Bayesian Basics:
A Gentle introduction

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What is Bayesian Statistics?

What is Bayesian Statistics?

What is Bayesian Statistics **Not**?

Frequentist Statistics

PHILOSOPHICAL
TRANSACTIONS:

An Essay towards Solving a Problem in the Doctrine of Chances. By the Late Rev. Mr. Bayes, F. R. S. Communicated by Mr. Price, in a Letter to John Canton, A. M. F. R. S.

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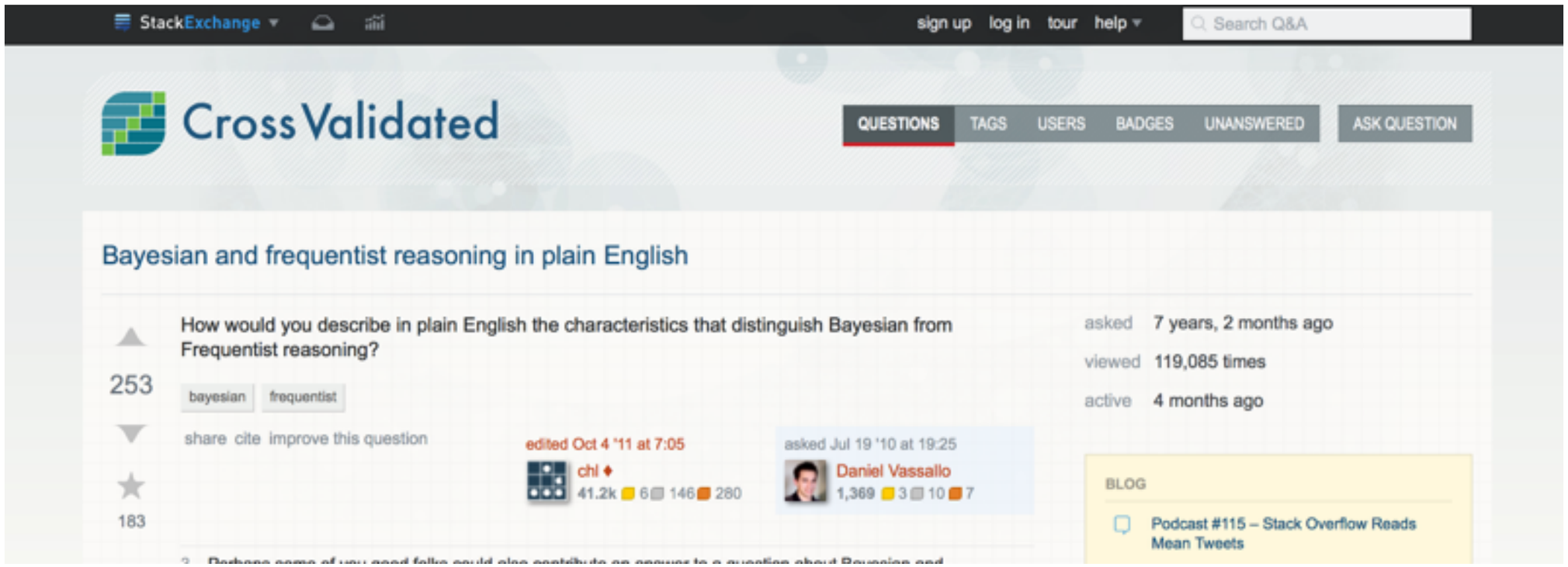
Modern Title:

A Method of Calculating the Exact Probability of All Conclusions founded on Induction

A Crowd-Sourced Intuitive Explanation

A Popular Question on **stackexchange.com:**



https://stats.stackexchange.com/questions/22/bayesian-and-frequentist-reasoning-in-plain-english

A Crowd-Sourced Intuitive Explanation

A Popular Question on **stackexchange.com:**

“How would you describe in plain English the characteristics that distinguish Bayesian from Frequentist reasoning?”

https://stats.stackexchange.com/questions/22/bayesian-and-frequentist-reasoning-in-plain-english

A Crowd-Sourced Intuitive Explanation

The Most Popular **Answer**:

Problem:

I have misplaced my phone somewhere in the home. I can use the phone locator on the base of the instrument to locate the phone and when I press the phone locator the phone starts beeping.

Which area of my home should I search?

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A Crowd-Sourced Intuitive Explanation

The Most Popular **Answer**:

Frequentist Reasoning

I can hear the phone beeping. I also have a mental model which helps me identify the area from which the sound is coming. Therefore, upon hearing the beep, I infer the area of my home I must search to locate the phone.

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A Crowd-Sourced Intuitive Explanation

The Most Popular **Answer**:

Bayesian Reasoning

I can hear the phone beeping. Now, apart from a mental model which helps me identify the area from which the sound is coming from, ***I also know the locations where I have misplaced the phone in the past.*** *So, I combine my inferences using the beeps and my prior information about the locations I have misplaced the phone in the past to identify an area I must search to locate the phone.*

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Example #2:
A Mystery Cough

Data: A Doctor’s patient keeps coughing

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Hypothesis #1:
The patient has a cold

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Inferential Question:
What is the cause of the cough (a cold or chewing gum?)

Example #2:
A Mystery Cough

Bayesian statistics divides inference into two main ingredients:

1. The **Prior** probability of a hypothesis

2. The **Likelihood** of data under a hypothesis

Bayes Theorem (see later slide) provides **a formula for combining these two source of information** when performing inference.

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The likelihood of observing the data if the hypothesis were true

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P(Data | Hypothesis)

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Likelihood:

P(Data | Hypothesis # 1) = High or Low?

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Likelihood:

P(Data | Hypothesis # 1) = ***High***

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A Mystery Cough

Data: A Doctor's patient keeps coughing

Hypothesis #1: The patient has a cold

Hypothesis #2: The patient has chewing gum in the lungs

Likelihood:

P(Data | Hypothesis # 2) = High or Low?

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Data: A Doctor's patient keeps coughing

Hypothesis #1: The patient has a cold

Hypothesis #2: The patient has chewing gum in the lungs

Likelihood:

P(Data | Hypothesis # 2) = ***High***

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A Mystery Cough

Data: A Doctor's patient keeps coughing

	Hypothesis #1: The patient has a cold	**Hypothesis #2:** The patient has chewing gum in the lungs
Likelihood: P(Data | Hypothesis)	*High*	*High*

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	+	+
Prior: P(Hypothesis)	*High*	*Low*
	=	=
	High	*Medium*

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Likelihood: P(Data | Hypothesis)	*High*	*High*
	+	+
Prior: P(Hypothesis)	*High*	*Low*
	=	=
Winner! →	*High*	*Medium*

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Two problems with the formulation above:

1: We're just adding highs and lows — is there a better formula?

2: We chose a single winner — is there a better selection technique?

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Bayes' Rule!

P(Hypothesis | Data) = P(Data | Hypothesis) x P(Hypothesis)

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↑

likelihood

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prior

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solution:
Sampling (repeated guessing)

Recap: In Bayesian Statistics:

- Every Hypothesis must be given a **Prior** probability: P(H)

- Every Hypothesis implies a **Likelihood** for the data: P(D | H)

- **Bayes Theorem** provides the recipe for combining **Prior** and **Likelihood** information optimally

- Bayesian statistics allows you to probabilistically **sample answers** to an inferential question, **rather than chose one single winner**

- **advanced exercises in the notebook —>**

Further Reading / Viewing / **Coding**:

How do you do a Bayesian Mixed Effects GLM in R?

Check out Bradley Rentz's Excellent tutorial on RSTANARM

SLIDES:

https://github.com/rentzb/bayesian-pape

R MARKDOWN:

https://github.com/rentzb/bayesian-pape/blob/master/presentation/intro_rstanarm.Rmd

Further Reading / Viewing / **Coding**:

How do you do a Bayesian inference in Python using PYMC3?

Check out the notebook:

Bayesian Inference with PYMC3 -- Inferring the Size - Pitch Relationship.ipynb

Further Reading / **Viewing** / Coding:

Looking for more intuitive explanations?

Check out:

A 10-minute intuitive video explainer of Bayes by Julia Galef (writer and founder of the Centre for Applied Rationality in New York):

A Visual Guide to Bayesian Thinking

https://www.youtube.com/watch?v=BrK7X_XlGB8

Further **Reading** / Viewing / Coding:

How is Bayesian inference used in the cognitive sciences?

Check out:

Tom Griffiths's Bayesian reading list:

http://cocosci.berkeley.edu/tom/bayes.html