

1. $\int u \, dv = uv - \int v \, du$
2. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{u} \, du = \ln|u| + C$
4. $\int e^u \, du = e^u + C$
5. $\int a^u \, du = \frac{a^u}{\ln a} + C$
6. $\int \sin u \, du = -\cos u + C$
7. $\int \cos u \, du = \sin u + C$
8. $\int \sec^2 u \, du = \tan u + C$
9. $\int \csc^2 u \, du = -\cot u + C$
10. $\int \sec u \tan u \, du = \sec u + C$
11. $\int \csc u \cot u \, du = -\csc u + C$
103. $\int \sinh u \, du = \cosh u + C$
104. $\int \cosh u \, du = \sinh u + C$

These integrals can be used without explanation

$$I_{12} = \int \tan u \, du$$

$$= \int \frac{\sin u}{\cos u} \, du$$

$$\text{Let } v = \cos u \longrightarrow dv = -\sin u \, du \longrightarrow -dv = \sin u \, du$$

$$= - \int \frac{1}{v} \, dv$$

$$= -\ln|v| + C$$

Reverse the substitution

$$= -\ln|\cos u| + C$$

$$I_{102} = \int \frac{1}{u \ln u} \, du$$

$$\text{Let } v = \ln u \longrightarrow dv = \frac{1}{u} \, du$$

$$= \int \frac{1}{v} \, dv$$

$$= \ln v + C$$

Reverse the substitution

$$= \ln|\ln u|$$

12

$$\int \tan u \, du = \ln|\cos u| + C$$

$$\int \frac{1}{u \ln u} \, du = \ln|\ln u| + C$$

102

$$I_{100} = \int \ln u \, du$$

$$\text{Integrate by parts} \left\{ \begin{array}{l} 1 = (u)' \\ (\ln u)' = \frac{1}{u} \end{array} \right.$$

$$= (u)(\ln u) - \int (u) \left(\frac{1}{u} \right) du$$

$$= u \ln u - \int 1 \, du$$

$$= u \ln u - u + C$$

$$\int \ln u \, du = u \ln u - u + C$$

$$I_{13} = \int \cot u \, du$$

$$= \int \frac{\cos u}{\sin u} \, du$$

$$\text{Let } v = \sin u \longrightarrow dv = \cos u \, du$$

$$= \int \frac{1}{v} \, dv$$

$$= \ln |v| + C$$

Reverse the substitution

$$= \ln |\sin u| + C$$

$$\int \cot u \, du = \ln |\sin u| + C$$

$$\begin{aligned}
 I_{14} &= \int \sec u \, du \\
 &= \int \sec u \cdot \frac{\sec u + \tan u}{\sec u + \tan u} \, du \\
 &= \int \frac{\sec^2 u + \sec u \cdot \tan u}{\sec u + \tan u} \, du
 \end{aligned}$$

$$\text{Let } v = \sec u + \tan u \longrightarrow dv = \sec^2 u + \sec u \tan u \, du$$

$$\begin{aligned}
 &= \int \frac{1}{v} \\
 &= \ln|v| + C
 \end{aligned}$$

Reverse the substitution

$$= \ln|\sec u + \tan u| + C$$

$$I_{96} = \int u e^{au} \, du$$

$$\text{Integrate by parts} \quad \begin{cases} (u)' = 1 \\ e^{au} = \left(\frac{e^{au}}{a}\right)' \end{cases}$$

$$\begin{aligned}
 &= (u) \left(\frac{e^{au}}{a} \right) - \int (1) \left(\frac{e^{au}}{a} \right) du \\
 &= \frac{1}{a} u e^{au} - \frac{1}{a} \int e^{au} \, du \\
 &= \frac{1}{a} u e^{au} - \frac{1}{a^2} e^{au} + C \\
 &= \left(\frac{1}{a} u - \frac{1}{a^2} \right) e^{au} + C
 \end{aligned}$$

Simplify

$$= \frac{1}{a^2} (au - 1) e^{au} + C$$

14

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\int u e^{au} \, du = \frac{1}{a^2} (au - 1) e^{au} + C$$

96

$$I_{92} = \int u \arctan u \, du$$

$$\begin{aligned} \text{Integrate by parts} \quad & \begin{cases} u = \left(\frac{1}{2}u^2\right)' \\ (\arctan u)' = \frac{1}{1+u^2} \end{cases} \\ & = \left(\frac{1}{2}u^2\right)(\arctan u) - \int \left(\frac{1}{2}u^2\right)\left(\frac{1}{1+u^2}\right) du \\ & = \frac{u^2}{2} \arctan u - \frac{1}{2} \int \frac{u^2 + 1 - 1}{1 + u^2} du \\ & = \frac{u^2}{2} \arctan u - \frac{1}{2} \int 1 - \frac{1}{1 + u^2} du \\ & = \frac{u^2}{2} \arctan u - \frac{1}{2} (u - \arctan u) + C \\ & = \frac{u^2 + 1}{2} \arctan u - \frac{u}{2} + C \end{aligned}$$

$$(92) \quad \int u \arctan u \, du = \frac{u^2+1}{2} \arctan u - \frac{u}{2} + C$$

$$\begin{aligned} I_{15} &= \int \csc u \, du \\ &= \int \csc u \cdot \frac{\csc u + \cot u}{\csc u + \cot u} du \\ &= \int \frac{\csc^2 u + \csc u \cdot \cot u}{\csc u + \cot u} du \end{aligned}$$

$$\begin{aligned} \text{Let } v &= \csc u + \cot u \longrightarrow dv = -\csc^2 u + \csc u \cot u \, du \\ &\longrightarrow -dv = \csc^2 u + \csc u \cot u \, du \end{aligned}$$

$$\begin{aligned} &= \int \frac{-dv}{v} \\ &= -\int \frac{1}{v} dv \\ &= -\ln|v| + C \end{aligned}$$

Reverse the substitution

$$= -\ln|\csc u + \cot u| + C$$

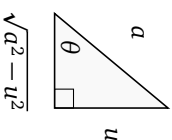
$$\int \csc u \, du = -\ln|\csc u + \cot u| + C$$

$$I_{16} = \int \frac{1}{\sqrt{a^2 - u^2}} du$$

$$\text{Let } \sin \theta = \frac{u}{a}, \quad \cos \theta = \frac{\sqrt{a^2 - u^2}}{a} \longrightarrow \tan \theta = \frac{u}{\sqrt{a^2 - u^2}}$$

$$\text{and so } u = a \sin \theta \longrightarrow du = a \cos \theta d\theta$$

$$\text{and } \sqrt{a^2 - u^2} = a \cos \theta$$



$$= \int \frac{a \cos \theta}{a \cos \theta} d\theta$$

$$= \int 1 d\theta$$

$$= \theta + C$$

Reverse the substitution of u , where $\sin \theta = \frac{u}{a} \longrightarrow \theta = \arcsin \frac{u}{a}$

$$= \arcsin \frac{u}{a} + C$$

$$I_{71} = \int \sec^3 u du$$

$$\text{Integrate by parts } \begin{cases} (\sec u)' = \sec u \tan u \\ \sec^2 u = (\tan u)' \end{cases}$$

$$= (\sec u)(\tan u) - \int (\sec u \tan u)(\tan u) du$$

$$= \sec u \tan u - \int \sec u \tan^2 u du$$

$$= \sec u \tan u - \int \sec u (\sec^2 u - 1) du$$

$$= \sec u \tan u - \int \sec^3 u du + \int \sec u du$$

$$2 \int \sec^3 u du = \sec u \tan u + \int \sec u du$$

$$\int \sec^3 u du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \int \sec u du$$

$$= \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

16

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$$

71

$$\int \sec^3 u du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

$$I_{65} = \int \tan^2 u \, du$$

Rewrite using trigonometric identity $\tan^2 u = \sec^2 u - 1$

$$\begin{aligned} &= \int \sec^2 u - 1 \, du \\ &= \tan u - u + C \end{aligned}$$

(65)

$$\int \tan^2 u \, du = \tan u - u + C$$

$$I_{17} = \int \frac{1}{a^2 + u^2} du$$

Let $v = \frac{u}{a} \longrightarrow av = u \longrightarrow a \, dv = du$

$$\begin{aligned} &= \int \frac{a}{a^2 + (av)^2} dv \\ &= \int \frac{a}{a^2(1 + v^2)} dv \\ &= \frac{1}{a} \int \frac{1}{1 + v^2} dv \\ &= \frac{1}{a} \arctan v \end{aligned}$$

Reverse the substitution of u

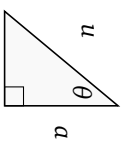
$$\begin{aligned} &= \frac{1}{a} \arctan \frac{u}{a} + C \end{aligned}$$

(17)

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$I_{18} = \int \frac{1}{u \sqrt{u^2 - a^2}} du$$

$$\text{Let } \sin \theta = \frac{\sqrt{u^2 - a^2}}{u}, \quad \cos \theta = \frac{a}{u} \quad \longrightarrow \quad \tan \theta = \frac{\sqrt{u^2 - a^2}}{a}$$



$$\text{and so } u = a \sec \theta \quad \longrightarrow \quad du = a \sec \theta \tan \theta \, d\theta$$

$$\text{and } \sqrt{u^2 - a^2} = a \tan \theta$$

$$I_{18} = \int \frac{a \sec \theta \tan \theta}{a \sec \theta \cdot a \tan \theta} d\theta$$

$$= \int \frac{1}{a} d\theta$$

$$= \frac{1}{a} \theta + C$$

Reverse the substitution, where $u = a \sec \theta \quad \longrightarrow \quad \theta = \operatorname{arcsec} \frac{u}{a}$

$$= \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

18

$$\int \frac{1}{u \sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

$$I_{64} = \int \cos^2 u \, du$$

$$= \int \frac{1 + \cos 2u}{2} du$$

$$= \frac{1}{2} \cdot \int 1 + \cos 2u \, du$$

$$= \frac{1}{2} \cdot \left(u + \frac{1}{2} \sin 2u \right) + C$$

$$= \frac{1}{2} u + \frac{1}{4} \sin 2u + C$$

$$\int \cos^2 u \, du = \frac{1}{2} u + \frac{1}{4} \sin 2u + C$$

64

$$\begin{aligned}
I_{63} &= \int \sin^2 u \, du \\
&= \int \frac{1 - \cos 2u}{2} \, du \\
&= \frac{1}{2} \cdot \int 1 - \cos 2u \, du \\
&= \frac{1}{2} \cdot \left(u - \frac{1}{2} \sin 2u \right) + C \\
&= \frac{1}{2} u - \frac{1}{4} \sin 2u + C
\end{aligned}$$

$$\int \sin^2 u \, du = \frac{1}{2} u - \frac{1}{4} \sin 2u + C \quad (63)$$

$$\begin{aligned}
I_{19} &= \int \frac{1}{a^2 - u^2} \, du \\
&= \int \frac{A}{a+u} + \frac{B}{a-u} \, du
\end{aligned}$$

$$\begin{aligned}
\text{Where } A(a-u) + B(a+u) &= 1 \longrightarrow A+B = \frac{1}{a} \text{ and } B-A=0 \\
\longrightarrow B=A \longrightarrow 2A &= \frac{1}{a} \longrightarrow A = \frac{1}{2a} \longrightarrow B = \frac{1}{2a}
\end{aligned}$$

$$\begin{aligned}
&= \int \frac{\frac{1}{2a}}{a+u} + \frac{\frac{-1}{2a}}{a-u} \, du \\
&= \frac{1}{2a} \cdot \int \frac{1}{a+u} + \frac{1}{a-u} \, du
\end{aligned}$$

Rearranging so u precedes a for aesthetic reasons

$$\begin{aligned}
&= \frac{1}{2a} \cdot \int \frac{1}{u+a} - \frac{1}{u-a} \, du \\
&= \frac{1}{2a} (\ln|u+a| - \ln|u-a|) + C \\
&= \frac{1}{2a} \cdot \ln \left| \frac{u+a}{u-a} \right| + C
\end{aligned}$$

$$\int \frac{1}{a^2 - u^2} \, du = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C \quad (19)$$

$$I_{20} = \int \frac{1}{u^2 - a^2} du$$

$$= \int \frac{A}{u - a} + \frac{B}{u + a} du$$

Where $A(u + a) + B(u - a) = 1 \longrightarrow A + B = 0$ and $A - B = \frac{1}{a}$

$\longrightarrow B = -A \longrightarrow 2A = \frac{1}{a} \longrightarrow A = \frac{1}{2a} \longrightarrow B = -\frac{1}{2a}$

$$= \int \frac{\frac{1}{2a}}{u - a} + \frac{-\frac{1}{2a}}{u + a} du$$

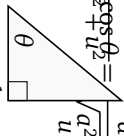
$$= \frac{1}{2a} \cdot \int \frac{1}{u - a} - \frac{1}{u + a} du$$

$$= \frac{1}{2a} (\ln|u - a| - \ln|u + a| + C)$$

$$= \frac{1}{2a} \cdot \ln \left| \frac{u - a}{u + a} \right| + C$$

$$I_{21} = \int \sqrt{a^2 + u^2} du$$

Let $\sin \theta = \frac{u}{\sqrt{a^2 + u^2}} \xrightarrow{\cos \theta = \frac{a}{\sqrt{a^2 + u^2}}} \tan \theta = \frac{u}{a}$



and so $u = a \tan \theta \xrightarrow{du = a \sec \theta \tan \theta d\theta}$

and $\sqrt{u^2 + a^2} = a \sec \theta$

$$= a \sec \theta a \sec^2 \theta d\theta$$

$$= a^2 \int \sec^3 \theta d\theta$$

Apply integral identity (71) :

$$\int \sec^3 u du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

$$= a^2 \left(\frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + C \right)$$

$$= \frac{a^2}{2} \frac{\sqrt{u^2 + a^2}}{a} \cdot \frac{u}{a} + \frac{a^2}{2} \ln \left| \frac{\sqrt{u^2 + a^2}}{a} + \frac{u}{a} \right| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln \left| \frac{u + \sqrt{u^2 + a^2}}{a} \right| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| + \frac{a^2}{2} \ln |a| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| + C$$

(20) $\int \frac{1}{u^2 - a^2} du = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$

$$\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| + C \quad (21)$$