Model setting:

$$y_{ij} \sim N(X_{ij}\theta_i, V^2)$$

$$\theta_i \sim N(\mu, \sum)$$

X_{ij} includes four elements: 1).log of price; 2). Display(0 or 1); 3). Display*(log of price); 4). Constant(1)

Prior:

$$V^2 \propto 1$$

$$(\mu, \sum) \sim NIW(m_0, \omega_0, c_0, d_0)$$

Posterior update and Gibbs sampling:

g: #groups; p_i: #observations within i[th] group; n: #total observations

$$n = \sum_i p_i = \sum_i \sum_j 1$$

$$rss = \sum_i \sum_j (y_{ij} - X_{ij}\theta_i)^2$$

$$P(V^2|\theta, y, \mu, \sum) \sim \Gamma(\tfrac{n}{2} - 1, \tfrac{1}{2}(rss)) \quad (1)$$

$$\Omega = \frac{1}{V^2} \times I_{(p_i \times p_i)}$$

$$D = \left(X_i^T \Omega X_i + \sum^{-1}\right)^{-1}$$

$$mean = D(X_i^T \Omega y_i + \sum^{-1}\mu)$$

$$P(\theta_i|y, V^2, \mu, \sum) \sim N(mean, D) \quad (2)$$

$$P\big((\mu, \sum)\big|\theta, y, V^2\big) = P\big((\mu, \sum)\big|\theta\big) \sim N(m_n, \omega_n, c_n, d_n) \quad (3)$$

$$\omega_n = \omega_0 + g$$

$$d_n = d_0 + g$$

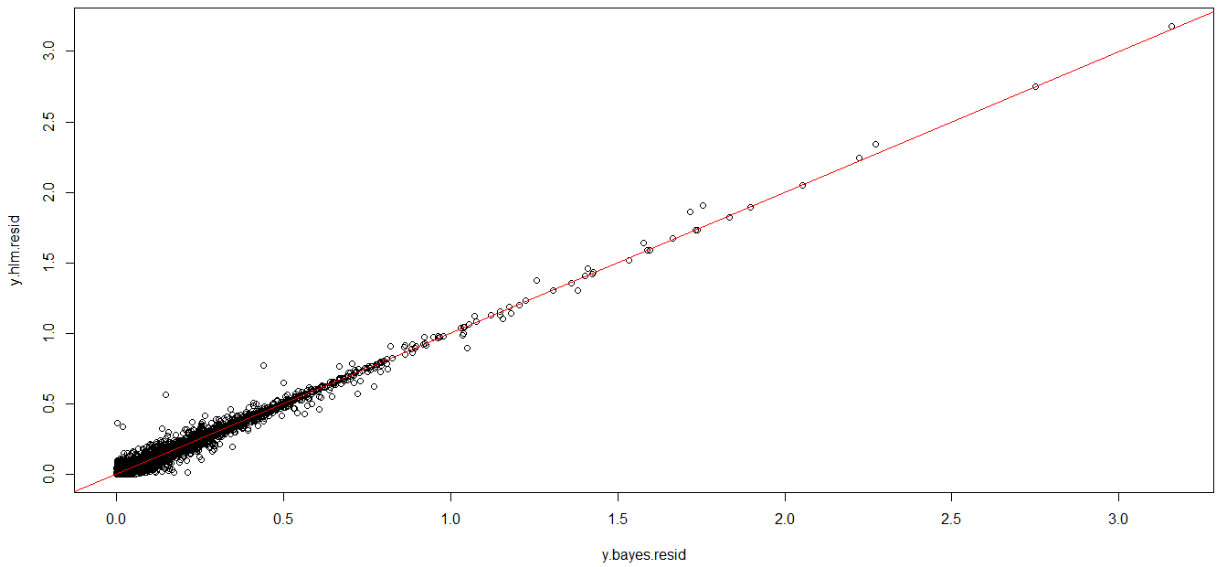
$$c_n = c_0 + S + \frac{\omega_0 g}{\omega_0 + g}(\beta_b - m_0)(\beta_b - m_0)^T$$

$$S = \sum_{i=1}^{g}(x_i - \beta_b)(x_i - \beta_b)^T$$

$$m_n = \frac{\omega_0 m_0 + g x_b}{\omega_0 + g}$$

Analyzing MCMC results:

1. Comparing residue by using lmer() and full Bayesian:



As can be seen, by using lmer() and full Bayesian way, the residue are quite similar, demonstrate the accuracy of MCMC.

2. Demand curve for each store: