不定积分王者百题S2 · 解

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康复训练

问题 1.

$$\int \frac{\sin x \pm \sinh x}{\cos x \pm \cosh x} \, \mathrm{d}x$$

问题 2.

$$\int \frac{\cos x \pm \cosh x}{\sin x \pm \sinh x} \, \mathrm{d}x$$

问题 3.

$$\int \frac{\sqrt{x^4 + kx^2 - 1}}{x^4 + 1} \, \mathrm{d}x$$

问题 4.

$$\int \frac{x \sin x}{(x - \sin x)^2} \, \mathrm{d}x$$

问题 5.

$$\int \frac{2 \arcsin x - \arcsin^5 x}{(1 - \arcsin^2 x)^{\frac{3}{2}}} \, \mathrm{d}x$$

问题 6.

$$\int \frac{x^2 + 2x - 4}{x^3 \sqrt{1-x}} \mathrm{e}^{\frac{1}{2}\arccos x} \, \mathrm{d}x$$

问题 7.

$$\int \frac{2x^2 \sqrt{1-x^2} + \cos(1-x^2)}{1 - x^2 + \sqrt{1-x^2}\sin(1-x^2)} \, \mathrm{d}x$$

问题 8.

$$\int \mathrm{e}^{\frac{2}{1-\sqrt{1+x}}} - \mathrm{e}^{\frac{1}{1-\sqrt{1+x}}}\left(1-\sqrt{1+x}\right) \, \mathrm{d}x$$

不散的宴席

问题 1.

$$\int \frac{\sqrt[3]{\cos x}}{3 + \sin x}\, \mathrm{d}x$$

问题 2.

$$\int \frac{\sqrt{\sin x}}{1 + \sin^2 x}\, \mathrm{d}x$$

问题 3.

$$\int \sqrt[3]{\frac{1 + \sin x}{1 - 2\sin x}}\, \mathrm{d}x$$

问题 4.

$$\int \sqrt[3]{\frac{1 + \sin x}{5 - 4\sin x}}\, \mathrm{d}x$$

问题 5.

$$\int \sqrt[4]{\frac{1 + \sin x}{3 - \sin x}}\, \mathrm{d}x$$

问题 6.

$$\int \frac{\sqrt[5]{\sin 2x - 2\cos x}}{3 + \sin x}\, \mathrm{d}x$$

问题 7.

$$\int \frac{\sqrt[4]{\cos x - \cos 3x}}{3 + \cos 2x}\, \mathrm{d}x$$

问题 8.

$$\int \frac{(\tan x - \cot x)^{\frac{7}{8}}}{3 + \cos 4x} \, \mathrm{d}x$$

问题 9.

$$\int \frac{1}{\sin x \sqrt[3]{\sin(x + \frac{\pi}{6})}} \, \mathrm{d}x$$

问题 10.

$$\int \frac{\sin^{\frac{n-1}{3}} x}{\sin(x - \frac{\pi}{6}) \sin^n(x + \frac{\pi}{6})} \, \mathrm{d}x$$

问题 11.

$$\int \frac{\sqrt{\cos x}}{(2 + \sin x)(7 - \sin x)} \, \mathrm{d}x$$

问题 12.

$$\int \frac{\sqrt{\sin^4 x + 7\sin^3 x + 7\sin x + 1}}{\sin 2x} \, \mathrm{d}x$$

问题 13.

$$\int \frac{\tan x}{\sqrt[3]{9\cos 4x + 7}} \, \mathrm{d}x$$

问题 14.

$$\int \frac{\sqrt{2\sqrt{3}\sin x + \cos^2 x}}{1 + \sin^2 x} \, \mathrm{d}x$$

问题 15.

$$\int \frac{\sqrt{9\sin x - \sin^3 x}}{3 + \sin^2 x}\,\mathrm{d}x$$

问题 16.

$$\int \frac{\sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\,\mathrm{d}x$$

问题 17.

$$\int \frac{\cos x\sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\,\mathrm{d}x$$

问题 18.

$$\int \frac{\sqrt{\sin x}\sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\,\mathrm{d}x$$

问题 19.

$$\int \frac{\csc x - \sin x}{\sqrt[4]{(3\sin^2 x - 2)^3(\sin^2 x + 2)}}\,\mathrm{d}x$$

问题 20.

$$\int \frac{(x^4 - 1)\sqrt{1 + x^4}}{x^8 + 1}\,\mathrm{d}x$$

问题 21.

$$\int \frac{1 + x^6}{(1 - x^6)\sqrt{1 + x^4}}\,\mathrm{d}x$$

问题 22.

$$\int \frac{1+x^6}{(1-x^6)\sqrt{1-x^2+x^4}}\,\mathrm{d}x$$

问题 23.

$$\int \frac{(x^3+2)\sqrt{1-x^3}}{(x^3-1)^2}\,\mathrm{d}x$$

问题 24.

$$\int \frac{(x^2-1)\sqrt{1+x^4}}{(x^2+1)(x^4+x^2+1)}\,\mathrm{d}x$$

问题 25.

$$\int \frac{(x^4-1)\sqrt{1+x^4}}{x^8+x^6+3x^4+x^2+1}\,\mathrm{d}x$$

问题 26.

$$\int \frac{(x^4-1)\sqrt{1+x^4}}{(x^4+3x^2+1)^2}\,\mathrm{d}x$$

问题 27.

$$\int \frac{(x^4+1)\sqrt{x^4-1}}{x^8-x^4+1}\,\mathrm{d}x$$

问题 28.

$$\int \frac{(x^3+2)\sqrt{1-x^3}}{(x^3-1)(x^3+x^2-1)}\,\mathrm{d}x$$

问题 29.

$$\int \frac{3x^5 - 2}{(x^5 - x^2 + 1)\sqrt{1 + x^5}} \, \mathrm{d}x$$

问题 30.

$$\int \frac{(2x^6 + 1)\sqrt{1 - x^6}}{x^{12} - 2x^6 + x^4 + 1} \, \mathrm{d}x$$

问题 31.

$$\int \frac{(2x^6 - 1)\sqrt{1 + x^6}}{(x^6 - x^2 + 1)^2} \, \mathrm{d}x$$

问题 32.

$$\int \frac{(2x^6 + 1)\sqrt{x^6 + 2x^2 - 1}}{(x^6 + x^2 - 1)^2} \, \mathrm{d}x$$

问题 33.

$$\int \frac{(x^6 - 2)(x^6 + 2x^4 + 1)^{\frac{3}{2}}}{x^3(x^6 + 1)^2} \, \mathrm{d}x$$

问题 34.

$$\int \frac{(2x^6 + 1)(x^6 - x^2 - 1)^{\frac{3}{2}}}{(x^6 - 1)^3} \, \mathrm{d}x$$

问题 35.

$$\int \frac{(x^3 - 2)\sqrt{2x^3 + x^2 + 2}}{(x^3 + 1)(x^3 + x^2 + 1)} \, \mathrm{d}x$$

问题 36.

$$\int \frac{(x^3 - 4)\sqrt{x^3 - x^2 + 2}}{(x^3 + 2)(x^3 + x^2 + 2)} \, \mathrm{d}x$$

问题 37.

$$\int \frac{(x^4 - 1)\sqrt{x^4 + x^2 + 1}}{(x^4 + 1)(2x^4 + x^2 + 2)} \, \mathrm{d}x$$

问题 38.

$$\int \frac{(x^4 - 2)\sqrt{x^4 + 2}}{(x^4 + x^2 + 2)(x^4 + 2x^2 + 2)} \, \mathrm{d}x$$

问题 39.

$$\int \frac{(3x^5 - 2)\sqrt{-1 + x^2 - x^5}}{(x^5 - 2x^2 + 1)(x^5 - x^2 + 1)} \, \mathrm{d}x$$

问题 40.

$$\int \frac{(3x^5 + 2)\sqrt{1 - x^2 - x^5}}{(x^5 - 1)(x^5 + x^2 - 1)} \, \mathrm{d}x$$

问题 41.

$$\int \frac{(2x^6 - 1)\sqrt{x^6 - x^2 + 1}}{(x^6 + 1)(x^6 + x^2 + 1)} \, \mathrm{d}x$$

问题 42.

$$\int \frac{(x^3 + 4)\sqrt{-x^3 + 2x^2 + 2}}{(x^3 - 2)(x^3 - 2x^2 - 2)} \, \mathrm{d}x$$

问题 43.

$$\int \frac{(2x^4 - x^2 + 1)\sqrt{1 - x^2 - x^4 - x^6}}{(x^4 - 1)(x^6 + x^4 - 1)} \, \mathrm{d}x$$

问题 44.

$$\int \frac{\sqrt{x^{12} + 33x^8 - 33x^4 - 1}}{x^8 - 14x^4 - 1} \, \mathrm{d}x$$

问题 45.

$$\int \frac{\cos^2 x \csc x}{(2\sin^2 x + \sqrt{3}\sin x - 3)\sqrt{2\sin x + \sqrt{3}}} \, \mathrm{d}x$$

问题 46.

$$\int \frac{\sqrt{x^6 + 45x^2 + 18}}{x^4 + 2x^2 - 3} \, \mathrm{d}x$$

问题 47.

$$\int \frac{4 - 5x^2}{\sqrt{4 - 8x^2 + 4x^4 - x^6}} \, \mathrm{d}x$$

问题 48.

$$\int \frac{(x + 2)\sqrt{1 - 3x + 4x^2 - x^3}}{(x - 1)^4} \, \mathrm{d}x$$

问题 49.

$$\int \frac{2x^2 + 3x - 2}{(x^2 + 1)\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}} \, \mathrm{d}x$$

问题 50.

$$\int \frac{x-3}{(x+1)\sqrt{x^3 - 15x^2 + 15x - 1}} \, \mathrm{d}x$$

问题 51.

$$\int \frac{(x^2+2)\sqrt{x^4 - 5x^2 + 4}}{x^2(x^2 + 2x - 2)} \, \mathrm{d}x$$

问题 52.

$$\int \frac{1}{x\sqrt[3]{x^2 - 6x + 12}} \, \mathrm{d}x$$

问题 53.

$$\int \frac{1}{x\sqrt[4]{x^3 - 3x^2 + 4x - 2}} \, \mathrm{d}x$$

问题 54.

$$\int \frac{x^2 + 3}{x\sqrt{-x^4 - 2x^3 + 6x^2 + 6x - 9}} \, \mathrm{d}x$$

问题 55.

$$\int \frac{-3x^2 + 4x + 7}{(x-1)^{\frac{4}{5}}\left((x+1)^{\frac{5}{2}} + 2(x-1)\right)} \, \mathrm{d}x$$

问题 56.

$$\int \frac{2x^2 + 6x - 1}{\left(x^2 - x - 1 + \sqrt{-x^4 + 6x^3 + 7x^2 + 2}\right)^2} \, \mathrm{d}x$$

问题 57.

$$\int \left(\frac{\sqrt[3]{x^3 - 3x^2 - 21x - 25}}{x^2 + 2x + 1} \right)^2 \mathrm{d}x$$

问题 58.

$$\int \frac{\tan^2 x \sqrt[3]{\tan x \sec x}}{1 + \sec^2 x} \, \mathrm{d}x$$

问题 59.

$$\int \frac{(\tan^2 x + 2)\sqrt[5]{\cot x \csc x}}{\tan x + 4 \csc x} \, \mathrm{d}x$$

问题 60.

$$\int \frac{\sqrt{x^2 + 8}}{(x^2 - 4)\sqrt[4]{x^2 - 1}} \, \mathrm{d}x$$

问题 61.

$$\int \frac{1}{x - 7} \sqrt[3]{\frac{x + 17}{x^2 - 1}} \, \mathrm{d}x$$

问题 62.

$$\int \sqrt{\frac{1 + \frac{4}{1-x}\sqrt{\frac{2}{7-x}}}{(1+x)(7-x)}} \, \mathrm{d}x$$

问题 63.

$$\int \frac{1}{\sqrt{1 + \sqrt{\tan^4 x + \sec^4 x}}} \, \mathrm{d}x$$

问题 64.

$$\int \sqrt{2 + \cos x + \sqrt{5 + 4\cos x}}\, \mathrm{d}x$$

问题 65.

$$\int \sqrt{2\csc^2 x + \sqrt{3 + \sin^2 x}}\, \mathrm{d}x$$

问题 66.

$$\int \frac{\sqrt{1 + \sqrt{1 + x^4}}}{1 + x^4}\, \mathrm{d}x$$

问题 67.

$$\int \frac{\sqrt{1 + \sqrt{1 + x^4} + \sqrt{2}\sqrt{1 + x^4 + \sqrt{1 + x^4}}}}{1 + x^4}\, \mathrm{d}x$$

问题 68.

$$\int \frac{\sqrt{x^4 + \sqrt{x^4 + 1} + 2\sqrt{x^4 - 1 + \sqrt{(x^8 - 1)(x^4 - 1)}}}}{1 + x^4}\, \mathrm{d}x$$

问题 69.

$$\int \frac{(x^4 + 1)^2}{(x^4 - 1)^2 \sqrt{x^2 + \sqrt{1 + x^4}}}\, \mathrm{d}x$$

问题 70.

$$\int \left(\sqrt{1 - k\sin^2 x} - \frac{1 - k}{(1 - k\sin^2 x)^{\frac{3}{2}}} \right) \mathrm{d}x$$

问题 71.

$$\int \frac{e^{-x} + 1 + \cot x - \cot^2 x}{e^{-x} + 2 + e^x \csc^2 x} \, dx$$

问题 72.

$$\int \arctan \frac{2}{\left(\sqrt{x^2 - x} - 1\right)\left(\sqrt{1 + \frac{1}{x}} - \sqrt{1 - \frac{1}{x}}\right)} \, dx$$

问题 73.

$$\int \frac{1 + \sin^2 x}{\cosh \cos x + \cos x \sinh \cos x} \, dx$$

问题 74.

$$\int \frac{1 - \sin x \cos x}{\cos x \sinh \sin x + \cosh \sin x} \, dx$$

问题 75.

$$\int \frac{\sin x + \cosh \cos x}{\cos x \sinh \cos x + \cosh \cos x - \sin x} \, dx$$

问题 76.

$$\int \frac{\sin x + \cosh \cos x}{\sqrt[4]{\cosh \cos x (\cos x \sinh \cos x - \sin x)(\cos x \sinh \cos x + \cosh \cos x - \sin x)^2}} \, dx$$

问题 77.

$$\int \frac{\sqrt[4]{5x^2 + 6x + 5}\left(\frac{p+q}{2}x^4 + px^3 + qx^2 + qx + p\right)}{\sqrt{(x-1)^3(x+1)^6}(3x^2 + 2x + 3)} \, dx$$

问题 78.

$$\int \frac{(x+1)^4}{(x^2+14x+1)(x^2-4x+1)\sqrt{x(1-x^2)}}\,\mathrm{d}x$$

问题 79.

$$\int \frac{\mathrm{e}^x+x-1-(x\tan x+1)(\tan x-1)}{\mathrm{e}^x+2x+x^2\mathrm{e}^{-x}\sec^2 x}\,\mathrm{d}x$$

问题 80.

$$\int \frac{\sqrt{x-1}}{(x+1)\sqrt{3x^2+1}}\,\mathrm{d}x$$

问题 81.

$$\int \sqrt[3]{\frac{\mathrm{e}^x\,(\mathrm{e}^x+1)}{(\mathrm{e}^x+3)(\mathrm{e}^x+4)}}\,\mathrm{d}x$$

问题 82.

$$\int \sqrt[3]{\frac{(\mathrm{e}^x-1)^2}{(\mathrm{e}^x+2)(2\mathrm{e}^x+1)}}\,\mathrm{d}x$$

问题 83.

$$\int \sqrt[3]{\frac{\mathrm{e}^{2x}+4\mathrm{e}^x-1}{\mathrm{e}^{2x}+22\mathrm{e}^x+125}}\,\mathrm{d}x$$

问题 84.

$$\int \frac{(\sinh x-1)\sqrt{2\sinh x+\cos 2x+1}}{\sinh x(\sinh x-3\cosh x+1)}\,\mathrm{d}x$$

问题 85.

$$\int \frac{\cos x}{\sqrt{\cos^2 x + 2\cos x - 2}} \, \mathrm{d}x$$

问题 86.

$$\int \frac{3\cos x + \lambda}{\sqrt{\cos^2 x + 2(\lambda - 1)\cos x + \lambda^2 - 2}} \, \mathrm{d}x$$

问题 87.

$$\int \frac{5\cos^2 x + 10\cos x + 2}{\sqrt{\cos^4 x + 6\cos^3 x + 10\cos^2 x + 2\cos x - 2}} \, \mathrm{d}x$$

问题 88.

$$\int \frac{2\sqrt{3}\sin x + 1}{\sqrt{\sin x \cos x + (\sqrt{2} + \sqrt{3})(\sin x + \cos x)}} \, \mathrm{d}x$$

问题 89.

$$\int \frac{\sin x}{\sqrt{\sin 2x + 2\sqrt{2}(\sin x + \cos x) - 3}} \, \mathrm{d}x$$

问题 90.

$$\int \frac{\sec x + \cos \tan x}{\sin x + \sin \tan x} \, \mathrm{d}x$$

问题 91.

$$\int \frac{\tan x + \cos \sec x}{1 + \sin x \sin \sec x} \, \mathrm{d}x$$

问题 92.

$$\int \frac{\left(\sqrt{x^2 - x + 1}\sqrt{x^2 + x + 1}\right)^{1009}(x^2 - 1)}{\left(\sqrt{x^2 - x + 1} + \sqrt{x^2 + x + 1}\right)^{2022}}\, \mathrm{d}x$$

问题 93.

$$\int x^{k-1}\left(x^n + \sqrt{1 + x^{2n}}\right)^{2 + \frac{k}{n}}\, \mathrm{d}x$$

问题 94.

$$\int \frac{\sqrt{x^{16} - 28x^{12} + 6x^8 - 28x^4 + 1}}{(3x^8 - 6x^4 - 1)\sqrt[4]{1 - x^4}}\, \mathrm{d}x$$

问题 95.

$$\int \frac{\sqrt[3]{(x^2 + 12)^2}}{(x^2 + 4)\sqrt[6]{x^2 + 3}}\, \mathrm{d}x$$

问题 96.

$$\int \frac{1}{\sqrt[3]{(x + 1)(3x^2 + 3x + 1)}}\, \mathrm{d}x$$

问题 97.

$$\int \frac{x}{\sqrt[3]{(x^3 - a)(x^3 - 2a)}}\, \mathrm{d}x$$

问题 98.

$$\int \frac{\sqrt{\sqrt{x + 1} + \sqrt{x - 1}}}{\sqrt{x^2 - 1}\sqrt[8]{2x^2 - 1}}\, \mathrm{d}x$$

问题 99.

$$\int \frac{11x^2 + 28x - 28}{(x+4)(x^2 + 24x + 44)\sqrt{x^3 + 1}} \, \mathrm{d}x$$

问题 100.

$$\int \frac{x^2 + ax - 1}{(x^2 + 1)\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}} \, \mathrm{d}x$$

题解说明

- 换元不会**回代**, 太懒了;
- 典题不会**继续**, 太显了;
- 结果不会**修正**, 太累了;
- 答案不会 +**C** , 淘汰了;
- 过程可能**有误**, 请指教;
- 欢迎读者**评论**, 谢谢啦.

康复训练 · 解

1. $\displaystyle \int \frac{\sin x \pm \sinh x}{\cos x \pm \cosh x}\,\mathrm{d}x$

解.

$$\int \frac{\sin x \pm \sinh x}{\cos x \pm \cosh x}\,\mathrm{d}x = 2\arctan\left(\tan\frac{x}{2}\tanh^{\pm 1}\frac{x}{2}\right), \quad \text{模板} \qquad \square$$

2. $\displaystyle \int \frac{\cos x \pm \cosh x}{\sin x \pm \sinh x}\,\mathrm{d}x$

解.

$$\int \frac{\cos x \pm \cosh x}{\sin x \pm \sinh x}\,\mathrm{d}x = \int \frac{1}{t}\,\mathrm{d}t, \quad t = \sin x \pm \sinh x \qquad \square$$

3. $\displaystyle \int \frac{\sqrt{x^4 + kx^2 - 1}}{x^4 + 1}\,\mathrm{d}x$

解.

$$\int \frac{\sqrt{x^4 + kx^2 - 1}}{x^4 + 1}\,\mathrm{d}x = \int \frac{1}{(k^2+4)t^4 - 2kt^2 + 1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4 + kx^2 - 1}} \qquad \square$$

4. $\displaystyle \int \frac{x\sin x}{(x - \sin x)^2}\,\mathrm{d}x$

解.

$$\begin{aligned}
\int \frac{x\sin x}{(x - \sin x)^2}\,\mathrm{d}x &= \int \frac{x\sin x}{\cos x - 1}\,\mathrm{d}\frac{1}{x - \sin x} \\
&= \cdots - \int \frac{1}{1 - \cos x}\,\mathrm{d}x, \quad \text{IBP} \\
&= \frac{1 + \cos x}{x - \sin x}
\end{aligned} \qquad \square$$

5. $\displaystyle \int \frac{2\arcsin x - \arcsin^5 x}{(1 - \arcsin^2 x)^{\frac{3}{2}}}\,\mathrm{d}x$

解.

$$\int \frac{2\arcsin x - \arcsin^5 x}{(1-\arcsin^2 x)^{\frac{3}{2}}}\,\mathrm{d}x = \Re\int \frac{2t-t^5}{(1-t^2)^{\frac{3}{2}}}\mathrm{e}^{\mathrm{i}t}\,\mathrm{d}t, \quad t = \sin x$$
$$= \Re\frac{-\mathrm{i}t^3+2t^2+\mathrm{i}t-1}{\sqrt{1-t^2}}\mathrm{e}^{\mathrm{i}t} \qquad \square$$

6. $\displaystyle\int \frac{x^2+2x-4}{x^3\sqrt{1-x}}\mathrm{e}^{\frac{1}{2}\arccos x}\,\mathrm{d}x$

解.

$$\int \frac{x^2+2x-4}{x^3\sqrt{1-x}}\mathrm{e}^{\frac{1}{2}\arccos x}\,\mathrm{d}x = -\frac{1}{\sqrt{2}}\int \frac{\cos 5t + 5\cos 3t - 10\cos t}{\cos^3 2t}\mathrm{e}^t, \quad t = \frac{1}{2}\arccos x$$
$$= -\frac{\cos 3t + \cos t - \sin 3t - 3\sin t}{\sqrt{2}\cos^2 2t}\mathrm{e}^t \qquad \square$$

7. $\displaystyle\int \frac{2x^2\sqrt{1-x^2}+\cos(1-x^2)}{1-x^2+\sqrt{1-x^2}\sin(1-x^2)}\,\mathrm{d}x$

解.

$$\int \frac{2x^2\sqrt{1-x^2}+\cos(1-x^2)}{1-x^2+\sqrt{1-x^2}\sin(1-x^2)}\,\mathrm{d}x$$
$$= 2\operatorname{artanh}\left(\tan\frac{\arccos\sqrt{1-x^2}}{2}\tan\frac{\frac{\pi}{2}-(1-x^2)}{2}\right), \quad \textcolor{blue}{模板} \qquad \square$$

8. $\displaystyle\int \mathrm{e}^{\frac{2}{1-\sqrt{1+x}}-\mathrm{e}^{\frac{1}{1-\sqrt{1+x}}}}\left(1-\sqrt{1+x}\right)\mathrm{d}x$

解.

$$\int \mathrm{e}^{\frac{2}{1-\sqrt{1+x}}-\mathrm{e}^{\frac{1}{1-\sqrt{1+x}}}}\left(1-\sqrt{1+x}\right)\mathrm{d}x = 2\int t\mathrm{e}^{-t}\,\mathrm{d}t, \quad t = \mathrm{e}^{\frac{1}{1-\sqrt{1+x}}}\left(1-\sqrt{1+x}\right)$$
$$= -\frac{2(1+t)}{\mathrm{e}^t} \qquad \square$$

不散的宴席 · 解

1. $\displaystyle \int \frac{\sqrt[3]{\cos x}}{3 + \sin x}\,\mathrm{d}x$

解.

$$\int \frac{\sqrt[3]{\cos x}}{3 + \sin x}\,\mathrm{d}x = -\frac{3}{4}\int \frac{1}{t^3 + 1}\,\mathrm{d}t, \quad t = \frac{1 - \sin x}{2\sqrt[3]{\cos^2 x}} \qquad \square$$

2. $\displaystyle \int \frac{\sqrt{\sin x}}{1 + \sin^2 x}\,\mathrm{d}x$

解.

$$\int \frac{\sqrt{\sin x}}{1 + \sin^2 x}\,\mathrm{d}x = -2\int \frac{1}{t^4 + 4}\,\mathrm{d}t, \quad t = \frac{\cos x}{\sqrt{\sin x}} \qquad \square$$

3. $\displaystyle \int \sqrt[3]{\frac{1 + \sin x}{1 - 2\sin x}}\,\mathrm{d}x$

解.

$$\int \sqrt[3]{\frac{1 + \sin x}{1 - 2\sin x}}\,\mathrm{d}x = -4\sqrt{3}\sqrt[3]{2}\int \frac{\sqrt[3]{\cos t}}{\cos 2t - 7}\,\mathrm{d}t, \quad \sin x = \frac{3 - \sin^2 t}{3 + \sin^2 t}$$
$$= \frac{\mathrm{i}}{4\sqrt{3}\sqrt[3]{4}}\int \frac{1}{1 + u^3}\,\mathrm{d}u - \frac{\mathrm{i}}{4\sqrt{3}\sqrt[3]{4}}\int \frac{1}{1 + v^3}\,\mathrm{d}v,$$
$$u = \frac{-\sqrt{3} + \mathrm{i}\sin t}{\sqrt{3}\sqrt[3]{2}\sqrt[3]{\cos^2 t}}, v = \frac{-\sqrt{3} - \mathrm{i}\sin t}{\sqrt{3}\sqrt[3]{2}\sqrt[3]{\cos^2 t}} \qquad \square$$

4. $\displaystyle \int \sqrt[3]{\frac{1 + \sin x}{5 - 4\sin x}}\,\mathrm{d}x$

解.

$$\int \sqrt[3]{\frac{1 + \sin x}{5 - 4\sin x}}\,\mathrm{d}x = 12\sqrt[3]{2}\int \frac{\sqrt[3]{\cos t}}{\cos 2t + 17}\,\mathrm{d}t, \quad \sin x = \frac{9 + \sin^2 t}{9 - \sin^2 t}$$
$$= \frac{3}{2\sqrt[3]{2}}\int \frac{1}{u^3 + 1}\,\mathrm{d}u - \frac{3}{2\sqrt[3]{2}}\int \frac{1}{v^3 + 1}\,\mathrm{d}v,$$
$$u = \frac{1 + \sin t}{2\sqrt[3]{\cos^2 t}}, v = \frac{1 - \sin t}{2\sqrt[3]{\cos^2 t}} \qquad \square$$

5. $\displaystyle\int \sqrt[4]{\dfrac{1+\sin x}{3-\sin x}}\,\mathrm{d}x$

解.

$$\int \sqrt[4]{\frac{1+\sin x}{3-\sin x}}\,\mathrm{d}x = -4\sqrt{2}\int \frac{t}{(t^4+2)\sqrt[4]{t^4+1}}\,\mathrm{d}t, \quad \sin x = \frac{2-t^4}{2+t^4}$$

$$= -4\sqrt{2}\int \frac{1}{s^4+4}\,\mathrm{d}s, \quad s = \frac{t^2}{\sqrt[4]{t^4+1}} \qquad \square$$

6. $\displaystyle\int \dfrac{\sqrt[5]{\sin 2x - 2\cos x}}{3+\sin x}\,\mathrm{d}x$

解.

$$\int \frac{\sqrt[5]{\sin 2x - 2\cos x}}{3+\sin x}\,\mathrm{d}x = 3\sqrt[3]{2}\int \frac{1}{2t^3+1}\,\mathrm{d}t, \quad t = \sqrt[3]{1+\sin x} \qquad \square$$

7. $\displaystyle\int \dfrac{\sqrt[4]{\cos x - \cos 3x}}{3+\cos 2x}\,\mathrm{d}x$

解.

$$\int \frac{\sqrt[4]{\cos x - \cos 3x}}{3+\cos 2x}\,\mathrm{d}x = -2\sqrt{2}\int \frac{t^4}{1+4t^8}\,\mathrm{d}t, \quad t = \frac{\sqrt[4]{\cos x}}{\sqrt{\sin x}} \qquad \square$$

8. $\displaystyle\int \dfrac{(\tan x - \cot x)^{\frac{7}{8}}}{3+\cos 4x}\,\mathrm{d}x$

解.

$$\int \frac{(\tan x - \cot x)^{\frac{7}{8}}}{3+\cos 4x}\,\mathrm{d}x = 2\int \frac{t^{14}}{2+t^{16}}\,\mathrm{d}t, \quad t = \sqrt[8]{\tan x - \cot x} \qquad \square$$

9. $\displaystyle\int \dfrac{1}{\sin x \sqrt[3]{\sin(x+\frac{\pi}{6})}}\,\mathrm{d}x$

解.

$$\int \frac{1}{\sin x \sqrt[3]{\sin(x+\frac{\pi}{6})}}\,\mathrm{d}x = 6\int \frac{1}{4t^3 - 3\sqrt{3}}\,\mathrm{d}t, \quad t = \frac{\sin(x+\frac{\pi}{3})}{\sqrt[3]{\sin(x+\frac{\pi}{6})}} \qquad \square$$

10. $\displaystyle\int \dfrac{\sin^{\frac{n-1}{3}} x}{\sin(x-\frac{\pi}{6})\sin^n(x+\frac{\pi}{6})}\,\mathrm{d}x$

解.

$$\int \frac{\sin^{\frac{n-1}{3}} x}{\sin(x - \frac{\pi}{6}) \sin^n (x + \frac{\pi}{6})} \, dx = \frac{2^{\frac{2n+1}{3}}}{3^{\frac{n}{2}}} \int \frac{(t^3 + 1)^{\frac{n-1}{3}}}{t} \, dt, \quad t = \frac{\sin(x - \frac{\pi}{6})}{\sin(x + \frac{\pi}{6})} \qquad \square$$

11. $\displaystyle\int \frac{\sqrt{\cos x}}{(2 + \sin x)(7 - \sin x)} \, dx$

解.

$$\int \frac{\sqrt{\cos x}}{(2 + \sin x)(7 - \sin x)} \, dx = -2 \int \frac{1}{t^4 + 27} \, dt, \quad t = \frac{1 - \sin x}{\sqrt{\cos x}} \qquad \square$$

12. $\displaystyle\int \frac{\sqrt{\sin^4 x + 7 \sin^3 x + 7 \sin x + 1}}{\sin 2x} \, dx$

解.

$$\int \frac{\sqrt{\sin^4 x + 7 \sin^3 x + 7 \sin x + 1}}{\sin 2x} \, dx = \frac{1}{4} \int \frac{\sqrt{4 - 3t - 3t^2}}{t(t - 1)} \, dt, \quad \sin x = \frac{1 - \sqrt{t}}{1 + \sqrt{t}} \qquad \square$$

13. $\displaystyle\int \frac{\tan x}{\sqrt[3]{9 \cos 4x + 7}} \, dx$

解.

$$\int \frac{\tan x}{\sqrt[3]{9 \cos 4x + 7}} \, dx = -\frac{3}{2\sqrt[3]{2}} \int \frac{1}{t^3 + 8} \, dt, \quad t = \sqrt[3]{\frac{(3 \cos 2x - 1)^2}{3 \cos 2x + 1}} \qquad \square$$

14. $\displaystyle\int \frac{\sqrt{2\sqrt{3} \sin x + \cos^2 x}}{1 + \sin^2 x} \, dx$

解.

$$\int \frac{\sqrt{2\sqrt{3} \sin x + \cos^2 x}}{1 + \sin^2 x} \, dx = -\sqrt{3} \int \frac{1}{4t^4 - 2t^2 + 1} \, dt, \quad t = \frac{\cos x}{\sqrt{2\sqrt{3} \sin x + \cos^2 x}} \qquad \square$$

15. $\displaystyle\int \frac{\sqrt{9 \sin x - \sin^3 x}}{3 + \sin^2 x} \, dx$

解.

$$\int \frac{\sqrt{9\sin x - \sin^3 x}}{3 + \sin^2 x}\, dx = -2\int \frac{1}{27t^4 + 1}\, dt, \quad t = \frac{\cos x}{\sqrt{9\sin x - \sin^3 x}} \qquad \square$$

16. $\displaystyle\int \frac{\sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\, dx$

解.

$$\int \frac{\sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\, dx = \frac{1}{2\sqrt[4]{2}} \int \left(1 + \frac{1}{t}\right) \frac{1}{\sqrt[4]{t^4 - 1}}\, dt, \quad t = \frac{1 + \mathrm{i}\sin x}{1 - \mathrm{i}\sin x} \qquad \square$$

17. $\displaystyle\int \frac{\cos x \sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\, dx$

解.

$$\int \frac{\cos x \sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\, dx = 4\int \frac{t^2}{4t^8 + 1}\, dt, \quad t = \frac{\sqrt[4]{\sin x}}{\sqrt{\cos x}} \qquad \square$$

18. $\displaystyle\int \frac{\sqrt{\sin x}\sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\, dx$

解.

$$\int \frac{\sqrt{\sin x}\sqrt[4]{\csc x - \sin x}}{1 + \sin^2 x}\, dx = 4\int \frac{t^4}{4t^8 + 1}\, dt, \quad t = \frac{\sqrt[4]{\sin x}}{\sqrt{\cos x}} \qquad \square$$

19. $\displaystyle\int \frac{\csc x - \sin x}{\sqrt[4]{(3\sin^2 x - 2)^3(\sin^2 x + 2)}}\, dx$

解.

$$\int \frac{\csc x - \sin x}{\sqrt[4]{(3\sin^2 x - 2)^3(\sin^2 x + 2)}}\, dx = -\frac{1}{3}\int \frac{t^2}{1 + t^4}\, dt, \quad t = \cos x \sqrt[4]{\frac{\sin^2 x + 2}{3\sin^2 x - 2}} \qquad \square$$

20. $\displaystyle\int \frac{(x^4 - 1)\sqrt{1 + x^4}}{x^8 + 1}\, dx$

解.

$$\int \frac{(x^4 - 1)\sqrt{1 + x^4}}{x^8 + 1}\, dx = \int \frac{1}{2t^4 - 1}\, dt, \quad t = \frac{x}{\sqrt{x^4 + 1}} \qquad \square$$

21. $\displaystyle\int \frac{1+x^6}{(1-x^6)\sqrt{1+x^4}}\,\mathrm{d}x$

解.

$$\int \frac{1+x^6}{(1-x^6)\sqrt{1+x^4}}\,\mathrm{d}x = \int \frac{t^2-1}{2t^4+t^2-1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4+1}} \qquad \square$$

22. $\displaystyle\int \frac{1+x^6}{(1-x^6)\sqrt{1-x^2+x^4}}\,\mathrm{d}x$

解.

$$\int \frac{1+x^6}{(1-x^6)\sqrt{1-x^2+x^4}}\,\mathrm{d}x = -\int \frac{1}{2t^4-t^2-1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4-x^2+1}} \qquad \square$$

23. $\displaystyle\int \frac{(x^3+2)\sqrt{1-x^3}}{(x^3-1)^2}\,\mathrm{d}x$

解.

$$\int \frac{(x^3+2)\sqrt{1-x^3}}{(x^3-1)^2}\,\mathrm{d}x = 2\int \mathrm{d}t, \quad t = \frac{x}{\sqrt{1-x^3}} \qquad \square$$

24. $\displaystyle\int \frac{(x^2-1)\sqrt{1+x^4}}{(x^2+1)(x^4+x^2+1)}\,\mathrm{d}x$

解.

$$\int \frac{(x^2-1)\sqrt{1+x^4}}{(x^2+1)(x^4+x^2+1)}\,\mathrm{d}x = -\int \frac{1}{2t^4+3t^2+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4+1}} \qquad \square$$

25. $\displaystyle\int \frac{(x^4-1)\sqrt{1+x^4}}{x^8+x^6+3x^4+x^2+1}\,\mathrm{d}x$

解.

$$\int \frac{(x^4-1)\sqrt{1+x^4}}{x^8+x^6+3x^4+x^2+1}\,\mathrm{d}x = -\int \frac{1}{t^4+t^2+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4+1}} \qquad \square$$

26. $\displaystyle\int \frac{(x^4-1)\sqrt{1+x^4}}{(x^4+3x^2+1)^2}\,\mathrm{d}x$

解.

$$\int \frac{(x^4-1)\sqrt{1+x^4}}{(x^4+3x^2+1)^2}\,\mathrm{d}x = -\int \frac{1}{(3t^2+1)^2}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4+1}} \qquad \square$$

27. $\displaystyle\int \frac{(x^4+1)\sqrt{x^4-1}}{x^8-x^4+1}\,\mathrm{d}x$

解.

$$\int \frac{(x^4+1)\sqrt{x^4-1}}{x^8-x^4+1}\,\mathrm{d}x = -\int \frac{1}{t^4+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4-1}} \qquad \square$$

28. $\displaystyle\int \frac{(x^3+2)\sqrt{1-x^3}}{(x^3-1)(x^3+x^2-1)}\,\mathrm{d}x$

解.

$$\int \frac{(x^3+2)\sqrt{1-x^3}}{(x^3-1)(x^3+x^2-1)}\,\mathrm{d}x = 2\int \frac{1}{1-t^2}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{1-x^3}} \qquad \square$$

29. $\displaystyle\int \frac{3x^5-2}{(x^5-x^2+1)\sqrt{1+x^5}}\,\mathrm{d}x$

解.

$$\int \frac{3x^5-2}{(x^5-x^2+1)\sqrt{1+x^5}}\,\mathrm{d}x = 2\int \frac{1}{t^2-1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^5+1}} \qquad \square$$

30. $\displaystyle\int \frac{(2x^6+1)\sqrt{1-x^6}}{x^{12}-2x^6+x^4+1}\,\mathrm{d}x$

解.

$$\int \frac{(2x^6+1)\sqrt{1-x^6}}{x^{12}-2x^6+x^4+1}\,\mathrm{d}x = \int \frac{1}{t^4+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{1-x^6}} \qquad \square$$

31. $\displaystyle\int \frac{(2x^6-1)\sqrt{1+x^6}}{(x^6-x^2+1)^2}\,\mathrm{d}x$

解.

$$\int \frac{(2x^6-1)\sqrt{1+x^6}}{(x^6-x^2+1)^2}\,\mathrm{d}x = -\int \frac{1}{(t^2-1)^2}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^6+1}} \qquad \square$$

32. $\displaystyle\int \frac{(2x^6 + 1)\sqrt{x^6 + 2x^2 - 1}}{(x^6 + x^2 - 1)^2}\, dx$

解.

$$\int \frac{(2x^6 + 1)\sqrt{x^6 + 2x^2 - 1}}{(x^6 + x^2 - 1)^2}\, dx = -\int \frac{1}{(t^2 - 1)^2}\, dt, \quad t = \frac{x}{\sqrt{x^6 + 2x^2 - 1}} \qquad \square$$

33. $\displaystyle\int \frac{(x^6 - 2)(x^6 + 2x^4 + 1)^{\frac{3}{2}}}{x^3 (x^6 + 1)^2}\, dx$

解.

$$\int \frac{(x^6 - 2)(x^6 + 2x^4 + 1)^{\frac{3}{2}}}{x^3 (x^6 + 1)^2}\, dx = -\int \frac{1}{t^2 (2t^2 - 1)^2}\, dt, \quad t = \frac{x^2}{\sqrt{x^6 + 2x^4 + 1}} \qquad \square$$

34. $\displaystyle\int \frac{(2x^6 + 1)(x^6 - x^2 - 1)^{\frac{3}{2}}}{(x^6 - 1)^3}\, dx$

解.

$$\int \frac{(2x^6 + 1)(x^6 - x^2 - 1)^{\frac{3}{2}}}{(x^6 - 1)^3}\, dx = -\int \frac{1}{(t^2 + 1)^3}\, dt, \quad t = \frac{x}{\sqrt{x^6 - x^2 - 1}} \qquad \square$$

35. $\displaystyle\int \frac{(x^3 - 2)\sqrt{2x^3 + x^2 + 2}}{(x^3 + 1)(x^3 + x^2 + 1)}\, dx$

解.

$$\int \frac{(x^3 - 2)\sqrt{2x^3 + x^2 + 2}}{(x^3 + 1)(x^3 + x^2 + 1)}\, dx = 4\int \frac{1}{t^4 - 1}\, dt, \quad t = \frac{x}{\sqrt{2x^3 + x^2 + 2}} \qquad \square$$

36. $\displaystyle\int \frac{(x^3 - 4)\sqrt{x^3 - x^2 + 2}}{(x^3 + 2)(x^3 + x^2 + 2)}\, dx$

解.

$$\int \frac{(x^3 - 4)\sqrt{x^3 - x^2 + 2}}{(x^3 + 2)(x^3 + x^2 + 2)}\, dx = -2\int \frac{1}{2t^4 + 3t^2 + 1}\, dt, \quad t = \frac{x}{\sqrt{x^3 - x^2 + 2}} \qquad \square$$

37. $\displaystyle\int \frac{(x^4 - 1)\sqrt{x^4 + x^2 + 1}}{(x^4 + 1)(2x^4 + x^2 + 2)}\, dx$

解.

$$\int \frac{(x^4-1)\sqrt{x^4+x^2+1}}{(x^4+1)(2x^4+x^2+2)}\,\mathrm{d}x = -\int \frac{1}{t^4-3t^2+2}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4+x^2+1}} \qquad \square$$

38. $\displaystyle\int \frac{(x^4-2)\sqrt{x^4+2}}{(x^4+x^2+2)(x^4+2x^2+2)}\,\mathrm{d}x$

解.

$$\int \frac{(x^4-2)\sqrt{x^4+2}}{(x^4+x^2+2)(x^4+2x^2+2)}\,\mathrm{d}x = -\int \frac{1}{2t^4+3t^2+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^4+2}} \qquad \square$$

39. $\displaystyle\int \frac{(3x^5-2)\sqrt{-1+x^2-x^5}}{(x^5-2x^2+1)(x^5-x^2+1)}\,\mathrm{d}x$

解.

$$\int \frac{(3x^5-2)\sqrt{-1+x^2-x^5}}{(x^5-2x^2+1)(x^5-x^2+1)}\,\mathrm{d}x = 2\int \frac{1}{t^2+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{-x^5+x^2-1}} \qquad \square$$

40. $\displaystyle\int \frac{(3x^5+2)\sqrt{1-x^2-x^5}}{(x^5-1)(x^5+x^2-1)}\,\mathrm{d}x$

解.

$$\int \frac{(3x^5+2)\sqrt{1-x^2-x^5}}{(x^5-1)(x^5+x^2-1)}\,\mathrm{d}x = 2\int \frac{1}{t^2+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{-x^5-x^2+1}} \qquad \square$$

41. $\displaystyle\int \frac{(2x^6-1)\sqrt{x^6-x^2+1}}{(x^6+1)(x^6+x^2+1)}\,\mathrm{d}x$

解.

$$\int \frac{(2x^6-1)\sqrt{x^6-x^2+1}}{(x^6+1)(x^6+x^2+1)}\,\mathrm{d}x = -\int \frac{1}{2t^4+3t^2+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{x^6-x^2+1}} \qquad \square$$

42. $\displaystyle\int \frac{(x^3+4)\sqrt{-x^3+2x^2+2}}{(x^3-2)(x^3-2x^2-2)}\,\mathrm{d}x$

解.

$$\int \frac{(x^3+4)\sqrt{-x^3+2x^2+2}}{(x^3-2)(x^3-2x^2-2)}\,\mathrm{d}x = 2\int \frac{1}{1-2t^2}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{-x^3+2x^2+2}} \qquad \square$$

43. $\displaystyle\int \frac{(2x^4-x^2+1)\sqrt{1-x^2-x^4-x^6}}{(x^4-1)(x^6+x^4-1)}\,\mathrm{d}x$

解.

$$\int \frac{(2x^4-x^2+1)\sqrt{1-x^2-x^4-x^6}}{(x^4-1)(x^6+x^4-1)}\,\mathrm{d}x = \int \frac{1}{2t^4+3t^2+1}\,\mathrm{d}t, \quad t = \frac{x}{\sqrt{1-x^2-x^4-x^6}} \qquad \square$$

44. $\displaystyle\int \frac{\sqrt{x^{12}+33x^8-33x^4-1}}{x^8-14x^4-1}\,\mathrm{d}x$

解.

$$\int \frac{\sqrt{x^{12}+33x^8-33x^4-1}}{x^8-14x^4-1}\,\mathrm{d}x = \int \frac{1}{108t^4-1}\,\mathrm{d}t, \quad t = \frac{x^5+x}{\sqrt{x^{12}+33x^8-33x^4-1}} \qquad \square$$

45. $\displaystyle\int \frac{\cos^2 x \csc x}{(2\sin^2 x + \sqrt{3}\sin x - 3)\sqrt{2\sin x + \sqrt{3}}}\,\mathrm{d}x$

解.

$$\int \frac{\cos^2 x \csc x}{(2\sin^2 x + \sqrt{3}\sin x - 3)\sqrt{2\sin x + \sqrt{3}}}\,\mathrm{d}x$$
$$= -2\int \frac{1}{t^4-9\sqrt{3}t^2+54}\,\mathrm{d}t, \quad t = \frac{(\sin x - \sqrt{3})\sqrt{2\sin x + \sqrt{3}}}{\cos x} \qquad \square$$

46. $\displaystyle\int \frac{\sqrt{x^6+45x^2+18}}{x^4+2x^2-3}\,\mathrm{d}x$

解.

$$\int \frac{\sqrt{x^6+45x^2+18}}{x^4+2x^2-3}\,\mathrm{d}x = 2\int \frac{1}{t^4-4t^2+3}\,\mathrm{d}t, \quad t = \frac{x^3-9x}{\sqrt{x^6+45x^2+18}} \qquad \square$$

47. $\displaystyle\int \frac{4-5x^2}{\sqrt{4-8x^2+4x^4-x^6}}\,\mathrm{d}x$

解.

$$\int \frac{4 - 5x^2}{\sqrt{4 - 8x^2 + 4x^4 - x^6}} \, dx = \arcsin \frac{x^5 - 3x^3 + 4x}{2}, \quad \text{PE} \qquad \square$$

48. $\displaystyle \int \frac{(x+2)\sqrt{1 - 3x + 4x^2 - x^3}}{(x-1)^4} \, dx$

解.

$$\int \frac{(x+2)\sqrt{1 - 3x + 4x^2 - x^3}}{(x-1)^4} \, dx = 2 \int \frac{1}{(t^2 - 1)^2} \, dt, \quad t = \frac{x}{\sqrt{1 - 3x + 4x^2 - x^3}} \qquad \square$$

49. $\displaystyle \int \frac{2x^2 + 3x - 2}{(x^2 + 1)\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}} \, dx$

解.

$$\int \frac{2x^2 + 3x - 2}{(x^2 + 1)\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}} \, dx$$
$$= \int \frac{6t^2 - 2 - \frac{22t^3 + t}{\sqrt{1 - 3t^2}}}{16t^4 + 1} \, dt, \quad t = \frac{x}{\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}} \qquad \square$$

50. $\displaystyle \int \frac{x - 3}{(x+1)\sqrt{x^3 - 15x^2 + 15x - 1}} \, dx$

解.

$$\int \frac{x - 3}{(x+1)\sqrt{x^3 - 15x^2 + 15x - 1}} \, dx = \frac{\sqrt{2}}{3} \arctan \frac{\sqrt{x^3 - 15x^2 + 15x - 1}}{\sqrt{2}(3x - 1)}, \quad \text{PE} \qquad \square$$

51. $\displaystyle \int \frac{(x^2 + 2)\sqrt{x^4 - 5x^2 + 4}}{x^2(x^2 + 2x - 2)} \, dx$

解.

$$\int \frac{(x^2 + 2)\sqrt{x^4 - 5x^2 + 4}}{x^2(x^2 + 2x - 2)} \, dx = -8 \int \frac{t^2}{(3t^2 - 1)(t^2 - 1)^2} \, dt, \quad t = \frac{x^2 + x - 2}{\sqrt{x^4 - 5x^2 + 4}} \qquad \square$$

52. $\displaystyle \int \frac{1}{x\sqrt[3]{x^2 - 6x + 12}} \, dx$

解.

$$\int \frac{1}{x\sqrt[3]{x^2 - 6x + 12}} \, dx = 3 \int \frac{1}{t^3 + 18} \, dt, \quad t = \frac{x - 6}{\sqrt[3]{x^2 - 6x + 12}} \qquad \square$$

53. $\displaystyle\int \frac{1}{x\sqrt[4]{x^3 - 3x^2 + 4x - 2}}\, dx$

解.

$$\int \frac{1}{x\sqrt[4]{x^3 - 3x^2 + 4x - 2}}\, dx = 4\int \frac{1}{t^4 + 8}\, dt, \quad t = \frac{x - 2}{\sqrt[4]{x^3 - 3x^2 + 4x - 2}} \qquad \square$$

54. $\displaystyle\int \frac{x^2 + 3}{x\sqrt{-x^4 - 2x^3 + 6x^2 + 6x - 9}}\, dx$

解.

$$\int \frac{x^2 + 3}{x\sqrt{-x^4 - 2x^3 + 6x^2 + 6x - 9}}\, dx = 2\arcsin \frac{\sqrt{x - 1}\sqrt{x + 3}}{\sqrt{2}\sqrt{x}}, \quad \text{PE} \qquad \square$$

55. $\displaystyle\int \frac{-3x^2 + 4x + 7}{(x - 1)^{\frac{4}{5}}\left((x + 1)^{\frac{5}{2}} + 2(x - 1)\right)}\, dx$

解.

$$\int \frac{-3x^2 + 4x + 7}{(x - 1)^{\frac{4}{5}}\left((x + 1)^{\frac{5}{2}} + 2(x - 1)\right)}\, dx = 10\int \frac{1}{2t^5 + 1}\, dt, \quad t = \frac{\sqrt[5]{x - 1}}{\sqrt{x + 1}} \qquad \square$$

56. $\displaystyle\int \frac{2x^2 + 6x - 1}{\left(x^2 - x - 1 + \sqrt{-x^4 + 6x^3 + 7x^2 + 2}\right)^2}\, dx$

解.

$$\int \frac{2x^2 + 6x - 1}{\left(x^2 - x - 1 + \sqrt{-x^4 + 6x^3 + 7x^2 + 2}\right)^2}\, dx$$
$$= \int \frac{1}{(t + 1)^2\sqrt{2t^2 + 1}}\, dt, \quad t = \frac{x^2 - x - 1}{\sqrt{-x^4 + 6x^3 + 7x^2 + 2}} \qquad \square$$

57. $\displaystyle\int \left(\frac{\sqrt[3]{x^3 - 3x^2 - 21x - 25}}{x^2 + 2x + 1}\right)^2\, dx$

解.

$$\int \left(\frac{\sqrt[3]{x^3 - 3x^2 - 21x - 25}}{x^2 + 2x + 1}\right)^2\, dx = -\int \frac{1}{(t^3 + 1)^2}\, dt, \quad t = \frac{x + 3}{\sqrt[3]{x^3 - 3x^2 - 21 - 25}} \qquad \square$$

58. $\displaystyle\int \frac{\tan^2 x\sqrt[3]{\tan x \sec x}}{1 + \sec^2 x}\, \mathrm{d}x$

解.

$$\int \frac{\tan^2 x\sqrt[3]{\tan x \sec x}}{1 + \sec^2 x}\, \mathrm{d}x = -3 \int \frac{1}{4t^6 + 1}\, \mathrm{d}t, \quad t = \sqrt[3]{\frac{\cos x}{\sin^2 x}} \qquad \square$$

59. $\displaystyle\int \frac{(\tan^2 x + 2)\sqrt[5]{\cot x \csc x}}{\tan x + 4\csc x}\, \mathrm{d}x$

解.

$$\int \frac{(\tan^2 x + 2)\sqrt[5]{\cot x \csc x}}{\tan x + 4\csc x}\, \mathrm{d}x = -5 \int \frac{1}{4t^5 + 1}\, \mathrm{d}t, \quad t = \sqrt[5]{\frac{\cos x}{\sin^2 x}} \qquad \square$$

60. $\displaystyle\int \frac{\sqrt{x^2 + 8}}{(x^2 - 4)\sqrt[4]{x^2 - 1}}\, \mathrm{d}x$

解.

$$\int \frac{\sqrt{x^2 + 8}}{(x^2 - 4)\sqrt[4]{x^2 - 1}}\, \mathrm{d}x = 2 \int \frac{1}{27t^4 - 1}\, \mathrm{d}t, \quad t = \frac{x}{\sqrt{x^2 + 8}\sqrt[4]{x^2 - 1}} \qquad \square$$

61. $\displaystyle\int \frac{1}{x - 7}\sqrt[3]{\frac{x + 17}{x^2 - 1}}\, \mathrm{d}x$

解.

$$\int \frac{1}{x - 7}\sqrt[3]{\frac{x + 17}{x^2 - 1}}\, \mathrm{d}x = 3 \int \frac{1}{54t^3 - 1}\, \mathrm{d}t, \quad t = \frac{\sqrt[3]{(x + 1)^2}}{\sqrt[3]{x - 1}\sqrt[3]{(x + 17)^2}} \qquad \square$$

62. $\displaystyle\int \sqrt{\frac{1 + \frac{4}{1-x}\sqrt{\frac{2}{7-x}}}{(1 + x)(7 - x)}}\, \mathrm{d}x$

解.

$$\int \sqrt{\frac{1 + \frac{4}{1-x}\sqrt{\frac{2}{7-x}}}{(1 + x)(7 - x)}}\, \mathrm{d}x = -\frac{4}{3} \int \frac{1}{\sqrt{4\sqrt{2} - t^2}}\, \mathrm{d}t, \quad t = \sqrt{1 - x}\sqrt[4]{7 - x} \qquad \square$$

63. $\displaystyle\int \frac{1}{\sqrt{1 + \sqrt{\tan^4 x + \sec^4 x}}}\, \mathrm{d}x$

解.

$$\int \frac{1}{\sqrt{1+\sqrt{\tan^4 x + \sec^4 x}}}\, dx = \int \frac{2t^4 + 2t^2 + 1}{(2t^2+1)(t^2+1)}\, dt, \quad t = \frac{\tan x}{\sqrt{1+\sqrt{\tan^4 x + \sec^4 x}}} \quad \square$$

64. $\displaystyle\int \sqrt{2 + \cos x + \sqrt{5 + 4\cos x}}\, dx$

解.

$$\int \sqrt{2 + \cos x + \sqrt{5 + 4\cos x}}\, dx = -\int \frac{t}{\sqrt{-t^2 + 4t - 3}}\, dt, \quad t = \sqrt{5 + 4\cos x} \quad \square$$

65. $\displaystyle\int \sqrt{2\csc^2 x + \sqrt{3 + \sin^2 x}}\, dx$

解.

$$\int \sqrt{2\csc^2 x + \sqrt{3 + \sin^2 x}}\, dx = 2\int \frac{t^4 - 3t^2 + 2}{t^4 - 4t^2 + 1}\, dt, \quad t = \sqrt{2 - \sqrt{3 + \sin^2 x}} \quad \square$$

66. $\displaystyle\int \frac{\sqrt{1 + \sqrt{1 + x^4}}}{1 + x^4}\, dx$

解.

$$\int \frac{\sqrt{1 + \sqrt{1 + x^4}}}{1 + x^4}\, dx = 2\int \frac{1}{t^4 + 1}\, dt, \quad t = \frac{x}{\sqrt{1 + \sqrt{1 + x^4}}} \quad \square$$

67. $\displaystyle\int \frac{\sqrt{1 + \sqrt{1 + x^4} + \sqrt{2}\sqrt{1 + x^4 + \sqrt{1 + x^4}}}}{1 + x^4}\, dx$

解.

$$\int \frac{\sqrt{1 + \sqrt{1 + x^4} + \sqrt{2}\sqrt{1 + x^4 + \sqrt{1 + x^4}}}}{1 + x^4}\, dx$$
$$= 4\int \frac{1}{t^4 + 1}\, dt, \quad t = \frac{x}{\sqrt{1 + \sqrt{1 + x^4} + \sqrt{2}\sqrt{1 + x^4 + \sqrt{1 + x^4}}}} \quad \square$$

68. $\displaystyle\int \frac{\sqrt{x^4 + \sqrt{x^4 + 1} + 2\sqrt{x^4 - 1 + \sqrt{(x^8 - 1)(x^4 - 1)}}}}{1 + x^4}\, dx$

解.

$$\int \frac{\sqrt{x^4 + \sqrt{x^4 + 1} + 2\sqrt{x^4 - 1 + \sqrt{(x^8 - 1)(x^4 - 1)}}}}{1 + x^4} \, \mathrm{d}x$$

$$= \int \frac{\sqrt{x^4 - 1}}{x^4 + 1} \, \mathrm{d}x + \int \frac{\sqrt{1 + \sqrt{x^4 + 1}}}{x^4 + 1} \, \mathrm{d}x$$

$$= -\int \frac{1}{4u^4 + 1} \, \mathrm{d}u + 2 \int \frac{1}{v^4 + 1} \, \mathrm{d}v, \quad u = \frac{x}{\sqrt{x^4 - 1}}, v = \frac{x}{1 + \sqrt{x^4 + 1}} \qquad \Box$$

69. $\displaystyle \int \frac{(x^4 + 1)^2}{(x^4 - 1)^2 \sqrt{x^2 + \sqrt{1 + x^4}}} \, \mathrm{d}x$

解.

$$\int \frac{(x^4 + 1)^2}{(x^4 - 1)^2 \sqrt{x^2 + \sqrt{1 + x^4}}} \, \mathrm{d}x = \int \frac{(t^2 - 1)^5}{(2t^2 - 1)(t^4 + 2t^2 - 1)^2} \, \mathrm{d}t, \quad t = \frac{x}{\sqrt{x^2 + \sqrt{x^4 + 1}}} \Box$$

70. $\displaystyle \int \left(\sqrt{1 - k \sin^2 x} - \frac{1 - k}{(1 - k \sin^2 x)^{\frac{3}{2}}} \right) \mathrm{d}x$

解.

$$\int \left(\sqrt{1 - k \sin^2 x} - \frac{1 - k}{(1 - k \sin^2 x)^{\frac{3}{2}}} \right) \mathrm{d}x = \frac{k}{2} \int \mathrm{d}t, \quad t = \frac{\sin 2x}{\sqrt{1 - k \sin^2 x}} \qquad \Box$$

71. $\displaystyle \int \frac{e^{-x} + 1 + \cot x - \cot^2 x}{e^{-x} + 2 + e^x \csc^2 x} \, \mathrm{d}x$

解.

$$\int \frac{e^{-x} + 1 + \cot x - \cot^2 x}{e^{-x} + 2 + e^x \csc^2 x} \, \mathrm{d}x = x - \int \frac{1}{t^2 + 1} \, \mathrm{d}t, \quad t = \frac{(e^x + 1)\tan x}{e^x} \qquad \Box$$

72. $\displaystyle \int \arctan \frac{2}{\left(\sqrt{x^2 - x} - 1 \right) \left(\sqrt{1 + \frac{1}{x}} - \sqrt{1 - \frac{1}{x}} \right)} \, \mathrm{d}x$

解.

$$\int \arctan \frac{2}{\left(\sqrt{x^2 - x} - 1\right)\left(\sqrt{1 + \frac{1}{x}} - \sqrt{1 - \frac{1}{x}}\right)} \, dx$$

$$= 4 \int \arctan \frac{2t(t^2 - 4)}{t^4 - 6t^2 + 4} \, d\frac{1}{t^2(t^2 - 4)}, \quad t = \sqrt{1 + \frac{1}{\sqrt{x}}} - \sqrt{1 - \frac{1}{\sqrt{x}}} \qquad \square$$

73. $\displaystyle \int \frac{1 + \sin^2 x}{\cosh \cos x + \cos x \sinh \cos x} \, dx$

解.

$$\int \frac{1 + \sin^2 x}{\cosh \cos x + \cos x \sinh \cos x} \, dx = \int \frac{1}{t^2 + 1} \, dt, \quad t = \frac{\sin x}{\sinh \cos x + \cos x \cosh \cos x} \qquad \square$$

74. $\displaystyle \int \frac{1 - \sin x \cos x}{\cos x \sinh \sin x + \cosh \sin x} \, dx$

解.

$$\int \frac{1 - \sin x \cos x}{\cos x \sinh \sin x + \cosh \sin x} \, dx = \int \frac{1}{t^2 + 1} \, dt, \quad t = \frac{\sin x}{\sinh \sin x + \cos x \cosh \sin x} \qquad \square$$

75. $\displaystyle \int \frac{\sin x + \cosh \cos x}{\cos x \sinh \cos x + \cosh \cos x - \sin x} \, dx$

解.

$$\int \frac{\sin x + \cosh \cos x}{\cos x \sinh \cos x + \cosh \cos x - \sin x} \, dx = \int dt, \quad t = \frac{1 + \sin x \cosh \cos x}{\sinh \cos x + \cos x \cosh \cos x} \qquad \square$$

76. $\displaystyle \int \frac{\sin x + \cosh \cos x}{\sqrt[4]{\cosh \cos x(\cos x \sinh \cos x - \sin x)(\cos x \sinh \cos x + \cosh \cos x - \sin x)^2}} \, dx$

解.

$$\int \frac{\sin x + \cosh \cos x}{\sqrt[4]{\cosh \cos x(\cos x \sinh \cos x - \sin x)(\cos x \sinh \cos x + \cosh \cos x - \sin x)^2}} \, dx$$

$$= \sqrt{2} \int \frac{1}{\sqrt[4]{1 - t^4}} \, dt, \quad t = \frac{1 + \sin x \cosh \cos x}{\sinh \cos x + \cos x \cosh \cos x} \qquad \square$$

77. $\displaystyle \int \frac{\sqrt[4]{5x^2 + 6x + 5}\left(\frac{p+q}{2} x^4 + px^3 + qx^2 + qx + p\right)}{\sqrt{(x - 1)^3 (x + 1)^6}(3x^2 + 2x + 3)} \, dx$

解.

$$\int \frac{\sqrt[4]{5x^2 + 6x + 5}\left(\frac{p+q}{2}x^4 + px^3 + qx^2 + qx + p\right)}{\sqrt{(x-1)^3(x+1)^6}\,(3x^2 + 2x + 3)}\,dx$$

$$= -\frac{1}{64}\int \frac{(5(p+q)t^4 + 2(9p+q)t^2 - 8(p-q)t + p+q)\sqrt[4]{4t^2 + 1}}{t^3(2t^2 + 1)}\,dt, \quad t = \frac{x+1}{x-1}$$

$$= -\frac{1}{64}\int \frac{u^4\left(5(p+q)u^8 + 2(31p - q)u^4 - 51p + 13q\right)}{(u^4 - 1)^2(u^4 + 1)}\,du - \frac{p-q}{8}\int \frac{v^4}{v^4 + 4}\,dv,$$

$$u = \sqrt[4]{4t^2 + 1},\, v = \frac{\sqrt[4]{4t^2 + 1}}{t} \qquad \square$$

78. $\displaystyle \int \frac{(x+1)^4}{(x^2 + 14x + 1)(x^2 - 4x + 1)\sqrt{x(1-x^2)}}\,dx$

解.

$$\int \frac{(x+1)^4}{(x^2 + 14x + 1)(x^2 - 4x + 1)\sqrt{x(1-x^2)}}\,dx = 2\int \frac{1}{1 - 108t^4}\,dt, \quad t = \frac{\sqrt{x(1-x^2)}}{(x+1)^2} \qquad \square$$

79. $\displaystyle \int \frac{e^x + x - 1 - (x\tan x + 1)(\tan x - 1)}{e^x + 2x + x^2 e^{-x}\sec^2 x}\,dx$

解.

$$\int \frac{e^x + x - 1 - (x\tan x + 1)(\tan x - 1)}{e^x + 2x + x^2 e^{-x}\sec^2 x}\,dx = x - \int \frac{1}{t^2 + 1}\,dt, \quad t = \frac{x\tan x}{e^x + x} \qquad \square$$

80. $\displaystyle \int \frac{\sqrt{x-1}}{(x+1)\sqrt{3x^2 + 1}}\,dx$

解.

$$\int \frac{\sqrt{x-1}}{(x+1)\sqrt{3x^2 + 1}}\,dx = \sqrt{2}\int \frac{t^2}{\sqrt{1 - t^6}}\,dt, \quad x = \frac{1 + t^2}{1 - t^2} \qquad \square$$

81. $\displaystyle \int \sqrt[3]{\frac{e^x(e^x + 1)}{(e^x + 3)(e^x + 4)}}\,dx$

解.

$$\int \sqrt[3]{\frac{e^x(e^x + 1)}{(e^x + 3)(e^x + 4)}}\,dx = \int \frac{1}{1 - t^3}\,dt, \quad t = \sqrt[3]{\frac{e^x(e^x + 3)^2}{(e^x + 1)^2(e^x + 4)}} \qquad \square$$

82. $\displaystyle\int \sqrt[3]{\frac{(e^x - 1)^2}{(e^x + 2)(2e^x + 1)}}\, dx$

解.

$$\int \sqrt[3]{\frac{(e^x - 1)^2}{(e^x + 2)(2e^x + 1)}}\, dx$$
$$= \sqrt[3]{4} \int \frac{1}{(t+3)\sqrt[3]{t^2 - 1}}\, dt - \sqrt[3]{4} \int \frac{1}{(t-3)\sqrt[3]{t^2 - 1}}\, dt, \quad t = 3\frac{e^x + 1}{e^x - 1}$$
$$= 3\sqrt[3]{4} \int \frac{1}{u^3 + 8}\, du - 3\sqrt[3]{4} \int \frac{1}{v^3 - 8}\, dv, \quad u = \sqrt[3]{\frac{(t-1)^2}{t+1}}, v = \sqrt[3]{\frac{(t+1)^2}{t-1}} \qquad \Box$$

83. $\displaystyle\int \sqrt[3]{\frac{e^{2x} + 4e^x - 1}{e^{2x} + 22e^x + 125}}\, dx$

解.

$$\int \sqrt[3]{\frac{e^{2x} + 4e^x - 1}{e^{2x} + 22e^x + 125}}\, dx = \frac{3}{5} \int \frac{1}{t^3 - 1}\, dt, \quad t = \frac{e^{2x} + 10e^x + 5}{\sqrt[3]{(e^{2x} + 22e^x + 125)(e^{2x} + 4e^x - 1)^2}} \Box$$

84. $\displaystyle\int \frac{(\sinh x - 1)\sqrt{2\sinh x + \cos 2x + 1}}{\sinh x(\sinh x - 3\cosh x + 1)}\, dx$

解.

$$\int \frac{(\sinh x - 1)\sqrt{2\sinh x + \cos 2x + 1}}{\sinh x(\sinh x - 3\cosh x + 1)}\, dx = \frac{1}{\sqrt{2}} \int \frac{\sqrt{1 + t^2}}{t(1 - 2t)}\, dt, \quad t = e^{-x}\tanh\frac{x}{2} \qquad \Box$$

85. $\displaystyle\int \frac{\cos x}{\sqrt{\cos^2 x + 2\cos x - 2}}\, dx$

解.

$$\int \frac{\cos x}{\sqrt{\cos^2 x + 2\cos x - 2}}\, dx = -\int \frac{t}{\sqrt{-t^4 - 2t^4 + 3t^2 + 2t - 2}}\, dt, \quad t = \cos x$$
$$= -\frac{2}{3} \arcsin\frac{\sqrt{t+1}\sqrt{t^2 + 2t - 2}}{\sqrt{2}}\, dt, \quad \text{PE} \qquad \Box$$

86. $\displaystyle\int \frac{3\cos x + \lambda}{\sqrt{\cos^2 x + 2(\lambda - 1)\cos x + \lambda^2 - 2}}\, dx$

解.

$$\int \frac{3\cos x + \lambda}{\sqrt{\cos^2 x + 2(\lambda-1)\cos x + \lambda^2 - 2}}\,\mathrm{d}x$$
$$= -\int \frac{3t + \lambda}{\sqrt{-t^4 - 2(\lambda-1)t^3 - (\lambda^2-3)t^2 + 2(\lambda-1)t + \lambda^2-2}}, \quad t = \cos x$$
$$= 2\arcsin \frac{\sqrt{1-t}\sqrt{t^2 + 2(\lambda-1)t + \lambda^2-2}}{\sqrt{2}(\lambda-1)}, \quad \text{PE} \qquad \square$$

87. $\displaystyle\int \frac{5\cos^2 x + 10\cos x + 2}{\sqrt{\cos^4 x + 6\cos^3 x + 10\cos^2 x + 2\cos x - 2}}\,\mathrm{d}x$

解.

$$\int \frac{5\cos^2 x + 10\cos x + 2}{\sqrt{\cos^4 x + 6\cos^3 x + 10\cos^2 x + 2\cos x - 2}}\,\mathrm{d}x$$
$$= -\int \frac{5t^2 + 10t + 2}{\sqrt{-t^6 - 6t^5 - 9t^4 + 4t^3 + 12t^2 + 2t - 2}}\,\mathrm{d}t, \quad t = \cos x$$
$$= \arcsin(t^5 + 5t^4 + 4t^3 - 8t^2 - 4t + 3), \quad \text{PE} \qquad \square$$

88. $\displaystyle\int \frac{2\sqrt{3}\sin x + 1}{\sqrt{\sin x \cos x + (\sqrt{2}+\sqrt{3})(\sin x + \cos x)}}\,\mathrm{d}x$

解.

$$\int \frac{2\sqrt{3}\sin x + 1}{\sqrt{\sin x \cos x + (\sqrt{2}+\sqrt{3})(\sin x + \cos x)}}\,\mathrm{d}x$$
$$= \sqrt{2}\int \frac{\sqrt{6}t + 1}{\sqrt{1-t^2}\sqrt{2t^2 + (4+2\sqrt{6})t - 1}} - 2\sqrt{3}\int \frac{1}{\sqrt{2t^2 + (4+2\sqrt{6})t - 1}}\,\mathrm{d}t,$$
$$t = \frac{\sin x + \cos x}{\sqrt{2}}$$
$$= \frac{2\sqrt{6}}{3}\arcsin \frac{((2-\sqrt{6})t + 1 - \sqrt{6})\sqrt{1-t}}{\sqrt{2}} + \cdots, \quad \text{PE} \qquad \square$$

89. $\displaystyle\int \frac{\sin x}{\sqrt{\sin 2x + 2\sqrt{2}(\sin x + \cos x) - 3}}\,\mathrm{d}x$

解.

$$\int \frac{\sin x}{\sqrt{\sin 2x + 2\sqrt{2}(\sin x + \cos x) - 3}}\, dx$$

$$= -\frac{1}{2}\int \frac{t}{\sqrt{1-t^2}\sqrt{t^2+2t-2}}\, dt - \frac{1}{2}\int \frac{1}{\sqrt{t^2+2t-2}}\, dt, \quad t = \frac{\sin x + \cos x}{\sqrt{2}}$$

$$= \frac{1}{3}\arcsin \frac{\sqrt{t+1}\sqrt{t^2+2t-2}}{\sqrt{2}} + \cdots, \quad \text{PE} \qquad \square$$

90. $\displaystyle \int \frac{\sec x + \cos\tan x}{\sin x + \sin\tan x}\, dx$

解.

$$\int \frac{\sec x + \cos\tan x}{\sin x + \sin\tan x}\, dx = -2\operatorname{artanh}\left(\tan\frac{\frac{\pi}{2}-x}{2}\tan\frac{\frac{\pi}{2}-\tan x}{2}\right), \quad \text{模板} \qquad \square$$

91. $\displaystyle \int \frac{\tan x + \cos\sec x}{1 + \sin x \sin\sec x}\, dx$

解.

$$\int \frac{\tan x + \cos\sec x}{1 + \sin x \sin\sec x}\, dx = -2\arctan\left(\tan\frac{\frac{\pi}{2}-x}{2}\tan\frac{\frac{\pi}{2}-\sec x}{2}\right), \quad \text{模板} \qquad \square$$

92. $\displaystyle \int \frac{\left(\sqrt{x^2-x+1}\sqrt{x^2+x+1}\right)^{1009}(x^2-1)}{\left(\sqrt{x^2-x+1}+\sqrt{x^2+x+1}\right)^{2022}}\, dx$

解.

$$\int \frac{\left(\sqrt{x^2-x+1}\sqrt{x^2+x+1}\right)^{1009}(x^2-1)}{\left(\sqrt{x^2-x+1}+\sqrt{x^2+x+1}\right)^{2022}}\, dx = \int \frac{t^{1010}}{(t+1)^{2022}}\, dt, \quad t = \frac{\sqrt{x^2-x+1}}{\sqrt{x^2+x+1}} \qquad \square$$

93. $\displaystyle \int x^{k-1}\left(x^n + \sqrt{1+x^{2n}}\right)^{2+\frac{k}{n}}\, dx$

解.

$$\int x^{k-1}\left(x^n + \sqrt{1+x^{2n}}\right)^{2+\frac{k}{n}}\, dx = \frac{1}{n2^{1+\frac{k}{n}}}\int t^{\frac{k}{n}-1}(t+2)\, dt, \quad t = \left(x^n + \sqrt{x^{2n}+1}\right)^2 - 1 \qquad \square$$

94. $\displaystyle \int \frac{\sqrt{x^{16}-28x^{12}+6x^8-28x^4+1}}{(3x^8-6x^4-1)\sqrt[4]{1-x^4}}\, dx$

解.

$$\int \frac{\sqrt{x^{16} - 28x^{12} + 6x^8 - 28x^4 + 1}}{(3x^8 - 6x^4 - 1)\sqrt[4]{1 - x^4}} \, dx = \frac{1}{3} \int \frac{1}{\sqrt[4]{1 - t^4}} \, dt, \quad t = \frac{x(x^8 + 6x^4 - 3)}{3x^8 - 6x^4 - 1}$$ □

95. $\displaystyle\int \frac{\sqrt[3]{(x^2 + 12)^2}}{(x^2 + 4)\sqrt[6]{x^2 + 3}} \, dx$

解.

$$\int \frac{\sqrt[3]{(x^2 + 12)^2}}{(x^2 + 4)\sqrt[6]{x^2 + 3}} \, dx = 3 \int \frac{1}{1 - t^6} \, dt, \quad t = \frac{x}{\sqrt[3]{x^2 + 12}\sqrt[6]{x^2 + 3}}$$ □

96. $\displaystyle\int \frac{1}{\sqrt[3]{(x + 1)(3x^2 + 3x + 1)}} \, dx$

解.

$$\int \frac{1}{\sqrt[3]{(x + 1)(3x^2 + 3x + 1)}} \, dx = \frac{1}{\sqrt[3]{3}} \int \frac{1}{\sqrt[3]{t^3 + 1}} \, dt, \quad t = 3x + 2$$ □

97. $\displaystyle\int \frac{x}{\sqrt[3]{(x^3 - a)(x^3 - 2a)}} \, dx$

解.

$$\int \frac{x}{\sqrt[3]{(x^3 - a)(x^3 - 2a)}} \, dx = \frac{1}{2} \int \frac{1}{t^3 - 1} \, dt, \quad t = \frac{\sqrt[3]{(x^3 - a)^2}}{x\sqrt[3]{x^3 - 2a}}$$ □

98. $\displaystyle\int \frac{\sqrt{\sqrt{x + 1} + \sqrt{x - 1}}}{\sqrt{x^2 - 1}\sqrt[8]{2x^2 - 1}} \, dx$

解.

$$\int \frac{\sqrt{\sqrt{x + 1} + \sqrt{x - 1}}}{\sqrt{x^2 - 1}\sqrt[8]{2x^2 - 1}} \, dx = 2\sqrt[8]{8} \int \frac{1}{\sqrt[8]{t^8 + 16}} \, dt, \quad t = \sqrt{x + 1} + \sqrt{x - 1}$$ □

99. $\displaystyle\int \frac{11x^2 + 28x - 28}{(x + 4)(x^2 + 24x + 44)\sqrt{x^3 + 1}} \, dx$

解.

$$\int \frac{11x^2 + 28x - 28}{(x + 4)(x^2 + 24x + 44)\sqrt{x^3 + 1}} \, dx = \frac{2}{3} \int \frac{1}{t^2 + 7} \, dt, \quad t = \frac{(x + 16)\sqrt{x^3 + 1}}{11x + 8}$$ □

100. $\displaystyle\int \frac{x^2 + ax - 1}{(x^2 + 1)\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}}\, \mathrm{d}x$

解.

$$\int \frac{x^2 + ax - 1}{(x^2 + 1)\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}}\, \mathrm{d}x$$
$$= \frac{a - 2}{4\sqrt{2}} \arctan \frac{(x^2 + 1)^2 - (x^2 + 2x - 1)\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}}{2\sqrt{2}x(x^2 - 1)}$$
$$+ \frac{a + 2}{4\sqrt{2}} \operatorname{artanh} \frac{(x^2 + 1)^2 - (x^2 - 2x - 1)\sqrt{x^4 - 2x^3 + 2x^2 + 2x + 1}}{2\sqrt{2}x(x^2 - 1)}, \quad \text{PE} \qquad \square$$