Exercise 16

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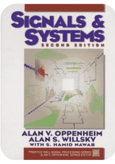

Signals and Systems

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Solution

Verified

Step 1

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(a) Given, $x_1[n] = (-1)^n$, it is clear that $x_1[n]$ is a periodic signal with period $N = 2$. Therefore, from $\mathbf{Section\ 3.6.1}$, if $x[n]$ is a periodic signal with period N, then

$$x[n] = \sum_{k=<N>} a_k e^{jk(2\pi/N)n}$$

and $k = 0, 1, ...N-1$, which implies,

$$\begin{aligned} x_1[n] = & \qquad (-1)^n \\ = & \sum_{k=<2>} a_k e^{jk(2\pi/2)n} \qquad (k = 0,1) \\ = & \qquad a_0(1) + a_1 e^{j\pi n} \\ x_1[n] = & \quad a_0 + a_1(-1)^n \qquad (\because e^{j\pi} = -1) \end{aligned}$$

Clearly, from eq(2) and eq(3), $a_0 = 0$ and $a_1 = 1$. Now form $\mathbf{Section\ 3.8}$, if $x[n]$ is a periodic signal with period N and is passed through a LTI system with frequency response $H(e^{j\omega})$, then the output $y[n]$ is given by,

$$\begin{aligned} y[n] = & \sum_{k=<N>} a_k H(e^{j\omega}) e^{jk(2\pi/N)n} \\ y[n] = & \sum_{k=<N>} a_k H(e^{j2\pi/N}) e^{jk(2\pi/N)n} \end{aligned}$$

Therefore, substitute the results of eq(3) in eq(5),

$$\begin{aligned} y_1[n] = & \quad 0 + (1)H(e^{j2\pi/2})e^{j(1)(2\pi/2)n} \\ y_1[n] = & \qquad H(e^{j\pi})e^{j\pi n} \end{aligned}$$

From eq(7), it is clear that $\omega = \pi$, and from the frequency response of $H(e^{j\omega})$ which is shown in $\mathbf{Figure\ P3.16}$, $H(e^{j\omega}) = 0$, Therefore,

$$y_1[n] = 0$$

Step 2

(b) Given, $x_2[n] = 1 + \sin\left(\frac{3\pi}{4}n + \frac{\pi}{4}\right)$, Therefore,

$$\begin{aligned} x_2[n+N] &= 1 + \sin\left(\frac{3\pi}{4}(n+N) + \frac{\pi}{4}\right) \\ x_2[n] &= 1 + \sin\left(\frac{3\pi}{4}n + \frac{3\pi}{4}N + \frac{\pi}{4}\right) \end{aligned}$$

Therefore, $\frac{3\pi}{4}N$ should be a multiple of 2π, i.e., $\frac{3\pi}{4}N = 2\pi m$. Therefore, $N = 16$ is the least possible value of N at a value of $m = 3$. Now $x_2[n]$ can be written as,

$$\begin{aligned} x_2[n] &= e^{j(2\pi/16)(0)n} + \frac{1}{2j}\left(e^{j(2\pi/16)(3)n}e^{j\pi/4} - e^{-j(2\pi/16)(3)n}e^{-j\pi/4}\right) \\ x_2[n] &= e^{j(2\pi/16)(0)n} - \frac{j}{2}\left(e^{j(2\pi/16)(3)n}e^{j\pi/4} - e^{-j(2\pi/16)(3)n}e^{-j\pi/4}\right) \\ x_2[n] &= e^{j(2\pi/16)(0)n} - \frac{j}{2}\left(e^{j(2\pi/16)(3)n}e^{j\pi/4} - e^{j(2\pi/16)(13)n}e^{-j\pi/4}\right) \end{aligned}$$

In the above equation $k = -3$ is changed to $k = 13$ by using the properties of exponential function. Now from **Section 3.6.1**, the non-zero Fourier series coefficients of $x_2[n]$ which are in the range $0 \leq k \leq 15$ are,

$$a_0 = 1, \quad a_3 = -\frac{je^{j\pi/4}}{2}, \quad a_{13} = \frac{je^{-j\pi/4}}{2}$$

Now form **Section 3.8**,

$$\begin{aligned} y_2[n] &= \sum_{k=0}^{15} a_k H(e^{j2\pi k/16})e^{jk(2\pi/16)n} \\ y_2[n] &= a_0H(0) + a_3H(e^{j3\pi/8})e^{j3\pi n/8} + a_{13}H(e^{j13\pi/8})e^{j13\pi n/8} \\ y_2[n] &= 0 - \frac{je^{j\pi/4}}{2}H(e^{j3\pi/8})e^{j3\pi n/8} + \frac{je^{-j\pi/4}}{2}H(e^{j13\pi/8})e^{j13\pi n/8} \end{aligned}$$

From eq(3), it is clear that $\omega_1 = 3\pi/8$ and $\omega_2 = 13\pi/8$, and from the frequency response of $H(e^{j\omega})$ which is shown in **Figure P3.16**, $H(e^{j\omega_1}) = 1 = H(e^{j\omega_2})$, Therefore,

$$\begin{aligned} y_2[n] &= -\frac{je^{j\pi/4}}{2}e^{j3\pi n/8} + \frac{je^{-j\pi/4}}{2}e^{j13\pi n/8} \\ y_2[n] &= -\frac{je^{j\pi/4}}{2}e^{j3\pi n/8} + \frac{je^{-j\pi/4}}{2}e^{-j3\pi n/8} \end{aligned}$$

$$y_2[n] = \sin\left(\frac{3\pi}{8}n + \frac{\pi}{4}\right)$$

Step 3

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$\mathbf{(c)}$ Given, $x_3[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{2}\right)^{n-4k} u[n-4k]$, Therefore,

$$x_3[n] = \left[\left(\frac{1}{2}\right)^n u[n]\right] * \sum_{k=-\infty}^{\infty} \delta[n-4k]$$

$$x_3[n] = d[n] * b[n]$$

where, $d[n] = \left(\frac{1}{2}\right)^n u[n]$ and $b[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k]$. therefore, the output of the filter $y_3[n]$ can be obtained by first passing the signal $b[n]$ through the filter with the filter response $H(e^{j\omega})$, and them convolving the result with $d[n]$.

From the signal $b[n]$, it is clear that the period of the signal is $N = 4$ Now from $\textbf{Section 3.6.1}$, the non-zero Fourier series coefficients of $x_3[n]$ which are in the range $0 \leq k \leq 3$ are,

$$a_k = \frac{1}{4}\sum_{n=0}^{3} b[n]e^{-jk(2\pi/4)n}$$

$$a_k = \frac{1}{4}\sum_{n=0}^{3}\sum_{k=-\infty}^{\infty} \delta[n-4k]e^{-jk(2\pi/4)n}$$

$$a_k = \frac{1}{4}\sum_{k=-\infty}^{\infty}\sum_{n=0}^{3} \delta[n-4k]e^{-jk(2\pi/4)n}$$

$$a_k = \frac{1}{4}, \quad \text{for all } k \quad (\because \delta[n] = 0 \quad \forall n \neq 0)$$

Step 4

Now form **Section 3.8**,

$$\begin{aligned} r[n] =& \sum_{k=0}^{3} a_k H(e^{j2\pi k/4}) e^{jk(2\pi/4)n} \\ r[n] =& \frac{1}{4}\big[H(e^{j0})e^{j0} + H(e^{j2\pi/4})e^{j(2\pi/4)n} + \\ & H(e^{j4\pi/4})e^{j2(2\pi/4)n} + H(e^{j6\pi/4})e^{j3(2\pi/4)n}\big] \\ r[n] =& \frac{1}{4}\big[H(e^{j0})e^{j0} + H(e^{j\pi/2})e^{j(\pi/2)n} + \\ & H(e^{j\pi})e^{j(\pi)n} + H(e^{j3\pi/2})e^{j3(\pi/2)n}\big] \end{aligned}$$

From the above equation, it is clear that $\omega_1 = 0$, $\omega_2 = \pi/2$, $\omega_3 = \pi$ and $\omega_4 = 3\pi/2$ and from the frequency response of $H(e^{j\omega})$ which is shown in **Figure P3.16**, $H(e^{j\omega_1}) = H(e^{j\omega_2}) = H(e^{j\omega_3}) = H(e^{j\omega_4}) = 0$, which implies,

$$r[n] = 0$$

Therefore, the final output,

$$\begin{aligned} y_3[n] =& \; d[n] * (b[n] * h[n]) \\ y_3[n] =& \; d[n] * r[n] \\ y_3[n] =& \; d[n] * 0 \\ y_3[n] =& \; 0 \end{aligned}$$

Result

$$\textbf{(a)}\; y_1[n] = 0$$

$$\textbf{(b)}\; y_2[n] = \sin\left(\tfrac{3\pi}{8}n + \tfrac{\pi}{4}\right)$$

$$\textbf{(c)}\; y_3[n] = 0$$

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Exercise 15

Exercise 17