Harvard University
Computer Science 121

Problem Set 0

Tuesday September 15, 2015 at 11:59pm

Problem set by **FILL IN YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)**

PART A (Graded by Cecilia and Madhu)

PROBLEM 1 (2+2 points, suggested length of 2 lines)

Let X and Y be sets. Using set notation, give formal descriptions of the following sets:

(A) The set of all nonempty subsets of X.

(B) The difference between X and Y, i.e. the set containing all elements of X that are not elements of Y. (This is denoted $X \setminus Y$.)

Solution.

PROBLEM 2 (3+3 points, suggested length of 1/4 page)

Let $\mathbb{N} = \{0, 1, 2, ...\}$ be the set of natural numbers. For each of the following functions $f : \mathbb{N} \to \mathbb{N}$, state whether f is (i) one-to-one/injective, (ii) onto/surjective, and/or (iii) bijective.

(A) $f(x) = x!$

(B) $f(x) = \begin{cases} x+1 & \text{if } x \text{ is even} \\ x-1 & \text{if } x \text{ is odd} \end{cases}$

Solution.

PROBLEM 3 (6 points, suggested length of 1/2 page)

Consider the binary relation $\lesssim$ defined by $A \lesssim B$ if there exists a one-to-one (injective) function $f : A \to B$. Is $\lesssim$ reflexive? symmetric? transitive? Briefly justify your answers. Explain in simple terms what it means if $A \lesssim B$ and $B \lesssim A$.
Solution.

PROBLEM 4 (0 points, suggested length of 1/4 page)

What is one key difference between classes you enjoy and classes you don't enjoy?

Solution.

PART B (Graded by Charles and Erin)

PROBLEM 5 (6 points, suggested length of 2/3 page)

Define the Fibonacci numbers as follows:

$$F_0 = 0$$

$$F_1 = 1$$

$$F_n = F_{n-1} + F_{n-2} \text{ for all } n > 1$$

Prove the following statement by induction:

(A) For $n > 1$, F_n equals the number of strings of length $n-2$ over alphabet $\Sigma = \{a, b\}$ that do not contain two consecutive a's.

Solution.

PROBLEM 6 (2+2+2 points, suggested length of 6 lines)

Let L_1 be the language $\{a^n : n \geq 0\}$ and L_2 be the language $\{x : x \in \{a,b\}^* \text{ and } |x| = 5\}$. Answer yes or no to the following questions.
(A) Do the following sets contain the empty string ε?

1. $L_1 \cap L_2$
2. $L_1 \cup L_2$

(B) Do the following sets have the empty set $\emptyset$ as a subset?

1. L_2
2. $L_1 \cap L_2$

(C) Do the following sets contain $\emptyset$ as an element?

1. L_1
2. $P(L_2)$

Solution.

PROBLEM 7 (Challenge!! 3 points, suggested length of 1/3 page)

Note: On every problem set we will provide a challenge problem, generally significantly more difficult than the other problems in the set, but worth only a few points. It is recommended that you attempt these problems, but only after completing the rest of the assignment.

Show that in any group of at least six people, either three of them are mutual friends (i.e. they all know each other) or three of them are mutual strangers (i.e., none of them know each other). You may assume that knowing is symmetric.

Solution.

PROBLEM 8 (0 points, suggested length of 1 line)

What is your favorite dining hall food?

Solution.