

1. a) $a \cdot b = 1 \times 2 + 6 \times 5 + 6 \times (-5) = 24$

$$\|a\| \|b\| \cos \theta = 24$$

$$\cos \theta = \frac{24}{\sqrt{1^2+6^2+6^2} \cdot \sqrt{2^2+5^2+(-5)^2}} = \frac{24}{\sqrt{26} \cdot \sqrt{54}} = \frac{24}{6\sqrt{6} \cdot 3\sqrt{6}} = \frac{24}{108} = \frac{2}{9}$$

b) $a \cdot b = 11 \times 11 + (-3) \times 5 + (-13) \times (-1) + 5 \times 5 = 100$

$$\cos \theta = \frac{100}{\sqrt{11^2+(-3)^2+(-13)^2+5^2} \cdot \sqrt{11^2+5^2+(-1)^2+5^2}} = \frac{100}{18\sqrt{100}} = \frac{100}{18 \times 10} = \frac{5}{9}$$

c) $a \cdot b = (-2) \times (-9) + 6 \times 5 + (-3) \times 2 + (-6) \times (-5) + 6 \times 3 = 90$

$$\cos \theta = \frac{90}{\sqrt{(-2)^2+6^2+(-3)^2+(-6)^2+6^2} \cdot \sqrt{(-9)^2+5^2+2^2+(-5)^2+3^2}} = \frac{90}{11 \cdot 12} = \frac{15}{22}$$

2. a) $AC = [2 \times 5 + (-5) \times 2, 2 \times 3 + (-5) \times 2, 2 \times 4 + (-5) \times 3] = [0, -4, -7]$

b) $AB^T = [2 \times 5 + (-5) \times 11, 2 \times 0 + (-5) \times (-5), 2 \times 3 + (-5) \times 1] = [-25, 25, 1]$

c) $BC = \begin{bmatrix} 5 \times 5 + 11 \times 2 & 5 \times 3 + 11 \times 2 & 5 \times 4 + 11 \times 3 \\ 0 \times 5 + (-5) \times 2 & 0 \times 3 + (-5) \times 2 & 0 \times 4 + (-5) \times 3 \\ 3 \times 5 + 1 \times 2 & 3 \times 3 + 1 \times 2 & 3 \times 4 + 1 \times 3 \end{bmatrix} = \begin{bmatrix} 39 & 29 & 41 \\ -10 & -10 & -15 \\ 17 & 11 & 15 \end{bmatrix}$

d) $CD = \begin{bmatrix} 5 \times (-2) + 3 \times (-11) + 4 \times 8 & 5 \times 5 + 3 \times (-4) + 4 \times (-2) & 5 \times 1 + 3 \times 3 + 4 \times (-3) & 5 \times 0 + 3 \times 2 + 4 \times (-1) \\ 2 \times (-2) + 2 \times (-11) + 3 \times 8 & 2 \times 5 + 2 \times (-4) + 3 \times (-2) & 2 \times 1 + 2 \times 3 + 3 \times (-3) & 2 \times 0 + 2 \times 2 + 3 \times (-1) \end{bmatrix}$

$$= \begin{bmatrix} 1 & 5 & 2 & 2 \\ 6 & -4 & -1 & 1 \end{bmatrix}$$

e) $B^T D = \begin{bmatrix} 5 \times (-2) + 0 \times (-11) + 3 \times 8 & 5 \times 5 + 0 \times (-4) + 3 \times (-2) & 5 \times 1 + 0 \times 3 + 3 \times (-3) & 5 \times 0 + 0 \times 2 + 3 \times (-1) \\ 11 \times (-2) + (-5) \times (-11) + 1 \times 8 & 11 \times 5 + (-5) \times (-4) + 1 \times (-2) & 11 \times 1 + (-5) \times 3 + 1 \times (-3) & 11 \times 0 + (-5) \times 2 + 1 \times (-1) \end{bmatrix} = \begin{bmatrix} 14 & 19 & -4 & -3 \\ 29 & 53 & -11 & -11 \end{bmatrix}$

$$3. a) \frac{d}{dx} f(x) = 4x^3 - 6x^2 + 4$$

$$b) \frac{d}{dt}(f(t)) = 5x^3t^4 - 21xt^2 + 5$$

$$c) \frac{d}{ds} f(s) = 5a^2bs^4 - 3bs^2 - 2acs + bc^2$$

$$4. a) F(x) = \sqrt{x^2+2x} \quad G(x) = x^2+2x, \quad F(x) = \sqrt{G(x)}$$

$$\frac{dF(x)}{dG(x)} = \frac{1}{2} (G(x))^{-\frac{1}{2}}, \quad \frac{dG(x)}{dx} = 2x+2$$

$$\frac{d}{dx} F(x) = \frac{dF(x)}{dG(x)} \cdot \frac{dG(x)}{dx} = \frac{2x+2}{2\sqrt{x^2+2x}} = \frac{x+1}{\sqrt{x^2+2x}}$$

$$b) f(x) = \left(\frac{1}{2}x+1\right)^2 (x^2-x+1)^4 \quad g(x) = \left(\frac{1}{2}x+1\right)^2, \quad h(x) = (x^2-x+1)^4$$

$$\frac{d}{dx} f(x) = \frac{d}{dx} g(x) \cdot h(x) = g(x) h'(x) + g'(x) h(x)$$

$$b(x) = \frac{1}{2}x+1, \quad g(x) = (b(x))^2, \quad \frac{d}{dx} b(x) = \frac{1}{2}, \quad g'(x) = \frac{d g(x)}{d b(x)} \cdot \frac{d b(x)}{dx} = 2 \cdot b(x) \cdot \frac{1}{2}$$

$$= b(x) = \frac{1}{2}x+1$$

$$c(x) = x^2-x+1, \quad h(x) = (c(x))^4, \quad \frac{d}{dx} c(x) = 2x-1, \quad \frac{d}{dx} h(x) = \frac{d h(x)}{d c(x)} \cdot \frac{d c(x)}{dx}$$

$$= 4(c(x))^3 (2x-1) = 4(x^2-x+1)^3 (2x-1)$$

$$g(x) h'(x) + g'(x) h(x) = 4 \cdot \left(\frac{1}{2}x+1\right)^2 \cdot (x^2-x+1)^3 (2x-1) + \left(\frac{1}{2}x+1\right) (x^2-x+1)^4$$

$$= \frac{1}{2} (x+2) (x^2-x+1)^3 (5x^2+5x-3)$$