Linear System Theory: HW3

2022/11/24

（Due: Dec. 1, 2022）

1. (20’) Consider the linear time invariant system (1).

$$\dot{x} = \begin{bmatrix} -1 & 5 \\ 0 & 2 \end{bmatrix} x + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$
$$y = \begin{bmatrix} -2 & 4 \end{bmatrix} x - 2u \tag{1}$$

(1). Is it BIBO stable?

(2). Is the state equation marginally stable or asymptotically stable?

2. (10’) For system $\dot{x} = f(x), x \in \mathbb{R}^n$ with $f(0) = 0$. Define stability and instability using $\varepsilon - \delta$ language.

3. (10’) Consider the system $\dot{x} = f(x), \quad x \in \mathbb{R}^n$ with $f(0) = 0$. Show that if the equilibrium point $x^* = 0$ is exponentially stable, then it is asymptotically stable.

4. (20’) Consider the system

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{x_1}{(1+x_1^2)^2} - \frac{x_2}{(1+x_1^2+x_2^2)^2}$$

(1). Find its equilibrium point.

(2). Show that $V(x) = x_2^2 + \dfrac{x_1^2}{1+x_1^2}$ is a Lyapunov function of this system.

5. (40’) Prove the Lyapunov stability theory, i.e., the Theorem 4.1 in Khalil “Nonlinear Systems”, the 3rd edition.

(This theorem is central to control theory. Please make sure that you truly understand the proof.)