```
   Macaulay2, version 1.15
   --loading configuration for package "FourTiTwo" from
   file /u03/heconn/.Macaulay2/init-FourTiTwo.m2
   --loading configuration for package "Topcom" from file
   /u03/heconn/.Macaulay2/init-Topcom.m2
   with packages: ConwayPolynomials, Elimination,
   IntegralClosure, InverseSystems,
                            LLLBases,
   PrimaryDecomposition, ReesAlgebra, TangentCone,
                            Truncations

i1 : load("div.m2")
i3 : R=QQ[x,y,z]

    o3 = R

    o3 : PolynomialRing

i4 : div(x*y, [x+z, y-z], 0)  -- 0 for Lex

   quotients = {y, -z}
    o4 = -z^2

    o4 : QQ[x, y, z, w]

i5 : div(x*y, [y-z, x+z], 0)

   quotients = {x, z}
    o5 = -z^2

    o5 : QQ[x, y, z, w]
```

So the remainders are the same, but the quotients are different.

Since the remainder is not 0, the polynomial $x\,y$ is not in the ideal.