

$$I_{12} = \int \tan u \, du$$

$$= \int \frac{\sin u}{\cos u} \, du$$

$$\text{Let } v = \cos u \longrightarrow dv = -\sin u \, du \longrightarrow -dv = \sin u \, du$$

$$= - \int \frac{1}{v} \, dv$$

$$= -\ln|v| + C$$

Reverse the substitution

$$= -\ln|\cos u| + C$$

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$$\int \tan u \, du = \ln|\cos u| + C$$

$$I_{13} = \int \cot u \, du$$

$$= \int \frac{\cos u}{\sin u} \, du$$

$$\text{Let } v = \sin u \longrightarrow dv = \cos u \, du$$

$$= \int \frac{1}{v} \, dv$$

$$= \ln|v| + C$$

Reverse the substitution

$$= \ln|\sin u| + C$$

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$$\int \cot u \, du = \ln|\sin u| + C$$

$$\begin{aligned}
 I_{14} &= \int \sec u \, du \\
 &= \int \sec u \cdot \frac{\sec u + \tan u}{\sec u + \tan u} \, du \\
 &= \int \frac{\sec^2 u + \sec u \cdot \tan u}{\sec u + \tan u} \, du
 \end{aligned}$$

Let $v = \sec u + \tan u \longrightarrow dv = \sec^2 u + \sec u \tan u \, du$

$$\begin{aligned}
 &= \int \frac{1}{v} \\
 &= \ln|v| + C
 \end{aligned}$$

Reverse the substitution

$$= \ln|\sec u + \tan u| + C$$

1. $\int u \, dv = uv - \int v \, du$
2. $\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{1}{u} \, du = \ln|u| + C$
4. $\int e^u \, du = e^u + C$
5. $\int a^u \, du = \frac{a^u}{\ln a} + C$
6. $\int \sin u \, du = -\cos u + C$
7. $\int \cos u \, du = \sin u + C$
8. $\int \sec^2 u \, du = \tan u + C$
9. $\int \csc^2 u \, du = -\cot u + C$
10. $\int \sec u \tan u \, du = \sec u + C$
11. $\int \csc u \cot u \, du = -\csc u + C$
103. $\int \sinh u \, du = \cosh u + C$
104. $\int \cosh u \, du = \sinh u + C$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

These integrals can be used without explanation

$$I_{16} = \int \frac{1}{\sqrt{a^2 - u^2}} du$$

$$\text{Let } \sin \theta = \frac{u}{a}, \cos \theta = \frac{\sqrt{a^2 - u^2}}{a} \longrightarrow \tan \theta = \frac{u}{\sqrt{a^2 - u^2}}$$

$$\text{and so } u = a \sin \theta \longrightarrow du = a \cos \theta d\theta$$

$$\text{and } \sqrt{a^2 - u^2} = a \cos \theta$$

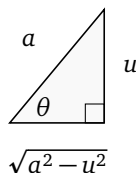
$$= \int \frac{a \cos \theta}{a \cos \theta} d\theta$$

$$= \int 1 d\theta$$

$$= \theta + C$$

$$\text{Reverse the substitution of } u, \text{ where } \sin \theta = \frac{u}{a} \longrightarrow \theta = \arcsin \frac{u}{a}$$

$$= \arcsin \frac{u}{a} + C$$



$$I_{17} = \int \frac{1}{a^2 + u^2} du$$

$$\text{Let } v = \frac{u}{a} \longrightarrow av = u \longrightarrow a dv = du$$

$$= \int \frac{a}{a^2 + (av)^2} dv$$

$$= \int \frac{a}{a^2(1 + v^2)} dv$$

$$= \frac{1}{a} \int \frac{1}{1 + v^2} dv$$

$$= \frac{1}{a} \arctan v$$

$$\text{Reverse the substitution of } u$$

$$= \frac{1}{a} \arctan \frac{u}{a} + C$$

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$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$$

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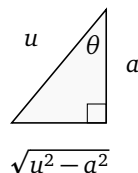
$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$I_{18} = \int \frac{1}{u\sqrt{u^2 - a^2}} du$$

Let $\sin \theta = \frac{\sqrt{u^2 - a^2}}{u}$, $\cos \theta = \frac{a}{u} \rightarrow \tan \theta = \frac{\sqrt{u^2 - a^2}}{a}$

and so $u = a \sec \theta \rightarrow du = a \sec \theta \tan \theta d\theta$

and $\sqrt{u^2 - a^2} = a \tan \theta$



$$I_{18} = \int \frac{a \sec \theta \tan \theta}{a \sec \theta \cdot a \tan \theta} d\theta$$

$$= \int \frac{1}{a} d\theta$$

$$= \frac{1}{a} \theta + C$$

Reverse the substitution, where $u = a \sec \theta \rightarrow \theta = \operatorname{arcsec} \frac{u}{a}$

$$= \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

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$$\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

$$I_{15} = \int \csc u du$$

$$= \int \csc u \cdot \frac{\csc u + \cot u}{\csc u + \cot u} du$$

$$= \int \frac{\csc^2 u + \csc u \cdot \cot u}{\csc u + \cot u} du$$

Let $v = \csc u + \cot u \rightarrow dv = -\csc^2 u + \csc u \cot u du$
 $\rightarrow -dv = \csc^2 u + \csc u \cot u du$

$$= \int \frac{-dv}{v}$$

$$= - \int \frac{1}{v} dv$$

$$= -\ln|v| + C$$

Reverse the substitution

$$= -\ln|\csc u + \cot u| + C$$

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$$\int \csc u du = -\ln|\csc u + \cot u| + C$$

$$I_{20} = \int \frac{1}{u^2 - a^2} du$$

$$= \int \frac{A}{u-a} + \frac{B}{u+a} du$$

Where $A(u+a) + B(u-a) = 1 \rightarrow A+B=0$ and $A-B = \frac{1}{a}$
 $\rightarrow B = -A \rightarrow 2A = \frac{1}{a} \rightarrow A = \frac{1}{2a} \rightarrow B = -\frac{1}{2a}$

$$= \int \frac{\frac{1}{2a}}{u-a} + \frac{-\frac{1}{2a}}{u+a} du$$

$$= \frac{1}{2a} \cdot \int \frac{1}{u-a} - \frac{1}{u+a} du$$

$$= \frac{1}{2a} (\ln|u-a| - \ln|u+a| + C)$$

$$= \frac{1}{2a} \cdot \ln \left| \frac{u-a}{u+a} \right| + C$$

$$\int \frac{1}{u^2 - a^2} du = \frac{1}{2a} \ln \left| \frac{u-a}{u+a} \right| + C$$

$$I_{63} = \int \sin^2 u du$$

$$= \int \frac{1 - \cos 2u}{2} du$$

$$= \frac{1}{2} \cdot \int 1 - \cos 2u du$$

$$= \frac{1}{2} \cdot \left(u - \frac{1}{2} \sin 2u \right) + C$$

$$= \frac{1}{2} u - \frac{1}{4} \sin 2u + C$$

$$\int \sin^2 u du = \frac{1}{2} u - \frac{1}{4} \sin 2u + C$$

$$\begin{aligned}
I_{64} &= \int \cos^2 u \, du \\
&= \int \frac{1 + \cos u}{2} \, du \\
&= \frac{1}{2} \cdot \int 1 + \cos u \, du \\
&= \frac{1}{2} \cdot \left(u + \frac{1}{2} \sin 2u \right) + C \\
&= \frac{1}{2} u + \frac{1}{4} \sin 2u + C
\end{aligned}$$

$$\int \cos^2 u \, du = \frac{1}{2} u + \frac{1}{4} \sin 2u + C$$

$$\begin{aligned}
I_{19} &= \int \frac{1}{a^2 - u^2} \, du \\
&= \int \frac{A}{a + u} + \frac{B}{a - u} \, du
\end{aligned}$$

Where $A(a - u) + B(a + u) = 1 \rightarrow A + B = \frac{1}{a}$ and $B - A = 0$
 $\rightarrow B = A \rightarrow 2A = \frac{1}{a} \rightarrow A = \frac{1}{2a} \rightarrow B = \frac{1}{2a}$

$$\begin{aligned}
&= \int \frac{\frac{1}{2a}}{a + u} + \frac{\frac{1}{2a}}{a - u} \, du \\
&= \frac{1}{2a} \cdot \int \frac{1}{a + u} + \frac{1}{a - u} \, du
\end{aligned}$$

Rearranging so u preceeds a for aesthetic reasons

$$\begin{aligned}
&= \frac{1}{2a} \cdot \int \frac{1}{u + a} - \frac{1}{u - a} \, du \\
&= \frac{1}{2a} (\ln|u + a| - \ln|u - a|) + C \\
&= \frac{1}{2a} \cdot \ln \left| \frac{u + a}{u - a} \right| + C
\end{aligned}$$

$$\int \frac{1}{a^2 - u^2} \, du = \frac{1}{2a} \ln \left| \frac{u + a}{u - a} \right| + C$$

$$I_{92} = \int u \arctan u \, du$$

$$\text{Integrate by parts } \begin{cases} u = (\frac{1}{2}u^2)' \\ (\arctan u)' = \frac{1}{1+u^2} \end{cases}$$

$$= (\frac{1}{2}u^2)(\arctan u) - \int (\frac{1}{2}u^2) \left(\frac{1}{1+u^2} \right) du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} \int \frac{u^2 + 1 - 1}{1+u^2} du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} \int 1 - \frac{1}{1+u^2} du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} (u - \arctan u) + C$$

$$= \frac{u^2 + 1}{2} \arctan u - \frac{u}{2} + C$$

$$\textcircled{92} \quad \int u \arctan u \, du = \frac{u^2 + 1}{2} \arctan u - \frac{u}{2} + C$$

$$I_{96} = \int u e^{au} \, du$$

$$\text{Integrate by parts } \begin{cases} (u)' = 1 \\ e^{au} = \left(\frac{e^{au}}{a} \right)' \end{cases}$$

$$= (u) \left(\frac{e^{au}}{a} \right) - \int (1) \left(\frac{e^{au}}{a} \right) du$$

$$= \frac{1}{a} u e^{au} - \frac{1}{a} \int e^{au} du$$

$$= \frac{1}{a} u e^{au} - \frac{1}{a^2} e^{au} + C$$

$$= \left(\frac{1}{a} u - \frac{1}{a^2} \right) e^{au} + C$$

Simplify

$$= \frac{1}{a^2} (au - 1) e^{au} + C$$

$$\textcircled{96} \quad \int u e^{au} \, du = \frac{1}{a^2} (au - 1) e^{au} + C$$

$$I_{100} = \int \ln u \, du$$

$$\text{Integrate by parts } \begin{cases} 1 = (u)' \\ (\ln u)' = \frac{1}{u} \end{cases}$$

$$= (u)(\ln u) - \int (u) \left(\frac{1}{u} \right) du$$

$$= u \ln u - \int 1 \, du$$

$$= u \ln u - u + C$$

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$$\int \ln u \, du = u \ln u - u + C$$

$$I_{102} = \int \frac{1}{u \ln u} \, du$$

$$\text{Let } v = \ln u \longrightarrow dv = \frac{1}{u} \, du$$

$$= \int \frac{1}{v} \, dv$$

$$= \ln v + C$$

Reverse the substitution

$$= \ln |\ln u|$$

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$$\int \frac{1}{u \ln u} \, du = \ln |\ln u| + C$$