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Science / Engineering / Signals and Systems (2nd Edition)

Exercise 22

Chapter 2, Page 141





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The convolution of x(t) and h(t) is given by,

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$
 (1)

Given,

$$x(t) = e^{-lpha t} u(t) \ h(t) = e^{-eta t} u(t)$$

(i) When $\alpha \neq \beta$:

The signals x(t) and h(t) ae as shown in Figure.2.22.1(a) and Figure.2.22.1(b).

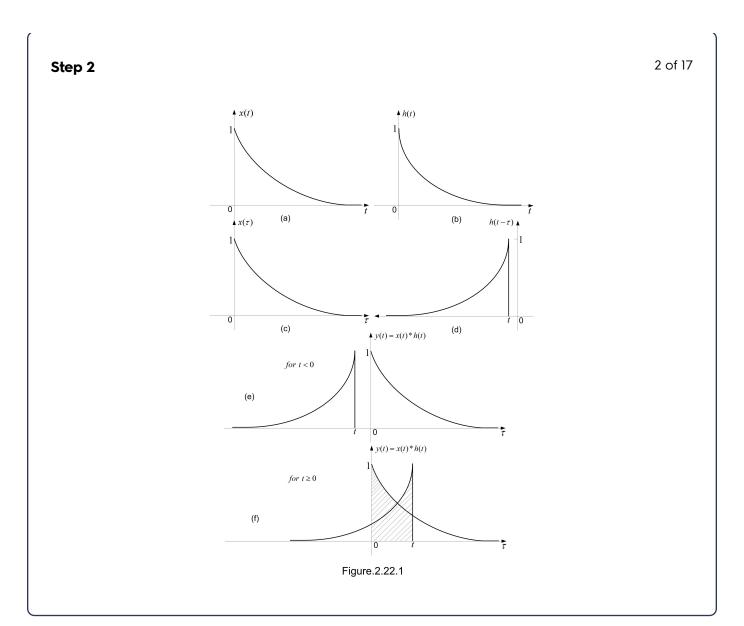
Figure.2.22.1(c) and Figure.2.22.1(d) represents $x(\tau)$ and $h(t-\tau)$ respectively. Figure.2.22.1(e) gives the convolution y(t) for t<0, but it can be observed from the figure that there is no overlapping of signals $x(\tau)$ and $h(t-\tau)$ for t<0. Therefore,

$$y(t) = 0$$
, if $t < 0$

From Figure.2.22.1(f), if $t\geq 0$, there will be overlapping of signals and the limits of overlapping or integration are from $\tau=0$ to $\tau=t$. Therefore, for $t\geq 0$,

$$egin{aligned} y(t) &= \int_0^t x(au)h(t- au)d au = \int_0^t (1)e^{-lpha au}e^{-eta(t- au)}d au = e^{-eta t}\int_0^t e^{-(lpha-eta) au}d au \ y(t) &= e^{-eta t}\int_0^t e^{-(lpha-eta) au}d au \ y(t) &= e^{-eta t}\left[-rac{e^{-(lpha-eta) au}}{lpha-eta}
ight]_0^t = e^{-eta t}\left[rac{1-e^{-(lpha-eta)t}}{lpha-eta}
ight] \ y(t) &= e^{-eta t}rac{1-e^{-(lpha-eta)t}}{lpha-eta}, & ext{if } t \geq 0 \ y(t) &= e^{-eta t}rac{(1-e^{-(lpha-eta)t})}{lpha-eta}u(t) \end{aligned}$$

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(ii) When $\alpha=\beta$: Figure.2.22.1(e) gives the convolution y(t) for t<0, but it can be observed from the figure that there is no overlapping of signals $x(\tau)$ and $h(t-\tau)$ for t<0. Therefore,

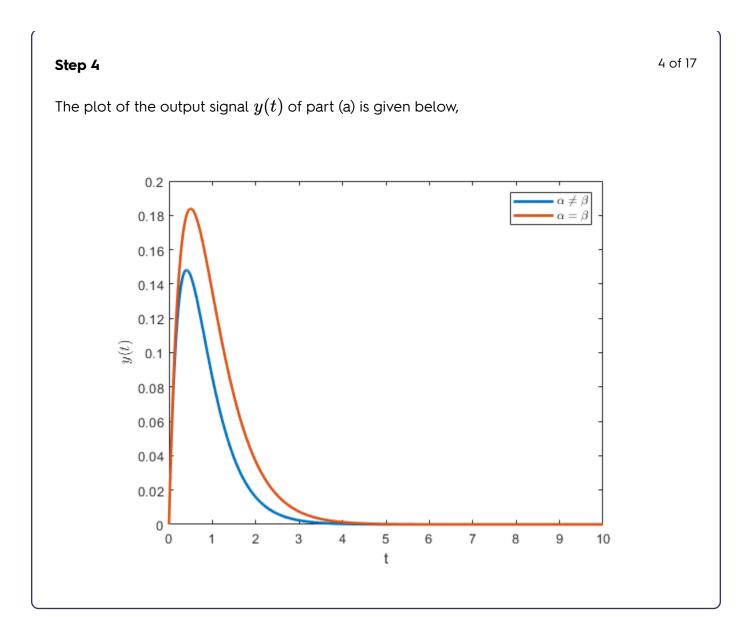
$$y(t) = 0, \quad \text{if } t < 0$$

From Figure.2.22.1(f), if $t\geq 0$, there will be overlapping of signals and the limits of overlapping or integration are from $\tau=0$ to $\tau=t$. Therefore, for $t\geq 0$,

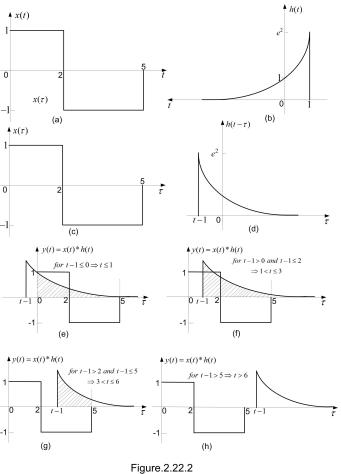
$$egin{aligned} y(t) &= \int_0^t x(au) h(t- au) d au = \int_0^t (1) e^{-lpha au} e^{-lpha(t- au)} d au = e^{-lpha t} \int_0^t e^{-(lpha-lpha) au} d au \ y(t) &= e^{-lpha t} \int_0^t e^{-(0) au} d au = e^{-lpha t} \int_0^t (1) au d au \ y(t) &= e^{-lpha t} \left[au
ight]_0^t = e^{-eta t} \left[t-0
ight] \ y(t) &= t e^{-lpha t}, \quad \text{if } t \geq 0 \ y(t) &= t e^{-lpha t} u(t) \end{aligned}$$

Therefore,

$$y(t) = egin{array}{ll} e^{-eta t \left(1-e^{-(lpha-eta)t}
ight)} u(t), & lpha
eq eta \ te^{-lpha t} u(t), & lpha = eta \end{array}$$



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\textbf{b) Given,

$$x(t) = u(t) - 2u(t-2) + u(t-5) \ h(t) = e^{2t}u(1-t)$$

The signals x(t) and h(t) ae as shown in Figure.2.22.2(a) and Figure.2.22.2(b).

Figure.2.22.2(c) and Figure.2.22.2(d) represents x(au) and h(t- au) respectively. Figure.2.22.2(e) gives the convolution y(t) for $t-1 \leq 0 \implies t \leq 1$, there will be overlapping of signals and the limits of overlapping or integration are from au=0 to au=5. Therefore,

$$egin{split} y(t) &= \int_0^5 x(au) h(t- au) d au = \int_0^2 (1) e^{2(t- au)} d au + \int_2^5 (-1) e^{2(t- au)} d au \ y(t) &= e^{2t} \int_0^2 e^{-2 au} d au - e^{2t} \int_2^5 e^{-2 au} d au \end{split}$$

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$$egin{align} y(t) &= e^{2t} \left[-rac{e^{-2 au}}{2}
ight]_0^2 - e^{2t} \left[-rac{e^{-2 au}}{2}
ight]_2^5 \ y(t) &= e^{2t} \left[-rac{e^{-2(2)}}{2} + rac{e^{-2(0)}}{2} + rac{e^{-2(5)}}{2} - rac{e^{-2(2)}}{2}
ight] \ y(t) &= \left[rac{e^{-10} - 2e^{-4} + 1}{2}
ight] e^{2t}, \qquad ext{for } t \leq 1 \ \end{cases}$$

From Figure.2.22.2(f), if $(t-1>0 \implies t>1$ and $t-1\leq 2 \implies t\leq 3$), there will be overlapping of signals and the limits of overlapping or integration are from $\tau=t-1$ to $\tau=5$. Therefore, for $1< t\leq 3$,

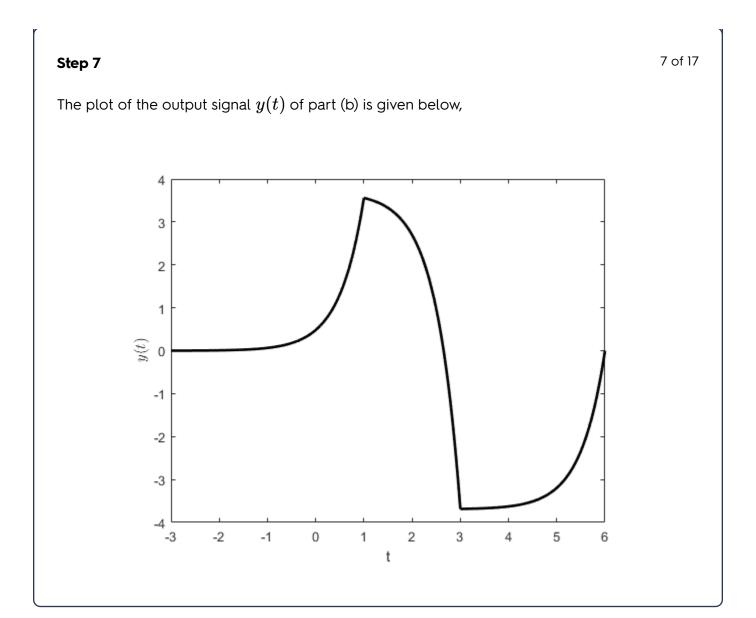
$$\begin{split} y(t) &= \int_{t-1}^5 x(\tau)h(t-\tau)d\tau = \int_{t-1}^2 (1)e^{2(t-\tau)}d\tau + \int_2^5 (-1)e^{2(t-\tau)}d\tau \\ y(t) &= e^{2t} \int_{t-1}^2 e^{-2\tau}d\tau - e^{2t} \int_2^5 e^{-2\tau}d\tau \\ y(t) &= e^{2t} \left[-\frac{e^{-2\tau}}{2} \right]_{t-1}^2 - e^{2t} \left[-\frac{e^{-2\tau}}{2} \right]_2^5 \\ y(t) &= e^{2t} \left[-\frac{e^{-2(2)}}{2} + \frac{e^{-2(t-1)}}{2} + \frac{e^{-2(5)}}{2} - \frac{e^{-2(2)}}{2} \right] \\ y(t) &= \left[\frac{e^{-10} - 2e^{-4} + e^{-2t}e^2}{2} \right] e^{2t} \\ y(t) &= \frac{e^{2(t-5)} - 2e^{2(t-2)} + e^2}{2}, \qquad \text{for } 1 < t \le 3 \end{split}$$

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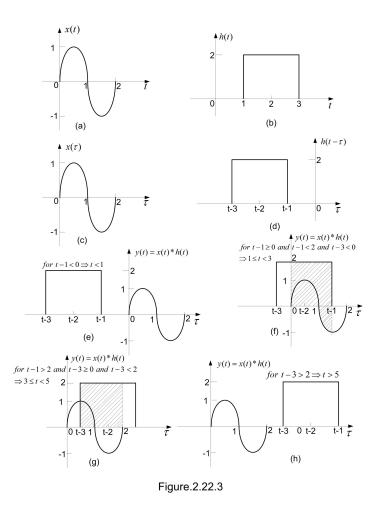
From Figure.2.22.2(g), if $(t-1>2\implies t>3$ and $t-1\le 5\implies t\le 6$), there will be overlapping of signals and the limits of overlapping or integration are from $\tau=t-1$ to $\tau=5$. Therefore, for $3< t\le 6$,

$$y(t) = \int_{t-1}^{5} x(\tau)h(t-\tau)d\tau = \int_{t-1}^{5} (-1)e^{2(t-\tau)}d\tau$$
 $y(t) = -e^{2t}\int_{t-1}^{5} e^{-2\tau}d\tau$
 $y(t) = -e^{2t}\left[-\frac{e^{-2\tau}}{2}\right]_{t-1}^{5}$

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The signals x(t) and h(t) ae as shown in Figure.2.22.3(a) and Figure.2.22.3(b).

Figure.2.22.3(c) and Figure.2.22.3(d) represents $x(\tau)$ and $h(t-\tau)$ respectively. Figure.2.22.3(e) gives the convolution y(t) for $t-1<0 \implies t<1$, but it can be observed from the figure that there is no overlapping of signals $x(\tau)$ and $h(t-\tau)$ for t<1. Therefore,

$$y(t) = 0$$
, if $t < 1$

From Figure.2.22.3(f), if $(t-1\geq 0\implies t\geq 1$ and $t-1<2\implies t<3$), there will be overlapping of signals and the limits of overlapping or integration are from $\tau=t-1$ to $\tau=5$. Therefore, for $1\leq t<3$,

$$y(t) = \int_0^{t-1} x(\tau)h(t-\tau)d au = \int_0^{t-1} (2)\sin(\pi\tau)d au$$

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$$egin{align} y(t) &= 2\int_0^{\pi}\sin(\pi au)d au = 2\left[-rac{\cos(\pi au)}{\pi}
ight]_0 \ y(t) &= 2\left[rac{\cos(0)-\cos(\pi(t-1))}{\pi}
ight] \ y(t) &= rac{2}{\pi}\left[1-\cos(\pi(t-1))
ight], \qquad ext{for } 1 \leq t < 3 \ \end{cases}$$

From Figure.2.22.3(g), if $(t-1\geq 2\implies t\geq 3$ and $t-3\leq 3\implies t\leq 5$), there will be overlapping of signals and the limits of overlapping or integration are from $\tau=t-1$ to $\tau=5$. Therefore, for $3\leq t\leq 5$,

$$egin{aligned} y(t) &= \int_{t-3}^2 x(au) h(t- au) d au = \int_{t-3}^2 (2) \sin(\pi au) d au \ y(t) &= 2 \int_{t-3}^2 \sin(\pi au) d au = 2 \left[-rac{\cos(\pi au)}{\pi}
ight]_{t-3}^2 \ y(t) &= 2 \left[rac{\cos(\pi(t-3)) - \cos(2\pi)}{\pi}
ight] \ y(t) &= rac{2}{\pi} \left[\cos(\pi(t-3)) - 1
ight], \qquad ext{for } 3 \leq t \leq 5 \end{aligned}$$

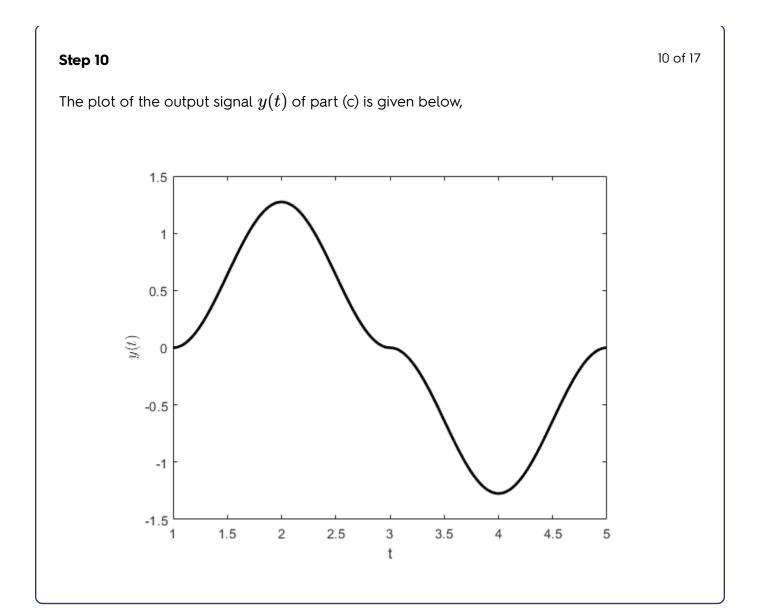
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Figure.2.22.3(h) gives the convolution y(t) for $t-3>2 \implies t>5$, but it can be observed from the figure that there is no overlapping of signals $x(\tau)$ and $h(t-\tau)$ for t>5. Therefore,

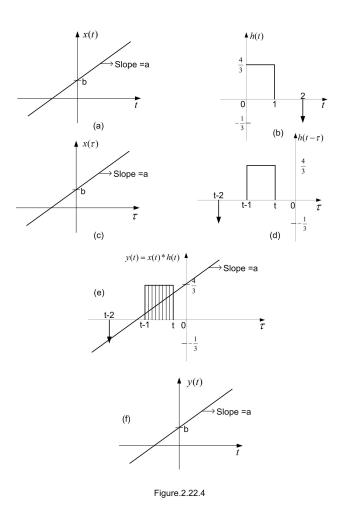
$$y(t) = 0, \text{ if } t > 5$$

Therefore,

$$y(t) = egin{array}{l} \left\{egin{array}{ll} 0, & t < 1 \ rac{2}{\pi}\left[1-\cos(\pi(t-1))
ight], & 1 \leq t < 3 \ rac{2}{\pi}\left[\cos(\pi(t-3))-1
ight], & 3 \leq t \leq 5 \ 0, & t > 5 \end{array}
ight.$$



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d) The signals x(t) and h(t) are shown in Figure.2.22.4(a) and Figure.2.22.4(b) respectively. From the Figures, the signals can be represented mathematically as,

$$x(t) = at + b$$
 $h(t) = rac{4}{3}\{u(t) - u(t-1)\} - rac{1}{3}\delta(t-2)$

Let us consider, $h_1(t)=\frac43\{u(t)-u(t-1)\}$ and $h_2(t)=\frac13\delta(t-2)$, then $h(t)=h_1(t)-h_2(t)$. Now consider, the convolution of x(t) and $h_1(t)$. From Figure.2.22.4(e), it is clear that the signals x(t) and $h_1(t)$ overlap and the limits of overlapping are from $\tau=t-1$ to $\tau=t$. Therefore,

$$egin{align} y_1(t) &= \int_{t-1}^t x(au) h_1(t- au) d au = \int_{t-1}^t rac{4}{3} (a au+b) d au \ y_1(t) &= rac{4}{3} \left[a rac{ au^2}{2} + b au
ight]^t &= rac{4}{3} \left[a rac{t^2}{2} + bt - a rac{(t-1)^2}{2} - b(t-1)
ight] \end{aligned}$$

$$y_1(t) = rac{4}{3} \left[a rac{t^2}{2} + bt - a rac{t^2}{2} + a rac{2t}{2} - a rac{1}{2} - bt + b
ight] \ y_1(t) = rac{4}{3} \left[at - rac{a}{2} + b
ight]$$

Now consider, the convolution of x(t) and $h_2(t)$. i.e.,

$$egin{align} y_2(t) &= x(t)*h_2(t) = x(t)*rac{1}{3}\delta(t-2) \ y_2(t) &= rac{1}{3}x(t-2) \ y_2(t) &= rac{1}{3}(a(t-2)+b) \ \end{array} \ egin{align} (\because x(t)*\delta(t-t_0) = x(t-t_0)) \ y_2(t) &= rac{1}{3}(a(t-2)+b) \ \end{cases}$$

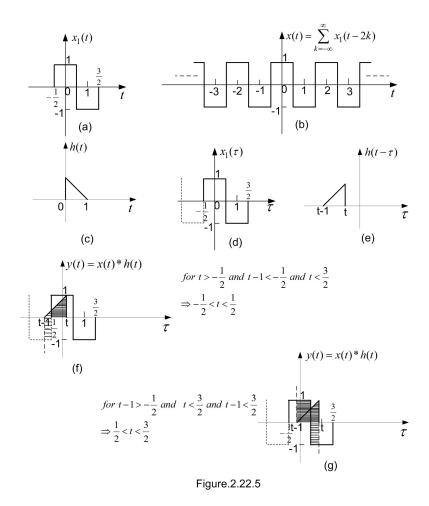
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Therefore, x(t) * h(t) is,

$$y(t) = x(t) * h(t) = x(t) * \{h_1(t) - h_2(t)\}$$
 $y(t) = x(t) * h(t) = [x(t) * h_1(t)] - [x(t) * h_2(t)]$ (: Distributive property of Convolute $y(t) = y_1(t) - y_2(t)$
 $y(t) = \frac{4}{3} \left[at - \frac{a}{2} + b \right] - \frac{1}{3} (a(t-2) + b)$
 $y(t) = at \left(\frac{4}{3} - \frac{1}{3} \right) + b \left(\frac{4}{3} - \frac{1}{3} \right) - \frac{2a}{3} + \frac{2a}{3} = at + b$
 $y(t) = x(t)$, for all t

Therefore, y(t)=x(t) for all $t \ y(t)$ is plotted in Figure.2.22.4(f)

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(e) The signals x(t) and h(t) are as shown in Figure.2.22.5(b) and Figure.2.22.5(c) respectively. It can be clearly seen that the signal x(t) is a periodic signal with period z and the fundamental signal to construct this periodic signal is $z_1(t)$ and is shown in Figure.2.22.5(a). Therefore,

$$egin{align} x(t) &= \sum_{k=-\infty}^\infty x_1(t-2k) \ x(t) &= \sum_{k=-\infty}^\infty \left[x_1(t) * \delta(t-2k)
ight] \ (\because x(t) * \delta(t-t_0) = x(t-t_0)) \ x(t) &= x_1(t) * \sum_{k=-\infty}^\infty \delta(t-2k) \ \end{cases}$$

Let us consider, $y_1(t) = x_1(t) st h(t)$ Now the convolution of x(t) and h(t) is given as,

 ∞

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$$y(t)=x(t)*h(t)=x_1(t)*\sum_{k=-\infty}\delta(t-2k)*h(t)$$
 $y(t)=x_1(t)*\left[\sum_{k=-\infty}^\infty\delta(t-2k)*h(t)
ight]$

$$y(t) = x_1(t) * \left[h(t) * \sum_{k=-\infty}^{\infty} \delta(t-2k)
ight]$$

$$y(t) = [x_1(t)*h(t)]*\sum_{k=-\infty}^\infty \delta(t-2k)$$

$$y(t) = y_1(t) * \sum_{k=-\infty}^{\infty} \delta(t-2k)$$

$$y(t) = \sum_{k=-\infty}^{\infty} y_1(t-2k)$$

(:: Associative property of Convolutio

(: Commutative property of Convolution

(: Associative property of Convolution)

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Therefore, y(t) is also a periodic signal with period 2. Let us now, try to obtain the convolution for one period of y(t).

The signals $x_1(t)$ and h(t) are as shown in Figure.2.22.5(a) and Figure.2.22.5(c).

Figure.2.22.5(d) and Figure.2.22.5(e) represents $x_1(au)$ and h(t- au) respectively.

Figure.2.22.5(f) gives the convolution of one period of y(t) if $(t>-\frac{1}{2}$ and $t-1<-\frac{1}{2} \implies t<\frac{1}{2}$), there will be overlapping of signals and the limits of overlapping or integration are from $\tau=-\frac{1}{2}$ to $\tau=t$ and $\tau=t-1$ to $\tau=-\frac{1}{2}$ (: Periodic Signal). Therefore, for $-\frac{1}{2}< t<\frac{1}{2}$,

$$egin{aligned} y(t) &= \int_{t-1}^t x(au)h(t- au)d au \ y(t) &= \int_{t-1}^{-rac{1}{2}} (-1)(1-t+ au)d au + \int_{-rac{1}{2}}^t (1)(1-t+ au)d au \ y(t) &= (t-1)\int_{t-1}^{-rac{1}{2}} 1d au - \int_{t-1}^{-rac{1}{2}} au d au + (1-t)\int_{-rac{1}{2}}^t 1d au + \int_{-rac{1}{2}}^t au d au \ y(t) &= (t-1)\left[au
ight]_{t-rac{1}{2}}^{-rac{1}{2}} - \left[rac{ au^2}{2}
ight]_{t-rac{1}{2}}^{-rac{1}{2}} + (1-t)\left[au^2
ight]_{t-rac{1}{2}}^t + \left[rac{ au^2}{2}
ight]_{t-rac{1}{2}}^t \end{aligned}$$

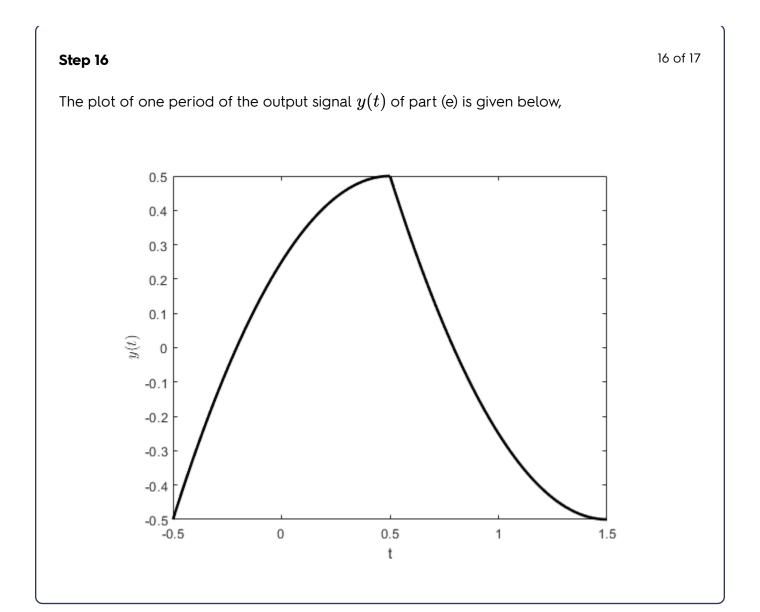
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Figure.2.22.5(g) gives the convolution of one period of y(t) if $(t-1>-\frac{1}{2}\implies t>\frac{1}{2}$ and $t<\frac{3}{2}$), there will be overlapping of signals and the limits of overlapping or integration are from $\tau=t-1$ to $\tau=\frac{1}{2}$ and $\tau=\frac{1}{2}$ to $\tau=t$. Therefore, for $\frac{1}{2}< t<\frac{3}{2}$,

$$\begin{split} y(t) &= \int_{t-1}^t x(\tau)h(t-\tau)d\tau \\ y(t) &= \int_{t-1}^{\frac{1}{2}} (1)(1-t+\tau)d\tau + \int_{\frac{1}{2}}^t (-1)(1-t+\tau)d\tau \\ y(t) &= (1-t)\int_{t-1}^{\frac{1}{2}} 1d\tau + \int_{t-1}^{\frac{1}{2}} \tau d\tau + (t-1)\int_{\frac{1}{2}}^t 1d\tau - \int_{\frac{1}{2}}^t \tau d\tau \\ y(t) &= (1-t)\left[\tau\right]_{t-1}^{\frac{1}{2}} + \left[\frac{\tau^2}{2}\right]_{t-1}^{\frac{1}{2}} + (t-1)\left[\tau\right]_{\frac{1}{2}}^t - \left[\frac{\tau^2}{2}\right]_{\frac{1}{2}}^t \\ y(t) &= (1-t)\left[\frac{1}{2}-t+1\right] + \left[\frac{(\frac{1}{2})^2}{2} - \frac{(t-1)^2}{2}\right] + (t-1)\left[t-\frac{1}{2}\right] - \left[\frac{t^2}{2} - \frac{(\frac{1}{2})^2}{2}\right] \\ y(t) &= (1-t)\left[\frac{3}{2}-t\right] + \frac{1}{8} - \frac{t^2}{2} - \frac{1}{2} + \frac{2t}{2} + (t-1)\left[t-\frac{1}{2}\right] - \frac{t^2}{2} + \frac{1}{8} \\ y(t) &= t^2 - \frac{5t}{2} + \frac{3}{2} + \frac{1}{8} - \frac{t^2}{2} - \frac{1}{2} + \frac{2t}{2} + t^2 + \frac{1}{2} - \frac{3t}{2} - \frac{t^2}{2} + \frac{1}{8} \\ y(t) &= t^2 - 3t + \frac{7}{4}, \qquad \text{for } \frac{1}{2} < t < \frac{3}{2} \end{split}$$

Therefore, y(t) is periodic signal with period 2 and one period of signal y(t) is,

$$y(t) = egin{cases} rac{1}{4} + t - t^2, & -rac{1}{2} < t < rac{1}{2} \ t^2 - 3t + rac{7}{4}, & rac{1}{2} < t < rac{3}{2} \end{cases}$$



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a)

$$y(t) = egin{array}{c} e^{-eta t \left(1-e^{-(lpha-eta)t}
ight)} u(t), & lpha
eq eta \ te^{-lpha t} u(t), & lpha = eta \end{array}$$

b)

$$y(t) = egin{array}{l} \left[rac{e^{-10}-2e^{-4}+1}{2}
ight]e^{2t}, & t \leq 1 \ rac{e^{2(t-5)}-2e^{2(t-2)}+e^2}{2}, & 1 < t \leq 3 \ rac{e^{2(t-5)}-e^2}{2}, & 3 < t \leq 6 \ 0, & Otherwise \end{array}$$

c)

$$y(t) = egin{array}{l} \left\{ egin{array}{ll} 0, & t < 1 \ rac{2}{\pi} \left[1 - \cos(\pi(t-1))
ight], & 1 \leq t < 3 \ rac{2}{\pi} \left[\cos(\pi(t-3)) - 1
ight], & 3 \leq t \leq 5 \ 0, & t > 5 \end{array}
ight.$$

d)

$$y(t) = x(t)$$
, for all t

e) One period of signal y(t) is,

$$y(t) = egin{array}{cccc} rac{1}{4} + t - t^2, & -rac{1}{2} < t < rac{1}{2} \ t^2 - 3t + rac{7}{4}, & rac{1}{2} < t < rac{3}{2} \end{array}$$

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Exercise 23

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Exercise 21

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