

COSC 4364 Spring 2016

Assignment 5

Due date: March 10. e-mail your answers and code. Hardcopy appreciated

Points				
Problem	a)	b)	c)	Total
1	2	2	2	6
2	12			12
3	2			2
4	8			8
5	16			16
6	8			8
7	2	2		4
8	8			8
9	20	20		40
Total				104

Problem 1. (3 x 2 points) Are the functions below quadratic splines? Explain why or why not.

a)

$$Q(x) = \begin{cases} 0.1x^2 & (0 \leq x \leq 1) \\ 9.3x^2 - 18.4x + 9.2 & (1 \leq x \leq 1.3) \end{cases}$$

b)

$$Q(x) = \begin{cases} -x^2 & (-100 \leq x \leq 0) \\ x & (0 \leq x \leq 100) \end{cases}$$

c)

$$Q(x) = \begin{cases} x & (-50 \leq x \leq 1) \\ x^2 & (1 \leq x \leq 2) \\ 4 & (2 \leq x \leq 50) \end{cases}$$

Problem 2. (12 points) Find by hand-calculation a quadratic spline interpolant for

x	-1	0	½	1	2	5/2
y	2	1	0	1	2	3

assuming $z_0=0$

Problem 3. (2 points) Is $|x|$ a first degree spline? Explain why or why not.

Problem 4. (8 points) Do there exist a, b, c and d such that

$$S(x) = \begin{cases} ax^3 + x^2 + cx & (-1 \leq x \leq 0) \\ bx^3 + x^2 + dx & (0 \leq x \leq 1) \end{cases}$$

is a natural cubic spline that interpolates $f(x) = |x|$ at the knots $-1, 0, 1$? Prove your answer.

Problem 5. (16 points) Find by hand-calculation the natural cubic spline interpolant for

x	1	2	3	4	5
y	0	1	0	1	0

Problem 6. (8 points) Find an expression for $B_i^2(x)$ and verify that it is piecewise quadratic. Show that $B_i^2(x)$ is 0 at every knot except

$$B_i^2(t_{i+1}) = (t_{i+1} - t_i) / (t_{i+2} - t_i) \text{ and } B_i^2(t_{i+2}) = (t_{i+3} - t_{i+2}) / (t_{i+3} - t_{i+1})$$

Problem 7. (2 x 2 points)

- Establish that $\sum_{i=-\infty}^{i=\infty} f(t_i) B_{i-1}^1(x)$ is a first degree spline that interpolates f at every knot.
- What is the zero-degree spline that interpolates f at every knot?

Problem 8. (8 points) Show that if $S(x) = \sum_{i=-\infty}^{i=\infty} A_i B_{i-2}^2(x)$ and $t_{j-1} \leq x \leq t_j$ then

$$\begin{aligned} S(x) &= [d(x - t_{j-1}) + e(t_j - x)] / (t_j - t_{j-1}) \text{ with} \\ d &= [A_{j+1}(x - t_{j-1}) + A_j(t_{j+1} - x)] / (t_{j+1} - t_{j-1}) \\ e &= [A_j(x - t_{j-2}) + A_{j-1}(t_j - x)] / (t_j - t_{j-2}) \end{aligned}$$

Problem 9. (2x20 points) Matlab programming. Let $S(x)$ be

- a quadratic spline (Section 6.1 in the book)
- a quadratic B-Spline (Section 6.3 in the book)

that interpolates $f(x) = 1/(1+(5x)^2)$ on the interval at 41 equally spaced in the interval $[-1, 1]$.

Evaluate $f(x) - S(x)$ at 200 equally spaced x values with $x_1 = -1$ and $x_{200} = 1$ and plot the error $f(x) - p(x)$.

What is the maximum positive error?

What is the maximum negative error?

What is the square root of the mean square error? ($\sqrt{\sum_{i=1}^{200} (f(x_i) - p(x_i))^2 / 200}$)

You should compare your results to those of Assignment 3 Problem 11.