multi-UAV formation. So the current focus of numerous research have been turned to multi-UAV formation trajectory planning. An overview about the auxiliary role of the multi-UAV formation trajectory design in a cooperative investigative process is given in [1]. In [2] an intelligent multi-agent system is introduced in multi-UAV formation modeling and this multi-agent system is represented the communication structure between various UAV. In [3] descriptions of many various methods of UAV path planning are summarized, such as the ant colony algorithm, genetic algorithm, PSO, cell decomposition, artificial potential field method roadmap method, the probability of road signs law. All these path planning methods have a common characteristic, that is, the single or multiple objectives and the different constraints during the implementation of their respective mandates are considered from a different point. Then through combing the objective and many constraints, an optimization problem is get. This optimization problem can be solved by any kind of intelligent algorithm. At present most of the research are concentrated on improving one section of the intelligence algorithms to solve that optimization problem. In [4] the weights which exist in many intelligent algorithms are adjusted based on the gray system theory. In [5] the methods about how to fuse the multiple sensor information are analyzed to obtain state estimates from the perspective of information fusion. In [6] the particle filter algorithm is applied to track path under the non-Gaussian condition. In [7] one improved interacting multiple model particle filter is proposed and applied in target tracking. In [8] the ant colony algorithm from the multi-objective optimization is studied. In [9] one consensus genetic algorithm is used to solve the multi-objective optimization problem.

Here in this paper the convex optimization algorithm is applied into the multi-UAV formation keeping. After establishing an optimization model of multi-UAV formation keeping trajectory, we combine the objective function and various constrained conditions to construct the corresponding Lagrangian function. Due to the presence of two types of optimization variables in the Lagrangian function. These two types of optimization variables are called primal variables and dual variables. When the optimal necessary condition is used to take minimize operation with respect to the primal variables, many equalities can be obtained. These equalities mean that not only all the primal variables but also the other dual variables can be represented as some explicit relationships of a certain dual variable. Then the process of determining the optimal decision variables in the original constrained optimization problem is changed to solve this special dual variable. This dual variable is solved by maximizing the dual function. Furthermore the usual gradient projection algorithm is proposed to maximize the dual function to solve that particular dual variable. When this dual variable is solved, all the primal variables and other dual variables are get by only substitution operations easily.

2 Multi-UAV formation keeping model

The formation control means some UAV which lie in the different initial states will converge to the specified geometric pattern. The optimal control problem in formations constitute can be divided into the free terminal state constraints and fixed terminal state constraints. To model the trajectory optimization problem in a free terminal

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condition, we choose the terminal state of UAV platforms suitably to reduce the energy loss during the formation process. Define the energy function of the i th UAV as:

$$E_i(t) = h_i(t) + \frac{v_i^2(t)}{2g} = s_i'(t)Qs_i(t) + c's_i(t) \quad (1)$$

Where $E_i(t)$ denotes the equivalent total energy of the i th UAV at time t. This total energy includes the potential energy and kinetic energy, $h_i(t)$ denotes the position coordinate components, $v_i(t)$ is the flight speed, $s_i(t)$ is the state variable after transforming the three freedoms particle motion in a coordinate, vector c and matrix Q are given as follows:

$$c = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}^T, Q = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2g} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2g} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2g} \end{bmatrix}$$

The corresponding index of the i th UAV is defined as follows:

$$J_i = \sum_{i=1}^{T} \left\{ \begin{array}{l} \alpha_i \left[s_i'(t)Qs_i(t) + c's_i(t) \right] \\ +\beta_i \left[z_i(t) - z_i^{ref}(t) \right]' \left[z_i(t) - z_i^{ref}(t) \right] \end{array} \right\} \quad (2)$$

Where scalar α_i, β_i are the weight coefficients, $z_i^{ref}(t)$ is the reference trajectory of the i th UAV which needs to track. In the limited central control structure, the formation route is planned by the lead aircraft and other UAV need to maintain the relative positions of the lead aircraft. When the i UAV is not the lead aircraft, we set $\beta_i = 0$. The collaborative index of the i th UAV may make each UAV keep the specified geometry shape. The collaborative index between the i th UAV and j th UAV is defined as follows:

$$\begin{aligned} J_{ij} = \sum_{i=1}^{T} \eta_i & \left[z_i(t) - z_j(t) - \Delta z_{ij}(t) \right]' \\ & \times \left[z_i(t) - z_j(t) - \Delta z_{ij}(t) \right] \end{aligned} \quad (3)$$

The scalar equation η_i is the weight coefficient. Combining the index functions (2), (3) and the kinematic model (1), the multi-UAV formation keeping optimization model is get.

$$\begin{aligned} & \min_{u_i(t), s_i(t)/i,t} \sum_{i=1}^{T} \left[J_i + \sum_{j=1}^{T} J_{ij} \right] \\ & \textit{subject to } s_i(t+1) = As_i(t) + Bu_i(t) \quad (4) \\ & \quad z_i(t) = Cs_i(t), \quad s_i(0) = s_i^0 \quad \forall i \in \{0 \cdots I\} \\ & \quad u_i(t) \in U_i, s_i(t) \in S_i, \forall t \in \{1 \cdots I\} \end{aligned}$$

Where $z_i(t)$ is the observed variable of the i th UAV, U_i and S_i are two feasible convex regions about the UAV control input and state variable. When sample period is Δt, the matrix A, B and C are the discrete state and observation matrix model respectively.

$$A = \begin{bmatrix} 1 & \Delta t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} \frac{\Delta t^2}{2} & 0 & 0 \\ \Delta t & 0 & 0 \\ 0 & \frac{\Delta t^2}{2} & 0 \\ 0 & \Delta t & 0 \\ 0 & 0 & \frac{\Delta t^2}{2} \\ 0 & 0 & \Delta t \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The collision avoidance can be achieved by specified different height layers which are determined by the feasible region S_i of each UAV state. From the optimization model (4), we see that the index function of each single UAV depends not only on its own state but also on other UAV states. If some appropriate auxiliary decision variables and constrained conditions are added, the coupling factor can be transferred into the constrained conditions, and the original problem is decoupled. As the coupling characteristic exists in the index function, we apply the indirect decomposition method to achieve the decoupling problem.

Denoting the state vector s_i and input u_i as follows:

$$\begin{cases} s_i = \left(s_i'(1) \quad \cdots \quad s_i'(T) \right)' \\ u_i = \left(u_i(0) \quad \cdots \quad u_i(T-1) \right)' \end{cases}$$

Defining the reference route as:

$$z_i^{ref} = \left(z_i^{ref}(1) \quad \cdots \quad z_i^{ref}(T) \right)'$$

Defining one unit matrix I_T as follows, and its dimension is set $T \times T$.

$$\tilde{Q} = I_T \otimes Q, \tilde{c} = \left(c' \quad \cdots \quad c' \right)', G = I_T \otimes C$$

The state equations (4) can rewritten in the recursion formula.

$$s_i = \begin{bmatrix} s_i(1) \\ s_i(2) \\ \vdots \\ s_i(T) \end{bmatrix} = \underbrace{\begin{bmatrix} A \\ A^2 \\ \vdots \\ A^T \end{bmatrix}}_{E} s_i^0 + \underbrace{\begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ A^{T-1}B & A^{T-2}B & \cdots & B \end{bmatrix}}_{F} u_i \quad (5)$$

Abbreviating the former (5) as:

$$s_i = Es_i^0 + Fu_i \quad (6)$$

After substituted (6) into the optimization model, the compact form of the multi UAV formation keeping model can be get.

$$\begin{aligned} & \min_{u_i(t), s_i(t)/i,t} \sum_{i=1}^{T} \left[J_i(s_i, u_i) + \sum_{j=1}^{T} J_{ij}(s_i, s_j) \right] \\ & \textit{subject to } s_i = Es_i^0 + Fu_i \quad (7) \\ & \quad u_i(t) \in U_i, s_i(t) \in S_i, \forall t \in \{1 \cdots I\} \end{aligned}$$

Where the individual indicator of the i th UAV is get:

$$J_i\left(s_i,u_i\right)=\alpha_i\left[s_i'\tilde{Q}s_i+\tilde{c}'s_i\right]+\beta_i\left[Gs_i-z_i^{ref}\right]'\left[Gs_i-z_i^{ref}\right] \quad (8)$$

Where the collaborative index between the i th UAV and j th UAV is also get:

$$J_{ij}\left(s_i,s_j\right)=\eta_i\left[Gs_i-Gs_j-\Delta z_{ij}\right]'\left[Gs_i-Gs_j-\Delta z_{ij}\right]$$

(9)

Where the column vector of the formula is set:

$$\Delta z_{ij}=\left(\Delta z_{ij}(1) \quad \cdots \quad \Delta z_{ij}(T)\right)'$$

From the trajectory optimization problem (7), each UAV has its own constrained condition. But the coupling term exist in its collaborative index $J_{ij}\left(s_i,s_j\right)$. To maintain the relative positional relationship between the UAV, the index function of the i UAV may contain the decision variable which comes from other UAV. In order to make each UAV independent we impose some transformations into the formula (7) . Then we find that the decision variable corresponding to each UAV only exists in its own individual index function. Introduce an auxiliary decision variable $\tilde{s}_i$ as follows

$$\tilde{\tilde{s}}_i=\left(\tilde{s}_{i1} \quad \cdots \quad \tilde{s}_{ij} \quad \cdots \quad \tilde{s}_{il}\right)'$$

The equivalent optimization problem can be obtained.

$$\begin{aligned}&\min_{u_i(t),s_i(t),\tilde{s}_i,\tilde{\tilde{s}}_i}\sum_{i=1}^{T}\left[J_i\left(s_i,u_i\right)+\sum_{j=1}^{T}J_{ij}\left(\tilde{s}_i,\tilde{\tilde{s}}_{ij}\right)\right]\\&\textit{subject to } s_i=Es_i^0+Fu_i \qquad (10)\\&\qquad u_i(t)\in U_i, s_i(t)\in S_i,\\&\qquad \tilde{s}_i=s_i, \quad \tilde{s}_{ij}=\tilde{s}_j \quad \forall i,j\in[0,I]\end{aligned}$$

Where in the optimization problem (10), there are four optimization decision variables $\left\{u_i,s_i,\tilde{s}_i,\tilde{\tilde{s}}_{ij}\right\}$. This constrained optimization problem can be solved by the next dual strategy.

3 Dual optimization design

Firstly we construct the Lagrangian function corresponding to (10) and introduce some appropriate Lagrange multiplier vector λ_i,μ_i,γ_i. Then the Lagrangian function corresponds to (10) is get:

$$L\left(s_i,u_i,\tilde{s}_i,\tilde{\tilde{s}}_{ij}\right)=\sum_{i=1}^{T}\begin{bmatrix}J_i\left(s_i,u_i\right)+\sum_{j=1}^{T}J_{ij}\left(\tilde{s}_i,\tilde{\tilde{s}}_{ij}\right)\\+\lambda_i\left(s_i-Es_i^0-Fu_i\right)+\mu_i\left(s_i-\tilde{s}_i\right)\\+\gamma_i\left(\tilde{s}_{ij}-\tilde{s}_j\right)\end{bmatrix}$$

$$\begin{aligned}&=\sum_{i=1}^{T}\begin{bmatrix}J_i\left(s_i,u_i\right)+\lambda_is_i-\lambda_iEs_i^0-\lambda_iFu_i+\mu_is_i\\+\sum_{j=1}^{T}J_{ij}\left(\tilde{s}_i,\tilde{\tilde{s}}_{ij}\right)-\mu_i\tilde{s}_i+\gamma_i\tilde{s}_{ij}-\gamma_i\tilde{s}_j\end{bmatrix} \qquad (11)\\&=\sum_{i=1}^{T}\left[L_i\left(s_i,u_i,\lambda_i,\mu_i\right)+\tilde{L}_i\left(\tilde{s}_i,\tilde{\tilde{s}}_{ij},\mu_i,\gamma_i\right)\right]\end{aligned}$$

Where the two quantities are defined by the formulas:

$$\begin{aligned}&L_i\left(s_i,u_i,\lambda_i,\mu_i\right)=J_i\left(s_i,u_i\right)+\lambda_is_i-\lambda_iEs_i^0-\lambda_iFu_i+\mu_is_i\\&\tilde{L}_i\left(\tilde{s}_i,\tilde{\tilde{s}}_{ij},\mu_i,\gamma_i\right)=\sum_{j=1}^{T}J_{ij}\left(\tilde{s}_i,\tilde{\tilde{s}}_{ij}\right)-\mu_i\tilde{s}_i+\gamma_i\tilde{s}_{ij}-\gamma_i\tilde{s}_j\end{aligned} \qquad (12)$$

The decision variables in (11) can be divided into one dual variables $\left\{\lambda_i,\mu_i,\gamma_i\right\}_{i=1}^{I}$ and primal variables $\left\{u_i,s_i,\tilde{s}_i,\tilde{s}_{ij}\right\}_{i=1}^{I}$. The dual function of original optimization problem (10) is get:

$$\min_{\left\{u_i,s_i,\tilde{s}_i,\tilde{\tilde{s}}_{ij}\right\}}\sum_{i=1}^{T}\left[L_i\left(s_i,u_i,\lambda_i,\mu_i\right)+\tilde{L}_i\left(\tilde{s}_i,\tilde{\tilde{s}}_{ij},\mu_i,\gamma_i\right)\right] \qquad (13)$$

The summation in (13) is similar to the separable variable. Because the former term contains only primal variables $\left\{u_i,s_i\right\}_{i=1}^{I}$ and the latter term contains only primal variable $\left\{\tilde{s}_i,\tilde{s}_{ij}\right\}_{i=1}^{I}$. As there is no any constrained condition in (13), we apply the optimal necessary conditions to solve this unconstrained minimization problem. Based on the optimal necessary conditions, we get :

$$\begin{cases}\dfrac{\partial L_i\left(s_i,u_i,\lambda_i,\mu_i\right)}{\partial s_i}=\dfrac{\partial J_i\left(s_i,u_i\right)}{\partial s_i}+\lambda_i+\mu_i=0\\[2ex]\dfrac{\partial L_i\left(s_i,u_i,\lambda_i,\mu_i\right)}{\partial u_i}=\dfrac{\partial J_i\left(s_i,u_i\right)}{\partial u_i}-\lambda_iF=0\end{cases} \qquad (14)$$

Expanding the partial derivatives in (14), we get:

$$\begin{aligned}&\frac{\partial J_i\left(s_i,u_i\right)}{\partial s_i}=2\alpha_i\tilde{Q}s_i+\alpha_i\tilde{c}+2\beta_iG\left(Gs_i-z_i^{ref}\right)\\&\frac{\partial J_i\left(s_i,u_i\right)}{\partial u_i}=\frac{\partial J_i\left(s_i,u_i\right)}{\partial s_i}\frac{\partial s_i}{\partial u_i} \qquad (15)\\&=\left[2\alpha_i\tilde{Q}s_i+\alpha_i\tilde{c}+2\beta_iG\left(Gs_i-z_i^{ref}\right)\right]F\end{aligned}$$

Substituting (15) into (14) , we get:

$$2\alpha_i\tilde{Q}s_i+\alpha_i\tilde{c}+2\beta_iG\left(Gs_i-z_i^{ref}\right)+\lambda_i+\mu_i=0 \qquad (16)$$

The variable s_i is solved as follows:

$$\left(2\alpha_i\tilde{Q}+2\beta_iGG\right)s_i=2\beta_iGz_i^{ref}-\alpha_i\tilde{c}-\lambda_i-\mu_i$$

As s_i is a state variable, so after taking inverse operation with the above equation, we get:

$$s_i=\left(2\alpha_i\tilde{Q}+2\beta_iGG\right)^{-1}\left(2\beta_iGz_i^{ref}-\alpha_i\tilde{c}-\lambda_i-\mu_i\right) \quad (17)$$

From the second equation in (14) , we get:

$$2\alpha_i\tilde{Q}s_i+\alpha_i\tilde{c}+2\beta_iG\left(Gs_i-z_i^{ref}\right)=\lambda_i \qquad (18)$$

Substituting (18) into the first equation of (14), we get:

$$\lambda_i+\lambda_i+\mu_i=0$$

That is:

$$\mu_i=-2\lambda_i \qquad (19)$$

Substituting (19) into (17) again, the first primal variable s_i is get as follows:

$$s_i=\left(2\alpha_i\tilde{Q}+2\beta_iGG\right)^{-1}\left(2\beta_iGz_i^{ref}-\alpha_i\tilde{c}+\lambda_i\right) \qquad (20)$$

Based on the state equation, the second primal variable u_i is get as follows:

$$u_i=F^{-1}\left(s_i-Es_i^0\right) \qquad (21)$$

From (19), (20), (21), we see the two primal variables $\left\{u_i,s_i\right\}_{i=1}^{I}$ and dual variables $\left\{\mu_i\right\}_{i=1}^{I}$ can be rewritten as the relationships with the dual variable $\left\{\lambda_i\right\}_{i=1}^{I}$. These relationships tell us that if the dual variable $\left\{\lambda_i\right\}_{i=1}^{I}$ is solved, then the variables $\{u_i, s_i, \mu_i\}_{i=1}^{I}$ can also be get easily by substituted the dual variable $\{\lambda_i\}_{i=1}^{I}$.

Applying the optimal necessary condition to the second objective function of (13), we get:

$$\frac{\partial \tilde{L}_i\left(\tilde{s}_i, \tilde{\tilde{s}}_{ij}, \mu_i, \gamma_i\right)}{\partial \tilde{s}_i} = 0, \quad \frac{\partial L_i\left(s_i, u_i, \lambda_i, \mu_i\right)}{\partial \tilde{s}_{ij}} = 0 \quad (22)$$

Reformulating and expanding (22), we get:

$$\sum_{j=1}^{T} \frac{\partial J_{ij}\left(\tilde{s}_i, \tilde{\tilde{s}}_{ij}\right)}{\partial \tilde{s}_i} - \mu_i - \gamma_i = 0, \ \sum_{j=1}^{T} \frac{\partial J_{ij}\left(\tilde{s}_i, \tilde{\tilde{s}}_{ij}\right)}{\partial \tilde{s}_{ij}} + \gamma_i = 0 \quad (23)$$

Respectively the two partial derivatives can be calculated:

$$\begin{cases} \dfrac{\partial J_{ij}\left(\tilde{s}_i, \tilde{\tilde{s}}_{ij}\right)}{\partial \tilde{s}_i} = \eta_i G\left[G\tilde{s}_i - G\tilde{s}_{ij} - \Delta z_{ij}\right] \\ \dfrac{\partial J_{ij}\left(\tilde{s}_i, \tilde{\tilde{s}}_{ij}\right)}{\partial \tilde{s}_{ij}} = -\dfrac{\partial J_{ij}\left(\tilde{s}_i, \tilde{\tilde{s}}_{ij}\right)}{\partial \tilde{s}_i} \end{cases} \quad (24)$$

Substituting (24) into (23) , we get:

$$\begin{cases} \sum_{j=1}^{T} \eta_i G\left[G\tilde{s}_i - G\tilde{s}_{ij} - \Delta z_{ij}\right] = \mu_i + \gamma_i \\ \eta_i G\left[G\tilde{s}_i - G\tilde{s}_{ij} - \Delta z_{ij}\right] = \gamma_i \end{cases} \quad (25)$$

From the above equation it is obvious to get:

$$\sum_{j=1}^{T} \gamma_i = \mu_i + \gamma_i$$

That is:

$$\gamma_i = \frac{\mu_i}{I-1} = -\frac{2}{I-1}\lambda_i \quad (26)$$

The above equation shows that the dual variables $\{\gamma_i\}_{i=1}^{I}$ can be also written as one relationship with $\{\lambda_i\}_{i=1}^{I}$. Expanding the second equality of (25), we get:

$$\eta_i GG\left(\tilde{s}_i - \tilde{s}_{ij}\right) = \gamma_i + \eta_i G\Delta z_{ij}$$

That is:

$$\tilde{s}_i - \tilde{s}_{ij} = \left(\eta_i GG\right)^{-1}\left(\gamma_i + \eta_i G\Delta z_{ij}\right) \quad (27)$$

It is that:

$$\tilde{s}_i = \tilde{s}_{ij} + \left(\eta_i GG\right)^{-1}\left(\gamma_i + \eta_i G\Delta z_{ij}\right) \quad (28)$$

The primal variables $\{\tilde{s}_i, \tilde{s}_{ij}\}_{i=1}^{I}$ are solved from the equations (25). After substituting (28) into the first equality of (25), we find that it is an identity so that we can not solve the two primal variables $\{\tilde{s}_i, \tilde{s}_{ij}\}_{i=1}^{I}$. To overcome this problem, we assume there is a linear affine relation holds between $\tilde{s}_i$ and $\tilde{s}_{ij}$ i.e., we have:

$$\tilde{s}_i = M\tilde{s}_{ij} + m \quad (29)$$

Substituting (28) into the linear affine relation (29), we get:

$$M\tilde{s}_{ij} + m = \tilde{s}_{ij} + \left(\eta_i GG\right)^{-1}\left(\gamma_i + \eta_i G\Delta z_{ij}\right)$$

Continuing to sort out the above equation, we get:

$$\tilde{s}_{ij} = \left(M - I\right)^{-1}\left[\left(\eta_i GG\right)^{-1}\left(\gamma_i + \eta_i G\Delta z_{ij}\right) - m\right] \quad (30)$$

The formulas (28) and (30) mean that the remaining two primal variables $\{\tilde{s}_i, \tilde{s}_{ij}\}_{i=1}^{I}$ and remaining dual variables $\{\gamma_i\}_{i=1}^{I}$ are all expressed as the relationships of the dual variables $\{\lambda_i\}_{i=1}^{I}$. The above analysis shows that all the primal variables and the remaining two dual variables are related with $\{\lambda_i\}_{i=1}^{I}$. The process of solving $\{\lambda_i\}_{i=1}^{I}$ is very important in the whole optimization problem. As long as the value of $\{\lambda_i\}_{i=1}^{I}$ is given, all the decision variables of the original optimization problem can be get by substituting the value of $\{\lambda_i\}_{i=1}^{I}$ into the above equation.

Substituting the above relations into the dual function (13), the dual optimization problem with respect to only $\lambda = \left(\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_I\right)'$ is get.

$$\min_{\lambda} f\left(\lambda\right) = \min_{\lambda} \sum_{i=1}^{I} f_i\left(\lambda\right) = \min_{\lambda} \sum_{i=1}^{I} \left[L_i\left(\lambda_i\right) + \tilde{L}_i\left(\lambda_i\right)\right] \quad (31)$$

When applied the usual gradient projection algorithm in the dual optimization problem (31), its main steps include:

$$\begin{cases} \theta^k = \lambda^k + \sigma^k\left(\lambda^k - \lambda^{k-1}\right) \\ \lambda^{k+1} = P\left(\theta^k - \alpha\nabla f\left(\theta^k\right)\right) \end{cases} \quad (32)$$

Where the first step is the construction operation and the second step is the projection operation, the symbols P denotes the projection operator. The initial iteration value is chosen as $\lambda^{-1} = \lambda^0$ and weights $\sigma^k \in \left(0,1\right)$. When a reasonable weight step σ^k is chosen, the iteration complexity of the gradient projection algorithm can be guaranteed to be $O\left(1/\sqrt{\varepsilon}\right)$. ε is a very small positive value. The weight step σ^k is usually chosen as follows:

$$\sigma^k = \frac{\tau^k\left(1 - \tau^{k-1}\right)}{\tau^{k-1}}, \ k = 0,1\cdots \quad (33)$$

Where the sequence $\{\tau^k\}$ meets that $\tau^0 = \tau^1 \in \left[0,1\right]$ and includes:

$$\frac{\left(1 - \tau^{k+1}\right)}{\left(\tau^{k+1}\right)^2} \le \frac{1}{\left(\tau^k\right)^2}, \ \tau^k \le \frac{2}{k+2}, \ k = 0,1\cdots \quad (34)$$

Where the selection strategy is generally used:

$$\sigma^k = \begin{cases} 0 & if\ k = 0 \\ \dfrac{k-1}{k+2} & if\ k = 1,2\cdots \end{cases} \quad \tau^k = \begin{cases} 1 & if\ k = -1 \\ \dfrac{2}{k+2} & if\ k = 0,1\cdots \end{cases}$$

Above we give a detailed analysis about the dual function of the original optimization problem (10) . The dual function is transformed into one optimization problem which contains only one dual variable. The usual gradient projection algorithm is proposed to solve it. When the dual variables that exist in the dual optimization problem are solved, then all the other two dual variables and four primal variables can be get by substitution operation

4 Simulation

To verify the efficiency of the dual optimization algorithm in multi-UAV formation keeping trajectory, we build the simulation environment under windows system. The simulation scenario includes three UAV which constitute formation in the two-dimensional plane. The connection relation between the three UAV can be expressed in a matrix form as follows:

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The element 1 in the above matrix denotes that there have been numerous exchanges of information between the two UAV. The sample interval which is used to discrete the formation keeping model is set as $\Delta T = 1s$. The maximum acceleration rate of each UAV is set as $35m/s^2$, the maximum speed is set as $250m/s$, the total execution time is set as $30s$. The three parameters in the objective function are chosen as follows:

$$\alpha_i = 0.01, \beta_i = 0.1, \eta_i = 0.1$$

When the three UAV move in the two-dimensional plane, the initial positions of the three UAV are respectively:

$$(0,2500),(0,2300),(0,2200)$$

The dual optimization results corresponding to the three UAV formation keeping trajectories are shown in Fig1. The black curve represents the reference tracking curve and the three color curves represent the flight trajectories of each UAV. From Fig1, we see that the three UAV can all achieve the goal of tracking the reference trajectory. The change process curve about objective function with the number of iteration is drawn in Fig2. From Fig2, we see that when the number of iteration is 50, the gradient projection algorithm will be terminated and the algorithm process has reached a stable convergence.

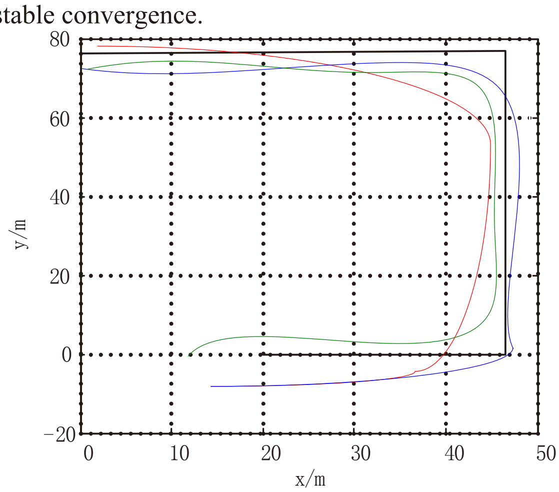

Fig1: The formation flight path

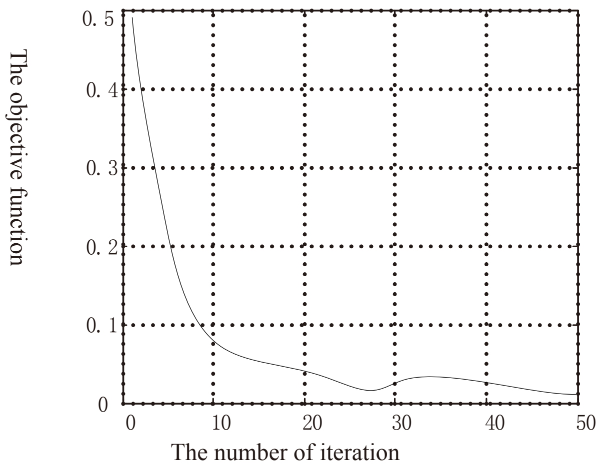

Fig 2: The iterative change case

5 Conclusion

After establishing the multi-UAV formation keeping model, the relationships between the primal variables and dual variables in the dual function are derived. So the original constrained optimization problem can be transformed to an unconstrained dual optimization problem. A gradient projection algorithm is proposed to solve the dual optimization problem. The next subject is to consider how to apply the dual optimization into the multi-UAV self-organization problems.

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