Chenguang He 8977866134 EE 112 HW#6

Problem 1.

(a) $N(t) = \sum_{n=0}^{\infty} I(0,t) \cdot S_n$, $S_n = \sum_{i=0}^{n} X_i$

$E(N(t)) = \sum_{n=1}^{\infty} P(N(t) \geq n) = \sum_{n=1}^{\infty} P(S_n \leq t) = \sum_{n=1}^{\infty} F_n(t)$. $F_n(t)$ is CDF that $P(X_n \leq t)$

(b) if $y > t$ and $X_n > t$ $P(N(t)=1) = P(S_1 = t)$

if $n=1$, $P(N(t)=1 \mid X_n = y) = P(S_1 = t \mid X_1 = y) = 0$

if $n > 1$ $P(N(t)=1 \mid X_n = y) = P(S_1 = t \mid X_n = y) = P(S_1 = t) = P(X_1 = t)$

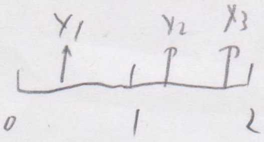
if $y < t$, $X_n < t$ $P(N(t)=1 \mid X_n = y) = P(S_1 = t \mid X_n = y) = P(S_1 = t)$

Problem 2 is a MATLAB programming problem. All procedure and result is shown in the end

Extra credit

(a) $P(X_1 \leq 1, X_2 > 1 \mid N(2) = 3) = P(X_1 \leq 1, X_2 > 1, X_2 < X_3 \leq 2)$

So $0 < X_1 \leq 1 < X_2 < X_3 \leq 2$



joint density for 3 first packet arrival

$P(0 \leq X_1 \leq 1) = \int_0^1 f(X_1 \mid N(1)=1) \cdot P(N(1)=1)\, dx$. $f(X_1 \mid N(1)=1) = 1$

$P(1 < X_2 < X_3) = \int_1^{X_2} f_X(X_1, X_2, X_3 \mid N(2)=3) \cdot P(N(2)=3)\, dx$ $f_X(X_1, X_2, X_3 \mid N(2)=3)$

$P(X_2 < X_3 < 2) = \int_{X_2}^{2} f_X(X_1, X_2, X_3 \mid N(2)=3) \, P(N(2)=3)\, dx$. $= \frac{3!}{2^3}$

So

$$P(X_1 \le 1 \cap 1 < X_2 < 2 \cap X_2 < X_3 \le 2) = \int_0^1 \int_1^2 \int_{x_2}^2 1 \cdot \frac{3!}{2^3}\, dx_3\, dx_2\, dx_1$$

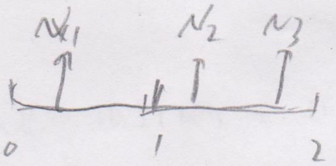
$$= \int_0^1 \int_1^2 \int_{x_2}^2 \frac{3}{4}\, dx_3\, dx_2\, dx_1$$

$$= \int_0^1 \int_1^2 \left(\frac{3}{2} - \frac{3}{4}x_2\right) dx_2\, dx_1 = \int_0^1 \frac{3}{8}\, dx_1 = \frac{3}{8}$$

(b) $P(N(1)=1 \mid N(2)=3)$. In the time-line diagram

that $P(N(1)=1 \mid N(2)=3) = P(X_1 \le 1 \cap 1 < X_2 < 2 \cap X_2 < X_3 < 2)$

So $P(N(1)=1 \mid N(2)=3) = \frac{3}{8}$



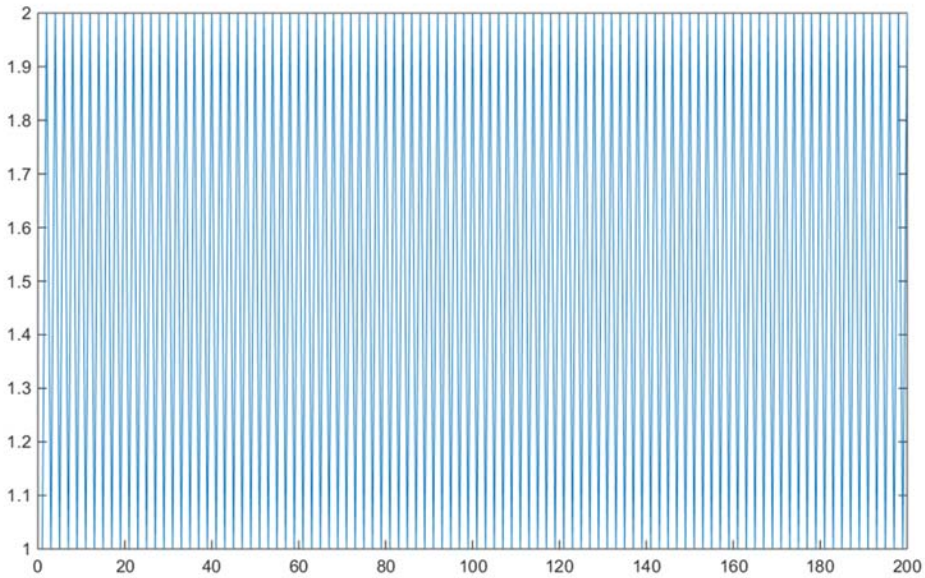
Problem 2

```
function chain = mc(probabilities_matrix, starting_value)
  chain_length = 200;
  chain = zeros(1,chain_length);
  chain(1)=starting_value;
  %sum = 0;
  for i=2:chain_length
    current_step = probabilities_matrix (chain(i-1),:);
    cumulative_distribution = cumsum(current_step);
    r = rand();
    chain(i) = find(cumulative_distribution>r,1);
  end
end
```

(a)

P = [0,1;1,0], start_value = 1

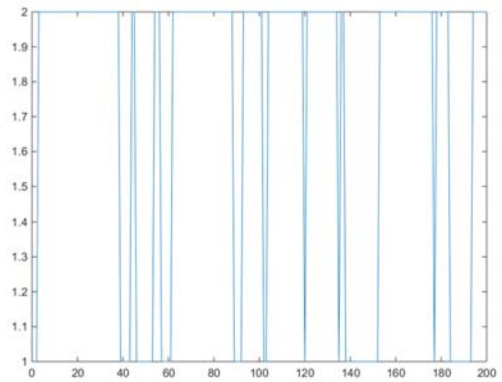
The Output diagram of state is as follow:



the state switch between 1 and 2. We repeat this 10 times and get same diagrams. Time average of this path and all paths is 1.5. Ensemble average is 1.5 as well. This is an ergodic MC.

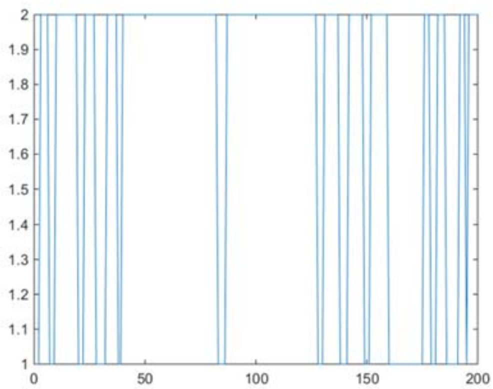
(b), P = [0.75,0.25;0.1,0.9], start_value = 1

Output state diagram of first path is shown as follow:

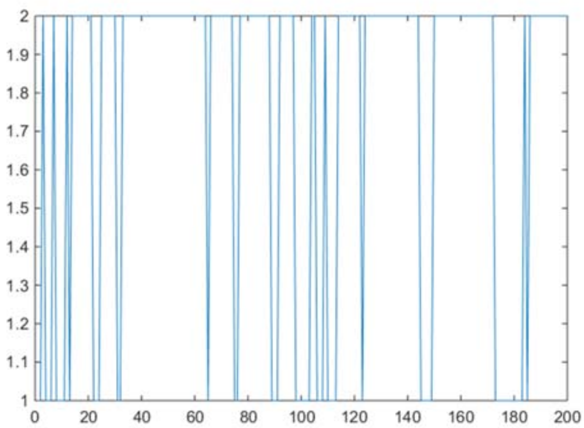


Time average of this path is 1.667.

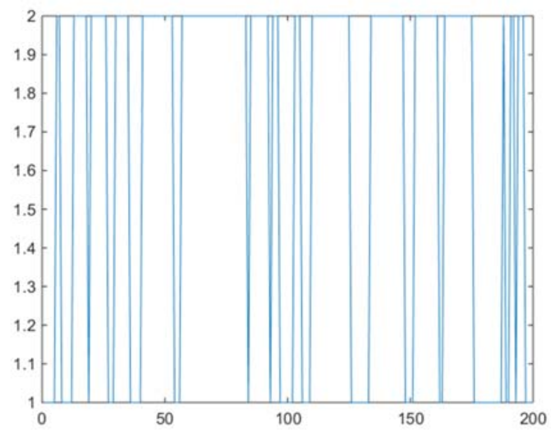
Output state diagram of second path is shown as follow, the time average is 1.5467:



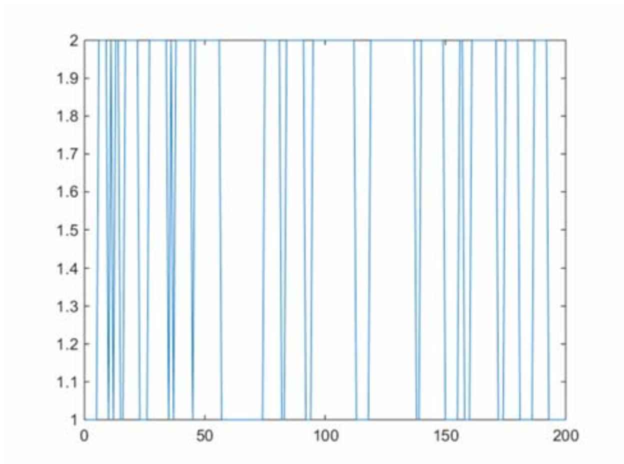
Output state transition diagram of third path with time average = 1.8:



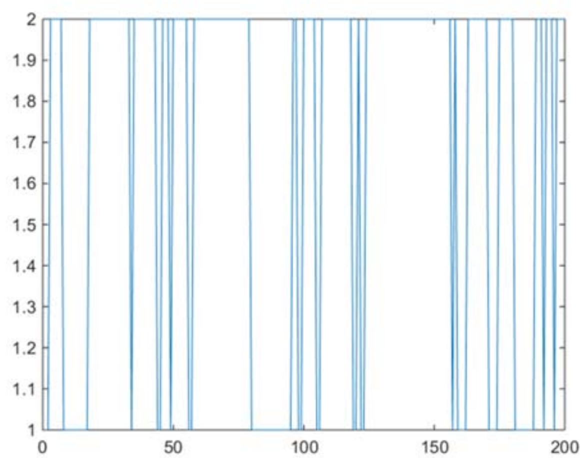
Output state transition diagram of fourth path with time average = 1.5867:



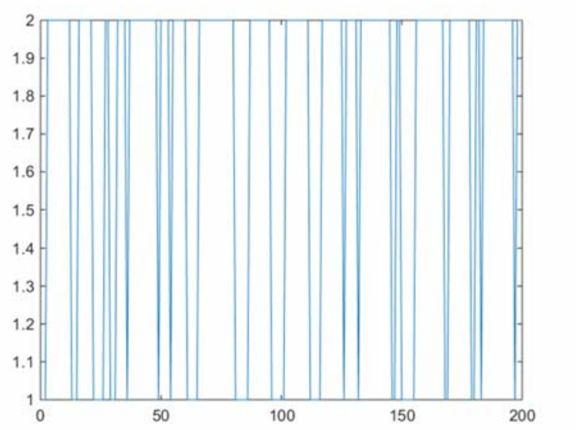
Output state transition diagram of fifth path with time average = 1.6533:



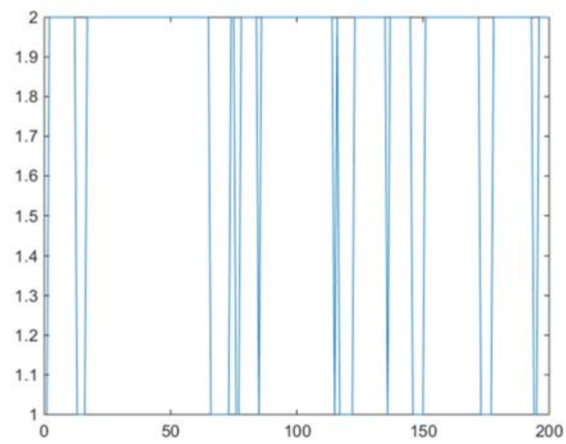
Output state transition diagram of sixth path with time average = 1.7733:



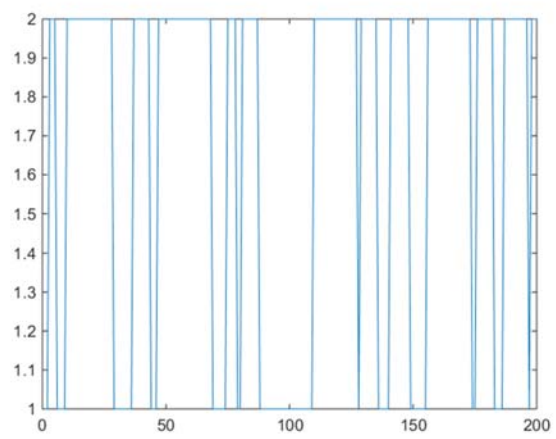
Output state transition diagram of seventh path with time average = 1.8133:



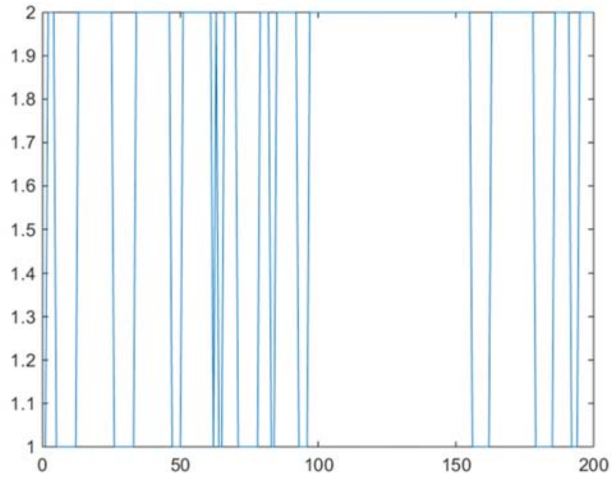
Output state transition diagram of eighth path with time average = 1.8533:



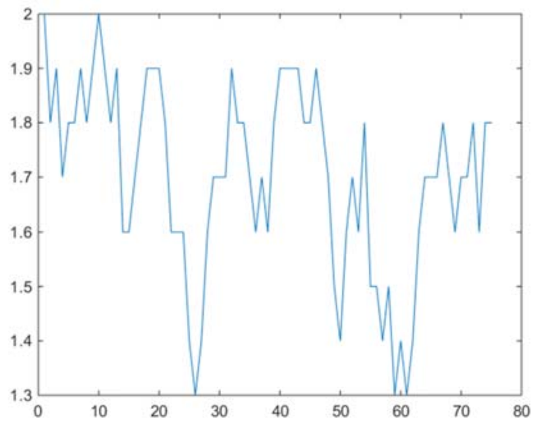
Output state transition diagram of ninth path with time average = 1.7600:



Output state transition diagram of tenth path with time average = 1.8000:



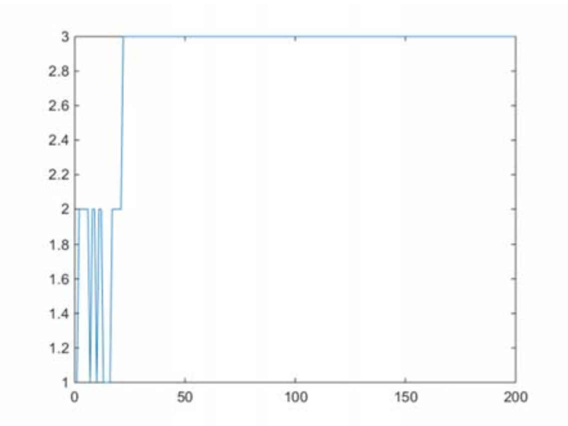
For above 10 paths, we calculate ensemble average from t = 75 to 200 and the diagram is shown as follow:



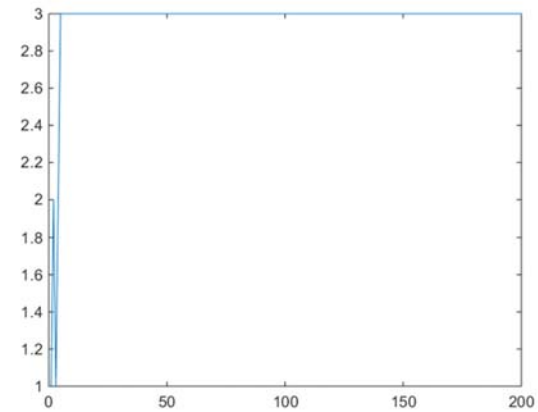
The ensemble average is 1.7013. This is not an ergodic MC.

(c), P = [0.48,0.48,0.04;0.22,0.7,0.08;0,0,1], start_value = 1

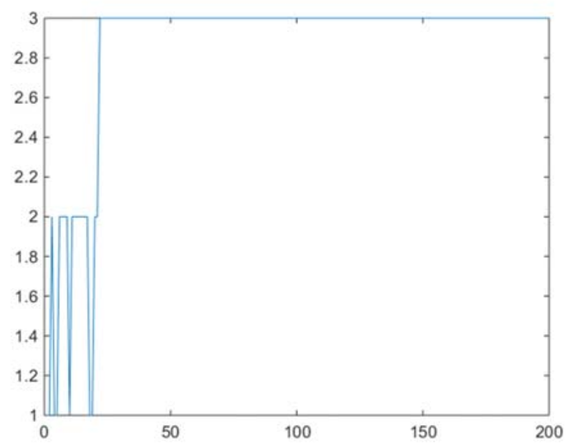
Output state diagram of first path is shown as follow with time average = 3.0000:



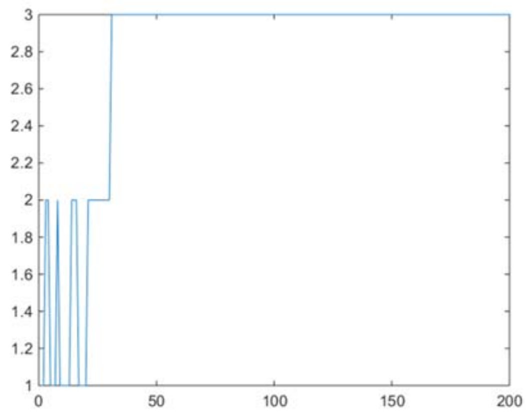
Output state diagram of second path is shown as follow with time average = 3.0000:



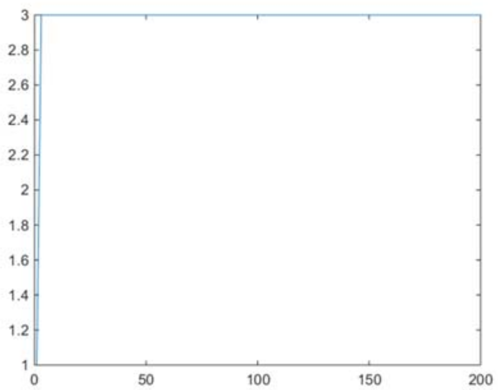
Output state diagram of third path is shown as follow with time average = 3.0000:



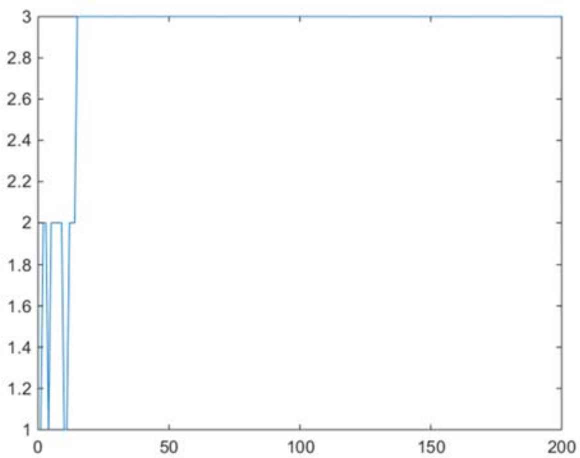
Output state diagram of fourth path is shown as follow with time average = 3.0000:



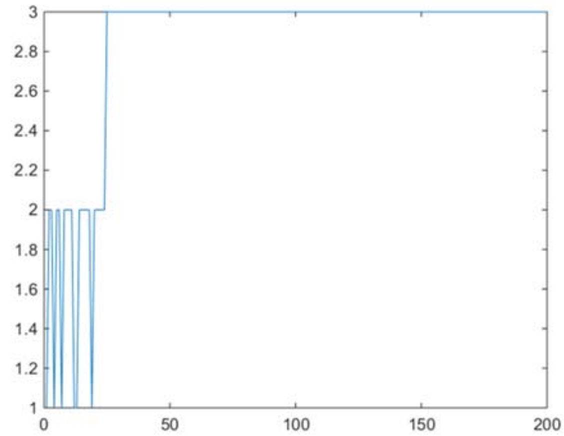
Output state diagram of fifth path is shown as follow with time average = 3.0000:



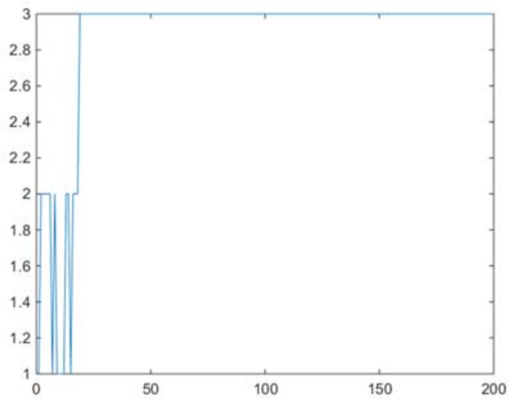
Output state diagram of sixth path is shown as follow with time average = 3.0000:



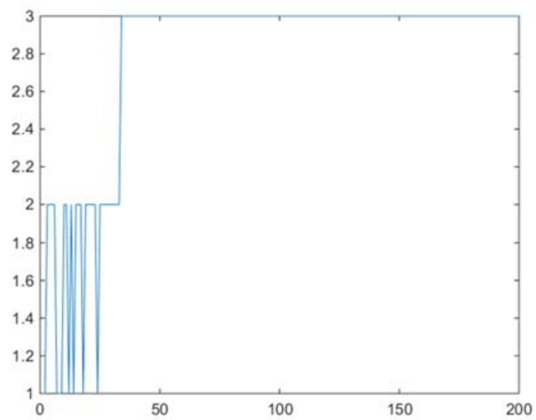
Output state diagram of seventh path is shown as follow with time average = 3.0000:



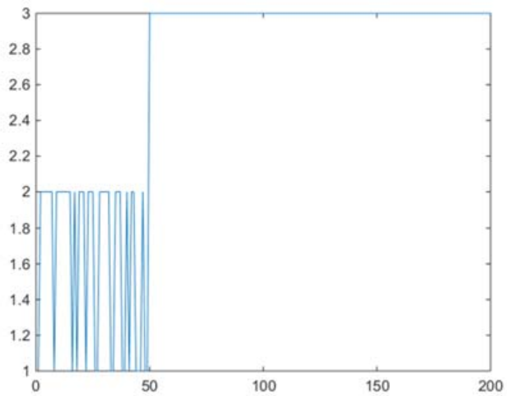
Output state diagram of eighth path is shown as follow with time average = 3.0000:



Output state diagram of ninth path is shown as follow with time average = 3.0000:



Output state diagram of tenth path is shown as follow with time average = 3.0000:



The ensemble average and time average for t >= 75 are same, both of them are 3. This MC is an ergodic markov chain.