

1.  $\int u \, dv = uv - \int v \, du$
2.  $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3.  $\int \frac{1}{u} \, du = \ln|u| + C$
4.  $\int e^u \, du = e^u + C$
5.  $\int a^u \, du = \frac{a^u}{\ln a} + C$
6.  $\int \sin u \, du = -\cos u + C$
7.  $\int \cos u \, du = \sin u + C$
8.  $\int \sec^2 u \, du = \tan u + C$
9.  $\int \csc^2 u \, du = -\cot u + C$
10.  $\int \sec u \tan u \, du = \sec u + C$
11.  $\int \csc u \cot u \, du = -\csc u + C$
103.  $\int \sinh u \, du = \cosh u + C$
104.  $\int \cosh u \, du = \sinh u + C$

These integrals can be used without explanation



$$\begin{aligned}
 I_{12} &= \int \tan u \, du \\
 &= \int \frac{\sin u}{\cos u} \, du
 \end{aligned}$$

$$\text{Let } v = \cos u \longrightarrow dv = -\sin u \, du \longrightarrow -dv = \sin u \, du$$

$$\begin{aligned}
 &= - \int \frac{1}{v} \, dv \\
 &= -\ln|v| + C
 \end{aligned}$$

Reverse the substitution

$$= -\ln|\cos u| + C$$

$$\int \tan u \, du = \ln|\cos u| + C$$

$$\begin{aligned}
 I_{13} &= \int \cot u \, du \\
 &= \int \frac{\cos u}{\sin u} \, du
 \end{aligned}$$

$$\text{Let } v = \sin u \longrightarrow dv = \cos u \, du$$

$$\begin{aligned}
 &= \int \frac{1}{v} \, dv \\
 &= \ln|v| + C
 \end{aligned}$$

Reverse the substitution

$$= \ln|\sin u| + C$$

$$\begin{aligned}
 I_{14} &= \int \sec u \, du \\
 &= \int \sec u \cdot \frac{\sec u + \tan u}{\sec u + \tan u} \, du \\
 &= \int \frac{\sec^2 u + \sec u \cdot \tan u}{\sec u + \tan u} \, du
 \end{aligned}$$

Let  $v = \sec u + \tan u \longrightarrow dv = \sec^2 u + \sec u \tan u \, du$

$$\begin{aligned}
 &= \int \frac{dv}{v} \\
 &= \ln|v| + C
 \end{aligned}$$

Reverse the substitution

$$= \ln|\sec u + \tan u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$\begin{aligned}
 I_{15} &= \int \csc u \, du \\
 &= \int \csc u \cdot \frac{\csc u - \cot u}{\csc u - \cot u} \, du \\
 &= \int \frac{\csc^2 u - \csc u \cdot \cot u}{\csc u - \cot u} \, du
 \end{aligned}$$

Let  $v = \csc u - \cot u \longrightarrow dv = \csc^2 u - \csc u \cot u \, du$

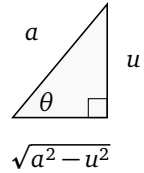
$$\begin{aligned}
 &= \int \frac{dv}{v} \\
 &= \ln|v| + C
 \end{aligned}$$

Reverse the substitution

$$= \ln|\csc u - \cot u| + C$$

$$I_{16} = \int \frac{1}{\sqrt{a^2 - u^2}} du$$

Let  $\sin \theta = \frac{u}{a}$ ,  $\cos \theta = \frac{\sqrt{a^2 - u^2}}{a} \longrightarrow \tan \theta = \frac{u}{\sqrt{a^2 - u^2}}$



and so  $u = a \sin \theta \longrightarrow du = a \cos \theta d\theta$

and  $\sqrt{a^2 - u^2} = a \cos \theta$

$$= \int \frac{a \cos \theta}{a \cos \theta} d\theta$$

$$= \int 1 d\theta$$

$$= \theta + C$$

Reverse the substitution of  $u$ , where  $\sin \theta = \frac{u}{a} \longrightarrow \theta = \arcsin \frac{u}{a}$

$$= \arcsin \frac{u}{a} + C$$

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$$

$$I_{17} = \int \frac{1}{a^2 + u^2} du$$

$$\text{Let } v = \frac{u}{a} \longrightarrow av = u \longrightarrow a dv = du$$

$$= \int \frac{a}{a^2 + (av)^2} dv$$

$$= \int \frac{a}{a^2(1 + v^2)} dv$$

$$= \frac{1}{a} \int \frac{1}{1 + v^2} dv$$

$$= \frac{1}{a} \arctan v$$

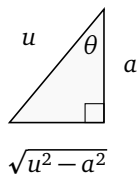
Reverse the substitution of  $u$

$$= \frac{1}{a} \arctan \frac{u}{a} + C$$



$$I_{18} = \int \frac{1}{u\sqrt{u^2 - a^2}} du$$

Let  $\sin \theta = \frac{\sqrt{u^2 - a^2}}{u}$ ,  $\cos \theta = \frac{a}{u} \longrightarrow \tan \theta = \frac{\sqrt{u^2 - a^2}}{a}$



and so  $u = a \sec \theta \longrightarrow du = a \sec \theta \tan \theta d\theta$

and  $\sqrt{u^2 - a^2} = a \tan \theta$

$$\begin{aligned} I_{18} &= \int \frac{a \sec \theta \tan \theta}{a \sec \theta \cdot a \tan \theta} d\theta \\ &= \int \frac{1}{a} d\theta \\ &= \frac{1}{a} \theta + C \end{aligned}$$

Reverse the substitution, where  $u = a \sec \theta \longrightarrow \theta = \operatorname{arcsec} \frac{u}{a}$

$$= \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

$$\begin{aligned}
 I_{19} &= \int \frac{1}{a^2 - u^2} du \\
 &= \int \frac{A}{a + u} + \frac{B}{a - u} du
 \end{aligned}$$

Where  $A(a - u) + B(a + u) = 1 \longrightarrow A + B = \frac{1}{a}$  and  $B - A = 0$   
 $\longrightarrow B = A \longrightarrow 2A = \frac{1}{a} \longrightarrow A = \frac{1}{2a} \longrightarrow B = \frac{1}{2a}$

$$\begin{aligned}
 &= \int \frac{\frac{1}{2a}}{a + u} + \frac{\frac{1}{2a}}{a - u} du \\
 &= \frac{1}{2a} \cdot \int \frac{1}{a + u} + \frac{1}{a - u} du
 \end{aligned}$$

Rearranging so  $u$  precedes  $a$  for aesthetic reasons

$$\begin{aligned}
 &= \frac{1}{2a} \cdot \int \frac{1}{u + a} - \frac{1}{u - a} du \\
 &= \frac{1}{2a} (\ln|u + a| - \ln|u - a|) + C \\
 &= \frac{1}{2a} \cdot \ln \left| \frac{u + a}{u - a} \right| + C
 \end{aligned}$$

$$\int \frac{1}{a^2 - u^2} du = \frac{1}{2a} \ln \left| \frac{u + a}{u - a} \right| + C$$

$$\begin{aligned}
 I_{20} &= \int \frac{1}{u^2 - a^2} du \\
 &= \int \frac{A}{u - a} + \frac{B}{u + a} du
 \end{aligned}$$

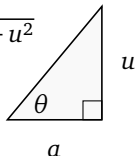
Where  $A(u + a) + B(u - a) = 1 \longrightarrow A + B = 0$  and  $A - B = \frac{1}{a}$   
 $\longrightarrow B = -A \longrightarrow 2A = \frac{1}{a} \longrightarrow A = \frac{1}{2a} \longrightarrow B = -\frac{1}{2a}$

$$\begin{aligned}
 &= \int \frac{\frac{1}{2a}}{u - a} + \frac{-\frac{1}{2a}}{u + a} du \\
 &= \frac{1}{2a} \cdot \int \frac{1}{u - a} - \frac{1}{u + a} du \\
 &= \frac{1}{2a} (\ln|u - a| - \ln|u + a| + C) \\
 &= \frac{1}{2a} \cdot \ln \left| \frac{u - a}{u + a} \right| + C
 \end{aligned}$$

$$\int \frac{1}{u^2 - a^2} du = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C$$

$$I_{21} = \int \sqrt{a^2 + u^2} du$$

Let  $\sin \theta = \frac{u}{\sqrt{a^2 + u^2}}, \cos \theta = \frac{a}{\sqrt{a^2 + u^2}} \rightarrow \tan \theta = \frac{u}{a}$



and so  $u = a \tan \theta \rightarrow du = a \sec^2 \theta \tan \theta d\theta$   
 and  $\sqrt{u^2 + a^2} = a \sec \theta$

$$= a \sec \theta a \sec^2 \theta d\theta$$

$$= a^2 \int \sec^3 \theta d\theta$$

Apply integral identity (71) :

$$\int \sec^3 u du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

$$= a^2 \left( \frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + C \right)$$

$$= \frac{a^2}{2} \frac{\sqrt{u^2 + a^2}}{a} \cdot \frac{u}{a} + \frac{a^2}{2} \ln \left| \frac{\sqrt{u^2 + a^2}}{a} + \frac{u}{a} \right| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln \left| \frac{u + \sqrt{u^2 + a^2}}{a} \right| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| - \frac{a^2}{2} \ln |a| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| + C$$

(21)  $\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| + C$

$$\begin{aligned}
 I_{63} &= \int \sin^2 u \, du \\
 &= \int \frac{1 - \cos 2u}{2} \, du \\
 &= \frac{1}{2} \cdot \int 1 - \cos 2u \, du \\
 &= \frac{1}{2} \cdot \left( u - \frac{1}{2} \sin 2u \right) + C \\
 &= \frac{1}{2} u - \frac{1}{4} \sin 2u + C
 \end{aligned}$$

$$\int \sin^2 u \, du = \frac{1}{2} u - \frac{1}{4} \sin 2u + C$$

$$\begin{aligned}
 I_{64} &= \int \cos^2 u \, du \\
 &= \int \frac{1 + \cos 2u}{2} \, du \\
 &= \frac{1}{2} \cdot \int 1 + \cos 2u \, du \\
 &= \frac{1}{2} \cdot \left( u + \frac{1}{2} \sin 2u \right) + C \\
 &= \frac{1}{2} u + \frac{1}{4} \sin 2u + C
 \end{aligned}$$

$$I_{65} = \int \tan^2 u \, du$$

Rewrite using trigonometric identity  $\tan^2 u = \sec^2 u - 1$

$$\begin{aligned} &= \int \sec^2 u - 1 \, du \\ &= \tan u - u + C \end{aligned}$$

$$\int \tan^2 u \, du = \tan u - u + C$$

$$I_{66} = \int \cot^2 u \, du$$

Rewrite using trigonometric identity  $\tan^2 u = \csc^2 u - 1$

$$\begin{aligned} &= \int \csc^2 u - 1 \, du \\ &= -\cot u - u + C \end{aligned}$$



$$I_{71} = \int \sec^3 u \, du$$

$$\text{Integrate by parts } \begin{cases} (\sec u)' = \sec u \tan u \\ \sec^2 u = (\tan u)' \end{cases}$$

$$= (\sec u)(\tan u) - \int (\sec u \tan u)(\tan u) \, du$$

$$= \sec u \tan u - \int \sec u \tan^2 u \, du$$

$$= \sec u \tan u - \int \sec u (\sec^2 u - 1) \, du$$

$$= \sec u \tan u - \int \sec^3 u \, du + \int \sec u \, du$$

$$2 \int \sec^3 u \, du = \sec u \tan u + \int \sec u \, du$$

$$\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \int \sec u \, du$$

$$= \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

$$\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C \quad (71)$$

$$I_{72} = \int \csc^3 u \, du$$

$$\text{Integrate by parts } \begin{cases} (\csc u)' = -\csc u \cot u \\ \csc^2 u = (-\cot u)' \end{cases}$$

$$= (\csc u)(-\cot u) - \int (-\csc u \cot u)(-\cot u) \, du$$

$$= -\csc u \cot u - \int \csc u \cot^2 u \, du$$

$$= -\csc u \cot u - \int \csc u (\csc^2 u - 1) \, du$$

$$= -\csc u \cot u - \int \csc^3 u \, du + \int \csc u \, du$$

$$2 \int \csc^3 u \, du = -\csc u \cot u + \int \csc u \, du$$

$$\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \int \csc u \, du$$

$$= \frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

$$(72) \quad \int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

$$I_{73} = \int \sin^n u \, du$$

$$\text{Integrate by parts } \begin{cases} (\sin^{n-1} u)' = (n-1) \sin^{n-2} u \cos u \\ \sin u = (-\cos u)' \end{cases}$$

$$= (\sin^{n-1} u)(-\cos u) - \int ((n-1) \sin^{n-2} u \cos u)(-\cos u) \, du$$

$$= -\sin^{n-1} u \cos u + (n-1) \int \sin^{n-2} u \cos^2 u \, du$$

$$= -\sin^{n-1} u \cos u + (n-1) \int \sin^{n-2} u (1 - \sin^2 u) \, du$$

$$= -\sin^{n-1} u \cos u + (n-1) \left( \int \sin^{n-2} u \, du - \int \sin^n u \, du \right)$$

$$1 + (n-1) \int \sin^n u \, du = -\sin^{n-1} u \cos u + (n-1) \int \sin^{n-2} u \, du$$

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{(n-1)}{n} \int \sin^{n-2} u \, du$$

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du \quad (73)$$

$$I_{74} = \int \sin^n u \, du$$

$$\text{Integrate by parts } \begin{cases} (\cos^{n-1} u)' = -(n-1) \cos^{n-2} u \sin u \\ \cos u = (\sin u)' \end{cases}$$

$$= (\cos^{n-1} u)(\sin u) - \int (-(n-1) \cos^{n-2} u \sin u)(\sin u) \, du$$

$$= \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \sin^2 u \, du$$

$$= \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u (1 - \cos^2 u) \, du$$

$$= \cos^{n-1} u \sin u + (n-1) \left( \int \cos^{n-2} u \, du - \int \cos^n u \, du \right)$$

$$1 + (n-1) \int \cos^n u \, du = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \, du$$

$$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{(n-1)}{n} \int \cos^{n-2} u \, du$$

$$(74) \quad \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

$$I_{82} = \int u \sin u \, du$$

$$\text{Integrate by parts} \quad \begin{cases} (u)' = 1 \\ \sin u = (-\cos u)' \end{cases}$$

$$= (u)(-\cos u) - \int (1)(-\cos u) \, du$$

$$= \int \cos u \, du - u \cos u$$

$$= \sin u - u \cos u$$

$$\int u \sin u \, du = \sin u - u \cos u + C$$

$$I_{83} = \int u \cos u \, du$$

$$\text{Integrate by parts } \begin{cases} (u)' = 1 \\ \cos u = (\sin u)' \end{cases}$$

$$= (u)(\sin u) - \int (1)(\sin u) \, du$$

$$= u \sin u - (-\cos u)$$

$$= \cos u + u \sin u$$

$$I_{92} = \int u \arctan u \, du$$

$$\text{Integrate by parts } \begin{cases} u = (\frac{1}{2}u^2)' \\ (\arctan u)' = \frac{1}{1+u^2} \end{cases}$$

$$= (\frac{1}{2}u^2)(\arctan u) - \int (\frac{1}{2}u^2) \left( \frac{1}{1+u^2} \right) du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} \int \frac{u^2 + 1 - 1}{1 + u^2} du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} \int 1 - \frac{1}{1 + u^2} du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} (u - \arctan u) + C$$

$$= \frac{u^2 + 1}{2} \arctan u - \frac{u}{2} + C$$

$$\int u \arctan u \, du = \frac{u^2+1}{2} \arctan u - \frac{u}{2} + C$$

$$I_{96} = \int u e^{au} \, du$$

$$\text{Integrate by parts } \begin{cases} (u)' = 1 \\ e^{au} = \left(\frac{e^{au}}{a}\right)' \end{cases}$$

$$= (u) \left(\frac{e^{au}}{a}\right) - \int (1) \left(\frac{e^{au}}{a}\right) \, du$$

$$= \frac{1}{a} u e^{au} - \frac{1}{a} \int e^{au} \, du$$

$$= \frac{1}{a} u e^{au} - \frac{1}{a^2} e^{au} + C$$

$$= \left(\frac{1}{a} u - \frac{1}{a^2}\right) e^{au} + C$$

Simplify

$$= \frac{1}{a^2} (au - 1) e^{au} + C$$



$$I_{100} = \int \ln u \, du$$

$$\text{Integrate by parts } \begin{cases} 1 = (u)' \\ (\ln u)' = \frac{1}{u} \end{cases}$$

$$= (u)(\ln u) - \int (u) \left( \frac{1}{u} \right) du$$

$$= u \ln u - \int 1 \, du$$

$$= u \ln u - u + C$$

$$\int \ln u \, du = u \ln u - u + C$$

$$I_{102} = \int \frac{1}{u \ln u} du$$

$$\text{Let } v = \ln u \longrightarrow dv = \frac{1}{u} du$$

$$= \int \frac{1}{v} dv$$

$$= \ln v + C$$

Reverse the substitution

$$= \ln |\ln u|$$

$$\int \frac{1}{u \ln u} du = \ln |\ln u| + C$$



12:  $\tan u$

20:  $\frac{1}{u^2 - a^2}$

82:  $u \sin u$

13:  $\cot u$

21:  $\sqrt{a^2 + u^2}$

83:  $u \cos u$

14:  $\sec u$

63:  $\sin^2 u$

92:  $u \arctan u$

15:  $\csc u$

64:  $\cos^2 u$

96:  $ue^{au}$

16:  $\frac{1}{\sqrt{a^2 - u^2}}$

65:  $\tan^2 u$

97:  $ue^{au}$

17:  $\frac{1}{a^2 + u^2}$

66:  $\cot^2 u$

100:  $\ln u$

18:  $\frac{1}{u\sqrt{u^2 - a^2}}$

71:  $\sec^3 u$

102:  $\frac{1}{u \ln u}$

19:  $\frac{1}{a^2 - u^2}$

72:  $\csc^3 u$

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