Exercise 2.7.1

Check that S(f i , f j) = 0 for all pairs $1 \leq i < j \leq 5$ in Example 1.

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   SageMath version 9.2, Release Date: 2020-10-24

>>> R.<x,y>=PolynomialRing(QQ,2,order='deglex')

>>> F=[x^3-2*x*y, x^2*y-2*y^2+x, -x^2, -2*x*y, -2*y^2+x]

>>>
>>> load("buch.sage")
>>> for i in [1..5] :
        for j in [i+1..5] :
          fi = F[i-1]
          fj = F[j-1]
          print("For ",(i,j)," Spoly = ", Spoly(fi,fj),
    end=" ")
          print(" \tremainder = ", div(Spoly(fi,fj), F))
   For  (1, 2)  Spoly =  -x^2      remainder =  0
   For  (1, 3)  Spoly =  -2*x*y    remainder =  0
   For  (1, 4)  Spoly =  -2*x*y^2          remainder =  0
   For  (1, 5)  Spoly =  2*x^2*y^2 - 2*x*y
   remainder =  0
   For  (2, 3)  Spoly =  x - 2*y^2         remainder =  0
   For  (2, 4)  Spoly =  x - 2*y^2         remainder =  0
   For  (2, 5)  Spoly =  2*x*y^3 + x - 2*y^2
   remainder =  0
   For  (3, 4)  Spoly =  0         remainder =  0
   For  (3, 5)  Spoly =  2*x*y^2   remainder =  0
   For  (4, 5)  Spoly =  2*y^3     remainder =  0

>>>
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