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Brief paper

Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics ☆

Hongwei Zhang [a], Frank L. Lewis [b,1]

[a] School of Electrical Engineering, Southwest Jiaotong University, Chengdu, Sichuan, 610031, PR China

[b] Automation and Robotics Research Institute, The University of Texas at Arlington, Fort Worth, TX 76118-7115, USA

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ABSTRACT

A practical design method is developed for cooperative tracking control of higher-order nonlinear systems with a dynamic leader. The communication network is a weighted directed graph with a fixed topology. Each follower node is modeled by a higher-order integrator incorporating with unknown nonlinear dynamics and an unknown disturbance. The leader node is modeled as a higher-order nonautonomous nonlinear system. It acts as a command generator giving commands only to a small portion of the networked group. A robust adaptive neural network controller is designed for each follower node such that all follower nodes ultimately synchronize to the leader node with bounded residual errors. Moreover, these controllers are distributed in the sense that the controller design for each follower node only requires relative state information between itself and its neighbors. A simulation example demonstrates the effectiveness of the algorithm.

1. Introduction

Research on networked cooperative systems (or multi-agent systems) has attracted much attention in the past two decades. Its widespread applications include spacecraft, mobile robots, sensor networks, etc. Some seminal works are Fax and Murray (2004), Jadbabaie, Lin, and Morse (2003), Olfati-Saber and Murray (2004), Ren and Beard (2005), and Tsitsiklis, Bertsekas, and Athans (1986), to name a few.

Considerable effort has focused on two control problems of networked systems, i.e., *cooperative regulation problem* and *cooperative tracking problem*. For cooperative regulation problem, controllers are designed to drive all the agents/nodes to a common value, i.e., consensus equilibrium, which is not prescribed and depends on initial conditions (Ren, Beard, & Atkins, 2007). This is also known as (leaderless) consensus in the literature. As for the cooperative tracking problem, there is a leader/control node that acts as a command generator, ignoring information from all the other nodes. The leader node only gives commands to a small portion of the networked group. All the follower nodes are trying to track the trajectory of the leader node. This is called leader–follower consensus, consensus with a virtual leader, or synchronization to a leader in the literature. Numerous results on these two topics have been published in the past few years and readers are referred to survey papers (Olfati-Saber, Fax, & Murray, 2007; Ren et al., 2007; Ren, Beard, & Atkins, 2005) and references therein.

This paper studies the cooperative tracking control of higher-order nonlinear systems. Our research is motivated by the following several observations. First, most existing works on multi-agent systems studied the first- and second-order systems. However, in engineering, many systems are modeled by higher-order dynamics. For example, a single link flexible joint manipulator is well modeled by a fourth-order nonlinear system (Khalil, 2002). The jerk (i.e., derivative of acceleration) systems, described by third-order differential equations, are of particular interest in mechanical engineering. Due to the challenges of designing cooperative controls for systems distributed on communication graphs, it is nontrivial to extend results for first- and second-order systems to systems with higher-order dynamics. Therefore, the literature in cooperative control has many papers dedicated to higher-order cooperative control, most of which deal with higher-order linear systems ((Jiang, Wang, & Jia, 2009; Ren, Moore, & Chen, 2007; Wang & Cheng, 2007; Zhang, Lewis, & Das, 2011) etc.).

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E-mail addresses: hwzhang@swjtu.edu.cn (H. Zhang), lewis@uta.edu (F.L. Lewis).

[1] Tel.: +1 817 272 5972; fax: +1 817 272 5989.

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Second, even for the first- and second-order problems with dynamics described by single or double integrators, cooperative control has not been fully investigated. However, almost all physical systems are inherently nonlinear, and cooperative control of nonlinear systems is more challenging. Pinning control was introduced for controlling synchronization of interconnected dynamic systems with identical nonlinear dynamics (Li, Wang, & Chen, 2004; Wang & Chen, 2002). But, in practice, the node dynamics may be non-identical or even unknown. Consensus of nonlinear systems was also studied recently in Lee and Ahn (2010) and Qu, Chunyu, and Wang (2007), where the nonlinear dynamics are assumed to be known precisely. Moreover, external disturbances such as white noise, often neglected by the current research, exist in almost every practical application. Most relevant to this paper are the works (Das & Lewis, 2010, 2011; Hou, Cheng, & Tan, 2009). Ref. (Hou et al., 2009) applied neural adaptive control to leaderless consensus problem of first-order nonlinear systems on undirected graphs. They also showed that the method can be extended to higher-order systems using the backstepping technique. As is well known, backstepping is a recursive design procedure whose complexity increases drastically with the order of the systems. In Das and Lewis (2010), cooperative tracking problems was solved for first-order nonlinear systems with unknown dynamics, and this result was generalized to second-order nonlinear systems in Das and Lewis (2011).

Motivated by the above observations, this paper deals with the cooperative tracking control problem of general higher-order nonlinear systems on directed graphs with a time-varying active leader, and thus further generalizes the results in Das and Lewis (2010, 2011). Each follower node is a higher-order integrator incorporating with unknown nonlinear dynamics and an unknown external disturbance. The node dynamics can all be different. The leader node is a higher-order nonautonomous nonlinear system whose dynamics is unknown to all the follower nodes. This paper proposes distributed neural adaptive controllers for networked higher-order systems, which guarantee the ultimate boundedness of the tracking errors.

Compared with Das and Lewis (2010, 2011), the main contributions of this paper are threefold. First, the node dynamics are extended to general higher-order nonlinear systems in the Brunovsky form, which include first- and second-order systems as special cases. Second, the requirement of graph topology is relaxed such that the augmented graph has a spanning tree. This means the original graph may be disconnected, as long as the leader node pins into the proper nodes in each disconnected component. This is a necessary condition and less stringent than strong connectedness. Finally, fewer assumptions are made. Thus the controller design is more flexible. It is worth mentioning that the above extensions are nontrivial, since the node dynamics get more involved with the graph topology. Also note that Lemma 2 in Das and Lewis (2011), which plays a fundamental role in the stability analysis, does not work for the higher-order case. This results in a Lyapunov stability analysis that is more complicated. In particular, two Lyapunov equations are used in this paper, i.e., one for graph topology and one for control design. This paper is also an improved work of our preliminary results (Zhang & Lewis, 2010; Zhang, Lewis, & Qu, 2012).

2. Basic graph theory and notations

A graph is expressed by $\mathcal{G} = (\mathcal{V}, E)$. $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a nonempty set of nodes/agents and $E \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges/arcs. $(v_i, v_j) \in E$ means there is an edge from node i to node j. The topology of a weighted graph is often represented by the adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$, and $a_{ij} > 0$ if $(v_j, v_i) \in E$; otherwise $a_{ij} = 0$. Throughout this paper, it is assumed that $a_{ii} = 0$ and the topology is fixed, i.e., A is time-invariant. A digraph is a directed graph. Define $d_i = \sum_{j=1}^{N} a_{ij}$ as the weighted in-degree of node i and $D = \mathrm{diag}(d_1, \ldots, d_N) \in \mathbb{R}^{N\times N}$ as the in-degree matrix. The graph Laplacian matrix is $L = [l_{ij}] = D - A$. Let $\underline{1} = [1, \ldots, 1]^T$ with appropriate dimension; then $L\underline{1} = 0$. The set of neighbors of node i is denoted as $N_i = \{j | (v_j, v_i) \in E\}$. If node j is a neighbor of node i, then node i can get information from node j, not necessarily vice versa for directed graph. For undirected graph, neighbor is a mutual relation. A direct path from node i to node j is a sequence of successive edges in the form $\{(v_i, v_k), (v_k, v_l), \ldots, (v_m, v_j)\}$. A digraph has a spanning tree, if there is a node (called the root), such that there is a directed path from the root to every other node in the graph. A digraph is strongly connected, if for any ordered pair of nodes $[v_i, v_j]$ with $i \neq j$, there is a direct path from node i to node j.

Throughout this paper, the following notations are used. $|\cdot|$ is the absolute value of a real number; $\|\cdot\|$ is the Euclidean norm of a vector; $\|\cdot\|_F$ is the Frobenius norm of a matrix; $tr\{\cdot\}$ is the trace of a matrix; $\sigma(\cdot)$ is the set of singular values of a matrix, with the maximum singular value $\bar{\sigma}(\cdot)$ and the minimum singular value $\underline{\sigma}(\cdot)$; matrix $P > 0$ ($P \geq 0$) means P is positive definite (positive semidefinite); I denotes the identity matrix with appropriate dimensions; and $\mathcal{N} = \{1, \ldots, N\}$.

3. Problem formulation

Consider $N (N \geq 2)$ agents with distinct dynamics. Dynamics of the ith ($i = 1, \ldots, N$) agent is described in the Brunovsky form

$$\begin{aligned} \dot{x}_{i,m} &= x_{i,m+1}, \quad m = 1, \ldots, M-1 \\ \dot{x}_{i,m} &= f_i(x_i) + u_i + \zeta_i, \quad m = M \end{aligned} \tag{1}$$

where $x_{i,m} \in \mathbb{R}$ is the mth state of node i; $x_i = [x_{i,1}, \ldots, x_{i,M}]^T$ is the state vector of node i; $f_i(\cdot) : \mathbb{R}^M \to \mathbb{R}$ is locally Lipschitz in $\mathbb{R}^M$ with $f_i(0) = 0$, and it is assumed to be unknown; $u_i \in \mathbb{R}$ is the control input/protocol; and $\zeta_i \in \mathbb{R}$ is an external disturbance, which is unknown but bounded. Define $x^m = [x_{1,m}, \ldots, x_{N,m}]^T$. Then one has

$$\begin{aligned} \dot{x}^m &= x^{m+1}, \quad m = 1, \ldots, M-1 \\ \dot{x}^m &= f(x) + u + \zeta, \quad m = M \end{aligned}$$

where $f(x) = [f_1(x_1), \ldots, f_N(x_N)]^T$, $u = [u_1, \ldots, u_N]^T$ and $\zeta = [\zeta_1, \ldots, \zeta_N]^T$. Specifically, when $M = 3$, x^1, x^2 and x^3 can be the global position vector, global velocity vector and global acceleration vector, respectively.

The time-varying dynamics of the leader node, labeled 0, is described by

$$\begin{aligned} \dot{x}_{0,m} &= x_{0,m+1}, \quad m = 1, \ldots, M-1 \\ \dot{x}_{0,m} &= f_0(t, x_0), \quad m = M \end{aligned} \tag{2}$$

where $x_{0,m} \in \mathbb{R}$ is the mth state; $x_0 = [x_{0,1}, \ldots, x_{0,M}]^T$ is the state vector; and $f_0(t, x_0) : [0, \infty) \times \mathbb{R}^M \to \mathbb{R}$ is piecewise continuous in t and locally Lipschitz in x_0 with $f_0(t, 0) = 0$ for all $t \geq 0$ and $x_0 \in \mathbb{R}^M$, and it is unknown to all other nodes. System (2) is assumed to be forward complete, i.e., for every initial condition, the solution $x_0(t)$ exists for all $t \geq 0$. In other words, there is no finite escape time. The leader node dynamics (2) can be considered as an exosystem that generates a desired command trajectory.

Define the mth order tracking error (or disagreement variable) for node i ($i \in \mathcal{N}$) as $\delta_{i,m} = x_{i,m} - x_{0,m}$. Let $\delta^m = [\delta_{1,m}, \ldots, \delta_{N,m}]^T$; then $\delta^m = x^m - \underline{x}_{0,m}$, where $\underline{x}_{0,m} = [x_{0,m}, \ldots, x_{0,m}]^T \in \mathbb{R}^N$. The objective of this paper is to design distributed controllers for all follower nodes, such that the tracking error δ^m converges to a small neighborhoods of zero, for all $m = 1, \ldots, M$. This is illustrated by the following definition, which extends the standard concept of uniform ultimate boundedness (Khalil, 2002; Lewis, Yeşildirek, & Liu, 1996) to cooperative control systems.

Definition 1 (*Cooperative Uniform Ultimate Boundedness*). For any m ($m = 1, \ldots, M$), the tracking error δ^m is said to be cooperatively uniformly ultimately bounded (CUUB) if there exists a compact set $\Omega^m \subset \mathbb{R}^N$ with the property that $\{0\} \subset \Omega^m$, so that $\forall \delta^m(t_0) \in \Omega^m$, there exists a bound B^m and time $T_m(B^m, \delta^1(t_0), \ldots, \delta^M(t_0))$, such that $\|\delta^m(t)\| \le B^m, \forall t \ge t_0 + T_m$. □

If the tracking error δ^m is CUUB, then $x_{i,m}(t)$ is bounded within a neighborhood of $x_{0,m}(t)$, for all $i \in \mathcal{N}$ and $t \ge t_0 + T_m$. This guarantees a practical notion of "close enough" synchronization.

In this paper, it is assumed that only *relative state information* can be used for the controller design. More precisely, for ith node, the only obtainable information is the neighborhood synchronization error (Khoo, Xie, & Man, 2009)

$$e_{i,m} = \sum_{j \in N_i} a_{ij}(x_{j,m} - x_{i,m}) + b_i(x_{0,m} - x_{i,m}), \tag{3}$$

where $m = 1, \ldots, M$ and $b_i \ge 0$ is the weight of edge from the leader node to node i $(i \in \mathcal{N})$. $b_i > 0$ if and only if there is an edge from the leader node to node i. Define $e^m = [e_{1,m}, \ldots, e_{N,m}]^T$, $\underline{f}_0 = [f_0(t, x_0), \ldots, f_0(t, x_0)]^T \in \mathbb{R}^N$, and $B = \mathrm{diag}(b_1, \ldots, b_N) \in \mathbb{R}^{N \times N}$. A straightforward derivation yields

$$\dot{e}^m = e^{m+1}, \quad m = 1, \ldots, M-1$$
$$\dot{e}^m = -(L+B)(f(x) + u + \zeta - \underline{f}_0), \quad m = M.$$

Define the augmented graph as $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{E}\}$, where $\bar{\mathcal{V}} = \{v_0, v_1, \ldots, v_N\}$ and $\bar{E} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. The following assumption on the graph topology is required for cooperative tracking problems.

Assumption 1. The augmented graph $\bar{\mathcal{G}}$ contains a spanning tree with the root node being the leader node 0. □

Suppose Assumption 1 does not hold and there are exactly a set of k $(1 \le k < N)$ nodes in graph $\mathcal{G}$, i.e., $S_k = \{n_1, \ldots, n_k\}$ and $S_k \subset \mathcal{N}$, which do not have directed paths from the leader node 0. This further implies that the set of nodes S_k do not have access to information of the rest set of nodes $\bar{S}_k = \mathcal{N} \setminus S_k$. It is obvious that there must exist some nodes in the set S_k, which are either isolated (i.e., isolated single node or isolated subgroup), or act as leader nodes which do not receive information from any other nodes. Obviously, in both cases, synchronization to the original leader node 0 cannot be achieved.

Remark 1. Assumption 1 includes the following graph topologies as special cases:

(a) Graph $\mathcal{G}$ is strongly connected and at least one node in graph $\mathcal{G}$ can get information from the leader node (Das & Lewis, 2010, 2011).
(b) Graph $\mathcal{G}$ contains a spanning tree and at least the root node can get access to the leader node (Li, Duan, Chen, & Huang, 2010; Zhang et al., 2011).
(c) The augmented graph $\bar{\mathcal{G}}$ is itself a spanning tree with the root being the leader node (Chen, Lewis, & Xie, 2011).
(d) The augmented graph $\bar{\mathcal{G}}$ has a hierarchical structure (Du, Li, & Qian, 2011).
(e) Graph $\mathcal{G}$ is disconnected and each separated subgroup is either a single node or contains a spanning tree. □

The following two technical lemmas play important roles in the proof of Theorem 1.

Lemma 1. *Define*

$$q = [q_1, \ldots, q_N]^T = (L+B)^{-1}\underline{1},$$
$$P = \mathrm{diag}\{p_i\} = \mathrm{diag}\{1/q_i\},$$
$$Q = P(L+B) + (L+B)^T P. \tag{4}$$

Then $P > 0$ and $Q > 0$.

Proof. First we show that $L + B$ is nonsingular. Under Assumption 1, at least one node in $\mathcal{G}$ can get information directly from the leader node, i.e., $b_i > 0$ for at least one i $(i \in \mathcal{N})$. Without loss of generality, we assume there are two nodes r_1 and r_2 such that $b_{r_1} > 0$ and $b_{r_2} > 0$. Then for matrix $L + B$, $|l_{ii} + b_i| \ge \sum_{j=1, j\neq i}^N l_{ij}$ for all $i \in \mathcal{N}$ and the strict inequality holds for $i \in \{r_1, r_2\}$. Assumption 1 implies that, for any other node i which do not have direct access to the leader node (i.e., $i \in \mathcal{N} \setminus \{r_i, r_2\}$), there must be a direct path either originated from node r_1 or node r_2. According to Shivakumar and Chew (1974), $L + B$ is nonsingular. Then $L + B$ is nonsingular M-matrix Qu (2009, Theorem 4.25). Lemma 1 follows from the same development as in Qu (2009, Theorem 4.25). □

Lemma 2.

$$\|\delta^m\| \le \|e^m\| / \underline{\sigma}(L+B), \quad m = 1, \ldots, M.$$

Proof. Under Assumption 1, $L + B$ is nonsingular. Noticing $e^m = -(L+B)\delta^m$, it follows that $\delta^m = -(L+B)^{-1}e^m$. Therefore $\|\delta^m\| = \|(L+B)^{-1}e^m\| \le \|e^m\| / \underline{\sigma}(L+B)$. □

4. Adaptive cooperative tracking control: Lyapunov design

In this section, we show how to design distributed neural network (NN) controllers for follower nodes, such that the cooperative tracking problem is solved.

4.1. Sliding mode error

Introduce sliding mode error r_i for node i $(i \in \mathcal{N})$ as

$$r_i = \lambda_1 e_{i,1} + \lambda_2 e_{i,2} + \cdots + \lambda_{M-1} e_{i,M-1} + e_{i,M}. \tag{5}$$

The design parameter $\lambda_1, \ldots, \lambda_{M-1}$ are chosen such that the polynomial $s^{M-1} + \lambda_{M-1}s^{M-2} + \cdots + \lambda_1$ is Hurwitz. Then on the sliding surface $r_i = 0$, $e_i \to 0$ exponentially as $t \to \infty$. The control objective in this paper is then to keep the sliding mode error r_i staying on, or near the sliding surface. For the latter case, e_i will be bounded. Define the global sliding mode error as $r = [r_1, \ldots, r_N]^T$; then $r = \lambda_1 e^1 + \cdots + \lambda_{M-1}e^{M-1} + e^M$.

Define $E_1 = [e^1, \ldots, e^{M-1}] \in \mathbb{R}^{N \times (M-1)}$, $E_2 = \dot{E}_1 = [e^2, \ldots, e^M]$, $l = [0, \ldots, 0, 1]^T \in \mathbb{R}^{M-1}$, and $\Lambda = \left[\begin{array}{c|c} 0 & I \\ \hline -\lambda_1 & -\lambda_2 \ldots -\lambda_{M-1} \end{array}\right]$. Then one has

$$E_2 = E_1\Lambda^T + rl^T. \tag{6}$$

Since Λ is Hurwitz, given any positive number β, there exists a matrix $P_1 > 0$, such that Lyapunov equation (7) holds.

$$\Lambda^T P_1 + P_1 \Lambda = -\beta I. \tag{7}$$

The dynamics of the sliding mode error r is

$$\dot{r} = \rho - (L+B)(f(x) + u + \zeta - \underline{f}_0),$$

where

$$\rho = \lambda_1 e^2 + \lambda_2 e^3 + \cdots + \lambda_{M-1}e^M = E_2\bar{\lambda} \tag{8}$$

with $\bar{\lambda} = [\lambda_1, \ldots, \lambda_{M-1}]^T$.

The next lemma shows that (ultimate) boundedness of r_i implies (ultimate) boundedness of e_i, $\forall i \in \mathcal{N}$.

Lemma 3. *For all $i = 1, \ldots, N$, suppose*

$$|r_i(t)| \le \psi_i, \quad \forall t \ge t_0$$
$$|r_i(t)| \le \xi_i, \quad \forall t \ge T_i$$

for some bounds $\psi_i > 0$, $\xi_i > 0$, and time $T_i > t_0$. Then there exist bounds $\Psi_i > 0$, $\Xi_i > 0$ and time $\Delta_i > t_0$, such that

$$\|e_i(t)\| \le \Psi_i, \quad \forall t \ge t_0$$
$$\|e_i(t)\| \le \Xi_i, \quad \forall t \ge \Delta_i.$$

Proof. Define $\varrho_i = [e_{i,1}, \dots, e_{i,M-1}]^T$. Then (5) implies

$$\dot{\varrho}_i = \Lambda \varrho_i + l r_i. \tag{9}$$

Using the similar development as in Ge and Zhang (2003, Lemma 2.1), it is straightforward to show that if $r_i(t)$ is bounded, i.e., $|r_i(t)| \le \psi_i$ for $t \ge t_0$, then $\varrho_i(t)$ is bounded. Eq. (9) further implies $\dot{\varrho}_i(t)$ is bounded, for $t \ge t_0$. Therefore, $e_i(t)$ is bounded, i.e., $\|e_i\| \le \Psi_i$, $\forall t \ge t_0$ for some $\Psi_i > 0$. Similarly, it can be shown that when $r_i(t)$ is ultimately bounded, $e_i(t)$ is ultimately bounded. □

4.2. Linear-in-parameter neural network

Assume the unknown nonlinearity $f_i(x_i)$ in (1) can be expressed on a prescribed compact set $\Omega \subset \mathbb{R}^M$ by

$$f_i(x_i) = W_i^T \phi_i(x_i) + \epsilon_i, \tag{10}$$

where $\phi_i(x_i) \in \mathbb{R}^{v_i}$ is a suitable set of v_i basis functions; $W_i \in \mathbb{R}^{v_i}$ is the ideal neural network weight vector; and $\epsilon_i \in \mathbb{R}$ is the NN approximation error. A variety of NN basis functions can be selected, including sigmoids, Gaussians, etc.

Remark 2. By Stone–Weierstrass approximation theorem (Stone, 1948), in the compact set Ω, given any positive number ε_i, there exists a large enough positive integer v_i^*, such that for any $v_i \ge v_i^*$, one can always find an ideal weight vector W_i and a suitable basis set $\phi_i(\cdot)$, which makes (10) satisfy the property that $\max_{x_i \in \Omega} |\epsilon_i| \le \varepsilon_i$. □

Define the approximation of $f_i(x_i)$ as

$$\hat{f}_i(x_i) = \hat{W}_i^T(t)\phi_i(x_i),$$

where $\hat{W}_i(t) \in \mathbb{R}^{v_i}$ (or $\hat{W}_i$ for notational convenience) is the current actual values of the NN weights for the ith node and must be provided using only the local information available to node i (in fact, by the NN weight tuning law (16)).

Define $W = \mathrm{diag}(W_1, \dots, W_N)$, $\hat{W} = \mathrm{diag}(\hat{W}_1, \dots, \hat{W}_N)$, $\epsilon = [\epsilon_1, \dots, \epsilon_N]^T$, $\phi(x) = [\phi_1^T(x_1), \dots, \phi_N^T(x_N)]^T$; then the global nonlinearity $f(x)$ can be written as

$$f(x) = W^T\phi(x) + \epsilon, \tag{11}$$

and its approximation is

$$\hat{f}(x) = \hat{W}^T\phi(x). \tag{12}$$

The error of the NN weights is defined as $\tilde{W} = W - \hat{W}$.

Remark 3. Denote $\phi_{iM} = \max_{x_i \in \Omega} \|\phi_i(x_i)\|$ and $W_{iM} = \|W_i\|$. According to the definitions of ϕ, W and ϵ, it is easy to observe that there exist positive numbers Φ_M, W_M and ϵ_M, such that $\|\phi\| \le \Phi_M$, $\|W\|_F \le W_M$ and $\|\epsilon\| \le \epsilon_M$. □

4.3. Distributed controller design

This section presents the distributed control law u_i and the distributed adaptive NN weight tuning law. Before proceeding, we make the following assumptions.

Assumption 2. (a) There exists a positive number $X_M > 0$ such that $\|x_0(t)\| \le X_M$, $\forall t \ge t_0$.
(b) There exists a continuous function $g(\cdot) : \mathbb{R}^M \to \mathbb{R}$, such that $|f_0(t, x_0)| \le |g(x_0)|$, $\forall x_0 \in \mathbb{R}^M$, $\forall t \ge t_0$.
(c) For each node i, the disturbance ζ_i is unknown but bounded. Or, equivalently, the overall disturbance vector ζ is also bounded by $\|\zeta\| \le \zeta_M$, where ζ_M can be unknown. □

Remark 4. Since node 0 acts as a command generator, Assumption 2(a) is reasonable in practical applications. According to Assumption 2(b), it is straightforward to show that there exists a positive number F_M such that $|f_0(t, x_0)| \le F_M$, $\forall x_0 \in \Omega_0$ and $\forall t \ge t_0$, where $\Omega_0 \triangleq \{x_0 \in \mathbb{R}^M | \|x_0\| \le X_M\}$. □

Remark 5. The "artificial" bounds X_M, ζ_M, F_M, W_M and ϵ_M (as in Assumption 2(a), (c), Remarks 3 and 4 respectively) are actually not used in the controller design, thus they do not have to be known. They are only required for the stability analysis. The bound Φ_M (as in Remark 3) is needed to choose the design parameter c as in (15). Since generally we can choose some squashing functions as the basis set, such as sigmoids, Gaussians, and hyperbolic tangents, Φ_M can be explicitly expressed. □

4.3.1. Distributed control law

According to Remark 2, v_i neurons are used for each node i. Design the distributed control law for each node i as

$$u_i = \frac{1}{d_i + b_i}(\lambda_1 e_{i,2} + \dots + \lambda_{M-1} e_{i,M}) - \hat{f}_i(x_i) + c r_i, \tag{13}$$

or collectively

$$u = (D + B)^{-1}\rho - \hat{f}(x) + cr. \tag{14}$$

The control gain c satisfies

$$c > \frac{2}{\underline{\sigma}(Q)}\left(\frac{\gamma^2}{\kappa} + \frac{2}{\beta}g^2 + h\right), \tag{15}$$

with $\gamma = -\frac{1}{2}\Phi_M\bar{\sigma}(P)\bar{\sigma}(A)$, $h = \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\bar{\lambda}\|$ and $g = -\frac{1}{2}\left(\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\Lambda\|_F\|\bar{\lambda}\| + \bar{\sigma}(P_1)\right)$, where P_1 is defined in (7) for any $\beta > 0$ and Q is defined as in Lemma 1.

Remark 6. When $M = 1, 2$, control law (13) reduces to the control law for the first-order nonlinear system (Das & Lewis, 2010) and the second-order nonlinear system (Das & Lewis, 2011), respectively. □

4.3.2. NN tuning law

The NN adaptive tuning law is designed to be

$$\dot{\hat{W}}_i = -F_i\phi_i r_i p_i(d_i + b_i) - \kappa F_i \hat{W}_i, \tag{16}$$

or collectively

$$\dot{\hat{W}} = -F\phi r P(D + B) - \kappa F\hat{W}, \tag{17}$$

where design parameters $F_i = F_i^T \in \mathbb{R}^{v_i \times v_i}$ are arbitrary positive definite matrices and $F = \mathrm{diag}(F_1, \dots, F_N)$; κ is a positive scalar tuning gain; and P is defined in Lemma 1. Note the control law (13) and NN tuning law (16) are implemented using only local information available for node i.

4.4. Main results

Theorem 1. *Consider the distributed system (1) and the leader node (2). Suppose Assumptions 1 and 2 hold. Using the distributed control law (13) and the distributed NN tuning law (16), one has the following results:*

(1) *The tracking errors $\delta^1, \dots, \delta^M$ are cooperative uniformly ultimately bounded, which implies all nodes in graph $\mathcal{G}$ synchronize to the leader node 0 with bounded residual errors.*
(2) *The states $x_i(t)$ $(i = 1, \dots, N)$ are bounded $\forall t \ge t_0$.*

Proof. (1) Given fixed positive numbers ε_i, there exist numbers of neurons v_i^* $(i = 1, \dots, N)$, such that for $v_i \ge v_i^*$, the NN approximation errors satisfy $\|\epsilon_i\| \le \varepsilon_i$. Then by Remark 3, one has $\|W\|_F \le W_M$, $\|\phi\| \le \Phi_M$ and $\|\epsilon\| \le \epsilon_M$.

Consider the Lyapunov function candidate

$$V = V_1 + V_2 + V_3, \tag{18}$$

where $V_1 = \frac{1}{2}r^T Pr$, $V_2 = \frac{1}{2}tr\{\tilde{W}^T F^{-1}\tilde{W}\}$ and $V_3 = \frac{1}{2}tr\{E_1 P_1 (E_1)^T\}$.

First we compute the derivative of V_1.

$$\dot{V}_1 = r^T P\dot{r} = r^T P[\rho - (L+B)(f(x) + u + \zeta - \underline{f_0})]. \tag{19}$$

To simplify the notations, denote $f(x)$ as f, $\hat{f}(x)$ as $\hat{f}$, and $\phi(x)$ as ϕ in the sequel. Substituting (14) into (19), and considering (11) and (12), one has

$$\dot{V}_1 = -r^T P(L+B)(\epsilon + \zeta - \underline{f_0}) - cr^T P(L+B)r - r^T P(D+B)\tilde{W}^T\phi + r^T PA\tilde{W}^T\phi + r^T PA(D+B)^{-1}\rho.$$

Noting the fact that $x^T y = tr\{yx^T\}$ if $x, y \in \mathbb{R}^N$, and considering (4), one has

$$\dot{V}_1 = -r^T P(L+B)(\epsilon + \zeta - \underline{f_0}) - \frac{1}{2}cr^T Qr - tr\{\tilde{W}^T\phi r^T P(D+B)\} + tr\{\tilde{W}^T\phi r^T PA\} + r^T PA(D+B)^{-1}\rho.$$

Since $\dot{\tilde{W}} = \dot{W} - \dot{\hat{W}} = -\dot{\hat{W}}$, considering (6), (8) and (17), one has

$$\begin{aligned}
&\dot{V}_1 + \dot{V}_2 \\
&= -\frac{1}{2}cr^T Qr - r^T P(L+B)(\epsilon + \zeta - \underline{f_0}) \\
&\quad + \kappa tr\{\tilde{W}^T W\} - \kappa\|\tilde{W}\|_F^2 + tr\{\tilde{W}^T\phi r^T PA\} \\
&\quad + r^T PA(D+B)^{-1}E_1\Lambda^T\bar{\lambda} + r^T PA(D+B)^{-1}rl^T\bar{\lambda} \\
&\le -\frac{1}{2}c\underline{\sigma}(Q)\|r\|^2 + \bar{\sigma}(P)\bar{\sigma}(L+B)T_M\|r\| - \kappa\|\tilde{W}\|_F^2 \\
&\quad + \Phi_M\bar{\sigma}(P)\bar{\sigma}(A)\|\tilde{W}\|_F\|r\| + \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|r\|^2\|l\|\|\bar{\lambda}\| \\
&\quad + \|r\|\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|E_1\|_F\|\Lambda\|_F\|\bar{\lambda}\| + \kappa W_M\|\tilde{W}\|_F,
\end{aligned}$$

where $T_M = \epsilon_M + \zeta_M + F_M$.

The derivative of V_3 is

$$\dot{V}_3 = tr\{E_2 P_1 (E_1)^T\}. \tag{20}$$

Substituting (6) into (20) and considering (7) gives

$$\begin{aligned}
\dot{V}_3 &= -\frac{\beta}{2}tr\{E_1(E_1)^T\} + tr\{rl^T P_1(E_1)^T\} \\
&\le -\frac{\beta}{2}\|E_1\|_F^2 + \bar{\sigma}(P_1)\|r\|\|E_1\|_F.
\end{aligned}$$

Therefore,

$$\begin{aligned}
\dot{V} &\le -\left(\frac{1}{2}c\underline{\sigma}(Q) - \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\bar{\lambda}\|\right)\|r\|^2 - \kappa\|\tilde{W}\|_F^2 \\
&\quad - \frac{\beta}{2}\|E_1\|_F^2 + \Phi_M\bar{\sigma}(P)\bar{\sigma}(A)\|\tilde{W}\|_F\|r\| \\
&\quad + \left(\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\Lambda\|_F\|\bar{\lambda}\| + \bar{\sigma}(P_1)\right)\|r\|\|E_1\|_F \\
&\quad + \bar{\sigma}(P)\bar{\sigma}(L+B)T_M\|r\| + \kappa W_M\|\tilde{W}\|_F.
\end{aligned} \tag{21}$$

Let $\gamma = -\frac{1}{2}\Phi_M\bar{\sigma}(P)\bar{\sigma}(A)$, $h = \frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\bar{\lambda}\|$ and $g = -\frac{1}{2}\left(\frac{\bar{\sigma}(P)\bar{\sigma}(A)}{\underline{\sigma}(D+B)}\|\Lambda\|_F\|\bar{\lambda}\| + \bar{\sigma}(P_1)\right)$. Then (21) is written as

$$\dot{V} \le -z^T Kz + \omega^T z = -V_z(z), \tag{22}$$

where

$$z = \left[\|E_1\|_F, \|\tilde{W}\|_F, \|r\|\right]^T, \tag{23}$$

$\omega = [0, \kappa W_M, \bar{\sigma}(P)\bar{\sigma}(L+B)T_M]^T$, $K = \begin{bmatrix} \frac{\beta}{2} & 0 & g \\ 0 & \kappa & \gamma \\ g & \gamma & \mu \end{bmatrix}$ and $\mu = \frac{1}{2}c\underline{\sigma}(Q) - h$.

$V_z(z)$ is positive definite if the following conditions C1 and C2 hold:

(C1) K is positive definite.

(C2) $\|z\| > \frac{\|\omega\|}{\underline{\sigma}(K)}$.

According to Sylvester's criterion, $K > 0$ if

$$\beta > 0,$$
$$\beta\kappa > 0,$$
$$\kappa(\beta\mu - 2g^2) - \beta\gamma^2 > 0.$$

Solving the above system of equations gives the equivalent condition (15). Since $\|\omega\|_1 > \|\omega\|$, condition (C2) holds if $\|z\| \ge Bd$ with

$$Bd = \frac{\|\omega\|_1}{\underline{\sigma}(K)} = \frac{\bar{\sigma}(P)\bar{\sigma}(L+B)T_M + \kappa W_M}{\underline{\sigma}(K)}. \tag{24}$$

Therefore, under condition (15), one has

$$\dot{V} \le -V_z(z), \quad \forall\|z\| \ge Bd,$$

with $V_z(z)$ being a continuous positive definite function.

Straightforward computation of (18) implies

$$\underline{\sigma}(\Gamma)\|z\|^2 \le V \le \bar{\sigma}(T)\|z\|^2, \tag{25}$$

where $\Gamma = \mathrm{diag}\left(\frac{\underline{\sigma}(P_1)}{2}, \frac{1}{2\bar{\sigma}(F)}, \frac{\underline{\sigma}(P)}{2}\right) \in \mathbb{R}^{3\times 3}$ and $T = \mathrm{diag}\left(\frac{\bar{\sigma}(P_1)}{2}, \frac{1}{2\underline{\sigma}(F)}, \frac{\bar{\sigma}(P)}{2}\right) \in \mathbb{R}^{3\times 3}$. Then following Theorem 4.18 in Khalil (2002), one can draw the conclusion that for any initial value $z(t_0)$ (or equivalently $V(t_0)$), there exists a time T_0 such that

$$\|z(t)\| \le \sqrt{\frac{\bar{\sigma}(T)}{\underline{\sigma}(\Gamma)}}Bd, \quad \forall t \ge t_0 + T_0. \tag{26}$$

Let $k = \min_{\|z\|\ge Bd} V_z(z)$. Then using essentially the same development as in Khalil (2002), it can be shown that

$$T_0 = \frac{V(t_0) - \bar{\sigma}(T)Bd^2}{k}. \tag{27}$$

By definition of z, (26) implies that $r(t)$ is ultimately bounded. Then $r_i(t)$ is ultimately bounded. By Lemma 3, $e_i(t)$ is ultimately bounded ($\forall i \in \mathcal{N}$), which implies $e^m(t)$ ($\forall m = 1, \ldots, M$) is ultimately bounded. Then, following Lemma 2, the tracking errors $\delta^1, \ldots, \delta^M$ are CUUB and all nodes in graph $\mathcal{G}$ synchronize, in the sense of Definition 1, to the trajectory $x_0(t)$ of the leader node.

(2) It can be shown that the state $x_i(t)$ is bounded $\forall i \in \mathcal{N}$ and $\forall t \ge t_0$. In fact, (22) implies

$$\dot{V} \le -\underline{\sigma}(K)\|z\|^2 + \|\omega\|\,\|z\|. \tag{28}$$

Then combination of (25) and (28) gives

$$\frac{d}{dt}(\sqrt{V}) \le -\frac{\underline{\sigma}(K)}{2\bar{\sigma}(T)}\sqrt{V} + \frac{\|\omega\|}{2\sqrt{\underline{\sigma}(\Gamma)}}.$$

Thus $V(t)$ is bounded for all $t \ge t_0$ by Corollary 1.1 in Ge and Wang (2004). Since (18) implies $\|r\|^2 \le \frac{2V(t)}{\underline{\sigma}(P)}$, there follows the boundedness of $r(t)$, $\forall t \ge t_0$. By Lemmas 2 and 3, it is straightforward that $\delta^m(t)$ is also bounded. Noting $\delta^m = x^m - \underline{x}_{0,m}$ and considering Assumption 2(a), one can see $x^m(t)$, $\forall m = 1, \ldots, M$, is bounded all the time, i.e., $x_i(t)$ is bounded. In other words, $x_i(t)$ is contained in a compact set Ω_i, $\forall i \in \mathcal{N}$ and $t \ge t_0$. $\square$

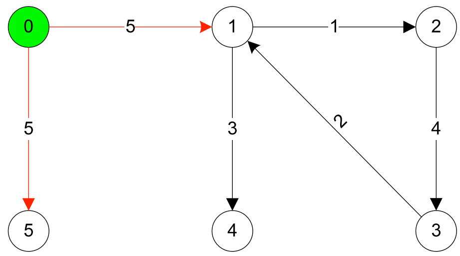

Fig. 1. Topology of the augmented graph $\bar{\mathcal{G}}$.

Remark 7. For cooperative control of higher-order systems, the interplay between the node dynamics and the graph structure is more complicated than the first- and second-order cases. This is reflected in the fact that two Lyapunov equations are involved in the stability analysis: one for the individual node controller design (i.e., (7)), and the other for the graph structure (i.e., (4)). $\square$

Remark 8 (*Finite-Time Convergence*)**.** Inequality (26) implies that the sliding mode error r_i reaches a bounded neighborhood of the origin in finite time T_0 given in (27). It then follows, from Lemmas 2 and 3, that the tracking error δ_i reaches the final bound in finite time. $\square$

Remark 9 (*Design Issues*)**.** The design parameter F_i is an arbitrary positive definite matrix, which implies p_iF_i is an arbitrary positive definite matrix. This further implies that one can pick an arbitrary positive number p_i for the NN tuning law. Therefore, the proposed controller u_i is completely distributed, as one can see from (13) and (16). It is also worth noting that although (24) is a rather conservative case for the ultimate error bound, it provides a practical design tool for higher-order synchronization problems. It shows relations between the graph topology and the control design parameters in reducing the error bound. $\square$

Remark 10. It is well known in the literature of neural network control (Ge & Wang, 2004; Lewis, Jagannathan, & Yeşildirek, 1999) that the size of the compact set Ω in Remark 2 is not needed for the controller design. It is commonly assumed to be as large as desired in practical applications, such that states x_i will stay inside Ω, i.e., $\Omega_i \subset \Omega$, $\forall i \in \mathcal{N}$ and $t \geq t_0$. In this sense, the results obtained in this paper is semi-global. In the case where Remarks 2 and 3 hold for all $x_i \in \mathbb{R}^M$, the results will be global. $\square$

5. Numerical example

Consider a 5-node digraph $\mathcal{G}$ and a leader node 0 described in Fig. 1. Note that this topology satisfies Assumption 1 and falls into case (e) of Remark 1. Let the dynamics of the leader node be

$$\dot{x}_{0,1} = x_{0,2}$$
$$\dot{x}_{0,2} = x_{0,3}$$
$$\begin{aligned}\dot{x}_{0,3} &= -x_{0,2} - 2x_{0,3} + 1 + 3\sin(2t) + 6\cos(2t) \\ &\quad - \frac{1}{3}(x_{0,1} + x_{0,2} - 1)^2(x_{0,1} + 4x_{0,2} + 3x_{0,3} - 1). \end{aligned} \tag{29}$$

The follower nodes are described by third-order nonlinear systems in the form of (1) with

$$\begin{aligned}
\dot{x}_{1,3} &= x_{1,2}\sin(x_{1,1}) + \cos(x_{1,3})^2 + u_1 + \zeta_1, \\
\dot{x}_{2,3} &= -x_{2,1}x_{2,2} + 0.01x_{2,1} - 0.01(x_{2,1})^3 + u_2 + \zeta_2, \\
\dot{x}_{3,3} &= x_{3,2} + \sin(x_{3,3}) + u_3 + \zeta_3, \\
\dot{x}_{4,3} &= -3(x_{4,1} + x_{4,2} - 1)^2(x_{4,1} + x_{4,2} + x_{4,3} - 1) \\
&\quad - x_{4,2} - x_{4,3} + 0.5\sin(2t) + \cos(2t) + u_4 + \zeta_4, \\
\dot{x}_{5,3} &= \cos(x_{5,1}) + u_5 + \zeta_5,
\end{aligned} \tag{30}$$

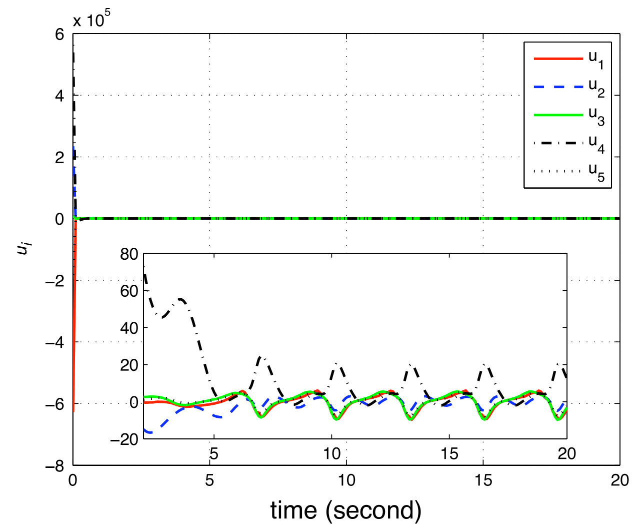

Fig. 2. Profiles of the controllers u^i.

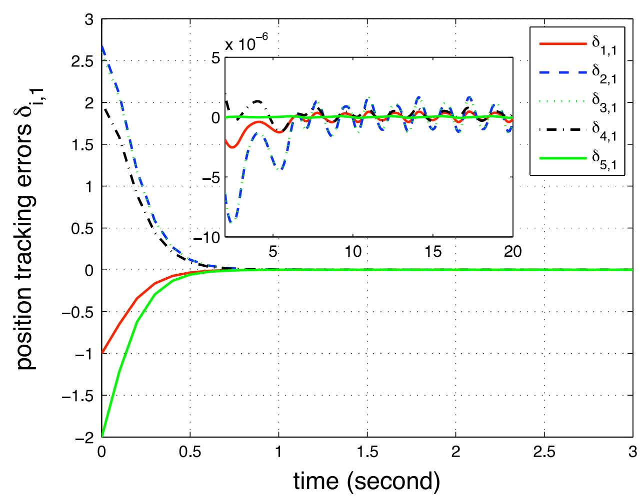

Fig. 3. Profiles of the position tracking errors $\delta_{i,1}$.

where disturbances ζ_i are taken as random and bounded by $|\zeta_i| \leq 2$. Note that open-loop system of node 2 (without disturbance) exhibits a chaotic behavior for the initial value $x_2 = [0.325, 0.5, 0]^T$; the open-loop system of node 3 has a finite escape time; the open-loop system of node 4 has the same structure as the leader node but with different parameters.

In this example, 6 neurons are used for each NN. The NN weights are initialized to be $\hat{W}_i(0) = [0, 0, 0, 0, 0, 0]^T$, $\forall i$. Sigmoid basis functions are used. Choose the design parameter as $\lambda_1 = 100$, $\lambda_2 = 20$, $c = 600$, $\kappa = 0.01$, $F_i = 2000I$, and $p_i > 0$. Profiles of the controllers are shown in Fig. 2. Profiles of the tracking errors are shown in Figs. 3–5. Note that the tracking errors $\delta_{i,1}$, $\delta_{i,2}$, and $\delta_{i,3}$ $(i = 1, \ldots, 5)$ do not converge to zero, but are bounded by small residual errors. These figures demonstrate the effectiveness of the proposed algorithm.

6. Conclusion

This paper considered the cooperative tracking control problem of networked higher-order nonlinear systems with distinct unknown dynamics and bounded external disturbances. The nonlinearities are only assumed to be locally Lipschitz. A robust

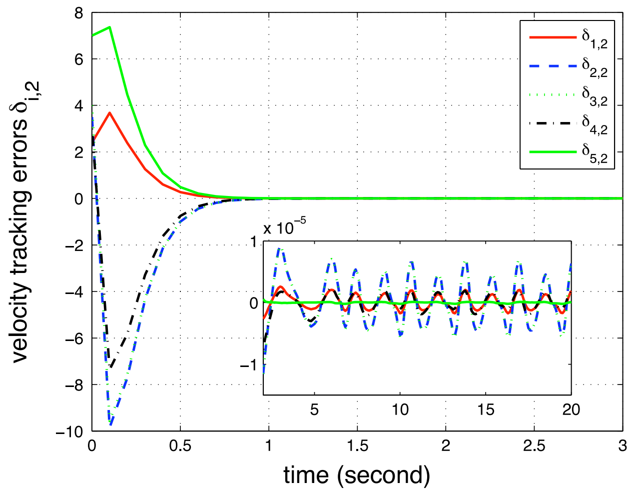

Fig. 4. Profiles of the velocity tracking errors $\delta_{i,2}$.

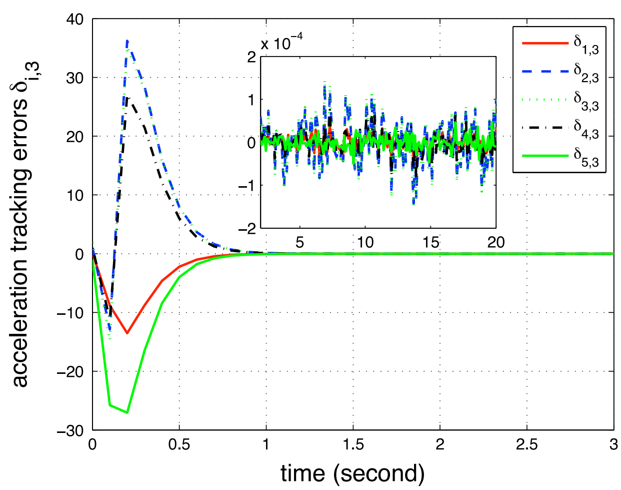

Fig. 5. Profiles of the acceleration tracking errors $\delta_{i,3}$.

neural adaptive control algorithm was proposed. This algorithm is completely distributed in the sense that, the controller for each follower node only uses information of itself and its neighbors. The cooperative tracking problem of more general nonlinear systems over a dynamic communication network is worthy of further investigation.

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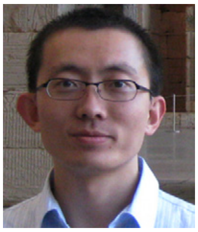

Hongwei Zhang received the B.E. and M.E. degrees from the Department of Automation, Tianjin University, China, in 2003 and 2006, respectively, and the Ph.D. degree from the Department of Mechanical and Automation Engineering, the Chinese University of Hong Kong, Hong Kong, in 2010.

From July 2009 to December 2010, he was a visiting scholar and subsequently a postdoctoral researcher at the University of Texas at Arlington, Texas, USA. From February 2011 to February 2012, he was with the City University of Hong Kong as a postdoctoral researcher. He is now an associate professor at Southwest Jiaotong University, Sichuan, China.

He serves as a reviewer for many refereed journals, including Automatica, Systems & Control Letters, International Journal of Robust and Nonlinear Control, IEEE Transactions on Neural Networks and Learning Systems, and IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics.

His current research interests are cooperative control, neural adaptive control, and optimal control.

Frank L. Lewis, Fellow IEEE, Fellow IFAC, Fellow UK Institute of Measurement & Control, PE Texas, UK Chartered Engineer, is Distinguished Scholar Professor and Moncrief-O'Donnell Chair at University of Texas at Arlington's Automation & Robotics Research Institute. He obtained the Bachelor's Degree in Physics/EE and the MSEE at Rice University, the M.S. in Aeronautical Engineering from University of West Florida, and the Ph.D. at Ga. Tech. He works in feedback control, intelligent systems, distributed control systems, and sensor networks. He is author of 6 US patents, 216 journal papers, 330 conference papers, 14 books, 44 chapters, and 11 journal special issues. He received the Fulbright Research Award, NSF Research Initiation Grant, ASEE Terman Award, Int. Neural Network Soc. Gabor Award 2009, UK Inst Measurement & Control Honeywell Field Engineering Medal 2009; received Outstanding Service Award from Dallas IEEE Section; was selected as Engineer of the year by Ft. Worth IEEE Section; was listed in Ft. Worth Business Press Top 200 Leaders in Manufacturing; received the 2010 IEEE Region 5 Outstanding Engineering Educator Award and the 2010 UTA Graduate Deans Excellence in Doctoral Mentoring Award. He served on the NAE Committee on Space Station in 1995. He is an elected Guest Consulting Professor at South China University of Technology and Shanghai Jiao Tong University. He is Founding Member of the Board of Governors of the Mediterranean Control Association and helped win the IEEE Control Systems Society Best Chapter Award (as Founding Chairman of DFW Chapter), the National Sigma Xi Award for Outstanding Chapter (as President of UTA Chapter), and the US SBA Tibbets Award in 1996 (as Director of ARRIs SBIR Program).