$$\int$$

$$1: \quad \int u \ \mathrm{d}v \ = \ uv - \int v \ \mathrm{d}u$$

$$2: \quad \int u^n \ \mathrm{d}u \ = \ \frac{u^{n+1}}{n+1} \ + \ C, \ n \neq -1$$

$$3: \quad \int \frac{1}{u} \ \mathrm{d}u \ = \ \ln|u| \ + \ C$$

$$4: \quad \int e^u \ \mathrm{d}u \ = \ e^u \ + \ C$$

$$5: \quad \int a^u \ \mathrm{d}u \ = \ \frac{a^u}{\ln a} \ + \ C$$

$$6: \quad \int \sin u \ \mathrm{d}u \ = \ -\cos u \ + \ C$$

$$7: \quad \int \cos u \ \mathrm{d}u \ = \ \sin u \ + \ C$$

$$8: \quad \int \sec^2 u \ \mathrm{d}u \ = \ \tan u \ + \ C$$

$$9: \quad \int \csc^2 u \ \mathrm{d}u \ = \ -\cot u \ + \ C$$

$$10: \quad \int \sec u \tan u \ \mathrm{d}u \ = \ \sec u \ + \ C$$

$$11: \quad \int \csc u \cot u \ \mathrm{d}u \ = \ -\csc u \ + \ C$$

$$103: \quad \int \sinh u \ \mathrm{d}u \ = \ \cosh u \ + \ C$$

$$104: \quad \int \cosh u \ \mathrm{d}u \ = \ \sinh u \ + \ C$$

$$I_{12} = \int \tan u \, du$$

$$= \int \frac{\sin u}{\cos u} \, du$$

Let $\quad v = \cos u \quad \longrightarrow \quad dv = -\sin u \, du \quad \longrightarrow \quad -dv = \sin u \, du$

$$= -\int \frac{1}{v} \, dv$$

$$= -\ln|v| + C$$

Reverse the substitution

$$= -\ln|\cos u| + C$$

$$\int \tan u \, du = \ln|\cos u| + C \qquad \qquad (12)$$

$$I_{13} = \int \cot u \, du$$

$$= \int \frac{\cos u}{\sin u} \, du$$

Let $v = \sin u \longrightarrow dv = \cos u \, du$

$$= \int \frac{1}{v} \, dv$$

$$= \ln|v| + C$$

Reverse the substitution

$$= \ln|\sin u| + C$$

(13) $\qquad \int \cot u \, du = \ln|\sin u| + C$

$$I_{14} = \int \sec u \, du$$

$$= \int \sec u \cdot \frac{\sec u + \tan u}{\sec u + \tan u} \, du$$

$$= \int \frac{\sec^2 u + \sec u \cdot \tan u}{\sec u + \tan u} \, du$$

Let $\quad v = \sec u + \tan u \quad \longrightarrow \quad dv = \sec^2 u + \sec u \tan u \, du$

$$= \int \frac{dv}{v}$$

$$= \ln|v| + C$$

Reverse the substitution

$$= \ln|\sec u + \tan u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C \qquad \text{(14)}$$

$$I_{15} = \int \csc u \, du$$

$$= \int \csc u \cdot \frac{\csc u - \cot u}{\csc u - \cot u} \, du$$

$$= \int \frac{\csc^2 u - \csc u \cdot \cot u}{\csc u - \cot u} \, du$$

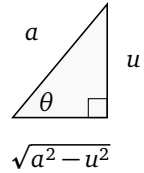Let $v = \csc u - \cot u \longrightarrow dv = \csc^2 u - \csc u \cot u \, du$

$$= \int \frac{dv}{v}$$

$$= \ln|v| + C$$

Reverse the substitution

$$= \ln|\csc u - \cot u| + C$$

(15) $\qquad\qquad \int \csc u \, du = \ln|\csc u - \cot u| + C$

$$I_{16} = \int \frac{1}{\sqrt{a^2 - u^2}}\, du$$

Let $\sin\theta = \dfrac{u}{a}, \quad \cos\theta = \dfrac{\sqrt{a^2 - u^2}}{a} \quad \longrightarrow \quad \tan\theta = \dfrac{u}{\sqrt{a^2 - u^2}}$

and so $\quad u = a\sin\theta \quad \longrightarrow \quad du = a\cos\theta\, d\theta$

and $\quad \sqrt{a^2 - u^2} = a\cos\theta$

$$= \int \frac{a\cos\theta}{a\cos\theta}\, d\theta$$

$$= \int 1\, d\theta$$

$$= \theta + C$$

Reverse the substitution of u, where $\quad \sin\theta = \dfrac{u}{a} \longrightarrow \theta = \arcsin\dfrac{u}{a}$

$$= \arcsin\frac{u}{a} + C$$

If $a = 1$ this solution can be used with explanation.

$$\int \frac{1}{\sqrt{a^2 - u^2}}\, du \;=\; \arcsin\frac{u}{a} + C \qquad (16)$$

$$I_{17} = \int \frac{1}{a^2 + u^2}\, du$$

$$\text{Let } v = \frac{u}{a} \quad \longrightarrow \quad av = u \quad \longrightarrow \quad a\, dv = du$$

$$= \int \frac{a}{a^2 + (av)^2}\, dv$$

$$= \int \frac{a}{a^2\left(1 + v^2\right)}\, dv$$

$$= \frac{1}{a} \int \frac{1}{1 + v^2}\, dv$$

$$= \frac{1}{a} \arctan v$$

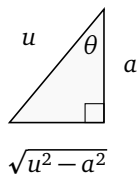Reverse the substitution of u

$$= \frac{1}{a} \arctan \frac{u}{a} + C$$

If $a = 1$ this solution can be used with explanation.

(17)
$$\int \frac{1}{a^2+u^2}\, du = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$I_{18} = \int \frac{1}{u\sqrt{u^2 - a^2}}\, du$$

Let $\sin\theta = \dfrac{\sqrt{u^2 - a^2}}{u}$, $\cos\theta = \dfrac{a}{u}$ $\longrightarrow$ $\tan\theta = \dfrac{\sqrt{u^2 - a^2}}{a}$

and so $u = a\sec\theta$ $\longrightarrow$ $du = a\sec\theta\tan\theta\, d\theta$

and $\sqrt{u^2 - a^2} = a\tan\theta$

$$I_{18} = \int \frac{a\sec\theta\tan\theta}{a\sec\theta \cdot a\tan\theta}\, d\theta$$

$$= \int \frac{1}{a}\, d\theta$$

$$= \frac{1}{a}\theta + C$$

Reverse the substitution, where $u = a\sec\theta$ $\longrightarrow$ $\theta = \operatorname{arcsec}\dfrac{u}{a}$

$$= \frac{1}{a}\operatorname{arcsec}\frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2-a^2}}\, du \;=\; \frac{1}{a}\operatorname{arcsec}\frac{u}{a} \;+\; C \qquad (18)$$

$$I_{19} = \int \frac{1}{a^2 - u^2} \, du$$

$$= \int \frac{A}{a+u} + \frac{B}{a-u} \, du$$

Where $A(a-u) + B(a+u) = 1 \quad \longrightarrow \quad A+B = \frac{1}{a}$ and $B-A=0$

$\longrightarrow \quad B=A \quad \longrightarrow \quad 2A = \frac{1}{a} \quad \longrightarrow \quad A = \frac{1}{2a} \quad \longrightarrow \quad B = \frac{1}{2a}$

$$= \int \frac{\frac{1}{2a}}{a+u} + \frac{\frac{-1}{2a}}{a-u} \, du$$

$$= \frac{1}{2a} \cdot \int \frac{1}{a+u} + \frac{1}{a-u} \, du$$

Rearranging so u preceeds a for aesthetic reasons

$$= \frac{1}{2a} \cdot \int \frac{1}{u+a} - \frac{1}{u-a} \, du$$

$$= \frac{1}{2a} \left(\ln|u+a| - \ln|u-a| \right) + C$$

$$= \frac{1}{2a} \cdot \ln \left| \frac{u+a}{u-a} \right| + C$$

(19) $\int \frac{1}{a^2-u^2} \, du = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$

$$I_{20} = \int \frac{1}{u^2 - a^2} \, du$$

$$= \int \frac{A}{u - a} + \frac{B}{u + a} \, du$$

Where $\quad A(u + a) + B(u - a) = 1 \quad \longrightarrow \quad A + B = 0 \quad$ and $\quad A - B = \frac{1}{a}$

$\longrightarrow \quad B = -A \quad \longrightarrow \quad 2A = \frac{1}{a} \quad \longrightarrow \quad A = \frac{1}{2a} \quad \longrightarrow \quad B = -\frac{1}{2a}$

$$= \int \frac{\frac{1}{2a}}{u - a} + \frac{-\frac{1}{2a}}{u + a} \, du$$

$$= \frac{1}{2a} \cdot \int \frac{1}{u - a} - \frac{1}{u + a} \, du$$

$$= \frac{1}{2a} \left(\ln|u - a| - \ln|u + a| + C \right)$$

$$= \frac{1}{2a} \cdot \ln\left| \frac{u - a}{u + a} \right| + C$$

$$\int \frac{1}{u^2 - a^2} \, du = \frac{1}{2a} \ln\left| \frac{u - a}{u + a} \right| + C \qquad \textcircled{20}$$

$$I_{21} = \int \sqrt{a^2 + u^2}\, du$$

Let $\sin\theta = \dfrac{u}{\sqrt{a^2+u^2}}, \quad \cos\theta = \dfrac{a}{\sqrt{a^2+u^2}} \quad \longrightarrow \quad \tan\theta = \dfrac{u}{a}$
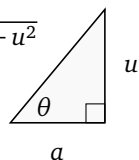
and so $\quad u = a\tan\theta \quad \longrightarrow \quad du = a\sec\theta\tan\theta\, d\theta$

and $\quad \sqrt{u^2 + a^2} = a\sec\theta$

$$= a\sec\theta\, a\sec^2\theta\, d\theta$$

$$= a^2 \int \sec^3\theta\, d\theta$$

Apply integal identity $\boxed{71}$:

$$\int \sec^3 u\, du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$$

$$= a^2 \left(\tfrac{1}{2}\sec\theta\tan\theta + \tfrac{1}{2}\ln|\sec\theta + \tan\theta| + C \right)$$

$$= \frac{a^2}{2}\frac{\sqrt{u^2+a^2}}{a}\cdot\frac{u}{a} + \frac{a^2}{2}\ln\left|\frac{\sqrt{u^2+a^2}}{a} + \frac{u}{a}\right| + C$$

$$= \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left|\frac{u+\sqrt{u^2+a^2}}{a}\right| + C$$

$$= \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2+a^2}\right| - \frac{a^2}{2}\ln|a| + C$$

$$= \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2+a^2}\right| + C$$

$\boxed{21}$ $\quad \int \sqrt{a^2+u^2}\, du = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$

$$I_{63} = \int \sin^2 u \, du$$

$$= \int \frac{1 - \cos 2u}{2} \, du$$

$$= \frac{1}{2} \cdot \int 1 - \cos 2u \, du$$

$$= \frac{1}{2} \cdot \left(u - \frac{1}{2} \sin 2u \right) + C$$

$$= \tfrac{1}{2} u - \tfrac{1}{4} \sin 2u + C$$

$$\int \sin^2 u \, du \;=\; \tfrac{1}{2} u - \tfrac{1}{4} \sin 2u + C \qquad \text{(63)}$$

$$I_{64} = \int \cos^2 u \, du$$

$$= \int \frac{1 + \cos 2u}{2} \, du$$

$$= \frac{1}{2} \cdot \int 1 + \cos 2u \, du$$

$$= \frac{1}{2} \cdot \left(u + \frac{1}{2} \sin 2u \right) + C$$

$$= \frac{1}{2} u + \frac{1}{4} \sin 2u + C$$

(64) $\quad \int \cos^2 u \, du \;=\; \frac{1}{2} u + \frac{1}{4} \sin 2u + C$

$$I_{65} = \int \tan^2 u \, du$$

Rewrite using trigonometric identity $\tan^2 u = \sec^2 u - 1$

$$= \int \sec^2 u - 1 \, du$$

$$= \tan u - u + C$$

$$\int \tan^2 u \ du \ = \ \tan u \ - \ u \ + \ C \qquad\qquad (65)$$

$$I_{66} = \int \cot^2 u \, du$$

Rewrite using trigonometric identity $\tan^2 u = \csc^2 u - 1$

$$= \int \csc^2 u - 1 \, du$$

$$= -\cot u - u + C$$

$$\boxed{66} \qquad \int \cot^2 u \, du \;=\; -\cot u \,-\, u \,+\, C$$

$$I_{67} = \int \sin^3 u \, du$$

$$= \int (1 - \cos^2 u) \sin u \, du$$

$$= \int \sin u \, du - \int \cos^2 u \, \sin u \, du$$

Let $\quad v = \cos u \quad \longrightarrow \quad dv = -\sin u \, du$

$$= \int v^2 \, dv - \cos u$$

$$= \frac{1}{3} v^3 - \cos u + C$$

Reverse the substitution

$$= \frac{1}{3} \cos^3 u - \cos u + C$$

(which we could leave right there)

$$= \left(\frac{1}{3} \cos^2 u - 1 \right) \cos u + C$$

$$= \left(\frac{1}{3} (1 - \sin^2 u) - 1 \right) \cos u + C$$

$$= -\frac{1}{3} \left(2 + \sin^2 u \right) \cos u + C$$

$$\int \sin^3 u \, du \;=\; -\tfrac{1}{3} \left(2 + \sin^2 u \right) \cos u \;+\; C \qquad \text{(67)}$$

$$I_{68} = \int \cos^3 u \, du$$

$$= \int (1 - \sin^2 u) \cos u \, du$$

$$= \int \cos u \, du - \int \sin^2 u \, \cos u \, du$$

Let $v = \sin u \quad \longrightarrow \quad dv = \cos u \, du$

$$= \sin u - \int v^2 \, dv$$

$$= \sin u - \frac{1}{3} v^3 + C$$

Reverse the substitution

$$= \sin u - \frac{1}{3} \sin^3 u + C$$

(which we could leave right there)

$$= \left(1 - \frac{1}{3} \sin^2 u \right) \sin u + C$$

$$= \left(1 - \frac{1}{3} (1 - \cos^2 u) \right) \sin u + C$$

$$= \frac{1}{3} \left(2 + \cos^2 u \right) \sin u + C$$

(68) $\int \cos^3 u \, du = \frac{1}{3} (2 + \cos^2 u) \sin u + C$

$$I_{69} = \int \tan^3 u \, du$$

$$= \int (\sec^2 u - 1) \tan u \, du$$

$$= \int (\sec^2 u \, \tan u) \, du - \int \tan u \, du$$

Let $\quad v = \tan u \quad \longrightarrow \quad dv = \sec^2 u \, du$

$$= \int v \, dv - \ln|\sec u|$$

$$= \frac{1}{2} v^2 - \ln|\sec u| + C$$

Reverse the substitution

$$= \frac{1}{2} \tan^2 u - \ln|\sec u| + C$$

(which we could leave right there)

$$= \frac{1}{2} \tan^2 u + \ln|\cos u| + C$$

$$\int \tan^3 u \, du \;=\; \tfrac{1}{2} \tan^2 u \,+\, \ln|\cos u| \,+\, C \qquad \text{(69)}$$

$$I_{70} = \int \cot^3 u \, du$$

$$= \int (\csc^2 u - 1) \cot u \, du$$

$$= \int (\csc^2 u \, \cot u) \, du - \int \cot u \, du$$

Let $v = \cot u \quad \longrightarrow \quad dv = -\csc^2 u \, du$

$$= -\int v \, dv - \ln|\sin u|$$

$$= -\frac{1}{2}v^2 - \ln|\sin u| + C$$

Reverse the substitution

$$= -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$$

(which we could leave right there)

$$= -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$$

(70) $\int \cot^3 u \, du \;=\; -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$

$$I_{71} = \int \sec^3 u \, du$$

Integrate by parts $\begin{cases} (\sec u)' = \sec u \tan u \\ \sec^2 u = (\tan u)' \end{cases}$

$$= (\sec u)(\tan u) - \int (\sec u \tan u)(\tan u) \, du$$

$$= \sec u \tan u - \int \sec u \tan^2 u \, du$$

$$= \sec u \tan u - \int \sec u (\sec^2 u - 1) \, du$$

$$= \sec u \tan u - \int \sec^3 u \, du + \int \sec u \, du$$

$$2 \int \sec^3 u \, du = \sec u \tan u + \int \sec u \, du$$

$$\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \int \sec u \, du$$

$$= \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln|\sec u + \tan u| + C$$

$$\int \sec^3 u \, du = \tfrac{1}{2} \sec u \tan u + \tfrac{1}{2} \ln|\sec u + \tan u| + C \quad (71)$$

$$I_{72} = \int \csc^3 u \, du$$

Integrate by parts $\begin{cases} (\csc u)' = -\csc u \cot u \\ \csc^2 u = (-\cot u)' \end{cases}$

$$= (\csc u)(-\cot u) - \int (-\csc u \cot u)(-\cot u) \, du$$

$$= -\csc u \cot u - \int \csc u \cot^2 du$$

$$= -\csc u \cot u - \int \csc u (\csc^2 u - 1) \, du$$

$$= -\csc u \cot u - \int \csc^3 u \, du + \int \csc u \, du$$

$$2 \int \csc^3 u \, du = -\csc u \cot u + \int \csc u \, du$$

$$\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \int \csc u \, du$$

$$= \frac{1}{2} \csc u \cot u + \frac{1}{2} \ln|\csc u - \cot u| + C$$

(72) $\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln|\csc u - \cot u| + C$

$$I_{73} = \int \sin^n u \, du$$

Integrate by parts
$$\begin{cases} \left(\sin^{n-1} u\right)' = (n-1)\sin^{n-2} u \cos u \\ \quad \sin u = (-\cos u)' \end{cases}$$

$$= \left(\sin^{n-1} u\right)(-\cos u) - \int \left((n-1)\sin^{n-2} u \cos u\right)(-\cos u)\, du$$

$$= -\sin^{n-1} u \, \cos u + (n-1) \int \sin^{n-2} u \cos^2 u \, du$$

$$= -\sin^{n-1} u \, \cos u + (n-1) \int \sin^{n-2} u(1 - \sin^2 u)\, du$$

$$= -\sin^{n-1} u \, \cos u + (n-1)\left(\int \sin^{n-2} u \, du - \int \sin^n u \, du\right)$$

$$1 + (n-1) \int \sin^n u \, du = -\sin^{n-1} u \, \cos u + (n-1) \int \sin^{n-2} u \, du$$

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \, \cos u + \frac{(n-1)}{n} \int \sin^{n-2} u \, du$$

$$\int \sin^n u \, du = -\tfrac{1}{n} \sin^{n-1} u \, \cos u + \tfrac{n-1}{n} \int \sin^{n-2} u \, du \quad (73)$$

$$I_{74} = \int \cos^n u \, du$$

Integrate by parts
$$\begin{cases} \left(\cos^{n-1} u\right)' = -(n-1)\cos^{n-2} u \sin u \\ \cos u = (\sin u)' \end{cases}$$

$$= \left(\cos^{n-1} u\right)(\sin u) - \int \left(-(n-1)\cos^{n-2} u \sin u\right)(\sin u) \, du$$

$$= \cos^{n-1} u \, \sin u + (n-1) \int \cos^{n-2} u \sin^2 u \, du$$

$$= \cos^{n-1} u \, \sin u + (n-1) \int \cos^{n-2} u (1 - \cos^2 u) \, du$$

$$= \cos^{n-1} u \, \sin u + (n-1) \left(\int \cos^{n-2} u \, du - \int \cos^n u \, du \right)$$

$$1 + (n-1) \int \cos^n u \, du = \cos^{n-1} u \, \sin u + (n-1) \int \cos^{n-2} u \, du$$

$$\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \, \sin u + \frac{(n-1)}{n} \int \cos^{n-2} u \, du$$

$$(74) \quad \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \, \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

$$I_{75} = \int \tan^n u \, du$$

$$= \int \tan^{n-2} u \tan^2 u \, du$$

$$= \int \tan^{n-2} u (\sec^2 u - 1) \, du$$

$$= \int \tan^{n-2} u \sec^2 u \, du - \int \tan^{n-2} u \, du$$

Let $v = \tan u \longrightarrow dv = \sec^2 u \, du$

$$= \int v^{n-2} u \, dv - \int \tan^{n-2} u$$

$$= \frac{1}{n-1} v^{n-1} - \int \tan^{n-2} u$$

$$= \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u$$

$$\int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du \qquad \text{(75)}$$

$$I_{76} = \int \cot^n u \, du$$

$$= \int \cot^{n-2} u \cot^2 u \, du$$

$$= \int \cot^{n-2} u (\csc^2 u - 1) \, du$$

$$= \int \cot^{n-2} u \, \csc^2 u \, du - \int \cot^{n-2} u \, du$$

Let $\quad v = \cot u \quad \longrightarrow \quad dv = -\csc^2 u \, du$

$$= -\int v^{n-2} u \, dv - \int \cot^{n-2} u$$

$$= \frac{-1}{n-1} v^{n-1} - \int \cot^{n-2} u$$

$$= \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u$$

(76) $\quad \int \cot^n u \, du \;=\; \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$

$$I_{77} = \int \sec^n u \, du$$

Integrate by parts
$$\begin{cases} \sec^2 u = (\tan u)' \\ \left(\sec^{n-2} u\right)' = (n-2)\sec^{n-3} u \sec u \tan u \end{cases}$$

$$= (\tan u)\left(\sec^{n-2} u\right) - \int (\tan u)\left((n-2)\sec^{n-3} u \sec u \tan u\right) du$$

$$= \tan u \sec^{n-2} u - (n-2) \int \sec^{n-2} \tan^2 u \, du$$

$$= \tan u \sec^{n-2} u - (n-2) \int \sec^{n-2}(\sec^2 u - 1) \, du$$

$$= \tan u \sec^{n-2} u - (n-2)\left(\int \sec^n u \, du - \int \sec^{n-2} du\right)$$

$$(1 + (n-2)) \int \sec^n u \, du = \tan u \sec^{n-2} u - (n-2) \int \sec^{n-2} du$$

$$\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u - \frac{n-2}{n-1} \int \sec^{n-2} du$$

$$\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du \tag{77}$$

$$I_{82} = \int u \sin u \, du$$

Integrate by parts
$$\begin{cases} (u)' &= 1 \\ \sin u &= (-\cos u)' \end{cases}$$

$$= (u)(-\cos u) - \int (1)(-\cos u) \, du$$

$$= \int \cos u \, du - u \cos u$$

$$= \sin u - u \cos u$$

(82) $$\int u \sin u \, du = \sin u - u \cos u + C$$

$$I_{83} = \int u \cos u \, du$$

Integrate by parts $\begin{cases} (u)' &= 1 \\ \cos u &= (\sin u)' \end{cases}$

$$= (u)(\sin u) - \int (1)(\sin u) \, du$$

$$= u \sin u - (-\cos u)$$

$$= \cos u + u \sin u$$

$$\int u \cos u \, du \;=\; \cos u + u \sin u + C \qquad (83)$$

$$I_{92} = \int u \arctan u \, du$$

Integrate by parts
$$\begin{cases} u = \left(\tfrac{1}{2}u^2\right)' \\ (\arctan u)' = \dfrac{1}{1+u^2} \end{cases}$$

$$= \left(\tfrac{1}{2}u^2\right)(\arctan u) - \int \left(\tfrac{1}{2}u^2\right)\left(\dfrac{1}{1+u^2}\right) du$$

$$= \frac{u^2}{2}\arctan u - \frac{1}{2}\int \frac{u^2 + 1 - 1}{1+u^2} \, du$$

$$= \frac{u^2}{2}\arctan u - \frac{1}{2}\int 1 - \frac{1}{1+u^2} \, du$$

$$= \frac{u^2}{2}\arctan u - \frac{1}{2}(u - \arctan u) + C$$

$$= \frac{u^2 + 1}{2}\arctan u - \frac{u}{2} + C$$

(92) $\int u \arctan u \, du = \frac{u^2+1}{2} \arctan u - \frac{u}{2} + C$

$$I_{96} = \int u e^{au}\, du$$

Integrate by parts
$$\begin{cases} (u)' &= 1 \\ e^{au} &= \left(\dfrac{e^{au}}{a}\right)' \end{cases}$$

$$= (u)\left(\frac{e^{au}}{a}\right) - \int (1)\left(\frac{e^{au}}{a}\right) du$$

$$= \frac{1}{a} u e^{au} - \frac{1}{a} \int e^{au}\, du$$

$$= \frac{1}{a} u e^{au} - \frac{1}{a^2} e^{au} + C$$

$$= \left(\frac{1}{a} u - \frac{1}{a^2}\right) e^{au} + C$$

Simplify

$$= \frac{1}{a^2}(au - 1) e^{au} + C$$

$$\int u e^{au}\, du = \tfrac{1}{a^2}(au - 1) e^{au} + C \qquad \text{(96)}$$

$$I_{97} = \int u^n e^{au} \, du$$

$$\text{Integrate by parts} \quad \begin{cases} (u^n)' & = \; n \cdot u^{n-1} \\[2mm] e^{au} & = \; \left(\dfrac{e^{au}}{a}\right)' \end{cases}$$

$$= (u^n)\left(\frac{e^{au}}{a}\right) - \int (n \cdot u^{n-1})\left(\frac{e^{au}}{a}\right) du$$

$$= \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} \, du$$

(97) $\int u^n e^{au} \; du \;\; = \;\; \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} \; du$

$$I_{100} = \int \ln u \, du$$

Integrate by parts
$$\begin{cases} 1 = (u)' \\ (\ln u)' = \dfrac{1}{u} \end{cases}$$

$$= (u)(\ln u) - \int (u)\left(\frac{1}{u}\right) du$$

$$= u \ln u - \int 1 \, du$$

$$= u \ln u - u + C$$

$$\int \ln u \, du = u \ln u - u + C \qquad \boxed{100}$$

$$I_{102} = \int \frac{1}{u \ln u} \, du$$

Let $\quad v = \ln u \quad \longrightarrow \quad dv = \frac{1}{u} \, du$

$$= \int \frac{1}{v} \, dv$$

$$= \ln v + C$$

Reverse the substitution

$$= \ln|\ln u|$$

(102) $\qquad \int \frac{1}{u \ln u} \, du \; = \; \ln|\ln u| \; + \; C$

12: $\tan u$

64: $\cos^2 u$

75: $\tan^n u$

13: $\cot u$

65: $\tan^2 u$

76: $\cot^n u$

14: $\sec u$

66: $\cot^2 u$

77: $\sec^n u$

15: $\csc u$

67: $\sin^3 u$

82: $u \sin u$

16: $\dfrac{1}{\sqrt{a^2 - u^2}}$

68: $\cos^3 u$

83: $u \cos u$

17: $\dfrac{1}{a^2 + u^2}$

69: $\tan^3 u$

92: $u \arctan u$

18: $\dfrac{1}{u\sqrt{u^2 - a^2}}$

70: $\cot^3 u$

96: $u e^{au}$

19: $\dfrac{1}{a^2 - u^2}$

71: $\sec^3 u$

97: $u^n e^{au}$

20: $\dfrac{1}{u^2 - a^2}$

72: $\csc^3 u$

100: $\ln u$

21: $\sqrt{a^2 + u^2}$

73: $\sin^n u$

102: $\dfrac{1}{u \ln u}$

63: $\sin^2 u$

74: $\cos^n u$

$$\left(102\right) \qquad \int \frac{1}{u \ln u}\ \mathrm{d}u\ =\ \ln|\ln u| + C$$