12: $\tan u$	20: $\dfrac{1}{u^2-a^2}$	82: $u\sin u$
13: $\cot u$	21: $\sqrt{a^2+u^2}$	83: $u\cos u$
14: $\sec u$	63: $\sin^2 u$	92: $u\ \arctan u$
15: $\csc u$	64: $\cos^2 u$	96: ue^{au}
16: $\dfrac{1}{\sqrt{a^2-u^2}}$	65: $\tan^2 u$	97: ue^{au}
17: $\dfrac{1}{a^2+u^2}$	66: $\cot^2 u$	100: $\ln u$
18: $\dfrac{1}{u\sqrt{u^2-a^2}}$	71: $\sec^3 u$	102: $\dfrac{1}{u\ln u}$
19: $\dfrac{1}{a^2-u^2}$	72: $\csc^3 u$	

1. $\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u$

2. $\int u^n\,\mathrm{d}u = \dfrac{u^{n+1}}{n+1} + C,\quad n \neq -1$

3. $\int \frac{1}{u}\,\mathrm{d}u = \ln|u| + C$

4. $\int e^u\,\mathrm{d}u = e^u + C$

5. $\int a^u\,\mathrm{d}u = \dfrac{a^u}{\ln a} + C$

6. $\int \sin u\,\mathrm{d}u = -\cos u + C$

7. $\int \cos u\,\mathrm{d}u = \sin u + C$

8. $\int \sec^2 u\,\mathrm{d}u = \tan u + C$

9. $\int \csc^2 u\,\mathrm{d}u = -\cot u + C$

10. $\int \sec u \tan u\,\mathrm{d}u = \sec u + C$

11. $\int \csc u \cot u\,\mathrm{d}u = -\csc u + C$

103. $\int \sinh u\,\mathrm{d}u = \cosh u + C$

104. $\int \cosh u\,\mathrm{d}u = \sinh u + C$

These integrals can be used without explanation

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$$I_{12} = \int \tan u \, \mathrm{d}u$$

$$= \int \frac{\sin u}{\cos u} \, \mathrm{d}u$$

Let $\quad v = \cos u \quad \longrightarrow \quad \mathrm{d}v = -\sin u \, \mathrm{d}u \quad \longrightarrow \quad -\mathrm{d}v = \sin u \, \mathrm{d}u$

$$= -\int \frac{1}{v} \, \mathrm{d}v$$

$$= -\ln|v| + C$$

Reverse the substitution

$$= -\ln|\cos u| + C$$

$$\int \tan u \, \mathrm{d}u = \ln|\cos u| + C$$

$$I_{13} = \int \cot u \, \mathrm{d}u$$

$$= \int \frac{\cos u}{\sin u} \, \mathrm{d}u$$

Let $\quad v = \sin u \quad \longrightarrow \quad \mathrm{d}v = \cos u \, \mathrm{d}u$

$$= \int \frac{1}{v} \, \mathrm{d}v$$

$$= \ln|v| + C$$

Reverse the substitution

$$= \ln|\sin u| + C$$

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13

$$\int \cot u \, \mathrm{d}u = \ln|\sin u| + C$$

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$$
\begin{aligned}
I_{14} &= \int \sec u \,\mathrm{d}u \\
&= \int \sec u \cdot \frac{\sec u + \tan u}{\sec u + \tan u} \,\mathrm{d}u \\
&= \int \frac{\sec^2 u + \sec u \cdot \tan u}{\sec u + \tan u} \,\mathrm{d}u
\end{aligned}
$$

Let $v = \sec u + \tan u \longrightarrow \mathrm{d}v = \sec^2 u + \sec u \tan u \,\mathrm{d}u$

$$
\begin{aligned}
&= \int \frac{\mathrm{d}v}{v} \\
&= \ln|v| + C
\end{aligned}
$$

Reverse the substitution

$$= \ln|\sec u + \tan u| + C$$

$$\int \sec u \,\mathrm{d}u = \ln|\sec u + \tan u| + C$$

$$
\begin{aligned}
I_{15} &= \int \csc u \, \mathrm{d}u \\
&= \int \csc u \cdot \frac{\csc u - \cot u}{\csc u - \cot u} \, \mathrm{d}u \\
&= \int \frac{\csc^2 u - \csc u \cdot \cot u}{\csc u - \cot u} \, \mathrm{d}u
\end{aligned}
$$

Let $\quad v = \csc u - \cot u \quad \longrightarrow \quad \mathrm{d}v = \csc^2 u - \csc u \cot u \, \mathrm{d}u$

$$
\begin{aligned}
&= \int \frac{\mathrm{d}v}{v} \\
&= \ln|v| + C
\end{aligned}
$$

Reverse the substitution

$$
= \ln|\csc u - \cot u| + C
$$

15 $\displaystyle\int \csc u \, \mathrm{d}u = \ln|\csc u - \cot u| + C$

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$$I_{102} = \int \frac{1}{u \ln u} \, \mathrm{d}u$$

Let $\quad v = \ln u \quad \longrightarrow \quad \mathrm{d}v = \frac{1}{u}\,\mathrm{d}u$

$$= \int \frac{1}{v}\,\mathrm{d}v$$

$$= \ln v + C$$

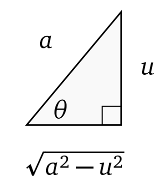
Reverse the substitution

$$= \ln|\ln u|$$

102 $\quad \int \frac{1}{u \ln u}\,\mathrm{d}u \;=\; \ln|\ln u| + C$

$$I_{16} = \int \frac{1}{\sqrt{a^2 - u^2}} \, \mathrm{d}u$$

Let $\quad \sin\theta = \frac{u}{a}, \;\; \cos\theta = \frac{\sqrt{a^2-u^2}}{a} \quad \longrightarrow \quad \tan\theta = \frac{u}{\sqrt{a^2-u^2}}$



and so $\quad u = a\sin\theta \quad \longrightarrow \quad \mathrm{d}u = a\cos\theta\,\mathrm{d}\theta$

and $\quad \sqrt{a^2-u^2} = a\cos\theta$

$$= \int \frac{a\cos\theta}{a\cos\theta}\,\mathrm{d}\theta$$

$$= \int 1\,\mathrm{d}\theta$$

$$= \theta + C$$

Reverse the substitution of u, where $\quad \sin\theta = \frac{u}{a} \longrightarrow \theta = \arcsin\frac{u}{a}$

$$= \arcsin\frac{u}{a} + C$$

$\int \frac{1}{\sqrt{a^2-u^2}}\,\mathrm{d}u \;=\; \arcsin\frac{u}{a} + C$ $\quad$ 16

$$I_{17} = \int \frac{1}{a^2 + u^2}\, \mathrm{d}u$$

Let $v = \frac{u}{a} \longrightarrow av = u \longrightarrow a\,\mathrm{d}v = \mathrm{d}u$

$$= \int \frac{a}{a^2 + (av)^2}\, \mathrm{d}v$$

$$= \int \frac{a}{a^2(1 + v^2)}\, \mathrm{d}v$$

$$= \frac{1}{a} \int \frac{1}{1 + v^2}\, \mathrm{d}v$$

$$= \frac{1}{a} \arctan v$$

Reverse the substitution of u

$$= \frac{1}{a} \arctan \frac{u}{a} + C$$

$$I_{100} = \int \ln u\, \mathrm{d}u$$

Integrate by parts $\begin{cases} 1 = (u)' \\ (\ln u)' = \frac{1}{u} \end{cases}$

$$= (u)(\ln u) - \int (u)\left(\frac{1}{u}\right) \mathrm{d}u$$

$$= u \ln u - \int 1\, \mathrm{d}u$$

$$= u \ln u - u + C$$

17 $\int \frac{1}{a^2+u^2}\, \mathrm{d}u = \frac{1}{a} \arctan \frac{u}{a} + C$

$\int \ln u\ \mathrm{d}u = u \ln u - u + C$ 100

$$I_{97} = \int u^n e^{au}\, \mathrm{d}u$$

Integrate by parts $\begin{cases} (u^n)' = n \cdot u^{n-1} \\ e^{au} = \left(\dfrac{e^{au}}{a}\right)' \end{cases}$

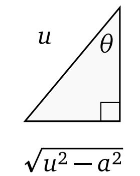
$$= (u^n)\left(\frac{e^{au}}{a}\right) - \int \left(n \cdot u^{n-1}\right)\left(\frac{e^{au}}{a}\right) \mathrm{d}u$$

$$= \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au}\, \mathrm{d}u$$

(97) $\int u^n e^{au}\, \mathrm{d}u = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au}\, \mathrm{d}u$

$$I_{18} = \int \frac{1}{u\sqrt{u^2 - a^2}}\, \mathrm{d}u$$

Let $\sin\theta = \dfrac{\sqrt{u^2 - a^2}}{u}$, $\cos\theta = \dfrac{a}{u} \longrightarrow \tan\theta = \dfrac{\sqrt{u^2 - a^2}}{a}$



and so $u = a\sec\theta \longrightarrow \mathrm{d}u = a\sec\theta\tan\theta\, \mathrm{d}\theta$

and $\sqrt{u^2 - a^2} = a\tan\theta$

$$I_{18} = \int \frac{a\sec\theta\tan\theta}{a\sec\theta \cdot a\tan\theta}\, \mathrm{d}\theta$$

$$= \int \frac{1}{a}\, \mathrm{d}\theta$$

$$= \frac{1}{a}\theta + C$$

Reverse the substitution, where $u = a\sec\theta \longrightarrow \theta = \operatorname{arcsec}\dfrac{u}{a}$

$$= \frac{1}{a} \operatorname{arcsec}\frac{u}{a} + C$$

(18) $\int \frac{1}{u\sqrt{u^2 - a^2}}\, \mathrm{d}u = \frac{1}{a} \operatorname{arcsec}\frac{u}{a} + C$

$$I_{19} = \int \frac{1}{a^2 - u^2}\,\mathrm{d}u$$

$$= \int \frac{A}{a+u} + \frac{B}{a-u}\,\mathrm{d}u$$

Where $A(a-u)+B(a+u)=1 \longrightarrow A+B=\frac{1}{a}$ and $B-A=0$
$\longrightarrow B=A \longrightarrow 2A=\frac{1}{a} \longrightarrow A=\frac{1}{2a} \longrightarrow B=\frac{1}{2a}$

$$= \int \frac{\frac{1}{2a}}{a+u} + \frac{\frac{-1}{2a}}{a-u}\,\mathrm{d}u$$

$$= \frac{1}{2a} \cdot \int \frac{1}{a+u} + \frac{1}{a-u}\,\mathrm{d}u$$

Rearranging so u preceeds a for aesthetic reasons

$$= \frac{1}{2a} \cdot \int \frac{1}{u+a} - \frac{1}{u-a}\,\mathrm{d}u$$

$$= \frac{1}{2a}\left(\ln|u+a| - \ln|u-a|\right) + C$$

$$= \frac{1}{2a} \cdot \ln\left|\frac{u+a}{u-a}\right| + C$$

$$I_{96} = \int u e^{au}\,\mathrm{d}u$$

Integrate by parts $\begin{cases} (u)' = 1 \\ e^{au} = \left(\frac{e^{au}}{a}\right)' \end{cases}$

$$= (u)\left(\frac{e^{au}}{a}\right) - \int (1)\left(\frac{e^{au}}{a}\right)\mathrm{d}u$$

$$= \frac{1}{a} u e^{au} - \frac{1}{a}\int e^{au}\,\mathrm{d}u$$

$$= \frac{1}{a} u e^{au} - \frac{1}{a^2} e^{au} + C$$

$$= \left(\frac{1}{a}u - \frac{1}{a^2}\right) e^{au} + C$$

Simplify

$$= \frac{1}{a^2}(au-1)\,e^{au} + C$$

$$I_{92} = \int u \arctan u \, \mathrm{d}u$$

Integrate by parts $\begin{cases} u = \left(\frac{1}{2}u^2\right)' \\ (\arctan u)' = \frac{1}{1+u^2} \end{cases}$

$$= \left(\tfrac{1}{2}u^2\right)(\arctan u) - \int \left(\tfrac{1}{2}u^2\right)\left(\frac{1}{1+u^2}\right) \mathrm{d}u$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} \int \frac{u^2+1-1}{1+u^2} \, \mathrm{d}u$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} \int 1 - \frac{1}{1+u^2} \, \mathrm{d}u$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} (u - \arctan u) + C$$

$$= \frac{u^2+1}{2} \arctan u - \frac{u}{2} + C$$

92 $\int u \arctan u \ \mathrm{d}u = \frac{u^2+1}{2} \arctan u - \frac{u}{2} + C$

$$I_{20} = \int \frac{1}{u^2 - a^2} \, \mathrm{d}u$$

$$= \int \frac{A}{u-a} + \frac{B}{u+a} \, \mathrm{d}u$$

Where $A(u+a) + B(u-a) = 1 \longrightarrow A + B = 0$ and $A - B = \frac{1}{a}$
$\longrightarrow B = -A \longrightarrow 2A = \frac{1}{a} \longrightarrow A = \frac{1}{2a} \longrightarrow B = -\frac{1}{2a}$

$$= \int \frac{\frac{1}{2a}}{u-a} + \frac{-\frac{1}{2a}}{u+a} \, \mathrm{d}u$$

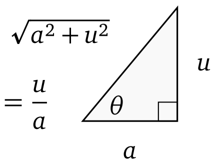
$$= \frac{1}{2a} \cdot \int \frac{1}{u-a} - \frac{1}{u+a} \, \mathrm{d}u$$

$$= \frac{1}{2a} \left(\ln|u-a| - \ln|u+a| + C \right)$$

$$= \frac{1}{2a} \cdot \ln\left|\frac{u-a}{u+a}\right| + C$$

$\int \frac{1}{u^2-a^2} \, \mathrm{d}u = \frac{1}{2a} \ln\left|\frac{u-a}{u+a}\right| + C$ 20

$$I_{21} = \int \sqrt{a^2+u^2}\,\mathrm{d}u$$



Let $\sin\theta = \dfrac{u}{\sqrt{a^2+u^2}}$, $\cos\theta = \dfrac{a}{\sqrt{a^2+u^2}}$ $\longrightarrow$ $\tan\theta = \dfrac{u}{a}$

and so $u = a\tan\theta \longrightarrow \mathrm{d}u = a\sec\theta\tan\theta\,\mathrm{d}\theta$

and $\sqrt{u^2+a^2} = a\sec\theta$

$$= a\sec\theta\, a\sec^2\theta\,\mathrm{d}\theta$$

$$= a^2 \int \sec^3\theta\,\mathrm{d}\theta$$

Apply integal identity 71 :

$$\int \sec^3 u\,\mathrm{d}u = \frac{1}{2}\sec u\tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$$

$$= a^2\left(\tfrac{1}{2}\sec\theta\tan\theta + \tfrac{1}{2}\ln|\sec\theta+\tan\theta| + C\right)$$

$$= \frac{a^2}{2}\frac{\sqrt{u^2+a^2}}{a}\cdot\frac{u}{a} + \frac{a^2}{2}\ln\left|\frac{\sqrt{u^2+a^2}}{a} + \frac{u}{a}\right| + C$$

$$= \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left|\frac{u+\sqrt{u^2+a^2}}{a}\right| + C$$

$$= \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2+a^2}\right| - \frac{a^2}{2}\ln|a| + C$$

$$= \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2+a^2}\right| + C$$

21 $\int\sqrt{a^2+u^2}\,\mathrm{d}u = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$

$$I_{83} = \int u\cos u\,\mathrm{d}u$$

Integrate by parts $\begin{cases} (u)' = 1 \\ \cos u = (\sin u)' \end{cases}$

$$= (u)(\sin u) - \int (1)(\sin u)\,\mathrm{d}u$$

$$= u\sin u - (-\cos u)$$

$$= \cos u + u\sin u$$

$\int u\cos u\;\mathrm{d}u = \cos u + u\sin u + C$ 83

$$I_{82} = \int u \sin u \, \mathrm{d}u$$

Integrate by parts $\begin{cases} (u)' & = 1 \\ \sin u & = (-\cos u)' \end{cases}$

$$= (u)(-\cos u) - \int (1)(-\cos u)\, \mathrm{d}u$$

$$= \int \cos u \, \mathrm{d}u - u \cos u$$

$$= \sin u - u \cos u$$

$$I_{63} = \int \sin^2 u \, \mathrm{d}u$$

$$= \int \frac{1-\cos 2u}{2} \, \mathrm{d}u$$

$$= \frac{1}{2} \cdot \int 1 - \cos 2u \, \mathrm{d}u$$

$$= \frac{1}{2} \cdot \left(u - \frac{1}{2} \sin 2u\right) + C$$

$$= \tfrac{1}{2} u - \tfrac{1}{4} \sin 2u + C$$

82 $\int u \sin u \; \mathrm{d}u \;=\; \sin u - u \cos u + C$

$\int \sin^2 u \; \mathrm{d}u \;=\; \tfrac{1}{2} u - \tfrac{1}{4} \sin 2u + C$ 63

$$
\begin{aligned}
I_{64} &= \int \cos^2 u \,\mathrm{d}u \\
&= \int \frac{1+\cos 2u}{2}\,\mathrm{d}u \\
&= \frac{1}{2} \cdot \int 1 + \cos 2u \,\mathrm{d}u \\
&= \frac{1}{2} \cdot \left(u + \frac{1}{2}\sin 2u \right) + C \\
&= \tfrac{1}{2}u + \tfrac{1}{4}\sin 2u + C
\end{aligned}
$$

$$
I_{77} = \int \sec^n u \,\mathrm{d}u
$$

Integrate by parts $\left\{ \begin{aligned} \sec^2 u &= (\tan u)' \\ \left(\sec^{n-2} u\right)' &= (n-2)\sec^{n-3} u \sec u \tan u \end{aligned} \right.$

$$
\begin{aligned}
&= (\tan u)\left(\sec^{n-2} u\right) - \int (\tan u)\left((n-2)\sec^{n-3} u \sec u \tan u\right)\mathrm{d}u \\
&= \tan u \ \sec^{n-2} u - (n-2)\int \sec^{n-2} \tan^2 u \,\mathrm{d}u \\
&= \tan u \ \sec^{n-2} u - (n-2)\int \sec^{n-2}(\sec^2 u - 1)\,\mathrm{d}u \\
&= \tan u \ \sec^{n-2} u - (n-2)\left(\int \sec^n u \,\mathrm{d}u - \int \sec^{n-2} \mathrm{d}u \right)
\end{aligned}
$$

$$
\begin{aligned}
(1+(n-2))\int \sec^n u \,\mathrm{d}u &= \tan u \ \sec^{n-2} u - (n-2)\int \sec^{n-2} \mathrm{d}u \\
\int \sec^n u \,\mathrm{d}u &= \frac{1}{n-1}\tan u \ \sec^{n-2} u - \frac{n-2}{n-1}\int \sec^{n-2} \mathrm{d}u
\end{aligned}
$$

64 $\int \cos^2 u \ \mathrm{d}u \ = \ \frac{1}{2}u \ + \ \frac{1}{4}\sin 2u \ + \ C$

$\int \sec^n u \ \mathrm{d}u \ = \ \frac{1}{n-1}\tan u \sec^{n-2} u \ + \ \frac{n-2}{n-1}\int \sec^{n-2} u \ \mathrm{d}u$ 77

$$\begin{aligned}
I_{76} &= \int \cot^n u \, \mathrm{d}u \\
&= \int \cot^{n-2} u \cot^2 u \, \mathrm{d}u \\
&= \int \cot^{n-2} u (\csc^2 u - 1) \, \mathrm{d}u \\
&= \int \cot^{n-2} u \ \csc^2 u \, \mathrm{d}u - \int \cot^{n-2} u \, \mathrm{d}u
\end{aligned}$$

Let $\quad v = \cot u \quad \longrightarrow \quad \mathrm{d}v = -\csc^2 u \, \mathrm{d}u$

$$\begin{aligned}
&= - \int v^{n-2} u \, \mathrm{d}v - \int \cot^{n-2} u \\
&= \frac{-1}{n-1} v^{n-1} - \int \cot^{n-2} u \\
&= \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u
\end{aligned}$$

(76) $\int \cot^n u \ \mathrm{d}u \ = \ \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \ \mathrm{d}u$

$$I_{65} = \int \tan^2 u \, \mathrm{d}u$$

Rewrite using trigonometric identity $\tan^2 u = \sec^2 u - 1$

$$\begin{aligned}
&= \int \sec^2 u - 1 \, \mathrm{d}u \\
&= \tan u - u + C
\end{aligned}$$

$\int \tan^2 u \ \mathrm{d}u \ = \ \tan u \ - \ u \ + \ C$ (65)

$$I_{66} = \int \cot^2 u \, \mathrm{d}u$$

Rewrite using trigonometric identity $\tan^2 u = \csc^2 u - 1$

$$= \int \csc^2 u - 1 \, \mathrm{d}u$$

$$= -\cot u - u + C$$

66 $\int \cot^2 u \; \mathrm{d}u \;=\; -\cot u - u + C$

$$I_{75} = \int \tan^n u \, \mathrm{d}u$$

$$= \int \tan^{n-2} u \tan^2 u \, \mathrm{d}u$$

$$= \int \tan^{n-2} u (\sec^2 u - 1) \, \mathrm{d}u$$

$$= \int \tan^{n-2} u \, \sec^2 u \, \mathrm{d}u - \int \tan^{n-2} u \, \mathrm{d}u$$

Let $v = \tan u \quad \longrightarrow \quad \mathrm{d}v = \sec^2 u \, \mathrm{d}u$

$$= \int v^{n-2} u \, \mathrm{d}v - \int \tan^{n-2} u$$

$$= \frac{1}{n-1} v^{n-1} - \int \tan^{n-2} u$$

$$= \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u$$

$\int \tan^n u \; \mathrm{d}u \;=\; \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \; \mathrm{d}u$ 75

$$I_{74} = \int \cos^n u \, \mathrm{d}u$$

Integrate by parts $\begin{cases} \left(\cos^{n-1} u\right)' = -(n-1)\cos^{n-2} u \sin u \\ \cos u = (\sin u)' \end{cases}$

$$= \left(\cos^{n-1} u\right)(\sin u) - \int \left(-(n-1)\cos^{n-2} u \sin u\right)(\sin u)\, \mathrm{d}u$$

$$= \cos^{n-1} u \, \sin u + (n-1) \int \cos^{n-2} u \sin^2 u \, \mathrm{d}u$$

$$= \cos^{n-1} u \, \sin u + (n-1) \int \cos^{n-2} u (1-\cos^2 u)\, \mathrm{d}u$$

$$= \cos^{n-1} u \, \sin u + (n-1) \left(\int \cos^{n-2} u \, \mathrm{d}u - \int \cos^n u \, \mathrm{d}u \right)$$

$$1 + (n-1) \int \cos^n u \, \mathrm{d}u = \cos^{n-1} u \, \sin u + (n-1) \int \cos^{n-2} u \, \mathrm{d}u$$

$$\int \cos^n u \, \mathrm{d}u = \frac{1}{n} \cos^{n-1} u \, \sin u + \frac{(n-1)}{n} \int \cos^{n-2} u \, \mathrm{d}u$$

$$I_{67} = \int \sin^3 u \, \mathrm{d}u$$

$$= \int (1-\cos^2 u) \sin u \, \mathrm{d}u$$

$$= \int \sin u \, \mathrm{d}u - \int \cos^2 u \, \sin u \, \mathrm{d}u$$

Let $v = \cos u \longrightarrow \mathrm{d}v = -\sin u \, \mathrm{d}u$

$$= \int v^2 \, \mathrm{d}v - \cos u$$

$$= \frac{1}{3} v^3 - \cos u + C$$

Reverse the substitution

$$= \frac{1}{3} \cos^3 u - \cos u + C$$

(which we could leave right there)

$$= \left(\frac{1}{3} \cos^2 u - 1 \right) \cos u + C$$

$$= \left(\frac{1}{3} (1 - \sin^2 u) - 1 \right) \cos u + C$$

$$= -\frac{1}{3} \left(2 + \sin^2 u\right) \cos u + C$$

$$I_{68} = \int \cos^3 u \, \mathrm{d}u$$

$$= \int (1 - \sin^2 u) \cos u \, \mathrm{d}u$$

$$= \int \cos u \, \mathrm{d}u - \int \sin^2 u \, \cos u \, \mathrm{d}u$$

Let $v = \sin u \longrightarrow \mathrm{d}v = \cos u \, \mathrm{d}u$

$$= \sin u - \int v^2 \, \mathrm{d}v$$

$$= \sin u - \frac{1}{3} v^3 + C$$

Reverse the substitution

$$= \sin u - \frac{1}{3} \sin^3 u + C$$

(which we could leave right there)

$$= \left(1 - \frac{1}{3} \sin^2 u\right) \sin u + C$$

$$= \left(1 - \frac{1}{3}(1 - \cos^2 u)\right) \sin u + C$$

$$= \frac{1}{3} \left(2 + \cos^2 u\right) \sin u + C$$

68 $\int \cos^3 u \ \mathrm{d}u \ = \ \frac{1}{3}(2 + \cos^2 u) \sin u \ + \ C$

$$I_{73} = \int \sin^n u \, \mathrm{d}u$$

Integrate by parts $\begin{cases} \left(\sin^{n-1} u\right)' = (n-1) \sin^{n-2} u \cos u \\ \sin u = (-\cos u)' \end{cases}$

$$= \left(\sin^{n-1} u\right)(-\cos u) - \int \left((n-1) \sin^{n-2} u \cos u\right)(-\cos u) \, \mathrm{d}u$$

$$= -\sin^{n-1} u \, \cos u + (n-1) \int \sin^{n-2} u \cos^2 u \, \mathrm{d}u$$

$$= -\sin^{n-1} u \, \cos u + (n-1) \int \sin^{n-2} u (1 - \sin^2 u) \, \mathrm{d}u$$

$$= -\sin^{n-1} u \, \cos u + (n-1) \left(\int \sin^{n-2} u \, \mathrm{d}u - \int \sin^n u \, \mathrm{d}u \right)$$

$$1 + (n-1) \int \sin^n u \, \mathrm{d}u = -\sin^{n-1} u \, \cos u + (n-1) \int \sin^{n-2} u \, \mathrm{d}u$$

$$\int \sin^n u \, \mathrm{d}u = -\frac{1}{n} \sin^{n-1} u \, \cos u + \frac{(n-1)}{n} \int \sin^{n-2} u \, \mathrm{d}u$$

$\int \sin^n u \ \mathrm{d}u \ = \ -\frac{1}{n} \sin^{n-1} u \ \cos u \ + \ \frac{n-1}{n} \int \sin^{n-2} u \ \mathrm{d}u$ 73

$$I_{72} = \int \csc^3 u \, du$$

Integrate by parts $\begin{cases} (\csc u)' = -\csc u \cot u \\ \csc^2 u = (-\cot u)' \end{cases}$

$$= (\csc u)(-\cot u) - \int (-\csc u \cot u)(-\cot u) \, du$$

$$= -\csc u \cot u - \int \csc u \cot^2 du$$

$$= -\csc u \cot u - \int \csc u (\csc^2 u - 1) \, du$$

$$= -\csc u \cot u - \int \csc^3 u \, du + \int \csc u \, du$$

$$2 \int \csc^3 u \, du = -\csc u \cot u + \int \csc u \, du$$

$$\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \int \csc u \, du$$

$$= \frac{1}{2} \csc u \cot u + \frac{1}{2} \ln|\csc u - \cot u| + C$$

72 $\int \csc^3 u \; du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln|\csc u - \cot u| + C$

$$I_{69} = \int \tan^3 u \, du$$

$$= \int (\sec^2 u - 1) \tan u \, du$$

$$= \int (\sec^2 u \tan u) \, du - \int \tan u \, du$$

Let $v = \tan u \longrightarrow dv = \sec^2 u \, du$

$$= \int v \, dv - \ln|\sec u|$$

$$= \frac{1}{2} v^2 - \ln|\sec u| + C$$

Reverse the substitution

$$= \frac{1}{2} \tan^2 u - \ln|\sec u| + C$$

(which we could leave right there)

$$= \frac{1}{2} \tan^2 u + \ln|\cos u| + C$$

$\int \tan^3 u \; du = \frac{1}{2} \tan^2 u + \ln|\cos u| + C$ 69

$$I_{70} = \int \cot^3 u \, \mathrm{d}u$$

$$= \int (\csc^2 u - 1) \cot u \, \mathrm{d}u$$

$$= \int (\csc^2 u \, \cot u) \, \mathrm{d}u - \int \cot u \, \mathrm{d}u$$

Let $v = \cot u \quad \longrightarrow \quad \mathrm{d}v = -\csc^2 u \, \mathrm{d}u$

$$= -\int v \, \mathrm{d}v - \ln|\sin u|$$

$$= -\frac{1}{2} v^2 - \ln|\sin u| + C$$

Reverse the substitution

$$= -\frac{1}{2} \cot^2 u - \ln|\sin u| + C$$

(which we could leave right there)

$$= -\frac{1}{2} \cot^2 u - \ln|\sin u| + C$$

$$I_{71} = \int \sec^3 u \, \mathrm{d}u$$

Integrate by parts $\begin{cases} (\sec u)' = \sec u \tan u \\ \sec^2 u = (\tan u)' \end{cases}$

$$= (\sec u)(\tan u) - \int (\sec u \tan u)(\tan u) \, \mathrm{d}u$$

$$= \sec u \tan u - \int \sec u \tan^2 \mathrm{d}u$$

$$= \sec u \tan u - \int \sec u (\sec^2 u - 1) \, \mathrm{d}u$$

$$= \sec u \tan u - \int \sec^3 u \, \mathrm{d}u + \int \sec u \, \mathrm{d}u$$

$$2 \int \sec^3 u \, \mathrm{d}u = \sec u \tan u + \int \sec u \, \mathrm{d}u$$

$$\int \sec^3 u \, \mathrm{d}u = \frac{1}{2} \sec u \tan u + \frac{1}{2} \int \sec u \, \mathrm{d}u$$

$$= \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln|\sec u + \tan u| + C$$

$\int \cot^3 u \, \mathrm{d}u = -\frac{1}{2} \cot^2 u - \ln|\sin u| + C$

$\int \sec^3 u \, \mathrm{d}u = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln|\sec u + \tan u| + C$