Albert Aparicio Isarn

394109230

CLP Lab 1. Estudio Previo

2 $\underline{\underline{C}} = \frac{1}{d}\sigma^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$. Obtener autovalores teóricos en función de ρ y σ^2

$\underline{d=2}$ $\underline{\det(A - \lambda I) = 0}$

$$\det(\underline{\underline{C}} - \lambda \underline{\underline{I}}) = \det\left(\frac{1}{2}\sigma^2\begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix}\right) = \begin{vmatrix} \frac{1}{2}\sigma^2 - \lambda & \frac{\rho\sigma^2}{2} \\ \frac{\sigma^2\rho}{2} & \frac{\sigma^2}{2} - \lambda \end{vmatrix} = 0$$

$$\left(\frac{\sigma^2}{2} - \lambda\right)^2 - \left(\frac{\sigma^2\rho}{2}\right)^2 = 0$$

$$\frac{\sigma^4}{4} + \lambda^2 - \lambda \cdot 2\frac{\sigma^2}{2} - \frac{\sigma^4\rho^2}{4} = 0$$

$$\lambda^2 - \lambda \cdot \sigma^2 + \left(\frac{\sigma^4}{4} - \frac{\sigma^4\rho^2}{4}\right) = 0$$

$$\underline{\lambda^2 - \lambda \cdot \sigma^2 + \frac{\sigma^4}{4}(1-\rho^2) = 0}$$

$$\lambda = \frac{\sigma^2 \pm \sqrt{(-\sigma^2)^2 - \cancel{4} \cdot 1 \cdot \left(\frac{\sigma^4}{\cancel{4}}(1-\rho^2)\right)}}{2 \cdot 1}$$

$$\lambda = \frac{\sigma^2 \pm \sqrt{\sigma^4 - \sigma^4(1-\rho^2)}}{2} = \frac{\sigma^2 \pm \sqrt{\sigma^4 \cdot \rho^2}}{2}$$

$$\underline{\lambda = \frac{\sigma^2 \pm \sigma^2\rho}{2}}$$

$$\boxed{\lambda_1 = \frac{\sigma^2 + \sigma^2\rho}{2} = \frac{\sigma^2}{2}(1+\rho)}$$

$$\boxed{\lambda_2 = \frac{\sigma^2 - \sigma^2\rho}{2} = \frac{\sigma^2}{2}(1-\rho)}$$

Obs.:

relación entre ejes de las elipses (cortes horizontales de la pdf gaussiana) es la relación entre los $\underline{\lambda_i}$