Problem Set 5

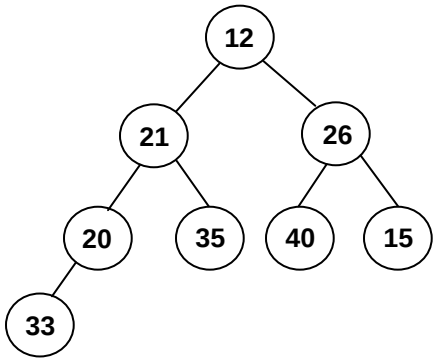
Due prior to lecture on Wednesday, December 9
No submissions will be accepted after Sunday, December 13, at 11:59 p.m.
so that we can post solutions before the final exam.

Notes:

- ***Because of the grad-credit projects, there are no extra grad-credit problems for this problem set.***
- ***There are no programming problems for this problem set.***

Written Exercises (100 points total)

Because we are asking you to draw diagrams, you may submit either a plain-text file or a PDF file. Give it the name `ps5.txt` *or* `ps5.pdf`*, and put your name and email address at the top of the file.*



1. **Heaps and heapsort** (10 points total)
   a. (3 points) Illustrate the process of turning the tree at right into a max-at-top heap. Show the tree after each sift operation.
   b. (2 points) What is the array representation of the max-at-top heap that you obtain in part a?
   c. (5 points) Heapsort begins by turning the array to be sorted into a heap. Assume that your answer to part b is the result of this process. Illustrate the remaining steps involved in applying heapsort to this array. Show the contents of the array after each element is put into its final position – i.e., at the end of each iteration of the while loop in the `heapSort` method covered in lecture.

2. **Hash tables** (12 points total; 4 points each part)
   The following sequence of keys is to be inserted in a hash table of size 8:

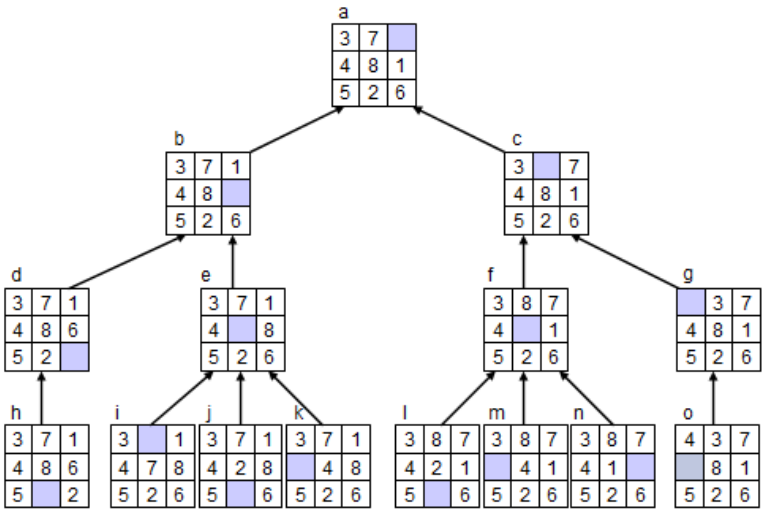
   cat, goat, dog, bird, bison, ant, flea, bat, duck

   The hash function assigns to each key the number of characters in the key.

   a. Assume that open addressing with linear probing is used to insert the keys. Determine which key causes the table to overflow, and show the table at the point at which it does so.
   b. Assume that open addressing with quadratic probing is used to insert the keys. Determine which key causes the table to overflow, and show the table at the point at which it does so.
   c. Now assume instead that open addressing with double hashing is used to

insert the keys, with the value of the second hash function based on the first character in the key: a = 1, b = 2, c = 3, d = 4, e = 5, f = 6, g = 7, etc. Determine which key causes the table to overflow, and show the table at the point at which it does so.

3. **Informed state-space search** (8 points total; 4 points each part)
Below is a portion of the state-space search tree for the Eight Puzzle whose initial configuration is shown at the root of the tree. The states have been labeled with lower-case letters to allow you to name them.



a. What are the priorities that greedy search would assign to the following states: state a, state c, state f, and state m? Assume that the Manhattan distance heuristic from lecture is used to estimate the remaining cost.
b. What are the priorities that A* search would assign to the same four states? Assume again that the Manhattan distance heuristic from lecture is used to estimate the remaining cost.

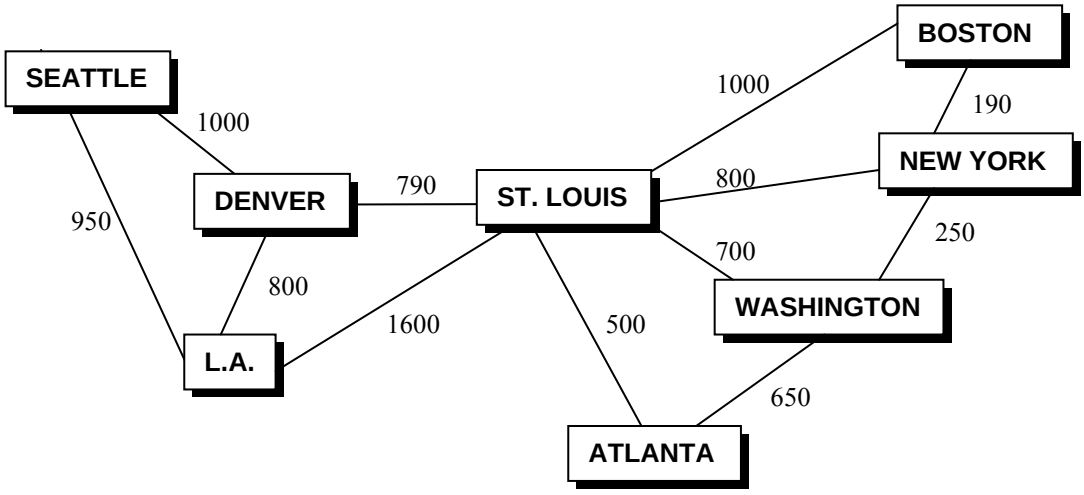
4. **List-based priority queue** (8 points total)
a. (5 points) A priority queue can be implemented using a list instead of a heap. Describe how you could use a list to implement a priority queue in which the remove operation would have a time complexity of $O(1)$.
b. (3 points) What would be the efficiency of the insert operation in this list-based implementation?

5. **Testing for a path between vertices** (10 points total; 5 points each part)
A graph may or may not include a path between a given pair of vertices.

a. Given the graph representation used in the `Graph` class from lecture, write an algorithm that takes references to two `Vertex` objects and determines if there is a path from the first vertex to the second vertex. The method should return `true` if there is a path and `false` otherwise.

b. For a graph with n vertices, what would be the best-case time efficiency of the algorithm from part a? What would be the worst-case time efficiency? Use big-O notation, and explain your answers briefly.

Questions 6-8 refer to the following graph:



6. **Graph traversals** (8 points; 2 points each part)
   Suppose that you have purchased a pass that allows you to fly anywhere you wish along the routes that are shown in the diagram above.
   a. List the order in which you will visit the cities if you start from L.A. and do a breadth-first traversal. You should assume that the edges of each vertex are stored in order of increasing distance, as we did in the examples in lecture.
   b. What is the path from L.A. to New York in the breadth-first spanning tree? Give the path in the form A -> B -> C -> etc., where A, B, and C are vertices.
   c. List the order in which you will visit the cities if you start from L.A. and do a depth-first traversal. You should assume that the edges of each vertex are stored in order of increasing distance, as we did in the examples in lecture.
   d. What is the path from L.A. to New York in the depth-first spanning tree? Give the path in the form A -> B -> C -> etc., where A, B, and C are vertices.

7. **Minimal spanning tree** (6 points)
   A cheaper version of the same pass requires that you confine yourself to flights that are part of a minimal spanning tree for the graph. List the order in which flights/edges will be added to this tree if you build it using Prim's algorithm, starting from L.A. Use the form (city1, city2) when specifying an edge.

8. **Dijkstra's shortest-path algorithm** (8 points total)
   Suppose you set out to use Dijkstra's algorithm to determine the shortest distance from L.A. to every other city in the graph.
   a. (5 points) Make a table showing the order in which the cities are finalized and the minimum distance to each.

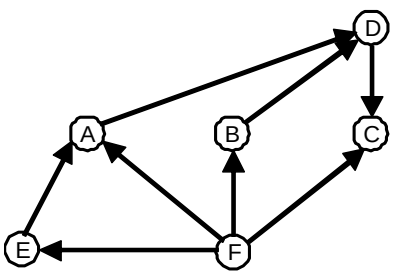
b. (3 points) What path(s) does the algorithm discover from L.A. to Boston? Include both the final shortest path and any temporary estimates for the shortest path. Give the paths in the form A -> B -> C -> etc., where A, B, and C are vertices.

9. **Routing packets** (8 points)
You have been asked to hard-code a portion of the network routing table for one of the servers that your company owns. The lengths of cable connecting pairs of your company's servers are given in the matrix below; a value of -1 in a given location indicates that there is no cable connecting that pair of servers.

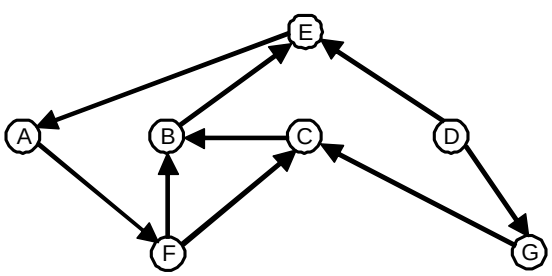
Assume that you are focusing on server #5. For each of the other servers, determine where server #5 should route packets destined for that server – i.e., to which other server should packets destined for that server be sent next? Your goal is to minimize the total length of cable over which the packet would need to travel, assuming that all of the other servers are also routing packets in this way. *Your answer should apply one of the graph algorithms that we covered in lecture.* **Show the steps** that you take in employing the algorithm to solve this problem, and **explain briefly** why it is an appropriate algorithm for this problem.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	-1	125	-1	-1	325	500	-1	-1
2	125	-1	-1	-1	150	-1	-1	-1
3	-1	-1	-1	140	120	-1	-1	-1
4	-1	-1	140	-1	275	-1	-1	285
5	325	150	120	275	-1	-1	-1	625
6	500	-1	-1	-1	-1	-1	115	-1
7	-1	-1	-1	-1	-1	115	-1	400
8	-1	-1	-1	285	625	-1	400	-1

10. **Directed graphs and topological sort** (10 points total; 5 points each part)
a. Is graph 10-1 a directed acyclic graph (a DAG)? If it isn't a DAG, specify *all* cycles that are present in the graph. If it is a DAG, use topological sort to find *one* of the possible topological orderings of the vertices in the graph.
b. Repeat the process outlined above on graph 10-2.



graph 10-1



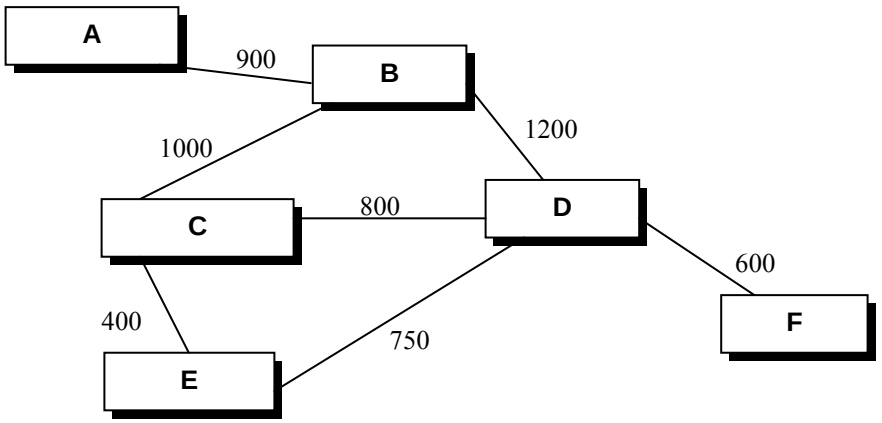
graph 10-2

11. **Alternative MST algorithm** (6 points)

Prim's algorithm is just one possible algorithm for finding a minimum spanning tree. Another such algorithm was developed by Kruskal:

```
kruskal_MST(Graph g) {
    put each of g's vertices in its own set
    while (there is more than one set) {
        let e = the minimum-cost edge that hasn't been considered
        if (e connects vertices that are in different sets) {
            add e to the MST
            merge the sets containing the vertices connected by e
        }
    }
}
```

Note that this algorithm considers the edges in order of increasing cost. In addition, at an intermediate stage of the algorithm, there may be multiple trees that are not connected to each other, although they will ultimately be joined together to form a single MST.



For example, if we applied Kruskal's algorithm to the graph above, we would start out with the following sets:

{ A }, { B }, { C }, { D }, { E }, { F }

We would consider the edge (C, E) first, because it has the lowest cost (400). Because it connects vertices in different sets, we would add this edge to the tree and merge the sets involved to get:

{ A}, { B }, { C, E }, { D }, { F }

We would next consider the edge (D, F), because it has the smallest remaining cost. Because it connects vertices in different sets, we would add this edge to the tree and merge the sets involved to get:

{ A }, { B }, { C, E }, { D, F }

We would next consider the edge (D, E), because it has the smallest remaining cost. Because it connects vertices in different sets, we would add this edge to the tree and merge the sets involved to get:

{ A }, { B }, { C, D, E, F }

We would next consider the edge (C, D), because it has the smallest remaining cost. Because it connects vertices that are *already in the same set*, we would *not* add this edge to the tree.

We would next consider the edge (A, B), because it has the smallest remaining cost. Because it connects vertices in different sets, we would add this edge to the tree and merge the sets involved to get:

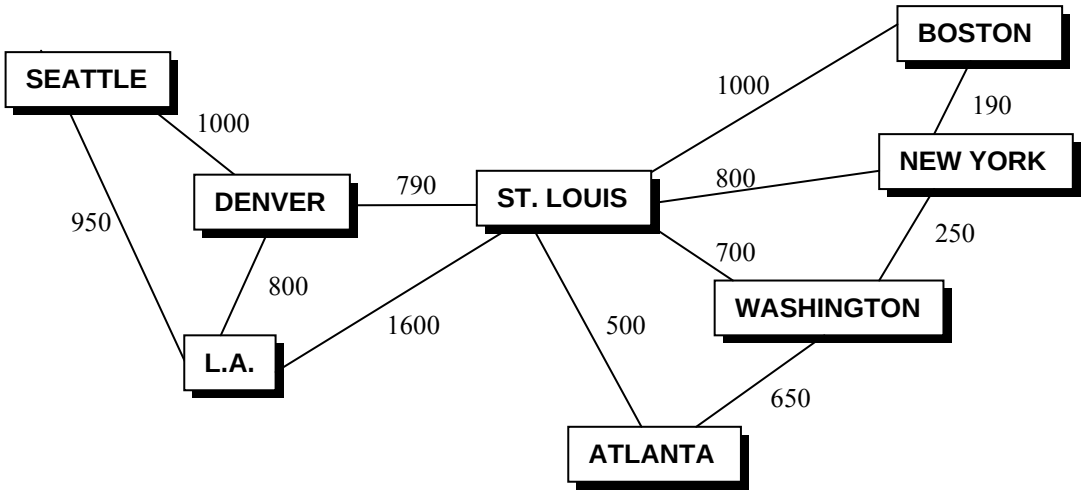
{ A, B }, { C, D, E, F }

We would next consider the edge (B, C), because it has the smallest remaining cost. Because it connects vertices in different sets, we would add this edge to the tree and merge the sets involved to get:

{ A, B, C, D, E, F }

Finally, we would consider the edge (B, D). Because it connects vertices in the same set, we would *not* add this edge to the tree.

Apply Kruskal's algorithm to the airline graph from earlier in the problem set, which is reproduced below. List the edges of the MST in the order in which the algorithm would add them, using the format (city1, city2) for an edge.



12. **Maximum-cost spanning tree** (6 points)
Consider the problem of constructing a *maximum*-cost spanning tree. Invent a variant of Prim's algorithm to solve this problem, and specify your algorithm using pseudocode.

Submitting Your Work

You should use Canvas to submit your `ps5_partI.txt` or `ps5_partI.pdf` file.

Here are the steps you should take to submit your work:

- Go to the page for submitting assignments (logging in as needed using the Login link in the upper-right corner, and entering your Harvard ID and PIN).
- Click on the **Problem Set 5** link.
- Click on the *Submit Assignment* link near the upper-right corner of the screen. (If you have already submitted something for this assignment, click on *Re-submit Assignment* instead.)
- Use the *Choose File* button to select the file to be submitted and click on the *Submit Assignment* button.
- After submitting the assignment, you should check your submission carefully. In particular, you should:
  - Check to make sure that you have a green checkmark symbol labeled *Turned In!* in the upper-right corner of the submission page (where the Submit Assignment link used to be), along with the name of the file that you are submitting.
  - Click on the link for each file to download it, and view the downloaded file so that you can ensure that you submitted the correct file.

  We will not accept any files after the fact, so please check your submission carefully!

Note: If you encounter problems submitting your files, close your browser and start again, or try again later if you still have time. If you are unable to submit and it is close to the deadline, email your homework before the deadline to `cscie22@fas.harvard.edu`