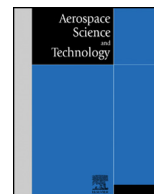




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Multivariable adaptive control based consensus flight control system for UAVs formation [☆]

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ABSTRACT

Formation flight contributes to improving the attack, reconnaissance and survival ability of the multiple unmanned aerial vehicles (UAVs). This paper studies a multivariable adaptive control based consensus flight method for UAVs formation. A majority of existing research is focused on the leader-following consensus problem assuming that only the parameters of followers are uncertain. However, they do not consider the leader dynamic uncertainty and the unknown external disturbances. Therefore, this paper addresses the problem of the UAVs consensus flight control with parametric uncertainties and unknown external disturbances for both the leader and follower. A multivariable model reference adaptive control (MRAC) based consensus flight control scheme is designed for UAVs formation, which enables the follower UAV to track the leader UAV. The stability of the multivariable MRAC based consensus flight control system is analyzed. Simulation results show that the proposed adaptive consensus flight control scheme has stronger robustness and adaptivity than the fixed control scheme.

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1. Introduction

With the wide use of high-tech science and technology in the field of aviation, development of unmanned aerial vehicles (UAVs) has made a breakthrough [1]. After decades of development, the UAVs have been widely used in military and civil applications. However, a single UAV has many limitations to complete the complex tasks. For example, when a single UAV is performing a reconnaissance mission, it may fail to observe the target from different directions, due to the sensor's detection limitation. Therefore, the cooperativity among UAVs is usually indispensable. In order to improve the efficiency and broaden the range of applications, the concept of UAVs formation flight was put forward, which enables UAVs to complete the consensus flight, aerial refueling, cooperative combat missions and so on [2]. In military applications, the UAVs formation flight can expand investigation view and improve the rate of task completion. In civil applications, it has wide prospect in the fields of meteorological exploration, forest fire prevention and emergency rescue. In these applications, several key technical problems should be addressed, such as cooperative path planning [3,4], cooperative mission planning [5], formation relative navigation [6], formation control [7] and collision avoidance [8].

At present, specific formation strategies of formation flight mainly include: leader-follower formation, behavior-based formation, virtual structure formation and artificial potential field formation. Several formation control algorithms have been proposed, such as: feedback linearization (Li, 2001) [9]; H_∞ control (Ren, 2003) [10]; PID control (Zuo, 2004) [11]; linear quadratic (LQ) control (Giulietti, 2000, 2005) [12,13]; adaptive control (Semsar, 2006) [14]; neural network based dynamic surface control (Peng, 2013) [15]. However, the formation control methods in [9–13] are based on the system model with known parameters, while the method in [14] only considers the unknown leader command and disturbance. Ref. [15] considered the uncertain local dynamics and uncertain leader dynamics, but it was mainly a neural network method for multiple autonomous surface vehicles formation. Refs. [16,17] designed some adaptive formation control algorithms, however, they were for the second-order agents in an unknown spatiotemporal flowfield.

When a group of UAVs are keeping a desired special distance and flying in a stable, specified structure, formation flight problem can be equal to a consensus problem. Consensus means that the followers eventually reach an agreement on the state or output of a leader. Recently, the leader-following system with general linear dynamics has been investigated. Ni et al. [18] studied a consensus leader-following problem for the identical followers and leader. This method can also be applied in the formation of multiple UAVs. Being different from [18], Liu et al. [19] solved a consensus

[☆] Fully documented templates are available in the elsarticle package on CTAN.

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problem for the leader-following systems with different dynamics under external disturbances. Peng et al. proposed the distributed adaptive control schemes respectively using the virtual structure strategy and the leader-following strategy for the uncertain nonlinear multi-agent systems in [20] and [21]. In many cases, the model parameters of both the leader and followers are unknown, thus it is difficult to control the formation flight by a fixed controller. Adaptive control is a control method capable of dealing with uncertain systems to ensure desired control performance under the matching conditions [22,23]. A majority of existing research is focused on the leader-following consensus problem assuming that the parameters of follower agents are uncertain, while few literature consider the leader and the follower dynamic uncertainty at the same time.

Therefore, we focus on the consensus flight control problem of the leader-following UAVs system with parametric uncertainties and unknown external disturbances. Different from the results in the literature, the main contributions of this paper are as follows: This problem has not been addressed in the literature. In the real engineering environment, it is difficult to get accurate UAV dynamic parameters. Being different from [11,14], both the parametric uncertainties of the lead UAV and wing UAV are considered here. Different with [24], the unknown external disturbances in both the lead UAV and wing UAV are considered. Therefore, a state feedback state tracking multivariable model reference adaptive control (MRAC) scheme is presented, which ensures the consensus flight control system stable, rejects the effect of the disturbances and makes the wing UAV track the lead UAV asymptotically.

The remainder of this paper is organized as follows. In Section 2 we consider the parametric uncertainties and unknown external disturbances, to establish dynamic models of consensus flight problem. In Section 3, we develop an adaptive control scheme for leader-following consensus flight problem of uncertain UAVs system. In Section 4, we present the simulation results of the UAVs in formation flight to illustrate the effectiveness of the adaptive control method. Finally, we discuss some conclusions and the future work in Section 5.

2. Consensus flight control problem of UAVs formation

In this section, the consensus flight control problem of leader-following UAVs formation with the parameter uncertainties and unknown external disturbances are formulated.

2.1. UAV dynamic model

The UAV is generally a nonlinear system with six degrees of freedom, which can be linearized according to the small-disturbance theory, expressed as [24,25]

$$E\dot{X} = \tilde{A}X + \tilde{B}U \quad (1)$$

where E , \tilde{A} and \tilde{B} are Jacobian matrices; $X = [V, \beta, \alpha, \theta, p, q, r]^T$ is the state vector. For $E = \text{diag}(m, mV^*, 1, 1, mV^*, 1, 1)$, $\det(E) = m^3V^{*2}$, here m and V^* denote the mass and balanced velocity of the UAV, and both of them are not equal to zero. Therefore, $\det(E) = m^3V^{*2} \neq 0$, E is nonsingular. Then (1) can be rewritten in the form of standard linear state equation

$$\dot{X} = \underline{A}X + \underline{B}U \quad (2)$$

where $\underline{A} = E^{-1}\tilde{A}$ and $\underline{B} = E^{-1}\tilde{B}$ contain the UAV's stability derivatives.

Generally, based on the earth-surface inertial reference frame, the linear model (2) is decoupled to a longitudinal model and a lateral model

$$\dot{X}_{lon} = A_{lon}X_{lon} + B_{lon}U_{lon} \quad (3)$$

$$\dot{X}_{lat} = A_{lat}X_{lat} + B_{lat}U_{lat} \quad (4)$$

where $X_{lon} = [\Delta V, \Delta \alpha, \Delta q, \Delta \theta]^T$ denote velocity, angle of attack, pitch angular rate and pitch angle, $U_{lon} = [\Delta \delta_e, \Delta \delta_T]^T$ represent the control surfaces deflections of elevator and throttle, $X_{lat} = [\Delta \beta, \Delta p, \Delta r]^T$ denote sideslip angle, roll angular rate and yaw angular rate, $U_{lat} = [\Delta \delta_a, \Delta \delta_r]^T$ represent the control surfaces deflections of aileron and rudder, respectively. A_{lon} and A_{lat} are unknown matrices which are decomposed from \underline{A} , while B_{lon} and B_{lat} are unknown matrices which are decomposed from \underline{B} . The detailed expressions of these matrices can be found in [24,25]. Δ denotes the deviation from the equilibrium point.

The longitudinal and lateral controllers are usually designed separately. Therefore, for design convenience of the controllers, an unified state-space model for UAV's longitudinal motion or lateral motion is established, given by

$$\dot{x} = Ax + Bu \quad (5)$$

Here, for the longitudinal controller design, x denotes X_{lon} , A denotes A_{lon} , B denotes B_{lon} and u denotes U_{lon} ; For the lateral controller design, x denotes X_{lat} , A denotes A_{lat} , B denotes B_{lat} and u denotes U_{lat} .

2.2. Consensus flight control problem

The consensus flight control problem of UAVs in formation flight can be described as the leader-following problem. In this problem, both the leader and the follower have uncertain parameters and unknown external disturbances.

Consider a linear time-invariant follower plant in state-space form

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Bd(t) \\ d(t) = d_0 + \sum_{\beta=1}^q d_{\beta} f_{\beta}(t) \end{cases} \quad (6)$$

where $x(t) \in R^n$, $u(t) \in R^p$, A and B are unknown parameter matrices, the disturbance $d(t)$ is matched to the control input $u(t)$ through the same matrix B , with $d_0 \in R^p$ and $d_{\beta} \in R^p$ being unknown constant vectors, and $f_{\beta}(t)$, $\beta = 1, 2, \dots, q$ are some known bounded basis functions, for some $q \geq 0$. The leader dynamic system is given by

$$\begin{cases} \dot{x}_m(t) = A_m x_m(t) + B_m u_m(t) + B_m d_m(t) \\ d_m(t) = d_{m0} + \sum_{\beta=1}^{q_m} d_{m\beta} f_{m\beta}(t) \end{cases} \quad (7)$$

where $x_m(t) \in R^n$, $u_m(t) \in R^m$, A_m and B_m are an unknown parameter matrices, and $x_m(t)$ and $u_m(t)$ are bounded and available for measurement, $d_m(t)$ is matched to the control input $u_m(t)$ through the same matrix B_m , with $d_{m0} \in R^m$ and $d_{m\beta} \in R^m$ being unknown constant vectors, and $f_{m\beta}(t)$, $\beta = 1, 2, \dots, q_m$ are some known bounded basis functions, for some $q_m \geq 0$. Here, $d(t)$ and $d_m(t)$ represent the unknown external disturbances. Some disturbance of UAVs is the interference of the airflow, which can be described as a constant matrix. More general disturbances can not be denoted as constant vectors, they are usually in the form of (6) and (7) [26].

Remark 1. In the leader-following UAVs system, $x_m(t)$ denotes the states of the lead UAV, $u_m(t)$ represents the control input of the lead UAV. They are all bounded and can be obtained by the wing UAV through the measurement or the communication.

The control objective is to design a bounded state feedback control law $u(t)$ including a disturbance compensator to make the follower system state $x(t)$ bounded and track the leader system $x_m(t)$ asymptotically, i.e., $\lim_{t \rightarrow \infty} (x(t) - x_m(t)) = 0$ and reject the effect of unknown external disturbances.

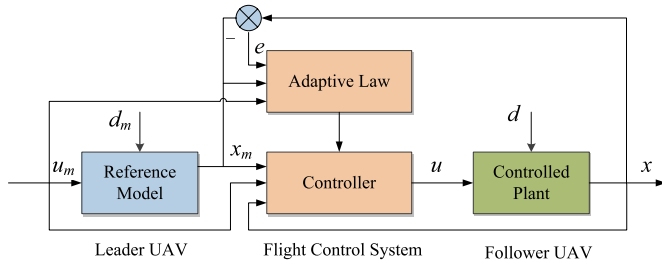


Fig. 1. A MRAC scheme for UAVs formation

2.3. MRAC framework

Given the consensus flight control problem of the UAVs in formation flight, an advanced control method is indispensable. MRAC methods have been applied in the aircraft flight control fields [27]. Therefore, in this paper, a novel MRAC scheme is proposed to solve this problem, whose schematic diagram is shown in Fig. 1. The control system is mainly composed of the reference model, the flight control system and the controlled object. The proposed MRAC scheme has an adaptation capacity to deal with parameter uncertainties and unknown external disturbances for both the leader and the follower.

3. Adaptive consensus flight control design

In order to meet the control objectives, we will solve the UAVs consensus flight control problem using the multivariable MRAC scheme.

3.1. Adaptive control scheme for consensus flight

For the leader-following UAVs with unknown system matrices and external disturbances, in order to reject the effect of the disturbances so that the desired consensus flight control performance can be achieved, we make some assumptions first.

Assumption 1. [24,28,29] There exist four parameter matrices $k_1^* \in R^{n \times p}$, $k_2^* \in R^{p \times m}$, $k_3^* \in R^{n \times p}$ and $k_4^* \in R^{p \times p}$ which satisfy:

$$\begin{aligned} A_e &= A + Bk_1^{*T}, & B_m &= Bk_2^* \\ A_m &= A + Bk_3^{*T}, & B_e &= Bk_4^* \end{aligned} \quad (8)$$

where $A_e \in R^{n \times n}$ is a stable matrix and $B_e \in R^{n \times p}$ is a matrix.

Assumption 2. [24,28,29] There is a known matrix $S \in R^{p \times p}$ such that $k_4^* S$ is symmetric and positive definite: $M_s = k_4^* S = (k_4^* S)^T > 0$.

Assumption 1 is the plant-model state matching condition. Assumption 2 is needed to design the adaptive law for the controller parameters.

A nominal controller with a disturbance compensator is chosen as

$$u^*(t) = k_1^{*T}(x(t) - x_m(t)) + k_2^* u_m(t) + k_3^{*T} x_m(t) + k_5^*(t) \quad (9)$$

where k_1^{*T} , k_2^* and k_3^{*T} satisfy the Assumption 1, and

$$\begin{aligned} k_5^*(t) &= k_2^* d_m(t) - d(t) \\ &= (k_2^* d_{m0} - d_0) + \sum_{\beta=1}^q k_2^* d_{m\beta} f_{m\beta}(t) - \sum_{\beta=1}^q d_{\beta} f_{\beta}(t) \quad (10) \\ &= k_{50}^* + \sum_{\beta=1}^q k_{5\beta}^* f_{m\beta}(t) + \sum_{\beta=1}^q k_{5\beta}^* f_{\beta}(t) \end{aligned}$$

with $k_{50}^* = k_2^* d_{m0} - d_0$, $k_{5\beta}^* = k_2^* d_{m\beta}$, and $k_{5\beta}^* = -d_{\beta}$. Then the tracking error satisfies

$$\dot{e}(t) = A_e e(t), \quad e(0) = x(0) - x_m(0) \quad (11)$$

which implies that $\lim_{t \rightarrow \infty} e(t) = 0$ exponentially.

When the parameters of A , B , A_m , B_m are unknown, an adaptive controller is designed as

$$u(t) = k_1^T(t)(x(t) - x_m(t)) + k_2(t)u_m(t) + k_3^T(t)x_m(t) + k_5(t) \quad (12)$$

where $k_1(t)$, $k_2(t)$, $k_3(t)$ and $k_5(t)$ are the estimates of k_1^* , k_2^* , k_3^* and k_5^* , respectively.

The design objective is to choose adaptive laws to update these estimates so that the control objective is still achievable even if there are parameter uncertainties and unknown external disturbances. Therefore, we choose the adaptive laws as

$$\dot{k}_1^T(t) = -S^T B_e^T P e(t) e^T(t) \quad (13)$$

$$\dot{k}_2(t) = -S^T B_e^T P e(t) u_m^T(t) \quad (14)$$

$$\dot{k}_3^T(t) = -S^T B_e^T P e(t) x_m^T(t) \quad (15)$$

$$\dot{k}_{50}(t) = -S^T B_e^T P e(t) \quad (16)$$

$$\dot{k}_{5\beta}(t) = -S^T B_e^T P e(t) f_{\beta}^T(t), \quad \beta = 0, 1, 2, \dots, q \quad (17)$$

$$\dot{k}_{5m\beta}(t) = -S^T B_e^T P e(t) f_{m\beta}^T(t), \quad \beta = 0, 1, 2, \dots, q_m \quad (18)$$

where $P = P^T > 0$, satisfies $A_e^T P + P A_e = -Q < 0$, for any chosen $Q \in R^{n \times n}$ being constant and $Q = Q^T > 0$. $S \in R^{p \times p}$ satisfies Assumption 2.

Theorem 1. The adaptive controller (12), with the adaptive laws (13)-(18), applied to the linear system (6) guarantees that all closed-loop signals are bounded and the tracking error $e(t) = x(t) - x_m(t)$ goes to zero as t goes to infinity.

Proof. According to (6), (7) and (12), we obtain the tracking error equation

$$\begin{aligned} \dot{e}(t) &= A_e e(t) + B_e k_4^{*-1} [\tilde{k}_1^T(t) e(t) + \tilde{k}_2(t) u_m(t) + \tilde{k}_3^T(t) x_m(t) \\ &\quad + \tilde{k}_5(t)] \end{aligned} \quad (19)$$

where the parameter errors are

$$\tilde{k}_i(t) = k_i(t) - k_i^*, \quad i = 1, 2, 3 \quad (20)$$

$$\tilde{k}_{50}(t) = k_{50}(t) - k_{50}^* \quad (21)$$

$$\tilde{k}_{5\beta}(t) = k_{5\beta}(t) - k_{5\beta}^*, \quad \beta = 1, 2, \dots, q \quad (22)$$

$$\tilde{k}_{5m\beta}(t) = k_{5m\beta}(t) - k_{5m\beta}^*, \quad \beta = 1, 2, \dots, q_m \quad (23)$$

For convenience of derivation, the time constant t is omitted in the following proof. In order to prove the adaptive controller (12) with adaptive laws (13)-(18) to ensure $e = x - x_m$ bounded, we choose a positive definite function as

$$\begin{aligned} V(e_c) &= e^T P e + \text{tr}[\tilde{k}_1 M_s^{-1} \tilde{k}_1^T] + \text{tr}[\tilde{k}_2^T M_s^{-1} \tilde{k}_2] + \text{tr}[\tilde{k}_3 M_s^{-1} \tilde{k}_3^T] \\ &\quad + \text{tr}[\tilde{k}_{50}^T M_s^{-1} \tilde{k}_{50}] + \sum_{\beta=1}^q \text{tr}[\tilde{k}_{5\beta}^T M_s^{-1} \tilde{k}_{5\beta}] \\ &\quad + \sum_{\beta=1}^{q_m} \text{tr}[\tilde{k}_{5m\beta}^T M_s^{-1} \tilde{k}_{5m\beta}] \end{aligned} \quad (24)$$

with $\text{tr}[M]$ denoting the trace of a square matrix M .

Calculate the time derivative of $V(e_c)$ as

$$\begin{aligned} \dot{V} &= 2e^T P \dot{e} + 2\text{tr}[\tilde{k}_1 M_s^{-1} \dot{\tilde{k}}_1^T] + 2\text{tr}[\tilde{k}_2^T M_s^{-1} \dot{\tilde{k}}_2] + 2\text{tr}[\tilde{k}_3 M_s^{-1} \dot{\tilde{k}}_3^T] \\ &\quad + 2\text{tr}[\tilde{k}_{50}^T M_s^{-1} \dot{\tilde{k}}_{50}] + 2 \sum_{\beta=1}^q \text{tr}[\tilde{k}_{5\beta}^T M_s^{-1} \dot{\tilde{k}}_{5\beta}] \\ &\quad + 2 \sum_{\beta=1}^{q_m} \text{tr}[\tilde{k}_{5m\beta}^T M_s^{-1} \dot{\tilde{k}}_{5m\beta}] \end{aligned} \quad (25)$$

Substituting (19) into (25), we have

$$\begin{aligned} \dot{V} = & -e^T Q e + 2e^T P B_e k_4^{*-1} \tilde{k}_1^T e + 2e^T P B_e k_4^{*-1} \tilde{k}_2 u_m \\ & + 2e^T P B_e k_4^{*-1} \tilde{k}_3 x_m + 2e^T P B_e k_4^{*-1} \tilde{k}_5 + 2tr[\tilde{k}_1 M_s^{-1} \dot{\tilde{k}}_1^T] \\ & + 2tr[\tilde{k}_2 M_s^{-1} \dot{\tilde{k}}_2] + 2tr[\tilde{k}_3 M_s^{-1} \dot{\tilde{k}}_3] + 2tr[\tilde{k}_{50} M_s^{-1} \dot{\tilde{k}}_{50}] \\ & + 2\sum_{\beta=1}^q tr[\tilde{k}_{5\beta}^T M_s^{-1} \dot{\tilde{k}}_{5\beta}] + 2\sum_{\beta=1}^{q_m} tr[\tilde{k}_{5m\beta}^T M_s^{-1} \dot{\tilde{k}}_{5m\beta}] \end{aligned} \quad (26)$$

Using the definition $M_s = k_4^* S = (k_4^* S)^T > 0$ and the properties that $tr[M_1 M_2] = tr[M_2 M_1]$, $tr[M_3] = tr[M_3^T]$, we obtain

$$e^T P B_e k_4^{*-1} \tilde{k}_1^T e = tr[\tilde{k}_1 M_s^{-1} S^T B_e^T P e e^T] = -tr[\tilde{k}_1 M_s^{-1} \dot{\tilde{k}}_1^T] \quad (27)$$

$$e^T P B_e k_4^{*-1} \tilde{k}_2 u_m = tr[\tilde{k}_2 M_s^{-1} S^T B_e^T P e u_m^T] = -tr[\tilde{k}_2 M_s^{-1} \dot{\tilde{k}}_2] \quad (28)$$

$$e^T P B_e k_4^{*-1} \tilde{k}_3 x_m = tr[\tilde{k}_3 M_s^{-1} S^T B_e^T P e x_m^T] = -tr[\tilde{k}_3 M_s^{-1} \dot{\tilde{k}}_3] \quad (29)$$

$$\begin{aligned} e^T P B_e k_4^{*-1} \tilde{k}_5 &= e^T P B_e k_4^{*-1} [\tilde{k}_{50} + \sum_{\beta=1}^{q_m} \tilde{k}_{5m\beta} f_{m\beta} \\ &+ \sum_{\beta=1}^q \tilde{k}_{5\beta} f_{\beta}] \\ &= -tr[\tilde{k}_{50} M_s^{-1} \dot{\tilde{k}}_{50}] - \sum_{\beta=1}^q tr[\tilde{k}_{5\beta}^T M_s^{-1} \dot{\tilde{k}}_{5\beta}] \\ &- \sum_{\beta=1}^{q_m} tr[\tilde{k}_{5m\beta}^T M_s^{-1} \dot{\tilde{k}}_{5m\beta}] \end{aligned} \quad (30)$$

Substituting (27)-(30) into (26), we get

$$\dot{V} = -e^T(t) Q e(t) \leq -q_m \|e(t)\|_2^2 \leq 0, \quad Q = Q^T > 0 \quad (31)$$

where $q_m > 0$ is the minimum eigenvalue of Q . From the above results, the desired properties of the proposed adaptive laws are obvious:

(i) $V > 0$ and $\dot{V} \leq 0$ imply that the equilibrium state ($e = 0$, $\tilde{k}_1 = 0$, $\tilde{k}_2 = 0$, $\tilde{k}_3 = 0$, $\tilde{k}_{50} = 0$, $\tilde{k}_{5\beta} = 0$, $\tilde{k}_{5m\beta} = 0$) of the closed-loop system (13)-(19) is uniformly stable and the system state ($e(t) = 0$, $\tilde{k}_1(t) = 0$, $\tilde{k}_2(t) = 0$, $\tilde{k}_3(t) = 0$, $\tilde{k}_{50}(t) = 0$, $\tilde{k}_{5\beta}(t) = 0$, $\tilde{k}_{5m\beta}(t) = 0$) is uniformly bounded, which gives the boundedness of $x(t)$, $k_1(t)$, $k_2(t)$, $k_3(t)$, $k_{50}(t)$, $k_{5\beta}(t)$ and $k_{5m\beta}(t)$ and in turn of the boundedness of $\dot{e}(t)$;

(ii) Eq. (31) implies $e(t) \in L^2$. With $e(t) \in L^2 \cap L^\infty$ and $\dot{e}(t) \in L^\infty$, applying Barbalat Lemma [22], we conclude that $\lim_{t \rightarrow \infty} e(t) = 0$.

Remark 2. The wing UAV in the leader-following system with uncertain parameters and unknown external disturbances can track the lead UAV asymptotically when using multivariable MRAC method. The effect of the unknown external disturbances can be rejected effectively when applying adaptive disturbance rejection strategy. Moreover, for the situation without external disturbances, an adaptive control $u(t) = k_1^T(t)(x(t) - x_m(t)) + k_2(t)u_m(t) + k_3^T(t)x_m(t)$ with adaptive laws (13)-(15) will also make the wing UAV track the lead UAV asymptotically.

Remark 3. For the proposed MRAC method, the convergence of control parameters estimation may not be guaranteed. However, the boundedness of control parameters estimation and the attenuation of their variations can be achieved.

3.2. Extension to multi-UAV system

This work focus on the consensus flight control problem for one lead UAV and one wing UAV formation. The information of leader states, control inputs and follower's states are necessary for the consensus controller design. When there exist more than two wing UAVs in the multi-UAV system, a suitable communication topology is necessary. The information interaction among UAVs can adopt the distributed control strategy, due to less information interactions and conflicts than the centralized control strategy. It means

that each wing UAV needs to interact its motion information with the adjacent UAVs and may not get the information from the lead UAV directly.

The distributed control strategy can be described by direct graphs, with different kinds of formation communication topology structures. Each follower in the direct graph can get information directly from a nominal leader, which can be an adjacent communicated UAV or the lead UAV. Thus, the proposed adaptive control scheme can be applied in the followers, to make each follower asymptotically track its nominal leader. The convergence of local tracking error for each UAV will contribute to achieving the convergent global consensus performance.

4. Simulation study

In this section, we use MATLAB R2014a to simulate the leader-following UAVs system, to verify the effectiveness of the multivariable MRAC scheme, by comparing with the fixed control.

4.1. Control problem and method

Controlled plant. A linear model of a real UAV called Silver Fox is developed, including the longitudinal and lateral dynamics, based on the aircraft parameters and the aerodynamic data in [30]. The fuselage of the Silver Fox is 1.8 m long and the aircraft weighs 8.6 kg. The Silver Fox can carry small cameras and global positioning system (GPS) receiver. It was originally developed by Advanced Ceramics Research (ACR). The linear model of the UAV which consists of the longitudinal and lateral equations are respectively given by

$$\begin{bmatrix} \Delta \dot{V} \\ \Delta \dot{\alpha} \\ \Delta \dot{q} \\ \Delta \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.0785 & 6.0293 & -1.6485 & -9.7783 \\ -0.0489 & -3.9919 & -0.7386 & 0.0326 \\ -0.0003 & -96.9781 & -260.2504 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta V \\ \Delta \alpha \\ \Delta q \\ \Delta \theta \end{bmatrix} + \begin{bmatrix} -2.1657 & 1.4976 \\ -0.575 & -0.0052 \\ -95.5596 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \delta_e \\ \Delta \delta_T \end{bmatrix} \quad (32)$$

$$\begin{bmatrix} \Delta \dot{\beta} \\ \Delta \dot{p} \\ \Delta \dot{r} \end{bmatrix} = \begin{bmatrix} -0.1798 & 0.069 & -0.9976 \\ -22.4565 & -8.213 & 2.0046 \\ 15.0747 & -0.6578 & -0.7095 \end{bmatrix} \begin{bmatrix} \Delta \beta \\ \Delta p \\ \Delta r \end{bmatrix} + \begin{bmatrix} 0.0000 & 0.0873 \\ 99.5144 & 2.4034 \\ -7.9397 & -10.1124 \end{bmatrix} \begin{bmatrix} \Delta \delta_a \\ \Delta \delta_r \end{bmatrix} \quad (33)$$

Control problem and objective. For the UAVs consensus flight control system with uncertainties for both the lead UAV and the wing UAV, we use the multivariable MRAC method to make the state of the wing UAV track the state of the lead UAV. The objective of the consensus flight control based on the multivariable MRAC is to design a bounded state feedback control law $u(t)$ to make the wing UAV state $x(t)$ bounded and track the lead UAV state $x_m(t)$ asymptotically, i.e. $\lim_{t \rightarrow \infty} (x(t) - x_m(t)) = 0$.

Matching condition. For the multivariable MRAC based consensus flight control method, the design conditions as specified by Assumption 1 and Assumption 2 need to be satisfied. The system matrices of lead UAV and the wing UAV are the same, thus $A_m = A$, $B_m = B$. However, they may have different initial states, different control inputs, or different external disturbances. In the aircraft flight control field, Assumption 1 and Assumption 2 are normally satisfied based on a linear quadratic (LQ) method, the details of which can be found in [31-34]. Therefore, the matching

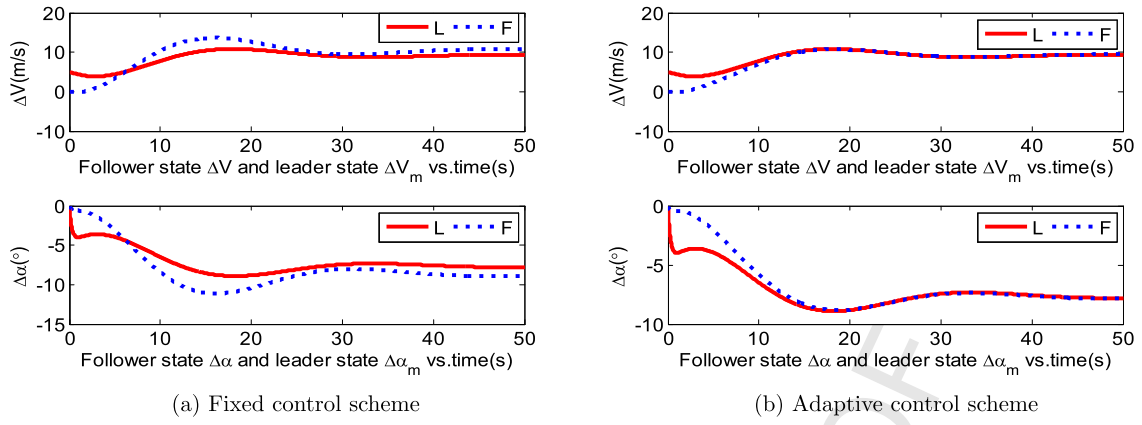


Fig. 2. Follower states ΔV and $\Delta \alpha$ vs. leader states with adaptive control vs. fixed control.

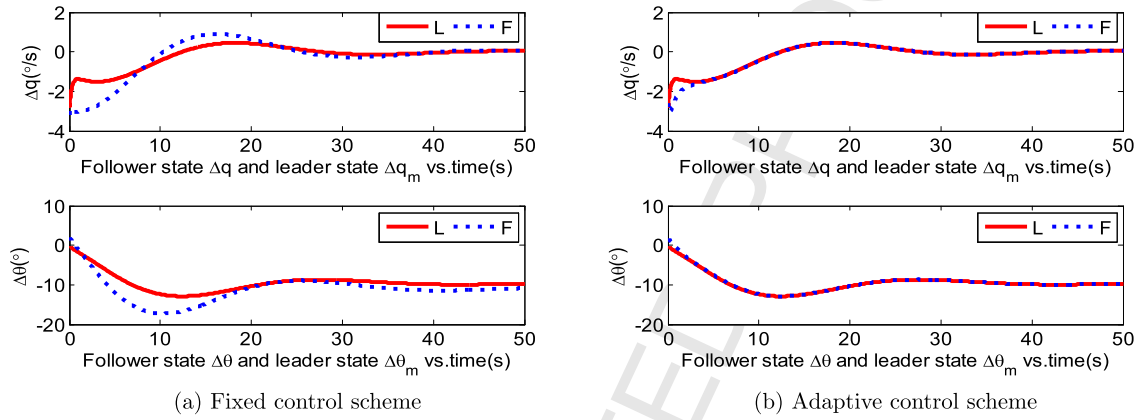


Fig. 3. Follower states Δq and $\Delta \theta$ vs. leader states with adaptive control vs. fixed control.

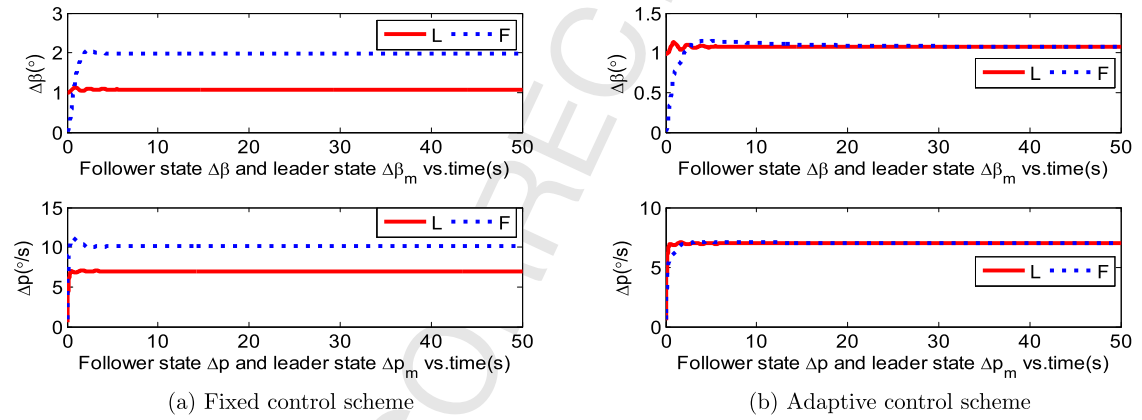


Fig. 4. Follower states $\Delta \beta$ and Δp vs. leader states with adaptive control vs. fixed control.

conditions are satisfied for the adaptive consensus flight control system design.

Control methods and initial parameters. For comparison, two control methods are adopted, one is the proposed adaptive control, the other is the fixed control which exactly is the nominal control described by (9). The fixed control parameters can be obtained according to the matching conditions, assuming that the system matrices are known. For the longitudinal control channel, the leader's initial state is $X_{mlon} = [5 \text{ m/s}, 0^\circ, 0^\circ/\text{s}, 0^\circ]^T$, the follower's initial state is $X_{lon} = [0 \text{ m/s}, 0^\circ, 0^\circ/\text{s}, 2^\circ]^T$, and the control input is $U_{mlon} = [5^\circ, 0.05]^T$, respectively. For the lateral control channel, they are $X_{mlat} = [1^\circ, 0^\circ/\text{s}, 0^\circ/\text{s}]^T$, $X_{lat} = [0^\circ, 0^\circ/\text{s}, 0^\circ/\text{s}]^T$, and $U_{mlat} = [-2^\circ, -1^\circ]^T$, respectively. The external disturbances

acting on the leader and the follower are $d_m(t) = [1.8, 0.05]^T + 1 * [1, 0.02]^T$ and $d(t) = [3, 0.1]^T + 5 * [2, 0.03]^T$, respectively. For the lateral control channel, the external disturbances acting on the leader and the follower are $d_m(t) = [0.8, 0.5]^T + 1 * [2, 1]^T$ and $d(t) = [1, 0.7]^T + 3 * [0, 2]^T$, respectively. The adaptive longitudinal and lateral controllers of the wing UAV are respectively designed by

$$U_{lon} = k_{1lon}^T (X_{lon} - X_{mlon}) + k_{2lon} U_{mlon} + k_{3lon}^T X_{mlon} + k_{5lon} \quad (34)$$

$$U_{lat} = k_{1lat}^T (X_{lat} - X_{mlat}) + k_{2lat} U_{mlat} + k_{3lat}^T X_{mlat} + k_{5lat} \quad (35)$$

where k_{i1on}, k_{i1at} ($i = 1, 2, 3, 5$) are updated by (13)-(18).

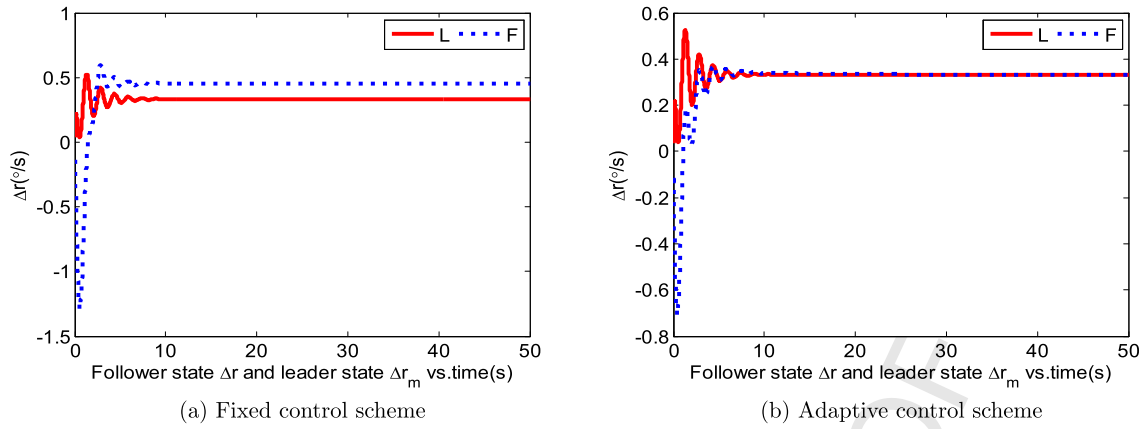


Fig. 5. Follower state Δr vs. leader state with adaptive control vs. fixed control.

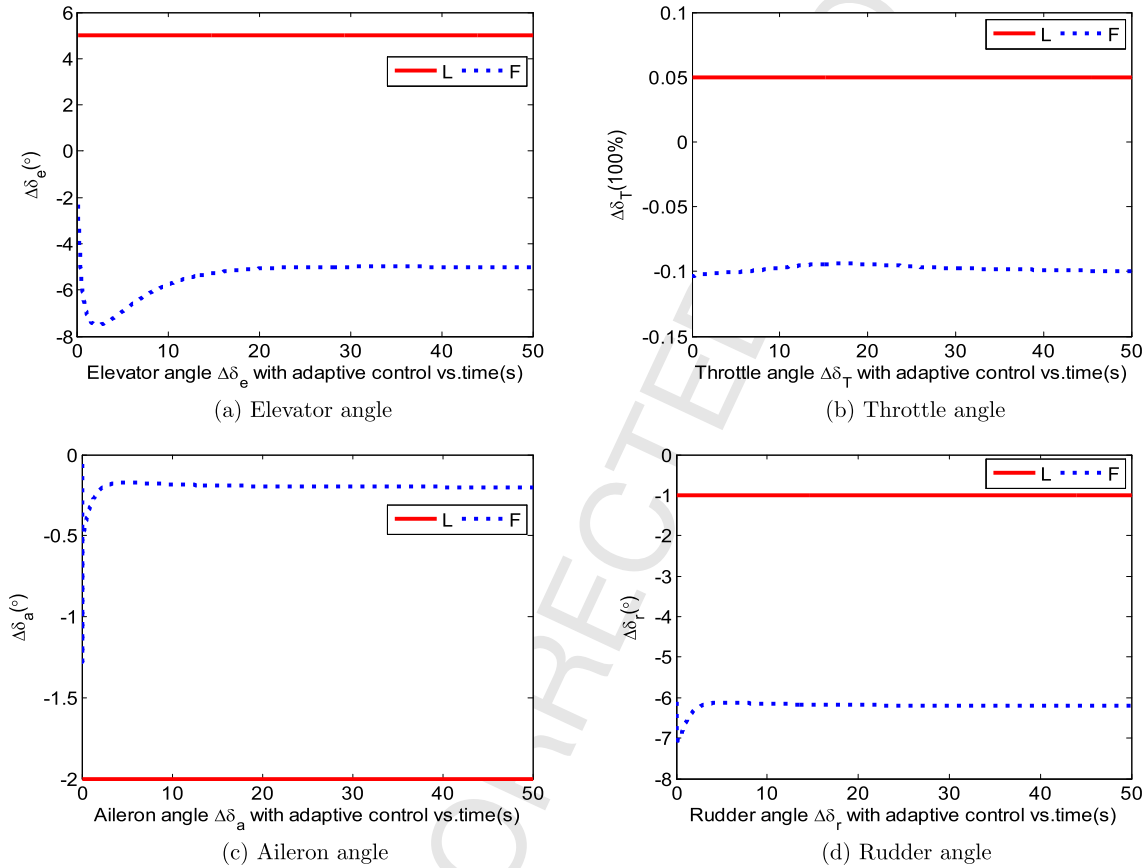


Fig. 6. Follower control input vs. leader control input with adaptive control.

4.2. Simulation results and discussion

The responses results of the flight states under the adaptive control and fixed control are shown in Fig. 2–4. Fig. 5 shows the control surfaces deflections of elevator, throttle, aileron and rudder under adaptive control (see also Fig. 6).

Performance analysis of consensus flight control. The final values of tracking errors are as follows: (i) Fixed control: $\Delta V_e = 1.3046$ m/s, $\Delta\alpha_e = -1.0830^\circ$, $\Delta\beta_e = 0.9053^\circ$, $\Delta p_e = 3.1228^\circ/\text{s}$, $\Delta q_e = 0.0226^\circ/\text{s}$, $\Delta\theta_e = -1.1288^\circ$, $\Delta r_e = 0.1203^\circ/\text{s}$; (ii) Adaptive control: $\Delta V_e = 0.2004$ m/s, $\Delta\alpha_e = 0^\circ$, $\Delta\beta_e = 0^\circ$, $\Delta p_e = 0^\circ/\text{s}$, $\Delta q_e = 0^\circ/\text{s}$, $\Delta\theta_e = 0^\circ$, $\Delta r_e = 0^\circ/\text{s}$. We get that almost state tracking errors converge to zero. It implies that although there are unknown external disturbances for both the lead UAV and the wing UAV, the flight states X_{lon} and X_{lat} of the wing UAV can

still asymptotically track the states of the lead UAV when using the adaptive control scheme. Therefore, it is found that the effect of the unknown external disturbances is rejected by applying the adaptive control scheme.

Comparison of the adaptive control and fixed control. We get that the proposed multivariable MRAC scheme provides substantially improved performance over the fixed control method under the same flight conditions. The reason is that the fixed control is obtained based on the known system parameters, which are not accurate enough under the system uncertainties. While the adaptive control is based on the UAV model with unknown parameters, whose control gains can be updated online through the adaptive laws. Furthermore, the final values of state tracking errors under fixed control cannot converge to zero. While the final values

of state tracking errors under adaptive control converge to zero asymptotically. It means that the proposed multivariable MRAC method can reject unknown external disturbances and achieve better robustness than the fixed control. Therefore, the proposed multivariable MRAC scheme will be more suitable for the real applications.

5. Conclusion

When a group of UAVs are flying in a stable and specified structure, formation flight can be equal to a consensus flight problem. Therefore, in order to address the consensus flight control problem of the multiple UAVs with parameter uncertainties and external disturbances, a multivariable adaptive control scheme is designed. Simulation study exhibits that the formation flight stability and tracking properties can be better achieved by the multivariable MRAC scheme, comparing with the fixed control method.

This work addresses the velocity and attitudes consensus control problem of the UAVs in formation flight. In fact, the control scheme is also suitable for the trajectory tracking control problem of the UAVs formation, due to the full-state tracking strategy. However, the consensus flight control problem under unknown external disturbances with general forms still needs to be investigated in future.

Declaration of competing interest

None declared.

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