

The Book of Integrals

<https://github.com/heckman/book-of-integrals>

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1:	$\int u \, dv = uv - \int v \, du$	12: $\tan u$	64: $\cos^2 u$	75: $\tan^n u$
2:	$\int u^n \, du = \frac{u^{n+1}}{n+1} + C, \, n \neq -1$	13: $\cot u$	65: $\tan^2 u$	76: $\cot^n u$
3:	$\int \frac{1}{u} \, du = \ln u + C$	14: $\sec u$	66: $\cot^2 u$	77: $\sec^n u$
4:	$\int e^u \, du = e^u + C$	15: $\csc u$	67: $\sin^3 u$	78: $\csc^n u$
5:	$\int a^u \, du = \frac{a^u}{\ln a} + C$	16: $\frac{1}{\sqrt{a^2 - u^2}}$	68: $\cos^3 u$	82: $u \sin u$
6:	$\int \sin u \, du = -\cos u + C$	17: $\frac{1}{a^2 + u^2}$	69: $\tan^3 u$	83: $u \cos u$
7:	$\int \cos u \, du = \sin u + C$	18: $\frac{1}{u\sqrt{u^2 - a^2}}$	70: $\cot^3 u$	92: $u \arctan u$
8:	$\int \sec^2 u \, du = \tan u + C$	19: $\frac{1}{a^2 - u^2}$	71: $\sec^3 u$	96: ue^{au}
9:	$\int \csc^2 u \, du = -\cot u + C$	20: $\frac{1}{u^2 - a^2}$	72: $\csc^3 u$	97: $u^n e^{au}$
10:	$\int \sec u \tan u \, du = \sec u + C$	21: $\sqrt{a^2 + u^2}$	73: $\sin^n u$	100: $\ln u$
11:	$\int \csc u \cot u \, du = -\csc u + C$	63: $\sin^2 u$	74: $\cos^n u$	102: $\frac{1}{u \ln u}$
103:	$\int \sinh u \, du = \cosh u + C$			
104:	$\int \cosh u \, du = \sinh u + C$			

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$$I_{12} = \int \tan u \, du$$

$$= \int \frac{\sin u}{\cos u} \, du$$

$$\text{Let } v = \cos u \longrightarrow dv = -\sin u \, du \longrightarrow -dv = \sin u \, du$$

$$= - \int \frac{1}{v} \, dv$$

$$= -\ln|v| + C$$

Reverse the substitution

$$= -\ln|\cos u| + C$$

$$\int \tan u \, du = \ln|\cos u| + C$$

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$$I_{13} = \int \cot u \, du$$

$$= \int \frac{\cos u}{\sin u} \, du$$

Let $v = \sin u \longrightarrow dv = \cos u \, du$

$$= \int \frac{1}{v} \, dv$$

$$= \ln|v| + C$$

Reverse the substitution

$$= \ln|\sin u| + C$$

$$\int \cot u \, du = \ln|\sin u| + C$$

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$$\begin{aligned}
 I_{14} &= \int \sec u \, du \\
 &= \int \sec u \cdot \frac{\sec u + \tan u}{\sec u + \tan u} \, du \\
 &= \int \frac{\sec^2 u + \sec u \cdot \tan u}{\sec u + \tan u} \, du
 \end{aligned}$$

$$\text{Let } v = \sec u + \tan u \longrightarrow dv = \sec^2 u + \sec u \tan u \, du$$

$$= \int \frac{dv}{v}$$

$$= \ln|v| + C$$

Reverse the substitution

$$= \ln|\sec u + \tan u| + C$$

$$\int \sec u \, du = \ln|\sec u + \tan u| + C$$

$$I_{15} = \int \csc u \, du$$

$$= \int \csc u \cdot \frac{\csc u - \cot u}{\csc u - \cot u} \, du$$

$$= \int \frac{\csc^2 u - \csc u \cdot \cot u}{\csc u - \cot u} \, du$$

$$\text{Let } v = \csc u - \cot u \longrightarrow dv = \csc^2 u - \csc u \cot u \, du$$

$$= \int \frac{dv}{v}$$

$$= \ln|v| + C$$

Reverse the substitution

$$= \ln|\csc u - \cot u| + C$$

$$I_{102} = \int \frac{1}{u \ln u} \, du$$

$$\text{Let } v = \ln u \longrightarrow dv = \frac{1}{u} \, du$$

$$= \int \frac{1}{v} \, dv$$

$$= \ln v + C$$

Reverse the substitution

$$= \ln|\ln u|$$

15

$$\int \csc u \, du = \ln|\csc u - \cot u| + C$$

$$\int \frac{1}{u \ln u} \, du = \ln|\ln u| + C$$

102

$$I_{100} = \int \ln u \, du$$

$$\text{Integrate by parts} \left\{ \begin{array}{l} 1 = (u)' \\ (\ln u)' = \frac{1}{u} \end{array} \right.$$

$$= (u)(\ln u) - \int (u) \left(\frac{1}{u} \right) du$$

$$= u \ln u - \int 1 \, du$$

$$= u \ln u - u + C$$

$$\int \ln u \, du = u \ln u - u + C$$

$$I_{16} = \int \frac{1}{\sqrt{a^2 - u^2}} du$$

$$\text{Let } \sin \theta = \frac{u}{a}, \quad \cos \theta = \frac{\sqrt{a^2 - u^2}}{a} \quad \longrightarrow \quad \tan \theta = \frac{u}{\sqrt{a^2 - u^2}}$$

$$\text{and so } u = a \sin \theta \quad \longrightarrow \quad du = a \cos \theta \, d\theta$$

$$\text{and } \sqrt{a^2 - u^2} = a \cos \theta$$

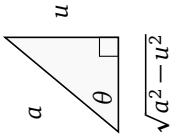
$$= \int \frac{a \cos \theta}{a \cos \theta} d\theta$$

$$= \int 1 \, d\theta$$

$$= \theta + C$$

Reverse the substitution of u , where $\sin \theta = \frac{u}{a} \longrightarrow \theta = \arcsin \frac{u}{a}$

$$= \arcsin \frac{u}{a} + C$$



If $a = 1$ this solution can be used with explanation.

$$\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$$

$$I_{17} = \int \frac{1}{a^2 + u^2} du$$

$$\text{Let } v = \frac{u}{a} \longrightarrow av = u \longrightarrow a \, dv = du$$

$$= \int \frac{a}{a^2 + (av)^2} dv$$

$$= \int \frac{a}{a^2 (1 + v^2)} dv$$

$$= \frac{1}{a} \int \frac{1}{1 + v^2} dv$$

$$= \frac{1}{a} \arctan v$$

Reverse the substitution of u

$$= \frac{1}{a} \arctan \frac{u}{a} + C$$

If $a = 1$ this solution can be used with explanation.

(17)

$$\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$$

$$I_{97} = \int u^n e^{au} du$$

$$\text{Integrate by parts} \quad \left\{ \begin{array}{l} (u^n)' = n \cdot u^{n-1} \\ e^{au} = \left(\frac{e^{au}}{a} \right)' \end{array} \right.$$

$$= (u^n) \left(\frac{e^{au}}{a} \right) - \int (n \cdot u^{n-1}) \left(\frac{e^{au}}{a} \right) du$$

$$= \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$\int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

(97)

$$I_{96} = \int ue^{au} \, du$$

Integrate by parts $\left\{ \begin{array}{l} (u)' = 1 \\ e^{au} = \left(\frac{e^{au}}{a}\right)' \end{array} \right.$

$$= (u) \left(\frac{e^{au}}{a} \right) - \int (1) \left(\frac{e^{au}}{a} \right) du$$

$$= \frac{1}{a} ue^{au} - \frac{1}{a} \int e^{au} \, du$$

$$= \frac{1}{a} ue^{au} - \frac{1}{a^2} e^{au} + C$$

$$= \left(\frac{1}{a} u - \frac{1}{a^2} \right) e^{au} + C$$

Simplify

$$= \frac{1}{a^2} (au - 1) e^{au} + C$$

$$\int ue^{au} \, du = \frac{1}{a^2} (au - 1) e^{au} + C \quad (96)$$

$$I_{18} = \int \frac{1}{u\sqrt{u^2 - a^2}} du$$

Let $\sin \theta = \frac{\sqrt{u^2 - a^2}}{u}$, $\cos \theta = \frac{a}{u} \longrightarrow \tan \theta = \frac{\sqrt{u^2 - a^2}}{a}$

and so $u = a \sec \theta \longrightarrow du = a \sec \theta \tan \theta \, d\theta$

and $\sqrt{u^2 - a^2} = a \tan \theta$

$$I_{18} = \int \frac{a \sec \theta \tan \theta}{a \sec \theta \cdot a \tan \theta} d\theta$$

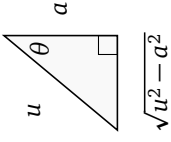
$$= \int \frac{1}{a} d\theta$$

$$= \frac{1}{a} \theta + C$$

Reverse the substitution, where $u = a \sec \theta \longrightarrow \theta = \operatorname{arcsec} \frac{u}{a}$

$$= \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{u}{a} + C \quad (18)$$



$$I_{19} = \int \frac{1}{a^2 - u^2} du$$

$$= \int \frac{A}{a+u} + \frac{B}{a-u} du$$

Where $A(a-u) + B(a+u) = 1 \longrightarrow A+B = \frac{1}{a}$ and $B-A = 0$
 $\longrightarrow B=A \longrightarrow 2A = \frac{1}{a} \longrightarrow A = \frac{1}{2a} \longrightarrow B = \frac{1}{2a}$

$$= \int \frac{\frac{1}{2a}}{a+u} + \frac{\frac{-1}{2a}}{a-u} du$$

$$= \frac{1}{2a} \cdot \int \frac{1}{a+u} + \frac{1}{a-u} du$$

Rearranging so u precedes a for aesthetic reasons

$$= \frac{1}{2a} \cdot \int \frac{1}{u+a} - \frac{1}{u-a} du$$

$$= \frac{1}{2a} (\ln|u+a| - \ln|u-a|) + C$$

$$= \frac{1}{2a} \cdot \ln \left| \frac{u+a}{u-a} \right| + C$$

$$I_{92} = \int u \arctan u du$$

Integrate by parts $\left\{ \begin{array}{l} u = (\frac{1}{2}u^2)' \\ (\arctan u)' = \frac{1}{1+u^2} \end{array} \right.$

$$= (\frac{1}{2}u^2)(\arctan u) - \int (\frac{1}{2}u^2) \left(\frac{1}{1+u^2} \right) du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} \int \frac{u^2+1-1}{1+u^2} du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} \int 1 - \frac{1}{1+u^2} du$$

$$= \frac{u^2}{2} \arctan u - \frac{1}{2} (u - \arctan u) + C$$

$$= \frac{u^2+1}{2} \arctan u - \frac{u}{2} + C$$

19

$$\int \frac{1}{a^2-u^2} du = \frac{1}{2a} \ln \left| \frac{u+a}{u-a} \right| + C$$

92

$$\int u \arctan u du = \frac{u^2+1}{2} \arctan u - \frac{u}{2} + C$$

$$I_{83} = \int u \cos u \, du$$

$$\text{Integrate by parts} \quad \begin{cases} (u)' = 1 \\ \cos u = (\sin u)' \end{cases}$$

$$= (u)(\sin u) - \int (1)(\sin u) \, du$$

$$= u \sin u - (-\cos u)$$

$$= \cos u + u \sin u$$

$$\int u \cos u \, du = \cos u + u \sin u + C \quad (83)$$

$$I_{20} = \int \frac{1}{u^2 - a^2} du$$

$$= \int \frac{A}{u - a} + \frac{B}{u + a} du$$

$$\begin{array}{ll} \text{Where} & A(u + a) + B(u - a) = 1 \longrightarrow A + B = 0 \quad \text{and} \quad A - B = \frac{1}{a} \\ \longrightarrow & B = -A \longrightarrow 2A = \frac{1}{a} \longrightarrow A = \frac{1}{2a} \longrightarrow B = -\frac{1}{2a} \end{array}$$

$$= \int \frac{\frac{1}{2a}}{u - a} + \frac{-\frac{1}{2a}}{u + a} du$$

$$= \frac{1}{2a} \cdot \int \frac{1}{u - a} - \frac{1}{u + a} du$$

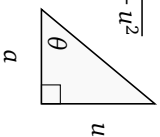
$$= \frac{1}{2a} (\ln|u - a| - \ln|u + a| + C)$$

$$= \frac{1}{2a} \cdot \ln \left| \frac{u - a}{u + a} \right| + C$$

$$\int \frac{1}{u^2 - a^2} du = \frac{1}{2a} \ln \left| \frac{u - a}{u + a} \right| + C \quad (20)$$

$$I_{21} = \int \sqrt{a^2 + u^2} \, du$$

$$\text{Let } \sin \theta = \frac{u}{\sqrt{a^2 + u^2}}, \quad \cos \theta = \frac{a}{\sqrt{a^2 + u^2}} \quad \longrightarrow \quad \tan \theta = \frac{u}{a}$$



$$\text{and so } u = a \tan \theta \longrightarrow du = a \sec^2 \theta \tan \theta \, d\theta$$

$$\text{and } \sqrt{u^2 + a^2} = a \sec \theta$$

$$= a \sec \theta a \sec^2 \theta \, d\theta$$

$$= a^2 \int \sec^3 \theta \, d\theta$$

Apply integral identity (71) :

$$\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln |\sec u + \tan u| + C$$

$$= a^2 \left(\frac{1}{2} \sec \theta \tan \theta + \frac{1}{2} \ln |\sec \theta + \tan \theta| + C \right)$$

$$= \frac{a^2}{2} \frac{\sqrt{u^2 + a^2}}{a} \cdot \frac{u}{a} + \frac{a^2}{2} \ln \left| \frac{\sqrt{u^2 + a^2}}{a} + \frac{u}{a} \right| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln \left| \frac{u + \sqrt{u^2 + a^2}}{a} \right| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| - \frac{a^2}{2} \ln |a| + C$$

$$= \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| + C$$

$$(21) \quad \int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}| + C$$

$$I_{82} = \int u \sin u \, du$$

$$\text{Integrate by parts } \begin{cases} (u)' = 1 \\ \sin u = (-\cos u)' \end{cases}$$

$$= (u)(-\cos u) - \int (1)(-\cos u) \, du$$

$$= \int \cos u \, du - u \cos u$$

$$= \sin u - u \cos u$$

$$\int u \sin u \, du = \sin u - u \cos u + C$$

$$I_{78} = \int \csc^n u \, du$$

$$\text{Integrate by parts} \left\{ \begin{array}{l} \csc^2 u = (-\cot u)' \\ (\csc^{n-2} u)' = -(n-2) \csc^{n-3} u \csc u \cot u \end{array} \right.$$

$$= (-\cot u) (\csc^{n-2} u) - \int (-\cot u) (-(n-2) \csc^{n-3} u \csc u \cot u) \, du$$

$$= -\cot u \csc^{n-2} u - (n-2) \int \csc^{n-2} \cot^2 u \, du$$

$$= -\cot u \csc^{n-2} u - (n-2) \int \csc^{n-2} (\csc^2 u - 1) \, du$$

$$= -\cot u \csc^{n-2} u - (n-2) \left(\int \csc^n u \, du - \int \csc^{n-2} u \, du \right)$$

$$(1 + (n-2)) \int \csc^n u \, du = -\cot u \csc^{n-2} u + (n-2) \int \csc^{n-2} u \, du$$

$$\int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$$

$$(78) \int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$$

$$\begin{aligned} I_{63} &= \int \sin^2 u \, du \\ &= \int \frac{1 - \cos 2u}{2} \, du \\ &= \frac{1}{2} \cdot \int 1 - \cos 2u \, du \\ &= \frac{1}{2} \cdot \left(u - \frac{1}{2} \sin 2u \right) + C \\ &= \frac{1}{2} u - \frac{1}{4} \sin 2u + C \end{aligned}$$

$$\int \sin^2 u \, du = \frac{1}{2} u - \frac{1}{4} \sin 2u + C$$

$$I_{64} = \int \cos^2 u \, du$$

$$= \int \frac{1 + \cos 2u}{2} \, du$$

$$= \frac{1}{2} \cdot \int 1 + \cos 2u \, du$$

$$= \frac{1}{2} \cdot \left(u + \frac{1}{2} \sin 2u \right) + C$$

$$= \frac{1}{2} u + \frac{1}{4} \sin 2u + C$$

$$I_{77} = \int \sec^n u \, du$$

$$\text{Integrate by parts} \quad \begin{cases} \sec^2 u = (\tan u)' \\ (\sec^{n-2} u)' = (n-2) \sec^{n-3} u \sec u \tan u \end{cases}$$

$$= (\tan u) (\sec^{n-2} u) - \int (\tan u) ((n-2) \sec^{n-3} u \sec u \tan u) \, du$$

$$= \tan u \sec^{n-2} u - (n-2) \int \sec^{n-2} \tan^2 u \, du$$

$$= \tan u \sec^{n-2} u - (n-2) \int \sec^{n-2} (\sec^2 u - 1) \, du$$

$$= \tan u \sec^{n-2} u - (n-2) \left(\int \sec^n u \, du - \int \sec^{n-2} u \, du \right)$$

$$(1 + (n-2)) \int \sec^n u \, du = \tan u \sec^{n-2} u + (n-2) \int \sec^{n-2} u \, du$$

$$\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$$

64

$$\int \cos^2 u \, du = \frac{1}{2} u + \frac{1}{4} \sin 2u + C$$

$$\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du \quad 77$$

$$I_{76} = \int \cot^n u \, du$$

$$= \int \cot^{n-2} u \cot^2 u \, du$$

$$= \int \cot^{n-2} u (\csc^2 u - 1) \, du$$

$$= \int \cot^{n-2} u \csc^2 u \, du - \int \cot^{n-2} u \, du$$

$$\text{Let } v = \cot u \quad \longrightarrow \quad dv = -\csc^2 u \, du$$

$$= - \int v^{n-2} u \, dv - \int \cot^{n-2} u$$

$$= -\frac{-1}{n-1} v^{n-1} - \int \cot^{n-2} u$$

$$= \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u$$

$$\int \cot^n u \, du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du \quad (76)$$

$$I_{65} = \int \tan^2 u \, du$$

$$\text{Rewrite using trigonometric identity } \tan^2 u = \sec^2 u - 1$$

$$= \int \sec^2 u - 1 \, du$$

$$= \tan u - u + C$$

$$\int \tan^2 u \, du = \tan u - u + C \quad (65)$$

$$I_{66} = \int \cot^2 u \, du$$

Rewrite using trigonometric identity $\tan^2 u = \csc^2 u - 1$

$$\begin{aligned} &= \int \csc^2 u - 1 \, du \\ &= -\cot u - u + C \end{aligned}$$

$$I_{75} = \int \tan^n u \, du$$

$$\begin{aligned} &= \int \tan^{n-2} u \tan^2 u \, du \\ &= \int \tan^{n-2} u (\sec^2 u - 1) \, du \\ &= \int \tan^{n-2} u \sec^2 u \, du - \int \tan^{n-2} u \, du \end{aligned}$$

Let $v = \tan u \longrightarrow dv = \sec^2 u \, du$

$$\begin{aligned} &= \int v^{n-2} u \, dv - \int \tan^{n-2} u \\ &= \frac{1}{n-1} v^{n-1} - \int \tan^{n-2} u \\ &= \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \end{aligned}$$

(66)

$$\int \cot^2 u \, du = -\cot u - u + C$$

$$\int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du$$

(75)

$$I_{74} = \int \cos^n u \, du$$

$$\begin{aligned} \text{Integrate by parts} \quad & \left\{ \begin{array}{l} (\cos^{n-1} u)' = -(n-1) \cos^{n-2} u \sin u \\ \cos u = (\sin u)' \end{array} \right. \\ & = (\cos^{n-1} u) (\sin u) - \int (-(n-1) \cos^{n-2} u \sin u) (\sin u) \, du \\ & = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \sin^2 u \, du \\ & = \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u (1 - \cos^2 u) \, du \\ & = \cos^{n-1} u \sin u + (n-1) \left(\int \cos^{n-2} u \, du - \int \cos^n u \, du \right) \end{aligned}$$

$$\begin{aligned} 1 + (n-1) \int \cos^n u \, du &= \cos^{n-1} u \sin u + (n-1) \int \cos^{n-2} u \, du \\ \int \cos^n u \, du &= \frac{1}{n} \cos^{n-1} u \sin u + \frac{(n-1)}{n} \int \cos^{n-2} u \, du \end{aligned}$$

$$(74) \quad \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$$

$$\begin{aligned} I_{67} &= \int \sin^3 u \, du \\ &= \int (1 - \cos^2 u) \sin u \, du \\ &= \int \sin u \, du - \int \cos^2 u \sin u \, du \\ \text{Let } v &= \cos u \quad \longrightarrow \quad dv = -\sin u \, du \\ &= \int v^2 \, dv - \cos u \\ &= \frac{1}{3} v^3 - \cos u + C \\ &\quad \text{Reverse the substitution} \\ &= \frac{1}{3} \cos^3 u - \cos u + C \\ &\quad \text{(which we could leave right there)} \\ &= \left(\frac{1}{3} \cos^2 u - 1 \right) \cos u + C \\ &= \left(\frac{1}{3} (1 - \sin^2 u) - 1 \right) \cos u + C \\ &= -\frac{1}{3} (2 + \sin^2 u) \cos u + C \end{aligned}$$

$$\int \sin^3 u \, du = -\frac{1}{3} (2 + \sin^2 u) \cos u + C$$

$$I_{68} = \int \cos^3 u \, du$$

$$= \int (1 - \sin^2 u) \cos u \, du$$

$$= \int \cos u \, du - \int \sin^2 u \cos u \, du$$

$$\text{Let } v = \sin u \longrightarrow dv = \cos u \, du$$

$$= \sin u - \int v^2 \, dv$$

$$= \sin u - \frac{1}{3} v^3 + C$$

Reverse the substitution

$$= \sin u - \frac{1}{3} \sin^3 u + C$$

(which we could leave right there)

$$= \left(1 - \frac{1}{3} \sin^2 u\right) \sin u + C$$

$$= \left(1 - \frac{1}{3}(1 - \cos^2 u)\right) \sin u + C$$

$$= \frac{1}{3}(2 + \cos^2 u) \sin u + C$$

$$I_{73} = \int \sin^n u \, du$$

$$\text{Integrate by parts} \left\{ \begin{array}{l} (\sin^{n-1} u)' = (n-1) \sin^{n-2} u \cos u \\ \sin u = (-\cos u)' \end{array} \right.$$

$$= (\sin^{n-1} u)(-\cos u) - \int ((n-1) \sin^{n-2} u \cos u)(-\cos u) \, du$$

$$= -\sin^{n-1} u \cos u + (n-1) \int \sin^{n-2} u \cos^2 u \, du$$

$$= -\sin^{n-1} u \cos u + (n-1) \int \sin^{n-2} u (1 - \sin^2 u) \, du$$

$$= -\sin^{n-1} u \cos u + (n-1) \left(\int \sin^{n-2} u \, du - \int \sin^n u \, du \right)$$

$$1 + (n-1) \int \sin^n u \, du = -\sin^{n-1} u \cos u + (n-1) \int \sin^{n-2} u \, du$$

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{(n-1)}{n} \int \sin^{n-2} u \, du$$

(68)

$$\int \cos^3 u \, du = \frac{1}{3}(2 + \cos^2 u) \sin u + C$$

(73)

$$\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$$

$$I_{72} = \int \csc^3 u \, du$$

$$\text{Integrate by parts} \quad \begin{cases} (\csc u)' = -\csc u \cot u \\ \csc^2 u = (-\cot u)' \end{cases}$$

$$= (\csc u)(-\cot u) - \int (-\csc u \cot u)(-\cot u) \, du$$

$$= -\csc u \cot u - \int \csc u \cot^2 u \, du$$

$$= -\csc u \cot u - \int \csc u (\csc^2 u - 1) \, du$$

$$= -\csc u \cot u - \int \csc^3 u \, du + \int \csc u \, du$$

$$2 \int \csc^3 u \, du = -\csc u \cot u + \int \csc u \, du$$

$$\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \int \csc u \, du$$

$$= \frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

$$(72) \quad \int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln |\csc u - \cot u| + C$$

$$I_{69} = \int \tan^3 u \, du$$

$$= \int (\sec^2 u - 1) \tan u \, du$$

$$= \int (\sec^2 u \tan u) \, du - \int \tan u \, du$$

$$\text{Let } v = \tan u \quad \longrightarrow \quad dv = \sec^2 u \, du$$

$$= \int v \, dv - \ln |\sec u|$$

$$= \frac{1}{2} v^2 - \ln |\sec u| + C$$

Reverse the substitution

$$= \frac{1}{2} \tan^2 u - \ln |\sec u| + C$$

(which we could leave right there)

$$= \frac{1}{2} \tan^2 u + \ln |\cos u| + C$$

$$\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln |\cos u| + C$$

$$I_{70} = \int \cot^3 u \, du$$

$$= \int (\csc^2 u - 1) \cot u \, du$$

$$= \int (\csc^2 u \cot u) \, du - \int \cot u \, du$$

$$\text{Let } v = \cot u \longrightarrow dv = -\csc^2 u \, du$$

$$= - \int v \, dv - \ln|\sin u|$$

$$= -\frac{1}{2}v^2 - \ln|\sin u| + C$$

Reverse the substitution

$$= -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$$

(which we could leave right there)

$$= -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$$

$$I_{71} = \int \sec^3 u \, du$$

$$\text{Integrate by parts} \quad \begin{cases} (\sec u)' = \sec u \tan u \\ \sec^2 u = (\tan u)' \end{cases}$$

$$= (\sec u)(\tan u) - \int (\sec u \tan u)(\tan u) \, du$$

$$= \sec u \tan u - \int \sec u \tan^2 u \, du$$

$$= \sec u \tan u - \int \sec u (\sec^2 u - 1) \, du$$

$$= \sec u \tan u - \int \sec^3 u \, du + \int \sec u \, du$$

$$2 \int \sec^3 u \, du = \sec u \tan u + \int \sec u \, du$$

$$\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \int \sec u \, du$$

$$= \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln|\sec u + \tan u| + C$$

(70)

$$\int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln|\sin u| + C$$

(71)

$$\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln|\sec u + \tan u| + C$$