Harvard University Extension School
Computer Science E-121

Problem Set 7

Due November 13, 2015 at 11:59 PM.
Submit your solutions electronically on the course website, located at `https://canvas.harvard.edu/courses/4896/assignments`. On the site, use the assignments tab to find the correct problem set, then with the "submit assignment" button, upload the PDF file of your solution.
LATE PROBLEM SETS WILL NOT BE ACCEPTED.
See the syllabus for the collaboration policy.

PROBLEM 1 (4+4+4 points, suggested length of 1 page)

Indicate whether each of the following languages is decidable. If it is decidable, give a **high-level**[1] (but complete) description of a Turing Machine that decides it. Otherwise, prove it is not decidable by contradiction: show that if a decider M_L existed for it, one could use M_L to construct a new decider M'_L that decides a problem we have proven to, in fact, be undecidable (such as HALT_{TM}). You do not need to define the mapping reduction function f for this problem.

In all cases, let T be a deterministic one-tape TM.

(A) $L = \{\langle T \rangle : T$ takes more than 53 steps on at least one input.$\}$

(B) $L = \{\langle T \rangle : T$ takes more than 77 steps on at least one input accepted by T.$\}$

(C) $L = \{\langle T \rangle : T$ takes more than 824 steps on all inputs accepted by T.$\}$

PROBLEM 2 (3+3 points, suggested length of 1/2 a page)

In the near future you're working as an engineer at Google/Microsoft/Facebook when your manager asks you to write the following two programs. Is this a problem? Provide a proof for your argument.

(A) Take another program's code as input and decide if that program is implemented with the shortest amount of code possible.

(B) Take another program's code and remove all inaccessible (dead) code from it.

PROBLEM 3 (4 points, suggested length of 1/3 a page)

Let $A = \{\langle R, S \rangle : R$ and S are regular expressions and $L(R) \subseteq L(S)\ \}$. Show that A is decidable by giving a high level description.

[1] See Piazza for an explanation of what a high-level description of a TM entails.

PROBLEM 4 (6 points, suggested length of 1/3 page)

Prove that any decidable language L is mapping reducible to any non-trivial decidable language L'. (Any decidable language other than Σ^*, $\emptyset$)

PROBLEM 5 (6 points, suggested length of 1/3 page)

For any Turing Machine M, the language accepted by M, $L(M)$ is always recursively enumerable. Let M_F be the machine created by flipping the accept and reject states of M. Prove that $L(M_F)$ is not always recursive whenever $L(M)$ is recursive.

PROBLEM 6 (6 points, suggested length of 1/3 page)

A computation of a Turing Machine *loops* if it repeats a configuration, that is, re-enters the same state with the identical tape and the head position. Prove that if every non-halting computation of M loops, then $L(M)$ is decidable. In particular, you need to give a high level description of a Turing machine that decides $L(M)$, and provide a short justification.