HW for Chapter 1

1. A company has 4 million US dollars to invest and it is required that all money need to be spent in a period of 4 years. For each year, if the amount of investment is x_i (i=1, 2, 3, 4) thousand, then the profit will be $x_i^{0.5}$ thousand (the invested money and profits CAN NOT be used anymore); the money not invested is deposited into the bank with the interest rate of 10% and the interest can be used again for investment. (10 pts)

Question: make an optimal investment plan such that the profits over this four-year period is maximized.

2. Relate with your own research and study, propose a real optimization problem, and provide the mathematical model for this problem. (10 pts)

NOTE: For both problems, you just need to write down the mathematical model, DO NOT solve the problem!!!

HW for Chapter 1 (Cont'd)

3. Solve the following problems with the graphic method:

(1) $\min f(X) = x_1^2 + x_2^2 - 12x_1 - 4x_2 + 40$ **(10 pts)**

$s.t.$ $x_1^2 + x_2^2 - 9 \le 0$

$-x_1 - x_2 + 2 \le 0$

$x_1, x_2 \ge 0$

(2) $\min f(X) = (x_1-2)^2 + (x_2-1)^2$ **(10 pts)**

$s.t.$ $x_1^2 + x_2 - 2 \le 0$

$-x_1 - x_2 + 1 \le 0$

$-x_1 \le 0$

HW for Chapter 1 (Cont'd)

4. For the depressed cubic equation:

$$f(x) = ax^3 + bx + c$$

(a) Discuss the roots of this equation in details. (10 pts)

(b) Find the real roots of the following equation: (10 pts)

$$f(x) = 4x^3 + 6x - 4$$

NOTE: For both parts, you need to find the roots analytically!

NOT numerically!

Due in class on Thursday 2023/09/07.