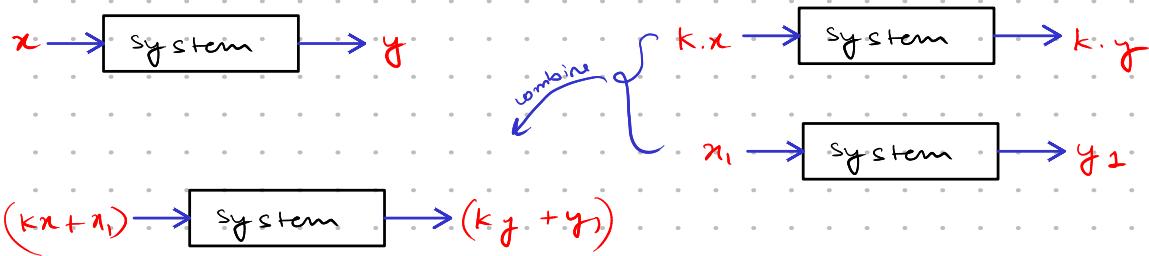


CTD L-1

- Linear circuit →



→ linear circuit consists of ① R, L, C linear element.

② independent voltage / current

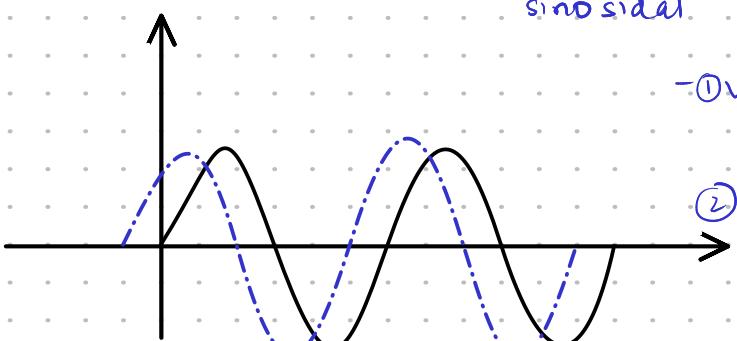
③ "linear dependent sources"

→ full response of circuit = transient + steady state.

- due to sudden change
- natural response
- RLC.

- due to source
- forced response
- V, I-

Sinusoidal & Complex Numbers



sinusoidal voltage source ↗

$$\textcircled{1} V = V_0 \sin \omega t \text{ or } V = V_0 \cos \omega t \quad \dots$$

$$\textcircled{2} V = V_0 \sin(\omega t + \phi)$$

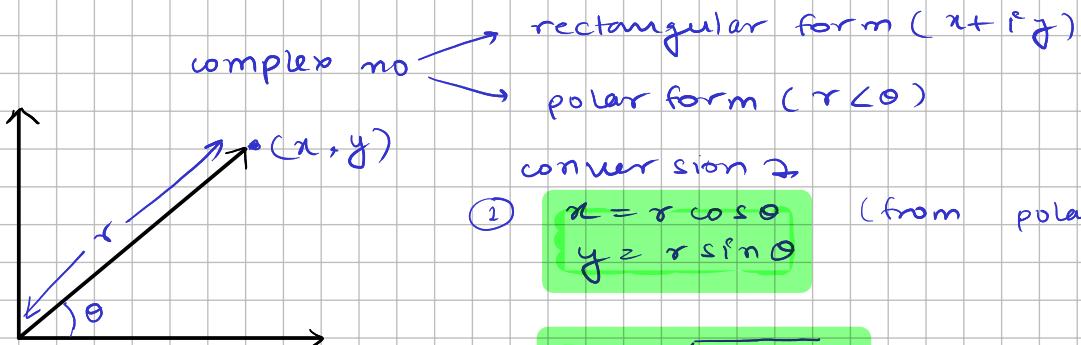
② is leading ① or ① is lagging ②

Trigonometry

- $\sin(-\theta) = -\sin \theta$
- $\sin(\theta + 90^\circ) = -\cos \theta$
- $\cos(-\theta) = \cos \theta$
- $\cos(\theta + 90^\circ) = \sin \theta$
- $\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$

- complex number $\rightarrow n + i\gamma$

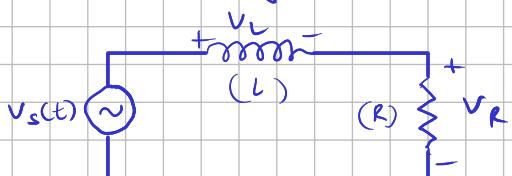
Euler's identity $\rightarrow e^{i\theta} = \cos\theta + i\sin\theta$



Steady State Analysis

\rightarrow assuming at $t \rightarrow \infty$ (transient finished)

consider γ



$$V_s(t) = V \cos(\omega t)$$

$$V_s(t) = iR + L \frac{di}{dt}$$

* consider $i(t) = A \cos \omega t + B \sin \omega t$

$$\begin{aligned} V \cos \omega t &= AR \cos \omega t + BR \sin \omega t + L(-Aw \sin \omega t + Bw \cos \omega t) \\ &= \cos \omega t (AR + wLB) + \sin \omega t (BR - AwL) \end{aligned}$$

$$\cancel{t=0}$$

$$\rightarrow V = AR + wLB \rightarrow A = \frac{V - wLB}{R}$$

$$\cancel{t=\pi/2\pi} \rightarrow 0 = BR - AwL \rightarrow B = \frac{AwL}{R}$$

$$AR = V - \frac{Aw^2L^2}{R} \rightarrow AR^2 = VR - Aw^2L^2$$

$$A = \frac{VR}{w^2L^2 + R^2}$$

$$B = \frac{wLV}{w^2L^2 + R^2}$$

$$i(t) = I_0 \cos(\omega t + \phi) \rightarrow \\ = I_0 (\cos \omega t \cos \phi - \sin \omega t \sin \phi) \quad \text{---(1)}$$

and from above statement

$$i(t) = \frac{VR}{\omega^2 L^2 + R^2} \cos(\omega t) + \frac{V\omega L}{\omega^2 L^2 + R^2} \sin \omega t \quad \text{---(11)}$$

compare

$$I_0 \cos \phi = \frac{VR}{\omega^2 L^2 + R^2}$$

$$I_0 \sin \phi = \frac{-V\omega L}{\omega^2 L^2 + R^2}$$

$$I_0 = \sqrt{\frac{V^2 R^2 + V^2 \omega^2 L^2}{(\omega^2 L^2 + R^2)^2}}$$

$$= \frac{V}{\sqrt{\omega^2 L^2 + R^2}} = Z_0$$

$$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$i(t) = \frac{V}{\sqrt{\omega^2 L^2 + R^2}} \cos(\omega t - \tan^{-1}(\omega L / R))$$

Phasor \rightarrow

$$i(t) = I_0 \cos(\omega t + \phi)$$

$$= R e^{j\phi} I_0 e^{j\omega t + j\phi}$$

\checkmark remove $e^{j\omega t}$

$$= I e^{j\phi}$$

$$I_0 e^{j\phi} = I_0 \angle \phi$$

I_0

phasor is a complex number
 \checkmark represented by capital letters.

Phasor representation in RLC circuit \rightarrow

$$v = v_m e^{j\omega t + \phi} \quad i = I_m e^{j\omega t + \phi}$$

① Resistor $\rightarrow v = iR \Rightarrow$

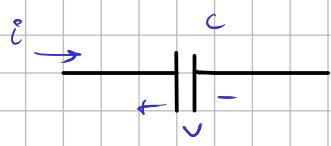


$$v_m e^{j(\omega t + \phi)} = R \times I_m e^{j(\omega t + \phi)}$$

$$v_m e^{j\phi} = R I_m e^{j\phi} \Rightarrow$$

$$V = RI$$

② Capacitor \rightarrow



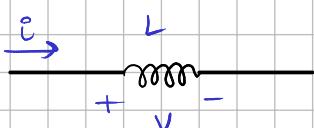
$$i = C \frac{dv}{dt}$$

$$I_m e^{j(\omega t + \phi)} = (C j\omega) V_m e^{j(\omega t + \phi)}$$

$$\Rightarrow \underbrace{I_m e^{j(\phi)}}_I = (C j\omega) \underbrace{V_m e^{j\phi}}_V$$

$$V = \frac{1}{j\omega C} \cdot I$$

③ Inductor \rightarrow



$$v = L \frac{di}{dt} \Rightarrow V_m e^{j(\omega t + \phi)} = L j\omega I_m e^{j(\omega t + \phi)}$$

$$= \underbrace{V_m e^{j\phi}}_V = j\omega L \underbrace{I_m e^{j\phi}}_I$$

$$V = j\omega L \cdot I$$

Impedance \rightarrow (Z)

$$Z = \frac{V}{I} (\Omega)$$

$$Z = R + jX$$

resistance

reactance

inductor or
capacitor

Admittance \rightarrow (Y)

$$Y = \frac{1}{Z} \text{ mho or siemens (s)}$$

Time domain \rightarrow frequency domain.
 ↓
 change with time ↓
 complex number

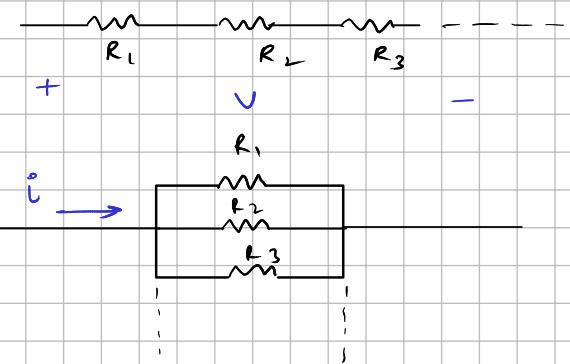
to solve circuit convert time domain to frequency domain

voltage domain \rightarrow

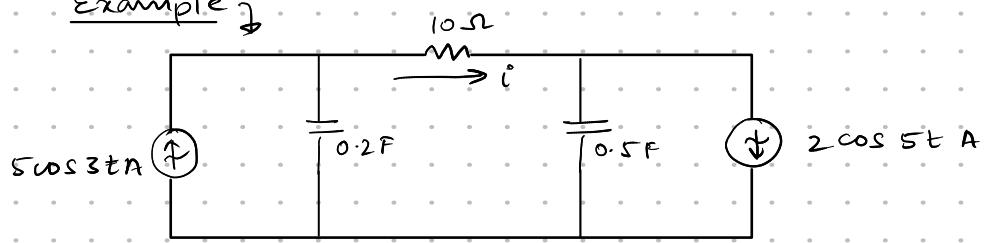
$$V_K = V \frac{R_K}{R_1 + R_2 + \dots + R_m}$$

current division \rightarrow

$$i_K = i \frac{Y_{R_K}}{Y_{R_1} + Y_{R_2} + \dots}$$



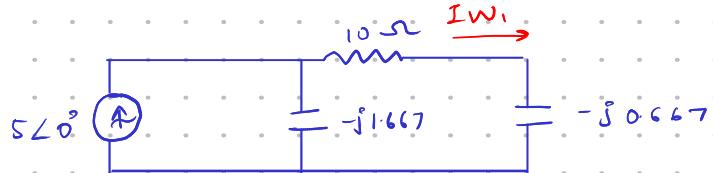
Example 3



↳ as 2 different frequencies we will use **superposition principle**

① consider left source

$$\omega_1 = 3 \text{ rad/s}$$



$$I_{\omega_1} = 5\angle 0^\circ \left[\frac{-j(1.667)}{-j(1.667) + 10 - j(0.667)} \right]$$

$$= 5 \left[\frac{-j(1.667)}{10 - j(2.334)} \right]$$

$$= 5 \left[\frac{-16.67 j + (3.89)}{100 - 5.44 j} \right]$$

$$= 5 (-0.17 j + 0.041)$$

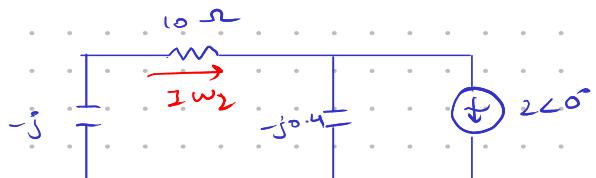
$$= 0.205 - 0.85 j$$

$$\text{in polar form } \rightarrow r = \sqrt{0.042 + 0.72} = 0.874$$

$$\theta = \tan^{-1} \left(\frac{0.72}{0.042} \right) = -76.4^\circ$$

$$= 0.874 \angle -76.4^\circ \text{ A}$$

→ right part



$$I_{\omega_2} = 2\angle 0^\circ \left(\frac{-j0.4}{-j0.4 + 10 - j0.4} \right)$$

$$= 79.2 \angle -82.03^\circ \text{ mA}$$

Applying superposition

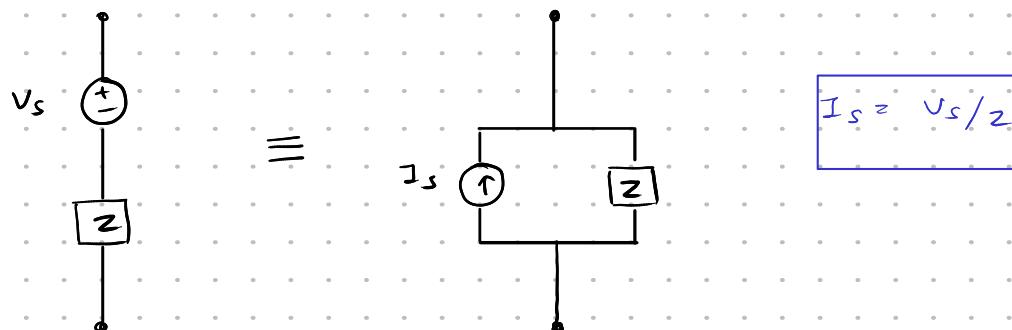
$$I = \cancel{I_{\omega_1}} + I_{\omega_2} \Rightarrow i(t) = i_{\omega_1}(t) + i_{\omega_2}(t)$$

$$i_{\omega_1}(t) = 811.7 \cos(3t - 76.8^\circ) \text{ mA}$$

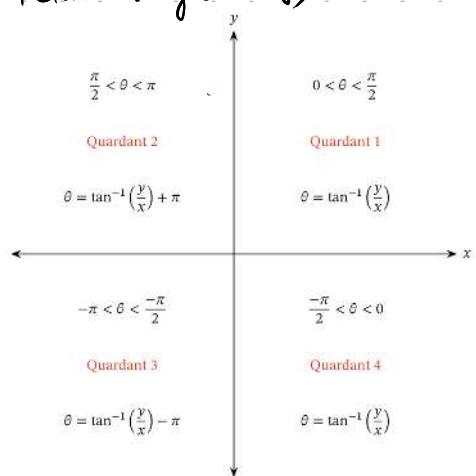
$$i_{\omega_2}(t) = 79.23 \cos(5t - 82.03) \text{ mA}$$

Phasors can only be when their corresponding frequency matches

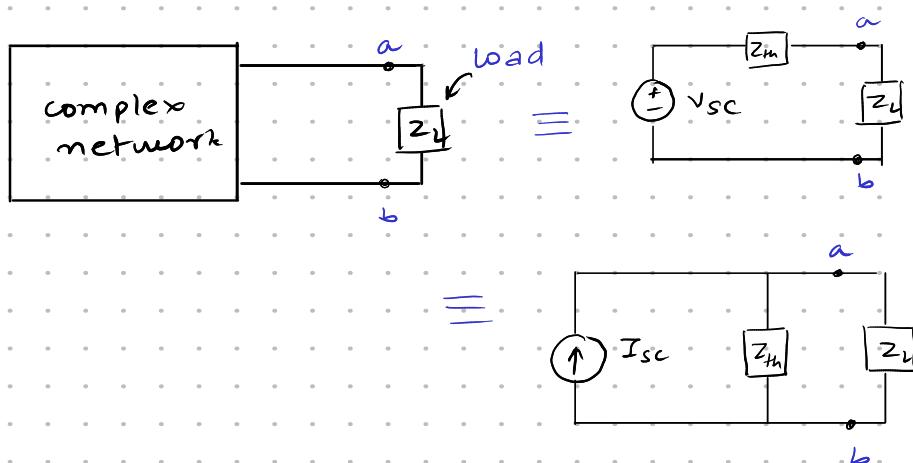
Source Transformation



Polar angles θ



Thevenin & Norton's Equivalent



→ Thevenin

- Remove load resistance in Thevenin but short for Norton
- calculate V_{sc}
- Remove all dependent sources. Thevenin
- calculate Z_{th} .

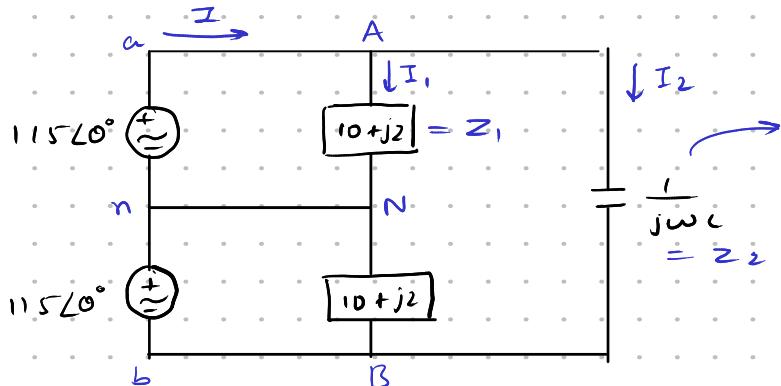


POWER ANALYSIS

$$PF = \cos(\theta - \phi)$$

if +ve \rightarrow lagging (inducting)
 -ve \rightarrow leading (capacitive)

* always write leading or lagging with P.F.



$$\theta = \phi$$

$$I_{AP} = I_1 + I_2$$

$$= \frac{115\angle 0^\circ}{10+2j} + \frac{230\angle 0^\circ}{(1/j\omega C)}$$

$$PF = \frac{\operatorname{Re}(S)}{|S|} = 1$$

$S = \text{complex power of total load} = \frac{1}{2} VI^*$

$$\Rightarrow \operatorname{Re}(S) = |S| \Rightarrow \operatorname{Im}(S) = 0$$

$$S = \frac{1}{2} V_{AN} I_{AN}^* + \frac{1}{2} V_{BN} I_{BN}^* + \frac{1}{2} V_{AB} I_{AB}^*$$

$$= \frac{1}{2} \left[115\angle 0^\circ \times \left(\frac{V_{AN}}{Z_1} \right)^* + 115\angle 0^\circ \left(\frac{V_{BN}}{Z_1} \right)^* + 230\angle 0^\circ \left(\frac{V_{AB}}{Z_2} \right)^* \right]$$

$$= \frac{1}{2} \left[\frac{|V_{AN}|^2}{|Z_1|^2} Z_1 + \frac{|V_{BN}|^2}{|Z_1|^2} Z_1 + \frac{|V_{AB}|^2}{|Z_2|^2} Z_2 \right]$$

Same

$$= \frac{1}{2} \left[\frac{2|V_{AN}|^2}{|Z_1|^2} Z_1 + 4 \frac{|V_{AN}|^2}{|Z_2|^2} |Z_2| \right]$$

$$= \frac{115^2}{104} (10 + 2j) + \frac{2(115)^2}{(1/j\omega C)^2} \left(\frac{-j}{\omega C} \right)$$

$$= 115^2 \left[\frac{10 + 2j}{104} - 2j (\omega C) \right]$$

$$= 115^2 \left[\frac{10}{104} + 2j \left[\frac{1}{104} - \omega C \right] \right]$$

$$\sqrt{104} =$$

$$\omega C = \frac{1}{104}$$

$$C = \frac{1}{104 \times \omega} = \frac{1}{104 \times 2\pi \times 500}$$

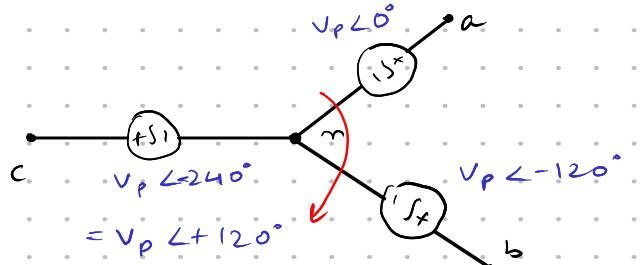
$$\Rightarrow$$

$$= 9.6 \times 10^{-3} \times 3.18 \times 10^{-3}$$

$$= 0.0 \underline{305} \times 10^{-3} = 30.5 \times 10^{-5}$$

$$= 30.5 \mu F.$$

Three Phase Sources



$$V_{an} = V_p < 0^\circ$$

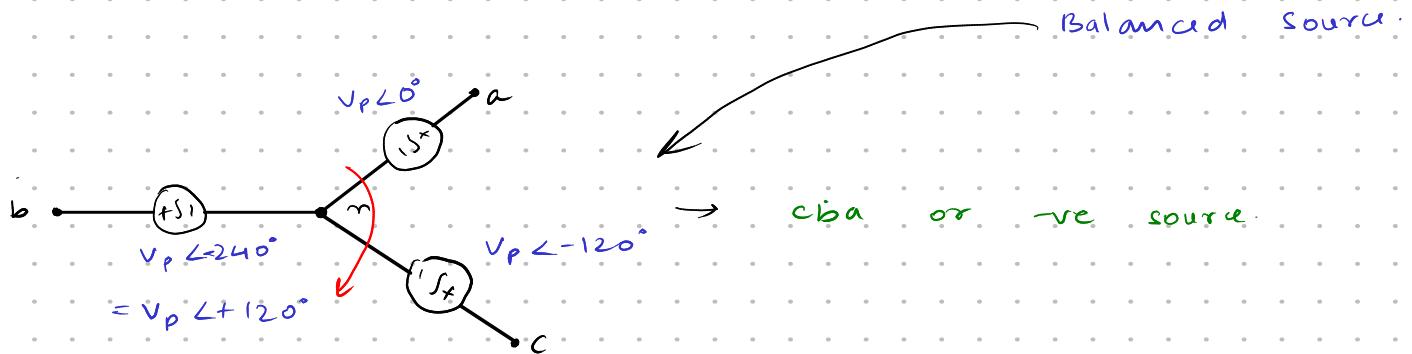
$$V_{bn} = V_p < -120^\circ$$

$$V_{cn} = V_p < -240^\circ$$

$$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p \quad []$$

$$V_{an} + V_{bn} + V_{cn} = 0$$

abc or positive sequence.



cba or -ve source.

$$V_{ab}, V_{ac}, V_{bc} \quad []$$

$$V_{ab} = V_{an} - V_{bn}$$

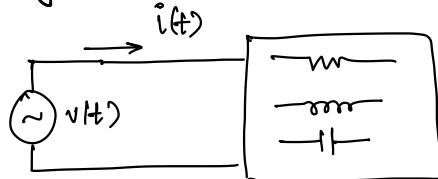
$$\begin{aligned} &= \sqrt{V_p^2 + V_p^2 - 2V_p^2 \cos(120^\circ)} \\ &= \sqrt{2V_p^2 - 2V_p^2 \times -\frac{1}{2}} \\ &= \sqrt{3}V_p = \sqrt{3}V_p < 30^\circ \end{aligned}$$

Question



Module 3

AC Power Analysis



$$\cos A \cdot \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

Instantaneous Power : $p(t) = v(t) i(t)$

$$v(t) = V_m \cos(\omega t + \theta)$$

$$i(t) = I_m \cos(\omega t + \phi)$$

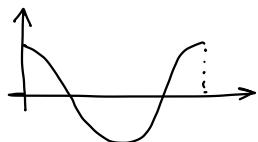
$$p(t) = V_m \cos(\omega t + \theta) \cdot I_m \cos(\omega t + \phi)$$

$$= \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\text{constant}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\text{twice frequency (harmonic)}}$$

Average Power :

$$\begin{aligned} P_{avg} &= \frac{1}{T} \int_0^T p(t) dt \\ &= \underbrace{\frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(\theta - \phi) dt}_{0} + \underbrace{\frac{1}{T} \int_0^T \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi) dt}_{0} \\ &= \frac{V_m I_m}{2} \cos(\theta - \phi) + 0 \end{aligned}$$

Average of \cos



$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$$

$$P_{avg} = \frac{1}{2} \operatorname{Re} \{ \mathbf{V} \mathbf{I}^* \}$$

$$\mathbf{I} = I_m e^{j\phi}$$

$$\mathbf{I}^* = I_m e^{-j\phi}$$

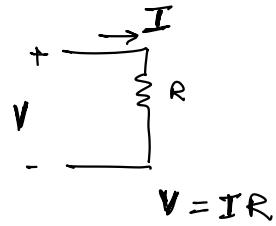
~~$P_{avg} = \mathbf{V} \mathbf{I}^*$~~

No phasor for power!

Average power absorbed by a
purely resistive element

$$P_{avg} = \frac{V_m I_m}{2} \cos(\Theta - \phi)$$

\uparrow \uparrow



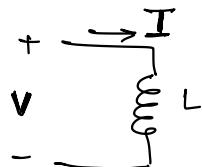
$$P_{avg} = \frac{V_m I_m}{2} = \frac{I_m^2 R}{2} = \frac{V_m^2}{2R}$$

Purely inductive element

$$P_{avg} = \frac{V_m I_m}{2} \cos(\Theta - \phi)$$

$$= \frac{V_m I_m}{2} \cos(90^\circ)$$

$$P_{avg} = 0$$



$$\mathbf{V} = j\omega L \mathbf{I}$$

$$(\Theta - \phi) =$$

$$\Rightarrow V_m e^{j\Theta} = j\omega L I_m e^{j\phi}$$

$$\Rightarrow \underbrace{\frac{V_m}{I_m}}_{e^{j\Theta-\phi}} = j\omega L \quad \Theta - \phi = 90^\circ$$

Purely capacitive element

$$P_{avg} = \frac{V_m I_m}{2} \cos(\Theta - \phi)$$

$$\Theta - \phi = -90^\circ$$

$$P_{avg} = 0$$

$p(t)$

$$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) \quad \checkmark$$

$$P_{avg} = \operatorname{Re} \left\{ \frac{V I^*}{2} \right\}$$

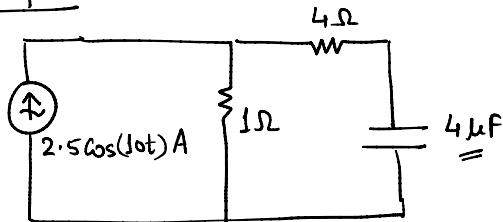
$$v(t) = V_m \cos(\omega t + \theta)$$

$$v(t) = \operatorname{Re} \left\{ V_m e^{j\omega t + j\theta} \right\}$$

$$e^{j\omega t}$$

$$V = V_m \angle \theta$$

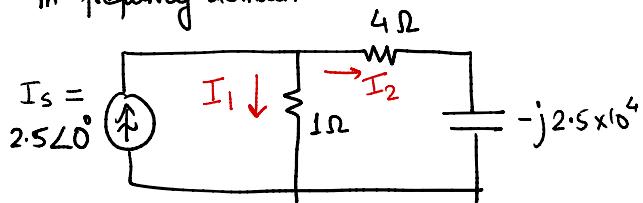
Example



① Find power delivered to each element at $t = 0, 10, 20$ ms

② Find average power delivered to each element.

In frequency domain



Power delivered to 1Ω

$$P_{1\Omega}(t) = V_{1\Omega}(t) i_p(t)$$

$$I_{1\Omega} = I_1 = I_S \frac{\frac{4-j2.5 \times 10^4}{1+4-j2.5 \times 10^4}}{\approx I_S \Rightarrow i_{1\Omega}(t) = 2.5 \cos(10t) A}$$

$$V_{1\Omega} = (I_1)(1\Omega) \Rightarrow V_{1\Omega}(t) = 2.5 \cos(10t) V$$

$$P_{1\Omega}(t) = 2.5 \cos(10t) \cdot 2.5 \cos(10t)$$

$$P_{1\Omega}(t) = 3.125 + 3.125 \cos(20t) \text{ Watts}$$

$$P_{1\Omega}(t=0) = 6.25 \text{ W}$$

$$P_{1\Omega}(t=10\text{ms}) = 6.1877 \text{ W}$$

$$P_{1\Omega}(t=20\text{ms}) = 6.0033 \text{ W}$$

Average power delivered to 1Ω

$$P_{avg, 1\Omega} = 3.125 \text{ W}$$

Power delivered to 4Ω :-

$$P_{4\Omega}(t) = V_{4\Omega}(t) \cdot i_{4\Omega}(t)$$

$$I_{4\Omega} = I_2 = I_s \frac{1}{S - j 2.5 \times 10^4} = 10^4 \angle 90^\circ A \rightarrow i_{4\Omega}(t)$$

$$V_{4\Omega} = 4(I_{4\Omega}) = 4 \times 10^4 \angle 90^\circ V \rightarrow v_{4\Omega}(t)$$

$$P_{4\Omega}(t) = 2 \times 10^{-8} + 2 \times 10^{-8} \cos(20t) W$$

$$P_{4\Omega}(t=0) = 4 \times 10^{-8} W$$

$$P_{4\Omega}(t=10ms) = 3.96 \times 10^{-8} W$$

$$P_{4\Omega}(t=20ms) = 3.84 \times 10^{-8} W$$

$$P_{avg, 4\Omega} = 2 \times 10^{-8} W$$

Power delivered to capacitor :-

$$P_C(t) =$$

$$P_{avg, C} =$$

$$I_c = I_2 = 10^4 \angle 90^\circ A \leftarrow \rightarrow i_c(t) \Rightarrow p(t) = i_c(t)v_c(t) \checkmark$$

$$V_c = (j 2.5 \times 10^4) I_2 = 2.5 V \rightarrow v_c(t)$$

$$\underbrace{i_c \times v_c}_{\text{Phasor of power}} \rightarrow p(t)$$

$$i_c(t) = 10^4 \cos(10t + 90^\circ)$$

$$v_c(t) = 2.5 \cos(10t)$$

$$P_C(t) = 1.25 \times 10^{-4} \cos(20t + 90^\circ) W$$

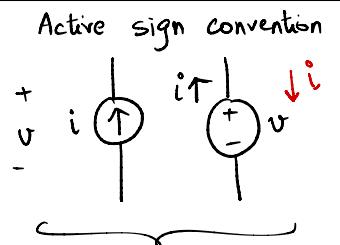
$$P_C(t=0) = 0 W$$

$$P_C(t=10ms) = 2.48 \times 10^{-5} W$$

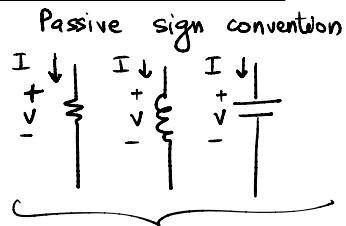
$$P_C(t=20ms) = 4.86 \times 10^{-5} W$$

$$P_{avg, C} = 0$$

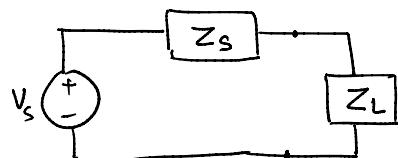
- ① $P_{1\Omega}(t) + P_{4\Omega}(t) + P_C(t) = \text{constant 1}$ X
- ② $P_{\text{avg}, 1\Omega} + P_{\text{avg}, 4\Omega} + P_{\text{avg}, C} = \text{constant 2}$ \checkmark $= P_{\text{avg, source}}$



$$v \cdot i = -20 \text{ W}$$



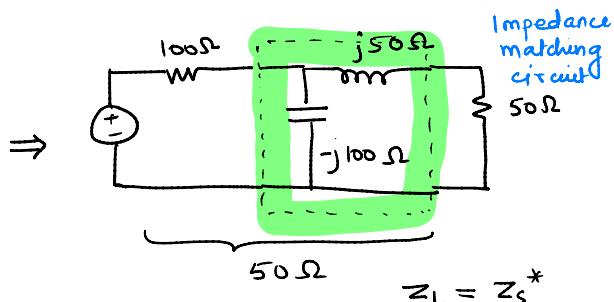
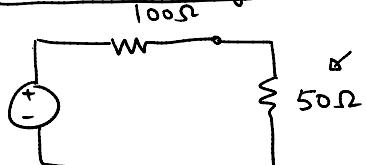
Maximum Power Transfer :



$$R_s = R_L$$

Max power delivered to load,
when $Z_L = Z_s^*$

Impedance Matching :-



$$Z_L = Z_s^*$$

Effective value (RMS value) of voltage and current:-

$$v(t) \xrightarrow{i(t)} R \quad \xrightarrow{V_{\text{eff}}} \quad I_{\text{eff}} \xrightarrow{R}$$

$$v(t) \leftarrow P_{\text{avg}} = P_{\text{avg}}$$

$$\Rightarrow \frac{1}{T} \int_0^T i^2(t) R dt = I_{\text{eff}}^2 R = \frac{V_{\text{eff}}^2}{R}$$

$$\Rightarrow I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt} = \text{Root mean square}$$

Sinusoidally varying sources

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_{\text{eff}} = \sqrt{\frac{1}{T} \int_0^T I_m^2 \cos^2(\omega t + \phi) dt}$$

$$I_{\text{eff}} = \frac{I_m}{\sqrt{2}}$$

$2 \cos^2 x = \cos 2x + 1$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

rms value

Suppose, $I = 10 \angle 90^\circ A$

$$\Rightarrow I = \underbrace{\frac{10}{\sqrt{2}}}_{\text{rms}} \angle 90^\circ A \text{ rms}$$

$$V_{\text{eff}} = \frac{V_m}{\sqrt{2}}$$

rms value

$$V_1 = 100 \angle -60^\circ V \text{ rms}$$

$$V_1 = 100\sqrt{2} \angle -60^\circ V$$

Apparent Power

$$P_{\text{apparent}} = I_{\text{eff}} V_{\text{eff}}$$

Power factor

$$\text{PF} = \frac{\text{average power}}{\text{apparent power}} = \frac{(V_m I_m / 2) \cos(\theta - \phi)}{V_{\text{eff}} I_{\text{eff}}} = \cos(\theta - \phi)$$

$$\text{PF} = \underbrace{\cos(\theta - \phi)}_{\text{Power factor angle}}$$

Review

Instantaneous Power : $p(t) = v(t) i(t)$

Average Power : $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{1}{2} \operatorname{Re}\{VI^*\}$

$$P_{avg, sources} = \sum P_{avg, elements}$$

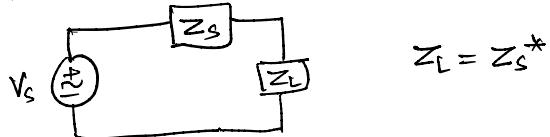
P_{avg} for resistive load : $P_{avg} \neq 0$

P_{avg} for reactive load : $P_{avg} = 0$

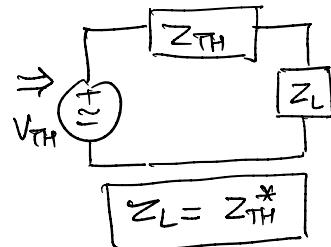
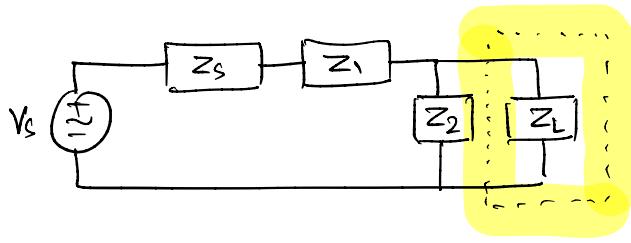
$$Z = R + jX$$

↑ resistive
↓ reactive

Max. power transfer :



$$Z_L = Z_S^*$$



$$Z_L = Z_{TH}^*$$

Effective V and I values (RMS values)

$$I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$i(t) = I_m \cos(\omega t + \phi)$$

$$I_{eff} = \frac{I_m}{\sqrt{2}}, \quad V_{eff} = \frac{V_m}{\sqrt{2}}$$

$$I = I_m \angle \phi = \sqrt{2} I_{eff} \angle \phi$$

Apparent Power

$$P_{apparent} = I_{eff} \cdot V_{eff}$$

$$\begin{array}{ll} I_{eff} & A_{rms} \\ V_{eff} & V_{rms} \\ \hline VA & (\text{volt-Ampere}) \end{array}$$

Power Factor

$$PF = \cos(\theta - \phi) = \frac{\text{Power}}{\text{Apparent}}$$

for purely resistive load: $PF = 1$ (max)

for purely reactive load: $PF = 0$ (min)

$$\begin{array}{ll} \theta - \phi & 90^\circ \\ \theta - \phi & -90^\circ \end{array}$$

Assume PF of a load = 0.5

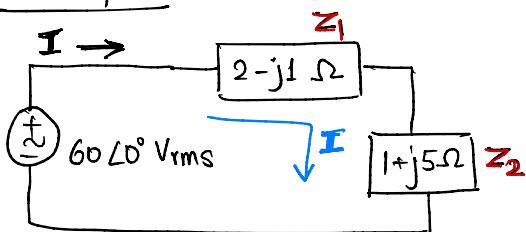
$$\cos(\theta - \phi) = 0.5$$

$$\Rightarrow (\theta - \phi) = \pm 60^\circ$$

$PF = 0.5$ leading \rightarrow capacitive $(\theta - \phi) < 0$
 0.5 lagging \rightarrow inductive $(\theta - \phi) > 0$

I w.r.t. V

Example

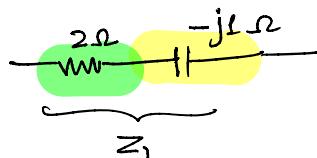


Find

- ① Average power delivered to each loads
- ② Average power supplied by source
- ③ Apparent power supplied by source
- ④ PF of combined load.

$$\text{① } P_{\text{avg}, z_1} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = \frac{1}{2} \text{Re}\{VI^*\}$$

$$= \underline{I_{\text{eff}}^2 R}$$



$$I =$$

Writing KVL:

$$- 60\angle 0^\circ V_{\text{rms}} + I (2 - j1 + 1 + j5) = 0$$

$$\Rightarrow I = \frac{60\angle 0^\circ V_{\text{rms}}}{3 + j4 \Omega} = \underbrace{12}_{I_{\text{eff}}} \angle -53.13^\circ \text{ A rms}$$

$$P_{\text{avg}, z_1} = (12)^2 2 = 288 \text{ W}$$

$$I = 12 \angle -53^\circ \text{ A rms}$$

$$i(t) = \frac{12}{I_{\text{eff}}} \cos(\omega t - 53^\circ) \text{ A rms}$$

I_{eff} (A rms) versus I_m (A)

$$I_{\text{eff}} = \frac{I_m}{\sqrt{2}} \Rightarrow I_m = I_{\text{eff}} \sqrt{2}$$

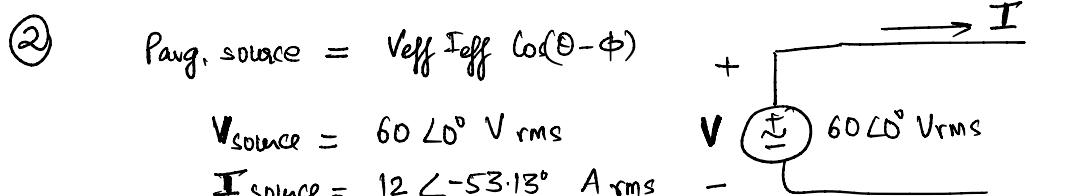
$$I_{\text{eff}} = 12$$

$$I_m = I_{\text{eff}} \sqrt{2} = \frac{12\sqrt{2}}{1}$$

$$i(t) = \frac{12\sqrt{2}}{I_m} \cos(\omega t - 53^\circ) \text{ A}$$

$$P_{\text{avg}} = \frac{1}{2} V_m I_m \cos(\theta - \phi) = V_{\text{eff}} I_{\text{eff}} \cos(\theta - \phi)$$

$$P_{\text{avg}, z_2} = I_{\text{eff}}^2 R = (12)^2 1 = 144 \text{ W}$$



$$P_{\text{avg, source}} = (60)(12) \cos(0 - (-53.13^\circ)) = 432 \text{ W}$$

$$P_{\text{avg}, z_1} = 288 \text{ W}, \quad P_{\text{avg}, z_2} = 144 \text{ W}$$

$$P_{\text{avg}, z_1} + P_{\text{avg}, z_2} = 432 \text{ W}$$

③ $P_{\text{apparent, source}} = I_{\text{eff}} V_{\text{eff}} = (60)(12) = 720 \text{ VA}$

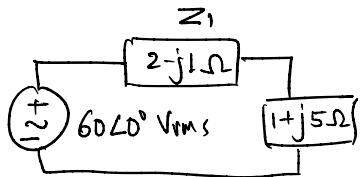
④ PF of combined loads = PF of source

$$\text{PF} = \cos(\theta - \phi) = \cos(\theta + 53.13^\circ) = 0.6 \text{ lagging}$$

$(\theta - \phi) > 0 \rightarrow \text{lagging}$

$(\theta - \phi) < 0 \rightarrow \text{leading}$

$$P_{avg, Z_1} = I_{eff}^2 R$$



$$P_{avg, Z_1} = V_{eff} I_{eff} \cos(\theta - \phi)$$

$$\begin{aligned} V_{Z_1} &= 60\angle 0^\circ \text{ V rms} \times \frac{2-j1}{3+j4} \\ &= 26.83 \angle -79.7^\circ \text{ V rms} \end{aligned}$$

$$I_{Z_1} = I = 12 \angle -53.13^\circ \text{ A rms}$$

$$P_{avg, Z_1} = 26.83 \times 12 \times \cos(-79.7^\circ - (-53.13^\circ)) = 2.88 \text{ W}$$

$$P_{avgage} = \frac{1}{2} \operatorname{Re} \{ V I^* \} ; \quad V = V_m \angle \theta \\ I = I_m \angle \phi$$

$$V_{eff} = V_{eff} \angle \theta = \frac{V_m}{\sqrt{2}} \angle \theta$$

$$I_{eff} = I_{eff} \angle \phi = \frac{I_m}{\sqrt{2}} \phi$$

$$P_{avgage} = \operatorname{Re} \{ V_{eff} I_{eff}^* \}$$

Complex Power

$$S = V_{eff} I_{eff}^* \quad (\text{unit: VA})$$

↙ { Not a phasor of power.
This is a complex number.

$$S = (V_{eff} \angle \theta) (I_{eff} \angle -\phi) = V_{eff} I_{eff} e^{j(\theta-\phi)}$$

$$\Rightarrow S = \underbrace{V_{eff} I_{eff} \cos(\theta - \phi)}_{\text{average power (VA)}} + j \underbrace{V_{eff} I_{eff} \sin(\theta - \phi)}_{\text{reactive power (VAR)}}$$

$$S = P + jQ$$

$$|S| = V_{\text{eff}} I_{\text{eff}} = \text{Papparent (VA)}$$

$$Z = R + jX$$

Purely resistive load $Z = R, X = 0$

$$S = P + jQ \rightarrow V_{\text{eff}} I_{\text{eff}} \frac{\sin(\theta - \phi)}{0}$$

\downarrow
 $V_{\text{eff}} I_{\text{eff}} \cos(\theta - \phi)$

$$P \neq 0$$

$$Q = 0$$

Purely reactive load $Z = jX, R = 0$

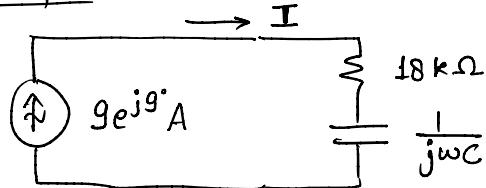
$$S = P + jQ$$

$$P = 0$$

$$Q \neq 0$$

Q signifies energy flow rate into or out of reactive component of load.

Example



Assume $C = 1 \mu F, \omega = 45 \text{ rad/s}$

Find:

- (1) Complex power provided by source
- (2) Time-average power absorbed by combined load.

Find

- (3) Reactive power absorbed by combined load
- (4) Apparent power absorbed by " "
- (5) Power factor of " "

$$S = 1.158 \times 10^6 \angle -50.99^\circ \text{ VA}$$

$$P_{\text{avg}} = 7.29 \times 10^5 \text{ W}$$

$$Q = -9 \times 10^5 \text{ VAR}$$

$$|S| = P_{\text{apparent}} = 1.158 \times 10^6 \text{ VA}$$

$$\text{PF} = 0.629 \text{ leading}$$

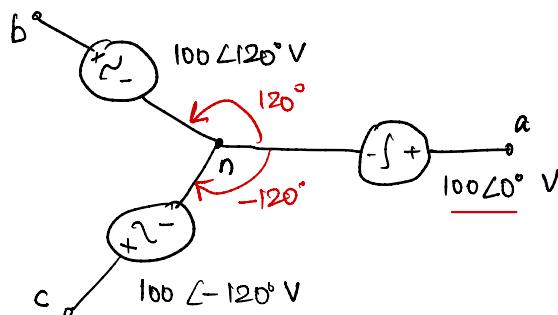
$$p(t) = 7.29 \times 10^5 + 1.158 \times 10^6 \cos(90t - 33^\circ) \text{ W}$$

Module 4

Polyphase Circuits

(Chapter 12 of textbook)

Three-phase source



$$V_{an} = 100\angle 0^\circ \text{ V}$$

$$V_{bn} = 100\angle 120^\circ \text{ V}$$

$$V_{cn} = 100\angle -120^\circ \text{ V}$$

$$\left. \begin{aligned} |V_{an}| &= |V_{bn}| = |V_{cn}| \\ \rightarrow V_{an} + V_{bn} + V_{cn} &= 0 \end{aligned} \right\} \text{Balanced Source}$$

Freq. Domain

$$100\angle 0^\circ$$

$$100\angle 120^\circ$$

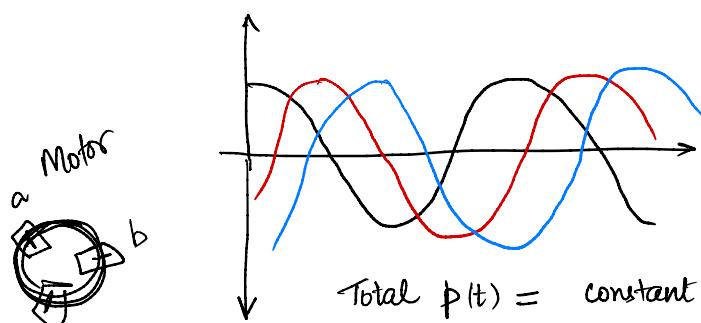
$$100\angle -120^\circ$$

Time Domain

$$100 \cos(\omega t)$$

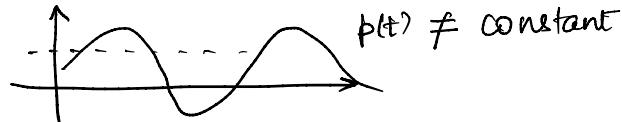
$$100 \cos(\omega t + 120^\circ)$$

$$100 \cos(\omega t - 120^\circ)$$



Total $p(t) = \text{constant}$ for three-phase sources

$$\text{for single phase source: } p(t) = \frac{1}{2} V_m I_m \cos(\theta - \phi) + \frac{1}{2} V_m I_m \cos(2\omega b + \theta - \phi)$$

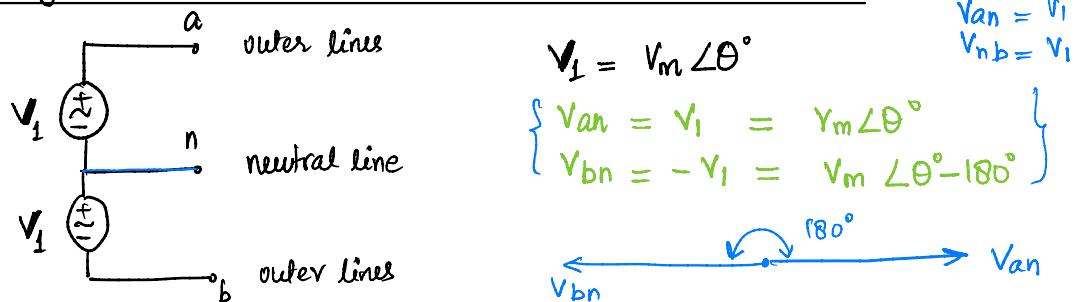


Advantages of three-phase sources are

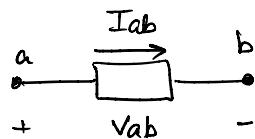
- constant power and constant torque to large motors.
- more economical since motors are more efficient.

$\theta^\circ, \theta - 180^\circ$

Single-Phase Three-Wire Source (Two-phase source)



Double Script Notations:



V_{an} = voltage at point a w.r.t. voltage at point n

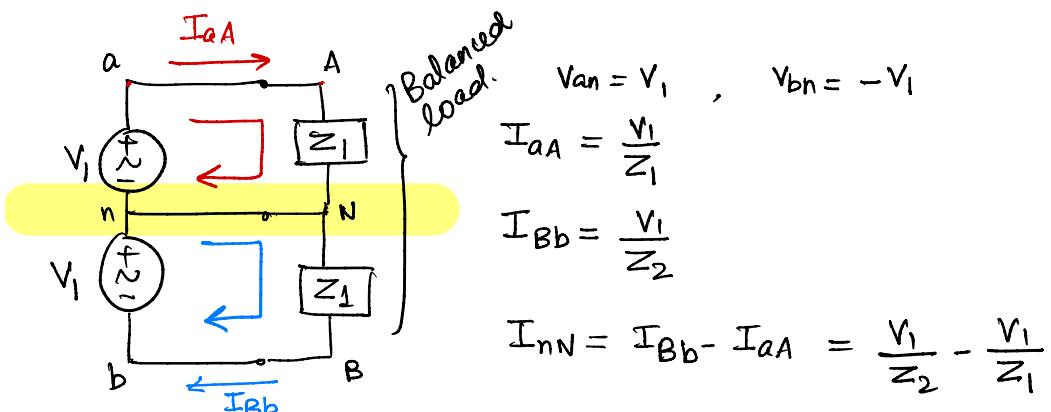
$$V_{ab} = V_{an} - V_{bn} = V_{an} + V_{nb}$$

$$V_{bn} = -V_{nb}$$

I_{ab} = current flowing from point a to point b.

Balanced Single-Phase Three-Wire Source:

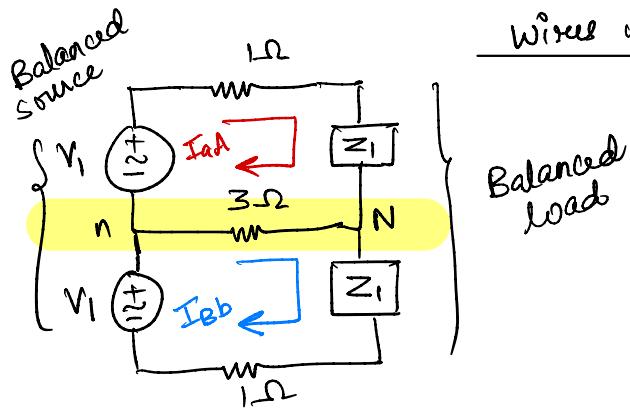
$$\left. \begin{array}{l} |V_{an}| = |V_{bn}| \\ V_{an} + V_{bn} = 0 \end{array} \right\} \text{Balanced source}$$



Assume $Z_1 = Z_2$ (Balanced load)

$$I_{nN} = \frac{V_1}{Z_1} - \frac{V_1}{Z_1} = 0$$

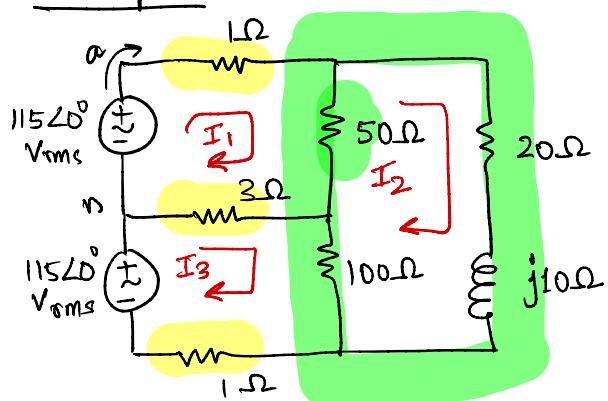
For balanced source } current in neutral line $I_{nN} = 0$
and balanced load }



Wires with non-zero resistance

$$I_{NN} = 0 = I_{Bb} - I_{aA}$$

Example



Find

- ① Average power delivered to each of three loads
- ② Avg power lost in outerlines and neutral line.
- ③ Avg power provided by source.

Line resistances

loads

Using mesh analysis

$$-115\angle 0^\circ + 1(I_1) + 50(I_1 - I_2) + 3(I_1 - I_3) = 0$$

$$50(I_2 - I_1) + (20 + j10)I_2 + 100(I_2 - I_3) = 0$$

$$-115\angle 0^\circ + (I_3 - I_1)3 + (I_3 - I_2)100 + I_3(1) = 0$$

$$\rightarrow I_1 = 11.24 \angle -19.83^\circ \text{ Arms}$$

$$I_2 = 9.389 \angle -24.47^\circ \text{ Arms}$$

$$I_3 = 10.37 \angle -21.80^\circ \text{ Arms}$$

$$I_1 - I_2 = 11.24 - 9.389$$

Avg power delivered to loads:

$$P_{avg, 50\Omega} = |I_2 - I_1|^2 (50) \\ = 206 \text{ W}$$

$$P_{avg, 100\Omega} = 117 \text{ W}$$

$$P_{avg, 20+j10\Omega} = I_2^2 (20) = 1763 \text{ W}$$

$$\text{Total power delivered to loads} = 2086 \text{ W}$$

Avg power lost in lines

$$P_{avg, aa} = |I_1|^2 1 = 126 \text{ W}$$

$$P_{avg, bb} = |I_3|^2 1 = 108 \text{ W}$$

$$P_{avg, nn} = |I_3 - I_1|^2 3 = 3 \text{ W}$$

$$\text{Total power lost in lines} = 237 \text{ W}$$

Power supplied by sources

$$P_{avg, an} = V_{an} I_{na} \cos(\theta - \phi) \\ = 115 \times 11.24 \cos(0 - (-19.83^\circ)) \\ = 1216 \text{ W}$$

$$V_{an} = 115 \angle 0^\circ \text{ V}_{rms}$$

$$I_{na} = I_1$$

$$P_{avg, nb} = 115 \times 10.37 \cos(21.80^\circ) \\ = 1107 \text{ W}$$

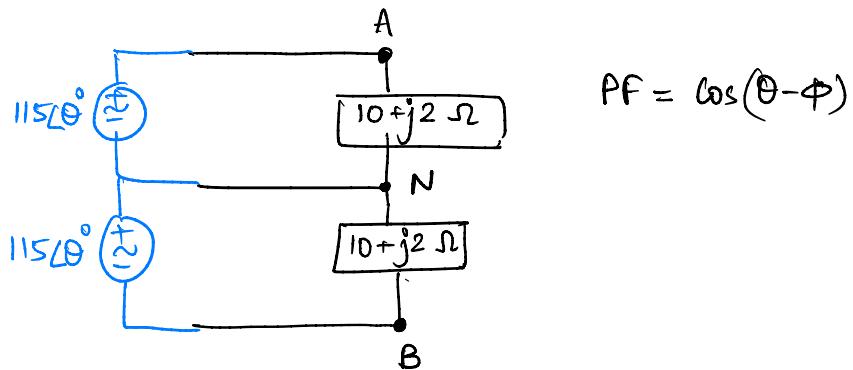
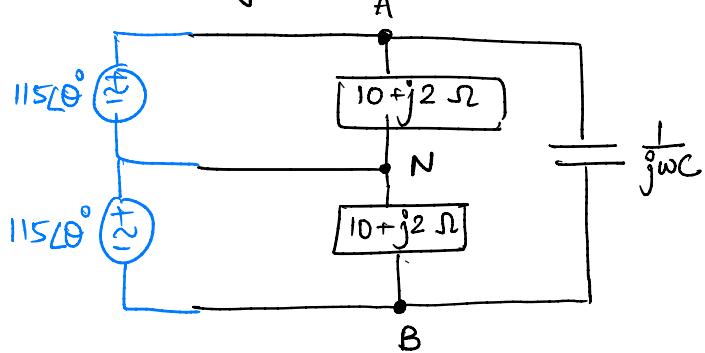
$$\text{Total power supplied} = 2323 \text{ W}$$

$$P_{supplied} = P_{delivered} + P_{loss}$$

① Example : Consider $V_{12} = 9 \angle 30^\circ$, $V_{32} = 3 \angle 130^\circ$
 find $V_{21} = 9 \angle -150^\circ$, $V_{13} = V_{12} + V_{23}$

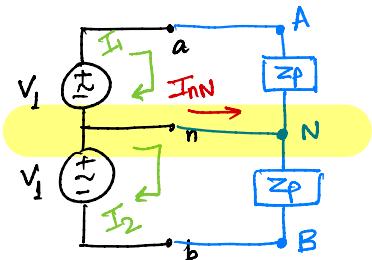
② Consider a single-phase three-wire balanced source connected to a balanced load. $f = 50 \text{ Hz}$, $V_{an} = 115 \angle 0^\circ \text{ V}$

- Find power factor of the load if capacitor is omitted.
- find value of C that will lead to a unity power factor of total load.



Review

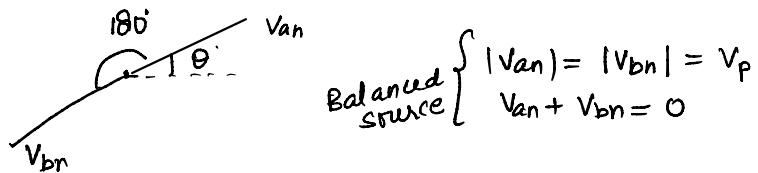
Single-phase Three-wire sources (Two-phase)



$$V_1 = V_p \angle \theta^\circ$$

$$V_{an} = V_a - V_n = V_1 = V_p \angle \theta^\circ$$

$$V_{bn} = -V_1 = -V_p \angle \theta^\circ = V_p \angle \theta^\circ - 180^\circ$$



$$I_{nN} = 0 \quad \leftarrow \text{when source and load are balance.}$$

Time-domain	freq. domain
$V_{an}(t) = V_p \cos(\omega t + \theta^\circ)$	$V_{an} = V_1 = V_p \angle \theta^\circ$
$V_{bn}(t) = V_p \cos(\omega t + \theta^\circ - 180^\circ)$	$V_{bn} = -V_1 = V_p \angle \theta^\circ - 180^\circ$

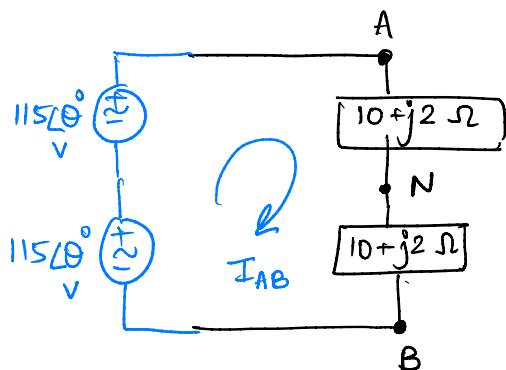
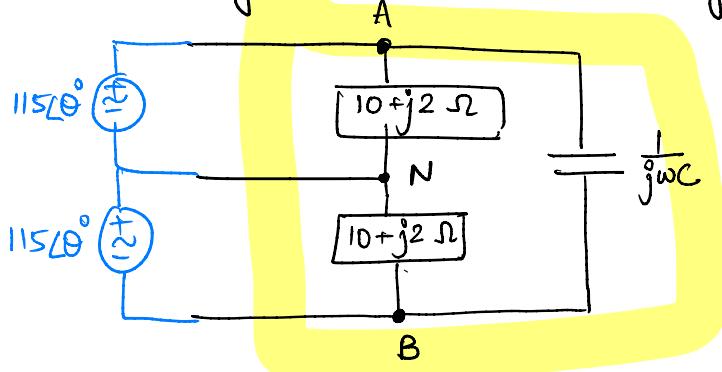
$$p(t) = V(t) i(t)$$

$$P_{avg} = V \cdot I \quad X$$

$$P_{avg} = \frac{1}{2} \operatorname{Re} \{ \underbrace{V \cdot I^*} \} \quad \checkmark$$

① Example : Consider $V_{12} = 9 \angle 30^\circ$, $V_{32} = 3 \angle 130^\circ$
 find $V_{21} = 9 \angle -150^\circ$, $V_{13} = V_{12} + V_{23}$

② Consider a single-phase three-wire balanced source connected to a balanced load. $f = 50 \text{ Hz}$, $V_{an} = 115 \angle 0^\circ \text{ V}$
 → • Find power factor of the load if capacitor is omitted.
 • find value of C that will lead to a unity power factor of total load.



$$PF = \cos(\theta - \phi)$$

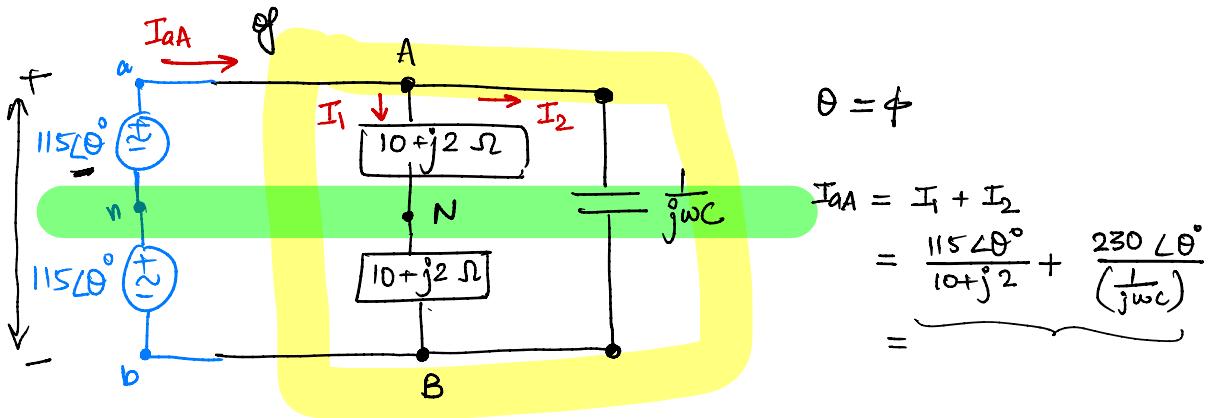
$$I_{AB} = \frac{115 \angle 0^\circ + 115 \angle 0^\circ}{2(10 + j2)} = \frac{230 \angle 0^\circ}{2(10 + j2)}$$

$$= 11.27 \angle 0^\circ - 11.3^\circ \text{ A}$$

$$\phi = \theta - 11.3^\circ$$

$$PF = \underbrace{\cos(\theta - \theta + 11.3^\circ)}_{\text{lagging}} = 0.98 \text{ lagging}$$

$$PF = 1 \Rightarrow \cos(\theta - \phi) = 1 \Rightarrow \theta - \phi = 0 \Rightarrow \theta = \phi$$



$$\theta = \phi$$

$$\begin{aligned} I_{AA} &= I_1 + I_2 \\ &= \frac{115 \angle 0^\circ}{10 + j2} + \frac{230 \angle 0^\circ}{\left(\frac{1}{j\omega C}\right)} \\ &= \underbrace{\quad \quad \quad}_{\text{Calculation}} \end{aligned}$$

$$S = \text{Complex power of total load} = \frac{1}{2} VI^*$$

$$\text{PF} = \frac{\text{Re}\{S\}}{|S|} \Rightarrow \text{PF} = 1 \Rightarrow |S| = \text{Re}\{S\} \Rightarrow \boxed{\text{Im}\{S\} = 0}$$

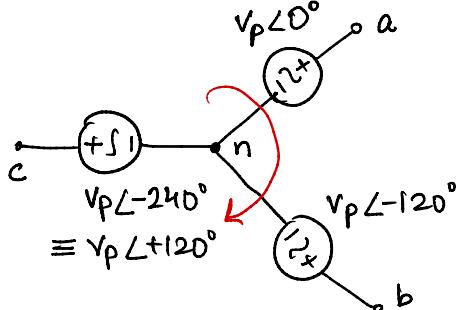
$$\begin{aligned} S &= \frac{1}{2} V_{AN} I_{AN}^* + \frac{1}{2} V_{NB} I_{NB}^* + \frac{1}{2} V_{AB} I_{AB}^* \\ &= \frac{1}{2} V_{AN} \left(\frac{V_{AN}}{Z_p}\right)^* + \frac{1}{2} V_{NB} \left(\frac{V_{NB}}{Z_p}\right)^* + \frac{1}{2} V_{AB} \left(\frac{V_{AB}}{Z_c}\right)^* \\ &= \underbrace{\frac{1}{2} \frac{|V_{AN}|^2}{|Z_p|^2} Z_p}_{} + \underbrace{\frac{1}{2} \frac{|V_{NB}|^2}{|Z_p|^2} Z_p}_{} + \underbrace{\frac{1}{2} \frac{|V_{AB}|^2}{|Z_c|^2} Z_c}_{} \\ &= \frac{|V_{AN}|^2}{|Z_p|^2} Z_p + \frac{1}{2} \frac{|V_{AB}|^2}{|Z_c|^2} Z_c \end{aligned}$$

$Z_p = 10 + j2$
$Z_c = \frac{1}{j\omega C}$

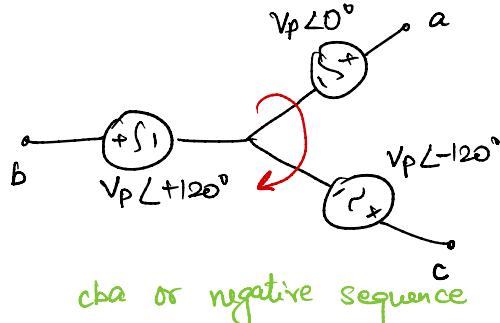
$$S = \frac{115^2}{104} (10 + j2) + \frac{1}{2} \frac{(230)^2}{(1/j\omega C)^2} \left(\frac{-j}{\omega C}\right)$$

$$\text{Im}(S) = 0 \Rightarrow C = \frac{1}{\omega \times 104} = \frac{1}{100\pi \times 104} = 30.6 \mu F$$

Three-phase Sources



→ abc or positive sequence



$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle 120^\circ$$

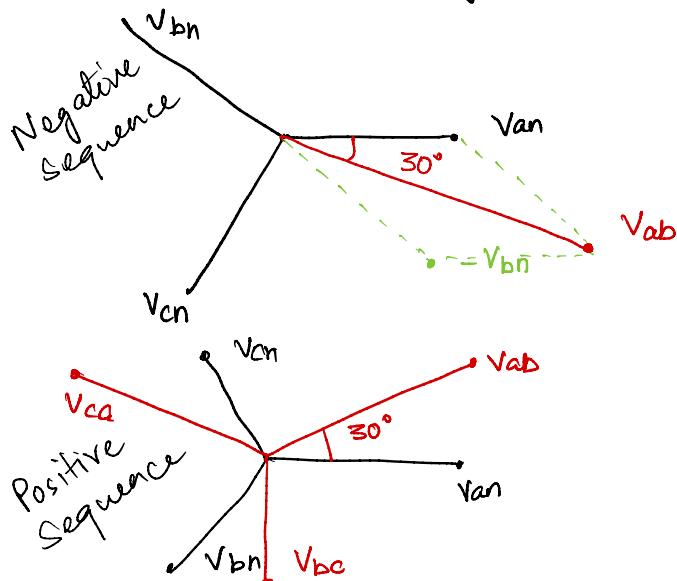
$$\left\{ \begin{array}{l} |V_{an}| = |V_{bn}| = |V_{cn}| = V_p \\ V_{an} + V_{bn} + V_{cn} = 0 \end{array} \right.$$

Balanced source.

Balanced source

Phase voltages: V_{an} , V_{bn} , V_{cn}

Line-to-line or line voltages: V_{ab} , V_{bc} , V_{ca}

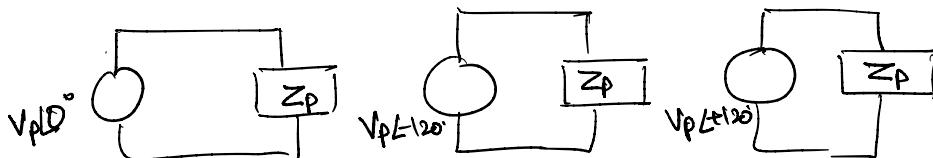
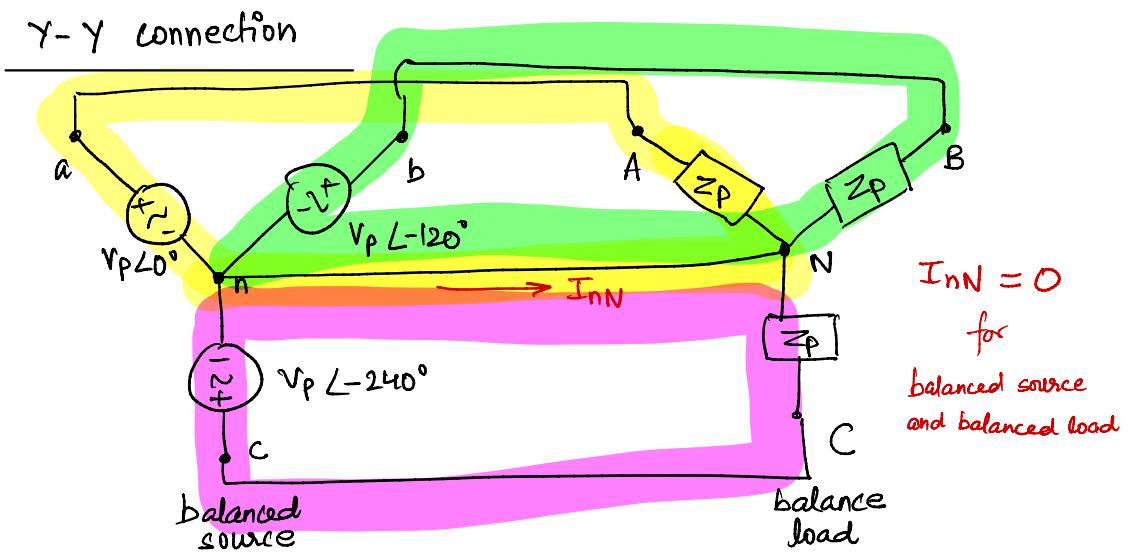


$$V_{ab} = V_{an} - V_{bn} = V_{an} + (-V_{bn})$$

$$V_L = \sqrt{3} V_p$$

$$\begin{aligned} V_{ab} &= V_{an} + V_{nb} \\ &= V_{an} - V_{bn} \\ &= \sqrt{3} V_p \angle 30^\circ \end{aligned}$$

$$\begin{aligned} V_{bc} &= \sqrt{3} V_p \angle -90^\circ \\ V_{ca} &= \underbrace{\sqrt{3} V_p \angle -210^\circ}_{V_L} \end{aligned}$$



"per-phase" analyses

$$I_{AA} = \frac{V_{an}}{Z_p}$$

$$I_{BB} = \frac{V_{bn}}{Z_p} = \frac{V_{an} \angle -120^\circ}{Z_p} = I_{AA} \angle -120^\circ$$

$$I_{CC} = \frac{V_{cn}}{Z_p} = \frac{V_{an} \angle +120^\circ}{Z_p} = I_{AA} \angle +120^\circ$$

$$I_{nn} = I_{AA} + I_{BB} + I_{CC} = 0$$

$$V_{an} = V_p \angle 0^\circ$$

$$\begin{aligned} V_{bn} &= V_p \angle -120^\circ \\ &= V_{an} \angle -120^\circ \end{aligned}$$

$$= V_{an} (1 \angle -120^\circ)$$

Example : Consider three-phase balanced $\gamma - \gamma$ connected system.

$$V_{an} = 200 \angle 0^\circ \text{ V rms}$$

$$Z_p = 100 \angle 60^\circ \Omega$$

find phase and line voltages .

find phase and line currents.

find total average power delivered to load.

Phase Voltages :

$$V_{an} =$$

$$V_{bn} = 200 \angle -120^\circ \text{ Vrms}$$

$$V_{cn} = 200 \angle +120^\circ \text{ Vrms}$$

$$\text{line voltages: } V_{ab} = \sqrt{3} V_p \angle 30^\circ = 200\sqrt{3} \angle 30^\circ \text{ Vrms}$$

$$V_{bc} = 200\sqrt{3} \angle -90^\circ \text{ Vrms}$$

$$V_{ca} = 200\sqrt{3} \angle -210^\circ \text{ Vrms}$$

Line currents:

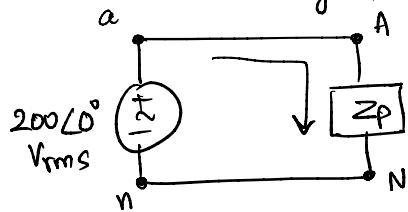
$$I_{aA} = \frac{V_{an}}{Z_p} = \frac{200 \angle 0^\circ}{100 \angle 60^\circ} = 2 \angle -60^\circ \text{ Arms}$$

$$I_{bB} = 2 \angle -120^\circ \text{ Arms}$$

$$I_{cC} = 2 \angle -300^\circ \text{ Arms}$$

Total Average power:

Average power in phase A



$$P_{avg, A} = \frac{1}{2} \operatorname{Re}\{VI^*\} = \operatorname{Re}\{V_{eff} I_{eff}^*\}$$

$$= \operatorname{Re}\{V_{an} I_{AN}\}$$

$$= \operatorname{Re}\{200 \angle 0^\circ \times 2 \angle -60^\circ\} \text{ W}$$

$$= \operatorname{Re}\{400 \angle -60^\circ\}$$

$$= 400 \cos(-60^\circ)$$

$$= 200 \text{ W}$$

$$P_{avg, B} = 200 \text{ W}$$

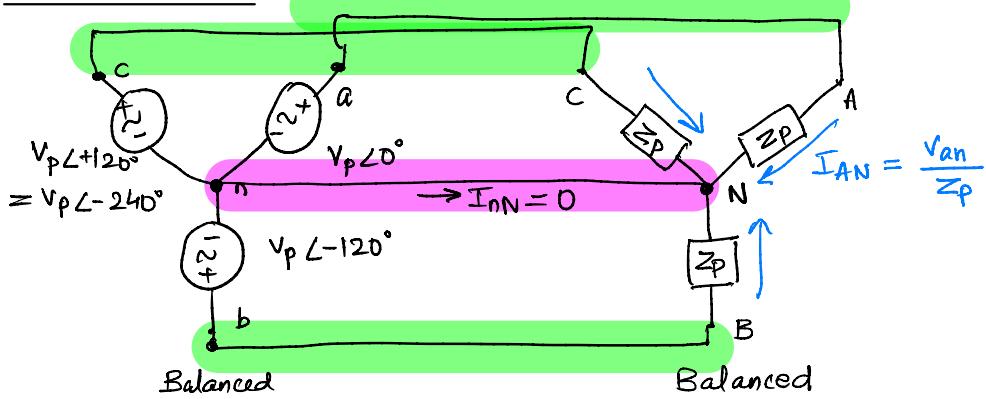
$$P_{avg, C} = 200 \text{ W}$$

$$P_{avg, \text{total}} = 600 \text{ W.}$$

Three phase systems

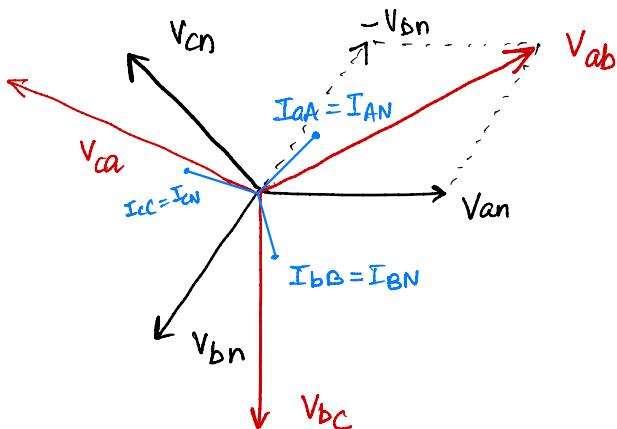
Y-Y connection

"star-star" connection



$$\left. \begin{aligned} V_{an} &= V_p \angle 0^\circ \\ V_{bn} &= V_p \angle -120^\circ \\ V_{cn} &= V_p \angle -240^\circ \end{aligned} \right\} \text{phase voltages}$$

$$\left. \begin{aligned} V_{ab} &= V_{an} - V_{bn} = \sqrt{3} V_p \angle 30^\circ \\ V_{bc} &= V_{bn} - V_{cn} = \sqrt{3} V_p \angle -90^\circ \\ V_{ca} &= V_{cn} - V_{an} = \sqrt{3} V_p \angle -210^\circ \end{aligned} \right\} \text{line voltages}$$



V_p = magnitude of phase voltages = $|V_{AN}| = |V_{BN}| = |V_{CN}| = V_p$
 V_L = magnitude of line voltages = $|V_{ab}| = |V_{bc}| = |V_{ca}| = \sqrt{3}V_p$

$$V_L = \sqrt{3}V_p$$

Line currents = phase currents.

$$I_{aA} = I_{AN} = \frac{V_{AN}}{Z_p} = \frac{V_p}{Z_p}$$

$$I_{bB} = I_{BN} = \frac{V_{BN}}{Z_p}$$

$$I_{cC} = I_{CN} = \frac{V_{CN}}{Z_p}$$

$$\begin{aligned} Z_p &= \frac{V_{AN}}{I_{AN}} \\ &= \frac{V_p \angle \theta^\circ}{I_p \angle \phi^\circ} \\ &= \frac{V_p}{I_p} \angle \theta - \phi^\circ \end{aligned}$$

I_p = magnitude of phase currents = $|I_{AN}| = |I_{BN}| = |I_{CN}|$

I_L = magnitude of line currents = $|I_{aA}| = |I_{bB}| = |I_{cC}|$

$$I_p = I_L$$

$$\begin{aligned} P_{avg} \text{ for any phase} &= \frac{1}{2} \operatorname{Re} \{ V I^* \} & V_{AN} &= V_p \angle \theta^\circ \\ P_{avg, A} &= \frac{1}{2} \operatorname{Re} \{ V_{AN} I_{AN}^* \} & I_{AN} &= I_p \angle \phi^\circ \\ &= \frac{1}{2} \operatorname{Re} \{ V_p \angle \theta^\circ (I_p \angle \phi^\circ)^* \} \\ &= \frac{1}{2} V_p I_p \cos(\theta - \phi) \\ &= \frac{1}{2} \frac{1}{\sqrt{3}} V_L I_L \cos(\theta - \phi) \end{aligned}$$

P_{avg} per phase = P_{avg} Aphase = P_{avg} Bphase = P_{avg} Cphase = $\frac{1}{2} V_p I_p \cos(\theta - \phi)$

$$P_{avg} \text{ per phase} = \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi), \text{ if } V_L, I_L \text{ are rms values.}$$

$$P_{avg} \text{ total} = \frac{3}{2} V_p I_p \cos(\theta - \phi)$$

$$= 3 V_p I_p \cos(\theta - \phi), \text{ if } V_p \& I_p \text{ are rms values.}$$

$$P_{avg} \text{ total} = \sqrt{3} V_L I_L \cos(\theta - \phi), \text{ if } V_L \& I_L \text{ are rms values.}$$

Total instantaneous power

$$P_{\text{total}}(t) = P_A(t) + P_B(t) + P_C(t)$$

$$P_A(t) = V_{AN}(t) i_{AN}(t)$$

$$V_{AN}(t) = V_p \cos(\omega t + \theta)$$

$$i_{AN}(t) = I_p \cos(\omega t + \phi)$$

$$P_A(t) = V_p I_p \cos(\omega t + \phi) \cos(\omega t + \theta)$$

$$= \frac{V_p I_p}{2} \left[\underbrace{\cos(\theta - \phi)}_{\beta} + \underbrace{\cos(2\omega t + \theta + \phi)}_{\alpha} \right]$$

$$= \frac{V_p I_p}{2} [\cos \alpha + \cos \beta]$$

$$P_B(t) = V_{BN}(t) i_{BN}(t) = \frac{V_p I_p}{2} [\cos \beta + \cos(\alpha - 240^\circ)]$$

$$P_C(t) = \frac{V_p I_p}{2} [\cos \beta + \cos(\alpha + 240^\circ)]$$

$$P_{\text{total}}(t) = \frac{V_p I_p}{2} \left[3 \cos \beta + \cos \alpha + \underbrace{\cos(\alpha - 240^\circ)}_{\alpha} + \underbrace{\cos(\alpha + 240^\circ)}_{\alpha} \right]$$

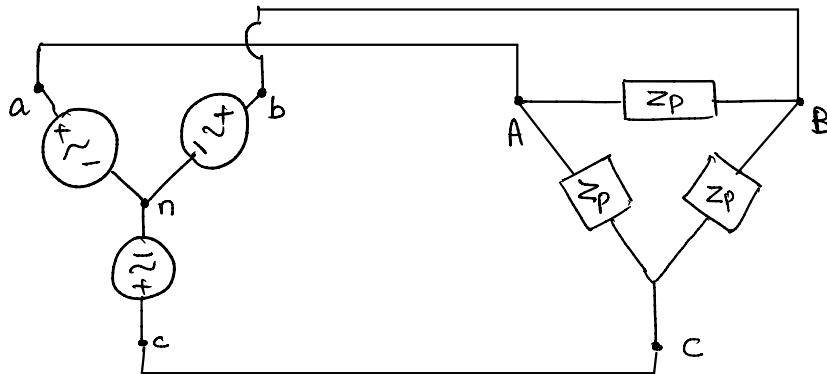
$$P_{\text{total}}(t) = 3 \frac{V_p I_p}{2} \cos \beta = \frac{3 V_p I_p}{2} \cos(\theta - \phi) - \cos \alpha$$

$$P_{\text{avg}} = \frac{1}{T} \int_0^T P(t) dt$$

Y-Δ Connections

Y-connected source

Δ-connected load (no neutral node)



Phase voltages: $V_{AB} = V_{ab}$, $V_{BC} = V_{bc}$, $V_{CA} = V_{ca}$

Line voltages: V_{ab} , V_{bc} , V_{ca} } ← same as phase voltages

$$\left. \begin{aligned} V_{AB} &= V_{ab} = V_{an} - V_{bn} = \sqrt{3} V_{an} \angle 30^\circ \\ V_{BC} &= V_{bc} = \sqrt{3} V_{an} \angle -90^\circ \\ V_{CA} &= V_{ca} = \sqrt{3} V_{an} \angle -210^\circ \end{aligned} \right\} \begin{array}{l} \text{Line voltages} \\ = \text{phase voltages} \end{array}$$

V_p = magnitude of phase voltage = $|V_{AB}| = |V_{BC}| = |V_{CA}| = \sqrt{3} |V_{an}|$

V_L = magnitude of line voltage = $(V_{ab}) = |V_{bc}| = |V_{ca}| = \sqrt{3} |V_{an}|$

$$V_p = V_L$$

Line currents: $I_{aA} = I_{AB} - I_{CA} = \sqrt{3} I_{AB} \angle -30^\circ$

$$I_{bB} = \sqrt{3} I_{BC} \angle -30^\circ$$

$$I_{cC} = \sqrt{3} I_{CA} \angle -30^\circ$$

Phase currents: $I_{AB} = V_{AB} / Z_P$

$$I_{BC} = V_{BC} / Z_P$$

$$I_{CA} = V_{CA} / Z_P$$

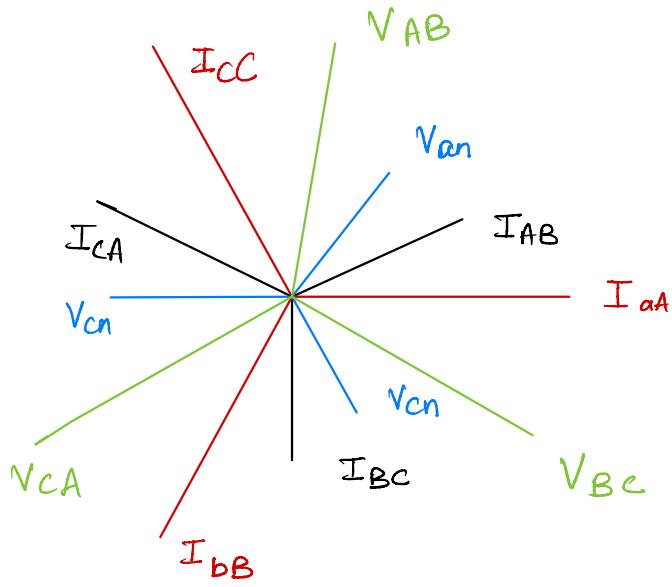
$$\begin{aligned} Z_P &= \frac{V_{AB}}{I_{AB}} \\ &= \frac{V_p \angle \theta^\circ}{I_p \angle \phi^\circ} \\ &= \frac{V_p}{I_p} \angle \theta - \phi^\circ \end{aligned}$$

I_p = magnitude of phase currents = $|I_{AB}| = |I_{BC}| = |I_{CA}|$

I_L = magnitude of line currents = $|I_{aA}| = |I_{bB}| = |I_{cC}|$

$$\Rightarrow I_L = \sqrt{3} \underbrace{|I_{AB}|}_{I_p}$$

$$\Rightarrow I_L = \sqrt{3} I_p$$



Similar to Y-Y connection:

$$P_{avg \text{ per phase}} = \frac{1}{2} R_e \left\{ V_{AB} I_{AB}^* \right\} = \frac{1}{2} V_p I_p \cos(\Theta - \phi)$$

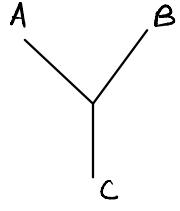
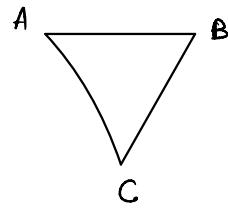
$$P_{avg \text{ per phase}} = \frac{V_L I_L}{\sqrt{3}} \cos(\Theta - \phi) , \text{ if } V_L, I_L \text{ are rms values.}$$

$$P_{avg \text{ total}} = \frac{3}{2} V_p I_p \cos(\Theta - \phi)$$

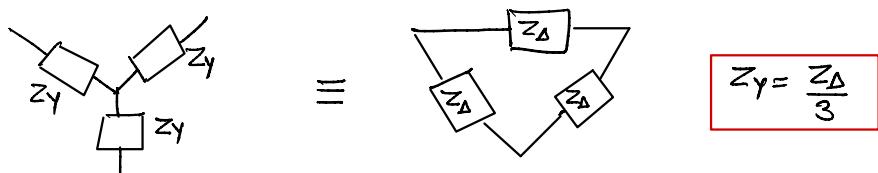
$$= 3 V_p I_p \cos(\Theta - \phi) , \text{ if } V_p \text{ & } I_p \text{ are rms values.}$$

$$P_{avg \text{ total}} = \sqrt{3} V_L I_L \cos(\Theta - \phi) , \text{ if } V_L \text{ & } I_L \text{ are rms values.}$$

Y-Y connection versus Y-Δ connection

		
Phase voltages Line voltages	V_{AN}, V_{BN}, V_{CN} V_{ab}, V_{bc}, V_{ca}	$V_{AB}, V_{BC}, V_{CA} \rightarrow$ same $V_{ab}, V_{bc}, V_{ca} \leftarrow$ same
Phase currents Line currents	$I_{AN}, I_{BN}, I_{CN} \leftarrow$ same $I_{aA}, I_{bB}, I_{cC} \leftarrow$ same	I_{AB}, I_{BC}, I_{CA} I_{aA}, I_{bB}, I_{cC}
	$V_L = \sqrt{3} V_p$ $I_L = I_p$	$V_L = V_p$ $I_L = \sqrt{3} I_p$
Z_p	$Z_p = \frac{V_p}{I_p} \angle \theta - \phi$	$Z_p = \frac{V_p}{I_p} \angle \theta - \phi$
Power per phase	$\frac{1}{2} V_p I_p \cos(\theta - \phi)$ $= \frac{1}{2} \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi)$	$\frac{1}{2} V_p I_p \cos(\theta - \phi)$ $= \frac{1}{2} \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi)$
Power total	$\frac{3}{2} V_p I_p \cos(\theta - \phi)$ $= \frac{1}{2} \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi)$	$\frac{3}{2} V_p I_p \cos(\theta - \phi)$ $= \frac{1}{2} \frac{V_L I_L}{\sqrt{3}} \cos(\theta - \phi)$

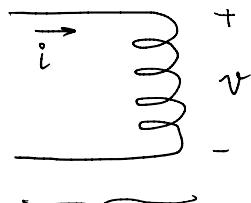
Star to delta load transformation :-



Module 5

Magnetically coupled circuits

(Chapter 13 of textbook)



"Pure linear inductors"
→ no resistance

$$v = L \frac{di}{dt}$$

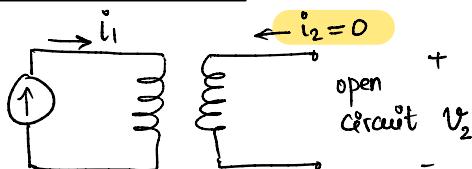
i leads to magnetic flux $i \propto \phi$

Magnetic flux leads to induced v $v \propto d\phi/dt \Rightarrow v \propto di/dt$

$$v = L \frac{di}{dt}$$

DC \rightarrow const $i \rightarrow$ const mag. flux $\rightarrow v = 0$

Mutual Inductance:



$$v_2 \propto \frac{di_1}{dt}$$

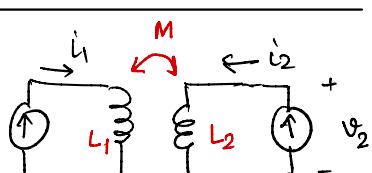
$$\Rightarrow v_2 = M \frac{di_1}{dt}$$

mutual inductance

L : self inductance

Henry

Now consider: $i_2 \neq 0$

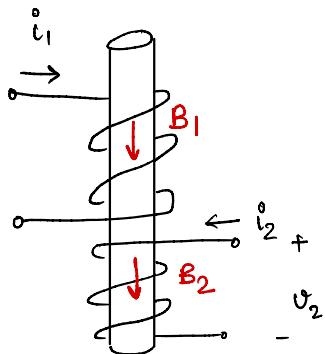


$$v_2 \propto \frac{di_1}{dt} , v_2 \propto \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Self inductance

Mutual inductance



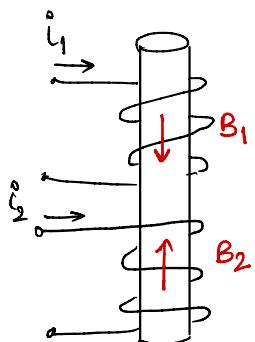
$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

B_2 B_1

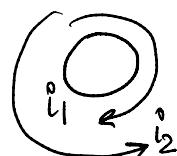
Top view



Additive magnetic flux



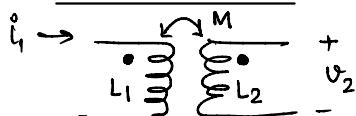
Top view



Subtractive magnetic flux

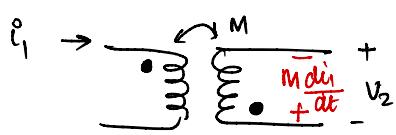
$$v_2 = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

Dot convention



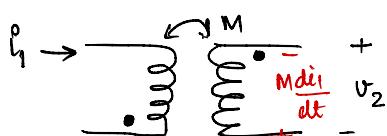
Assume $i_2 = 0$

$$v_2 = M \frac{di_1}{dt}$$

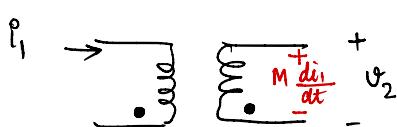


$$v_2 = -M \frac{di_1}{dt}$$

} Current entering at dotted terminal leads to +ve voltage reference at dotted terminal.



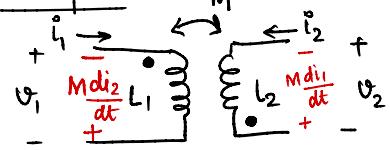
$$v_2 = -M \frac{di_1}{dt}$$



$$v_2 = M \frac{di_1}{dt}$$

} Current entering at un-dotted terminal leads to +ve voltage reference at un-dotted terminal.

Example :



(A) find v_2 if $i_1 = -8e^{-t} A$, $i_2 = 0$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

0

$$v_2 = -M \frac{di_1}{dt} = -2 \frac{d}{dt} (-8e^{-t}) = -2 \times (-8) \times (-1) e^{-t}$$

$$= -16 e^{-t} V$$

(B) find v_1 if $i_1 = 0$ and $i_2 = 5 \sin 45t A$.

$$v_1 = \underbrace{L_1 \frac{di_1}{dt}}_0 - M \frac{di_2}{dt}$$

$$v_1 = -M \frac{d}{dt} (5 \sin 45t) = -2 \times 5 \times 45 \cos 45t$$

$$= -450 \cos 45t V$$

Sign Convention for self inductance:

$$v = L \frac{di}{dt}$$

(current entering from
+ve terminal)

$$v = -L \frac{di}{dt}$$

(current leaving from
+ve terminal)

Phasor Representation :

$$V_1 = L_1 \frac{di^*}{dt} + M \frac{di^*}{dt}$$

$$V_2 = L_2 \frac{di^*}{dt} + M \frac{di^*}{dt}$$

$$v = L \frac{di}{dt}$$

time domain

$$V = j\omega L I_1$$

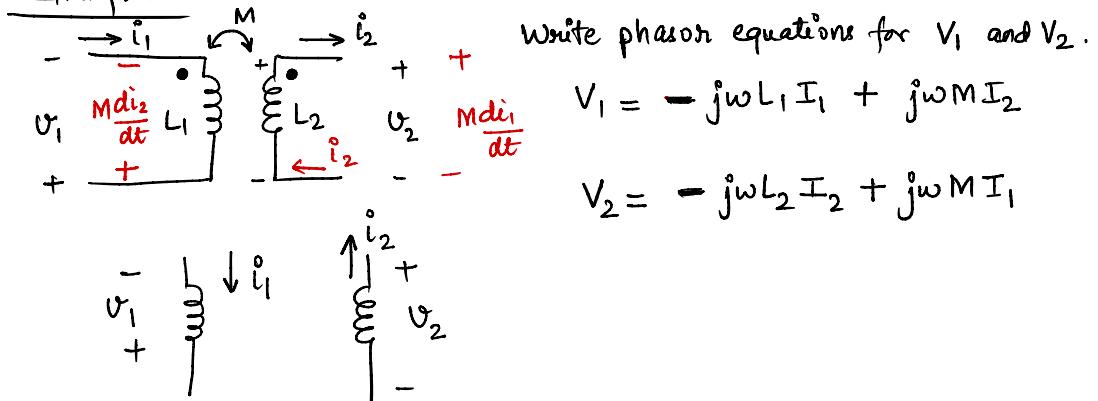
freq.
domain

for sinusoidally varying sources

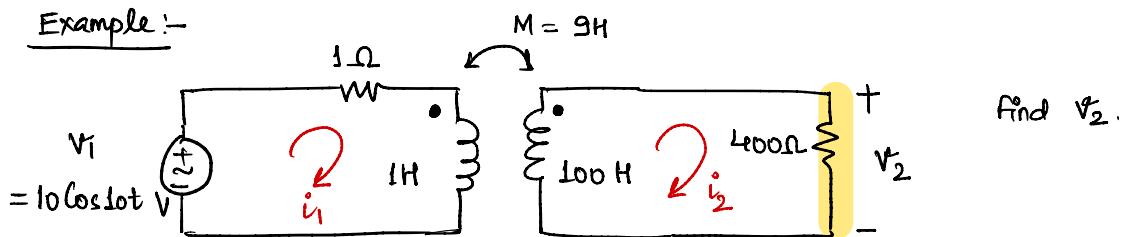
$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = j\omega L_2 I_2 + j\omega M I_1$$

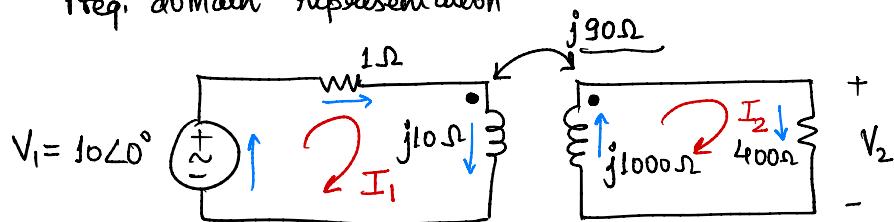
Example



Example :-

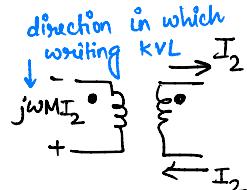


freq. domain representation



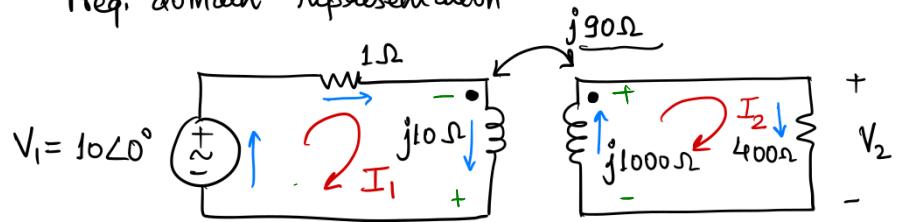
$$\text{Mesh 1: } -10 \angle 0^\circ + I_1 (1) + (j10) I_1 - (j90) I_2 = 0$$

$$\Rightarrow 10 \angle 0^\circ = (1 + j10) I_1 - j90 I_2$$



Last term

freq. domain representation



in direction of current there is potential drop across inductor.

→ In this manner take potential drop instead of potential gain in KVL

$$-10\angle 0^\circ + I_1(1) + j10I_1 - j90I_2 = 0$$

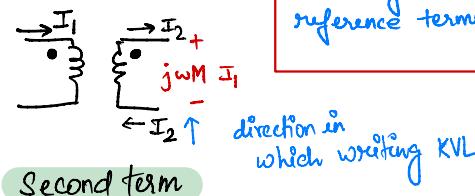
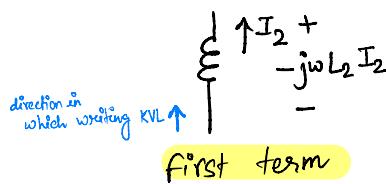
$$+j1000I_2 - j90I_1 + 400I_1 = 0$$

$$I_2(400 + j1000) = j90I_1$$

Ignore this normal convention will be followed

$$\text{Mesh 2: } -(-j1000)I_2 - (j90)I_1 + (400)I_2 = 0$$

$$\Rightarrow -j90I_1 + (400 + j1000)I_2 = 0$$



Always remember to put a minus sign if while writing KVL terms you encounter a -ve reference terminal first.

$$(1 + j10)I_1 - j90I_2 = 10$$

$$(-j90)I_1 + (400 + j1000)I_2 = 0$$

$$\text{Cramer's rule: } I_1 = \frac{D_1}{D_0}, \quad I_2 = \frac{D_2}{D_0}$$

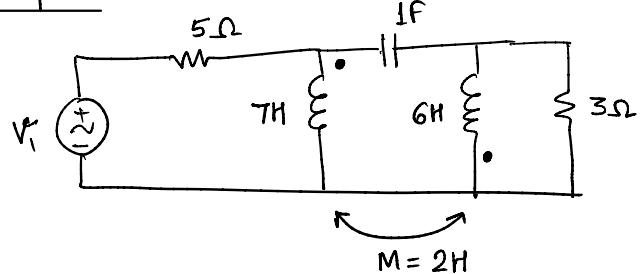
$$D_0 = \begin{vmatrix} 1+j10 & -j90 \\ -j90 & 400 + j1000 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} 10 & -j90 \\ 0 & 400 + j1000 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} 1+j10 & 10 \\ -j90 & 0 \end{vmatrix}$$

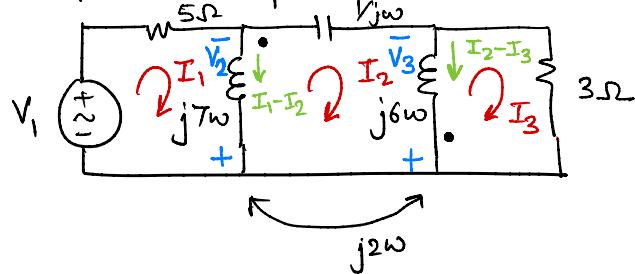
$$V_2 = 400I_2$$

Example

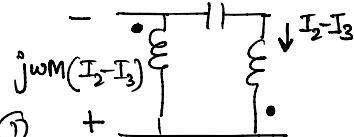


Write phasor mesh equations.

Phasor equivalent (freq. domain)



Mesh 1: $-V_1 + 5I_1 - V_2 = 0$

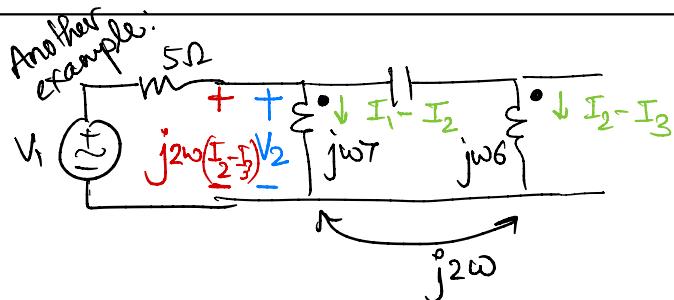


$$V_2 = -j7\omega(I_1 - I_2) + j2\omega(I_2 - I_3)$$

Mesh 3: $+V_3 + 3I_3 = 0$

$$V_3 = -j6\omega(I_2 - I_3) + j2\omega(I_1 - I_2)$$

Mesh 2: $+V_2 + \frac{1}{j\omega} I_2 - V_3 = 0$



$$-V_1 + 5I_1 + V_2 = 0$$

$$V_2 = +j\omega 7(I_1 - I_2)$$

$$+ j2\omega(I_2 - I_3)$$

Mesh equations:

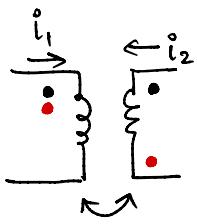
$$V_1 = (5 + j\omega 7)I_1 - j9\omega I_2 + j2\omega I_3$$

$$0 = 2j\omega I_1 - 8j\omega I_2 + (3 + j6\omega) I_3$$

$$0 = -9j\omega I_1 + (17j\omega + \frac{1}{j\omega}) I_2 - 8j\omega I_3$$

Energy stored :-

$$w(t) = \frac{1}{2} L(i(t))^2 \quad \text{for only 1 inductor}$$



$$w(t) = \frac{1}{2} L_1(i_1(t))^2 + \frac{1}{2} L_2(i_2(t))^2 \pm M i_1(t) i_2(t)$$

+ve if both i_1 & i_2 are entering at dotted (or undotted)
-ve if one of i_1 , i_2 is at undotted & another is at dotted.

Coupling coefficient :-

$$M \leq \sqrt{L_1 L_2}$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

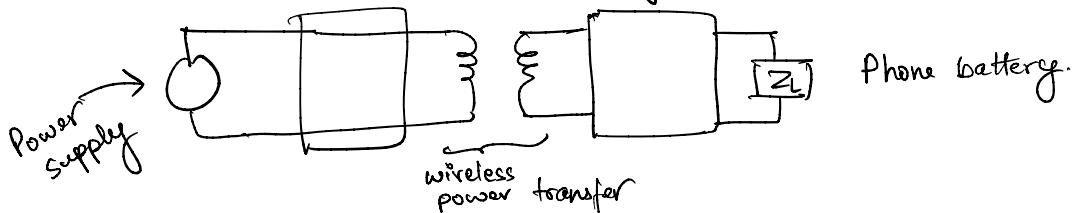
$$0 \leq k \leq 1$$

$k \rightarrow 0$ poor coupling or no coupling

$k \rightarrow 1$ tight coupling

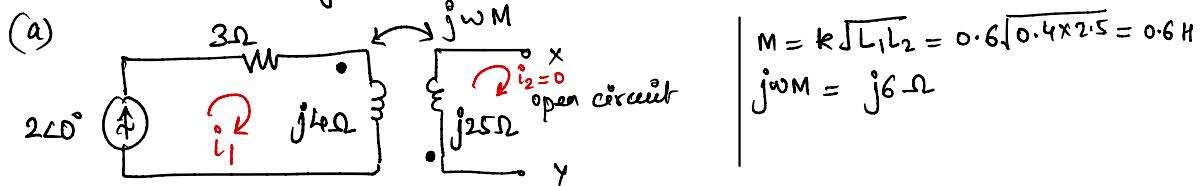
k depends on: distance between coils, size of coils, ferrite core

wireless power transfer \leftarrow inductively coupled
electric vehicle charging



Example: Let $i_s = 2 \cos \omega t$ A. Find total energy stored in network at $t=0$ if $K = 0.6$ and

- (a) xy terminals are open circuited.
- (b) xy terminals are short circuited.

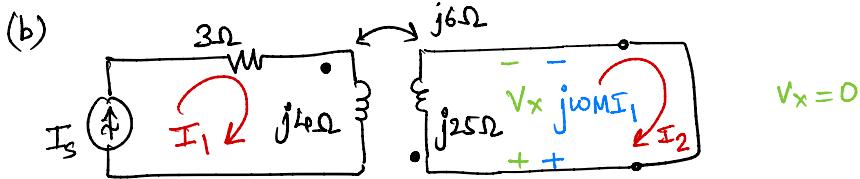


$$w(t) = \frac{1}{2} L_1 (i_1(t))^2 + \frac{1}{2} L_2 (i_2(t))^2 + M i_1(t) i_2(t)$$

$$j\omega L_1 = j4 \Rightarrow L_1 = \frac{4}{\omega} = 0.4$$

$$w(t) = \frac{1}{2} (0.4) (2 \cos \omega t)^2 = 0.8 \cos^2 \omega t$$

$$\text{at } t=0 \Rightarrow w(t=0) = 0.8 \text{ J}$$



$$w(t) = \frac{1}{2} L_1 (i_1(t))^2 + \frac{1}{2} L_2 (i_2(t))^2 + M i_1(t) i_2(t)$$

+ve sign because both I_1 and I_2 are entering from dotted terminals.

$$I_1 = I_s = 2 \angle 0^\circ$$

$$V_x = + (j25) I_2 + j\omega M I_1 = 0$$

$$\Rightarrow I_2 = - \frac{j6 I_1}{j25} = -0.24 I_1$$

V_x comes from two terms:

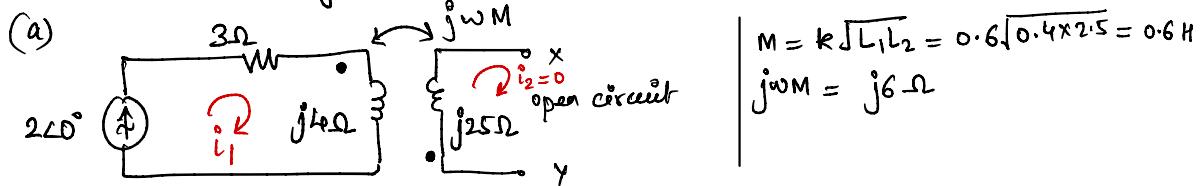
- ① self inductance of L_2
- ② mutual inductance M

$$\Rightarrow I_1 = -0.48 \angle 0^\circ \text{ A} = 0.48 \cos(10t + 180^\circ) \text{ A}$$

$$\begin{aligned} w(t=0) &= \frac{1}{2} (0.4) (2^2) + \frac{1}{2} (0.48)^2 (2.5) + (0.6)(2)(-0.48) \\ &= 0.512 \text{ J} \end{aligned}$$

Example: Let $i_s = 2 \cos \omega t$ A. Find total energy stored in network at $t=0$ if $K = 0.6$ and

- (a) xy terminals are open circuited.
- (b) xy terminals are short circuited.

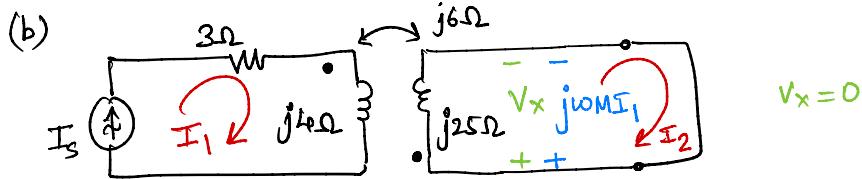


$$w(t) = \frac{1}{2} L_1 (i_1(t))^2 + \frac{1}{2} L_2 (i_2(t))^2 + M i_1(t) i_2(t)$$

$$j\omega L_1 = j4 \Rightarrow L_1 = \frac{4}{\omega} = 0.4$$

$$w(t) = \frac{1}{2} (0.4) (2 \cos \omega t)^2 = 0.8 \cos^2 \omega t$$

$$\text{at } t=0 \Rightarrow w(t=0) = 0.8 \text{ J}$$



$$w(t) = \frac{1}{2} L_1 (i_1(t))^2 + \frac{1}{2} L_2 (i_2(t))^2 + M i_1(t) i_2(t)$$

+ve sign because both I_1 and I_2 are entering from dotted terminals.

$$I_1 = I_s = 2 \angle 0^\circ$$

$$V_x = + (j25) I_2 + j\omega M I_1 = 0$$

$$\Rightarrow I_2 = - \frac{j6 I_1}{j25} = -0.24 I_1$$

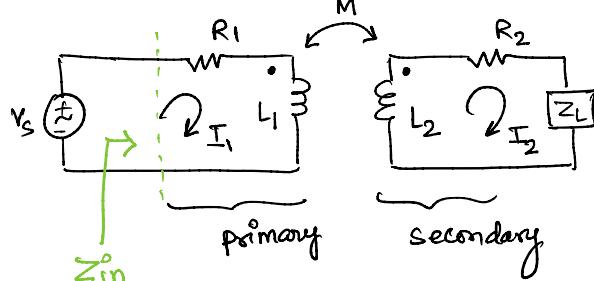
V_x comes from two terms:

- ① self inductance of L_2
- ② mutual inductance M

$$\Rightarrow I_1 = -0.48 \angle 0^\circ \text{ A} = 0.48 \cos(10t + 180^\circ) \text{ A}$$

$$\begin{aligned} w(t=0) &= \frac{1}{2} (0.4) (2^2) + \frac{1}{2} (0.48)^2 (2.5) + (0.6)(2)(-0.48) \\ &= 0.512 \text{ J} \end{aligned}$$

Linear Transformer



$$v \propto di/dt \quad \text{"linear"}$$

↪ no magnetic core

Mesh equations:

$$\begin{aligned} -V_s + I_1 R_1 + (j\omega L_1) I_1 - (j\omega M) I_2 &= 0 \\ (j\omega L_2) I_2 + R_2 I_2 + Z_L I_2 - (j\omega M) I_1 &= 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} V_s &= (R_1 + j\omega L_1) I_1 - j\omega M I_2 \\ 0 &= (-j\omega M) I_1 + (R_2 + j\omega L_2 + Z_L) I_2 \end{aligned}$$

$$\begin{aligned} V_s &= Z_{11} I_1 - j\omega M I_2 \\ 0 &= -j\omega M I_1 + Z_{22} I_2 \end{aligned} \quad \left. \begin{array}{l} \\ \end{array} \right\} \quad \frac{I_2}{I_1} = \frac{j\omega M}{Z_{22}}$$

$$\text{Input impedance : } Z_{in} = \frac{V_s}{I_1} = Z_{11} - j\omega M \frac{I_2}{I_1}$$

$$Z_{in} = Z_{11} - \frac{j\omega M \times j\omega M}{Z_{22}}$$

$$Z_{in} = Z_{11} + \underbrace{\frac{\omega^2 M^2}{Z_{22}}}_{\text{Reflected impedance}}$$

Reflected impedance

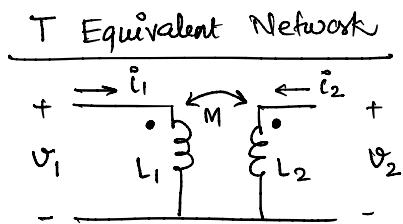
$$Z_{11} = R_1 + j\omega L_1$$

$$Z_{22} = R_2 + j\omega L_2 + Z_L$$

$$k = M/\sqrt{L_1 L_2}$$

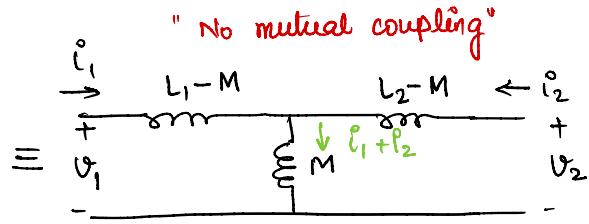
$$k=0$$

$$Z_{in} = Z_{11}$$



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

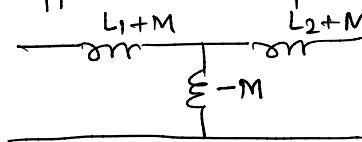


$$v_1 = (L_1 - M) \frac{di_1}{dt} + M \left(\frac{di_1}{dt} + \frac{di_2}{dt} \right)$$

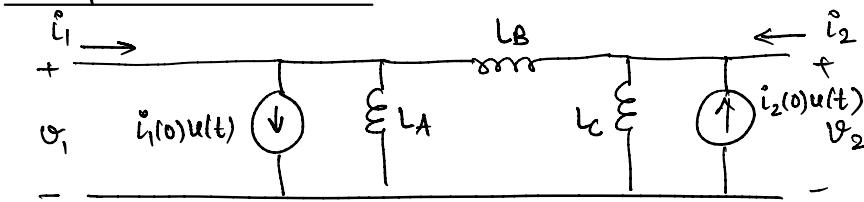
$$\Rightarrow v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Similarly $v_2 =$

If dots are on opposite sides; replace M by $-M$.



TT Equivalent Network



$$L_A = \frac{L_1 L_2 - M^2}{L_2 - M}$$

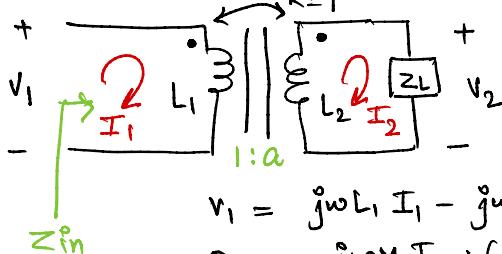
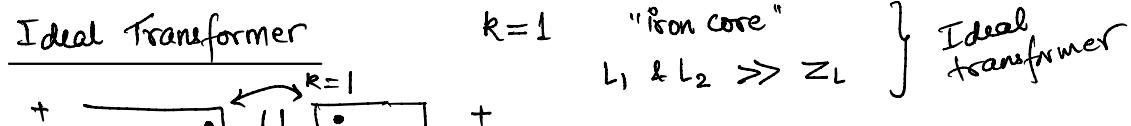
$$L_B = \frac{L_1 L_2 - M^2}{M}$$

$$L_C = \frac{L_1 L_2 - M^2}{L_1 - M}$$

Try at home.
Nodal analysis.

$$v = L \frac{di}{dt} \Rightarrow i = \frac{1}{L} \int_{-\infty}^t v dt \\ \Rightarrow i = \underbrace{i(0) u(t)}_{\uparrow \text{Please verify!}} + \frac{1}{L} \int_{-\infty}^t v(t) dt$$

If dots are on opposite sides, Replace M by $-M$



$$v_1 = j\omega L_1 I_1 - j\omega M I_2$$

$$0 = -j\omega M I_1 + (j\omega L_2 + Z_L) I_2$$

$$\begin{aligned} Z_{in} &= \frac{V_1}{I_1} = j\omega L_1 - j\omega M \left(\frac{I_2}{I_1} \right) \\ &= j\omega L_1 - j\omega M \left(\frac{j\omega M}{j\omega L_2 + Z_L} \right) \end{aligned}$$

$$L_2 \propto N_2^2$$

$$L_1 \propto N_1^2$$

Turn Ratio

$$a = \frac{L_2}{L_1}$$

$$a = \frac{N_2^2}{N_1^2}$$

$$Z_{in} = j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2}$$

Simplifying

$$Z_{in} = \frac{Z_L}{\underbrace{(Z_L/j\omega L_1) + a^2}_{(Z_L/j\omega L_1) + a^2}}$$

$$j\omega L_1 \gg Z_L$$

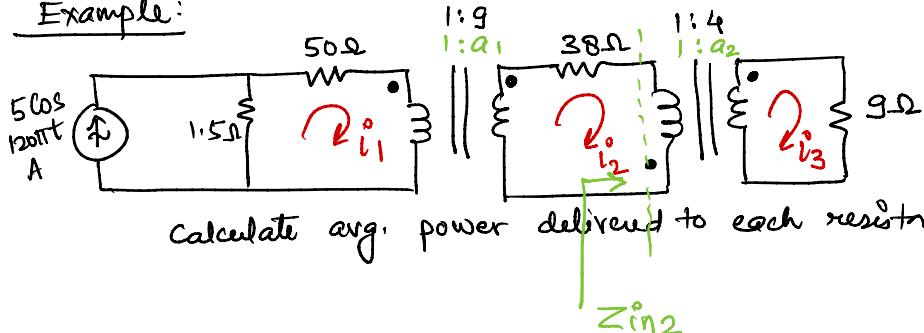
$$Z_{in} = \frac{Z_L}{a^2}$$

Current Ratio $\frac{I_2}{I_1} = \frac{1}{a}$

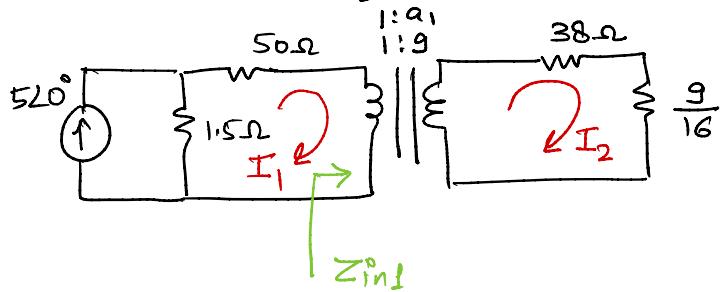
Voltage Ratio $\frac{V_2}{V_1} = a$

Replace a by $-a$ if
dots are on the opposite
sides.

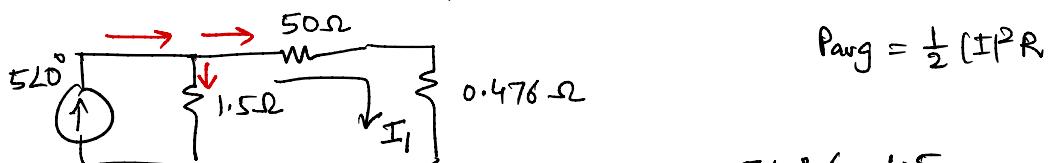
Example:



$$Z_{in2} = \frac{Z_L}{a_2^2} = \frac{9}{16} \Omega$$



$$Z_{in1} = \frac{(9/16) + 38}{a_1^2} = \frac{(9/16) + 38}{81} = 0.476 \Omega$$



current division: $I_1 = 5\angle 0^\circ \left(\frac{1.5}{1.5 + 50 + 0.476} \right)$

$$\Rightarrow I_1 = 0.144 \angle 0^\circ A$$

$$\frac{I_2}{I_1} = \frac{1}{a_1} \Rightarrow I_2 = \frac{0.144 \angle 0^\circ}{9} = 0.016 \angle 0^\circ A$$

$$\frac{I_3}{I_2} = \frac{1}{-a_2} \Rightarrow I_3 = \frac{0.016 \angle 0^\circ}{-4} = -4 \angle 0^\circ mA$$

$$P_{avg, 1.5\Omega} = \frac{1}{2} (5 - 0.144)^2 \times 1.5 = 17.67W$$

$$P_{avg, 50\Omega} = \frac{1}{2} \times 0.144^2 \times 50 = 0.518 W$$

$$P_{avg, 38\Omega} = \frac{1}{2} \times 0.016^2 \times 38 = 0.0048 W$$

$$P_{avg, 9\Omega} = \frac{1}{2} \times (-4)^2 \times 10^{-6} \times 9 = 72 \mu W$$