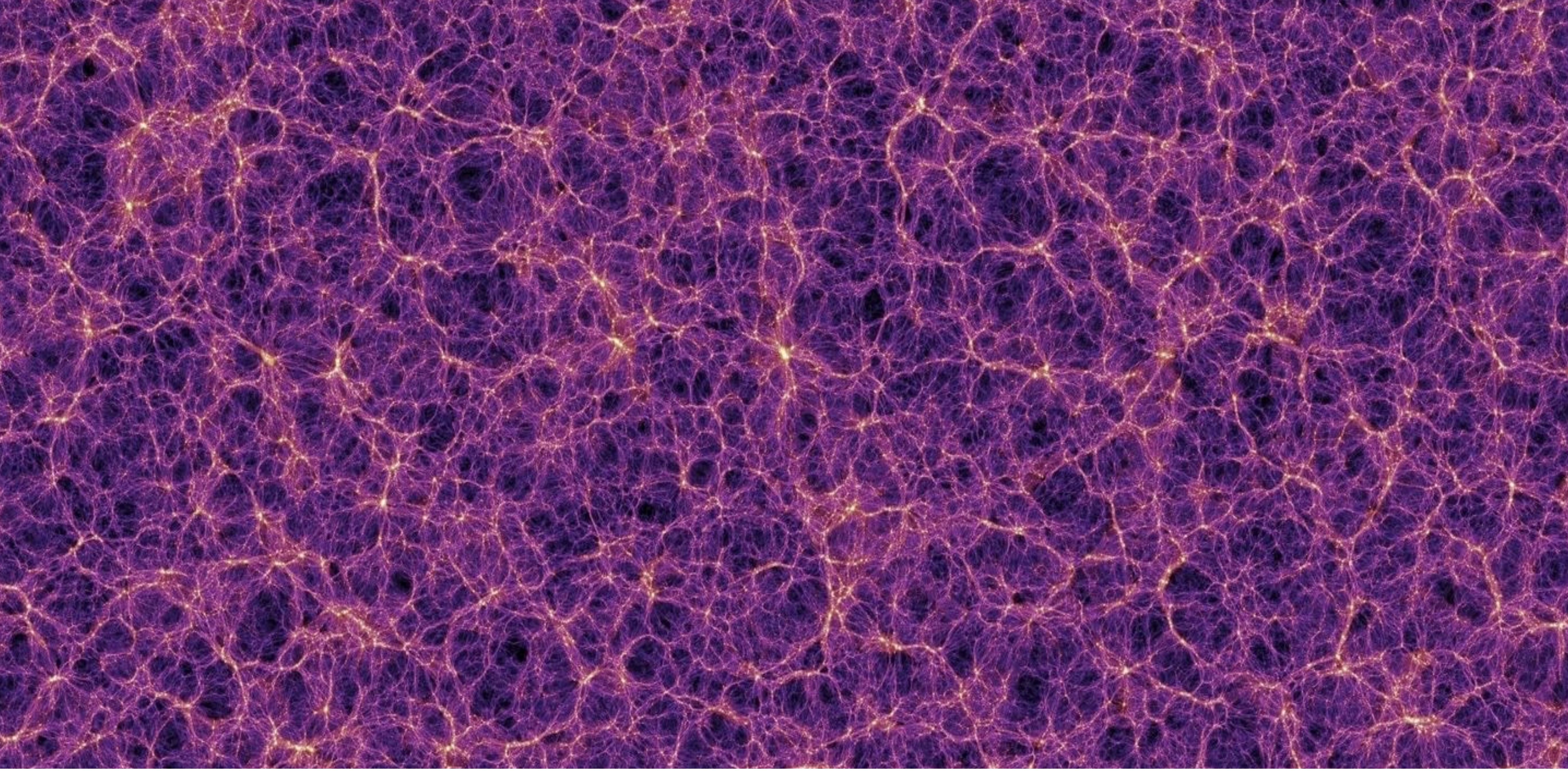

Computing inflationary predictions in general scalar-tensor theories

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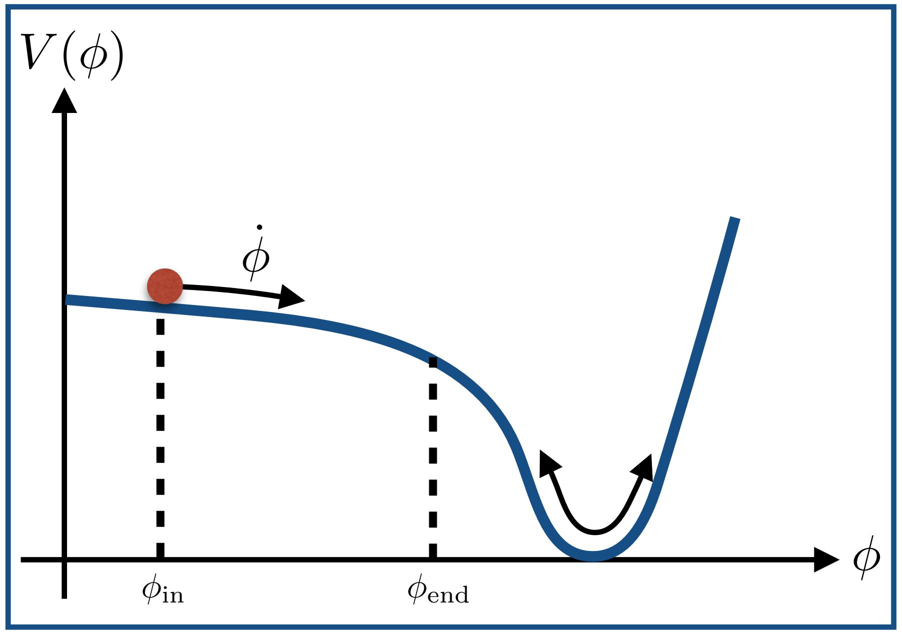
Slow-roll inflation in one slide



- $$\epsilon_H = -\frac{\dot{H}}{H^2}$$
- $$\eta_H = \frac{1}{2}\frac{\mathrm{d}\,\ln\epsilon_H}{\mathrm{d}N} - \epsilon_H$$

Slow-roll conditions:

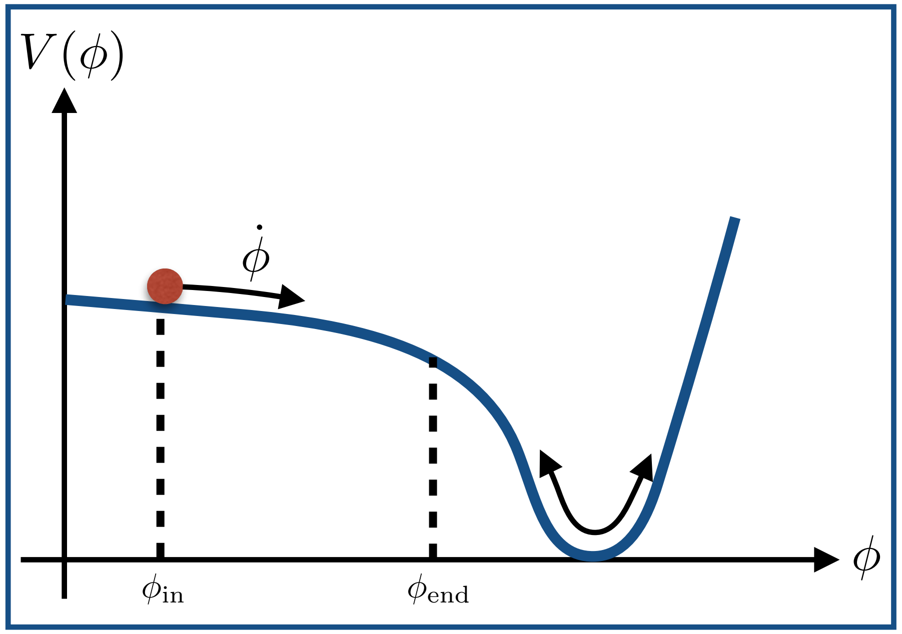
$$\epsilon_H \ll 1$$

$$|\eta_H| \ll 1$$

Number of *e*-folds of inflation:

$$N_{\rm CMB} = \int_{\phi_{\rm CMB}}^{\phi_{\rm end}} \frac{\mathrm{d}\phi}{M_{\rm pl}\sqrt{2\epsilon}} \approx 40 - 60$$

Slow-roll inflation in one slide



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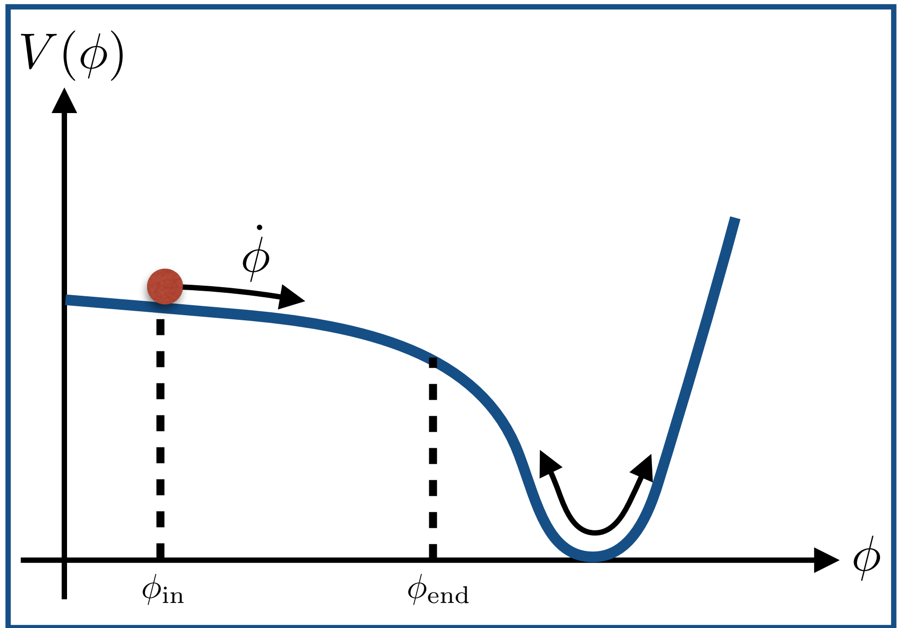
$$N_{\rm CMB} = \int_{\phi_{\rm CMB}}^{\phi_{\rm end}} \frac{{\rm d}\phi}{M_{\rm pl}\sqrt{2\epsilon}} \approx 40 - 60$$

- $$S_\zeta^{(2)} = \int {\rm d}^4 x a^3 \epsilon_H \left(\dot\zeta^2 - \frac{k^2}{a^2}\zeta^2\right)$$ Scalars
- $$S_\gamma^{(2)} = \sum_{\lambda=+,\times} \int {\rm d}^4 x \frac{a^3}{4}\left(\dot\gamma_\lambda^2 - \frac{k^2}{a^2}\gamma_\lambda^2\right)$$ Tensors

Mukhanov - Sasaki equation:

$$\boxed{v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0}$$

Slow-roll inflation in one slide



- $$\epsilon_H = -\frac{\dot H}{H^2}$$
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$\longleftrightarrow$

Slow-roll conditions:

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Number of *e*-folds of inflation:

$$N_{\rm CMB} = \int_{\phi_{\rm CMB}}^{\phi_{\rm end}} \frac{\mathrm{d}\phi}{M_{\rm pl}\sqrt{2\epsilon}} \approx 40-60$$

- $$S^{(2)}_\zeta = \int \mathrm{d}^4x a^3 \epsilon_H \left(\dot\zeta^2 - \frac{k^2}{a^2}\zeta^2\right)$$ Scalars
- $$S^{(2)}_\gamma = \sum_{\lambda=+,\times}\int \mathrm{d}^4x \frac{a^3}{4}\left(\dot\gamma_\lambda^2 - \frac{k^2}{a^2}\gamma_\lambda^2\right)$$ Tensors

Mukhanov - Sasaki equation:

$$\boxed{v_k'' + \left(k^2 - \frac{z''}{z}\right)v_k = 0}$$

Inflationary parameters:

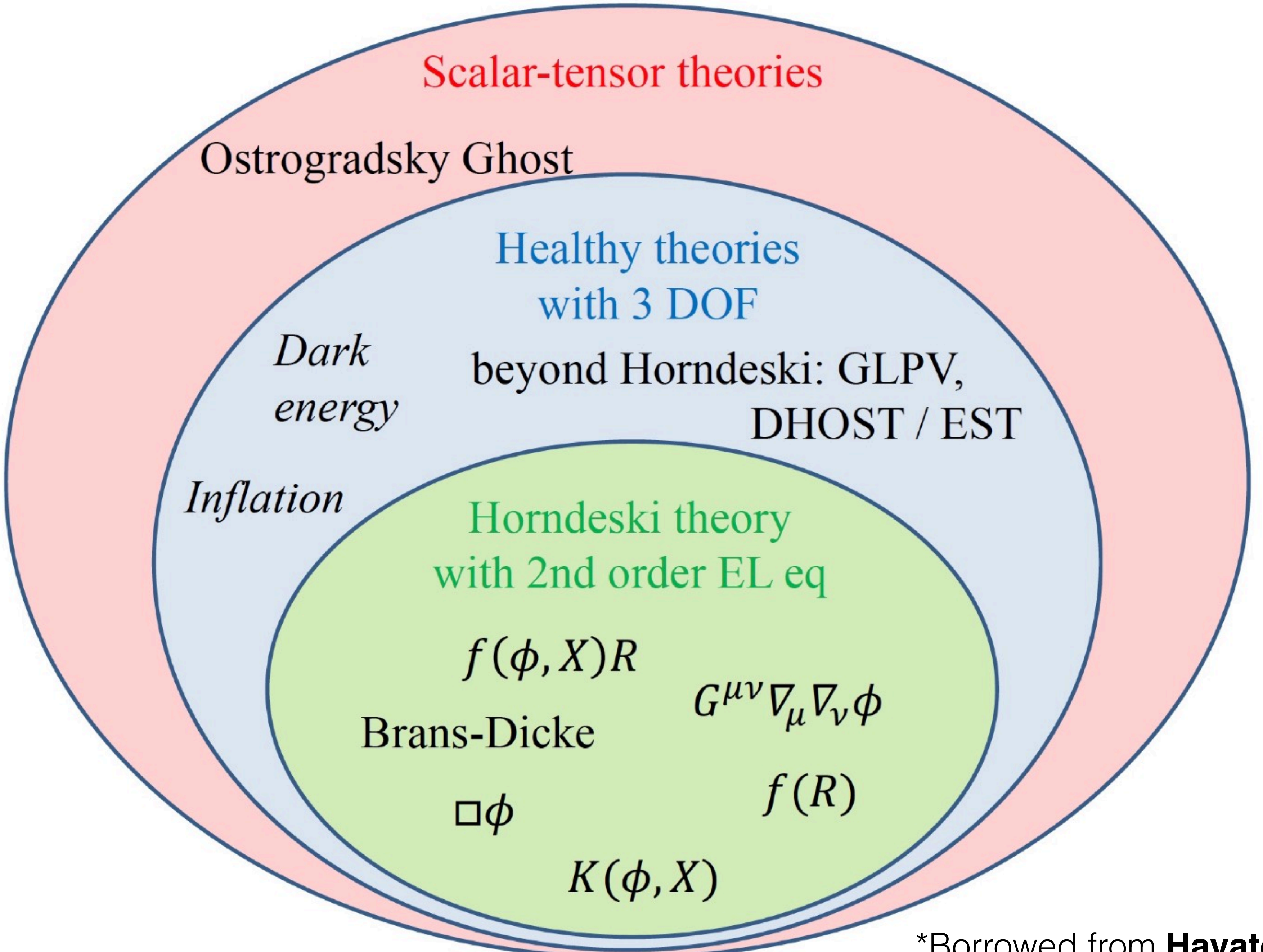
- $$r \equiv \frac{\Delta^2_\gamma}{\Delta^2_\zeta}$$
- $$n_s - 1 \equiv \frac{\mathrm{d}\,\ln\Delta^2_\zeta}{\mathrm{d}\,\ln k}$$

Outline:

1. Scalar - Tensor theories.

2. Generalized slow-roll (GSR) techniques.

3. G-inflation.



*Borrowed from **Hayato Motohashi**'s talks

Ostrogradski's theorem:

"Higher-derivative theories contain extra degrees of freedom, and are usually plagued by negative energies and related instabilities."

Horndeski theory:

- G. Horndeski (1974)
- A. Nicolis *et al.*; 0811.2197
- C. Deffayet *et al.*; 1103.3260

$$S = \sum_{i=2}^{5} \int \mathrm{d}^4 x \sqrt{-g} \mathcal{L}_i$$

$$X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

$$G_i = G_i(\phi, X)$$

$$\mathcal{L}_2 = G_2 \ ,$$

$$\mathcal{L}_3 = G_3 \Box \phi \ ,$$

$$\mathcal{L}_4 = G_4 R - 2G_{4,X} [(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \ ,$$

$$\mathcal{L}_5 = G_5 G^{\mu\nu}\phi_{;\mu\nu} + \frac{G_{5,X}}{3} [(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}] \ .$$

Applications to:

- Late-time Cosmology (dark energy).
- Early-universe Cosmology (inflation).

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Canonical inflation:

$$G_2 = -\frac{X}{2} - V(\phi) \ , \qquad G_4 = \frac{M_{\mathrm{pl}}^2}{2} \ ,$$

$$G_3 = 0 \ , \qquad G_5 = 0 \ .$$

Inflation

- T. Kobayashi *et al.*; 1105.5723
- H. Motohashi, W. Hu; 1704.01128

$$S_\zeta^{(2)} = \int \mathrm{d}^4 x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$S_\gamma^{(2)} = \sum_{\lambda=+,\times} \int \mathrm{d}^4 x \frac{a^3 b_t}{4 c_t^2} \left(\dot{\gamma}_\lambda^2 - \frac{c_t^2 k^2}{a^2} \gamma_\lambda^2 \right)$$

In canonical inflation:

$$b_s = b_t = c_s = c_t = 1$$

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$\frac{\mathrm{d}^2 v}{\mathrm{d}\tau^2} + \left(c_s^2 k^2 - \frac{1}{z} \frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2} \right) v = 0$$

* Mukhanov - Sasaki equation

- $v = z\zeta$
- $z = a\sqrt{\frac{2 b_s \epsilon_H}{c_s^2}}$

- Assume slow-roll approximation.
- Solve numerically.
- Use GSR techniques.

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \, \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

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Outline:

1. Scalar - Tensor theories.

2. Generalized slow-roll (GSR) techniques.

3. G-inflation.

Generalized Slow-Roll

- E. Stewart; 0110322
- C. Dvorkin, W. Hu; 0910.2237
- W. Hu; 1104.4500
- W. Hu; 1405.2020
- H. Motohashi, W. Hu; 1503.04810
- H. Motohashi, W. Hu; 1704.01128

… and others.

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

de Sitter background

deviations from dS

- $y \equiv \sqrt{2 c_s k} v$
- $x \equiv k s_s$
- $s_s \equiv \int c_s \mathrm{d}\tau$

- $f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2}} \frac{a H s_s}{c_s}$
- $\Delta_\zeta^2(k) = \lim_{x \to 0} \left| \frac{xy}{f} \right|^2$

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

3. Apply Green function techniques (GSR).

$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

- $$G = -2\ln f + \frac{2}{3}\left(\ln f\right)'$$ Source function
- $$W(u) = \frac{3\sin(2u)}{2u^3} - \frac{3\cos(2u)}{u^2} - \frac{3\sin(2u)}{2u}$$

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
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$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

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$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

4. Taylor expand GSR formula and write down analytic equations (OSR).

Optimized SR for Horndeski (leading order):

- H. Motohashi, W. Hu; 1704.01128

$$\ln \Delta_\zeta^2 \approx \ln\left(\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 - \frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1}\Big|_{x=x_1}$$

$$n_s - 1 \approx -4\epsilon_H - 2\delta_1 - \sigma_{s1} - \xi_{s1} - \frac{2}{3}\delta_2 - \frac{7}{3}\sigma_{s2} - \frac{1}{3}\xi_{s2}\Big|_{x=x_1}$$

$$\alpha_s \approx -2\delta_2 - \sigma_{s2} - \xi_{s2} - \frac{2}{3}\delta_3 - \frac{7}{3}\sigma_{s3} - \frac{1}{3}\xi_{s3} - 8\epsilon_H^2 - 10\epsilon_H\delta_1 + 2\delta_1^2\Big|_{x=x_1}$$

Scalars

$$\ln \Delta_\gamma^2 \approx \ln\left(\frac{H^2}{2\pi^2 b_t c_t}\right) - \frac{8}{3}\epsilon_H - \frac{7}{3}\sigma_{t1} - \frac{1}{3}\xi_{t1}\Big|_{x=x_1}$$

$$n_t \approx -2\epsilon_H - \sigma_{t1} - \xi_{t1} - \frac{7}{3}\sigma_{t2} - \frac{1}{3}\xi_{t2}\Big|_{x=x_1}$$

$$\alpha_t \approx -\sigma_{t2} - \xi_{t2} - \frac{7}{3}\sigma_{t3} - \frac{1}{3}\xi_{t3} - 4\epsilon_H^2 - 4\epsilon_H\delta_1\Big|_{x=x_1}$$

Tensors

$$\boxed{\ln x_1 \equiv \frac{7}{3} - \ln 2 - \gamma_{\rm E}}$$

Optimized SR for Horndeski (leading order):

- H. Motohashi, W. Hu; 1704.01128

$$r \equiv \frac{4\Delta_\gamma^2}{\Delta_\zeta^2} \approx 16\epsilon_H \frac{b_s c_s}{b_t c_t} \approx -\frac{8 b_s c_s}{b_t c_t} n_t,$$

Deviations from the standard consistency relation in the observations could be checked in this context.

Outline:

1. Scalar - Tensor theories.

2. Generalized slow-roll (GSR) techniques.

3. G-inflation.

$$G_3 = \mathrm{const}$$

K. Kamada, T. Kobayashi, M. Yamaguchi, J. Yokoyama; 1012.4238

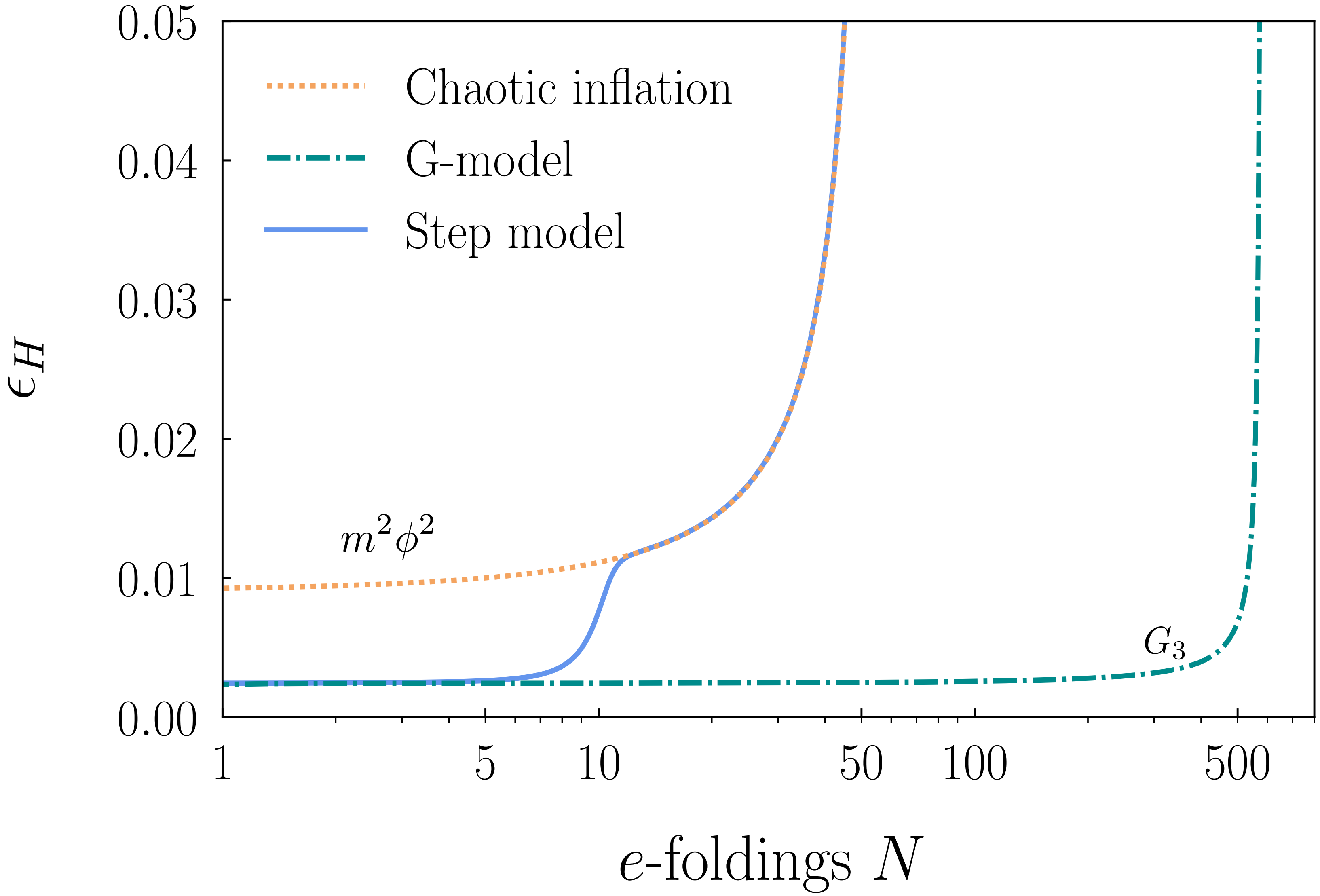
J. Ohashi, S. Tsujikawa; 1207.4879

HR, S. Passaglia, H. Motohashi, W. Hu, O. Mena;
JCAP 1804 (2018) no.04, 039

G_3 + *tanh* + chaotic inflation = transient G-inflation

$$\mathcal{L}_2 = G_2\ ,$$
$$\mathcal{L}_3 = G_3 \Box\phi\ ,$$
$$\mathcal{L}_4 = G_4 R - 2G_{4,X}[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}]\ ,$$
$$\mathcal{L}_5 = G_5 G^{\mu\nu}\phi_{;\mu\nu} + \frac{G_{5,X}}{3}[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}]\ .$$

$$G_2 = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2\ ,\ \boxed{G_3 = M^{-3}\left[1 + \tanh\left(\frac{\phi - \phi_r}{d}\right)\right]}\ ,\ G_4 = \frac{1}{2}M_{\rm pl}^2\ ,\ \ G_5 = 0$$



Step size ~ 4 *e*-folds

Ideal for OSR

HR, S. Passaglia, H. Motohashi, W. Hu, O. Mena;
JCAP 1804 (2018) no.04, 039

G_3 + *tanh* + chaotic inflation
= transient G-inflation

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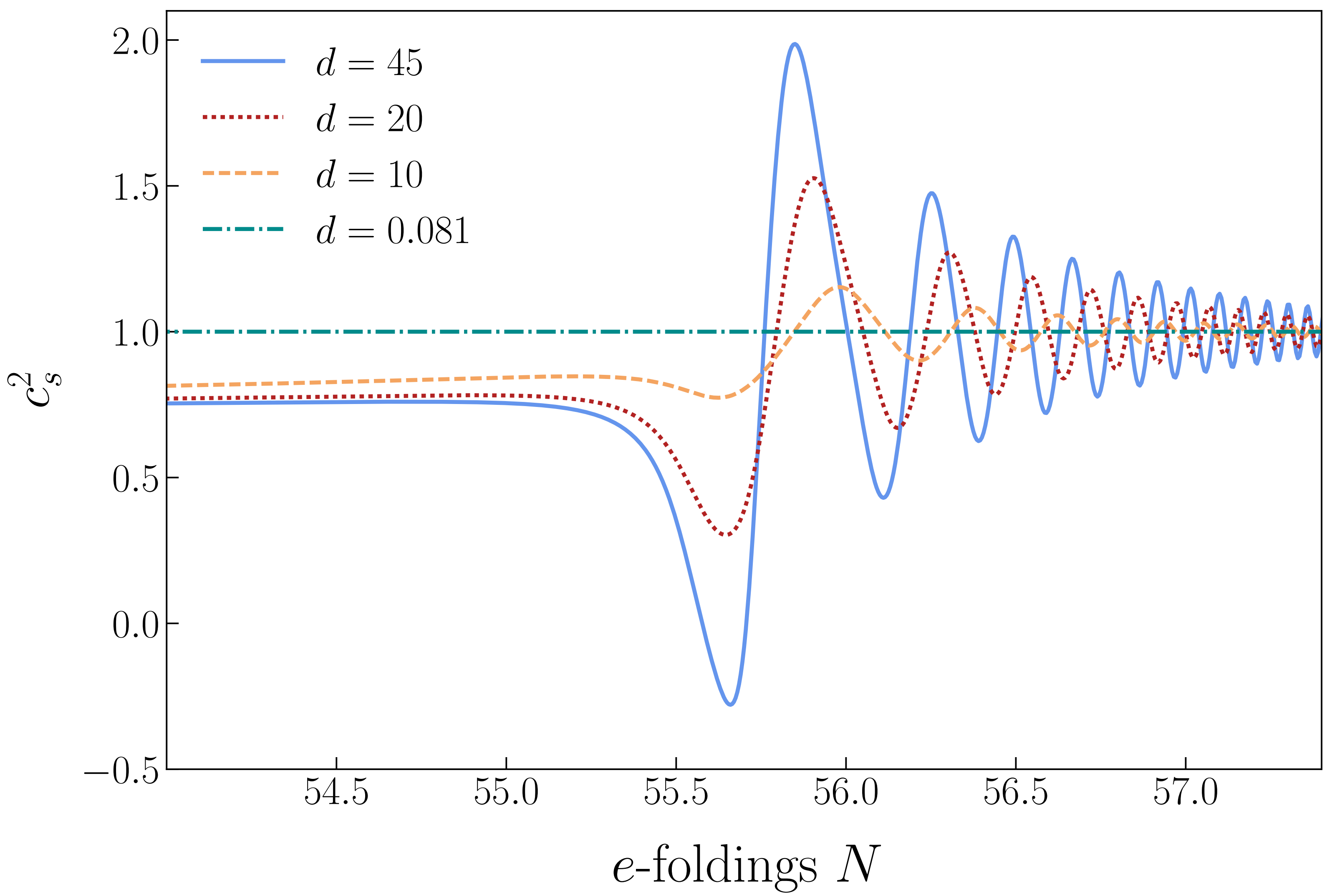
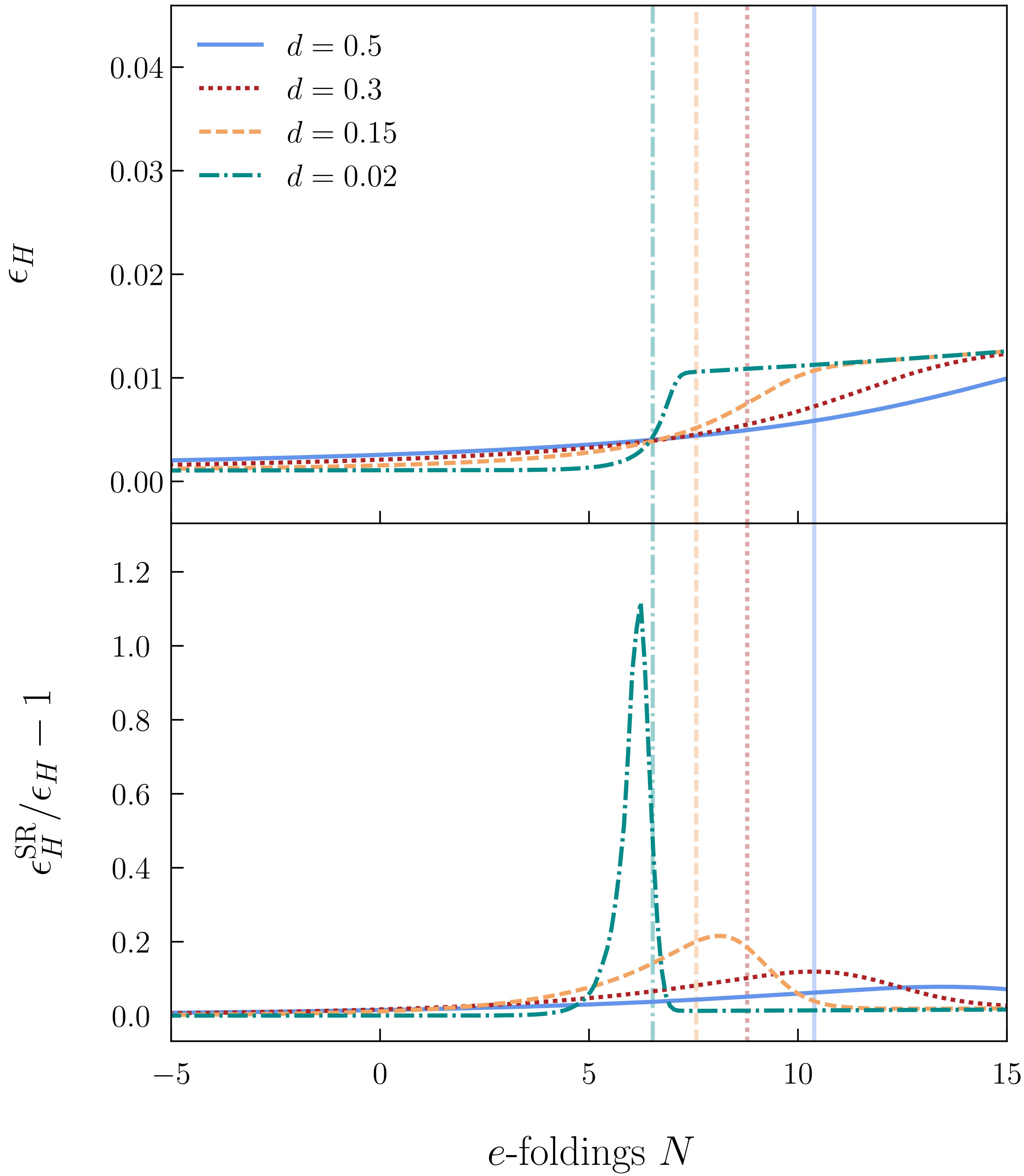


$$\phi_r = 13$$
$$M = 1.3 \times 10^{-4}$$

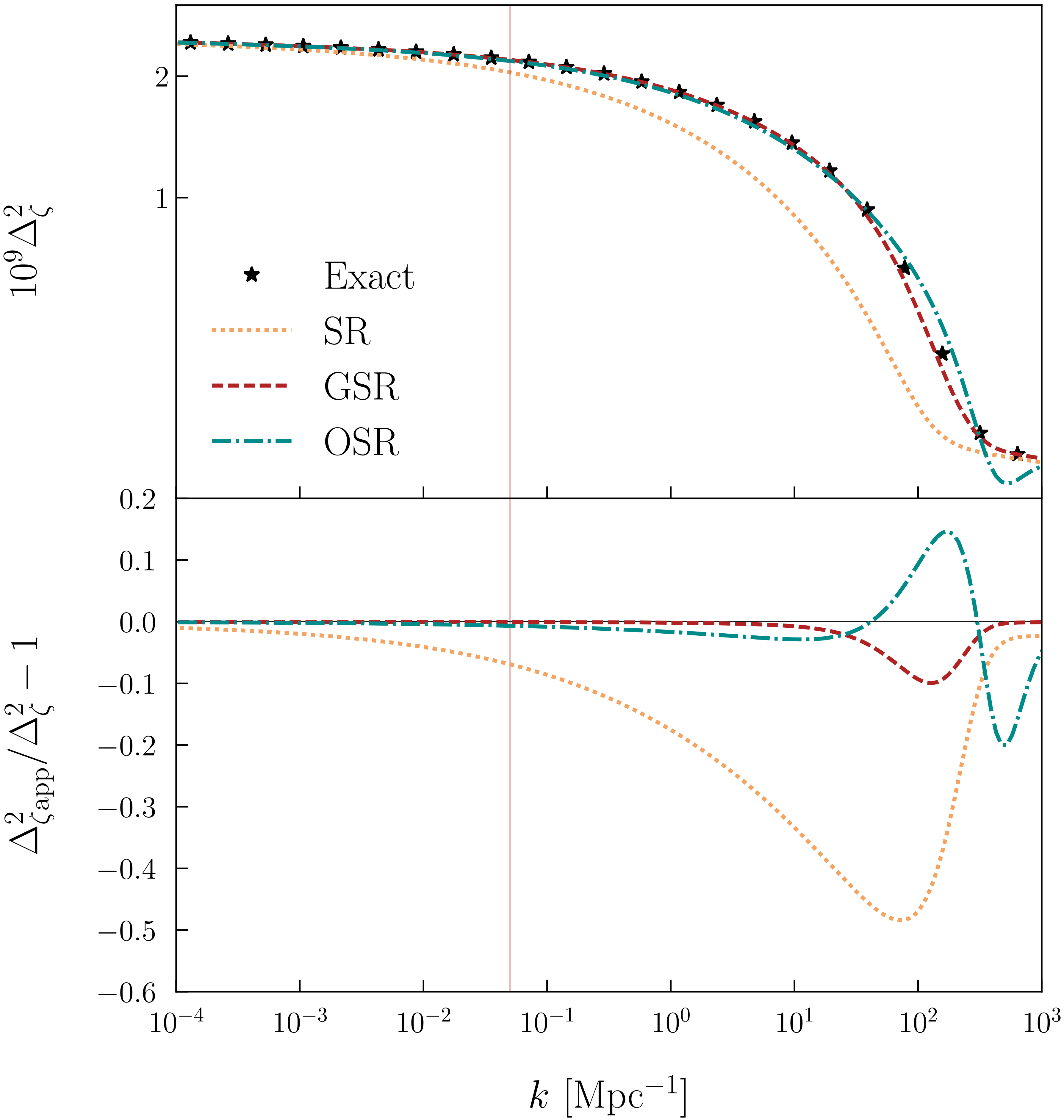
No gradient instabilities for any mass scale M.



Slow - roll violation:

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

- $N = 0$: CMB scales.
- Vertical lines: where the transition occurs.
- SR violation is maximal around the transition.



Different approximations

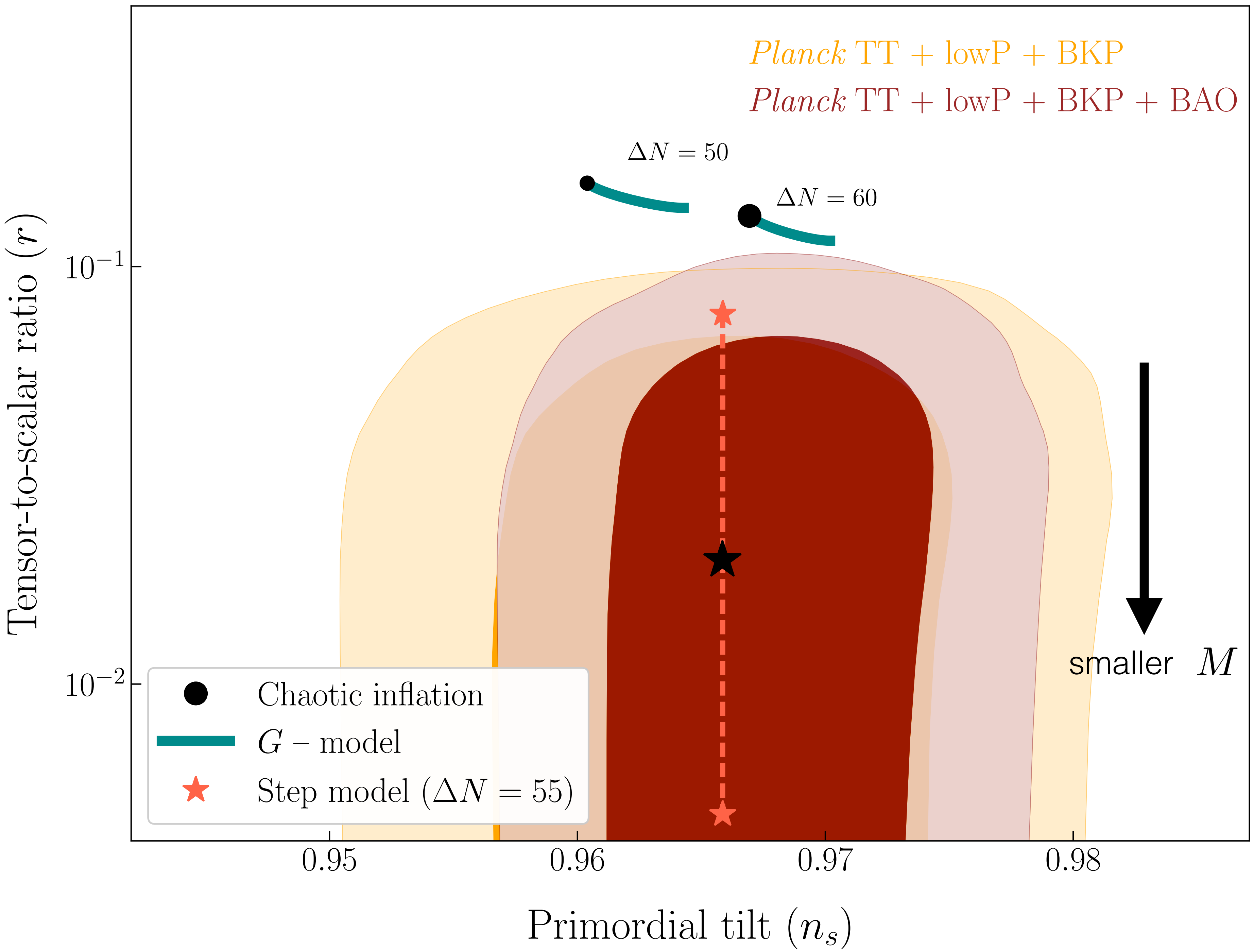
$d = 0.086$

$M = 1.3 \times 10^{-4}$

SR ✗

GSR ✓

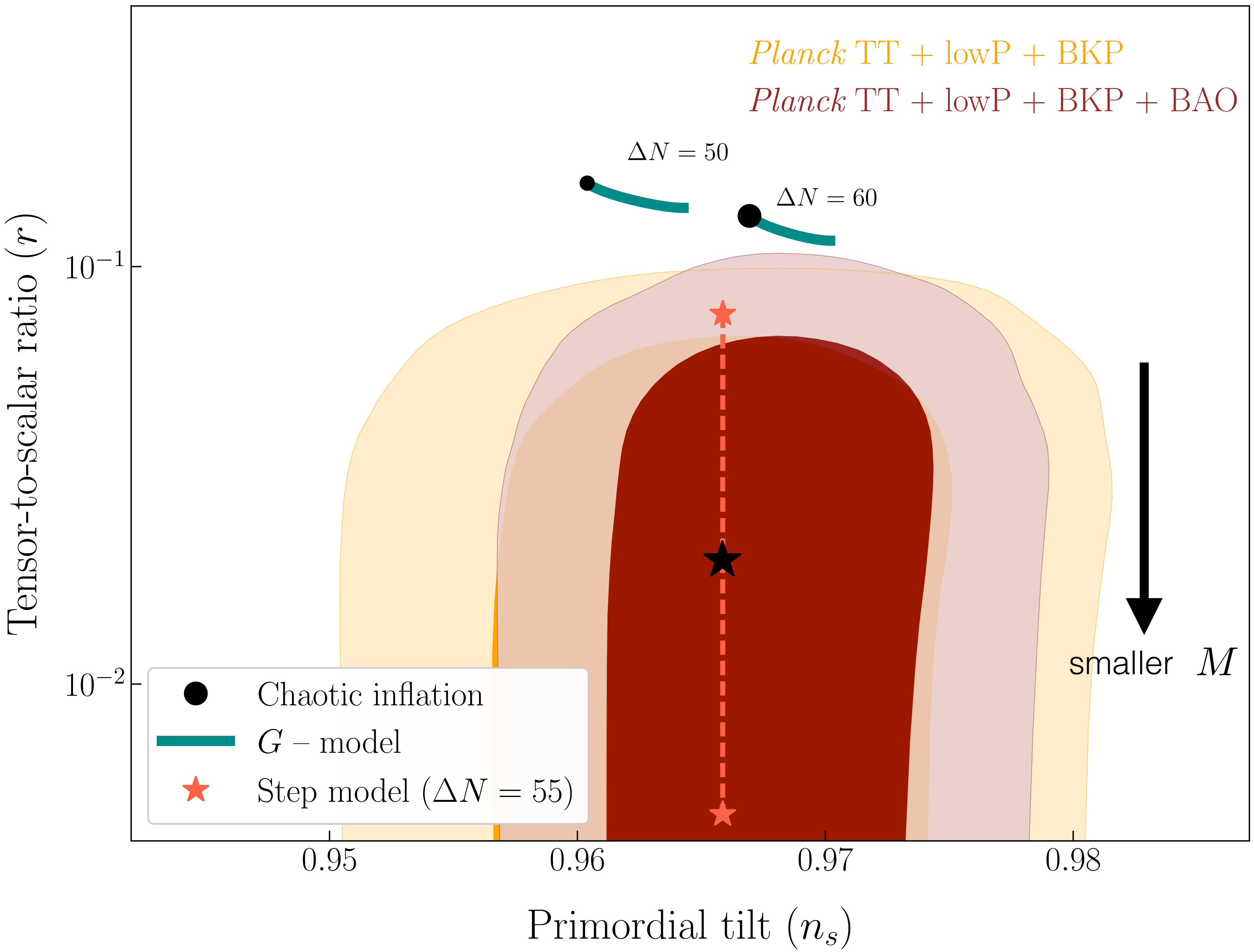
OSR ✓



Observables

$n_s = 9.66$
$\alpha_s = -0.011$

- n_s and α_s fixed.
- Find a set of values for d and ϕ_r.
- This places lower and upper bounds on r.



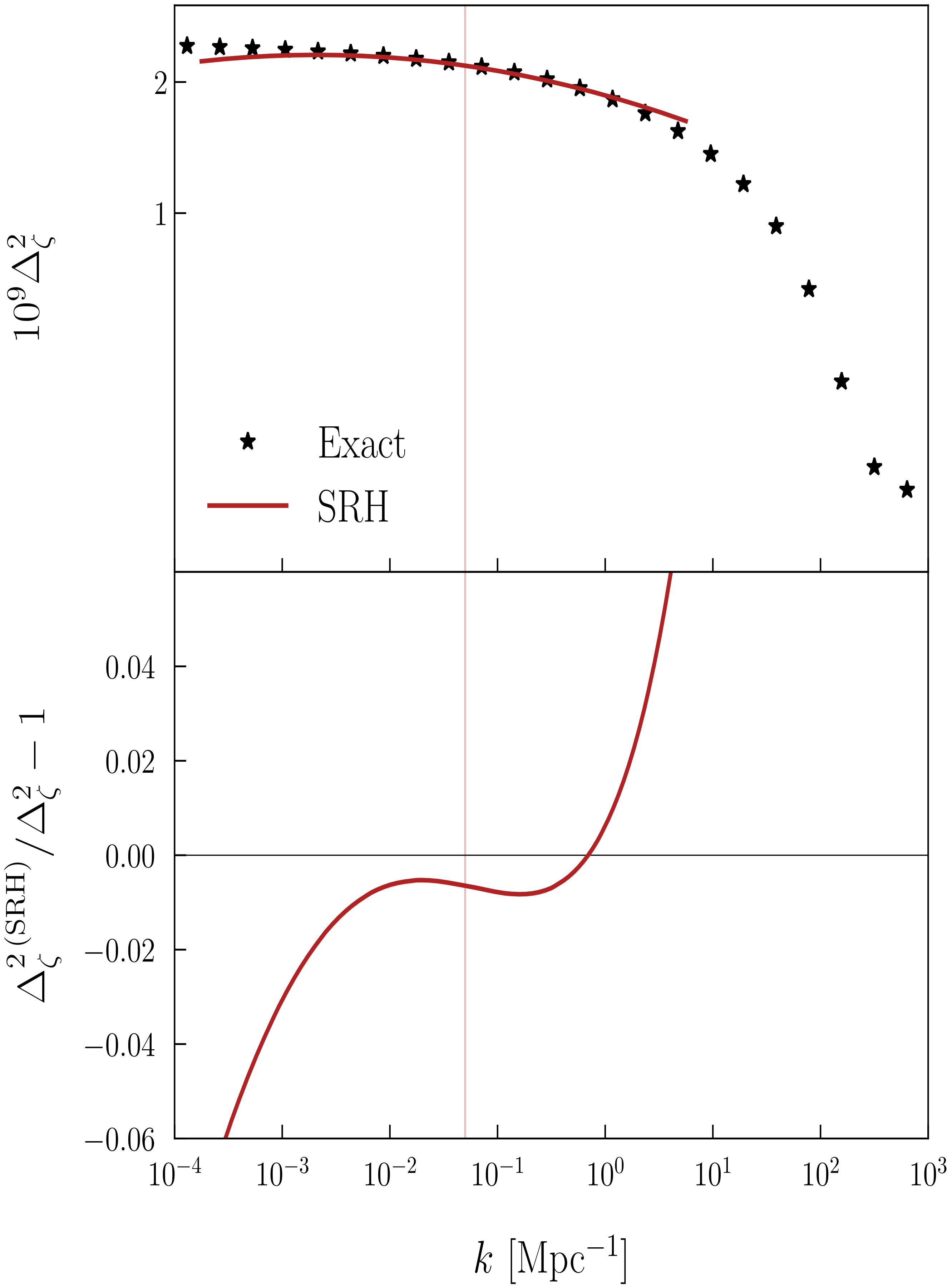
Observables

$$n_s = 9.66$$
$$\alpha_s = -0.011$$

$$\boxed{r \gtrsim 0.005}$$

Potentially observable with next-generation surveys.

- A smaller α_s would shift the line upwards because the step gets wider.
- A larger α_s would be in tension with measurements.



Slow - roll hierarchy formula:

$$\Delta_\zeta^{2(\mathrm{SRH})}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k_*)}$$

- Using OSR parameters

Deviations of less than 1%

Summary

- Inflation in the Horndeski framework is viable and can *cure* some popular models.

- The transient G-inflation model allows us to compute observables during a G-inflation period and to end inflation as canonical.

- Generalized slow-roll and Optimized slow-roll techniques are efficient tools for this type of models to compute the power spectra an also the bispectrum.