**Harvard University Extension School**
**Computer Science E-121**

**Problem Set 6**

Due November 6, 2015 at 11:59 PM.
Submit your solutions electronically on the course website, located at `https://canvas.harvard.edu/courses/4896/assignments`. On the site, use the assignments tab to find the correct problem set, then with the "submit assignment" button, upload the PDF file of your solution.
**LATE PROBLEM SETS WILL NOT BE ACCEPTED.**
See the syllabus for the collaboration policy.

PROBLEM 1 (6 points, suggested length of 1/3 page)

Prove that the class of decidable languages is closed under union.

PROBLEM 2 (6 points, suggested length of 1/3 page)

Prove that the class of Turing-recognizable languages is closed under intersection.

PROBLEM 3 (3 + 5 points, suggested length of 5 lines, 1/3 page)

(A) Consider $L = \{\langle M \rangle : M$ is a TM that accepts no strings shorter than 42 characters in length$\}$. Prove that L is not decidable. Hint: Consider Rice's Theorem.

(B) Use that the fact that $L$ is not decidable to prove that $L$ is not recognizable.

PROBLEM 4 (3 points, suggested length of 5 lines)

Define $\textsc{Prefix}(L) = \{x : xy \in L$ for some $y \in \Sigma^*\}$. Show that if $L$ is Turing-recognizable, then $\textsc{Prefix}(L)$ is Turing-recognizable.

PROBLEM 5 (8 points, suggested length of 2/3 page)

Let $L = \{\langle M \rangle : M$ moves left on the tape at some point when run on $\varepsilon\}$. Show that $L$ is decidable.

PROBLEM 6 (10 points, suggested length of 3/4 page)

Show that a language $L_1$ is Turing-recognizable if and only if there exists some decidable language $L_2$ such that $L_1 = \{x : \text{there exists } y \text{ such that } \langle x, y \rangle \in L_2\}$.

Note that the $\langle\rangle$ notation signifies that you are "stringify-ing" the contents within the bracket, since Turing Machines take strings as input.

(Hint: Imagine that $y$ gives you information about an accepting computation on $x$, if one exists.)

PROBLEM 7 (Challenge!! (3) points, suggested length of 1/3 page)

Show that every infinite regular language has a subset that is Turing-recognizable but undecidable.