Harvard University Extension School
Computer Science E-121

Problem Set 5

Due October 30, 2015 at 11:59 PM.
Submit your solutions electronically on the course website, located at `https://canvas.harvard.edu/courses/4896/assignments`. On the site, use the assignments tab to find the correct problem set, then with the "submit assignment" button, upload the PDF file of your solution.
LATE PROBLEM SETS WILL NOT BE ACCEPTED.
See the syllabus for the collaboration policy.

PROBLEM 1 (4+2 points, suggested length of 1/2 page)

Consider the Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$ where

- $Q = \{q_0, q_1, q_2, q_{\text{accept}}, q_{\text{reject}}\}$,
- $\Sigma = \{a, b\}$ and $\Gamma = \{a, b, \sqcup\}$,
- The start, accept and reject states are q_0, q_{accept}, and q_{reject} respectively.
- The function δ is given by:

q	σ	$\delta(q, \sigma)$
q_0	a	(q_0, b, R)
q_0	b	(q_1, a, R)
q_0	$\sqcup$	$(q_{\text{reject}}, \sqcup, R)$
q_1	a	(q_2, b, R)
q_1	b	(q_1, a, R)
q_1	$\sqcup$	$(q_{\text{reject}}, \sqcup, R)$
q_2	a	(q_2, b, R)
q_2	b	(q_2, a, R)
q_2	$\sqcup$	$(q_{\text{accept}}, \sqcup, R)$

(A) Give the sequence of configurations describing M's computation on the string $aabba$

(B) Describe $L(M)$.

PROBLEM 2 (4 points, suggested length of 1/4 page)

A Modern Turing Machine (MTM), instead of $\{L, R\}$, has $\{L_k, R_k\}$ (move the head left or right k spaces on the tape, for any integer k encoded in the rule in the MTM). Prove that the MTMs are equivalent in power to the TMs. In other words, any MTM can be converted to a TM with an equivalent language, and vice versa.

PROBLEM 3 (2 + 2 + 1 points, suggested length of 1/2 page)

A Democratic rule of a CFG (V, Σ, R, S) is one whose right-hand side is a member of $V\Sigma^* \cup \Sigma^*$, that is, only the leftmost symbol can be a nonterminal. Republican rules are defined analogously where only the rightmost symbol can be a nonterminal. Show that a grammar with only Democratic rules, or only Republican rules, generates a regular language, but a grammar with a mixture of Democratic and Republican rules may generate a non-regular language.

PROBLEM 4 (7 points, suggested length of 3/4 page)

Let us define EvenPalindrome$(L) = \{ww^R : w \in L\}$. Show that if L is regular, then EvenPalindome(L) is context-free. Note that this is not the same as the language of all even palindromes, which is actually just EvenPalindrome(Σ^*). Explain briefly how your solution works. You need not provide a formal proof of correctness.

PROBLEM 5 (CHALLENGE!! (3) points, suggested length of 1/2 page)

A Boring Turing Machine (BTM) can only write # on the tape (assume $\# \notin \Sigma$). Prove that the BTMs are equivalent in power to the TMs.

PROBLEM 6 (6 points, suggested length of 1/3 page)

Show that every infinite Turing-recognizable language has an infinite decidable sublanguage.

PROBLEM 7 (4+2+4 points, suggested length of 1 page)

Let $G = (V, \Sigma, R, S)$ where $V = \{S, V\}$, $\Sigma = \{a, b\}$, and R is the set of rules:

$$S \to bSS \mid aS \mid aV$$
$$V \to aVb \mid bVa \mid VV \mid \varepsilon$$

(A) Transform G into an equivalent grammar G' in Chomsky normal form. Show your work for each step of this conversion, but long justifications for each step are not necessary.

(B) Explain in English what language the grammar G generates. (One clearly worded sentence is fine; thorough justification is not necessary, but it might get you some partial credit if your answer is wrong.)

(C) Check whether the strings *abaab* and *bbabaa* are generated by G, using the recognition algorithm for grammars in Chomsky normal form given in class. Show the complete filled-in matrix for each of the two strings.