FORMATION OF PLANETARY SYSTEMS BY AGGREGATION: A COMPUTER SIMULATION

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ABSTRACT

Planetary systems that display the major regularities and irregularities of the solar system have been produced in a series of computer experiments employing a Monte Carlo technique. It is hypothesized that stars and planets form within cold, dark globules of dust and gas through aggregation of grains and inelastic collisions of particles. A computer program simulates the processes by which planets grow in accordance with this hypothesis from preplanetary nuclei on random orbits within the cloud of dust and gas surrounding a newly formed star. Each planetary system generated by using a different series of random numbers inputs is unique, but in all cases the orbital spacings have patterns of regularity suggestive of Bode's law, and the planetary mass distributions are similar to the solar system's. Binary star systems are produced in the same program by increasing the value of one parameter, the coefficient of density in the cloud.

I. INTRODUCTION

A satisfactory theory of the formation of the solar system should account for its major observed characteristics, particularly the following:

1. The distribution of mass and angular momentum. The sun contains almost 99.9 percent of the total mass, while the planets possess 99.5 percent of the total angular momentum of the system.

2. The distribution of mass among the planets. The innermost planets are small, those farther out are large, and the outermost are small again. Also, planetary masses are distributed fairly evenly on a logarithmic scale over a wide range, mainly from 10^{-7} to 10^{-3} times the mass of the sun.

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3. The differences in composition between the close-in terrestrial bodies and the predominantly gaseous giant planets farther out.

4. The near-constancy of the spacing ratio for orbital distances (where "spacing ratio" denotes the mean radius of one planet's orbit divided by that of the next innermost). Bode's law is an empirical expression of this.

5. The fact that all the planets orbit close to the same plane, revolving around the sun in the same direction, and that most of them rotate about their own axes with the same sense. Also, the invariable plane of the solar system does not coincide with the plane of the sun's equator.

These requirements, or a similar set, are generally acknowledged as minimal tests of an acceptable theory. In addition it is pertinent to note that other planetary systems are known to exist. These cannot be viewed directly, but the presence of dark companions of planetary size near certain nearby stars can be inferred from observed perturbations in the motions of the visible components. Thus it is generally believed that planetary systems are very prevalent in our galaxy, being common phenomena rather than rare. This paper presents a general theory of the formation of planetary systems. It is assumed that the solar system may be taken as a representative example of a planetary system.

The theory is made up of two separable parts, the first being concerned with the formation of the central star and a surrounding nebula of gas and dust, and the second with the formation of planets within the surrounding nebula. Each part may be considered independently, although both parts are needed to account for the present state of the solar system. Only the second part has been tested in a computerized simulation.

II. THE AGGREGATION THEORY

a) Formation of Central Star

My starting configuration is a roughly spherical "small globule," a dark nebula of gas and dust of a type that is extremely common in the Milky Way. A catalogue compiled by Schoenberg and cited by Lynds

(1968), for example, lists 1456 dark nebulae, of which the majority are relatively small. Generally speaking, small globules have masses that embrace the mass range of individual stars, and densities of the order of 10^{-20} g cm^{-3} (Spitzer 1968). Also, in general, the smaller the globules the denser, although their masses and dimensions are not known with much accuracy.

The assumed cloud has a mass slightly greater than one solar mass, a gas/dust mass ratio in the range 50 to 100 (i.e., the cloud contains 1 to 2 percent of dust, by mass), and is gravitationally self-contained. ("Gas" denotes hydrogen and helium; "dust" includes all the rest of the elements.) Whether isolated in space or embedded in a larger cloud, a body must have certain relationships among its mass, dimensions, and temperature if it is to continue to exist with any permanence. If its mass is too small, or its mean density too low at the temperature of the gas, atoms of hydrogen can escape from the cloud, both depleting its mass and reducing its mean density, and the cloud will eventually dissipate. For long-term stability, the ratio $r_o^2\rho_c/T$ must be equal to or greater than $9k_1k_3^2/8\pi Gk_2w$, where r_o = the radius of the cloud to the outer edge, ρ_c = the mean density of the cloud, T = the absolute temperature of the gas, k_1 = the Boltzmann constant, k_2 = the mass of one molecular weight, G = the gravitational constant, w = the molecular weight of the gas, and k_3 = the ratio of the escape velocity at the periphery to the mean molecular velocity of the gas. Using Jeans' criterion that k_3 must be of the order of 5 or greater for long-term stability, the ratio $r_o^2\rho_c/T$ must be equal to or greater than about 1×10^{16} w^{-1} g cm^{-1} deg^{-1}.

The cloud is assumed to be located in space at a great distance from any hot star so that it can lose heat by radiation to space, mainly from the dust particles, until the surface temperature of the grains approaches the temperature of their heat sink, the surrounding space, which may be as low as 3^oK. The cold grains provide a means for cooling the gas atoms and molecules that impinge upon them. The grains also provide surfaces upon which molecular compounds such as H_2 and NH_3 can form by surface catalysis. Solid hydrogen will eventually condense on the surface of the grains if the hydrogen pressure is sufficiently high, i.e., greater than approximately 10^{-13} dyne cm^{-2} (Wickramasinghe 1968). After this has occurred and the gas is in equilibrium with the solid hydrogen, the gas phase would be predominately helium and the grains would consist mainly of cores of ices (water, ammonia, etc.) and hydrated metallic silicates and oxides surrounded by mantles of solid hydrogen.

Initially, the density of the cloud is assumed to be uniform, and the particles are assumed to be moving in every conceivable direction. Such a cloud has a definite net angular momentum, which is a conserved property; an invariable plane of revolution (unless the net angular momentum is exactly zero); and a center of mass which, if we regard the cloud as being an isolated or closed system, remains fixed in space. Under these conditions all the particles will be moving around the center of mass on elliptical orbits (to a first approximation) within the cloud, at least during the time intervals between collisions with one another. It is well known that within a cloud of uniform density the gravitational field varies directly with distance, orbits of particles are symmetrically disposed around the center of mass (that is, the center of mass is not at one of the foci but at the geometrical center of each ellipse), and the orbital periods of all particles are identical (see Table 1).

Thus particles with large semimajor axes have high linear velocities, while those with small semimajor axes move very slowly indeed. For particles on circular orbits, the velocity is directly proportional to distance from the center of mass. The net angular momentum of the cloud is the vector sum of the angular momenta of all the particles of which it is composed. Since the particles are assumed to be moving in every direction, the angular momenta of some particles are canceled by the angular momenta of particles moving in the opposite direction. Generally, if the total number of particles in the cloud is 2N + n (N going one way and N + n going the other way), the net angular momentum reposes entirely in the n particles, which are all going in the same direction around the center of mass.

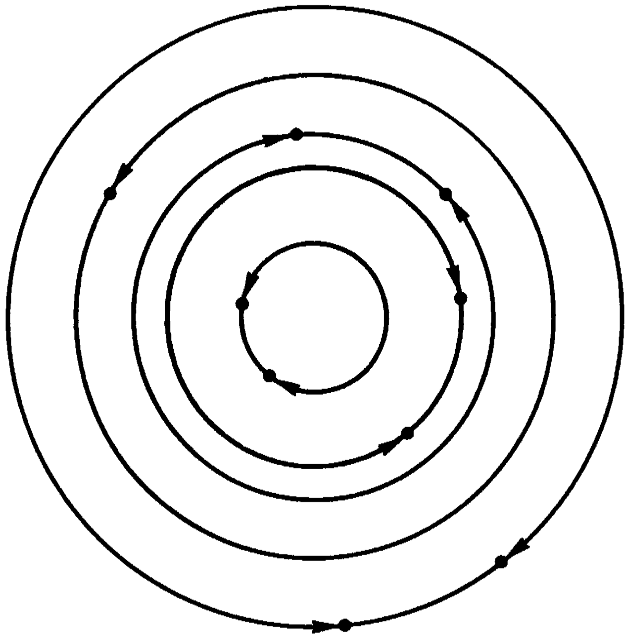

Fig. 1—Before inelastic collisions

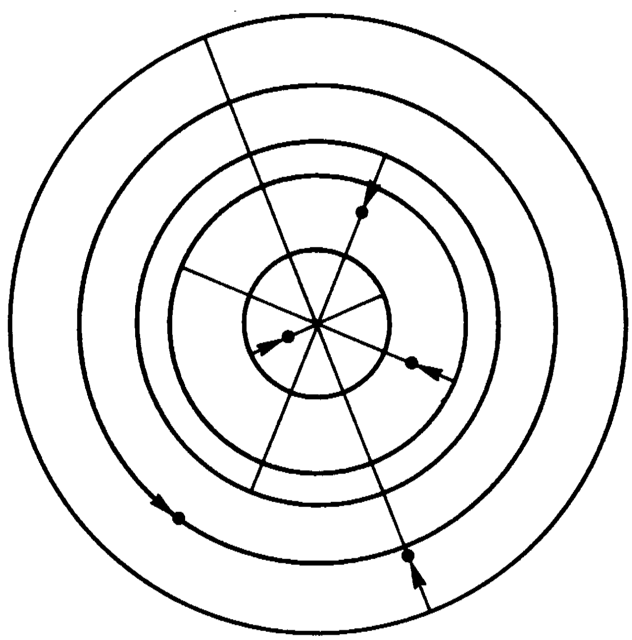

Fig. 2—After inelastic collisions

TABLE 1

MOTIONS OF PARTICLES IN A CLOUD OF UNIFORM DENSITY
CONTRASTED WITH THOSE AROUND A POINT MASS*

Parameter	Motion of Particles: Within Cloud of Uniform Density	Motion of Particles: Around Point Mass
Force field, F	$F \propto r^{+1}$	$F \propto r^{-2}$
Shape of orbits	Elliptical, symmetrical around center of mass	Elliptical with center of mass at one focus
Orbital periods, P	All identical	$P \propto r^{3/2}$
Circular orbital velocities, v	$v \propto r$	$v \propto r^{-1/2}$
Velocities along elliptical orbits, v	$v \propto \left(a^2 + b^2 - r^2\right)^{1/2}$	$v \propto [(2/r) - (1/a)]^{1/2}$
Angular velocity on circular orbit, ω	$\omega = (GM)^{1/2}/r_o^{3/2}$	$\omega = (GM)^{1/2}/r^{3/2}$

*Notation: r = distance from center of mass, M = total mass of system, a = semimajor axis of elliptical orbit, b = semiminor axis of elliptical orbit, r_o = radius of cloud to outer edge.

In the cloud there will be many collisions between the particles going in opposite directions. Particles going in the same direction will also collide, but more gently, since their relative velocities would tend to be low. Thus when particles going the same way touched very gently, they could tend to stick together because of any one of a number of mechanisms: crystal growth, melting and freezing, van der Waals forces, or some other unspecified mechanism. In any event, some natural mechanism enabling particles to stick together and form aggregates is assumed to exist. The possibility of inelastic collisions is also assumed. Two aggregates of equal mass, for example, but with equal and opposite velocity vectors, would completely cancel each other's angular momentum and fall toward the center of mass. Similar collisions between the 2N particles would produce a dense traffic situation near the center of mass (Figures 1 and 2), where further aggregation could proceed rapidly. The net result would be a massive body with very little angular momentum at the center of mass; the remaining n particles, carrying practically all the original net angular momentum of the system, would continue to orbit around the center of mass, all moving in the same general direction.

However, if practically all the mass falls inward to collect in a massive body at the center, the gravitational field is converted into one in which the force varies inversely with the square of the distance. For those particles remaining in orbit, the resultant effect is to shrink the size of their orbits, which can readily be demonstrated for particles on circular orbits. A given particle of mass m has an angular momentum $m\omega r^2$ which must be conserved. If the orbit of a particle remains circular during the gradual transition from $F_1 \propto r$, to $F_2 \propto r^{-2}$, then $\omega_1 r_1{}^2 = \omega_2 r_2{}^2$, where ω = angular velocity, r = orbital radius, and subscripts 1 and 2 designate the parameters before and after the transition.

Since $\omega_1 = (GM)^{1/2}/r_o{}^{3/2}$ and $\omega_2 = (GM)^{1/2}/r_2{}^{3/2}$, $r_2 = r_1{}^4/r_o{}^3$ or, setting $r_o = 1$, $r_2 = r_1{}^4$.

Thus, all orbits shrink in accordance with this relationship, creating a density gradient in the cloud, which now has a high concentration of particle orbits near the center of mass and a density decreasing with distance. It can be shown that for an instantaneous transition, if the initial density was ρ_i (constant), the final density ρ_f would be $\rho_f = \rho_i/4r^{9/4}$.

This result neglects other effects such as aerodynamic drag (interactions between the dust particles and the gas) and the effects of a gradual transition which would tend to modify the density function $\rho_f(r)$ but which cannot be treated analytically. Nevertheless, it is qualitatively acceptable that a density gradient would be established in the cloud.

Two separate sequences of events will now take place concurrently, one at the center of mass and one within the surrounding cloud. At the center of mass, the body formed from the original 2N particles will grow large enough to retain and capture hydrogen and helium. It will develop very high internal temperatures, pressures, and densities as a result of gravitational compression. If it grows massive enough, its

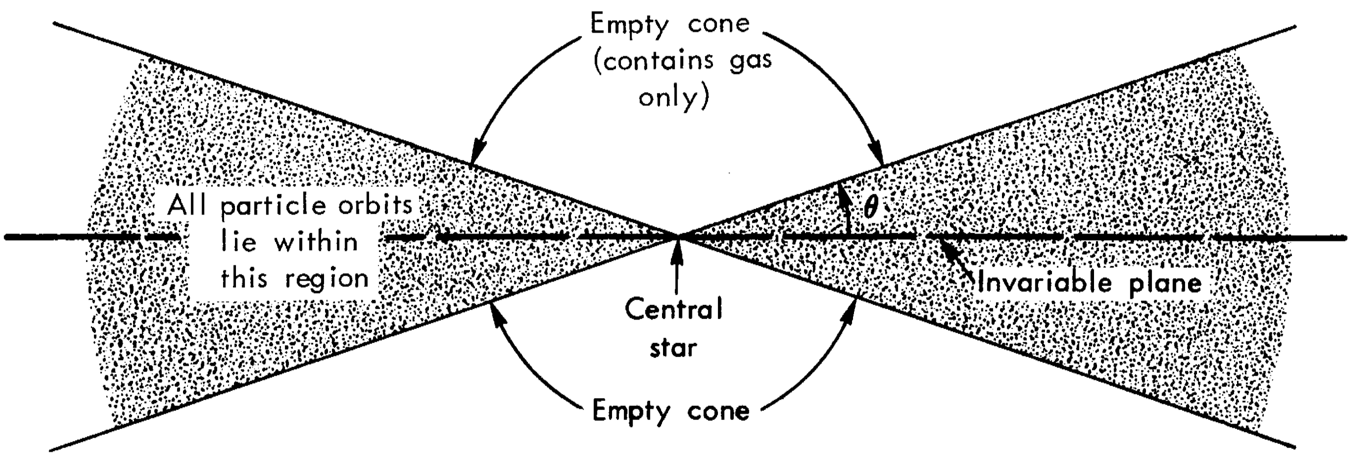

Fig. 3—Cross-section of exocone, perpendicular to invariable plane

internal temperature will become high enough to initiate self-sustaining thermonuclear fusion reactions, and the central mass will eventually become a main-sequence star; however, this would not happen immediately, since a finite length of time is required for the nuclear reactions to reach equilibrium.

b) Formation of Planets

Concurrently, within the surrounding cloud, aggregation of particles by nuclei will continue to take place. As has been recognized since the days of Poincaré (1911), in a cloud of particles with a nonzero net angular momentum where inelastic collisions can occur, particle orbits that are highly inclined to the invariable plane of revolution are gradually eliminated by being converted through collisions into lower-inclination orbits. More recently, McCrea (1960) has alluded to this same process. The net effect of the conversion of high-inclination orbits into low-inclination orbits is a gradual and continuing flattening of the cloud, so that for the particulate matter the spherical shape is lost, and the volume containing particulate orbits takes on a shape approaching that of an exocone,[1] at least in its inner regions

[1]Term adopted here to designate a sphere with cone-shaped voids centered on the axis; the shape produced by rotating an acute angle around an axis which passes through the vertex and is perpendicular to its bisector.

(Figure 3). The term exocone, although only suggestive, is preferred to disk, as being more descriptive of the general shape of the region occupied by the orbits of particles. Poincaré, in discussing a hypothesis of Du Ligondès, also showed that inelastic collisions between particles moving in the same direction decrease the eccentricities of their orbits. Specifically, when two particles moving on eccentric orbits in the same direction in the same plane collide and stick together, the resultant eccentricity of the combined bodies is generally lower than either of the original eccentricities.

Now we have arrived at the point where we can examine the formation of planets by aggregation within the cloud. The term aggregation is used instead of accretion because, in some theories of the origin of the solar system, accretion has been employed to mean the capturing by the sun of material from outside this solar system. This usually involves the sun's passing through an interstellar cloud and picking up mass from the cloud. The term accretion is avoided because of its prior use in this sense.

All during the preceding events particles of dust have been aggregating around nuclei within the cloud. Many of these would be broken up again through collisions with one another, but here and there a nucleus would be able to grow to such size that it could begin to sweep in particles of dust and grow more rapidly as it developed an appreciable gravitational field of its own. As it orbited about it would gradually sweep out a clear dust-free lane in the exocone. If it became massive enough, it would begin to collect gas as well as dust and to grow very rapidly. In any event, at some point when it had depleted the region from which it could sweep out dust or gas, its growth would cease.

Simultaneously other nuclei would also be growing within the exocone. As long as the orbit of the second nucleus (pretending for purposes of description that the nuclei grow one at a time, or sequentially, but realizing that many nuclei may be growing at the same time) is far removed from that of the first, it can grow independently as though the first did not exist. Similarly with other nuclei on nonintersecting paths through the cloud of dust and gas. However, as the process continues, some nuclei, having grown to planetary size, will inevitably collide with planets that have grown earlier and will merge with them to form a larger single planet. The product of the fusion process may or may not be able to grow still larger, depending on its mass and the type of growth material encountered along the new orbit that has resulted from the inelastic collision. Nuclei will continue to form and grow and merge and regrow until all the dust in the exocone has been incorporated into planetary objects. At this point the process of formation of the planetary system is complete, and there remain a number of planets on noninterfering orbits, all orbiting in the same direction around the central star, and a certain amount of leftover gas. The leftover gas, mainly hydrogen, eventually is driven entirely out of the system by the solar wind.

To recapitulate in outline form:

1. A spherical cloud of uniform density in space is postulated. Solid grains (dust) provide a mechanism for cooling the gas to the temperature of the surrounding space, $\sim 3^{\circ}K$. The solid grains consist of cores (of ice, solid ammonia, methane, metallic oxides and silicates) surrounded by mantles of solid hydrogen. The gas consists of helium and molecular hydrogen. The grains are of assorted sizes, some quite large, some small.

2. As required by a central force field in which the force varies directly with distance, the solid grains are moving on elliptical orbits, all with the same period, symmetrically disposed around the unoccupied center of mass. Velocity vectors are nearly isotropic, but the cloud has a net angular momentum and an invariable plane of revolution. Particles moving in one direction, counterclockwise say, slightly outnumber those moving clockwise.

3. The spherical cloud gradually becomes an exocone as a result of inelastic collisions between particles. Particles meeting head on cancel each other's angular momentum and fall inward, creating an increased density near the center of mass. Particles aggregate at the center of mass, forming a large concentrated body.

4. The central body warms up by gravitational compaction and by the infall of particles and becomes large enough to retain hydrogen at its surface temperatures. Through head-on collisions of particles in the cloud, practically all the particles moving in the clockwise direction are eliminated from the cloud by falling into the central body, leaving an exoconical nebula composed almost entirely of particles moving counterclockwise. The central body (protostar) has very little angular momentum, since it aggregated from particles in which the angular momentum had been canceled. Most of the original net angular momentum of the cloud remains with the particles of the nebula.

5. The gravitational force field has now been transformed into one in which the force varies inversely with the square of the distance. Accordingly, all particulate orbits contract in such a manner as to produce a density distribution in the nebula with density decreasing with distance from the center of mass.

6. The central body is of stellar size, but a finite length of time is required for the various thermonuclear reactions to reach equilibrium rates and for the body to start radiating like a star, possibly several thousands of years. During this period preplanetary nuclei aggregate from particles within the nebula. As these become large enough to sweep in particles by gravitational attraction, the particles heat up on impact and hydrogen escapes. The aggregation process is slow at first and continues to take place after the central body commences to radiate like a star and eventually reaches the main sequence. The exocone continues to get flatter.

7. The sweeping-up process by preplanetary nuclei continues. Hydrogen is evaporated from the grains by solar heating. Temperatures of planetary escape layers now become a function of distance from the sun. Whether or not a planet can accumulate and retain gas as well as dust becomes a function of its mass and the intensity of the radiation field at its closest approach to the sun. Planets continue to sweep in matter, dust only if below critical mass, and dust plus gas if above critical mass.

8. The process ends when all the dust is swept from the nebula. After formation of the planets is complete, the gas remaining in the nebula, mainly hydrogen, is heated by radiation and the solar wind and gradually escapes from the system.

III. EXPERIMENTAL SIMULATION

The aggregation hypothesis for the formation of the solar system was tested in a computerized model. In essence the model simulates an experiment in which planetary nuclei are injected randomly one at a time into a cloud of dust and gas and allowed to grow by sweeping in smaller particles. When one planet has ceased to grow, another nucleus is injected. Planets will coalesce if their orbits cross or come sufficiently close to one another; growth may continue after coalescence. As the planets grow they sweep out cleared paths of an annular, washer-like shape. At first a growing planet sweeps up dust only, but if it becomes massive enough, it can begin to gather in gas as well. Nuclei are injected sequentially into the cloud until all the dust (between two arbitrary extreme boundary radii) has been swept away. At this time the experiment is over and the planetary system is considered complete.

Certain parameters must be specified to obtain quantitative results: the density distribution within the cloud; the ratio of gas to dust; a definition of critical mass, or the planetary mass above which a planet can begin to accumulate gas in addition to dust; and the orbital eccentricity of particles within the cloud. Also, a few rules for the coalescence and growth of planets must be established.

When these parameters and rules have been set forth in a suitable manner, planetary systems very much like the solar system can be created. Multiple-star systems can be created by changing a single parameter, the density level within the cloud.

In the current computer program (code name: ACRETE) the general nature of the results is not highly sensitive to the selected forms or values of the parameters involved. That is, planetary or multiple-star systems still result when the parameters are altered over a wide range. The primary objective of devising these experiments was to provide a realistic test of the aggregation hypothesis. Thus, emphasis was placed on finding at least one set of conditions that results in the creation of planetary systems having the general pattern of the solar system; it is not implied that these duplicate the actual set of conditions that prevailed when the solar system was formed.

a) Initial Conditions in the Cloud

At the beginning of the experiment the following conditions are assumed:

There is a spherically symmetrical cloud of dust and gas with a constant ratio of gas to dust, the density decreasing with distance from the center.

The center of mass is occupied by a star with a mass of one unit (one solar mass).

All particles in the cloud are moving on elliptical orbits, with the center of mass at one focus.

The density of dust (ρ_1) within the cloud depends on a function of the form $\rho_1 = A \exp(-\alpha r^{1/n})$.

The overall density of gas and dust (ρ_2) within the cloud equals $K\rho_1$, where r is distance from the center of mass (in astronomical units, a.u.) and A, α, n, and K (the gas/dust ratio) are constants.

This density distribution was chosen for several reasons:

1. It depends only on r and decreases monotonically with increasing r.

2. The density at the center of the cloud is finite; $\rho_1 = A$ at $r = 0$.

3. The total mass M of the cloud is finite;

$$M = \frac{4\pi K A n \Gamma(3n)}{\alpha^{3n}} \cos\left(\frac{\pi}{2} - \theta_{max}\right)$$

where θ_{max} is the maximum inclination of orbits with respect to the invariable plane of the system.

4. The function $r^3\rho_1$ peaks in midrange--that is, the mass collected by a growing nucleus at a certain mean distance from the center, r, is approximately a function of $r^3 \exp(-\alpha r^{1/n})$. This function rises to a maximum value at $r = (3n/\alpha)^n$ then diminishes, approaching zero as r approaches infinity.

5. This density distribution is similar to those observed in globular clusters; that is, somewhat similar density distributions may be found in nature (Kurth 1957).

As shown below, planetary systems closely resembling the solar system were obtained experimentally when using the following constants: $A = 1.5 \times 10^{-3}$ (solar masses per cubic a.u.), $\alpha = 5$, $n = 3$, and $K = 50$. For these values, the total mass of the cloud in terms of solar mass M_s, is $M = 0.0584 \cos(\pi/2 - \theta_{max})M_s$, and $r^3\rho_1$ reaches a maximum at $r = 5.83$ a.u. A few other combinations of α and n were tried experimentally in preliminary runs but were abandoned when they failed to produce satisfactory (in terms of the focus on generating solar-system-like patterns) distributions of planetary sizes and numbers. It was decided to retain the values $\alpha = 5$ and $n = 3$ for the balance of the study, since this combination gave satisfactory results. It is apparent that great diversities in patterns of synthetic planetary systems can be produced by altering the form of the density distribution as well as the gas/dust ratio, the critical mass relationships, and the other input parameters. Some input combinations, for example, produce numerous planets, all of which are very small; others produce very small inner and outer planets but very large midrange planets. The main point of the exercise is that one can find certain combinations of input parameters that produce synthetic planetary systems bearing a close family resemblance to the solar system.

To simplify the model, all the particles making up the cloud are assumed to have the same orbital eccentricity, W, but the inclinations of their orbits and the orientations of the long axes of their orbits are assumed to be distributed randomly. It is not necessary to specify a maximum allowable inclination, θ_{max}, of particle orbits. For purposes of visualization one might imagine the cloud as having a shape similar to that suggested by Figure 3.

b) The Planetary Nuclei

Planetary nuclei, each having a mass m_o, are injected into the cloud one at a time and allowed to grow to completion before another is injected.

All planetary nuclei are injected into the invariable plane with inclination zero but with semimajor axis a_i and eccentricity ε_j chosen at random, using the internal random-number generator of the IBM 7044 computer. The semimajor axis is simply $a_i = 50Y_i$, where Y_i is a random number between zero and one, while the eccentricity is $\varepsilon_j = 1 - (1 - Y_j)^Q$, where Y_j is another random number and Q is given the value of 0.077 to conform to an empirical probability function for the distribution of orbital eccentricities of planets in the solar system (Dole 1964).

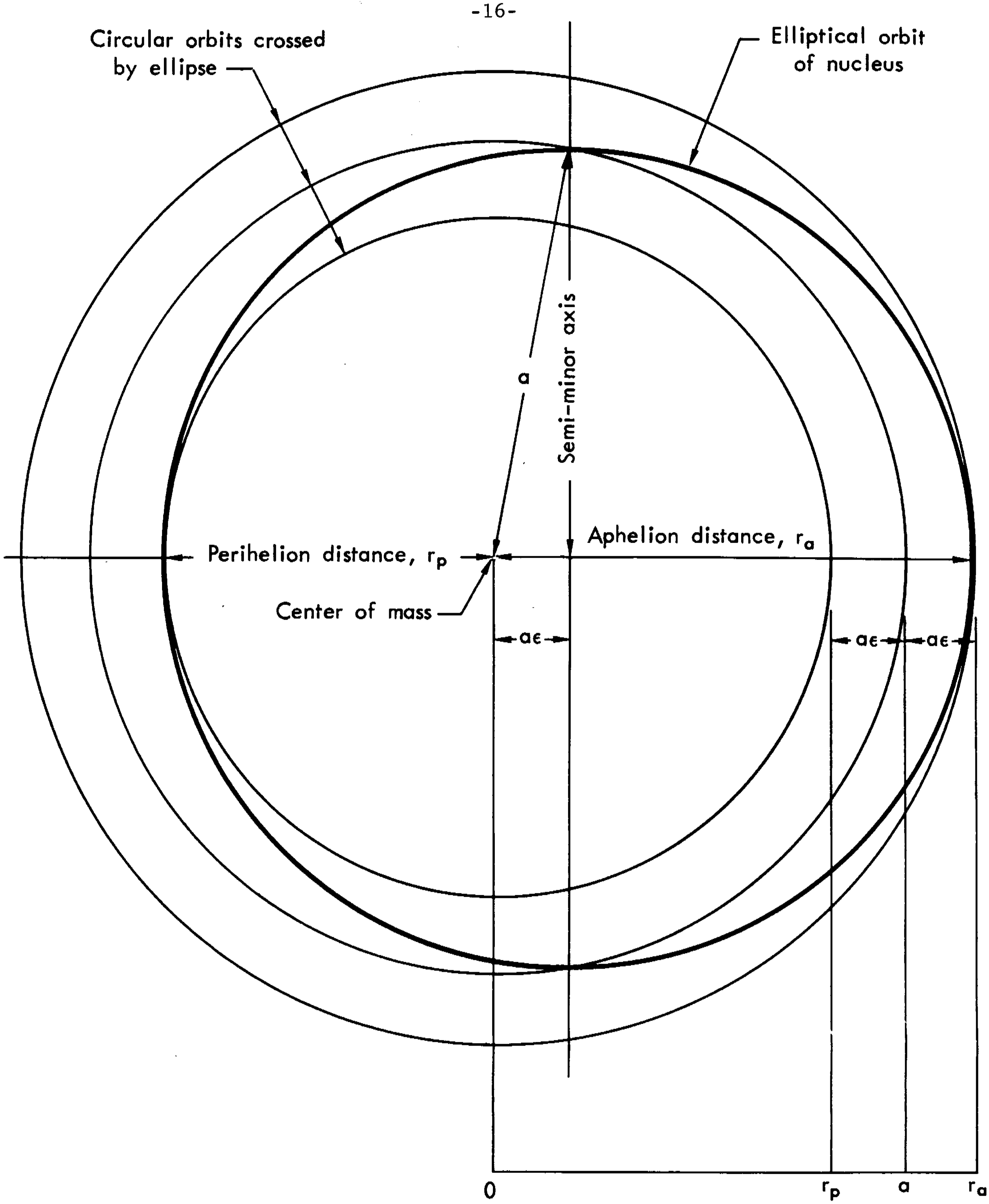

Fig. 4—Plan view of cloud with orbit of one nucleus shown. Length of semimajor axis = a

In actual practice the first random number of a series was used to establish the semimajor axis of the first particle, the second random number established the eccentricity of the first particle, the third random number established the semimajor axis of the second particle, the fourth random number the eccentricity of the second particle, and so on.

As indicated above ($a_i = 50Y_i$), the semimajor axes of planetary nuclei can never be greater than 50 distance units, which effectively sets an outer boundary to the problem. An inner boundary was also established, arbitrarily at 0.3 distance unit. (More than 92 percent of the total cloud mass lies between these bounds.)

c) Aggregation

To visualize the aggregation process, assume for the moment that all the particles in the cloud are moving around the center of mass on circular orbits, but that a planetary nucleus has a nonzero eccentricity ε and a semimajor axis a. As the nucleus moves around the center of mass its orbit crosses the orbits of all particles having orbital radii between $(a - a\varepsilon)$ and $(a + a\varepsilon)$, as shown in Figure 4. In time it will collide with all such particles, and if we assume that the particles stick to the planetary nucleus and add to its mass, the planetary nucleus will sweep out a clear annular path of width $2a\varepsilon$. Particles orbiting within the band but in different orbital planes will also tend to be collected, since their orbits also cross that of the nucleus. It is assumed that all orbits will precess, as do those of the planets under the influence of mutual gravitational perturbations, thus swinging their semimajor axes through all directions within the invariable plane.

In addition to capturing particles that cross its orbit, the planetary nucleus will capture particles by gravitational attraction if their orbits come sufficiently close, which is defined as the distance x, a function of the mass of the nucleus m, relative to the central body (of unit mass), and its distance r from the center of mass: $x = r[m/(1 + m)]^{1/4} = r\mu^{1/4}$. This function was chosen because

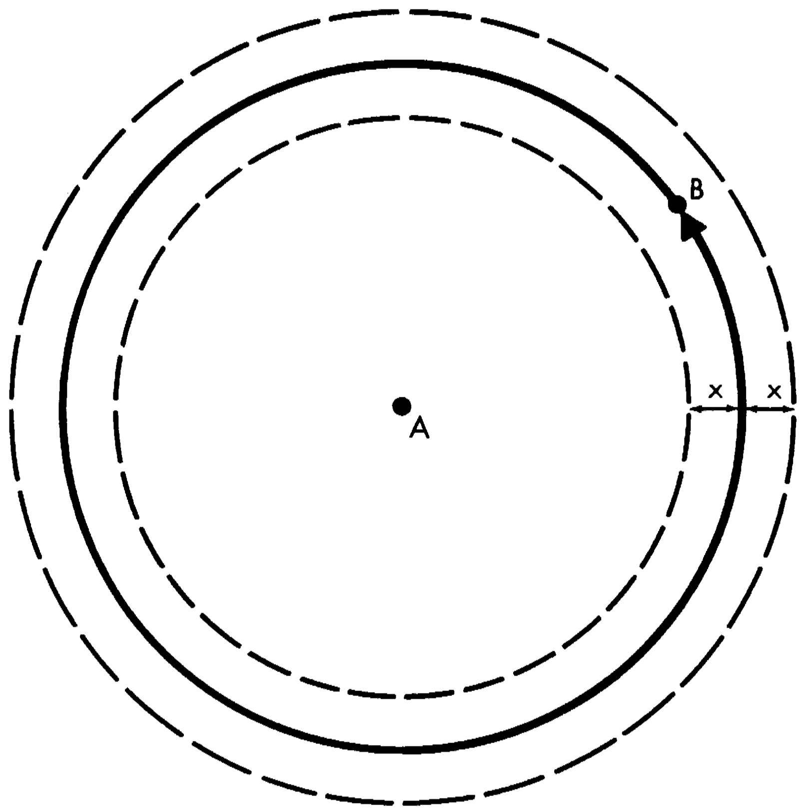

Fig. 5—Particles orbiting around body A inside dashed lines have unstable orbits and can be swept by body B

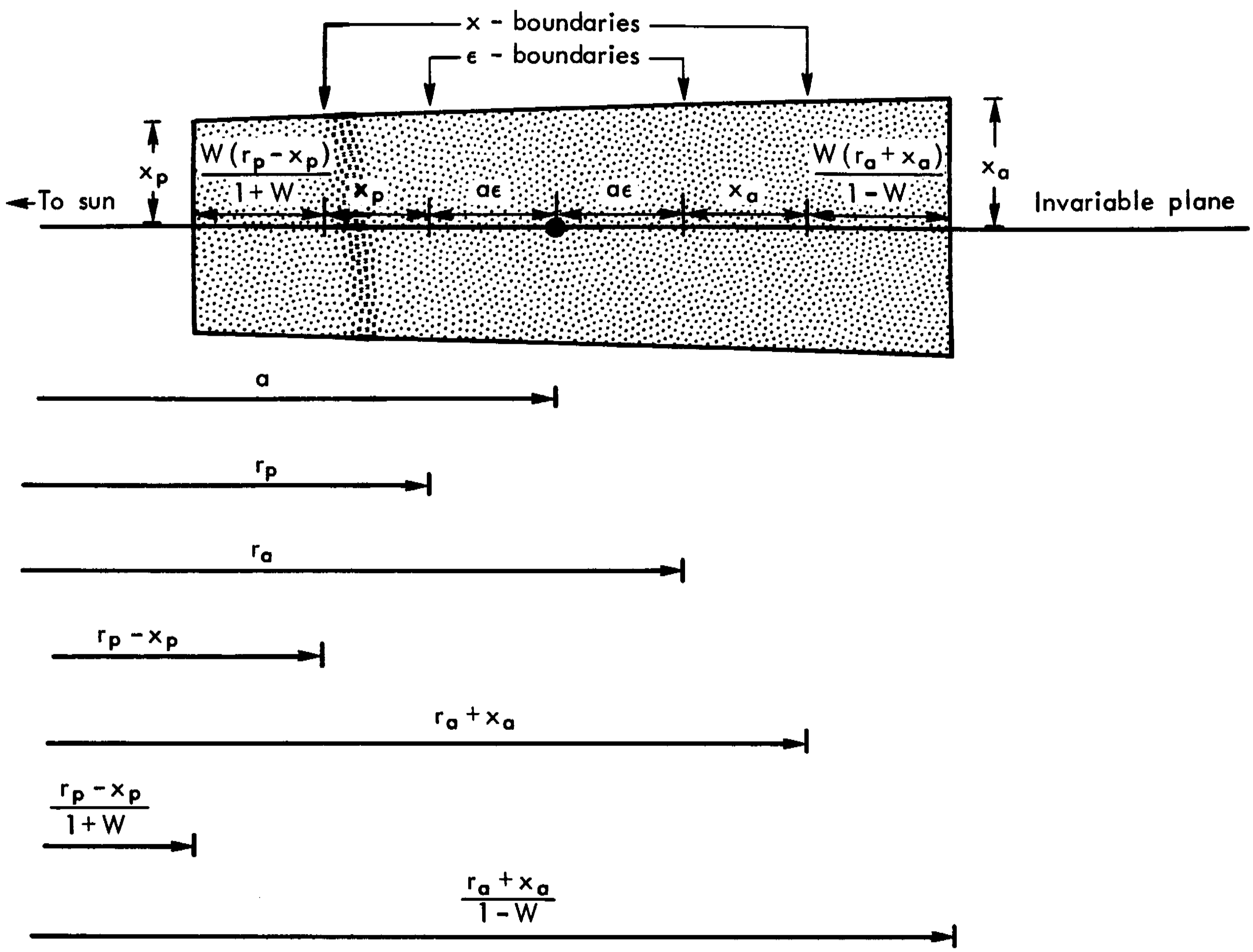

Fig. 6—Cross-section of one side of annular "washer" swept out by a growing planet

it provides simplicity and a reasonably close approximation to certain limits derived from the restricted three-body problem (Dole 1961). That is, in the restricted three-body problem, orbits of prograde particles moving around the larger mass are unstable if their orbital radii differ from that of the smaller of the two finite masses by less than a certain distance, which is of the approximate magnitude of x as given above (see Figure 5).

In the two paragraphs immediately above it was assumed, for simplicity, that the cloud particles were moving on circular orbits. If they are moving instead on orbits of nonzero eccentricity (the more general case), then an additional group of particles on either side of the band will be captured and swept out by the growing planetary nucleus. If all the particles in the cloud are assumed to have the same eccentricity W, the total band width swept out by the planetary nucleus will be

$$\text{Band width} = 2a\varepsilon + x_a + x_p + \frac{W\left(r_a + x_a\right)}{1 - W} + \frac{W\left(r_p - x_p\right)}{1 + W}$$

where the subscripts a and p indicate aphelion and perihelion positions. The swept region was assumed to have the cross-sectional shape indicated in Figure 6. Its volume is $2\pi(\text{bandwidth})(x_a + x_p)$. Its mass is approximated by the product, volume times cloud density ρ at a. The instantaneous mass at the (i + 1)th iteration is[2]

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[2]Somewhat more complicated expressions were required in the computer programs to take prior events into consideration—overlappings of swept bands, for example—and also to take account of coalescences.

$$m_{i+1} = \frac{8\pi a^3 \rho \mu_i^{1/4}}{1 - W^2}\left(\varepsilon + \mu_i^{1/4} + W + W\varepsilon\mu_i^{1/4}\right)$$

where μ_i represents the relative mass at the previous iteration: $\mu_i = [m_i/(1 + m_i)]$.

In trial computer runs, W (which is an approximation of the average eccentricity of the cloud particles) was varied from zero to 0.25, with resulting planetary systems most closely resembling the solar system in general pattern when W was set equal to 0.20 or 0.25.

The planetary nucleus grows iteratively. Starting with the original mass m_o, the distances x_a and x_p are computed, the swept mass is computed and added to the original mass, and the above steps are repeated. Growth is stopped and assumed to be completed when the mass increase between any two serial iterations falls below one ten-thousandth of the planetary mass. During the growth process only dust is accumulated as long as the planetary mass is less than a critical mass m_c (a function of r); that is, the cloud density is taken as ρ_1. However, if the planetary mass surpasses m_c, then some gas is assumed accumulated along with the dust, the effective spatial density of the accreted material being a function of the instantaneous mass m_i of the planetary nucleus. In the program described here the effective density, ρ, was arbitrarily assumed to have the form

$$\rho = \frac{K\rho_1}{1 + \left(\frac{m_c}{m_i}\right)^{0.5} (K - 1)}$$

For very large values of m_i, ρ approaches ρ_2, the overall density of gas and dust within the cloud; that is, very large masses are assumed to collect dust and gas from the cloud in nearly the same ratio in which they are present.

Following the reasoning of II. a), in order to retain a gas a planet must have $R^2\rho_m/T \geq 1 \times 10^{16}\ w^{-1}\ g\ cm^{-1}\ deg^{-1}$, where R = radius of planet to escape layer, ρ_m = mean density of planet, T = absolute temperature at escape layer. For simplicity, assuming $T = k_4 r_p^{-1/2}$, and that all planets have the same mean density, ρ_m, it can be shown that the critical mass may be defined as $m_c = Br_p^{-3/4}$. Depending on the values selected for w, k_4, and ρ_m, the proportionality factor B will be in the neighborhood of 1×10^{-5} to 2×10^{-5} solar mass.

In the computations B was given the value 1.2×10^{-5} solar mass. (At earth's distance from the sun, 1 a.u., the critical mass would be approximately four times the mass of the earth; 1 earth mass = 3×10^{-6} solar mass.)

The injected mass m_o was given the nominal value 10^{-15} solar mass. However, experimental runs have demonstrated that the final planetary mass does not depend on the value assumed for m_o. Far smaller values of m_o would have given the same results.

d) Coalescence of Planets

It sometimes occurs during the course of a run that two planets come within a distance x of each other. When this happens they are allowed to collide inelastically and to coalesce, forming a single planet. The body formed from the combined masses may continue to grow if conditions are suitable. In the computer simulation, the processes of growth-coalescence-growth take place sequentially; that is, growth is allowed to reach completion before coalescence can take place.

There has been some recent discussion on the question whether a planetary body (the moon, for example) gains mass or loses mass when struck by a smaller body (a large meteroid, for example). Analytically it is difficult to demonstrate this rigorously one way or the other for an atmosphereless body like the moon. However, Gilvarry (1964) has pointed out that if the larger body has an atmosphere, even one held only transiently, aerodynamic drag will prevent much of the ejecta from escaping and there will be a mass gain. It is assumed here that there is no mass lost from either body during coalescence. As a physical justification it is postulated that planets, even those too small to hold an atmosphere permanently, are surrounded by a thin atmosphere during the process of active formation.

Since two bodies on overlapping elliptical orbits, where the angle between their semimajor axes (a_1, a_2) is not specified, may collide at any point where intersections are possible, the orbital parameters of the single coalesced body are not uniquely determinable. Thus it was necessary to adopt a convention for assigning values to the resultant orbital eccentricity ε_3 and semimajor axis a_3 of the body after coalescence has occurred.

The maximum value that a_3 can have is determinable from the principle of conservation of energy:

$$a_3(\max) = \frac{m_1 + m_2}{\dfrac{m_1}{a_1} + \dfrac{m_2}{a_2}}$$

The corresponding eccentricity ε_3 is obtained from the principle of conservation of angular momentum:

$$\varepsilon_3 = \left\{ 1 - \left[\frac{m_1 a_1^{1/2} (1-\varepsilon_1^2)^{1/2} + m_2 a_2^{1/2} (1-\varepsilon_2^2)^{1/2}}{(m_1 + m_2) a_3^{1/2}} \right]^2 \right\}^{1/2}$$

In the present program the parameters of the coalesced planet were obtained by assuming $a_3 = a_3(\max)$, and computing the corresponding eccentricity ε_3.

e) Sweeping of Dust

As each planetary nucleus grows, it sweeps out cleared lanes of dust in the cloud. Some of the gas also is swept up in the vicinity of orbits of planets having masses greater than m_c. The computer program keeps an accounting of the types of bands remaining in the system at any given step: type 0 bands contain dust and gas in the original proportions; type 1 bands contain no dust but have all the original gas; type 2 bands have had some of the gas swept away as well as all of the dust. The run is terminated when there are no type 0 bands remaining between 0.3 and 50 a.u. A planetary nucleus cannot grow, of course, when it is injected into a region of space containing no dust.

f) An Example of One Run

In order to conduct a series of simulated syntheses of planetary systems in the ACRETE program it is necessary to input values for A, K, and W, and to provide a series of starting numbers (X_o) for the random-number generator, a different one for each run. The time required to generate one planetary system is of the order of 15 sec.

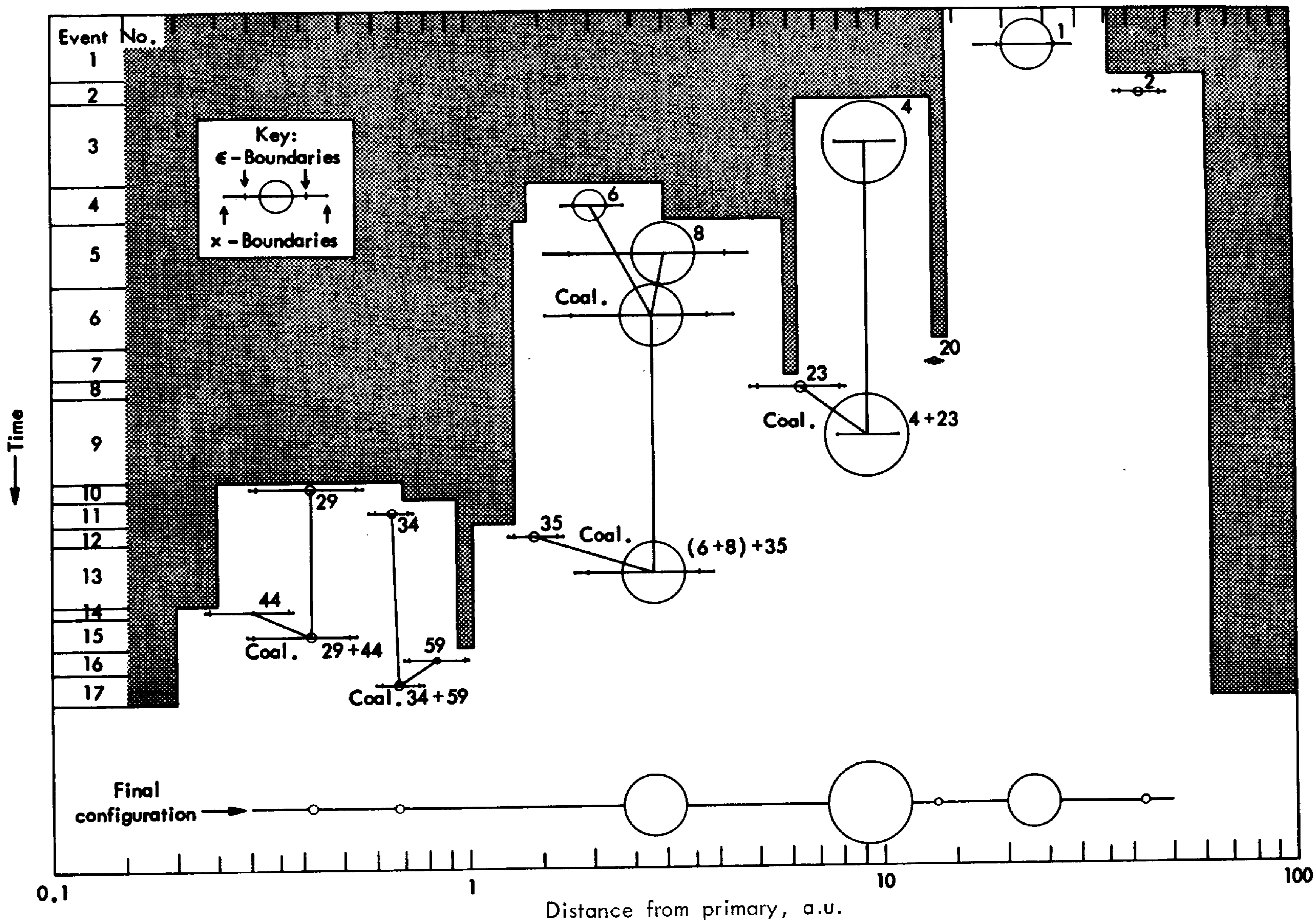

Fig. 7 —Sequential development of a planetary system (Set 3; $X_o = 41$)

Before presenting the results from a series of computer runs, an example of a single run is given below to clarify the sequential events taking place in the ACRETE program.

The example is from Set 3 ($A = 1.5 \times 10^{-3}$, $K = 50$, $W = 0.20$) for the run having the starting random number $X_o = 41$. Run 41 of Set 3 is shown in Figure 7 as a sequence of 17 events reading from top to bottom. Orbital distances are shown on a logarithmic scale, and planets are depicted as circles with radii proportional to the cube root of their masses. The shaded areas represent space containing dust, i.e., regions from which the dust of the original cloud has not yet been swept by being incorporated into a planetary body.

Event 1: The first nucleus is injected into the cloud at mean distance, $a = 22.8$ a.u., orbital eccentricity $\varepsilon = 0.149$. After growing iteratively and exceeding the critical mass, it becomes a giant planet, having swept in all the dust and much of the gas between 14.4 and 36.5 a.u.

Event 2: The second nucleus injected into the cloud at $a = 43.1$ a.u., $\varepsilon = 0.112$, does not exceed critical mass when growth is completed; thus it remains a terrestrial-type body after having swept in all the dust out to 61.6 a.u.

Event 3: The third nucleus injected at $a = 33.0$ a.u., $\varepsilon = 0.132$, finds itself in a region already swept clean of dust and so is unable to grow (becomes a "dud"). The fourth nucleus at $a = 9.29$ a.u., $\varepsilon = 0.007$, becomes a giant planet, sweeping in all the dust and some of the gas between 6.45 and 13.6 a.u., but leaving a narrow lane of original cloud between 13.6 and 14.4 a.u.

Event 4: The fifth nucleus injected is a dud like the third. The sixth nucleus at $a = 2.02$ a.u., $\varepsilon = 0.092$, grows to become a gas giant, having swept in all the dust between 1.14 and 2.97 a.u.

Event 5: The seventh nucleus is a dud. The eighth at $a = 2.97$ a.u., $\varepsilon = 0.408$ (highly eccentric), becomes a gas giant, sweeping in all the remaining dust between 1.28 and 5.91 a.u.

Event 6: Since the planet resulting from the growth of nucleus 8 has x-boundaries overlapping the ε-boundaries of the planet resulting from the growth of nucleus 6, the two bodies coalesce to form a single planet at $a = 2.82$ a.u., $\varepsilon = 0.361$. No further growth takes place.

Event 7: All nuclei from 9 to 19 are duds. Nucleus 20 at $a = 13.5$ a.u., $\varepsilon = 0.016$, manages to sweep in the narrow lane of dust left after the growth of nuclei 1 and 4 to planetary size, but having little material to draw on, becomes a very small planet.

Event 8: Nucleus 23 at $a = 6.43$ a.u., $\varepsilon = 0.229$, similarly sweeps in the narrow dust band near 6 a.u., growing to only small size.

Event 9: The planets resulting from nuclei 4 and 23 coalesce and regrow slightly to form a single body at $a = 9.28$ a.u., $\varepsilon = 0.007$.

Event 10: Nucleus 29 at $a = 0.421$ a.u., $\varepsilon = 0.269$, forms a small terrestrial planet and sweeps the region from 0.248 to 0.690 a.u.

Event 11: Nucleus 34 at $a = 0.661$ a.u., $\varepsilon = 0.093$, forms a small terrestrial body.

Event 12: Nucleus 35 at $a = 1.46$ a.u., $\varepsilon = 0.125$, forms a terrestrial body and leaves a narrow lane of unswept cloud material between 0.930 and 1.03 a.u.

Event 13: Coalescence of planets from nuclei 35 and 6 plus 8.

Event 14: Nucleus 44 at 0.304 a.u., $\varepsilon = 0.209$, forms a small terrestrial body.

Event 15: Coalescence of planets from nuclei 44 and 29.

Event 16: Nucleus 59 at $a = 0.837$ a.u., $\varepsilon = 0.161$, forms a small terrestrial body, sweeping in the last remaining dust band between 0.3 and 50 a.u.

Event 17: Coalescence of planets from nuclei 59 and 34. End of run. Final configuration contains seven planets.

The final output from the computer is a tabulation of the pertinent characteristics of the planets in the final configuration: semimajor axis, swept boundaries, x-boundaries, ε-boundaries, orbital eccentricity, and mass.

The series of runs discussed in subsequent sections were all formed by sequences of events similar to the one outlined above, although usually with fewer coalescences. The total number of nuclei injected ranged from about 40 to over 500 per run. Typically, less than 150 nuclei were injected, the majority of which were duds, of course.

VI. COMPUTATIONAL RESULTS

The computer runs, carried out on an IBM 7044 computer, produced planetary systems with all the general characteristics of the solar system; small rocky planets close in, large planets composed principally of gas in the middle distance range, and small planets again in the outer orbits; the numbers of planets in each system ranged generally from 7 to 14; the orbital radii of the planets followed a pattern generally similar to that of Bode's law; the total mass of the planets in each system was comparable to that of the solar system; the largest planet in each system was similar to Jupiter in mass.

a) Planetary Systems

The following parameters were fixed in the ACRETE program: $\alpha = 5$, $n = 3$, $m_o = 10^{-15} M_s$, $Q = 0.077$, and $B = 1.2 \times 10^{-5}$. Parameters A, K, and W were varied to find combinations of conditions that produced planetary systems most similar to the solar system in general character. Similarity to the solar system was judged on the criteria of number of planets, N, and the total mass of the planets, Σm. (For the solar system $N = 9$ and $\Sigma m = 1.345 \times 10^{-3} M_s$.)[3] Four sets of runs (of 40 runs each) have been carried out, in which A, K, and W were assigned the values in Table 2.

[3] Or N = 10 if Ceres is counted as a planet.

TABLE 2

VALUES OF A, K, AND W

Set Number	A	K	W
1	0.00125	100	0.15
2	0.00125	100	0.20
3	0.00150	50	0.20
4	0.00150	50	0.25

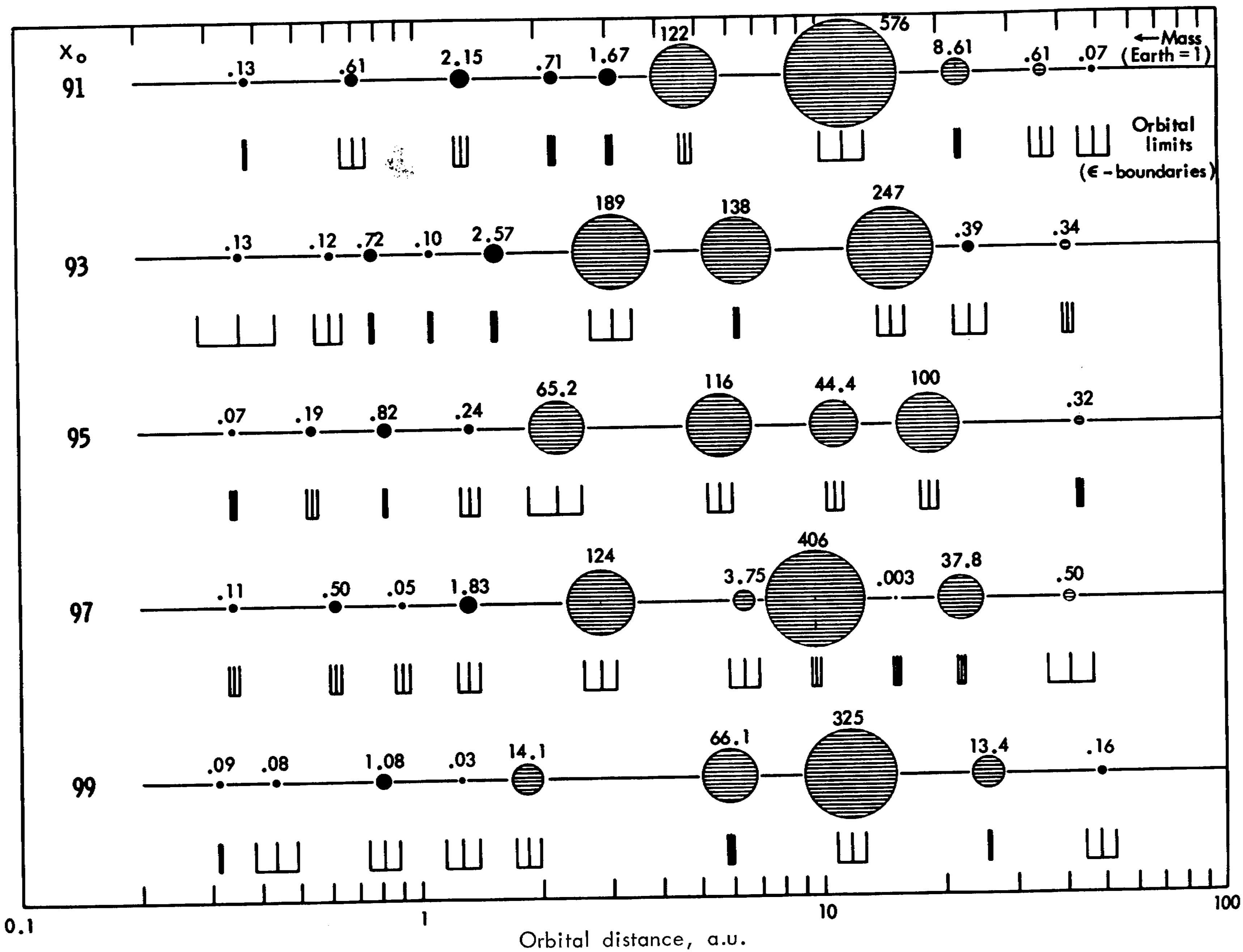

Fig. 8 — Planetary systems generated in ACRETE program: Set 4

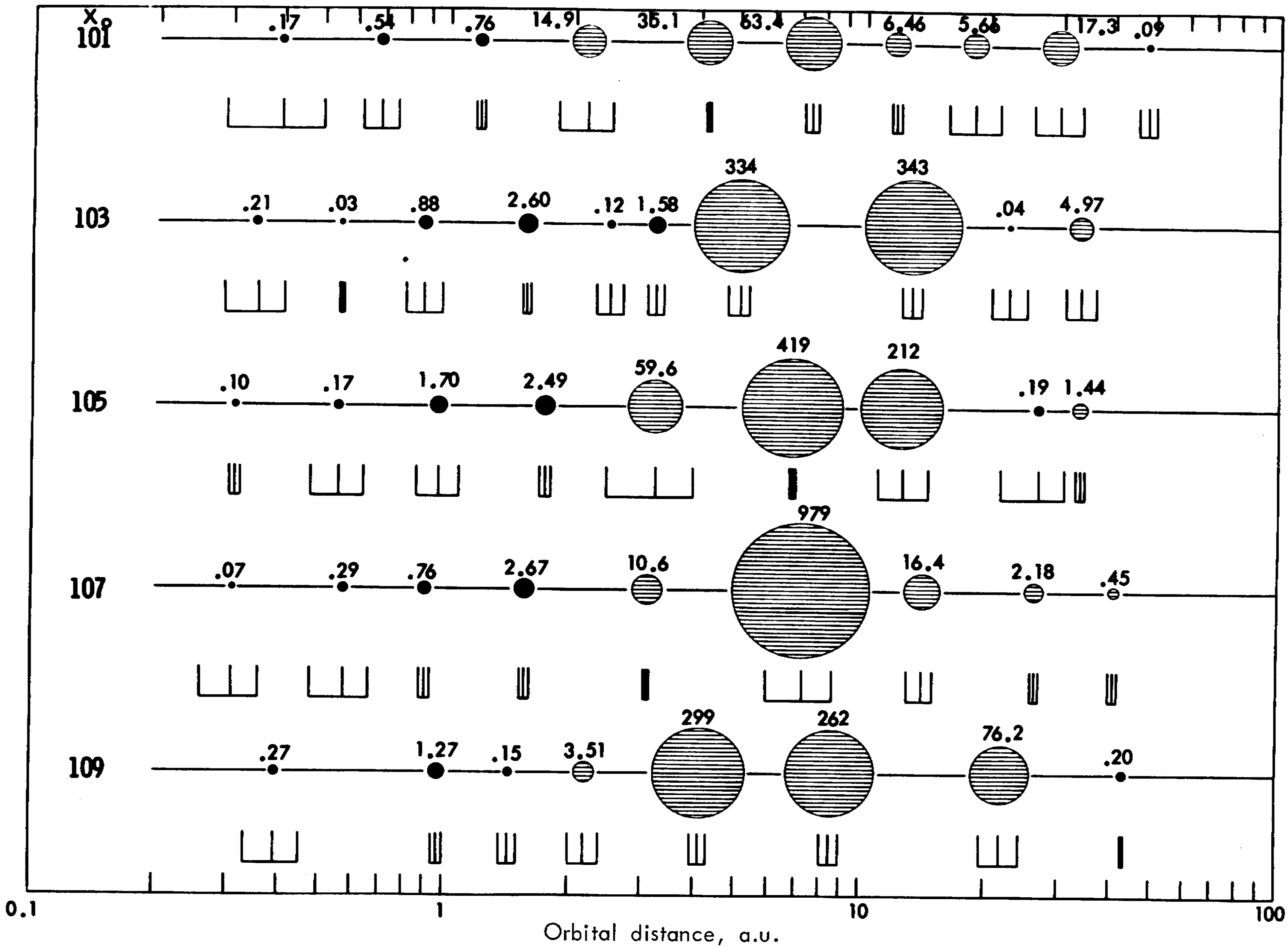

Fig. 9 —Planetary systems generated in ACRETE program: Set 4

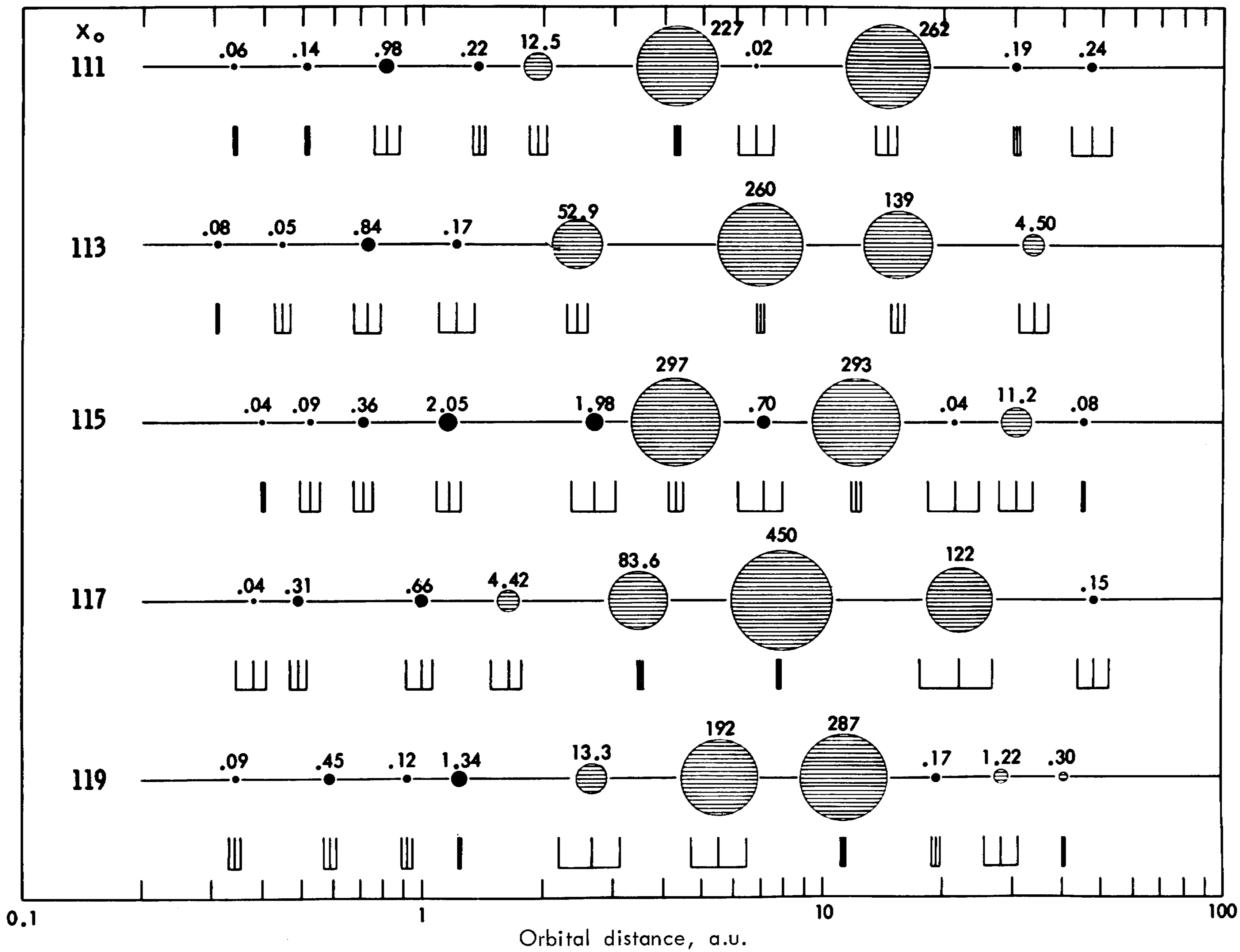

Fig.10—Planetary systems generated in ACRETE program: Set 4

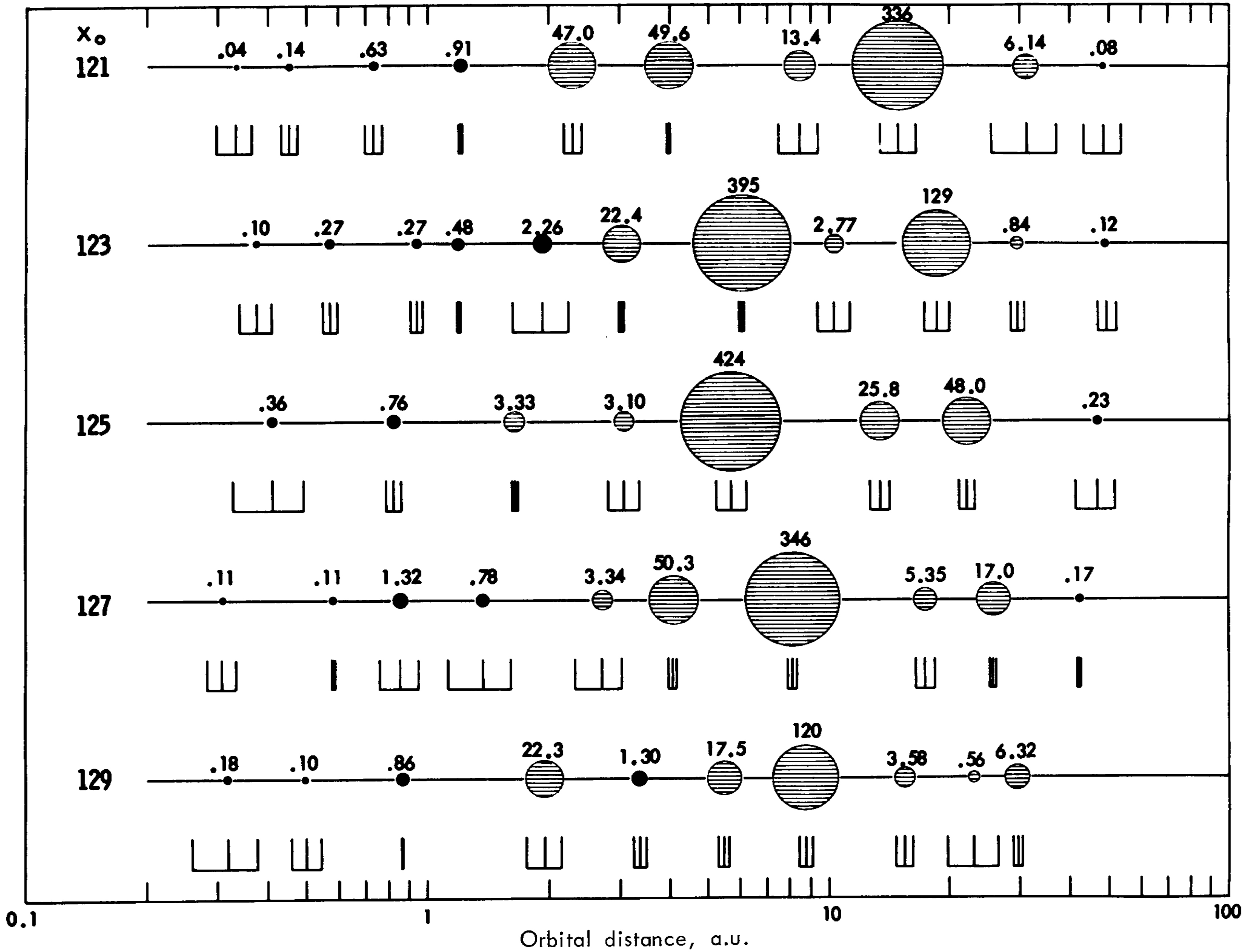

Fig. 11—Planetary systems generated in ACRETE program: Set 4

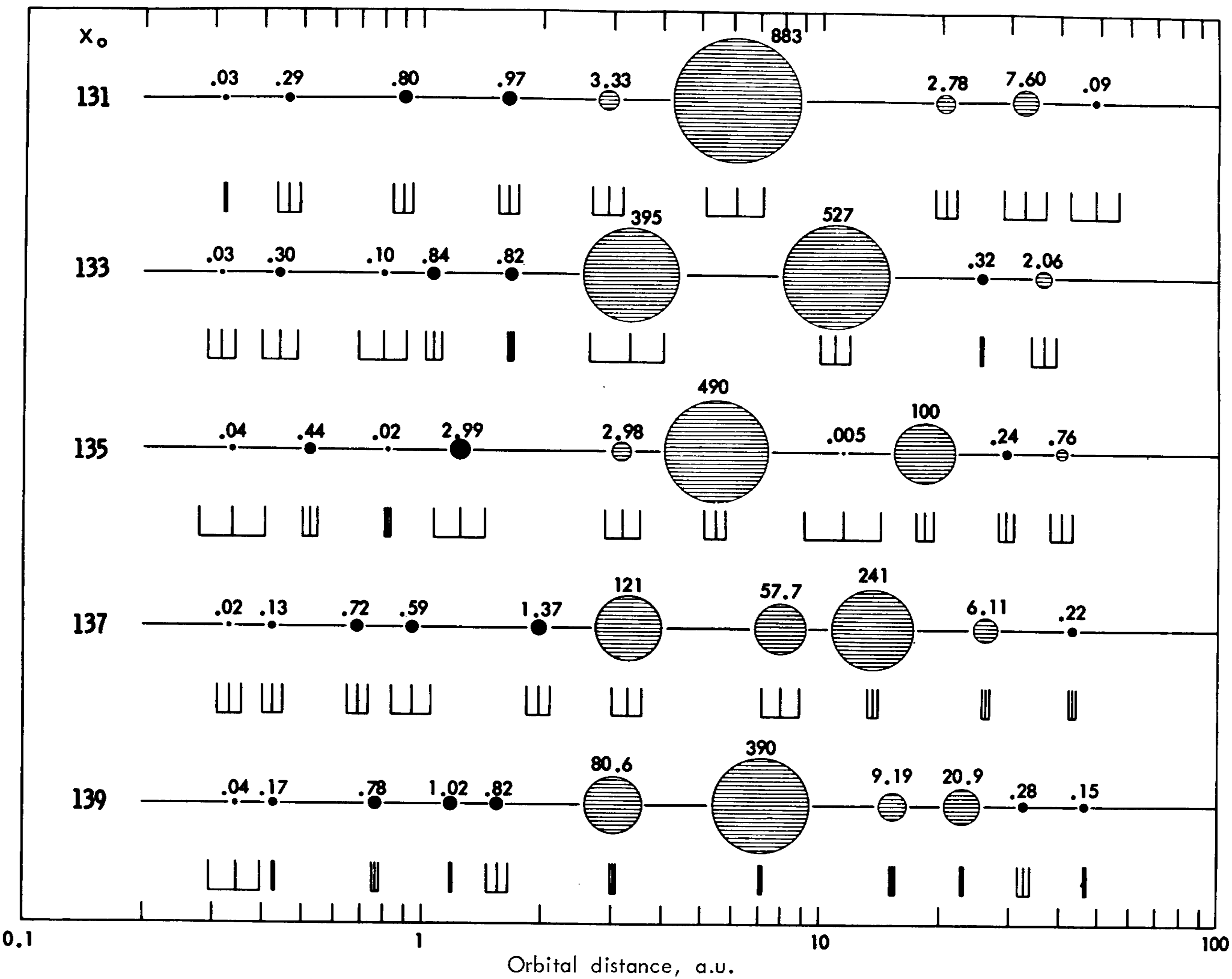

Fig. 12—Planetary systems generated in ACRETE program: Set 4

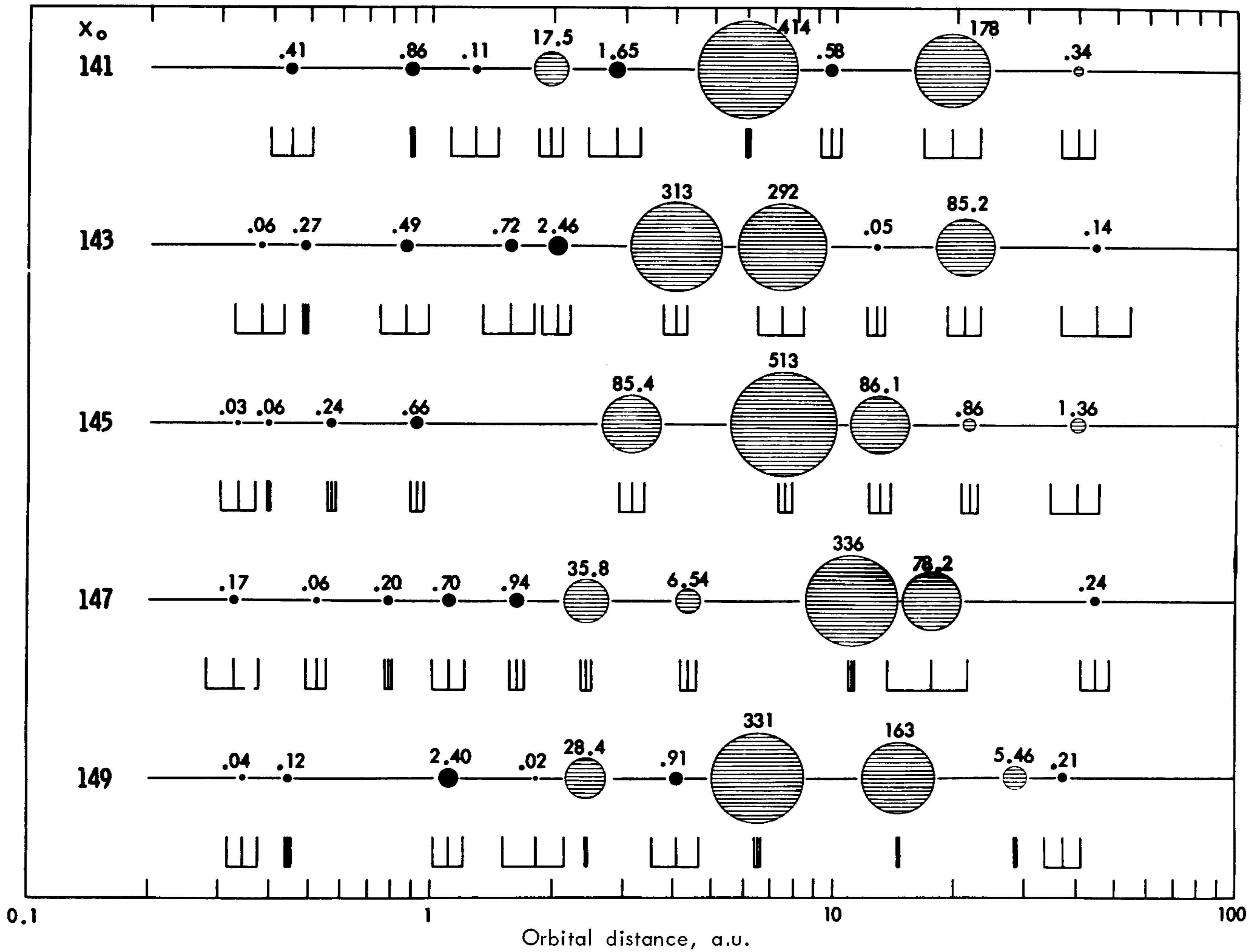

Fig.13—Planetary systems generated in ACRETE program: Set 4

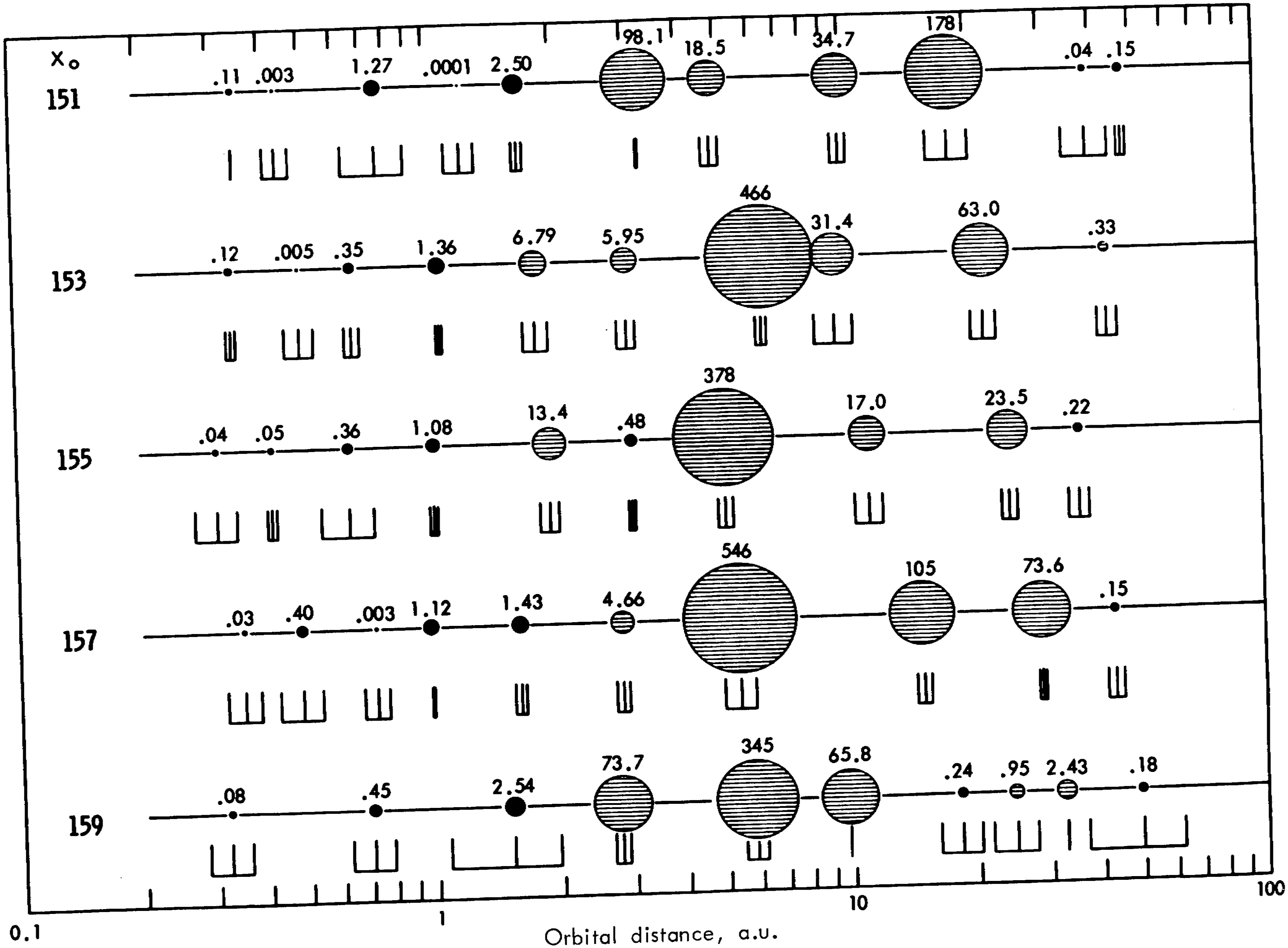

Fig.14—Planetary systems generated in ACRETE program: Set 4

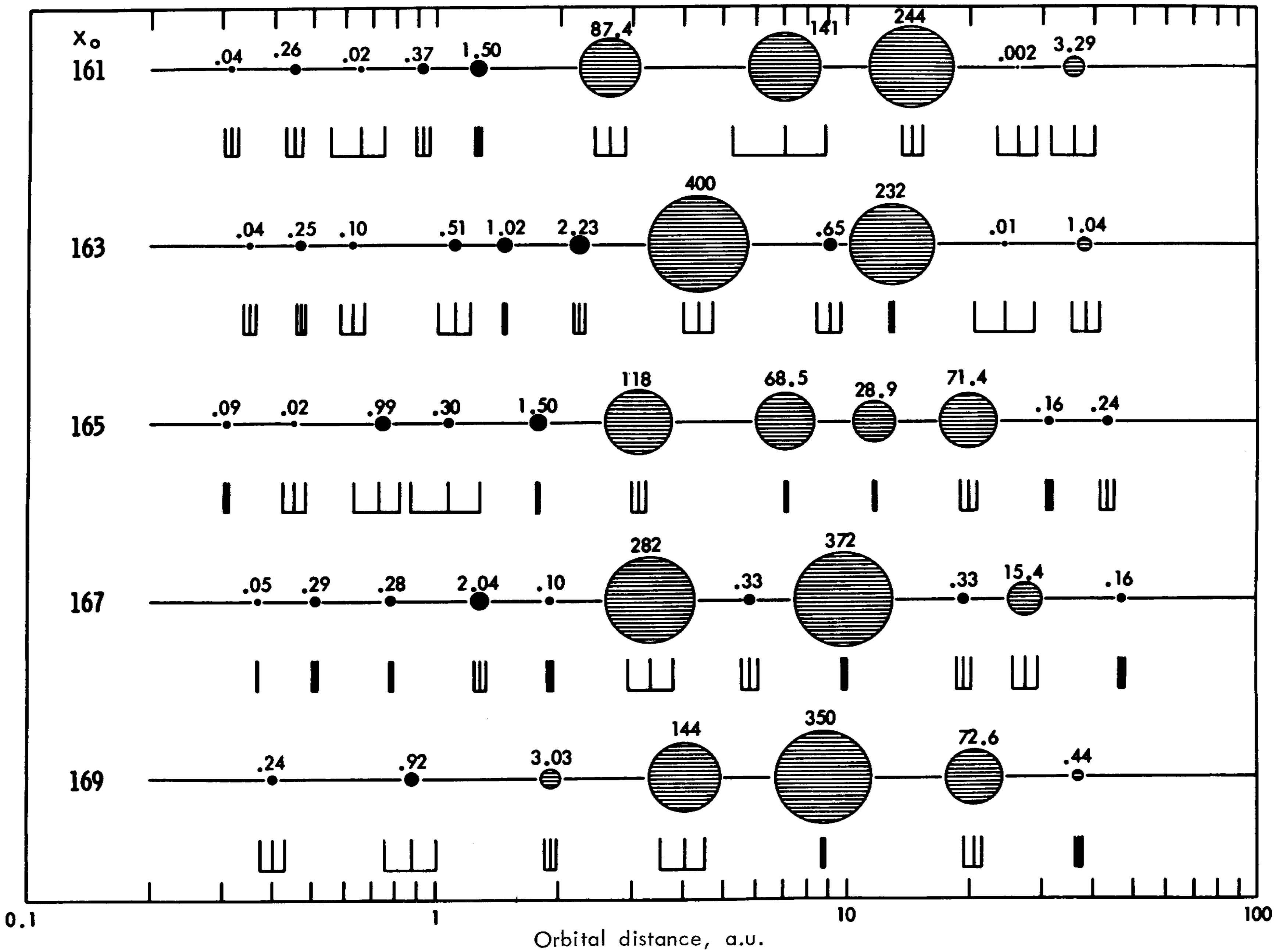

Fig. 15— Planetary systems generated in ACRETE program: Set 4

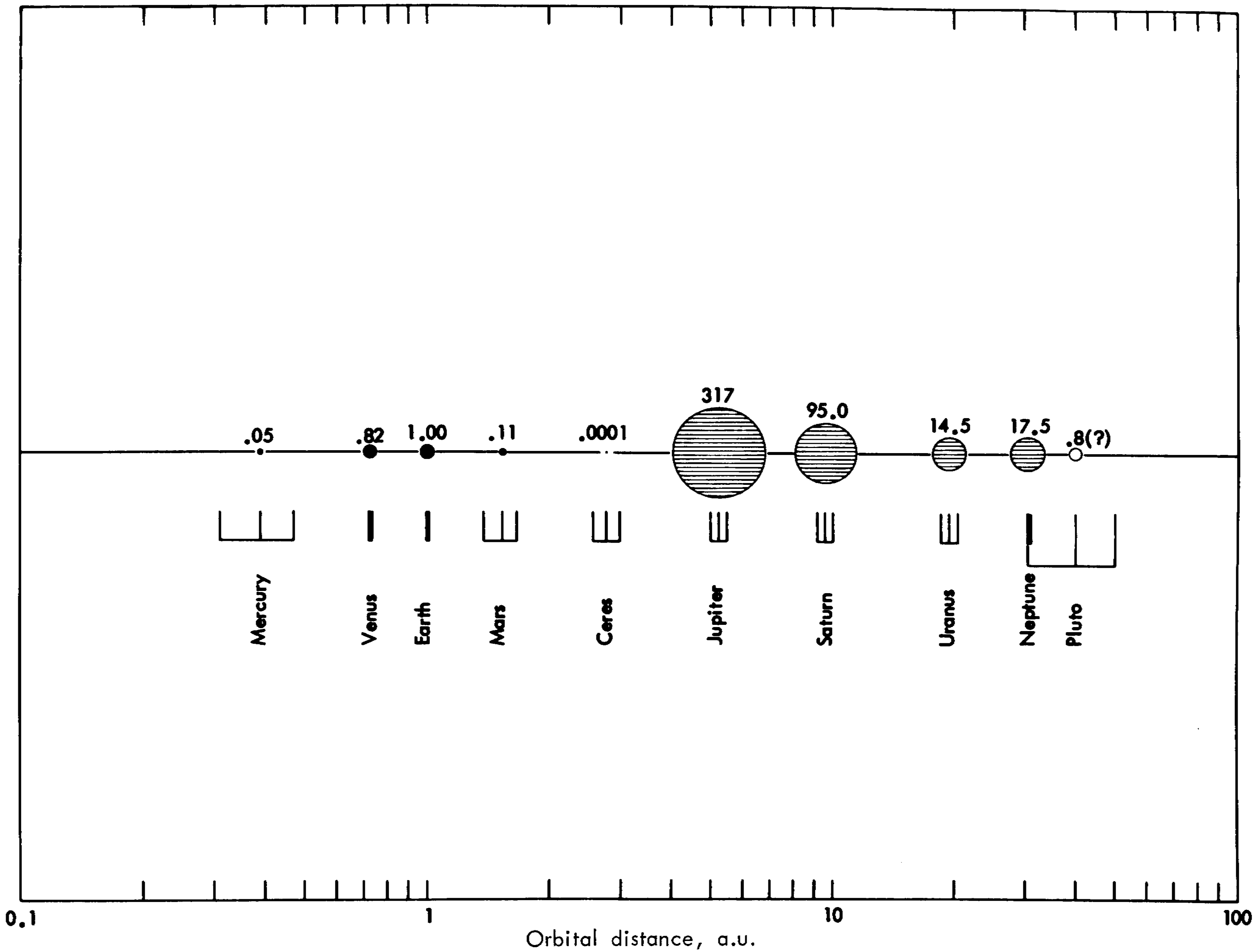

Fig. 16—The solar system

Only the results of the Set 4 runs are illustrated herein, but the results of the runs of Sets 3 and 4, in both of which planetary systems closely similar to the solar system were obtained, are summarized in Table 3. The similarities to the solar system in spacing of orbits and sizes of individual planets may be seen in Figures 8 through 15, in which the orbital radii are depicted on a logarithmic scale, and the planetary masses are indicated by the sizes of the circles (radius of circle proportional to $m^{1/3}$). Terrestrial bodies are shown as solid circles, gas giants by horizontal shading. Figure 16 shows the solar system represented in the same manner.

From the schematic diagrams of the systems produced in Set 4, as well as from the summary data, it may be seen that these planetary systems bear many marked resemblances to the solar system. It is not to be expected that any of the systems so produced would be identical to the solar system in all respects; this is far too much to expect from so small a sample. Yet the solar system could be intermingled with the other 40 and not be recognized as not being a member of the same set. For example, planets very similar to Mercury in mass and mean distance may be found in Runs 115, 117, and 143; planets similar to

TABLE 3

COMPUTER-GENERATED SYSTEMS COMPARED WITH SOLAR SYSTEM

Item	Solar System	Set 4	Set 3
Number of planets			
Average	9	9.2	10.1
Range	....	7-11	7-12
Total mass of planets			
Average ($\times 10^3$)	1.34	1.56	1.16
Range ($\times 10^3$)	....	0.43-3.04	0.58-1.92
Mass of largest planet*			
Average	317	305	258
Range	....	63-979	90-594
Spacing ratio			
Average	1.86	1.84	1.73
Range	1.31-3.41	1.22-3.37	1.17-4.09

*Earth = 1.

Venus may be found in Runs 95 and 113; Earth counterparts in Runs 155 and 157; Mars in Run 141; Jupiter in Run 103; Uranus in Run 99; Neptune in Runs 101 and 127; Pluto in Run 163.

The total masses of the planets in the systems are similar to that of the solar system, averaging $1.56 \times 10^{-3} M_s$ versus $1.34 \times 10^{-3} M_s$ for the solar system. The masses of the largest bodies in the systems average $0.92 \times 10^{-3} M_s$ versus $0.96 \times 10^{-3} M_s$ for Jupiter. The orbital spacing ratios are also very similar to those in the solar system, averaging 1.77 (1.84) versus 1.69 (1.86) for the solar system, the figures in parentheses being the averages when all bodies of mass less than 10^{-7} are excluded. Some systems contain spacing ratios smaller than any in the solar system; some contain spacing ratios comparable to that between Mars and Jupiter.

As shown in Table 4, the mass distributions have general similarities to that of the solar system planets, with most of the bodies falling into the mass range $10^{-7} M_s$ to $10^{-3} M_s$ and being distributed rather evenly within this range.

TABLE 4

COMPUTER-GENERATED SYSTEMS COMPARED WITH SOLAR SYSTEM

	Mass Range								
	10^{-10}	10^{-9}	10^{-8}	10^{-7}	10^{-6}	10^{-5}	10^{-4}	10^{-3}	10^{-2}
	Number of planets in indicated mass range								
Solar System .	1	0	0	2	3	2	2	0	
Set 4 av	0.03	0.08	0.40	2.9	2.8	1.1	1.7	0.65	
Set 3 av	0.03	0.03	0.53	3.2	3.4	1.3	1.9	0.33	

b) Multiple Star Systems

The effects of changing the factor A (coefficient of density in the cloud) were investigated in a separate series of computer runs. Five runs were made at each condition, employing the same starting random numbers (X_o = 23, 25, 29, 39, 41) to reduce the effects of changing too many variables at the same time.

Factors held constant were K = 100, W = 0.15. Parameter A was varied from 0.001 to 0.015. Some of the results are summarized in

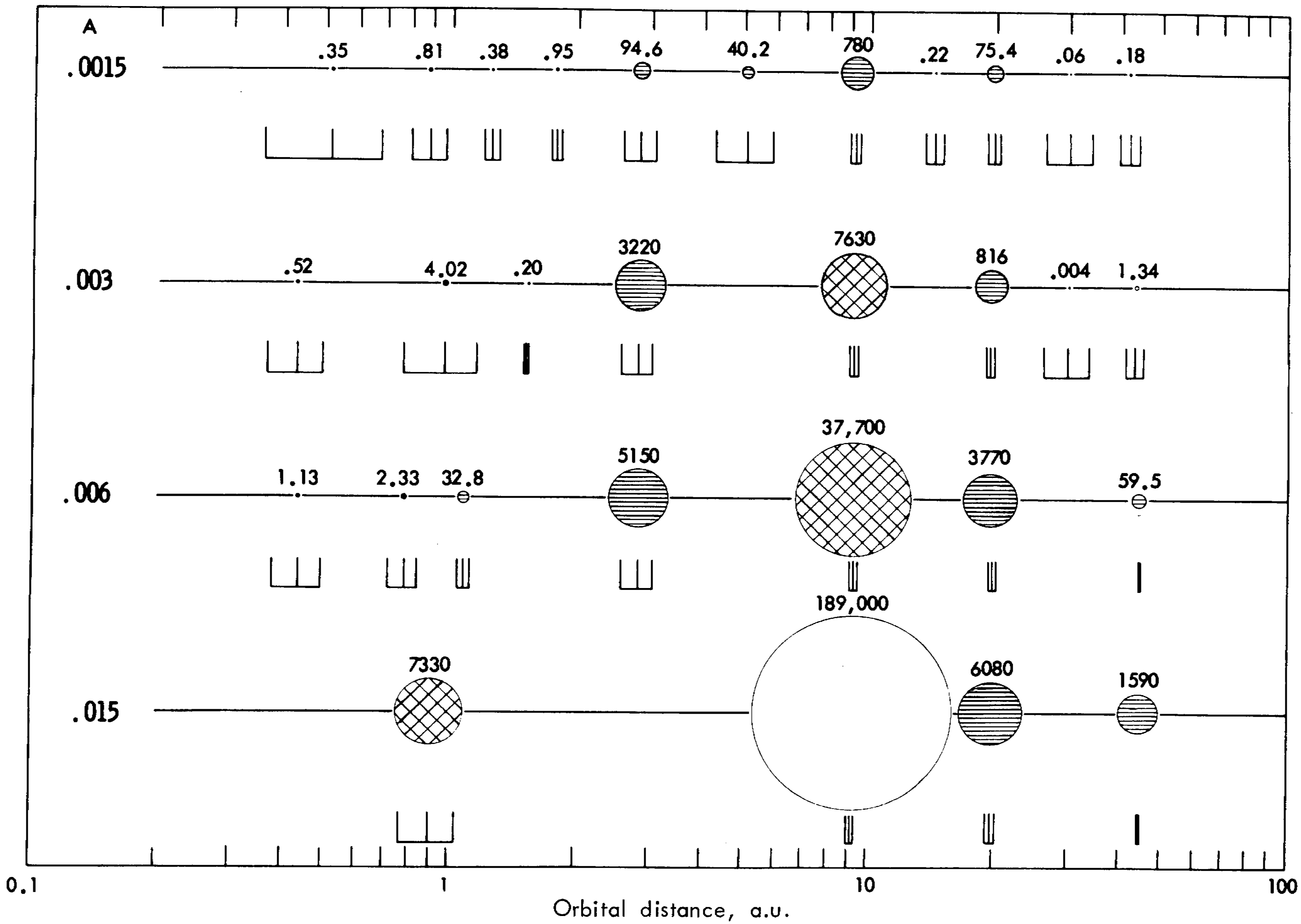

Fig. 17—Synthetic planetary systems: Effect of increasing A (X_o = 25)
(Scale of diameters not the same as in Figs. 8 to 16)

Figure 17, where red dwarf stars are identified by cross-hatchings; the open circle represents an orange dwarf star.

Even small increases in A result in large increases in the total mass of the systems produced; increasing A also decreases the average number of planets per system. As may be seen in Figure 17, for A = 0.003 and 0.006 the planetary system has become a binary star system, the body near 9 a.u. having grown large enough to be considered a red dwarf star. Observationally, the two stars of smallest mass now known are members of a binary system designated L726-8; each star has a mass estimated at about $0.04M_s$ (about 40 times the mass of Jupiter) or $13{,}000M_e$. The lower theoretical limit to the mass of a star is believed to be near $0.02M_s$. It will be noticed that the binary star systems still contain numerous planetary bodies. As A is increased still more the systems become multiple-star systems and the number of planetary companions diminishes. Actually, the results at the higher values of A should be considered only suggestive of the general trend, since the total mass of the "planetary" bodies is now becoming fairly high with respect to that of the central body, so that the original simplifying assumptions, which were adequate when the total planetary mass was well below $0.01M_s$, no longer apply so satisfactorily. The gravitational attractions of the several large masses for each other can no longer be considered to have negligible effects on the secular stability of the systems. This is pushing the ACRETE program somewhat beyond its original intent (to create planetary systems similar to the solar system). However, it would be readily possible to modify the program slightly to provide more rigorously for cases in which some of the planetary bodies grow to stellar mass. In any event, the general trend is clear. Simply increasing the value assigned to one parameter makes it possible to generate widely spaced binary and multiple-star systems.

V. CONCLUSIONS

The theory of the formation of the solar system presented here depends upon aggregation, some mechanism by which small particles can cling together to form prestellar and preplanetary nuclei. If such processes did take place during the early days of the solar system, then the major properties of the solar system emerge. Recalling the requirements for a satisfactory theory of origin, the present theory accounts for the distribution of mass and angular momentum between sun and planets, the differences in composition between the close-in terrestrial planets and the giant planets farther out, the near-constancy of spacing ratio for orbital distances, and the fact that all the planets orbit in almost the same plane and with the same sense. (The fact that the freely rotating (not tidally braked) bodies also rotate with the same sense has already been explained qualitatively by Guili (1968), who showed that the impacting of many small bodies from elliptical prograde orbits onto a planetary body results in a net prograde rotation of the body.)

The theory also accounts for the fact that the sun's equatorial plane does not necessarily lie exactly in the invariable plane of the solar system, which would be required in some theories. It provides an explanation for the fact that the planetary axes of rotation are tilted out of normal to the invariable plane, the result of chance inelastic collisions and coalescences with a few rather massive bodies in addition to the many small bodies being collected. While the computer program does not make provisions for the generation of satellites, it is implied by the aggregation theory that the nuclei of the large satellites (including the Moon) formed in the vicinity of their primary planets very early in the process (local overriding of r^{+1} field due to cloud by r^{-2} field due to a growing preplanetary nucleus) and that they grew by aggregation at the same time their primary planets were growing but at a slower rate, since they were initially smaller and competing for mass. It is implied they were originally in orbits more remote from their primary planets and that the orbital distances decreased as the primaries grew in mass. In other words, the large satellites have accompanied their primaries "since the beginning."

The present theory is compatible with the fact that the earth (presumably all the other planets too) is still sweeping up mass in the form of meteorites and micrometeroites at a rate of several thousand tons per day. The rates must be diminishing as the system becomes cleaner with the passage of time. Thus the rates of mass aggregation must have been much greater in the past than they are now and all the planets were subjected to a heavy bombardment of infalling material, as evinced by the pock-marked face of the Moon, the craters of Mars, and the ancient large meteorite craters of the earth (now much weathered for the most part).

The theory is also compatible with the presence of comets in the solar system, which may be considered primordial matter that has not yet been swept up by the planets.

It is not implied that the mode of star formation presented in this paper is the only process by which stars can originate within clouds of dust and gas. A competing process, gravitational collapse, may well be the primary mechanism by which stars of mass significantly greater than the sun's come into being, since the rate of gravitational collapse increases strongly with mass. The times required for the gravitational collapse of stars of small mass, as generally estimated, however, are so long that competing processes (i.e., nucleation and aggregation within the cloud) may take place more rapidly, before contraction as a whole can go to completion. This concept is consistent with the observed fact that class O, B, A, and early F stars frequently have large rotational velocities, while in late F and later classes, rapid rotation is observed only in close spectroscopic binaries.

The theory here proposed contains a number of implications about the universe beyond the solar system:

1. Practically all stars of small mass (less than about $2M_s$) have planetary companions.

2. Where one planet is detected by indirect means in the vicinity of a nearby star, numerous other planetary bodies should also be present.

3. Most binary star systems also contain planets.

4. Earth-like planets should be extremely abundant, occurring in the vicinity of a large proportion of main-sequence stars (since about 98 percent of all stars have masses less than $2M_s$).

5. Close binaries are formed when central aggregations occur at two separated centers, possibly because of asymmetry in the original dark globule.

6. Widely separated binaries, of one type at least, can result from the growth of one planet to stellar size.

The simulation program that has been discussed here was deliberately simplified for exploratory purposes; for example, by injecting all preplanetary nuclei into the cloud with zero inclination (orbits lie in the invariable plane), by omitting provisions for generating satellites, and by treating all planets as having the same bulk density in spite of differences in composition. I do not believe, however, that the overall results, as presented, would be changed significantly by making the model more complicated.

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FIGURES

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FORMATION OF PLANETARY SYSTEMS BY AGGREGATION:
A COMPUTER SIMULATION

S. H. Dole