Harvard University
Computer Science 121

Problem Set 2

Due September 29, 2015 11:59pm

Problem set by **FILL IN YOUR NAME HERE**

Collaboration Statement: **FILL IN YOUR COLLABORATION STATEMENT HERE
(See the syllabus for information)**

PART A (Graded by Juan, Sam)

PROBLEM 1 (1+1+1+1+1+1+1 points, suggested length of 3/4 page)

Determine the cardinality of each of the following sets (countably infinite, uncountably infinite, finite). If finite determine number of elements. Briefly justify each of your answers. You can assume $\Sigma = \{a, b\}$.

(A) $\{D : D \text{ is a DFA that accepts the string } ab\}$

(B) Set of all languages over the alphabet $\{a\}$

(C) $\{P(\Sigma^*)\}$

(D) $\emptyset \times \mathbb{N}$

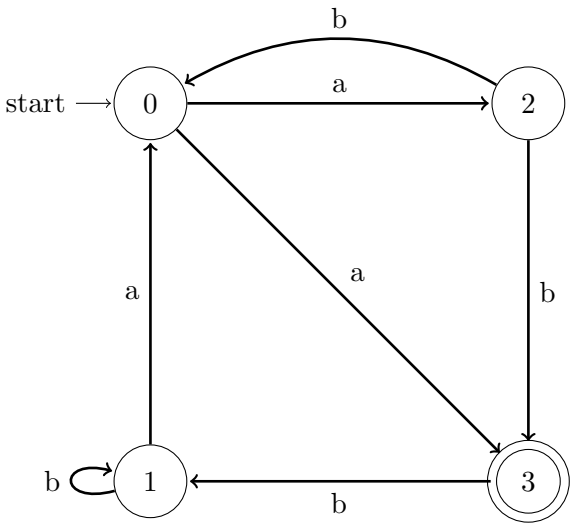
(E) Set of all syntactically correct OCaml programs.

(F) $P(\Sigma^*) \setminus REG$ where $REG = \{L : L \text{ is a regular language}\}$

(G) $REG \setminus P(\Sigma^*)$
Solution.

PROBLEM 2 (6 points, suggested length of 1/4 a page)

Convert the following NFA into a DFA using subset construction. Provide a formal description (5-tuple) and diagram for full credit.



Solution.

PROBLEM 3 (3+2+3 points, suggested length of 3/4 page)

(A) Prove that if A is an uncountable set and B is a countable set, then $A \setminus B$ is uncountable.

(B) Find 2 examples: one where the intersection of two countably infinite sets is finite and another where is countably infinite.

(C) Find 3 examples: one where the intersection of two uncountably infinite sets is finite, another where it is countably infinite, and another where it is uncountable.
Solution.

PART B (Graded by Cecilia and Charles)

PROBLEM 4 (each part 1 points, suggested length of 1/2 page)

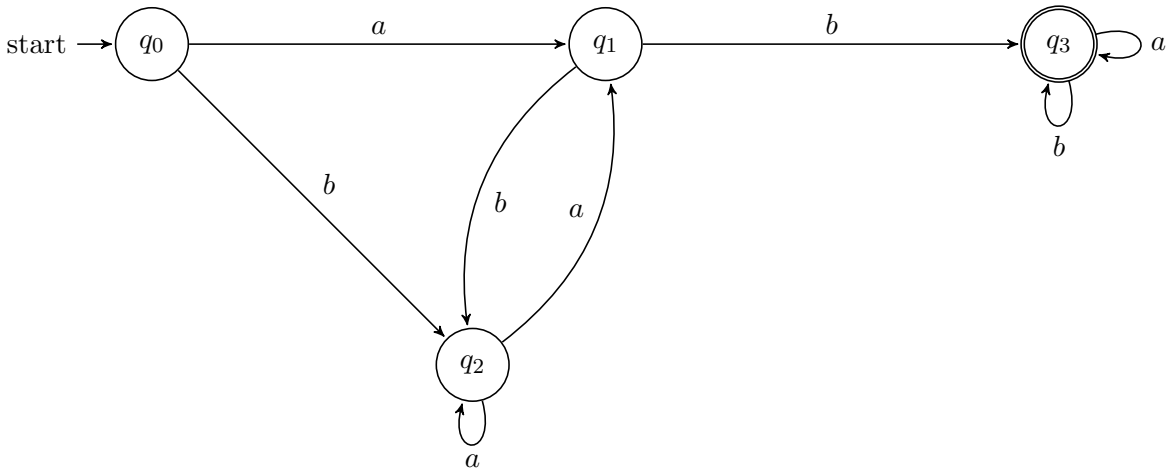
Translate the following languages over $\Sigma = \{a, b\}$ from English description to regular expressions, or vice versa.
(A) $L = \{w \in \Sigma^* : \text{the second letter of } w \text{ is the same as the last letter}, |w| \geq 2\}$
(B) $L = \{w \in \Sigma^* : \text{every } a \text{ is followed by an odd number of } b\text{'s}\}$
(C) $(a(a \cup b)^*b) \cup (b(a \cup b)^*a)$
(D) $L = \{w \in \Sigma^* : w \text{ has at least two } b\text{'s and at most one } a\}$
(E) $L = \{w \in \Sigma^* : \text{every } b \text{ in } w \text{ has a } b \text{ directly before it}\}$
(F) $(b^* \cup ba)^*$

Solution.

PROBLEM 5 (9 points, suggested length of 1 page)

Using the formal construction from the lecture notes or Sipser, convert the shown NFA to a regular expression. Please remove only one state per step. Show all steps or give explanation for steps not shown.

Note: There is no need to add an additional final state and add epsilon transitions to that state. If you are texing your solutions, you can just copy paste the NFA from the problem statement below and modify it. You can also use http://madebyevan.com/fsm/.



Solution.