ECE250: Signals and Systems
Practice sheet 1

August 28, 2024

1. Show that $B = \{\mathbf{v}_1, \mathbf{v}_2, ...\mathbf{v}_n\}$ is a basis of the vector space V if and only if every vector $\mathbf{v} \in V$ is uniquely expressible as a linear combination of the elements of B.

2. Given the following vectors in $\mathbf{R}^3$: $\mathbf{u}$=(1,3,5), $\mathbf{v}$=(1,4,6), $\mathbf{w}$=(2,-1,3) and $\mathbf{b}$=(6,5,17)
   (a) Does $\mathbf{b} \in W$=Span$\{\mathbf{u,v,w}\}$
   (b) If the answer to the first part is yes, express $\mathbf{b}$ as a linear combination of $\mathbf{u,v,w}$.

3. For $a, b \in \mathbf{R}$, let
   $p(x, y) = a^2 x_1 y_1 + ab x_2 y_1 + ab x_1 y_2 + b^2 x_2 y_2$, $x = (x_1, x_2)$, $y = (y_1, y_2) \in \mathbf{R}^2$ For what values of a and b does p: $\mathbf{R}^2 \times \mathbf{R}^2 \to \mathbf{R}$ define an inner product? Justify
   (a) $a > 0, b > 0$.
   (b) $ab \geq 0$.
   (c) $a = 0, b = 0$.
   (d) For no values of a, b.

4. A unit rectangular function is as follows:
   $$x(t) = \begin{cases} 1 & \text{for } |t| \leq 1/2 \\ 0 & \text{for } |t| > 1/2 \end{cases}$$
   Plot
   (a) $x(t)$
   (b) $3x(t + 1)/4$
   (c) $-4x(-t)$

5. Determine whether these signals are energy signals or power signals and calculate their energy or power

   (a) $x(t) = sin^2(\omega_0 t)$

   (b) $x(t) = tu(t)$

   (c) $x(t) = e^{j[3t+(\pi/2)]}$

   where,

   $$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

6. Determine and sketch the even and odd parts of he signals depicted in Figure 1. Label your sketches carefully.

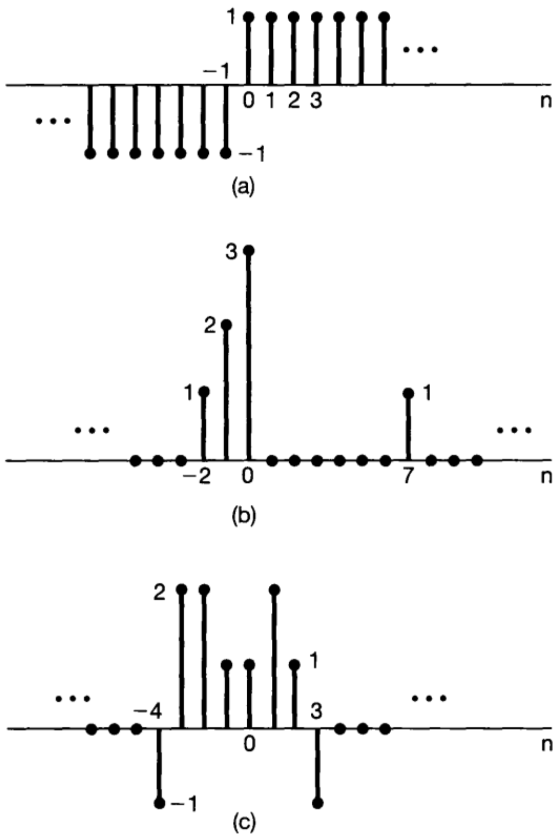

Figure 1: Figure for Q6.

7. Find and sketch the even and odd components of the following signal:

   $$x(t) = \begin{cases} t & \text{for } 0 \leq t \leq 1 \\ 2-t & \text{for } 1 < t \leq 2 \end{cases}$$

8. Find whether the following time signals are periodic or not? If periodic, determine the fundamental period.:

   (a) $x_1[n] = \sin(0.02\pi n)$

   (b) $x_2[n] = \cos(\frac{\pi}{2} + 0.3n)$

   (c) $x_3[n] = 1 - e^{j2\pi n/5} + e^{j4\pi n/7}$

9. For each signal given below, determine all the the values of the independent variable at which the even part of the signal is guaranteed to be zero.

   (a) $x_1(t) = sin(\frac{1}{2}t)$

   (b) $x_2(t) = e^{-5t}u(t+2)$

   (c) $x_3[n] = u[n] - u[n-4]$

10. A descrete- time signal is shown in Figure 2. Sketch and label carefully each of the following signals:

   (a) $x[n-4]$

   (b) $x[n-2]\delta[n-2]$

   (c) $x[(n-1)^2]$

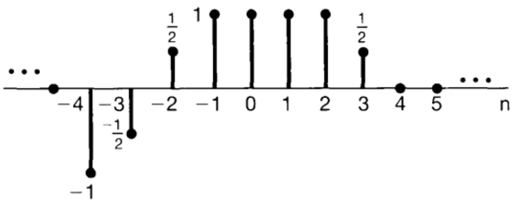

Figure 2: Figure for Q10.

11. What is the simplified value of $y[n]$, if:

    $y[n] = \sum_{n=-5}^{5} \sin(2n)\delta[n+7]$?

12. consider a signal $x(t) = u(t-2) - u(t-4)$, evaluate $\int_{-\infty}^{\infty} x(t)\delta(t)dt$.

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Practice Sheet 2

1. Evaluate the following integrals: (CO1)

   (a) $\int_{-1}^{1}(3t^2+1)\delta(t)dt$
   (b) $\int_{-\infty}^{\infty}(t^2+cos(\pi t))\delta(t-1)dt$
   (c) $\int_{-\infty}^{\infty}e^{-t}\delta(2t-2)dt$

   where, $\delta(t) = 0 \quad for \quad t \neq 0$

   and, $\int_{-\infty}^{\infty}\delta(t)dt = 1$

2. Find and sketch the following signals and their first derivatives: (CO1)

   (a) $x(t) = u(t) - u(t-a), a > 0$
   (b) $x(t) = t[u(t) - u(t-a)], a > 0$
   (c) $x(t) = sgn(t) = \begin{cases} 1 & for \quad t > 0 \\ -1 & for \quad t < 0 \end{cases}$

   where, $u(t) = \begin{cases} 1 & for \quad t \geq 0 \\ 0 & for \quad t < 0 \end{cases}$

3. A discrete-time signal x[n] is shown in Figure 1. Sketch and label each of the following signals. (CO1)

   (a) $x[n]u[1-n]$
   (b) $x[n]\{u[n+2] - u[n]\}$
   (c) $x[n]\delta[n-1]$

   where, $u[n] = \begin{cases} 1 & for \quad n \geq 0 \\ 0 & for \quad n < 0 \end{cases}$

   and, $\delta[n] = \begin{cases} 1 & for \quad n = 0 \\ 0 & for \quad otherwise \end{cases}$ also, $\sum_{n=-\infty}^{\infty}\delta[n] = 1$

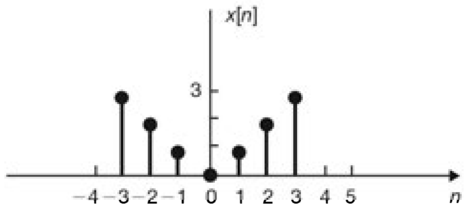

Figure 1: Figure for Question number 3

4. A continuous-time signal x(t) is shown in Figure 2. Sketch and label each of the following signals. (CO1)

   (a) $x(t)u(1-t)$
   (b) $x(t)[u(t)-u(t-1)]$
   (c) $x(t)\delta(t-\frac{3}{2})$

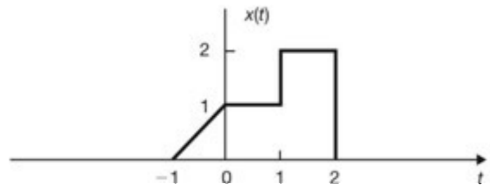

Figure 2: Figure for Question number 4

where, $u(t) = \begin{cases} 1 & for \quad t \geq 0 \\ 0 & for \quad t < 0 \end{cases}$

and, $\delta(t) = 0 \quad for \quad t \neq 0$ also, $\int_{-\infty}^{\infty} \delta(t)dt = 1$

5. Determine whether the following signals are energy signals, power signals, or neither. (CO1)

   (a) $x(t) = e^{-at}u(t), a > 0$
   (b) $x[n] = (-0.5)^n u[n]$
   (c) $x[n] = 2e^{j3n}$

where, $u(t) = \begin{cases} 1 & for \quad t \geq 0 \\ 0 & for \quad t < 0 \end{cases}$ and, $u[n] = \begin{cases} 1 & for \quad n \geq 0 \\ 0 & for \quad n < 0 \end{cases}$

6. Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period. (CO1)

   (a) $x(t) = cos\frac{\pi}{3}t + sin\frac{\pi}{4}t$
   (b) $x[n] = cos\frac{\pi}{3}n + sin\frac{\pi}{4}n$
   (c) $x[n] = cos^2\frac{\pi}{8}n$

7. Determine whether or not each of the following discrete-time signals is periodic. If the signal is periodic, determine its fundamental period. (CO1)

   (a) $x[n] = cos(\frac{\pi}{8}n^2)$
   (b) $x[n] = 2cos(\frac{\pi}{4}n) + sin(\frac{\pi}{8}n) - 2cos(\frac{\pi}{2}n + \frac{\pi}{6})$

ECE250: Signals and Systems
Practice sheet 3

September 6, 2024

1. (CO1,CO2)Compute and plot $y[n] = x[n] * h[n]$, where

$$x[n] = \begin{cases} 1 & \text{for } 3 \leq n \leq 8 \\ 0, & otherwise \end{cases}$$

$$h[n] = \begin{cases} 1 & \text{for } 4 \leq n \leq 15 \\ 0, & otherwise \end{cases}$$

2. (CO1,CO2)A linear system S has the relationship

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$$

between its input $x[n]$ and its output $y[n]$, where $g[n] = u[n] - u[n-4]$.

   (a) Determine $y[n]$ when $x[n] = \delta[n-1]$.

   (b) Determine $y[n]$ when $x[n] = \delta[n-2]$.

   (c) Is S LTI?

   (d) Determine $y[n]$ when $x[n] = u[n]$.

   (e) What is the system response?

3. (CO1,CO2)Let

$$h(t) = e^{2t}u(-t+4) + e^{-2t}u(t-5).$$

Determine A and B such that

$$h(t-\tau) = \begin{cases} e^{-2(t-\tau)} & \text{for } \tau < A \\ 0 & \text{for } A < \tau < B \\ e^{2(t-\tau)} & \text{for } B < \tau \end{cases}$$

4. (CO1,CO2)Which of the following impulse responses correspond(s) to stable LTI systems?
   (a) $h[n] = 3^n u[-n+10]$
   (b) $h(t) = e^{-t} cos(2t)u(t)$

5. (CO1,CO2)Let h(t) be the triangular pulse shown in figure-1(a) and let x(t) be the impulse train depicted in figure-1(b). That is,

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT).$$

   Determine and sketch y(t)=x(t)*h(t) for the following values of T:
   (a) $T = \frac{3}{2}$
   (b) $T = 4$
   (c) $T = 1$

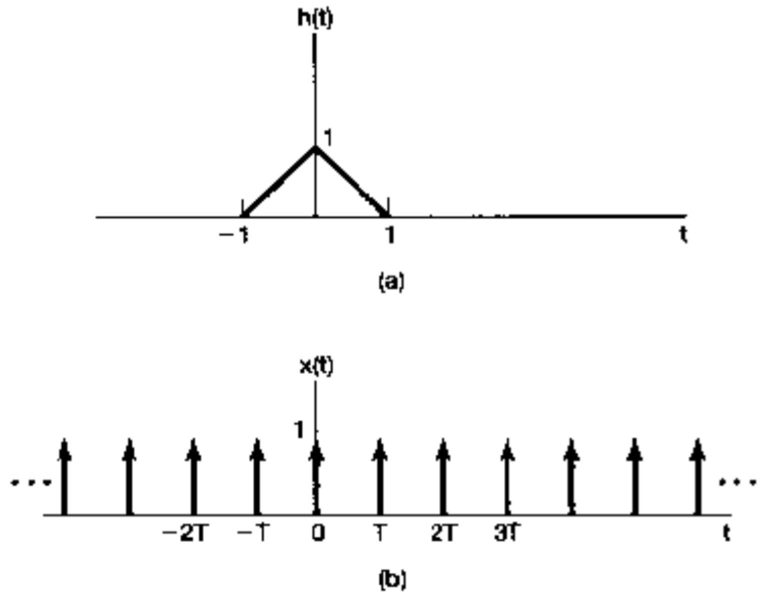

Figure 1: Figure2 for Q5.

6. (CO1,CO2)Determine whether the following systems are time-invariant or not;
   (a) $y(t) = \int_{t-T}^{t} x(u)\, du$
   (b) $y(t) = x(t) \sin \omega t$

(c) $y[n] = x[-n+2]$.

7. (CO1,CO2)Following are the impulse responses of LTI systems. Determine whether each system is causal and/or stable. Justify your answers.
   (a) $h[n] = (\frac{-1}{2})^n u[n] + (0.01)^n u[1-n]$
   (b) $h(t) = e^{-2t} u[t+50]$
   (c) $h(t) = te^{-t} u[t]$

8. (CO1,CO2)Consider an LTI system S and a signal $x(t) = 2e^{-3t}u(t-1)$. If

$$x(t) \rightarrow y(t)$$

and

$$\frac{dx(t)}{dt} \rightarrow -3y(t) + e^{-2t}u(t),$$

determine the impulse response $h(t)$ of S.

ECE250: Signals and Systems

Practice Sheet 4

1. (CO1, CO2) Let y(t) = x(t)*h(t). Then show that :

   $x(t-t_1) * h(t-t_2) = y(t-t_1-t_2)$

2. (CO2) Let $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$.

   (a) Compute $y(t) = x(t) * h(t)$.
   (b) Compute $g(t) = (\frac{dx(t)}{dt}) * h(t)$.
   (c) How is $g(t)$ relaled to $y(t)$?

3. (CO2) Consider a discrete-time system S_1 with impulse response $h[n] = (\frac{1}{5})^n u[n]$.

   (a) Find the integer A such that $h[n] - Ah[n-1] = \delta[n]$.
   (b) Using the result from part (a), determine the impulse response g[n] of an LTI system S_2 which is the inverse system of S_1.

4. (CO1, CO2) Consider a continuous-time LTI system whose step response is given by $s(t) = e^{-t}u(t)$ Determine and sketch the output of this system to the input $x(t)$ shown in Fig. 1.

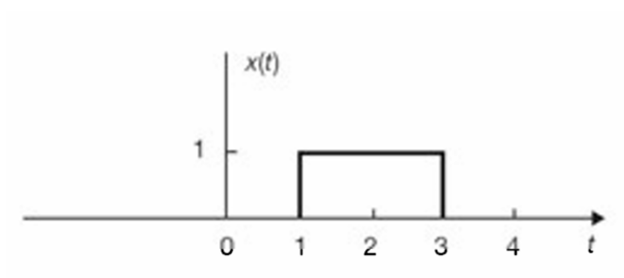

Figure 1:

Where step response is defined by the output of a system when a step input is applied to it.

5. (CO1, CO2) The system shown in Fig. 2 is formed by connecting two systems in cascade. The impulse responses of the systems are given by $h_1(t)$ and $h_2(t)$, respectively, and $h_1(t) = e^{-2t}u(t)$ and $h_2(t) = 2e^{-t}u(t)$

(a) Find the impulse response h(t) of the overall system shown in Fig. 3.
(b) Determine if the overall system is BIBO stable.

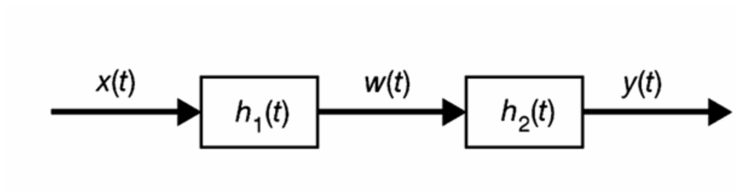

Figure 2:

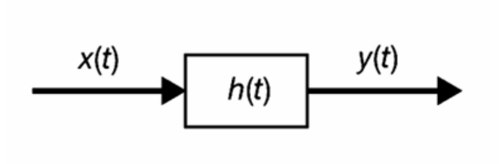

Figure 3:

<u>Q6</u>: For each of the following pairs of waveforms, use the convolution integral to find the response $y(t)$ of the LTI system with impulse response $h(t)$ to the input $x(t)$.

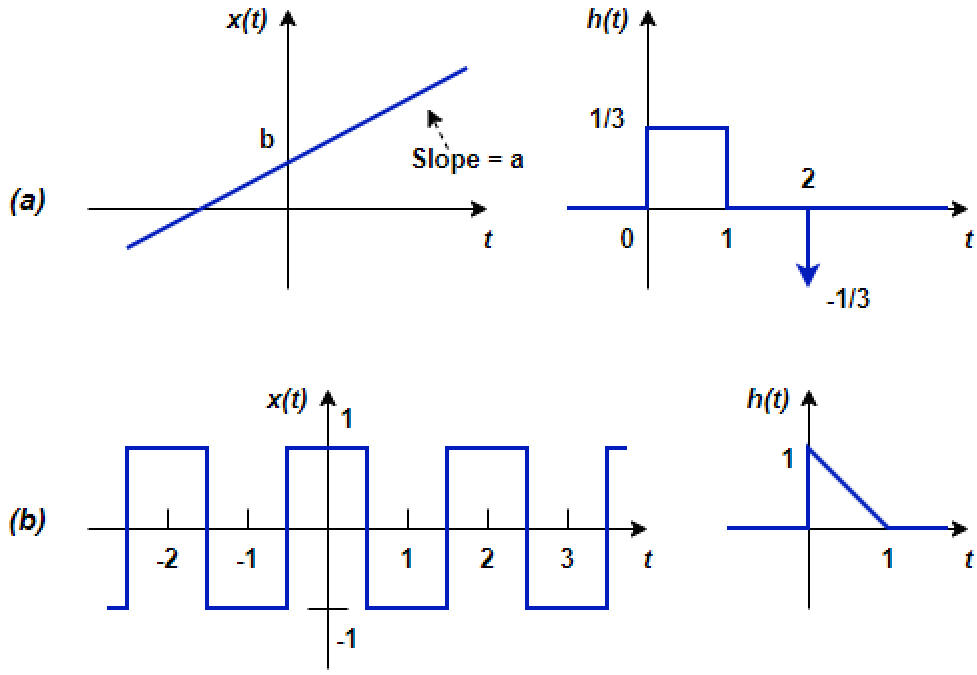

Figure 4

ECE250: Signals and Systems
Practice sheet 5

September 25, 2024

1. (CO1,CO2,CO3) Let $x[n]$ be a real and odd periodic signal with period $N = 7$ and Fourier coefficients a_k. Given that

$$a_{15} = j,\ a_{16} = 2j,\ a_{17} = 3j$$

determine the values of a_0, a_{-1}, a_{-2} and a_{-3}.

2. (CO1,CO2,CO3) Consider a discrete-time LTI system with impulse response

$$h[n] = \begin{cases} 1 & 0 \leq n \leq 2 \\ -1 & -2 \leq n \leq -1 \\ 0 & otherwise \end{cases}$$

Given that the input to this system is

$$x[n] = \sum_{k=-\infty}^{\infty} \delta[n - 4k],$$

determine the Fourier series coefficients of the output y[n].

3. (CO1,CO2,CO3) Consider a causal continuous-time LTI system whose input $x(t)$ and output $y(t)$ are related by the following differential equation:

$$\frac{d}{dt}y(t) + 4y(t) = x(t),$$

Find the Fourier series representation of the output $y(t)$ for each of the following inputs:

(a) $x(t) = \cos 2\pi t$.

(b) $x(t) = \sin 4\pi t + \cos(6\pi t + \pi/4)$.

4. (CO1,CO2,CO3) In each of the following, we specify the Fourier series coefficients of a signal that is periodic with period 8. Determine the signal $x[n]$ in each case.
   (a) a_k in Figure1(a).
   (b) a_k in Figure1(b).

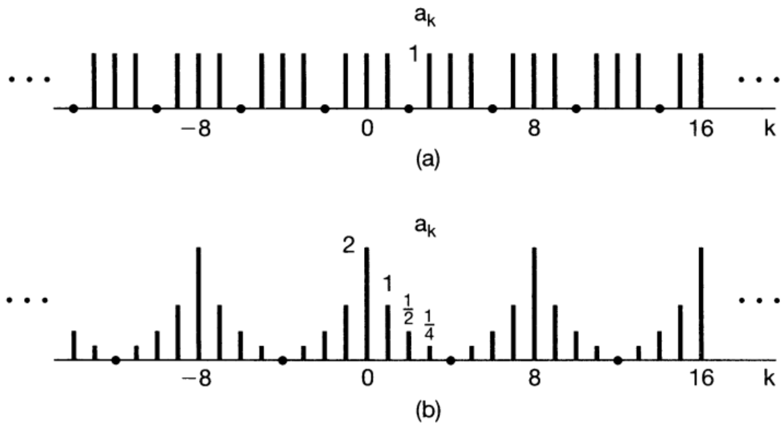

Figure 1: Figure for Q4.

5. (CO1,CO2,CO3) In each of the following, we specify the Fourier series coefficients of a continuous-time signal that is periodic with period 4. Determine the signal $x(t)$ in following case.
   (a) $a_k = \begin{cases} jk, & |k| < 3 \\ 0 & otherwise \end{cases}$

6. (CO1,CO2,CO3) A continuous-time periodic signal $x(t)$ is real valued and has a fundamental period $T = 8$. The nonzero Fourier series coefficients for $x(t)$ are specified as

$$a_1 = a_{-1}^* = j,\ a_5 = a_{-5} = 2.$$

Express $x(t)$ in the form

$$x(t) = \sum_{k=0}^{\infty} A_k \cos(\omega_k t + \phi_k).$$

7. (CO1,CO2,CO3) Considering a causal LTI system implemented as the RL circuit shown in Figure2. A current source produces an input current $x(t)$, and the system output is considered to be the current $y(t)$ flowing through the inductor. The differential equation relating $x(t)$ and $y(t)$ is given as

$$\frac{dy(t)}{dt} + y(t) = x(t)$$

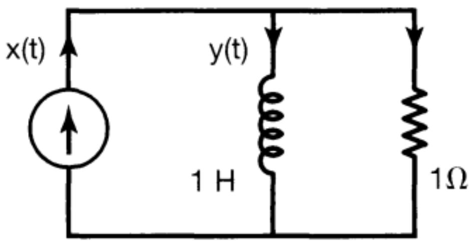

Figure 2: Figure for Q7.

(a) Determine the frequency response of this system by considering the output of the system to input of the form $x(t) = e^{j\omega t}$

8. (CO1,CO2,CO3) For the continuous-time periodic signal

$$x(t) = 2 + \cos\left(\tfrac{2\pi}{3}t\right) + 4\sin\left(\tfrac{5\pi}{3}t\right)$$

determine the fundamental frequency ω_0 and the Fourier series coefficients a_k such that

$$x(t) = \textstyle\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$