第三章　　不定积分

利用最简积分表，求下列积分：

1628. $\int \left(3-x^2\right)^3 dx$.

1629. $\int x^2(5-x)^4 dx$.

1630. $\int (1-x)(1-2x)(1-3x)dx$.

1631. $\int \left(\dfrac{1-x}{x}\right)^2 dx$.

1632. $\int \left(\dfrac{a}{x}+\dfrac{a^2}{x^2}+\dfrac{a^3}{x^3}\right)dx$.

1633. $\int \dfrac{x+1}{\sqrt{x}}dx$.

1634. $\int \dfrac{\sqrt{x}-2\sqrt[3]{x^2}+1}{\sqrt[4]{x}}dx$.

1635. $\int \dfrac{\left(1-x\right)^3}{x\sqrt[3]{x}}dx$.

1636. $\int \left(1-\dfrac{1}{x^2}\right)\sqrt{x\sqrt{x}}\,dx$.

1637. $\int \dfrac{\left(\sqrt{2x}-\sqrt[3]{3x}\right)^2}{x}\,dx$.

1638. $\int \dfrac{\sqrt{x^4+x^{-4}+2}}{x^3}dx$.

1639. $\int \dfrac{x^2\,dx}{1+x^2}$.

1640. $\int \dfrac{x^2\,dx}{1-x^2}$.

1641. $\int \dfrac{x^2+3}{x^2-1}dx$.

1642. $\int \dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1-x^4}}dx$.

1643. $\int \dfrac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}dx$.

1644. $\int \left(2^x+3^x\right)^2 dx$.

1645. $\int \dfrac{2^{x+1}-5^{x-1}}{10^x}dx$.

1646. $\int \dfrac{e^{3x}+1}{e^x+1}dx$.

1647. $\int (1+\sin x+\cos x)dx$.

1648. $\int \sqrt{1-\sin 2x}\,dx\,(0\leqslant x\leqslant \pi)$.

1649. $\int \cot^2 x dx$.

1650. $\int \tan^2 x dx$.

1651. $\int (a\sinh x+b\cosh x)dx$.

1652. $\int \tanh^2 x dx$.

1653. $\int \coth^2 x dx$.

1654. 证明：若

$$\int f(x)\mathrm{d}x = F(x)+C\ ,$$

则

$$\int f(ax+b)\mathrm{d}x = \frac{1}{a}F(ax+b)+C \quad (a\neq 0)\ .$$

求下列积分：

1655. $\int \dfrac{dx}{x+a}$.

1656. $\int (2x-3)^{10}\mathrm{d}x$.

1657. $\int \sqrt[3]{1-3x}\,\mathrm{d}x$.

1658. $\int \dfrac{dx}{\sqrt{2-5x}}$.

1659. $\int \dfrac{dx}{(5x-2)^{\frac{5}{2}}}$.

1660. $\int \dfrac{\sqrt[5]{1-2x+x^2}}{1-x}dx$.

1661. $\int \dfrac{dx}{2+3x^2}$.

1662. $\int \dfrac{dx}{2-3x^2}$.

1663. $\int \dfrac{dx}{\sqrt{2-3x^2}}$.

1664. $\int \dfrac{dx}{\sqrt{3x^2-2}}$.

1665. $\int \left(e^{-x} + e^{-2x} \right) dx$.

1666. $\int \left(\sin 5x - \sin 5\alpha \right) dx$.

1667. $\int \dfrac{dx}{\sin^2 \left(2x + \dfrac{\pi}{4} \right)}$.

1668. $\int \dfrac{dx}{1 + \cos x}$.

1669. $\int \dfrac{dx}{1 - \cos x}$.

1670. $\int \dfrac{dx}{1 + \sin x}$.

1671. $\int \left[\sinh(2x + 1) + \cosh(2x - 1) \right] dx$.

1672. $\int \dfrac{dx}{\cosh^2 \dfrac{x}{2}}$.

1673. $\int \dfrac{dx}{\sinh^2 \dfrac{x}{2}}$.

用适当变换被积函数的方法求下列积分：

1674. $\int \dfrac{x dx}{\sqrt{1 - x^2}}$.

1675. $\int x^2 \sqrt[3]{1 + x^3} \, dx$.

1676. $\int \dfrac{x dx}{3 - 2x^2}$.

1677. $\int \dfrac{x dx}{\left(1 + x^2 \right)^2}$.

1678. $\int \dfrac{x dx}{4 + x^2}$.

1679. $\int \dfrac{x^3 dx}{x^8 - 2}$.

1680. $\int \dfrac{dx}{\left(1 + x \right) \sqrt{x}}$.　　　提示：$\dfrac{dx}{\sqrt{x}} = 2 d\sqrt{x}$.

1681. $\int \sin \dfrac{1}{x} \cdot \dfrac{dx}{x^2}$.

1682. $\int \dfrac{dx}{x \sqrt{x^2 + 1}}$.

1683. $\int \dfrac{dx}{x \sqrt{x^2 - 1}}$.

1684. $\int \dfrac{dx}{\left(x^2 + 1 \right)^{\frac{3}{2}}}$.

1685. $\int \dfrac{x dx}{\left(x^2 - 1 \right)^{\frac{3}{2}}}$.

1686. $\int \dfrac{x^2 dx}{\left(8x^3 + 27 \right)^{\frac{2}{3}}}$.

1687. $\int \dfrac{dx}{\sqrt{x(1 + x)}}$.

1688. $\int \dfrac{dx}{\sqrt{x(1 - x)}}$.

1689. $\int x e^{-x^2} dx$.

1690. $\int \dfrac{e^x dx}{2 + e^x}$.

1691. $\int \dfrac{dx}{e^x + e^{-x}}$.

1692. $\int \dfrac{dx}{\sqrt{1 + e^{2x}}}$.

1693. $\int \dfrac{\ln^2 x}{x} dx$.

1694. $\int \dfrac{dx}{x \ln x \ln(\ln x)}$.

1695. $\int \sin^5 x \cos x dx$.

1696. $\int \dfrac{\sin x}{\sqrt{\cos^3 x}} dx$.

1697. $\int \tan x dx$.

1698. $\int \cot x dx$.

1699. $\int \dfrac{\sin x + \cos x}{\sqrt[3]{\sin x - \cos x}} dx$.

1700. (a) $\int \dfrac{\sin x \cos x}{\sqrt{a^2 \sin^2 x + b^2 \cos^2 x}} dx$;

(b) $\int \dfrac{\sin x}{\sqrt{\cos 2x}} dx$;

(c) $\int \dfrac{\cos x}{\sqrt{\cos 2x}} dx$;

(d) $\int \dfrac{\sinh x}{\sqrt{\cosh 2x}} dx$.

1701. $\int \dfrac{dx}{\sin^2 x \sqrt[4]{\cot x}}$.

1702. $\int \dfrac{dx}{\sin^2 x + 2 \cos^2 x}$.

1703. $\int \dfrac{dx}{\sin x}$.

1704. $\int \dfrac{dx}{\cos x}$.

1705. $\int \dfrac{dx}{\sinh x}$.

1706. $\int \dfrac{dx}{\cosh x}$.

1707. $\int \dfrac{\sinh x \cosh x}{\sqrt{\sinh^4 x + \cosh^4 x}} dx$.

1708. $\int \dfrac{dx}{\cosh^2 x \sqrt[3]{\tanh^2 x}}$.

1709. $\int \dfrac{\arctan x}{1 + x^2} dx$.

1710. $\int \dfrac{dx}{(\arcsin x)^2 \sqrt{1 - x^2}}$.

1711. $\int \sqrt{\dfrac{\ln\left(x + \sqrt{1 + x^2}\right)}{1 + x^2}} dx$.

1712. $\int \dfrac{x^2 + 1}{x^4 + 1} dx$.   提示：$\left(1 + \dfrac{1}{x^2}\right)dx = d\left(x - \dfrac{1}{x}\right)$.

1713. $\int \dfrac{x^2 - 1}{x^4 + 1} dx$.

1714. $\int \dfrac{x^4 dx}{\left(x^5 + 1\right)^4}$.

1715. $\int \dfrac{x^{\frac{n}{2}} dx}{\sqrt{1 + x^{n+2}}}$.

1716. $\int \dfrac{1}{1 - x^2} \ln \dfrac{1 + x}{1 - x} dx$.

1717. $\int \dfrac{\cos x \, dx}{\sqrt{2 + \cos 2x}}$.

1718. $\int \dfrac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx$.

1719. $\int \dfrac{2^x \cdot 3^x}{9^x - 4^x} dx$.

1720. $\int \dfrac{x \, dx}{\sqrt{1 + x^2 + \sqrt{\left(1 + x^2\right)^3}}}$.

用分项积分法计算下列积分：

1721. (a) $\int x^2 \left(2 - 3x^2\right)^2 dx$;

(b) $\int x(1 - x)^{10} dx$.

1722. $\int \dfrac{1 + x}{1 - x} dx$.

1723. $\int \dfrac{x^2}{1 + x} dx$.

1724. $\int \dfrac{x^3}{3 + x} dx$.

1725. $\int \dfrac{\left(1 + x\right)^2}{1 + x^2} dx$.

1726. $\int \dfrac{\left(2 - x\right)^2}{2 - x^2} dx$.

1727. $\int \dfrac{x^2}{\left(1 - x\right)^{100}} dx$.

1728. $\int \dfrac{x^5}{x + 1} dx$.

1729. $\int \dfrac{dx}{\sqrt{x + 1} + \sqrt{x - 1}}$.

1730. $\int x\sqrt{2 - 5x} \, dx$.   提示：$x \equiv -\dfrac{1}{5}(2 - 5x) + \dfrac{2}{5}$.

1731. $\int \dfrac{x \, dx}{\sqrt[3]{1 - 3x}}$.

1732. $\int x^3 \sqrt[3]{1 + x^2} \, dx$.

1733. $\int \dfrac{dx}{(x - 1)(x + 3)}$.   提示：$1 \equiv \dfrac{1}{4}\left[(x + 3) - (x - 1)\right]$.

1734. $\int \dfrac{dx}{x^2 + x - 2}$.

1735. $\int \dfrac{dx}{\left(x^2 + 1\right)\left(x^2 + 2\right)}$.

1736. $\int \dfrac{dx}{\left(x^2 - 2\right)\left(x^2 + 3\right)}$.

1737. $\int \dfrac{x \, dx}{(x + 2)(x + 3)}$.

1738. $\int \dfrac{x \, dx}{x^4 + 3x^2 + 2}$.

1739. $\int \dfrac{dx}{\left(x + a\right)^2 \left(x + b\right)^2}$ $(a \neq b)$.

1740. $\int \dfrac{dx}{\left(x^2 + a^2\right)\left(x^2 + b^2\right)}$ $\left(a^2 \neq b^2\right)$.

1741. $\int \sin^2 x \, dx$.

1742. $\int \cos^2 x \, dx$.

1743. $\int \sin x \sin(x + \alpha) dx$.

1744. $\int \sin 3x \sin 5x \mathrm{d}x$.

1745. $\int \cos \dfrac{x}{2} \cos \dfrac{x}{3} \mathrm{d}x$.

1746. $\int \sin\left(2x - \dfrac{\pi}{6}\right) \cos\left(3x + \dfrac{\pi}{4}\right) \mathrm{d}x$.

1747. $\int \sin^3 x \mathrm{d}x$.

1748. $\int \cos^3 x \mathrm{d}x$.

1749. $\int \sin^4 x \mathrm{d}x$.

1750. $\int \cos^4 x \mathrm{d}x$.

1751. $\int \cot^2 x \mathrm{d}x$.

1752. $\int \tan^3 x \mathrm{d}x$.

1753. $\int \sin^2 3x \sin^3 2x \mathrm{d}x$.

1754. $\int \dfrac{dx}{\sin^2 x \cos^2 x}$.   提示：$1 \equiv \sin^2 x + \cos^2 x$.

1755. $\int \dfrac{dx}{\sin^2 x \cos x}$.

1756. $\int \dfrac{dx}{\sin x \cos^3 x}$.

1757. $\int \dfrac{\cos^3 x}{\sin x} \mathrm{d}x$.

1758. $\int \dfrac{dx}{\cos^4 x}$.

1759. $\int \dfrac{dx}{1 + e^x}$.

1760. $\int \dfrac{\left(1 + e^x\right)^2}{1 + e^{2x}} \mathrm{d}x$.

1761. $\int \sinh^2 x \mathrm{d}x$.

1762. $\int \cosh^2 x \mathrm{d}x$.

1763. $\int \sinh x \sinh 2x \mathrm{d}x$.

1764. $\int \cosh x \cosh 3x \mathrm{d}x$.

1765. $\int \dfrac{dx}{\sinh^2 x \cosh^2 x}$.

用适当的代换求下列积分：

1766. $\int x^2 \sqrt[3]{1 - x}\, \mathrm{d}x$.

1767. $\int x^3 \left(1 - 5x^2\right)^{10} \mathrm{d}x$

1768. $\int \dfrac{x^2}{\sqrt{2 - x}} \mathrm{d}x$.

1769. $\int \dfrac{x^5}{\sqrt{1 - x^2}} \mathrm{d}x$.

1770. $\int x^5 \left(2 - 5x^3\right)^{\frac{2}{3}} \mathrm{d}x$.

1771. $\int \cos^5 x \sqrt{\sin x}\, \mathrm{d}x$.

1772. $\int \dfrac{\sin x \cos^3 x}{1 + \cos^2 x} dx$.

1773. $\int \dfrac{\sin^2 x}{\cos^6 x} \mathrm{d}x$.

1774. $\int \dfrac{\ln x \, dx}{x \sqrt{1 + \ln x}}$.

1775. $\int \dfrac{dx}{e^{\frac{x}{2}} + e^x}$.

1776. $\int \dfrac{dx}{\sqrt{1 + e^x}}$.

1777. $\int \dfrac{\arctan \sqrt{x}}{\sqrt{x}} \cdot \dfrac{dx}{1 + x}$.

利用 $x = a \sin t$ ，$x = a \tan t$ ，$x = a \sin^2 t$ 等三角函数代换，求下列积分（参数为正）：

1778. $\int \dfrac{dx}{\left(1 - x^2\right)^{\frac{3}{2}}}$.

1779. $\int \dfrac{x^2 \, dx}{\sqrt{x^2 - 2}}$.

1780. $\int \sqrt{1 - x^2}\, \mathrm{d}x$.

1781. $\int \dfrac{dx}{\left(x^2 + a^2\right)^{\frac{3}{2}}}$.

1782. $\int \sqrt{\dfrac{a + x}{a - x}}\, \mathrm{d}x$.

1783. $\int x \sqrt{\dfrac{x}{2a - x}}\, \mathrm{d}x$.

1784. $\int \dfrac{dx}{\sqrt{(x - a)(b - x)}}$.   提示：利用代换 $x - a = (b - a) \sin^2 t$.

1785. $\int \sqrt{(x-a)(b-x)} \, dx$.

利用 $x = a \sinh t$ ，$x = a \cosh t$ 等双曲函数代换，求下列积分(参数为正)：

1786. $\int \sqrt{a^2 + x^2} \, dx$.

1787. $\int \dfrac{x^2 \, dx}{\sqrt{a^2 + x^2}}$.

1788. $\int \sqrt{\dfrac{x-a}{x+a}} \, dx$.

1789. $\int \dfrac{dx}{\sqrt{(x+a)(x+b)}}$.   提示：令 $x + a = (b-a)\sinh^2 t$.

1790. $\int \sqrt{(x+a)(x+b)} \, dx$.

用分部积分法求下列积分：

1791. $\int \ln x \, dx$.

1792. $\int x^n \ln x \, dx \, (n \neq -1)$.

1793. $\int \left(\dfrac{\ln x}{x} \right)^2 dx$.

1794. $\int \sqrt{x} \ln^2 x \, dx$.

1795. $\int x e^{-x} \, dx$.

1796. $\int x^2 e^{-2x} \, dx$.

1797. $\int x^3 e^{-x^2} \, dx$.

1798. $\int x \cos x \, dx$.

1799. $\int x^2 \sin 2x \, dx$.

1800. $\int x \sinh x \, dx$.

1801. $\int x^3 \cosh 3x \, dx$.

1802. $\int \arctan x \, dx$.

1803. $\int \arcsin x \, dx$.

1804. $\int x \arctan x \, dx$.

1805. $\int x^2 \arccos x \, dx$.

1806. $\int \dfrac{\arcsin x}{x^2} \, dx$.

1807. $\int \ln \left(x + \sqrt{1+x^2} \right) dx$.

1808. $\int x \ln \dfrac{1+x}{1-x} \, dx$.

1809. $\int \arctan \sqrt{x} \, dx$.

1810. $\int \sin x \ln(\tan x) \, dx$.

求下列积分：

1811. $\int x^5 e^{x^3} \, dx$.

1812. $\int (\arcsin x)^2 \, dx$.

1813. $\int x (\arctan x)^2 \, dx$.

1814. $\int x^2 \ln \dfrac{1-x}{1+x} \, dx$.

1815. $\int \dfrac{x \ln \left(x + \sqrt{1+x^2} \right)}{\sqrt{1+x^2}} \, dx$.

1816. $\int \dfrac{x^2}{\left(1+x^2 \right)^2} \, dx$.

1817. $\int \dfrac{dx}{\left(a^2 + x^2\right)^2}$.

1818. $\int \sqrt{a^2 - x^2}\, dx$.

1819. $\int \sqrt{x^2 + a}\, dx$.

1820. $\int x^2 \sqrt{a^2 + x^2}\, dx$.

1821. $\int x \sin^2 x\, dx$.

1822. $\int e^{\sqrt{x}}\, dx$.

1823. $\int x \sin \sqrt{x}\, dx$.

1824. $\int \dfrac{x e^{\arctan x}}{\left(1 + x^2\right)^{\frac{3}{2}}}\, dx$.

1825. $\int \dfrac{e^{\arctan x}}{\left(1 + x^2\right)^{\frac{3}{2}}}\, dx$.

1826. $\int \sin(\ln x)\, dx$.

1827. $\int \cos(\ln x)\, dx$.

1828. $\int e^{ax} \cos bx\, dx$.

1829. $\int e^{ax} \sin bx\, dx$.

1830. $\int e^{2x} \sin^2 x\, dx$.

1831. $\int \left(e^x - \cos x\right)^2 dx$.

1832. $\int \dfrac{\operatorname{arccot} e^x}{e^x}\, dx$.

1833. $\int \dfrac{\ln(\sin x)}{\sin^2 x}\, dx$.

1834. $\int \dfrac{x\, dx}{\cos^2 x}$.

1835. $\int \dfrac{x e^x}{\left(x + 1\right)^2}\, dx$.

在求下面的积分时，需要把二次三项式化成标准形式，并利用下列公式：

Ⅰ． $\int \dfrac{dx}{a^2 + x^2} = \dfrac{1}{a} \arctan \dfrac{x}{a} + C \ (a \neq 0)$.

Ⅱ． $\int \dfrac{dx}{a^2 - x^2} = \dfrac{1}{2a} \ln\left|\dfrac{a + x}{a - x}\right| + C \ (a \neq 0)$.

Ⅲ． $\int \dfrac{x\, dx}{a^2 \pm x^2} = \pm \dfrac{1}{2} \ln\left|a^2 \pm x^2\right| + C$.

Ⅳ． $\int \dfrac{dx}{\sqrt{a^2 - x^2}} = \arcsin \dfrac{x}{a} + C \ (a > 0)$.

Ⅴ． $\int \dfrac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \ (a > 0)$.

Ⅵ． $\int \dfrac{x\, dx}{\sqrt{a^2 \pm x^2}} = \pm \sqrt{a^2 \pm x^2} + C \ (a > 0)$.

Ⅶ． $\int \sqrt{a^2 - x^2}\, dx = \dfrac{x}{2} \sqrt{a^2 - x^2} + \dfrac{a^2}{2} \arcsin \dfrac{x}{a} + C \ (a > 0)$.

Ⅷ． $\int \sqrt{x^2 \pm a^2}\, dx = \dfrac{x}{2} \sqrt{x^2 \pm a^2} \pm \dfrac{a^2}{2} \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \ (a > 0)$.

求下列积分：

1836. $\int \dfrac{dx}{a+bx^2}\ (ab\neq 0)$.

1837. $\int \dfrac{dx}{x^2-x+2}$.

1838. $\int \dfrac{dx}{3x^2-2x-1}$.

1839. $\int \dfrac{xdx}{x^4-2x^2-1}$.

1840. $\int \dfrac{x+1}{x^2+x+1}\mathrm{d}x$.

1841. $\int \dfrac{xdx}{x^2-2x\cos\alpha+1}$.

1842. $\int \dfrac{x^3dx}{x^4-x^2+2}$.

1843. $\int \dfrac{x^5dx}{x^6-x^3-2}$.

1844. $\int \dfrac{dx}{3\sin^2 x-8\sin x\cos x+5\cos^2 x}$.

1845. $\int \dfrac{dx}{\sin x+2\cos x+3}$.

1846. $\int \dfrac{dx}{\sqrt{a+bx^2}}\ (b\neq 0)$.

1847. $\int \dfrac{dx}{\sqrt{1-2x-x^2}}$.

1848. $\int \dfrac{dx}{\sqrt{x+x^2}}$.

1849. $\int \dfrac{dx}{\sqrt{2x^2-x+2}}$.

1850. 证明：若

$$y=ax^2+bx+c\left(a\neq 0\right),$$

则

$$\int \dfrac{dx}{\sqrt{y}}=\begin{cases}\dfrac{1}{\sqrt{a}}\ln\left|\dfrac{y'}{2}+\sqrt{ay}\right|+C, & a>0,\\[3mm] \dfrac{1}{\sqrt{-a}}\arcsin\dfrac{-y'}{\sqrt{b^2-4ac}}+C, & a<0.\end{cases}$$

1851. $\int \dfrac{xdx}{\sqrt{5+x-x^2}}$.

1852. $\int \dfrac{x+1}{\sqrt{x^2+x+1}}\mathrm{d}x$.

1853. (a) $\int \dfrac{xdx}{\sqrt{1-3x^2-2x^4}}$.

(b) $\int \dfrac{\cos xdx}{\sqrt{1+\sin x+\cos^2 x}}$.

1854. $\int \dfrac{x^3dx}{\sqrt{x^4-2x^2-1}}$.

1855. $\int \dfrac{x+x^3}{\sqrt{1+x^2-x^4}}\mathrm{d}x$.

1856. $\int \dfrac{dx}{x\sqrt{x^2+x+1}}$.

1857. $\int \dfrac{dx}{x^2\sqrt{x^2+x-1}}$.

1858. $\int \dfrac{dx}{(x+1)\sqrt{x^2+1}}$.

1859. $\int \dfrac{dx}{(x-1)\sqrt{x^2-x}}$.

1860. $\int \dfrac{dx}{(x+2)^2\sqrt{x^2+2x-5}}$.

1861. $\int \sqrt{2+x-x^2}\,\mathrm{d}x$.

1862. $\int \sqrt{2+x+x^2}\,\mathrm{d}x$.

1863. $\int \sqrt{x^4+2x^2-1}\,xdx$.

1864. $\int \dfrac{1-x+x^2}{x\sqrt{1+x-x^2}}\mathrm{d}x$.

1865. $\int \dfrac{x^2+1}{x\sqrt{x^4+1}}\mathrm{d}x$.

利用待定系数法，求下列积分：

1866. $\int \dfrac{2x+3}{(x-2)(x+5)}dx$.

1867. $\int \dfrac{xdx}{(x+1)(x+2)(x+3)}$.

1868. $\int \dfrac{x^{10}dx}{x^2+x-2}$.

1869. $\int \dfrac{x^3+1}{x^3-5x^2+6x}dx$.

1870. $\int \dfrac{x^4}{x^4+5x^2+4}dx$.

1871. $\int \dfrac{xdx}{x^3-3x+2}$.

1872. $\int \dfrac{x^2+1}{(x+1)^2(x-1)}dx$.

1873. $\int \left(\dfrac{x}{x^2-3x+2}\right)^2 dx$.

1874. $\int \dfrac{dx}{(x+1)(x+2)^2(x+3)^3}$.

1875. $\int \dfrac{dx}{x^5+x^4-2x^3-2x^2+x+1}$

1876. $\int \dfrac{x^2+5x+4}{x^2+5x+4}dx$.

1877. $\int \dfrac{dx}{(x+1)(x^2+1)}$.

1878. $\int \dfrac{dx}{(x^2-4x+4)(x^2-4x+5)}$.

1879. $\int \dfrac{xdx}{(x-1)^2(x^2+2x+2)}$.

1880. $\int \dfrac{dx}{x(1+x)(1+x+x^2)}$.

1881. $\int \dfrac{dx}{x^3+1}$.

1882. $\int \dfrac{xdx}{x^3-1}$.

1883. $\int \dfrac{dx}{x^4-1}$.

1884. $\int \dfrac{dx}{x^4+1}$.

1885. $\int \dfrac{dx}{x^4+x^2+1}$.

1886. $\int \dfrac{dx}{x^6+1}$.

1887. $\int \dfrac{dx}{(1+x)(1+x^2)(1+x^3)}$.

1888. $\int \dfrac{dx}{x^5-x^4+x^3-x^2+x-1}$.

1889. $\int \dfrac{x^2dx}{x^4+3x^3+\dfrac{9}{2}x^2+3x+1}$.

1890. 在什么条件下，积分

$$\int \dfrac{ax^2+bx+c}{x^3(x-1)^2}dx$$

为有理函数？

利用奥斯特罗格拉茨基方法求积分：

1891. $\int \dfrac{xdx}{(x-1)^2(x+1)^3}$.

1892. $\int \dfrac{dx}{(x^3+1)^2}$.

1893. $\int \dfrac{dx}{(x^2+1)^3}$.

1894. $\int \dfrac{x^2dx}{(x^2+2x+2)^2}$.

1895. $\int \dfrac{dx}{\left(x^4+1\right)^2}$.

1896. $\int \dfrac{x^2+3x-2}{(x-1)\left(x^2+x+1\right)^2}dx$.

1897. $\int \dfrac{dx}{\left(x^4-1\right)^3}$.

分出下列积分的代数部分：

1898. $\int \dfrac{x^2+1}{\left(x^4+x^2+1\right)^2}dx$.

1899. $\int \dfrac{dx}{\left(x^3+x+1\right)^3}$.

1900. $\int \dfrac{4x^5-1}{\left(x^5+x+1\right)^2}dx$.

1901. 计算积分

$$\int \dfrac{dx}{x^4+2x^3+3x^2+2x+1} .$$

1902. 在什么条件下，积分

$$\int \dfrac{\alpha x^2+2\beta x+\gamma}{\left(ax^2+2bx+c\right)^2}dx$$

为有理函数？

利用不同方法计算下列积分：

1903. $\int \dfrac{x^3}{\left(x-1\right)^{100}}dx$.

1904. $\int \dfrac{xdx}{x^8-1}$.

1905. $\int \dfrac{x^3dx}{x^8+3}$.

1906. $\int \dfrac{x^2+x}{x^6+1}dx$.

1907. $\int \dfrac{x^4-3}{x\left(x^8+3x^4+2\right)}dx$.

1908. $\int \dfrac{x^4dx}{\left(x^{10}-10\right)^2}$.

1909. $\int \dfrac{x^{11}dx}{x^8+3x^4+2}$.

1910. $\int \dfrac{x^9dx}{\left(x^{10}+2x^5+2\right)^2}$.

1911. $\int \dfrac{x^{2n-1}}{x^n+1}dx$.

1912. $\int \dfrac{x^{3n-1}}{\left(x^{2n}+1\right)^2}dx$.

1913. $\int \dfrac{dx}{x\left(x^{10}+2\right)}$.

1914. $\int \dfrac{dx}{x\left(x^{10}+1\right)^2}$.

1915. $\int \dfrac{1-x^7}{x\left(1+x^7\right)}dx$.

1916. $\int \dfrac{x^4-1}{x\left(x^4-5\right)\left(x^5-5x+1\right)}dx$.

1917. $\int \dfrac{x^2+1}{x^4+x^2+1}dx$.

1918. $\int \dfrac{x^2-1}{x^4+x^3+x^2+x+1}dx$.

1919. $\int \dfrac{x^5-x}{x^8+1}dx$.

1920. $\int \dfrac{x^4+1}{x^6+1}dx$.

1921. 试导出用于计算积分

$$I_n=\int \dfrac{dx}{\left(ax^2+bx+c\right)^n} \quad (a\neq 0)$$

的递推公式. 利用这个公式计算

$$I_3=\int \dfrac{dx}{\left(x^2+x+1\right)^3} .$$

提示：利用恒等式 $4a\left(ax^2+bx+c\right)\equiv \left(2ax+b\right)^2+\left(4ac-b^2\right)$.

1922. 利用代换 $t = \dfrac{x+a}{x+b}$ 计算积分

$$I = \int \frac{dx}{(x+a)^m (x+b)^n} \quad (m \text{ 和 } n \text{ 为正整数}).$$

利用这个代换，求

$$\int \frac{dx}{(x-2)^2 (x+3)^3} .$$

1923. 若 $P_n(x)$ 为 x 的 n 次多项式，计算

$$\int \frac{P_n(x)}{(x-a)^{n+1}} dx .$$

提示：利用泰勒公式.

1924. 设 $R(x) = R^*(x^2)$，其中 R^* 为有理函数. 函数 $R(x)$ 分解为有理分式时有什么特点？

1925. 计算

$$\int \frac{dx}{1+x^{2n}} ,$$

式中 n 为正整数.

利用化被积函数为有理函数的方法，求下列积分：

1926. $\displaystyle\int \frac{dx}{1+\sqrt{x}} .$

1927. $\displaystyle\int \frac{dx}{x\left(1 + 2\sqrt{x} + \sqrt[3]{x}\right)} .$

1928. $\displaystyle\int \frac{x\sqrt[3]{2+x}}{x + \sqrt[3]{2+x}} dx .$

1929. $\displaystyle\int \frac{1 - \sqrt{x+1}}{1 + \sqrt[3]{x+1}} dx .$

1930. $\displaystyle\int \frac{dx}{\sqrt{x}\left(1 + \sqrt[4]{x}\right)^3} .$

1931. $\displaystyle\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx .$

1932. $\displaystyle\int \frac{dx}{\sqrt[3]{(x+1)^2 (x-1)^4}} .$

1933. $\displaystyle\int \frac{x\,dx}{\sqrt[4]{x^3(a-x)}} \ (a>0) .$

1934. $\displaystyle\int \frac{dx}{\sqrt[n]{(x-a)^{n+1}(x-b)^{n-1}}}$ (n 为正整数).

1935. $\displaystyle\int \frac{dx}{1 + \sqrt{x} + \sqrt{1+x}} .$ 提示：令 $x = \left(\dfrac{u^2-1}{2u}\right)^2$.

1936. 考虑积分

$$\int R\left[x, (x-a)^{\frac{p}{n}} (x-b)^{\frac{q}{n}}\right] dx ,$$

其中 R 为有理函数，p，q，n 为整数. 证明：若 $p+q = kn$，其中 k 为整数，则该积分为初等函数.

求最简单二次无理式的积分：

1937. $\displaystyle\int \frac{x^2}{\sqrt{1+x+x^2}} dx .$

1938. $\displaystyle\int \frac{dx}{(x+1)\sqrt{x^2+x+1}} .$

1939. $\displaystyle\int \frac{dx}{(1-x)^2 \sqrt{1-x^2}} .$

1940. $\displaystyle\int \frac{\sqrt{x^2+2x+2}}{x} dx .$

1941. $\displaystyle\int \frac{x\,dx}{(1+x)\sqrt{1-x-x^2}} .$

1942. $\displaystyle\int \frac{1-x+x^2}{\sqrt{1+x-x^2}} dx .$

利用公式

$$\int \frac{P_n(x)}{y}\mathrm{d}x = Q_{n-1}(x)y + \lambda \int \frac{\mathrm{d}x}{y} ,$$

式中 $y = \sqrt{ax^2 + bx + c}$，$P_n(x)$ 为 n 次多项式，$Q_{n-1}(x)$ 为 $n-1$ 次多项式，λ 为常数，求下列积分：

1943. $\int \frac{x^3}{\sqrt{1 + 2x - x^2}}\mathrm{d}x$.

1944. $\int \frac{x^{10}dx}{\sqrt{1 + x^2}}$.

1945. $\int x^4 \sqrt{a^2 - x^2}\,\mathrm{d}x$.

1946. $\int \frac{x^3 - 6x^2 + 11x - 6}{\sqrt{x^2 + 4x + 3}}\mathrm{d}x$.

1947. $\int \frac{dx}{x^3 \sqrt{x^2 + 1}}$.

1948. $\int \frac{dx}{x^4 \sqrt{x^2 - 1}}$.

1949. $\int \frac{dx}{(x - 1)^3 \sqrt{x^2 + 3x + 1}}$.

1950. $\int \frac{dx}{(x + 1)^5 \sqrt{x^2 + 2x}}$.

1951. 在什么条件下，积分

$$\int \frac{a_1 x^2 + b_1 x + c_1}{\sqrt{ax^2 + bx + c}}\mathrm{d}x$$

是代数函数？

分解有理函数 $\frac{P(x)}{Q(x)}$ 为最简分式，求积分 $\int \frac{P(x)}{Q(x)y}dx$ ，式中 $y = \sqrt{ax^2 + bx + c}$ ：

1952. $\int \frac{xdx}{(x - 1)^2 \sqrt{1 + 2x - x^2}}$.

1953. $\int \frac{xdx}{(x^2 - 1)\sqrt{x^2 - x - 1}}$.

1954. $\int \frac{\sqrt{x^2 + x + 1}}{(x + 1)^2}\mathrm{d}x$.

1955. $\int \frac{x^3}{(1 + x)\sqrt{1 + 2x - x^2}}\mathrm{d}x$.

1956. $\int \frac{xdx}{(x^2 - 3x + 2)\sqrt{x^2 - 4x + 3}}$.

1957. $\int \frac{dx}{(1 + x^2)\sqrt{1 - x^2}}$.

1958. $\int \frac{dx}{(x^2 + 1)\sqrt{x^2 - 1}}$.

1959. $\int \frac{dx}{(1 - x^4)\sqrt{1 + x^2}}$.

1960. $\int \frac{\sqrt{x^2 + 2}}{x^2 + 1}\mathrm{d}x$.

化二次三项式为标准形式，计算下列积分：

1961. $\int \frac{dx}{(x^2 + x + 1)\sqrt{x^2 + x - 1}}$.

1962. $\int \frac{x^2 dx}{(4 - 2x + x^2)\sqrt{2 + 2x - x^2}}$.

1963. $\int \frac{(x + 1)dx}{(x^2 + x + 1)\sqrt{x^2 + x + 1}}$.

1964. 利用分式线性代换 $x = \frac{\alpha + \beta t}{1 + t}$ 计算积分

$$\int \frac{dx}{(x^2 - x + 1)\sqrt{x^2 + x + 1}} .$$

1965. 求

$$\int \frac{dx}{(x^2 + 2)\sqrt{2x^2 - 2x + 5}} .$$

利用欧拉代换：

$\quad$ (1)若 $a > 0$, $\sqrt{ax^2 + bx + c} = \pm\sqrt{a}\,x + z$ ；

$\quad$ (2)若 $c > 0$, $\sqrt{ax^2 + bx + c} = xz \pm \sqrt{c}$ ；

$\quad$ (3) $\sqrt{a(x - x_1)(x - x_2)} = z(x - x_1)$ ，

求下列积分：

1966. $\displaystyle\int\frac{dx}{x + \sqrt{x^2 + x + 1}}$.

1967. $\displaystyle\int\frac{dx}{1 + \sqrt{1 - 2x - x^2}}$

1968. $\displaystyle\int x\sqrt{x^2 - 2x + 2}\,dx$.

1969. $\displaystyle\int\frac{x - \sqrt{x^2 + 3x + 2}}{x + \sqrt{x^2 + 3x + 2}}dx$.

1970. $\displaystyle\int\frac{dx}{\left[1 + \sqrt{x(1 + x)}\right]^2}$.

利用不同方法计算下列积分：

1971. $\displaystyle\int\frac{dx}{\sqrt{x^2 + 1} - \sqrt{x^2 - 1}}$.

1972. $\displaystyle\int\frac{x\,dx}{\left(1 - x^3\right)\sqrt{1 - x^2}}$.

1973. $\displaystyle\int\frac{dx}{\sqrt{2} + \sqrt{1 - x} + \sqrt{1 + x}}$.

1974. $\displaystyle\int\frac{x + \sqrt{1 + x + x^2}}{1 + x + \sqrt{1 + x + x^2}}dx$.

1975. $\displaystyle\int\frac{\sqrt{x(x + 1)}}{\sqrt{x} + \sqrt{x + 1}}dx$.

1976. $\displaystyle\int\frac{\left(x^2 - 1\right)dx}{\left(x^2 + 1\right)\sqrt{x^4 + 1}}$.

1977. $\displaystyle\int\frac{\left(x^2 + 1\right)dx}{\left(x^2 - 1\right)\sqrt{x^4 + 1}}$.

1978. $\displaystyle\int\frac{dx}{x\sqrt{x^4 + 2x^2 - 1}}$.

1979. $\displaystyle\int\frac{\left(x^2 + 1\right)dx}{x\sqrt{x^4 + x^2 + 1}}$.

1980. 证明：积分

$$\int R\left(x, \sqrt{ax + b}, \sqrt{cx + d}\right)dx \quad （ R \text{ 为有理函数}）$$

的求法归结为有理函数的积分法：

$\quad$ 二项微分式的积分

$$\int x^m\left(a + bx^n\right)^p dx \quad （ m , n \text{ 和 } p \text{ 为有理数}）$$

仅在下列三种情形下可化为有理函数的积分(切比雪夫定理)：

$\quad$ 情形 1 , p 为整数,此时令 $x = z^N$,其中 N 为分数 m 和 n 的公分母.

$\quad$ 情形 2 , $\dfrac{m + 1}{n}$ 为整数,此时令 $a + bx^n = z^N$,其中 N 为分数 p 的分母.

$\quad$ 情形 3 , $\dfrac{m + 1}{n} + p$ 为整数. 此时利用代换 $ax^{-n} + b = z^N$,其中 N 为分数 p 的分母..

$\quad$ 若 $n = 1$,则这些情形等价于：(1) p 为整数；(2) m 为整数；(3) $m + p$ 为整数.

$\quad$ 计算下列积分：

1981. $\displaystyle\int\sqrt{x^3 + x^4}\,dx$.

1982. $\displaystyle\int\frac{\sqrt{x}}{\left(1 + \sqrt[3]{x}\right)^2}dx$.

1983. $\displaystyle\int\frac{dx}{\sqrt{1 + \sqrt[3]{x^2}}}$.

1984. $\displaystyle\int\frac{x^5\,dx}{\sqrt{1 - x^2}}$.

1985. $\int \dfrac{dx}{\sqrt[3]{1+x^3}}$.

1986. $\int \dfrac{dx}{\sqrt[4]{1+x^4}}$.

1987. $\int \dfrac{dx}{x\sqrt[6]{1+x^6}}$.

1988. $\int \dfrac{dx}{x^3\sqrt[5]{1+\dfrac{1}{x}}}$.

1989. $\int \sqrt[3]{3x-x^3}\,dx$.

1990. 在什么情形下，积分 $\int \sqrt{1+x^m}\,dx$ （ m 为有理数）为初等函数？

形如
$$\int \sin^m x \cos^n x\,dx \qquad （m \text{ 及 } n \text{ 为整数}）$$
的积分可利用巧妙的变换或运用递推公式计算．

求下列积分：

1991. $\int \cos^5 x\,dx$.

1992. $\int \sin^6 x\,dx$.

1993. $\int \cos^6 x\,dx$.

1994. $\int \sin^2 x \cos^4 x\,dx$.

1995. $\int \sin^4 x \cos^5 x\,dx$.

1996. $\int \sin^5 x \cos^5 x\,dx$.

1997. $\int \dfrac{\sin^3 x}{\cos^4 x}\,dx$.

1998. $\int \dfrac{\cos^4 x}{\sin^3 x}\,dx$.

1999. $\int \dfrac{dx}{\sin^3 x}$.

2000. $\int \dfrac{dx}{\cos^3 x}$.

2001. $\int \dfrac{dx}{\sin^4 x \cos^4 x}$.

2002. $\int \dfrac{dx}{\sin^3 x \cos^5 x}$.

2003. $\int \dfrac{dx}{\sin x \cos^4 x}$.

2004. $\int \tan^5 x\,dx$.

2005. $\int \cot^6 x\,dx$.

2006. $\int \dfrac{\sin^4 x}{\cos^6 x}\,dx$.

2007. $\int \dfrac{dx}{\sqrt{\sin^3 x \cos^5 x}}$.

2008. $\int \dfrac{dx}{\cos x \sqrt[3]{\sin^2 x}}$.

2009. $\int \dfrac{dx}{\sqrt{\tan x}}$.

2010. $\int \dfrac{dx}{\sqrt[3]{\tan x}}$.

2011. 推出下列积分的递推公式：

(a) $I_n = \int \sin^n x\,dx$;

(b) $K_n = \int \cos^n x\,dx \qquad (n>2)$.

利用这些公式计算
$$\int \sin^6 x\,dx \quad 和 \quad \int \cos^8 x\,dx .$$

2012. 推出下列积分的递推公式：

(a) $I_n = \int \dfrac{dx}{\sin^n x}$,

(b) $K_n = \int \dfrac{dx}{\cos^n x} \qquad (n>2)$.

利用这些公式计算
$$\int \dfrac{dx}{\sin^5 x} \quad 和 \quad \int \dfrac{dx}{\cos^7 x} .$$

为了计算下面的积分，可以运用公式：

I ． $\sin\alpha\sin\beta=\dfrac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right]$ ；

II ． $\cos\alpha\cos\beta=\dfrac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$ ；

III. $\sin\alpha\cos\beta=\dfrac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right]$ ．

求积分：

2013. $\displaystyle\int\sin5x\cos x\,\mathrm{d}x$ ．

2014. $\displaystyle\int\cos x\cos 2x\cos 3x\,\mathrm{d}x$ ．

2015. $\displaystyle\int\sin x\sin\frac{x}{2}\sin\frac{x}{3}\,\mathrm{d}x$ ．

2016. $\displaystyle\int\sin x\sin(x+a)\sin(x+b)\,\mathrm{d}x$ ．

2017. $\displaystyle\int\cos^2 ax\cos^2 bx\,\mathrm{d}x$ ．

2018. $\displaystyle\int\sin^3 2x\cos^2 3x\,\mathrm{d}x$ ．

为了计算下面的积分，可以运用恒等式：

$$\sin(\alpha-\beta)\equiv\sin\left[(x+\alpha)-(x+\beta)\right]，$$
$$\cos(\alpha-\beta)\equiv\cos\left[(x+\alpha)-(x+\beta)\right]．$$

求积分：

2019. $\displaystyle\int\frac{dx}{\sin(x+a)\sin(x+b)}$ ．

2020. $\displaystyle\int\frac{dx}{\sin(x+a)\cos(x+b)}$ ．

2021. $\displaystyle\int\frac{dx}{\cos(x+a)\cos(x+b)}$ ．

2022. $\displaystyle\int\frac{dx}{\sin x-\sin a}$ ．

2023. $\displaystyle\int\frac{dx}{\cos x+\cos a}$ ．

2024. $\displaystyle\int\tan x\tan(x+a)\,\mathrm{d}x$ ．

形如

$$\int R(\sin x,\cos x)\mathrm{d}x\quad（R\text{ 为有理函数})$$

的积分，在一般情形下可利用代换 $\tan\dfrac{x}{2}=t$ 化为有理函数的积分．

（a）若等式

$$R(-\sin x,\cos x)\equiv-\mathrm{R}(\sin x,\cos x)$$

或

$$R(\sin x,-\cos x)\equiv-\mathrm{R}(\sin x,\cos x)$$

成立，则最好利用相应的代换 $\cos x=t$ 或 $\sin x=t$ ．

（b）若等式

$$R(-\sin x,-\cos x)\equiv\mathrm{R}(\sin x,\cos x)$$

成立，则最好利用代换 $\tan x=t$ ．

求积分：

2025. $\displaystyle\int\frac{dx}{2\sin x-\cos x+5}$ ．

2026. $\displaystyle\int\frac{dx}{(2+\cos x)\sin x}$ ．

2027. $\displaystyle\int\frac{\sin^2 x}{\sin x+2\cos x}\mathrm{d}x$ ．

2028. $\displaystyle\int\frac{dx}{1+\varepsilon\cos x}$ ，(a) $0<\varepsilon<1$ ；(b) $\varepsilon>1$ ．

2029. $\displaystyle\int\frac{\sin^2 x}{1+\sin^2 x}\mathrm{d}x$ ．

2030. $\displaystyle\int\frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$ ．

2031. $\displaystyle\int\frac{\cos^2 x\,dx}{\left(a^2\sin^2 x+b^2\cos^2 x\right)^2}$ ．

2032. $\displaystyle\int\frac{\sin x\cos x}{\sin x+\cos x}\mathrm{d}x$ ．

2033. $\int \dfrac{dx}{\left(a\sin x + b\cos x\right)^2}$.

2034. $\int \dfrac{\sin x\, dx}{\sin^3 x + \cos^3 x}$.

2035. $\int \dfrac{dx}{\sin^4 x + \cos^4 x}$.

2036. $\int \dfrac{\sin^2 x \cos^2 x}{\sin^8 x + \cos^8 x}\, dx$.

2037. $\int \dfrac{\sin^2 x - \cos^2 x}{\sin^4 x + \cos^4 x}\, dx$.

2038. $\int \dfrac{\sin x \cos x}{1 + \sin^4 x}\, dx$.

2039. $\int \dfrac{dx}{\sin^6 x + \cos^6 x}$.

2040. $\int \dfrac{dx}{\left(\sin^2 x + 2\cos^2 x\right)^2}$.

2041. 把分母化为对数的形式，求积分

$$\int \frac{dx}{a\sin x + b\cos x} .$$

2042. 证明：

$$\int \frac{a_1\sin x + b_1\cos x}{a\sin x + b\cos x}\, \mathrm{d}x = Ax + B\ln\left|a\sin x + b\cos x\right| + C ,$$

式中 A ，B ，C 为常数．

提示：令 $a_1\sin x + b_1\cos x = A(a\sin x + b\cos x) + B(a\cos x - b\sin x)$ ，式中 A 和 B 为常数．

求积分：

2043. (a) $\int \dfrac{\sin x - \cos x}{\sin x + 2\cos x}\, \mathrm{d}x$;

(b) $\int \dfrac{\sin x}{\sin x - 3\cos x}\, \mathrm{d}x$.

2044. $\int \dfrac{dx}{3 + 5\tan x}$.

2045. $\int \dfrac{a_1\sin x + b_1\cos x}{\left(a\sin x + b\cos x\right)^2}\, \mathrm{d}x$.

2046. 证明：

$$\int \frac{a_1\sin x + b_1\cos x + c_1}{a\sin x + b\cos x + c}\, \mathrm{d}x$$

$$= Ax + B\ln\left|a\sin x + b\cos x + c\right| + C\int \frac{dx}{a\sin x + b\cos x + c} ,$$

式中 A ，B ，C 是某些常系数．

求积分：

2047. $\int \dfrac{\sin x + 2\cos x - 3}{\sin x - 2\cos x + 3}\, \mathrm{d}x$.

2048. $\int \dfrac{\sin x}{\sqrt{2} + \sin x + \cos x}\, \mathrm{d}x$.

2049. $\int \dfrac{2\sin x + \cos x}{3\sin x + 4\cos x - 2}\, \mathrm{d}x$.

2050. 证明：

$$\int \frac{a_1\sin^2 x + 2b_1\sin x\cos x + c_1\cos^2 x}{a\sin x + b\cos x}\, \mathrm{d}x = A\sin x + B\cos x + C\int \frac{dx}{a\sin x + b\cos x} ,$$

式中 A ，B ，C 是常系数．

求积分：

2051. $\int \dfrac{\sin^2 x - 4\sin x\cos x + 3\cos^2 x}{\sin x + \cos x}\, \mathrm{d}x$.

2052. $\int \dfrac{\sin^2 x - \sin x\cos x + 2\cos^2 x}{\sin x + 2\cos x}\, \mathrm{d}x$.

2053. 证明：若 $(a-c)^2 + b^2 \neq 0$，则

$$\int \frac{a_1 \sin x + b_1 \cos x}{a \sin^2 x + 2b \sin x \cos x + c \cos^2 x} dx = A\int \frac{du_1}{k_1 u_1^2 + \lambda_1} B\int \frac{du_2}{k_2 u_2^2 + \lambda_2} \ ,$$

式中 A，B 为待定系数，λ_1，λ_2 为方程

$$\begin{vmatrix} a-\lambda & b \\ b & c-\lambda \end{vmatrix} = 0 \quad (\lambda_1 \neq \lambda_2)$$

的根，而

$$u_i = (a-\lambda_i)\sin x + b\cos x \ , \qquad k_i = \frac{1}{a-\lambda_i} \quad (i=1,2) \ .$$

求积分：

2054. $\int \dfrac{2\sin x - \cos x}{3\sin^2 x + 4\cos^2 x} dx$.

2055. $\int \dfrac{(\sin x + \cos x)dx}{2\sin^2 x - 4\sin x \cos x + 5\cos^2 x}$.

2056. $\int \dfrac{\sin x - 2\cos x}{1 + 4\sin x \cos x} dx$.

2057. 证明：

$$\int \frac{dx}{(a\sin x + b\cos x)^n} = \frac{A\sin x + B\cos x}{(a\sin x + b\cos x)^{n-1}} + C\int \frac{dx}{(a\sin x + b\cos x)^{n-2}} \ ,$$

式中 A，B，C 为待定系数．

2058. 求 $\int \dfrac{dx}{(\sin x + 2\cos x)^3}$.

2059. 若 n 为大于1的正整数，证明：

$$\int \frac{dx}{(a+b\cos x)^n} = \frac{A\sin x}{(a+b\cos x)^{n-1}} + B\int \frac{dx}{(a+b\cos x)^{n-1}} + C\int \frac{dx}{(a+b\cos x)^{n-2}} \ ,$$

其中 $|a| \neq |b|$，并求出系数 A，B 和 C．

求积分：

2060. $\int \dfrac{\sin x \, dx}{\cos x \sqrt{1 + \sin^2 x}}$.

2061. $\int \dfrac{\sin^2 x}{\cos^2 x \sqrt{\tan x}} dx$.

2062. $\int \dfrac{\sin x \, dx}{\sqrt{2 + \sin 2x}}$.

2063. $\int \dfrac{dx}{(1 + \varepsilon \cos x)^2}$ $(0 < \varepsilon < 1)$.

2064. $\int \dfrac{\cos^{n-1}\frac{x+a}{2}}{\sin^{n+1}\frac{x-a}{2}} dx$.   提示：令 $t = \dfrac{\cos\frac{x+a}{2}}{\sin\frac{x-a}{2}}$.

2065. 推出积分

$$I_n = \int \left(\frac{\sin\frac{x-a}{2}}{\sin\frac{x+a}{2}} \right)^n dx \quad （n \text{ 为正整数}）$$

的递推公式．

2066. 证明：若 $P(x)$ 为 n 次多项式，则

$$\int P(x)e^{ax} dx = e^{ax}\left[\frac{P(x)}{a} - \frac{P'(x)}{2} + \cdots + (-1)^n \frac{P^{(n)}(x)}{a^{n+1}} \right] + C \ .$$

2067. 证明：若 $P(x)$ 为 n 次多项式，则

$$\int P(x)\cos ax\,dx = \frac{\sin ax}{a}\left[P(x) - \frac{P''(x)}{a^2} + \frac{P^{(4)}(x)}{a^4} - \cdots\right]$$

$$+ \frac{\cos ax}{a^2}\left[P'(x) - \frac{P''(x)}{a^2} + \frac{P^{(5)}(x)}{a^4} - \cdots\right] + C\,,$$

$$\int P(x)\sin ax\,dx = -\frac{\cos ax}{a}\left[P(x) - \frac{P''(x)}{a^2} + \frac{P^{(4)}(x)}{a^4} - \cdots\right]$$

$$+ \frac{\sin ax}{a^2}\left[P'(x) - \frac{P''(x)}{a^2} + \frac{P^{(5)}(x)}{a^4} - \cdots\right] + C\,.$$

求积分：

2068. $\int x^3 e^{3x}\,dx$.

2069. $\int (x^2 - 2x + 2)e^{-x}\,dx$.

2070. $\int x^5 \sin 5x\,dx$.

2071. $\int (1 + x^2)^2 \cos x\,dx$.

2072. $\int x^7 e^{-x^2}\,dx$.

2073. $\int x^2 e^{\sqrt{x}}\,dx$.

2074. $\int e^{ax}\cos^2 bx\,dx$.

2075. $\int e^{ax}\sin^3 bx\,dx$.

2076. $\int x e^x \sin x\,dx$.

2077. $\int x^2 e^x \cos x\,dx$.

2078. $\int x e^x \sin^2 x\,dx$.

2079. $\int (x - \sin x)^3\,dx$.

2080. $\int \cos^2 \sqrt{x}\,dx$.

2081. 证明：若 R 为有理函数，数 $a_1, a_2, \cdots, a_n$ 为可公约的，则积分

$$\int R\left(e^{a_1 x}, e^{a_2 x}, \cdots, e^{a_n x}\right)dx$$

可表示为初等函数的形式.

求积分：

2082. $\int \frac{dx}{(1 + e^x)^2}$.

2083. $\int \frac{e^{2x}}{1 + e^x}\,dx$.

2084. $\int \frac{dx}{e^{2x} + e^x - 2}$.

2085. $\int \frac{dx}{1 + e^{\frac{x}{2}} + e^{\frac{x}{3}} + e^{\frac{x}{6}}}$.

2086. $\int \frac{1 + e^{\frac{x}{2}}}{\left(1 + e^{\frac{x}{4}}\right)^2}\,dx$.

2087. $\int \frac{dx}{\sqrt{e^x - 1}}$.

2088. $\int \sqrt{\frac{e^x - 1}{e^x + 1}}\,dx$.

2089. $\int \sqrt{e^{2x} + 4e^x - 1}\,dx$.

2090. $\int \frac{dx}{\sqrt{1 + e^x} + \sqrt{1 - e^x}}$.

2091. 证明：若 R 为有理函数，其分母仅有实根，则积分
$$\int R(x)e^{ax}\,dx$$

可用初等函数和超越函数
$$\int \frac{e^{ax}}{x}\,dx = li\left(e^{ax}\right) + C, \quad \text{式中 } lix = \int \frac{dx}{\ln x}$$

来表示.

2092. 若 $P\left(\dfrac{1}{x}\right) = a_0 + \dfrac{a_1}{x} + \cdots + \dfrac{a_n}{x^n}$，$a_0, a_1, \cdots, a_n$ 为常数，则在什么情形下，积分
$$\int P\left(\frac{1}{x}\right)e^x\,dx$$

为初等函数？

求积分：

2093. $\displaystyle\int\left(1-\frac{2}{x}\right)^2 e^x\,dx$.

2094. $\displaystyle\int\left(1-\frac{1}{x}\right)e^{-x}\,dx$.

2095. $\displaystyle\int \frac{e^{2x}}{x^2-3x+2}\,dx$.

2096. $\displaystyle\int \frac{xe^x}{(x+1)^2}\,dx$.

2097. $\displaystyle\int \frac{x^4 e^{2x}}{(x-2)^2}\,dx$.

求含有 $\ln f(x)$，$\arctan f(x)$，$\arcsin f(x)$，$\arccos f(x)$ 等函数的积分，其中 $f(x)$ 为代数函数：

2098. $\displaystyle\int \ln^n x\,dx$（$n$ 为正整数）.

2099. $\displaystyle\int x^3 \ln^3 x\,dx$.

2100. $\displaystyle\int\left(\frac{\ln x}{x}\right)^3 dx$.

2101. $\displaystyle\int \ln\left[(x+a)^{x+a}(x+b)^{x+b}\right]\frac{dx}{(x+a)(x+b)}$.

2102. $\displaystyle\int \ln^2\left(x+\sqrt{1+x^2}\right)dx$.

2103. $\displaystyle\int \ln\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$.

2104. $\displaystyle\int \frac{\ln x}{(1+x^2)^{\frac{3}{2}}}\,dx$.

2105. $\displaystyle\int x\arctan(x+1)\,dx$.

2106. $\displaystyle\int \sqrt{x}\arctan\sqrt{x}\,dx$.

2107. $\displaystyle\int x\arcsin(1-x)\,dx$.

2108. $\displaystyle\int \arcsin\sqrt{x}\,dx$.

2109. $\displaystyle\int x\arccos\frac{1}{x}\,dx$

2110. $\displaystyle\int \arcsin\frac{2\sqrt{x}}{1+x}\,dx$.

2111. $\displaystyle\int \frac{\arccos x}{(1-x^2)^{\frac{3}{2}}}\,dx$.

2112. $\displaystyle\int \frac{x\arccos x}{(1-x^2)^{\frac{3}{2}}}\,dx$.

2113. $\displaystyle\int x\arctan x\ln\left(1+x^2\right)dx$.

2114. $\displaystyle\int x\ln\frac{1+x}{1-x}\,dx$.

2115. $\displaystyle\int \frac{\ln\left(x+\sqrt{1+x^2}\right)dx}{(1+x^2)^{\frac{3}{2}}}$.

求含有双曲函数的积分：

2116. $\displaystyle\int \sinh^2 x\cosh^2 x\,dx$.

2117. $\displaystyle\int \cosh^4 x\,dx$.

2118. $\displaystyle\int \sinh^3 x\,dx$.

2119. $\displaystyle\int \sinh x\sinh 2x\sinh 3x\,dx$.

2120. $\displaystyle\int \tanh x\,dx$.

2121. $\displaystyle\int \coth^2 x\,dx$.

2122. $\int \sqrt{\tanh x}\, dx$.

2123. (a) $\int \dfrac{dx}{\sinh x + 2\cosh x}$; (b) $\int \dfrac{dx}{\sinh^2 x - 4\sinh x \cosh x + 9\cosh^2 x}$;

(c) $\int \dfrac{dx}{0.1 + \cosh x}$; (d) $\int \dfrac{\cosh x\, dx}{3\sinh x - 4\cosh x}$.

2124. $\int \sinh ax \sin bx\, dx$. 2125. $\int \sinh ax \cos bx\, dx$.

求积分：

2126. $\int \dfrac{dx}{x^6\left(1+x^2\right)}$.

2127. $\int \dfrac{x^2 dx}{\left(1-x^2\right)^3}$.

2128. $\int \dfrac{dx}{1+x^4+x^8}$.

2129. $\int \dfrac{dx}{\sqrt{x}+\sqrt[3]{x}}$.

2130. $\int x^2 \sqrt{\dfrac{x}{1-x}}\, dx$.

2131. $\int \dfrac{x+2}{x^2\sqrt{1-x^2}}\, dx$.

2132. $\int \sqrt{\dfrac{x}{1-x\sqrt{x}}}\, dx$.

2133. $\int \dfrac{x^5 dx}{\sqrt{1+x^2}}$.

2134. $\int \dfrac{dx}{\sqrt[3]{x^2(1-x)}}$.

2135. $\int \dfrac{dx}{x\sqrt{1+x^3+x^6}}$.

2136. $\int \dfrac{dx}{x\sqrt{x^4-2x^2-1}}$.

2137. $\int \dfrac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\, dx$.

2138. $\int \dfrac{(1+x)dx}{x+\sqrt{x+x^2}}$.

2139. $\int \dfrac{\ln\left(1+x+x^2\right)}{\left(1+x\right)^2}\, dx$.

2140. $\int (2x+3)\arccos(2x-3)\, dx$.

2141. $\int x\ln\left(4+x^4\right)dx$.

2142. $\int \dfrac{\arcsin x}{x^2}\cdot \dfrac{1+x^2}{\sqrt{1-x^2}}\, dx$.

2143. $\int \dfrac{x\ln\left(1+\sqrt{1+x^2}\right)}{\sqrt{1+x^2}}\, dx$.

2144. $\int x\sqrt{x^2+1}\,\ln\sqrt{x^2-1}\, dx$.

2145. $\int \dfrac{x}{\sqrt{1-x^2}}\ln\dfrac{x}{\sqrt{1-x}}\, dx$.

2146. $\int \dfrac{dx}{\left(2+\sin x\right)^2}$.

2147. $\int \dfrac{\sin 4x}{\sin^8 x + \cos^8 x}\, dx$.

2148. $\int \dfrac{dx}{\sin x\sqrt{1+\cos x}}$.

2149. $\int \dfrac{ax^2+b}{x^2+1}\arctan x\, dx$.

2150. $\int \dfrac{ax^2+b}{x^2-1}\ln\left|\dfrac{x-1}{x+1}\right|dx$.

2151. $\int \dfrac{x\ln x}{\left(1+x^2\right)^2}\, dx$.

2152. $\int \dfrac{x\arctan x}{\sqrt{1+x^2}}\, dx$.

2153. $\int \dfrac{\sin 2x}{\sqrt{1+\cos^4 x}}\, dx$.

2154. $\int \dfrac{x^3 \arccos}{\sqrt{1-x^2}}\, dx$.

2155. $\int \dfrac{x^4 \arctan x}{1+x^2}\, dx$.

2156. $\int \dfrac{x\, arccot\, x}{\left(1+x^2\right)^2}\, dx$.

2157. $\int \dfrac{x\ln\left(x+\sqrt{1+x^2}\right)}{\left(1-x^2\right)^2}\, dx$.

2158. $\int \sqrt{1-x^2}\,\arcsin x\, dx$.

2159. $\int x\left(1+x^2\right)arccot\, x\, dx$.

2160. $\int x^n(1+\ln x)\mathrm{d}x$.

2161. $\int \dfrac{\arcsin e^x}{e^x}\mathrm{d}x$.

2162. $\int \dfrac{\arctan e^{\frac{x}{2}}}{e^{\frac{x}{2}}\left(1+e^x\right)}\mathrm{d}x$.

2163. $\int \dfrac{\mathrm{d}x}{\left(e^{x+1}+1\right)^2-\left(e^{x-1}+1\right)^2}$.

2164. $\int \sqrt{\tanh^2 x+1}\,\mathrm{d}x$.

2165. $\int \dfrac{1+\sin x}{1+\cos x}e^x\mathrm{d}x$.

2166. $\int |x|\mathrm{d}x$.

2167. $\int x|x|\mathrm{d}x$.

2168. $\int \left(x+|x|\right)^2\mathrm{d}x$.

2169. $\int \{|1+x|-|1-x|\}\mathrm{d}x$.

2170. $\int e^{-|x|}\mathrm{d}x$.

2171. $\int \max\{1,x^2\}\mathrm{d}x$.

2172. $\int \varphi(x)\mathrm{d}x$,其中 $\varphi(x)$ 为数 x 到最近整数的距离.

2173. $\int [x]|\sin \pi x|\mathrm{d}x$ $(x \geq 0)$.

2174. $\int f(x)\mathrm{d}x$,其中 $f(x)=\begin{cases}1-x^2, & |x|\leq 1,\\ 1-|x|, & |x|>1.\end{cases}$

2175. $\int f(x)\mathrm{d}x$,其中 $f(x)=\begin{cases}1, & -\infty<x<0,\\ x+1, & 0\leq x\leq 1,\\ 2x, & 1<x<+\infty\end{cases}$.

2176. 求 $\int xf''(x)\mathrm{d}x$.

2177. 求 $\int f'(2x)\mathrm{d}x$.

2178. 设 $f'\left(x^2\right)=\dfrac{1}{x}(x>0)$,求 $f(x)$.

2179. (a) 设 $f'\left(\sin^2 x\right)=\cos^2 x$,求 $f(x)$.

　　(b) 设 $f'(\ln x)=\begin{cases}1, & 0<x\leq 1,\\ x, & 1<x<+\infty,\end{cases}$ 且 $f(0)=0$,求 $f(x)$.

2180. 设 $f(x)$ 为严格单调的连续函数 $f^{-1}(x)$ 为其反函数,证明:若
$$\int f(x)\mathrm{d}x=F(x)+C ,$$

则
$$\int f^{-1}(x)\mathrm{d}x=xf^{-1}(x)-F\left(f^{-1}(x)\right)+C .$$

研究例子:
(a) $f(x)=x^n(n>0)$;

(b) $f(x)=e^x$;

(c) $f(x)=\arcsin x$;

(d) $f(x)=\operatorname{arctan} hx$.