COSC 4364 Spring 2017

Assignment 9

Out: April 18. Due: April 27. e-mail your report and code.

	Points				
Problem	a)	b)	c)	d)	Total
1	3				3
2	3				3
3	3				3
4	6				6
5	3	6			9
6	5				5
7	30				30
8	40				40
9	30				30
Total					129

Problem 1. (3 points) Vector norms are used to create so-called subordinate matrix norms. Which of the following conditions, if any, is satisfied by every subordinate matrix norm?

a. $||\mathbf{A}x|| \geq ||\mathbf{A}||\ ||x||$ b. $||\mathbf{I}||=1$. c. $||\mathbf{AB}|| \geq ||\mathbf{A}||\ ||\mathbf{B}||$ d. $||\mathbf{A}+\mathbf{B}|| \geq ||\mathbf{A}||+||\mathbf{B}||$
e. Non of these

Problem 2. (3 points) Which of the following conditions, if any, defines diagonal dominance of a matrix A?

a. $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|$ b. $|a_{ii}| \geq \sum_{j=1,j\neq I}^{n} |a_{ij}|$ c. $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ji}|$ d. $|a_{ii}| \geq \sum_{j=1,j\neq i}^{n} |a_{ji}|$ e. None of these

Problem 3. (3 points) A general procedure for solving a system of equations $(\mathbf{I}-\mathcal{G})\mathbf{x}=\mathbf{b}$ by an iterative method is to form an iteration formula $\mathbf{x}^{k+1} = \mathcal{G}\mathbf{x}^{k}+\mathbf{b}$. What is a necessary and sufficient condition that guarantees that the sequence $\mathbf{x}^{k}$ converges to the solution to $(\mathbf{I}-\mathcal{G})\mathbf{x}=\mathbf{b}$?

a. $\mathcal{G}$ is diagonally dominant. b. $\mathcal{G}$ is diagonally dominant. c. The spectral radius of $\mathcal{G}<1$.
d. None of these

Problem 4. (6 points) If a set of m+1 data points (x_k,y_k) k=0,1,…,m are to be represent by a least squares fit of $y=x^2-x+c$ derive an expression for c in terms of x_k and y_k.

Problem 5. (a) 3 points, b) 6 points) The Gram-Schmidt process is used on the vectors $\mathbf{x}_1 = [2,2,1]^T$, $\mathbf{x}_2 = [1,1,5]^T$ and $\mathbf{x}_3 = [-3,2,1]^T$ to create the orthonormal vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$.

a) Which one of the following vectors is the correct representation of $\mathbf{u}_1$?
   i) $[2/3,2/3,1/3]^T$ ii) $[2,2,1]^T$ iii) $[2/5,2/5,1/5]^T$ iv) None of these
b) Which one of the following vectors is the correct representation of $\mathbf{u}_2$?
   i) $(1/\text{sqrt}(27))[1,1,5]^T$ ii) $(1/\text{sqrt}(18))[-1,-1,4]^T$ iii) $[1,1,-4]^T$ iv) None of these

Problem 6. (5 points) A set of data points (x_k,y_k) are to be approximated by $y=1/(a+bx)$. A direct use of the least squares method results in a non-linear problem. How would you reformulate the problem to apply the least squares method?

Matlab problems

Problem 7. (Exercise 8.4.11) (30 points) Write your own Matlab code for the conjugate gradient method to solve the system Ax=b where A and b are as below

$$A = \begin{bmatrix} 3 & -1 & & & & & & & \frac{1}{2} \\ -1 & 3 & -1 & & & & & \frac{1}{2} & \\ & -1 & 3 & -1 & & & \frac{1}{2} & & \\ & & \ddots & \ddots & \ddots & & & & \\ & & & -1 & 3 & -1 & & & \\ & & & & \ddots & \ddots & \ddots & & \\ & & \frac{1}{2} & & & -1 & 3 & -1 & \\ & \frac{1}{2} & & & & & -1 & 3 & -1 \\ \frac{1}{2} & & & & & & & -1 & 3 \end{bmatrix} \qquad b = \begin{bmatrix} 2.5 \\ 1.5 \\ 1.5 \\ \vdots \\ 1.0 \\ 1.5 \\ \vdots \\ 1.5 \\ 2.5 \end{bmatrix}$$

(diag(A) = 3, 1st superdiag =-1, 1st subdiagonal =-1, antidiagonal=1/2 where it does not conflict with the diagonal and 1st super and subdiagonal. True solution is $x=(1,1,\ldots\ldots,1)^T$). For increasing values of the size *n* of the matrix compute the root mean square (RMS) error in your conjugate gradient solution by comparing to the true solution. Make a table and plot of how the number of iterations for convergence depend on *n* for an RMS error of 10^{-7}.

Problem 8. (Exercise 9.2.1). (40 points)
Step 1 - generating a noisy data set . For a 7th degree polynomial of your choice on the interval [-1,1] for 100 randomly selected values in [-1,1] compute the corresponding polynomial values. Then, find the maximum absolute value of the polynomial points, p_{max}. Then, perturb each polynomial value by randomly adding a value from $[-0.1p_{max},0.1p_{max}]$.

Step 2. Using the data set from step 1 find a least squares fit using the polynomial regression algorithm described on pages 440-443 in the book. Progressively specify an error tolerance 10^{-k} for k=3, 4, 5, 6, and 7. Report the polynomial you are getting for each error tolerance. Plot the polynomial you used to generate the noisy data and the polynomials generated for each error tolerance.

Problem 9. (Exercise 9.3.4) (30 points) The world population is given by the following table.

Year	Population (billions)
1000	0.340
1650	0.545
1800	0.907
1900	1.61
1950	2.56
1960	3.15
1970	3.65
1980	4.30
1990	5.30
2000	6.12
2010	6.98

The function $f(x) = (a+bx)/(1+cx)$ is perceived as providing a good representation of the table data for proper values of a, b and c. To be able to find those values consider the function $(1+cx)f(x) = a+bx$ and apply the lest squares method to find a, b, and c by forming an 11x3 matrix A such that $A[a,b,c]^T$ approximates $f(x)$. Use the SVD method from Matlab to find a, b, and c.