

# Advanced Binary Search 3

# Binary Search on Decimals

Find a real number between L and R following certain properties.

- How many iterations?
- Precision issues?
- Ways to tackle

$$O(\log(s:s)) \rightarrow O(\log [\int s:s])$$

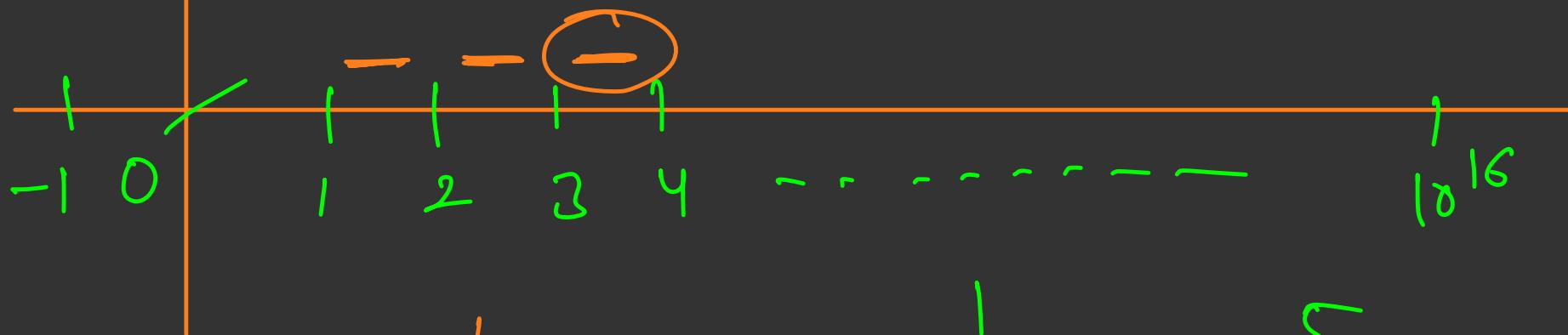
precision

$$\underline{\underline{Sqt(2)}}$$

$$\underline{\underline{10}}$$

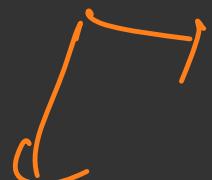
$$\underline{\underline{Sqt(x)}}$$

$$y^2 \leq x$$



$$\underline{\underline{\frac{1}{3}}}$$

=



$$\underline{\underline{\frac{1}{3}}}$$

0.3333

$$\underline{\underline{\frac{5}{2}}} = 2.5$$

$$\underline{\underline{0.33}}$$

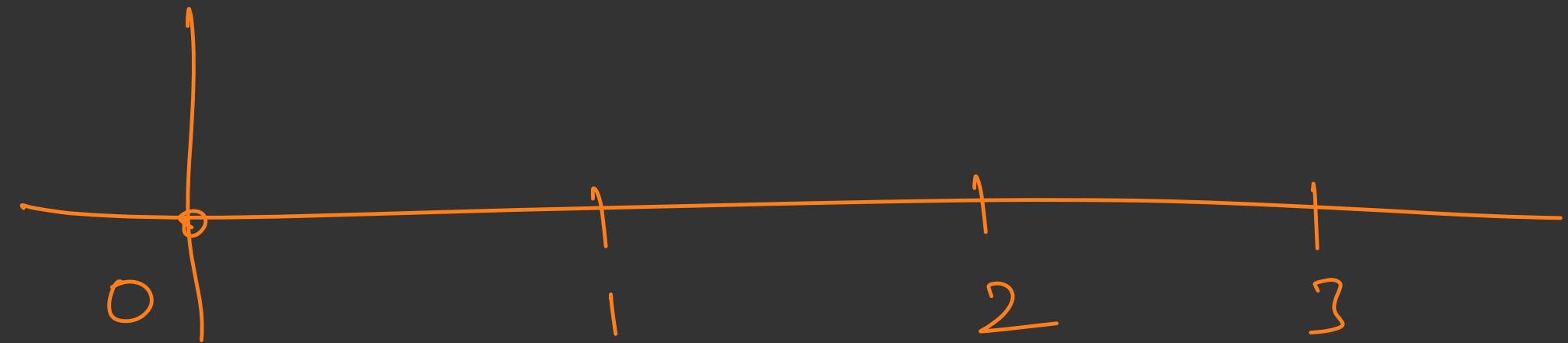
Print the answer which has a relative error of  $\leq 10^{-6}$  with the original answer

Yours ans =  $x$

Original ans =  $y$

$$|x - y| \leq 10^{-6}$$

$$y + 10^{-6} \geq x \geq y - 10^{-6}$$



$$\begin{array}{c} 1.6000001 \\ \swarrow \quad \searrow \\ 1.0000002 \end{array}$$

Original ans = 1.000123123

Ans fint =  $\begin{array}{r} 1.000123 \\ \hline 1.000124 \end{array}$

orig  $\rightarrow$  ~~x~~

your ans  $\rightarrow$  ~~y~~

$$|x - y| \leq 10^{-6}$$

~~.....~~

orig  $\rightarrow$

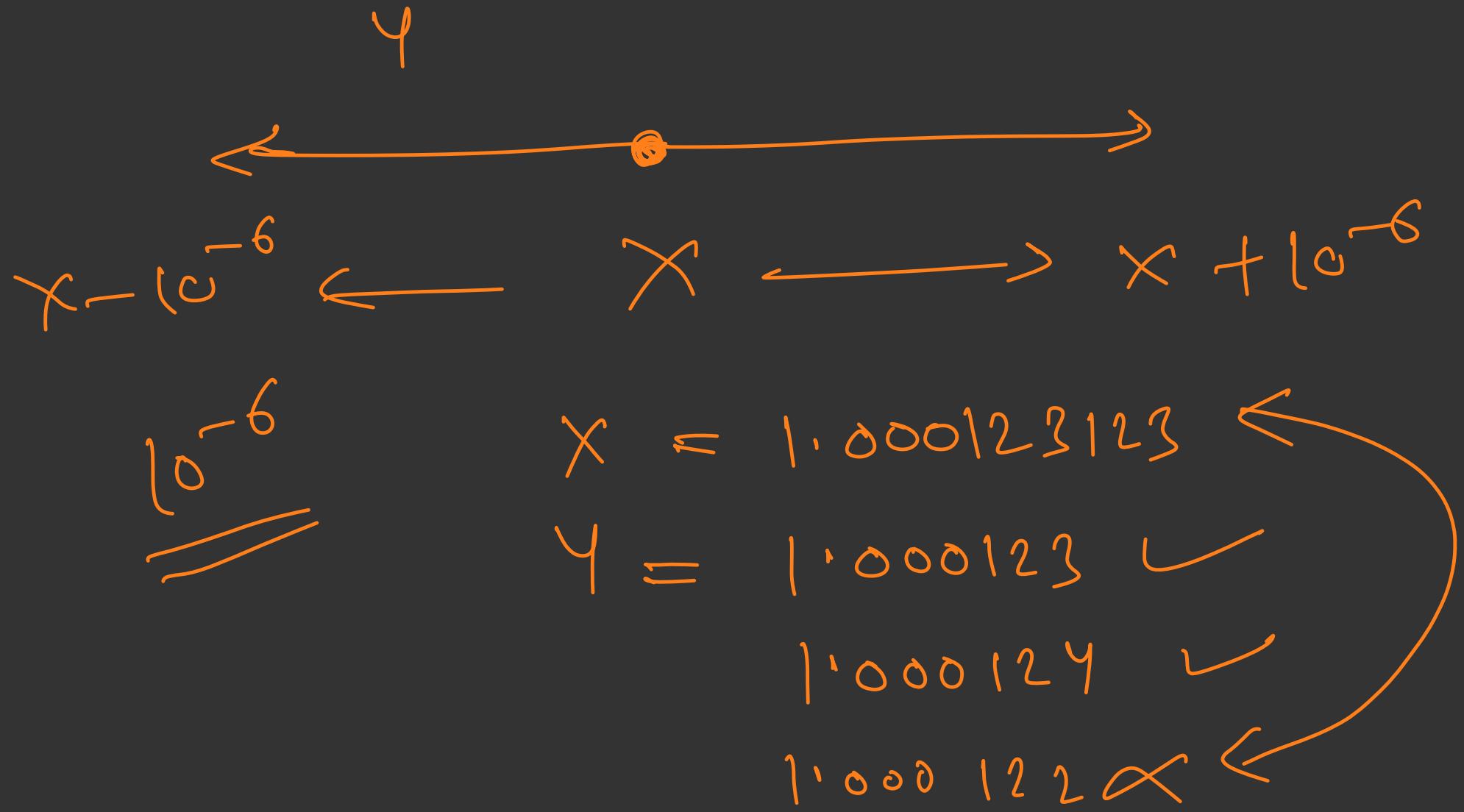
printed

ans =

1.000123123

1.000124

1.000122



$$\begin{aligned}
 X - Y &= 1.000123123 \\
 &\quad - 1.000122 \\
 &= 0.00000123 \Rightarrow \underline{10^{-6}}
 \end{aligned}$$

$y = 1.800124000$

$x = 1.000123123$

$0.000000877$

$x = 1.800123999$

$$y^2 \leq [x]$$

Act Ans = 1.000123123

1.000 122  
1000123 ]  
1000124

T T T T O T C S S S f f f

Size of the search space

$$0 \longrightarrow 10^{16}$$

8 decimal places

$$10^{16} \text{ integer} \times 10^8$$

$$\text{Search space} = \underbrace{\text{Integer Search Space}}_{\text{precision}}$$

$10^{16}$  int

8 decimal

$$\xrightarrow{10^{16}} = \xleftarrow{10^{-8}} 10^{24}$$

---

$$0 \xrightarrow{\text{8 decimal places}} 10^{16}$$

$$\underbrace{10^{16} \times 10^8}$$

$$10^{16} + 10^8$$

is binary searching on  $10^{24}$   
and

Search

first binary search on  $10^{16}$   
and then linear on  $10^8$

$$T.C \rightarrow O(\log(S \cdot S))$$

$$S \cdot S = 10^{24}$$

$$\rightarrow O(\log(10^{24}))$$

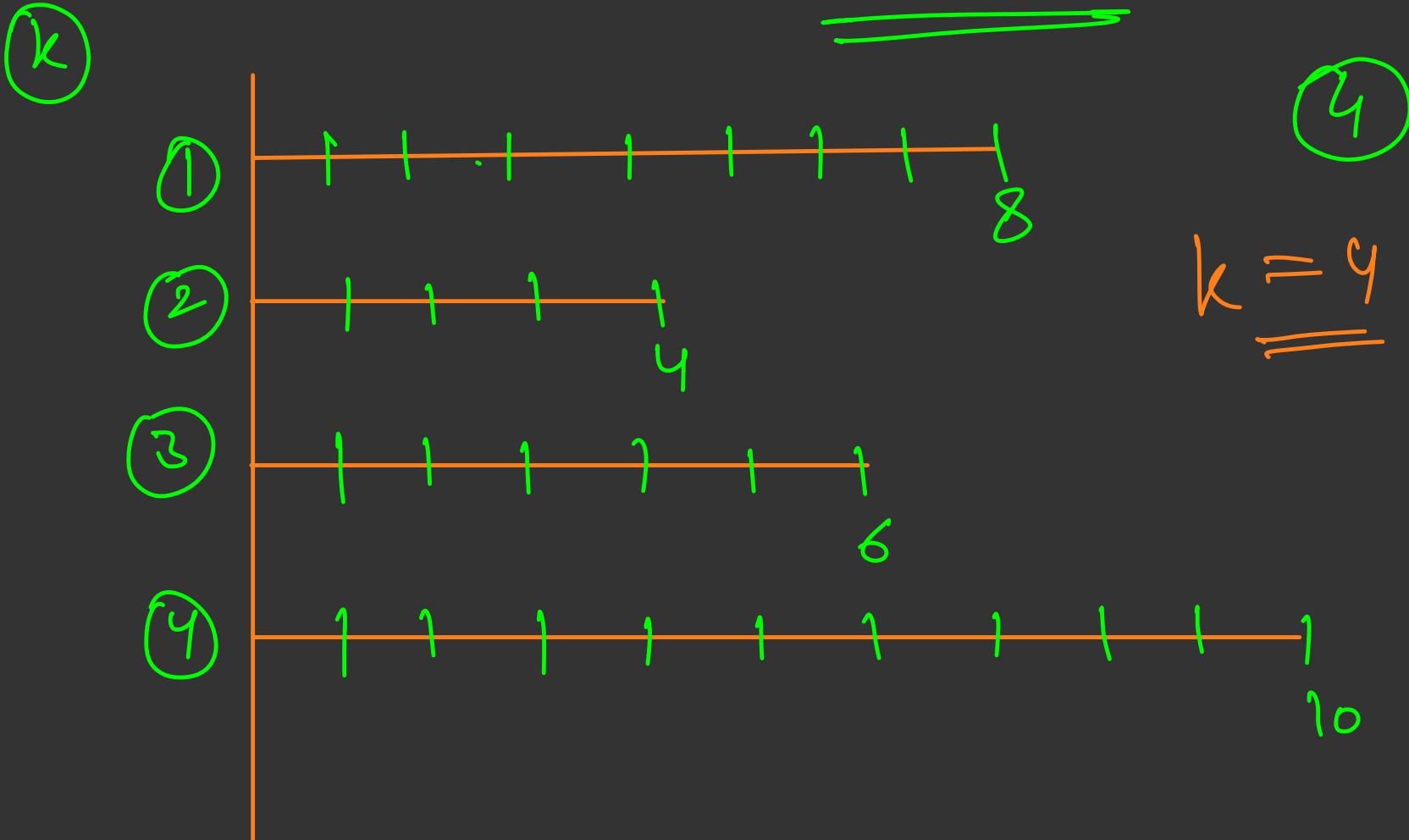
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$$O(\log(10^{16})) + O(\log(10^8))$$

$$\log(a) + \log(b) = \log(ab)$$

# Problem 1: Ropes

$n$  ropes  $\rightarrow$  1 to  $10^7$



Binary search on the maximum length

$f(x) \rightarrow T$  if we can cut out  $\geq k$  pieces  
of  $x$  length

$\rightarrow$  F o/w

$\min = 0$ ,  $\max = 10^7$

$n \rightarrow 10^4$

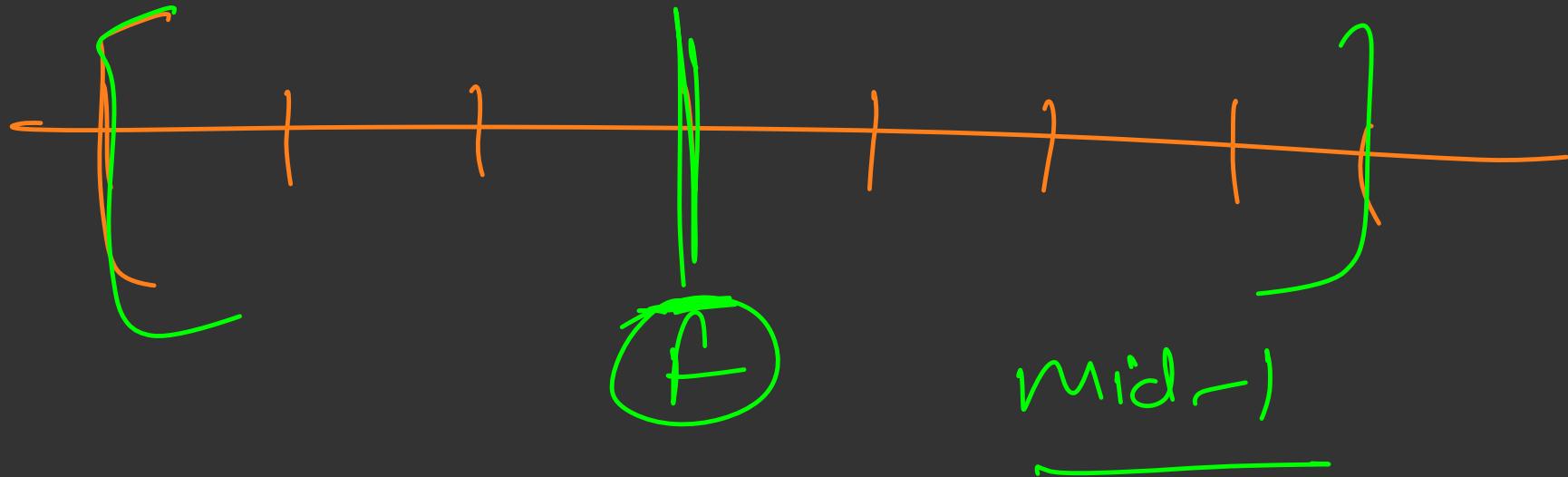
$\equiv$

$1 \leq q_i \leq 10^7$   $1 \leq k \leq 10^7$

$\equiv$

$\min = 0$ ,  $\max = 10^{18}$

1  2



TTTTTTT f f f f f f f

error

mid - error

mid + error

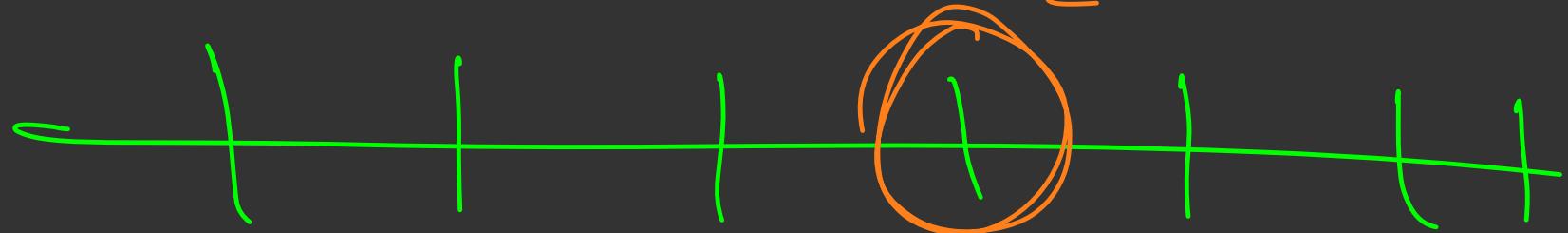
$10^{-6}$

mid -  $10^{-6}$

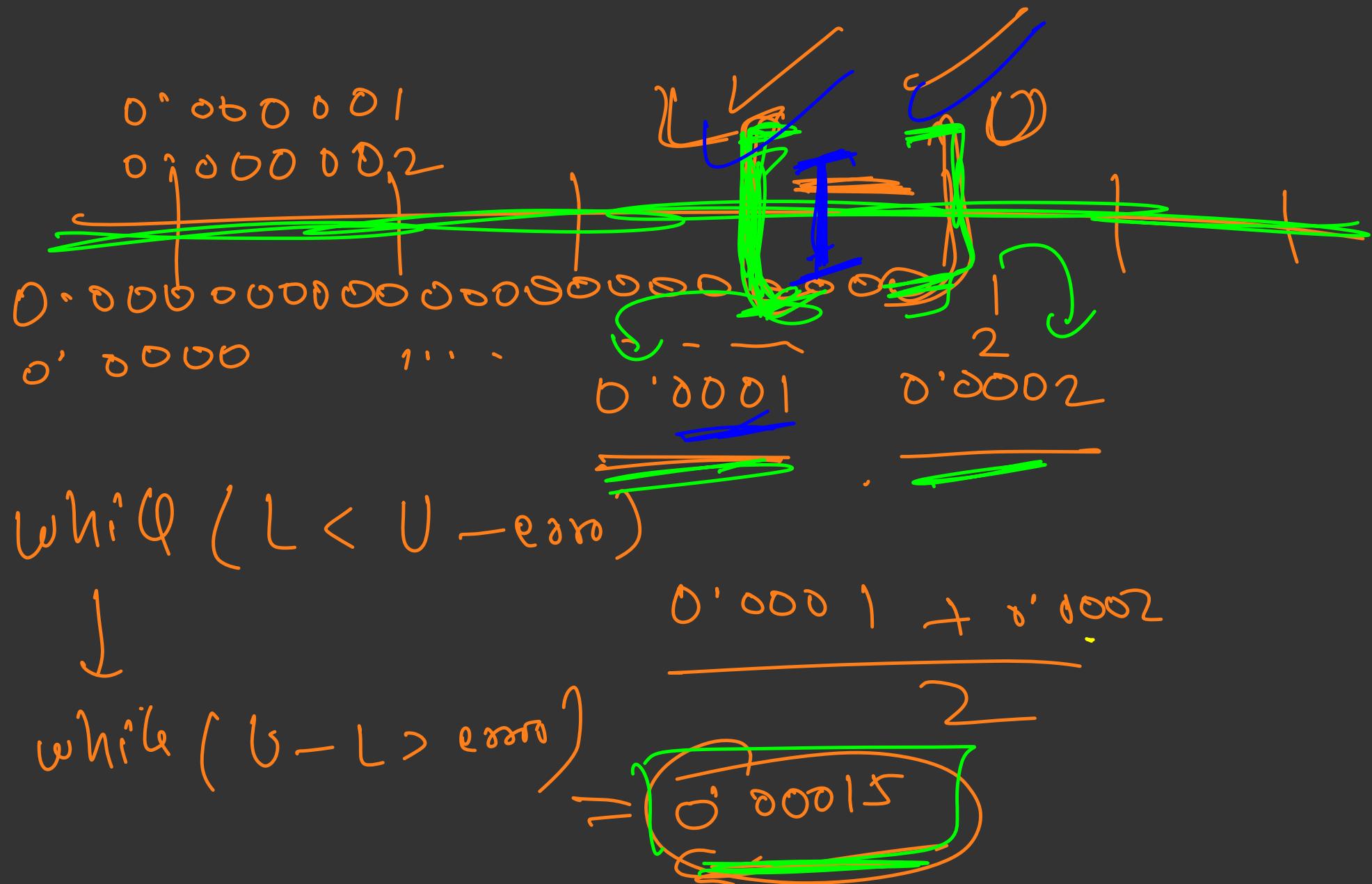
mid →

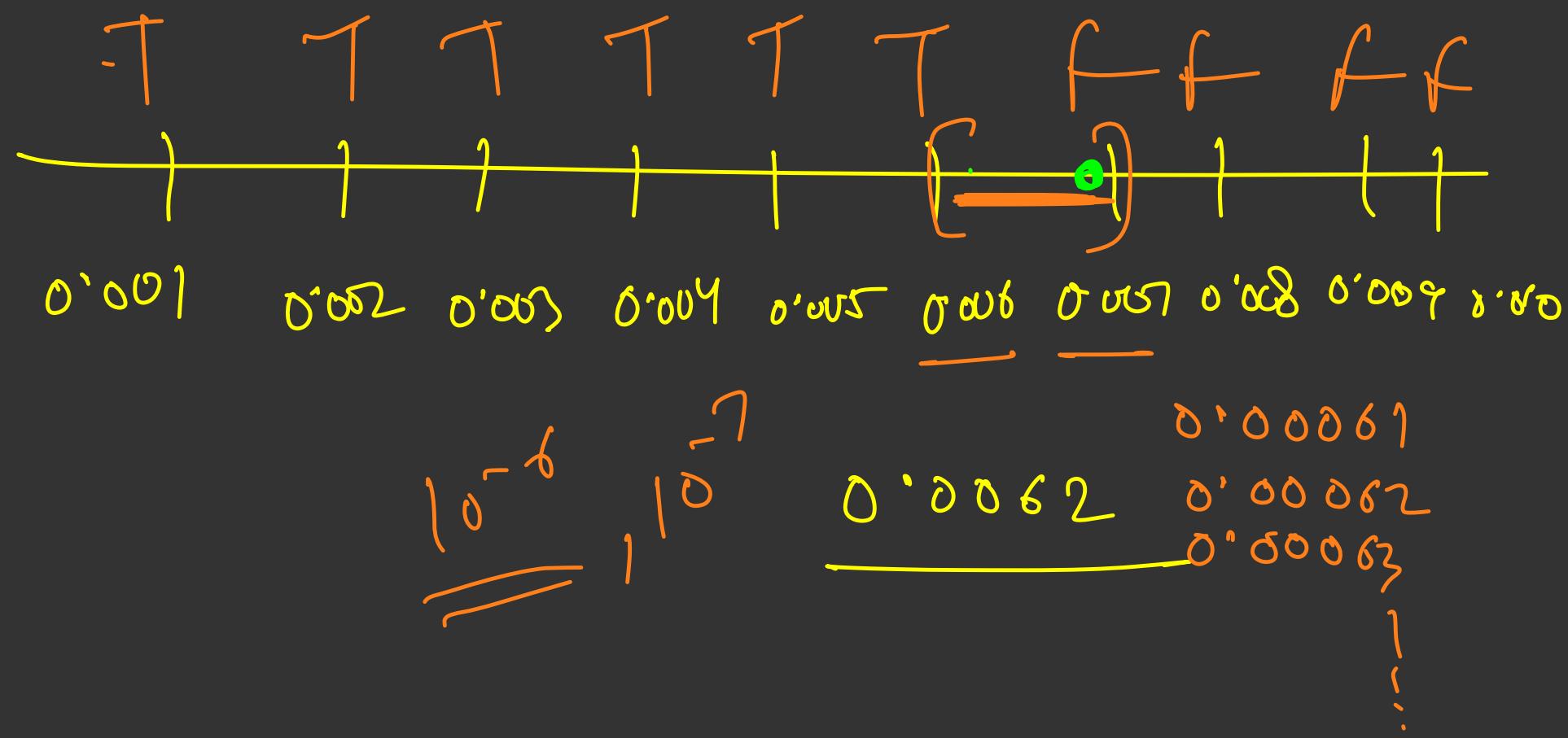
while ( $l \leq u$ ) {

mid  $\neq (l+u)/2$



y





$f(n) \rightarrow T$  if we can cut k

$$\boxed{a_0 | n}$$

$$\xrightarrow{\quad} a_1$$

pieces from all ropes  
contained  
of length n

$$a_2$$

$$a_3$$

$$a_7$$

- - - -

$$a_1 + a_2 + a_3 + \dots \geq k$$

```

bool checker(double x, vector<int>& arr, int k){
    long long total = 0; ← int
    for(auto i : arr){
        total += i / x; ← int
        if(total >= k){
            return true;
        }
    }
    return false;
}

void solve(){
    int n, k;
    cin >> n >> k;
    vector<int> arr(n);
    for(int i = 0; i < n; i++){
        cin >> arr[i];
    }
    double left = 0, right = 1e7, error = 1e-7;
    double ans = 0;
    while(right - left > error){
        double mid = (left + right) / 2;
        if(checker(mid, arr, k)){
            ans = max(ans, mid);
            left = mid + error;
        }else{
            right = mid - error;
        }
    }
    cout << fixed << ans;
}

```

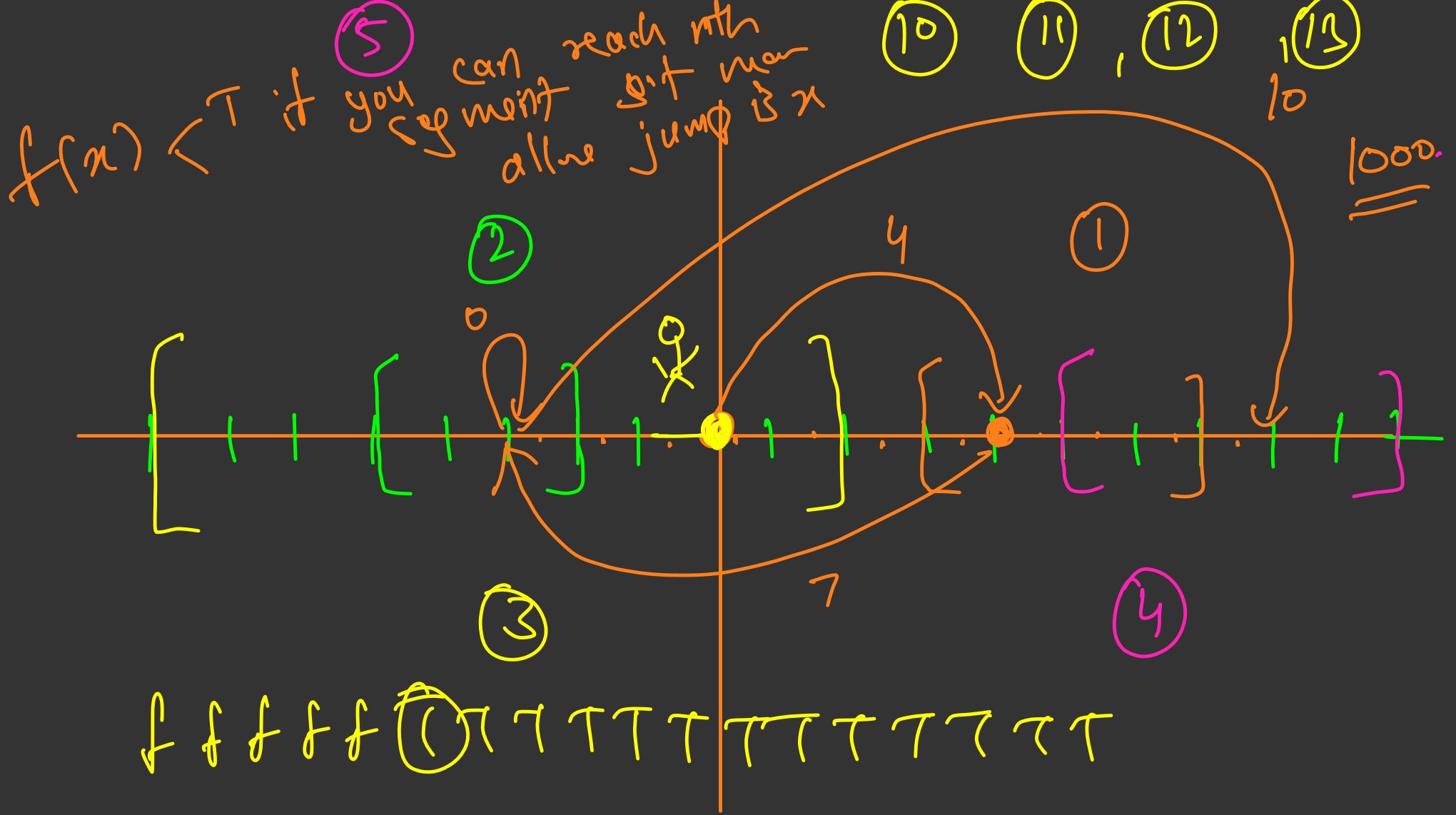
$O(\log(n))$  ✓  $O(n)$  ✓  $O(n \log(n))$   
 $O(\log(10^7))$  ✓  $O(\log(10^7))$  ✓  $O(\log(10^7))$   
 $O(\log(10^7))$  ←  $\log(10^7)$

Any problems with this code?

$4.5 \times 10^4$

# Problem 2: Jumping Through Segment

[CP-31 1400: P3]



$f(x)$



