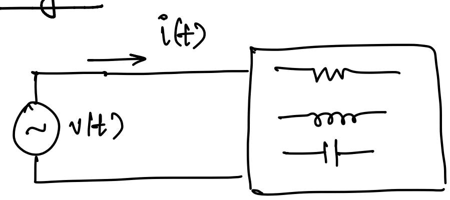
Module 3

AC Power Analysis



$\cos A . \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$

Instantaneous Power: $p(t) = v(t)\, i(t)$

$$v(t) = V_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$p(t) = V_m \cos(\omega t + \theta) . I_m \cos(\omega t + \phi)$$

$$= \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\substack{\text{constant} \\ \text{(DC term)}}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\substack{\text{twice frequency} \\ \text{(harmonic)}}}$$

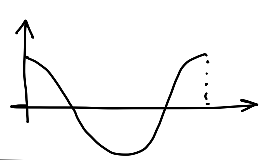
Average Power:

$$P_{avg} = \frac{1}{T} \int_0^T p(t)\, dt$$

$$= \underbrace{\frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi) . dt}_{} + \underbrace{\frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi) dt}_{0}$$

$$= \frac{V_m I_m}{2} \cos(\theta - \phi) + \quad 0$$

Average of Cos



$$\boxed{P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)}$$

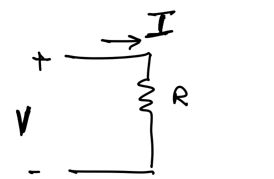
$$P_{avg} = \frac{1}{2}\,\mathrm{Re}\{\mathbf{V}\mathbf{I}^*\}$$

$$\mathbf{I} = I_m e^{j\phi}$$
$$\mathbf{I}^* = I_m e^{-j\phi}$$

$$\cancel{P_{avg} = \mathbf{V}\mathbf{I}^*}$$

No phasor for power !

Average power absorbed by a purely resistive element

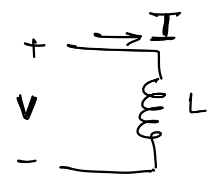


$$P_{avg} = \frac{V_m I_m}{2} \cos\underbrace{(\theta - \phi)}_{0^\circ}$$

$$\mathbf{V} = \mathbf{I}R$$

$$P_{avg} = \frac{V_m I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

Purely inductive element



$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

$$= \frac{V_m I_m}{2} \cos(90^\circ)$$

$$\boxed{P_{avg} = 0}$$

$$\mathbf{V} = j\omega L\,\mathbf{I}$$

$$(\theta - \phi) =$$

$$V_m e^{j\theta} = j\omega L\, I_m e^{j\phi}$$

$$\Rightarrow \frac{V_m}{I_m} e^{j\theta - \phi} = j\omega L$$

$$\theta - \phi = 90^\circ$$

Purely capacitive element

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

$$\theta - \phi = -90^\circ$$

$$\boxed{P_{avg} = 0}$$

$p(t)$

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) \quad ✓$$

$$P_{avg} = Re\left\{\frac{V I^*}{2}\right\}$$

$$v(t) = V_m \cos(\omega t + \theta)$$

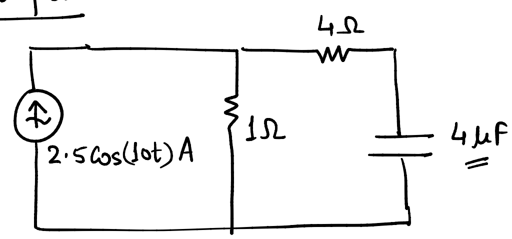
$$\downarrow$$

$$v(t) = Re\left\{V_m e^{j\omega t + j\theta}\right\}$$

$$\downarrow \quad e^{j\omega t}$$

$$V = V_m \angle \theta$$

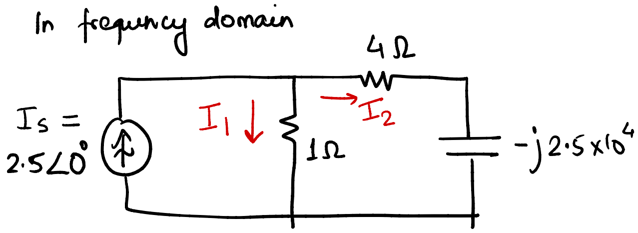
Example



① Find power delivered to each element at $t = 0, 10, 20$ ms

② Find average power delivered to each element.

In frequency domain



Power delivered to 1Ω

$$p_{1\Omega}(t) = v_{1\Omega}(t)\, i_{1\Omega}(t)$$

$$I_{1\Omega} = I_1 = I_s \frac{4 - j2.5\times10^4}{1 + 4 - j2.5\times10^4} \approx I_s \quad \Rightarrow i_{1\Omega}(t) = 2.5\cos(10t) \text{ A}$$

$$V_{1\Omega} = (I_1)(1\Omega) \quad \Rightarrow \quad v_{1\Omega}(t) = 2.5\cos(10t) \text{ V}$$

$$p_{1\Omega}(t) = 2.5\cos(10t) \cdot 2.5\cos(10t)$$

$$p_{1\Omega}(t) = 3.125 + 3.125\cos(20t) \text{ Watts}$$

$$p_{1\Omega}(t=0) = 6.25 \text{ W}$$

$$p_{1\Omega}(t=10\text{ms}) = 6.1877 \text{ W}$$

$$p_{1\Omega}(t=20\text{ms}) = 6.0033 \text{ W}$$

Average power delivered to 1Ω

$$P_{avg\,1\Omega} = 3.125 \text{ W}$$

Power delivered to 4Ω :-

$$p_{4\Omega}(t) = v_{4\Omega}(t) \cdot i_{4\Omega}(t)$$

$$I_{4\Omega} = I_2 = I_s \frac{1}{5 - j2.5\times10^4} = 10^{-4}\angle 90^\circ \ A \rightarrow i_{4\Omega}(t)$$

$$V_{4\Omega} = 4(I_{4\Omega}) = 4\times10^{-4}\angle 90^\circ \ V \rightarrow v_{4\Omega}(t)$$

$$p_{4\Omega}(t) = 2\times10^{-8} + 2\times10^{-8}\cos(20t) \ W$$

$$p_{4\Omega}(t=0) = 4\times10^{-8} \ W$$

$$p_{4\Omega}(t=10ms) = 3.96\times10^{-8} \ W$$

$$p_{4\Omega}(t=20ms) = 3.84\times10^{-8} \ W$$

$$P_{avg\,4\Omega} = 2\times10^{-8} \ W$$

Power delivered to capacitor :

$$p_c(t) =$$

$$P_{avg.c} =$$

$$I_c = I_2 = 10^{-4}\angle 90^\circ \ A \leftarrow \longrightarrow i_c(t) \qquad \Rightarrow p(t) = i_c(t)v_c(t)$$

$$V_c = (-j2.5\times10^4)I_2 = 2.5 \ V \longrightarrow v_c(t)$$

$$\cancel{\underbrace{\underline{I_c} \times \underline{V_c}}_{\text{Phasor of power}} \longrightarrow p(t)}$$

$$i_c(t) = 10^{-4}\cos(10t + 90^\circ)$$

$$v_c(t) = 2.5\cos(10t)$$

$$p_c(t) = 1.25\times10^{-4}\cos(20t + 90^\circ) \ W$$

$$p_c(t=0) = 0 \ W$$

$$p_c(t=10ms) = 2.48\times10^{-5} \ W$$

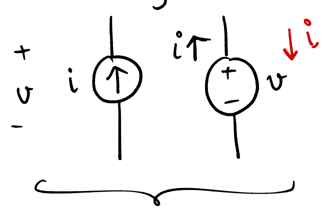
$$p_c(t=20ms) = 4.86\times10^{-5} \ W$$

$$P_{avg.c} = 0$$

① $p_{1\Omega}(t) + p_{4\Omega}(t) + p_c(t) = \text{Constant } 1$ ✗

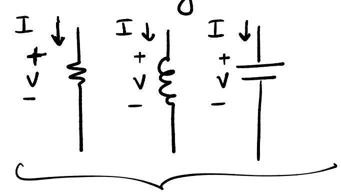
② $P_{avg,1\Omega} + P_{avg,4\Omega} + P_{avg,c} = \text{Constant } 2$ ✓ $= P_{avg,\,source}$

Active sign convention

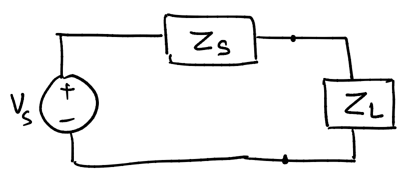


$v \cdot i = -20\,W$

Passive sign convention



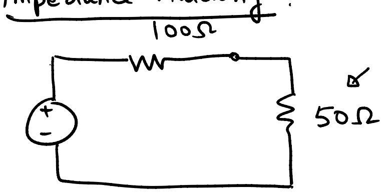
Maximum Power Transfer :



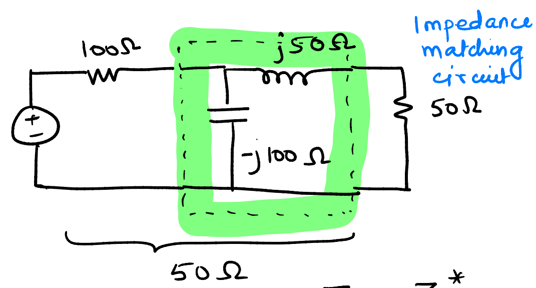
$R_S = R_L$

Max power delivered to load, when $\boxed{Z_L = Z_S^*}$ ✓

Impedance Matching :-

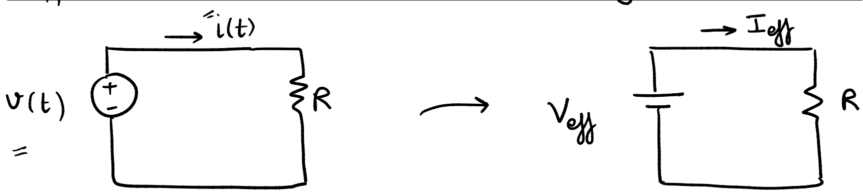


⇒



$Z_L = Z_S^*$

Effective value (RMS value) of voltage and current :—



v(t) ← P_{avg} = P_{avg}

$$\Rightarrow \frac{1}{T}\int_0^T i^2(t)\,R\,dt = I_{eff}^2 R = \frac{V_{eff}^2}{R}$$

$$\Rightarrow I_{eff} = \underbrace{\sqrt{\frac{1}{T}\int_0^T i^2(t)\,.\,dt}} =$$

Root mean square

Sinusoidally varying sources

$$i(t) = I_m \cos(\omega t + \phi)$$

$$2\cos^2 x = \cos 2x + 1$$

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t + \phi)\,dt}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\boxed{I_{eff} = \frac{I_m}{\sqrt{2}}}$$ rms value

Suppose, $\mathbf{I} = 10\angle 90^\circ$ A

$$\Rightarrow \mathbf{I} = \frac{10}{\sqrt{2}}\angle 90^\circ \text{ A rms}$$

$$\boxed{V_{eff} = \frac{V_m}{\sqrt{2}}}$$ rms value

$$V_1 = 100\angle -60^\circ \text{ V rms}$$

$$V_1 = 100\sqrt{2}\ \angle -60^\circ \text{ V}$$

Apparent Power

$$P_{apparent} = I_{eff} V_{eff}$$

Power Factor

$$PF = \frac{\text{average power}}{\text{apparent power}} = \frac{(V_m I_m / 2) \cos(\theta - \phi)}{V_{eff} I_{eff}} = \cos(\theta - \phi)$$

$$PF = \cos \underbrace{(\theta - \phi)}_{\text{Power factor angle}}$$

Review

Instantaneous Power : $p(t) = v(t)\, i(t)$

Average Power : $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} \mathrm{Re}\{ \mathbf{V}\mathbf{I}^* \}$

$$P_{avg,\,sources} = \sum P_{avg,\,elements}$$

$$Z = R + jX$$

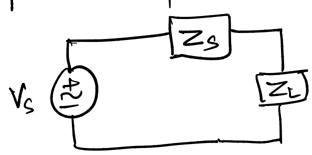
resistive

reactive

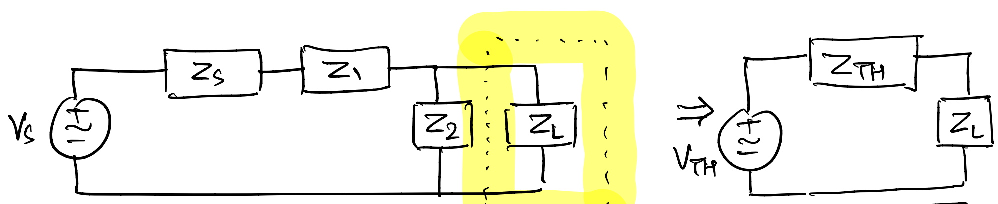
P_{avg} for resistive load : $P_{avg} \neq 0$

P_{avg} for reactive load : $P_{avg} = 0$

Max. power Transfer :



$$Z_L = Z_S^*$$



$$\boxed{Z_L = Z_{TH}^*}$$

Effective V and I values (RMS values)

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2(t)\, dt}$$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_{eff} = \frac{I_m}{\sqrt{2}} \quad , \quad V_{eff} = \frac{V_m}{\sqrt{2}}$$

$$\mathbf{I} = I_m \angle \phi = \sqrt{2}\, I_{eff} \angle \phi$$

I_{eff} A rms

V_{eff} V rms

Apparent Power

$P_{apparent} = I_{eff} \cdot V_{eff}$ VA (Volt-Ampere)

Power Factor

$$PF = \cos(\theta - \phi) = \frac{P_{average}}{P_{apparent}}$$

For purely resistive load: $PF = 1$ (max) $\quad \theta-\phi \;\; 90^\circ$

For purely reactive load: $PF = 0$ (min) $\quad \theta-\phi \;\; -90^\circ$

Assume PF of a load = 0.5

$$\cos(\theta-\phi) = 0.5$$

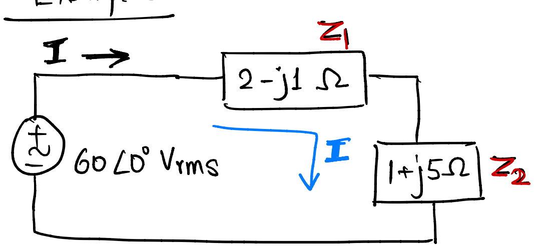
$$\Rightarrow (\theta-\phi) = \pm 60^\circ$$

$PF = 0.5$ leading $\rightarrow$ capacitive $(\theta-\phi) < 0$

0.5 lagging $\rightarrow$ inductive $(\theta-\phi) > 0$

$\boxed{\mathbf{I} \text{ w.r.t. } \mathbf{V}}$

Example

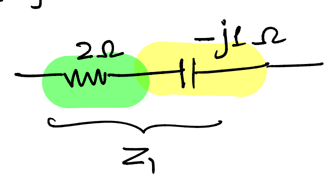


Find

1. Average power delivered to each loads
2. Average power supplied by source
3. Apparent power supplied by source
4. PF of combined load.

(1) $P_{avg,z_1} = \frac{1}{2} V_m I_m \cos(\theta-\phi) = \frac{1}{2} Re\{\mathbf{V}\mathbf{I}^*\}$

$$= \underline{I_{eff}^2 R}$$



$\mathbf{I} =$

Writing KVL;

$$-60\angle 0^\circ \text{ Vrms} + \mathbf{I}(2-j1 + 1+j5) = 0$$

$$\Rightarrow \mathbf{I} = \frac{60\angle 0^\circ \text{ Vrms}}{3+j4\ \Omega} = \underbrace{12}_{I_{eff}}\angle -53.13^\circ \text{ A rms}$$

$$P_{avg,z_1} = (12)^2\, 2 = 288 \text{ W}$$

$$\mathbf{I} = 12 \angle -53^\circ \quad A \text{ rms}$$

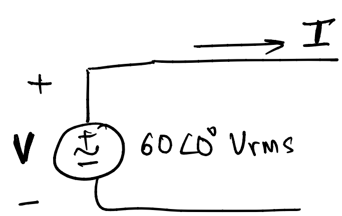
$$i(t) = \underbrace{12}_{I_{eff}} \cos(\omega t - 53^\circ) \quad \underline{A \text{ rms}}$$

$\boxed{\begin{array}{l} I_{eff}\ (A \text{ rms}) \quad \text{versus} \quad I_m\ (A) \\ \hline I_{eff} = \dfrac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{eff}\sqrt{2} \\ I_{eff} = 12 \\ I_m = I_{eff}\sqrt{2} = \underline{12\sqrt{2}} \\ i(t) = \underbrace{12\sqrt{2}}_{I_m} \cos(\omega t - 53^\circ) \quad A \end{array}}$

$$P_{avg} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{eff} I_{eff} \cos(\theta - \phi)$$

$$P_{avg, Z_2} = I_{eff}^2 R = (12)^2 1 = 144 \quad W$$

② $P_{avg,\ source} = V_{eff} I_{eff} \cos(\theta - \phi)$



$\mathbf{V}_{source} = 60 \angle 0^\circ \ V \text{ rms}$

$\mathbf{I}_{source} = 12 \angle -53.13^\circ \ A \text{ rms}$

$$P_{avg, source} = (60)(12) \cos(0 - (-53.13^\circ)) = 432 \ W$$

$$P_{avg, Z_1} = 288 \ W, \quad P_{avg, Z_2} = 144 \ W$$

$$P_{avg, Z_1} + P_{avg, Z_2} = 432 \ W$$

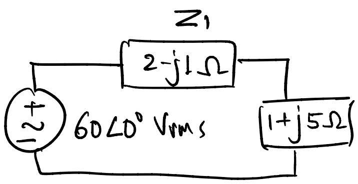
③ $P_{apparent,\ source} = I_{eff} V_{eff} = (60)(12) = 720 \quad VA$

④ PF of combined loads = PF of source

$$PF = \cos(\theta - \phi) = \cos(\underbrace{0^\circ + 53.13^\circ}) = 0.6 \ \text{lagging}$$

$(\theta - \phi) > 0 \rightarrow$ lagging

$(\theta - \phi) < 0 \rightarrow$ leading



$$P_{avg,Z_1} = I_{eff}^2 R$$

$$P_{avg,Z_1} = V_{eff}\, I_{eff} \cos(\theta - \phi)$$

$$\mathbf{V}_{Z_1} = 60\angle 0^\circ \text{ Vrms} \times \frac{2-j1}{3+j4}$$

$$= \underbrace{26.83}\ \angle -79.7^\circ \quad \text{Vrms}$$

$$\mathbf{I}_{Z_1} = \mathbf{I} = 12\angle -53.13^\circ \ \text{Arms}$$

$$P_{avg,Z_1} = 26.83 \times 12 \times \cos(-79.7^\circ - (-53.13^\circ)) = 288\ \text{W}$$

$$P_{average} = \frac{1}{2} \mathrm{Re}\{\mathbf{V}\mathbf{I}^*\} \quad ; \quad \mathbf{V} = V_m \angle \theta, \quad \mathbf{I} = I_m \angle \phi$$

$$\mathbf{V}_{eff} = V_{eff} \angle \theta = \frac{V_m}{\sqrt{2}} \angle \theta$$

$$\mathbf{I}_{eff} = I_{eff} \angle \phi = \frac{I_m}{\sqrt{2}}\, \phi$$

$$P_{average} = \mathrm{Re}\{\mathbf{V}_{eff}\, \mathbf{I}_{eff}^*\}$$

Complex Power

$$\mathbf{S} = \mathbf{V}_{eff}\, \mathbf{I}_{eff}^* \qquad \text{(unit: VA)}$$

Not a phasor of power.
This is a complex number.

$$\mathbf{S} = (V_{eff} \angle \theta^\circ)(I_{eff} \angle -\phi) = V_{eff} \cdot I_{eff}\, e^{j(\theta - \phi)}$$

$$\Rightarrow \mathbf{S} = \underbrace{V_{eff} I_{eff} \cos(\theta - \phi)}_{\substack{P \\ \text{average power}}} + j\, \underbrace{V_{eff} I_{eff} \sin(\theta - \phi)}_{\substack{Q \\ \text{reactive power}}}$$

(VA) (W) (VAR)

$$\mathbf{S} = P + jQ$$

$$|\mathbf{S}| = V_{eff} I_{eff} = P_{apparant} \quad (VA)$$

$Z = R + jX$

Purely resistive load $Z = R$, $X = 0$

$$\mathbf{S} = P + jQ \longrightarrow V_{eff} I_{eff} \underbrace{\sin(\theta - \phi)}_{0}$$

$$P \downarrow V_{eff} I_{eff} \cos(\theta - \phi)$$

$P \neq 0$

$Q = 0$

Purely reactive load $Z = jX$, $R = 0$

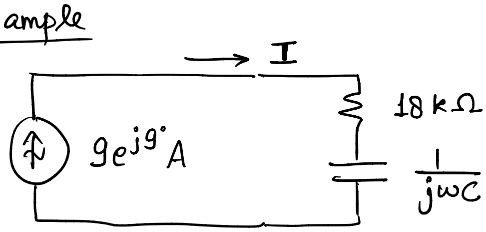
$\mathbf{S} = P + jQ$

$P = 0$

$Q \neq 0$

Q signifies energy flow rate into or out of reactive component of load.

Example



Assume $C = 1\,\mu f$, $\omega = 45$ rad/s

Find:

1. Complex power provided by source
2. Time-average power absorbed by combined load.

Find

3. Reactive power absorbed by combined load
4. Apparent power absorbed by " "
5. Power factor of " "

$S = 1.158 \times 10^{6} \angle -50.99^{\circ}$ VA

$P_{avg} = 7.29 \times 10^{5}$ W

$Q = -9 \times 10^{5}$ VAR

$|S| = P_{apparent} = 1.158 \times 10^{6}$ VA

$pf = 0.629$ leading

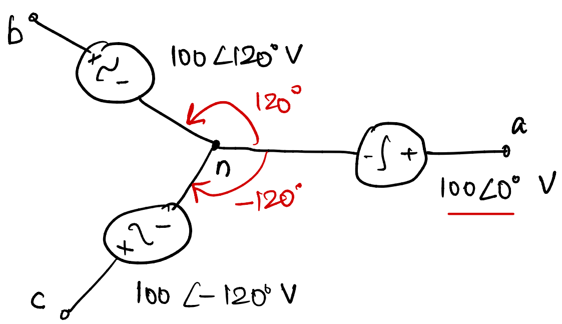
$p(t) = 7.28 \times 10^{5} + 1.158 \times 10^{6} \cos(90t - 33^{\circ})$ W

Module 4

Polyphase Circuits

(Chapter 12 of textbook)

Three-phase source



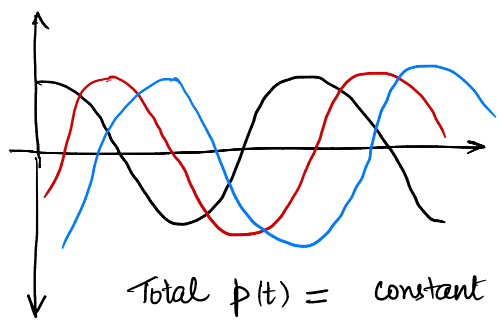
$V_{an} = 100\angle 0^\circ$ V

$V_{bn} = 100\angle 120^\circ$ V

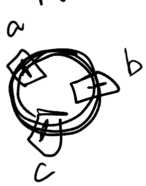
$V_{cn} = 100\angle -120^\circ$ V

$$\left.\begin{aligned} |V_{an}| = |V_{bn}| = |V_{cn}| \\ \rightarrow \; V_{an} + V_{bn} + V_{cn} = 0 \end{aligned}\right\}\ \text{Balanced Source}$$

Freq. Domain	Time Domain
$100\angle 0^\circ$	$100\cos(\omega t)$
$100\angle 120^\circ$	$100\cos(\omega t + 120^\circ)$
$100\angle -120^\circ$	$100\cos(\omega t - 120^\circ)$



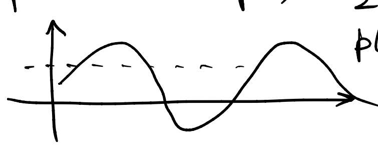
Motor



Total $p(t)$ = constant for three-phase sources

For single phase source: $p(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega t + \theta - \phi)$

$p(t) \neq$ constant

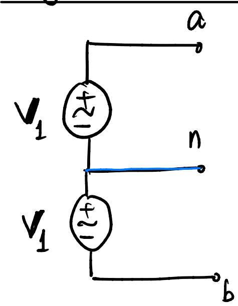


Advantages of three-phase sources are

- constant power and constant torque to large motors.
- more economical since motors are more efficients.

Single-Phase Three-Wire Source (Two-phase source)

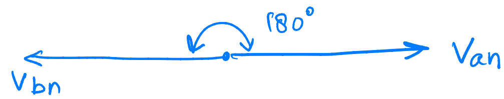
$\theta^\circ,\ \theta^\circ - 180^\circ$



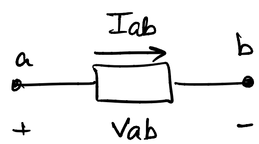
$V_1 = V_m \angle \theta^\circ$

$V_{an} = V_1$
$V_{nb} = V_1$

$$\left\{ \begin{aligned} V_{an} &= V_1 = V_m \angle \theta^\circ \\ V_{bn} &= -V_1 = V_m \angle \theta^\circ - 180^\circ \end{aligned} \right\}$$



Double script Notations:



V_{an} = voltage at point a w.r.t. voltage at point n

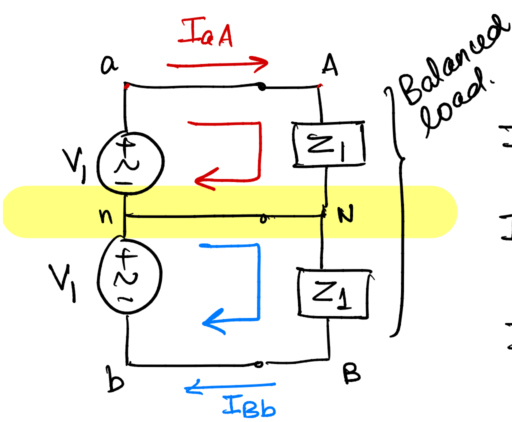
$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$

$V_{bn} = -V_{nb}$

I_{ab} = current flowing from point a to point b.

Balanced Single-Phase Three-Wire Source:

$$\left. \begin{aligned} |V_{an}| &= |V_{bn}| \\ V_{an} + V_{bn} &= 0 \end{aligned} \right\} \text{ Balanced source}$$



$V_{an} = V_1$, $V_{bn} = -V_1$

$I_{aA} = \dfrac{V_1}{Z_1}$

$I_{Bb} = \dfrac{V_1}{Z_2}$

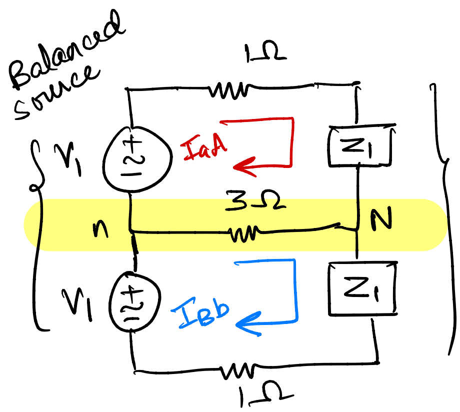
$I_{nN} = I_{Bb} - I_{aA} = \dfrac{V_1}{Z_2} - \dfrac{V_1}{Z_1}$

Assume $Z_1 = Z_2$ (Balanced load)

$I_{nN} = \dfrac{V_1}{Z_1} - \dfrac{V_1}{Z_1} = 0$

$\left.\begin{array}{l}\text{For balanced source} \\ \text{and balanced load}\end{array}\right\}$ current in neutral line $I_{nN} = 0$

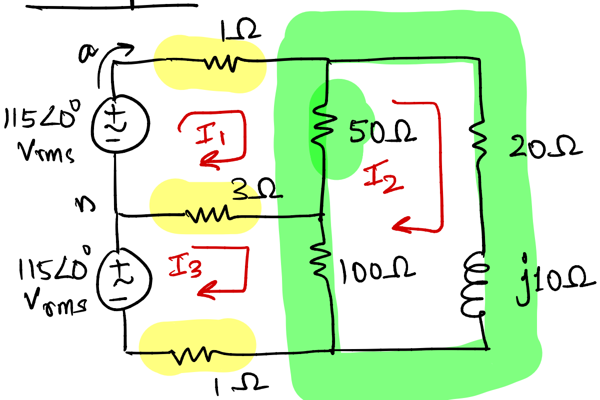
Wires with non-zero resistance



Balanced load

$$I_{nN} = 0 = I_{Bb} - I_{aA}$$

Example



line resistances

loads

Find

1. Average power delivered to each of three loads
2. Avg power lost in outerlines and neutral line.
3. Avg power provided by source.

Using mesh analysis

$$-115\angle 0^\circ + 1(I_1) + 50(I_1 - I_2) + 3(I_1 - I_3) = 0$$

$$50(I_2 - I_1) + (20 + j10) I_2 + 100(I_2 - I_3) = 0$$

$$-115\angle 0^\circ + (I_3 - I_1)3 + (I_3 - I_2)100 + I_3(1) = 0$$

$\longrightarrow$ $I_1 = 11.24 \angle -19.83^\circ$ Arms

$I_2 = 9.389 \angle -24.47^\circ$ Arms

$I_3 = 10.37 \angle -21.80^\circ$ Arms

$I_1 - I_2 = \cancel{11.24} - 9.389$

Avg power delivered to loads:

$$P_{avg,50\Omega} = |I_2 - I_1|^2 (50)$$
$$= 206 \text{ W}$$
$$P_{avg,100\Omega} = 117 \text{ W}$$
$$P_{avg,\underbrace{20+j10\Omega}} = I_2^2 (20) = 1763 \text{ W}$$

Total power delivered to loads = 2086 W

$P_{avg} = V_{eff} I_{eff} \cos(\theta - \phi)$

$P_{avg} = I_{eff}^2 R$

$P_{avg} = \frac{1}{2} I_m^2 R$

Avg power lost in lines

$$P_{avg.\,aA} = |I_1|^2 1 = 126 \text{ W}$$
$$P_{avg.\,Bb} = (I_3)^2 1 = 108 \text{ W}$$
$$P_{avg.\,nN} = |I_3 - I_1|^2 3 = 3 \text{ W}$$

Total power lost in lines = 237 W

Power supplied by sources

$V_{an} = 115\angle 0^\circ \text{ V}_{rms}$

$I_{na} = I_1$

$$P_{avg,an} = V_{an} I_{na} \cos(\theta - \phi)$$
$$= 115 \times 11.24 \cos(0 - (-19.83^\circ))$$
$$= 1216 \text{ W}$$

$$P_{avg,nb} = 115 \times 10.37 \cos(21.80^\circ)$$
$$= 1107 \text{ W}$$

Total power supplied = 2323 W

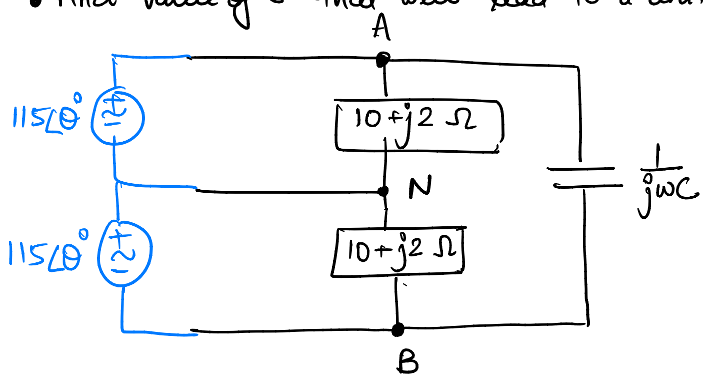
$$\boxed{P_{supplied} = P_{delivered} + P_{loss}}$$

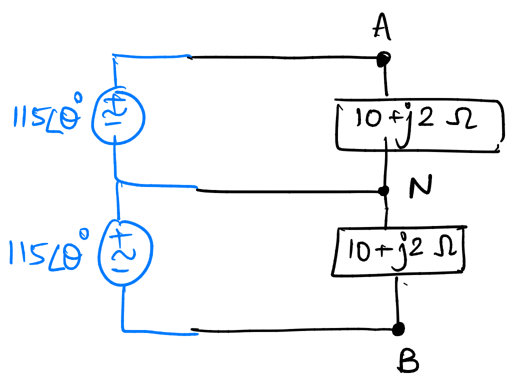
① <u>Example</u> : Consider $V_{12} = 9\angle 30^\circ$, $V_{32} = 3\angle 130^\circ$

Find $V_{21} = 9\angle -150^\circ$, $V_{13} = V_{12} + V_{23}$

② Consider a single-phase three-wire balanced source connected to a balanced load. $f = 50\,Hz$, $V_{an} = 115\angle 0^\circ\,V$

- Find power factor of the load if capacitor is omitted.
- Find value of C that will lead to a unity power factor of total load.





$PF = \cos(\theta - \phi)$