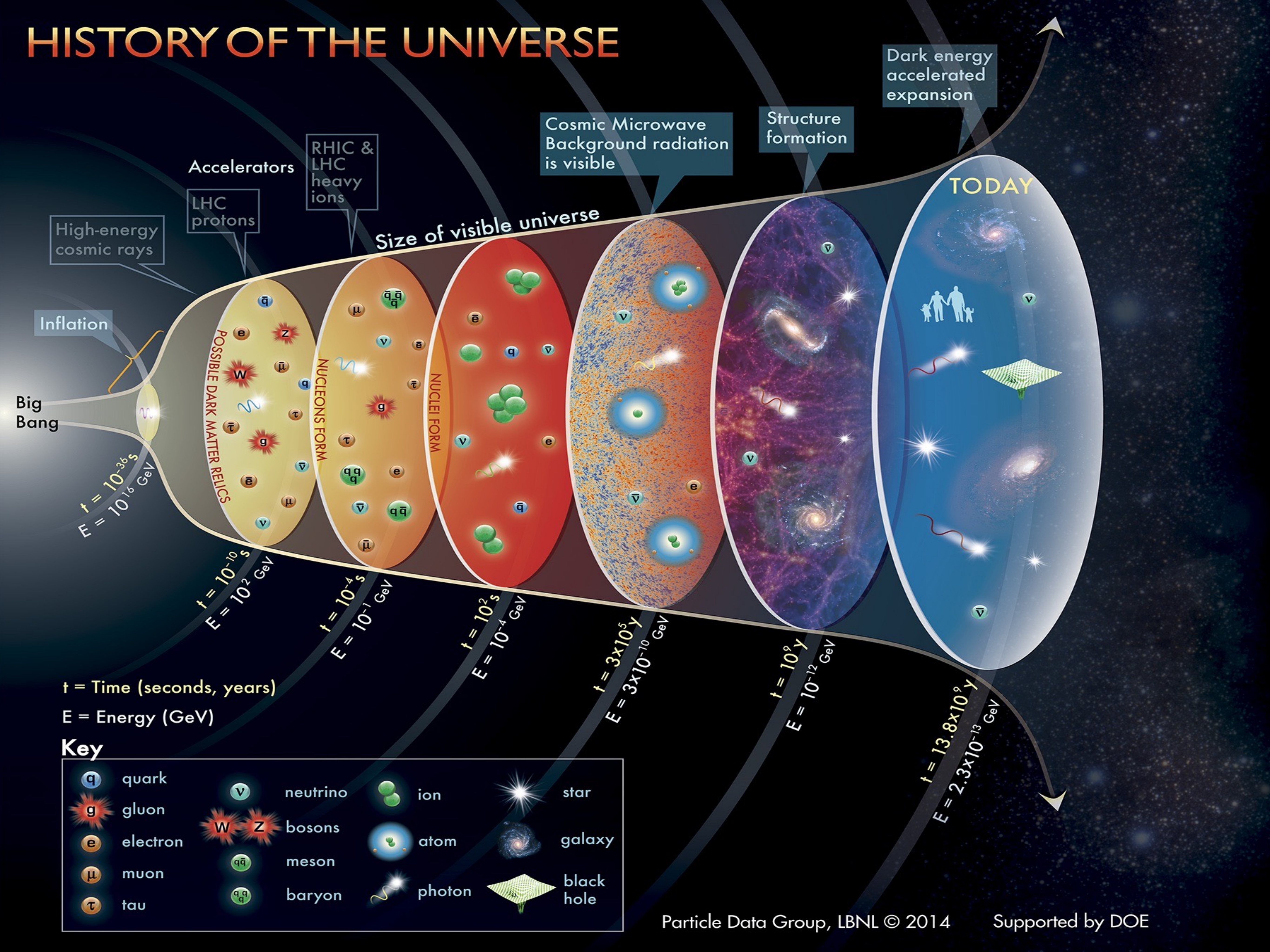
Single-Field Slow-Roll Inflation

Héctor A. Ramírez R.

Student Seminar

Outline

1. Problems of the Standard Big Bang Theory.
2. Classical dynamics of single-field slow-roll inflation.
3. Quantum fluctuations during inflation.
4. The present of the most-favoured models.
5. The non-minimally coupled chaotic model.
6. Future constraints.



HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Inflation
Big Bang
Size of visible universe
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^{-36} s
E = 10^{16} GeV
t = 10^{-10} s
E = 10^{2} GeV
t = 10^{-4} s
E = 10^{-1} GeV
t = 10^{2} s
E = 10^{-4} GeV
t = $3x10^{5}$ y
E = $3x10^{-10}$ GeV
t = 10^{9} y
E = 10^{-12} GeV
t = $13.8x10^{9}$ y
E = $2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
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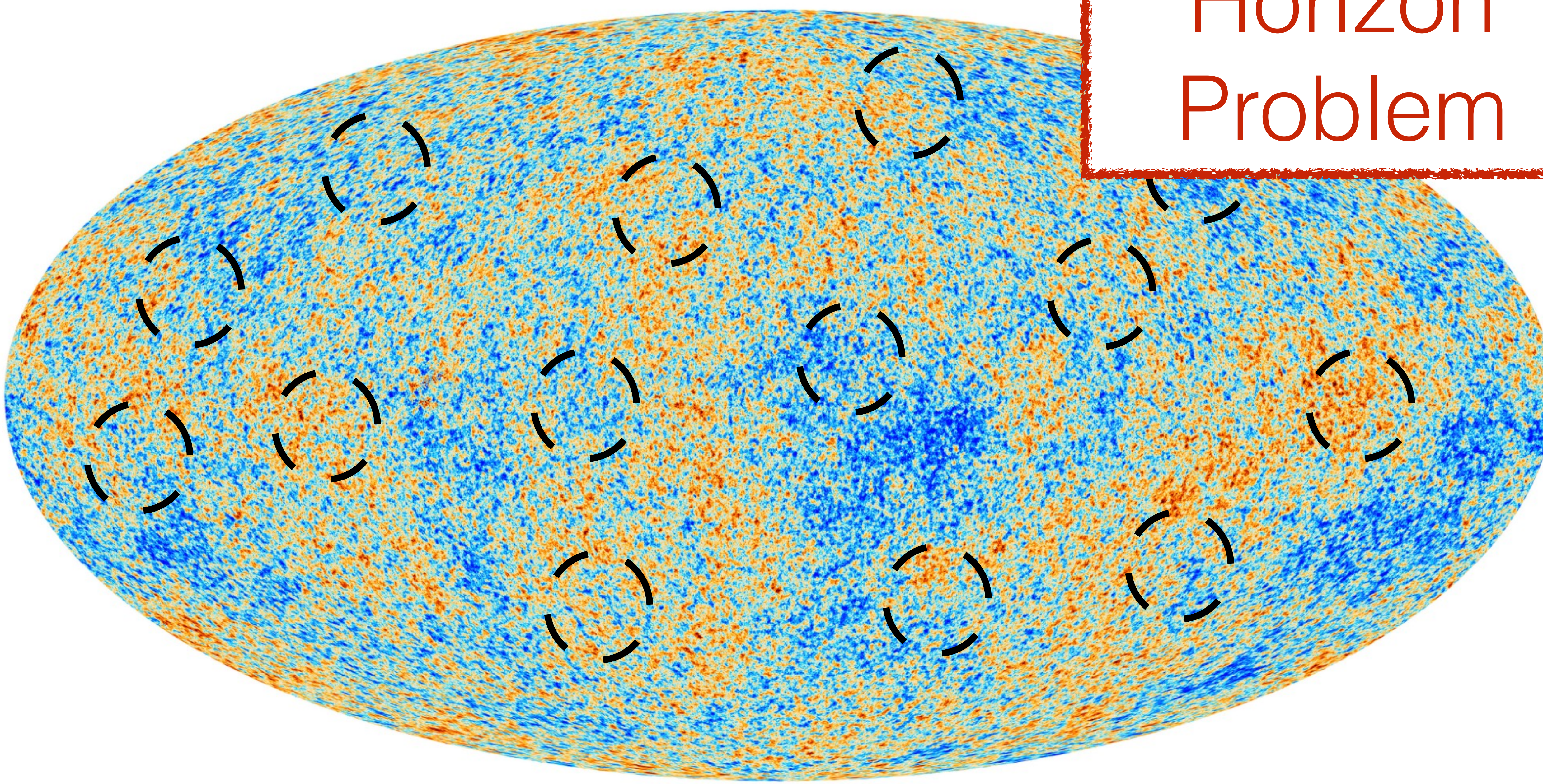
Two points in the sky with an angular separation exceeding 2 degrees should never have been in causal contact, yet they are observed to have the same temperature to high

Cosmic Microwave Background

Horizon Problem



Anisotropies at $\sim 10^{-5}$

10^4 Causally independent patches

Friedmann Equation:

$$H^2 = \frac{1}{3}\rho(a) - \frac{k}{a^2}$$

$$\Omega(a) = \frac{\rho(a)}{\rho_{\mathrm{crit}}(a)}$$

$$|1 - \Omega(a)| = \frac{k}{(aH)^2}$$

<0.01 right now!

$$(aH)^{-1}$$

Grows with time in ΛCDM

extreme fine-tuning

$$|1 - \Omega(a_{\mathrm{pl}})| \leq \mathcal{O}(10^{-61})$$

Why $\Omega(a_0) \sim \mathcal{O}(1)$ and not smaller or larger?

Flatness Problem

Inflationary universe: A possible solution to the horizon and flatness problems

Alan H. Guth*

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 11 August 1980)

The standard model of hot big-bang cosmology requires initial conditions which are problematic in two ways: (1) The early universe is assumed to be highly homogeneous, in spite of the fact that separated regions were causally disconnected (horizon problem); and (2) the initial value of the Hubble constant must be fine tuned to extraordinary accuracy to produce a universe as flat (i.e., near critical mass density) as the one we see today (flatness problem). These problems would disappear if, in its early history, the universe supercooled to temperatures 28 or more orders of magnitude below the critical temperature for some phase transition. A huge expansion factor would then result from a period of exponential growth, and the entropy of the universe would be multiplied by a huge factor when the latent heat is released. Such a scenario is completely natural in the context of grand unified models of elementary-particle interactions. In such models, the supercooling is also relevant to the problem of monopole suppression. Unfortunately, the scenario seems to lead to some unacceptable consequences, so modifications must be sought.

PHYSICAL REVIEW D VOLUME 23, NUMBER 2 15 JANUARY 1981

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Volume 108B, number 6 PHYSICS LETTERS 4 February 1982

A NEW INFLATIONARY UNIVERSE SCENARIO: A POSSIBLE SOLUTION OF THE HORIZON, FLATNESS, HOMOGENEITY, ISOTROPY AND PRIMORDIAL MONOPOLE PROBLEMS

A.D. LINDE
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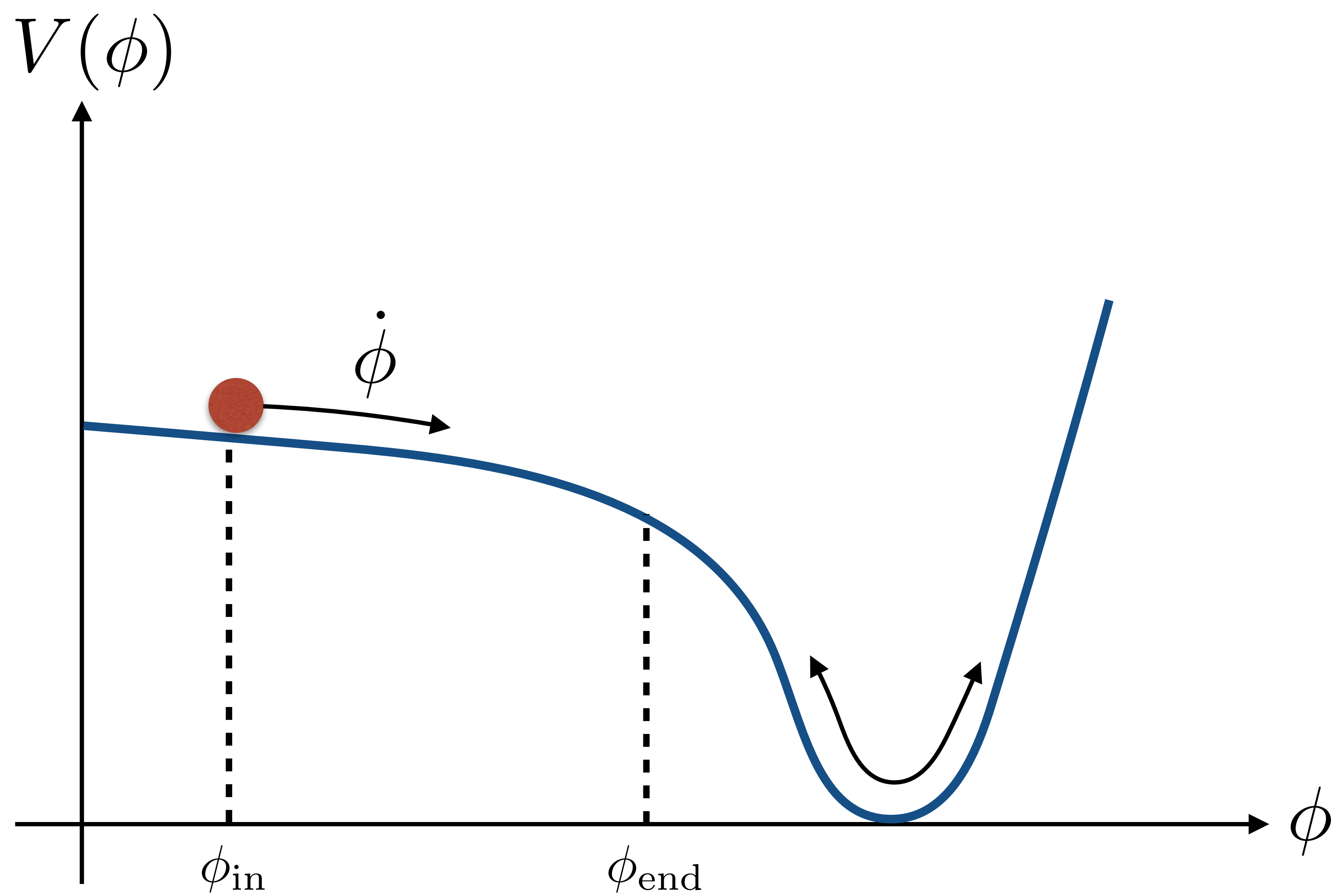
Received 29 October 1981

Slow-Roll inflation

Single field Slow-Roll inflation



$$S = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Classical dynamics of inflation

An accelerated universe

FLRW Cosmology:

$$ds^2 = a^2(\tau)\eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu$$

$$H \equiv \frac{\mathrm{d}\ln a}{\mathrm{d}t}$$

Scale factor

Hubble parameter

$$\mathrm{d}\tau = \frac{\mathrm{d}t}{a(t)} = \boxed{(aH)^{-1}}\mathrm{d}\, ln\, a$$

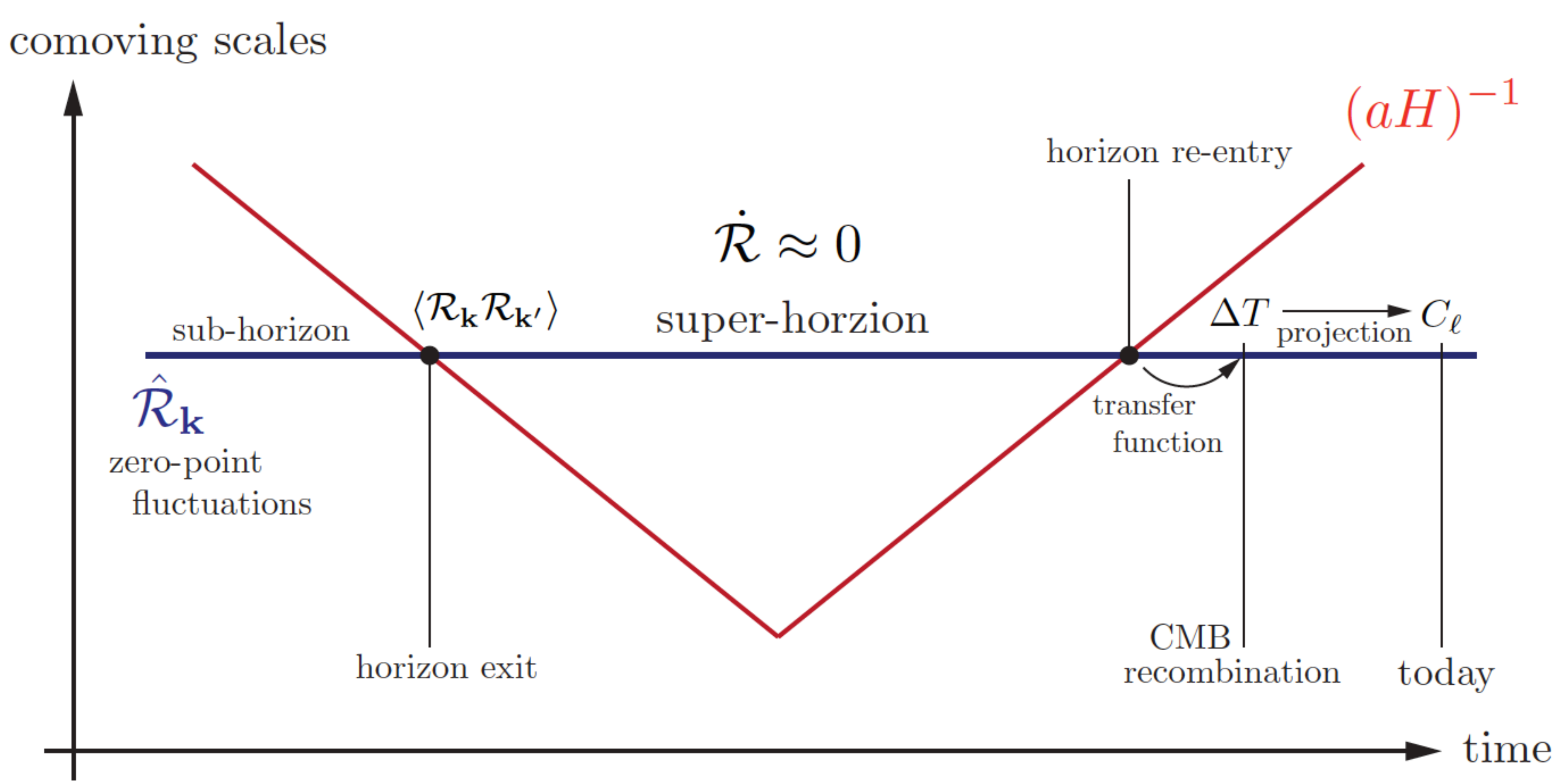
Comoving Hubble radius

Condition for inflation:

$$\frac{\mathrm{d}}{\mathrm{d}t}(aH)^{-1} = -\frac{(1-\epsilon)}{a} = -\frac{\ddot{a}}{\dot{a}^2} < 0 \longleftrightarrow \ddot{a} > 0$$

Accelerated expansion

Solution to the horizon and flatness problems



Solved!

$$|1 - \Omega(a)| = \frac{k}{(aH)^2}$$

Solved!

If the comoving Hubble radius decreases this drives towards flatness!

Slow-Roll inflation

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} = -V'(\phi) \qquad\qquad H^2 = -\frac{1}{3M_{\rm pl}^2}\left[\cancel{\frac{\dot{\phi}^2}{2}} + V(\phi)\right]$$

Slow-roll approximation

$$-\frac{\dot{H}}{H^2} \simeq \frac{M_{\rm pl}^2}{2}\left[\frac{V'(\phi)}{V(\phi)}\right]^2 \equiv \epsilon(\phi)$$

$$M_{\rm pl}^2\left[\frac{V''(\phi)}{V(\phi)}\right] \equiv \eta(\phi)$$

$$\boxed{\begin{array}{c}\epsilon(\phi) \ll 1 \\ |\eta(\phi)| \ll 1\end{array}}$$

Slow-roll conditions

Slow-Roll inflation

$$N \equiv \int_{a_i}^{a_f} \mathrm{d}\ln a = \int_{t_i}^{t_f} H(t)\mathrm{d}t \simeq \int_{\phi_{\rm in}}^{\phi_{\rm end}} \frac{\mathrm{d}\phi}{M_{\rm pl}\sqrt{2\epsilon}}$$

Number of *e*-folds

The largest scales observed in the CMB are produced some 40 to 60 *e*-folds before the end of inflation:

$$N_{\rm CMB} = \int_{\phi_{\rm CMB}}^{\phi_{\rm end}} \frac{\mathrm{d}\phi}{M_{\rm pl}\sqrt{2\epsilon}} \approx 40 - 60$$

A successful solution to the horizon problem requires at least $N_{\rm CMB}$ *e*-folds of inflation.

One example: Chaotic inflation.

$$\boxed{V(\phi) = \frac{1}{2} m^2 \phi^2}$$

- $\epsilon(\phi) = \eta(\phi) = 2\left(\frac{M_{\rm pl}}{\phi}\right)^2 \quad \longleftrightarrow \quad \phi_{\rm end} = \sqrt{2} M_{\rm pl}$
- $N = \int_{\phi_{\rm cmb}}^{\sqrt{2}M_{\rm pl}} \frac{\phi}{2M_{\rm pl}^2} {\rm d}\phi = -60 \quad \longleftrightarrow \quad \phi_{\rm cmb} \sim 15 M_{\rm pl}$

We need to compute some observables

Super-Planckian values!

The theory of quantum fluctuations

Quantum fluctuations during inflation

$$S = \int \mathrm{d}^4x \sqrt{-g}\left[\frac{1}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Perturbation theory in the comoving gauge:

$$\delta\phi = 0 \qquad\qquad \delta g_{ij} = a^2(1-2\zeta)\delta_{ij} + a^2 h_{ij}$$

$$S_{(2)} = \frac{1}{2}\int \mathrm{d}^4x\, a^3 \frac{\dot{\phi}}{H^2}\left[\dot{\zeta}^2 - a^{-2}(\partial_i\zeta)^2\right] + \ldots$$

$$= \frac{1}{2}\int \mathrm{d}^4x \left[(v')^2 - (\partial_i v)^2 + \frac{z''}{z}v^2\right]$$

$$v \equiv z\zeta$$

$$z^2 \equiv a^2\frac{\dot{\phi}^2}{H^2} = 2a^2\epsilon$$

Mukhanov - Sasaki equation:

$$v''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right) v_{\vec{k}} = 0$$

$$v_{\vec{k}}(\tau) \equiv \int \mathrm{d}^3\vec{x}\, e^{-i\vec{k}\cdot\vec{x}} v(\tau, \vec{x})$$

$$v_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) \overset{\lim_{k\tau\to 0}}{=} \frac{1}{i\sqrt{2}}\left(\frac{1}{k^{\frac{3}{2}}\tau}\right)$$

Gravitational waves:

$$S_{(2)} = \frac{M_{\rm pl}}{8}\int \mathrm{d}^4x\, a^2 \left[(h'_{ij})^2 - (\nabla h_{ij})^2\right]$$

$$h_{ij}(\tau, \mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{x}}{(2\pi^{\frac{3}{2}})} \sum_{\gamma=+,\times} \epsilon^{\gamma}_{ij}(k) h_{\mathbf{k},\gamma}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}}$$

$$v_{\mathbf{k},\gamma} \equiv \frac{a}{2} M_{\rm pl} h_{\mathbf{k},\gamma}$$

The observables:

$$\langle \hat{v}_{\vec{k}} \hat{v}_{\vec{k}'} \rangle \equiv P_v(k)\delta(\vec{k}+\vec{k}')$$

$$P_v(k) \equiv |v_k|^2$$

$$P_\zeta(k) = \frac{1}{z^2} P_v(k) = \frac{1}{z^2}\frac{1}{2k^3}\frac{1}{\tau^2} = \frac{1}{4k^3}\frac{H^2}{\epsilon}$$

Power spectra

$$\Delta_s^2(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k) = \frac{1}{8\pi^2}\frac{H^2}{\epsilon}\bigg|_{k=aH}$$

$$\Delta_t^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\rm pl}}\bigg|_{k=aH}$$

Tensor-to-scalar ratio

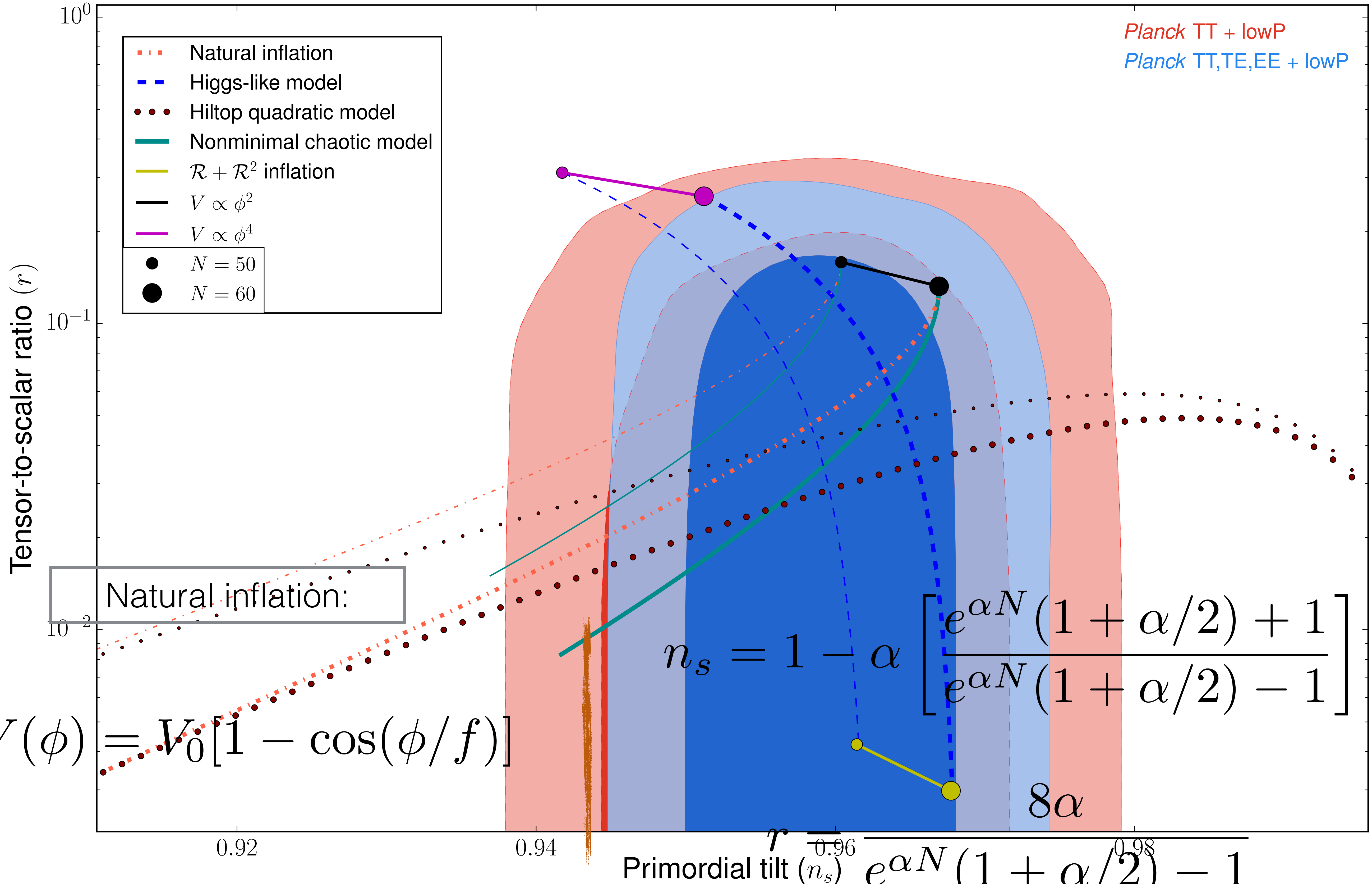
$$\boxed{r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon}$$

Primordial tilt

$$\boxed{n_s - 1 \equiv \frac{\mathrm{d}\ln \Delta_s^2}{\mathrm{d}\ln k} = -2\epsilon - \eta}$$

The most-favoured inflationary models

The present of slow-roll models:



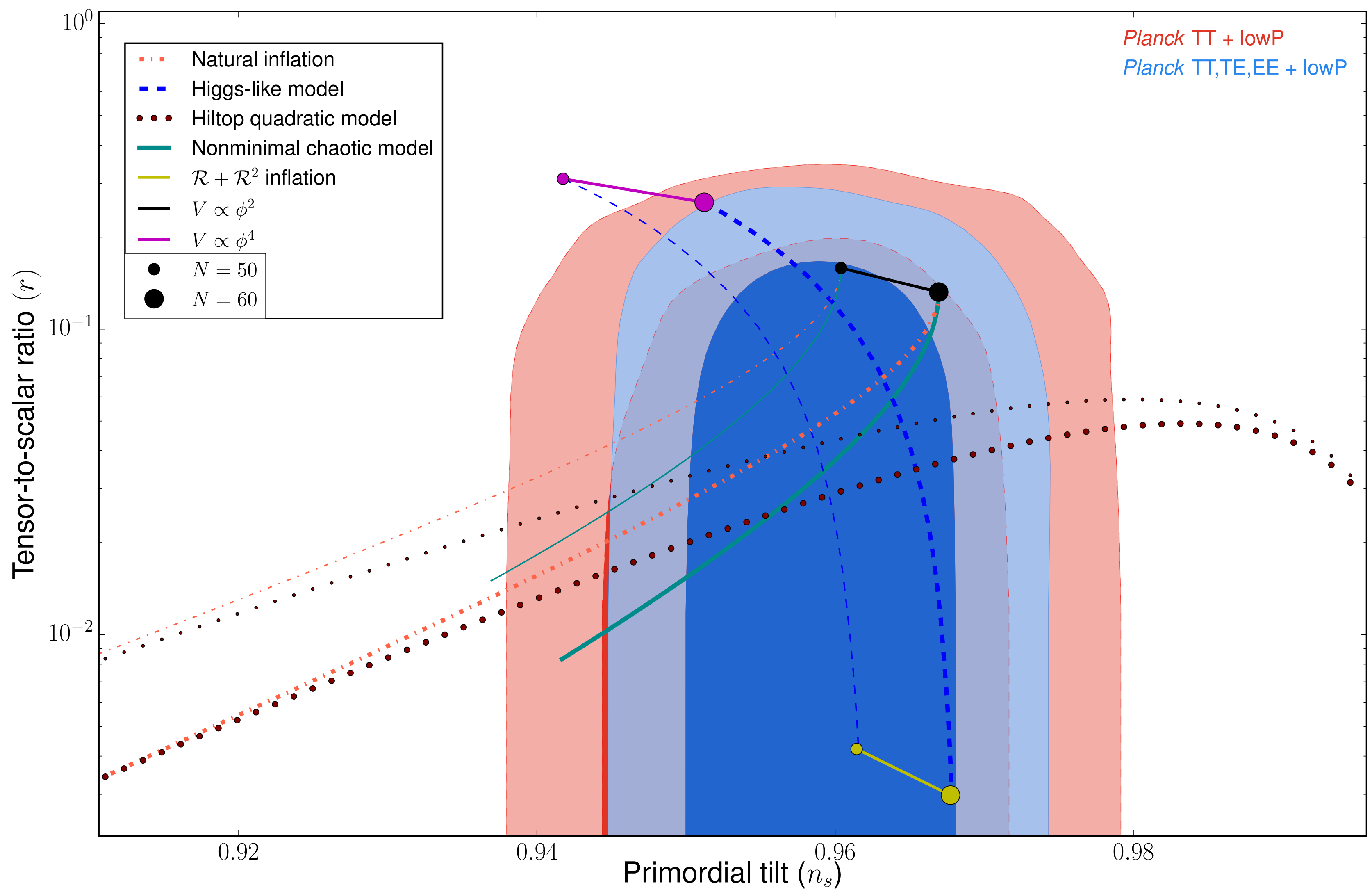
Natural inflation:

$$V(\phi) = V_0[1-\cos(\phi/f)]$$

$$n_s = 1 - \alpha\left[\frac{e^{\alpha N}(1+\alpha/2)+1}{e^{\alpha N}(1+\alpha/2)-1}\right]$$

$$r = \frac{8\alpha}{e^{\alpha N}(1+\alpha/2)-1}$$

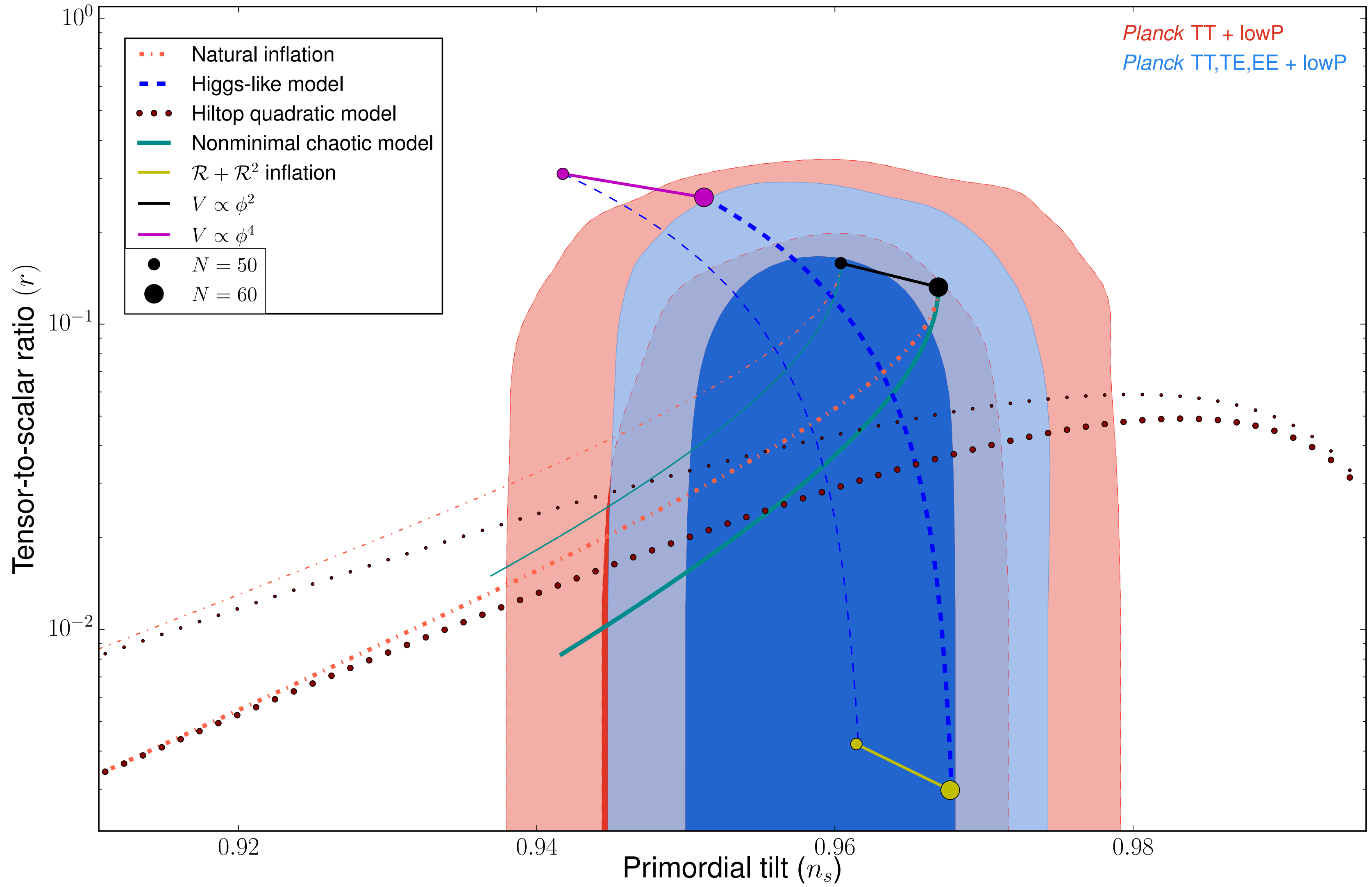
The present of slow-roll models:



Higgs-like models:

$$V(\phi) = \frac{\lambda(\phi^2 - v^2)^2}{4(1 - \xi\phi^2/M_{\rm pl}^2)^2}$$

The present of slow-roll models:



Hiltop inflation:

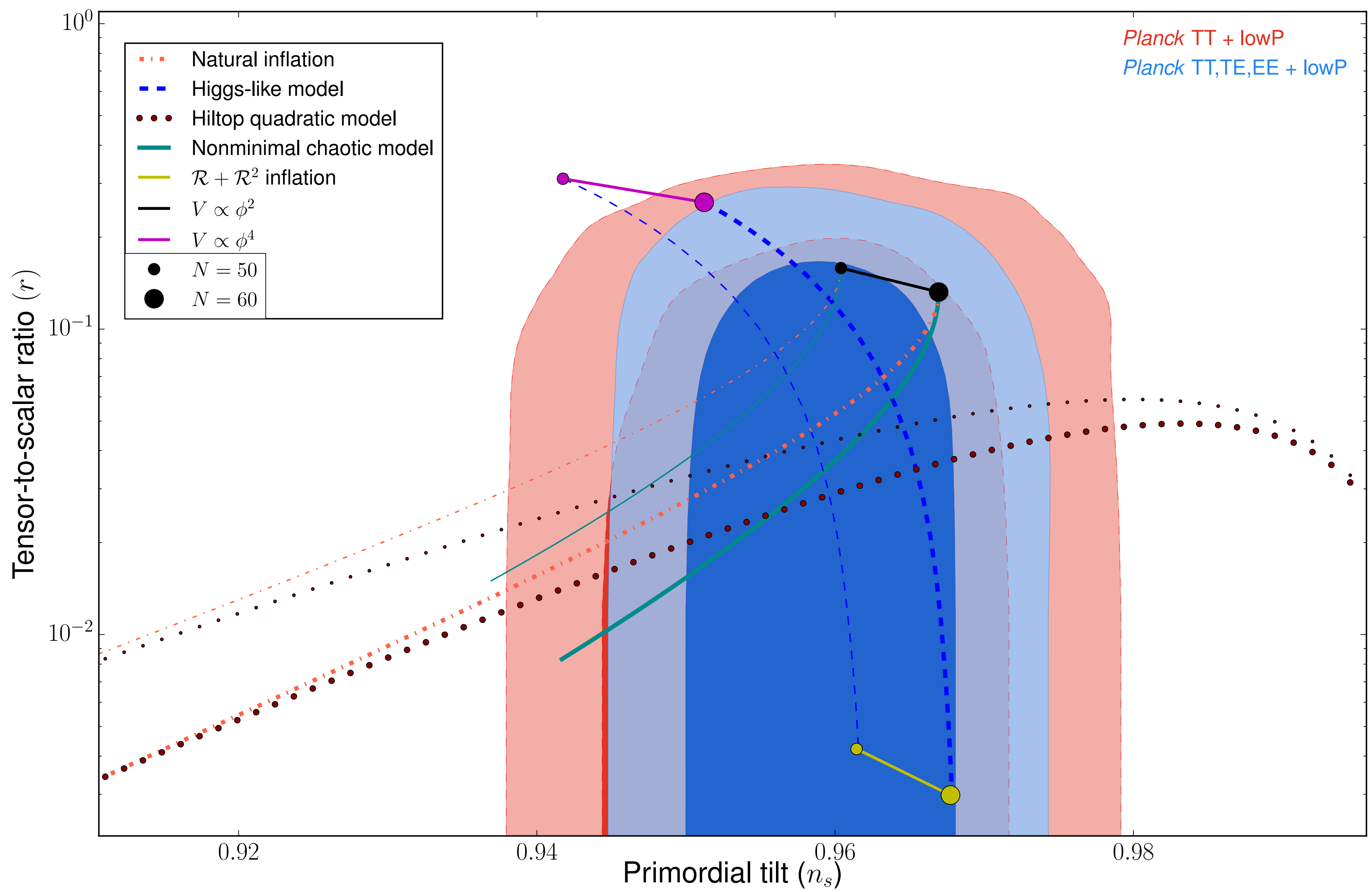
$$V(\phi) = V_0[1 - (\phi/\mu)^p]$$

$$\boxed{p = 2:}$$

$$n_s = 1 - 4|\eta_0|$$

$$r = 2(1 - n_s)^2 e^{N(n_s - 1)} |\eta_0|^{-1}$$

The present of slow-roll models:



Chaotic scenarios:

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_{\rm pl}^2\mathcal{R} + \xi\mathcal{R}\phi^2 - (\partial\phi)^2 - m^2\phi^2\right]$$

Non-minimall coupling

Do current data prefer a non-minimally coupled inflaton?

L. Boubekeur, E. Giusarma, O. Mena and H.R.,

arXiv:1502.05193 [astro-ph.CO].
Phys. Rev. D 91 (2015) 103004.

Motivation

- Non-Renormalizable terms are usually neglected for convenience.

- $\xi \mathcal{R} \phi^2$ is allowed in the renormalizable d=4 Lagrangian.

- Since the inflaton is coupled to light degrees of freedom (during reheating),

$$\mathcal{L}_{\text{reheating}} \simeq \lambda \phi^2/4! + y_\psi \phi \bar{\psi} \psi + \lambda_\chi \chi^2 \phi^2 + ...$$

the RGE of ξ is non-trivial. One can make it vanish at some scale, but it will be non-zero at some point because of its running:

$$\beta_\xi = \frac{\xi - \frac{1}{6}}{(4\pi)^2} \left[\lambda + \lambda_\xi + 4 y_\psi^2 + ... \right]$$

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}\mathcal{R} + \frac{\xi}{2}\mathcal{R}\phi^2 - \frac{1}{2}(\partial\phi)^2 - U(\phi)\right]$$

Conformal (Weyl) transformation

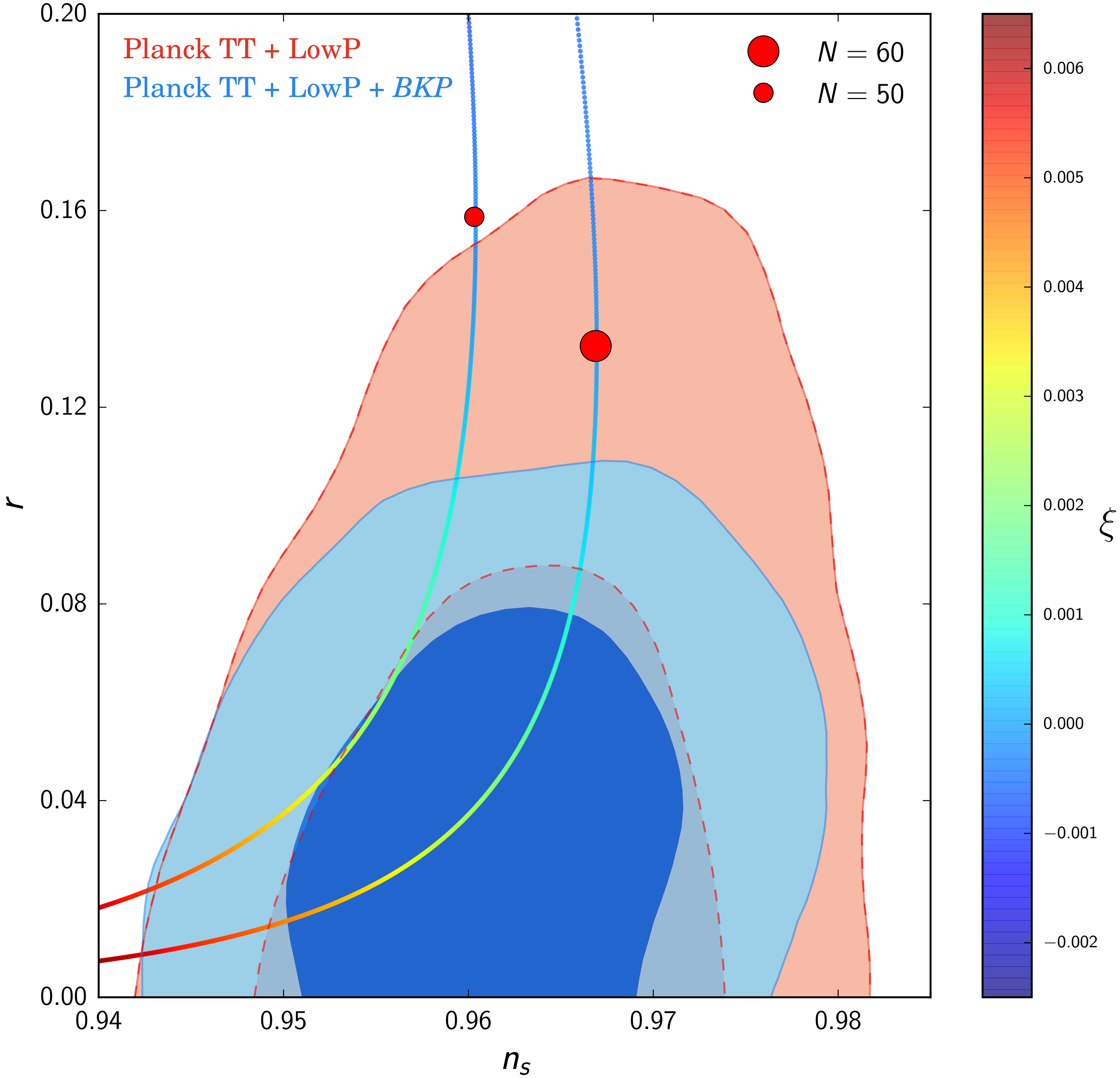
$$g_{\mu\nu}^{\rm E} = \Omega(\phi)g_{\mu\nu}$$

$$\boxed{\Omega(\phi) \equiv 1 + \frac{\xi\phi^2}{M_{\rm pl}^2}}$$

$$S = \int \mathrm{d}^4x\sqrt{-g_{\rm E}}\left[\frac{M_{\rm pl}^2}{2}\mathcal{R}_{\rm E} - \frac{1}{2}g_{\rm E}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V[\phi(\varphi)]\right]$$

$$\boxed{V[\phi(\varphi)] = \frac{U(\phi)}{\Omega^2(\phi)}} \longrightarrow \boxed{U(\phi) = \frac{1}{2}m^2\phi^2}$$

Results



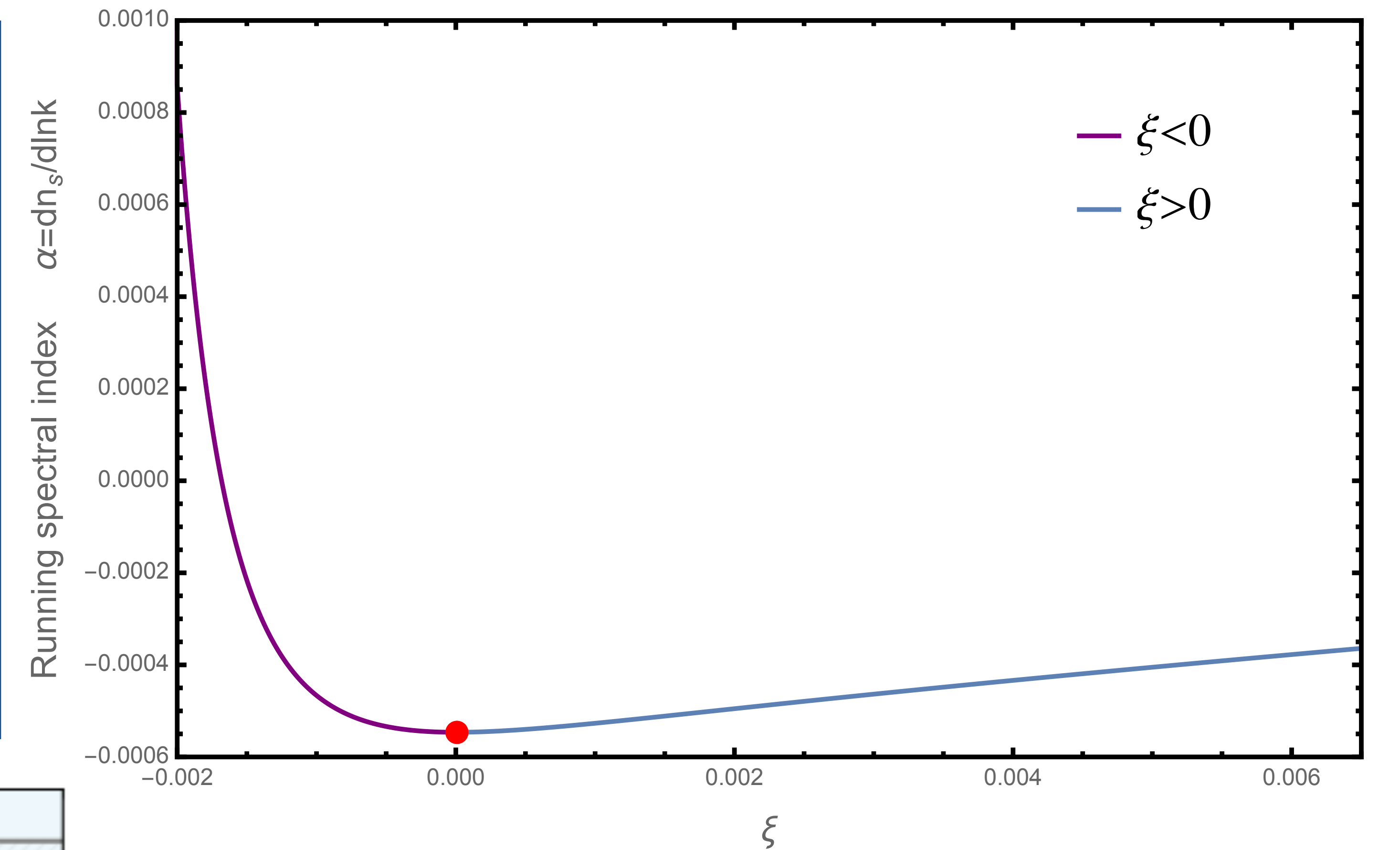
- We perform a MCMC analisys using COSMOMC.
- Consider various Planck'15 data sets (BKP=BICEP2/KECK + Planck).
- $\Lambda\mathrm{CDM}$ Cosmology + $\xi \neq 0$
- A non-vanishing ξ , in this context is preferred ($\xi \sim 10^{-3}$).
- A non-vanishing r is also favored ($r \simeq 0.04 - 0.06$).

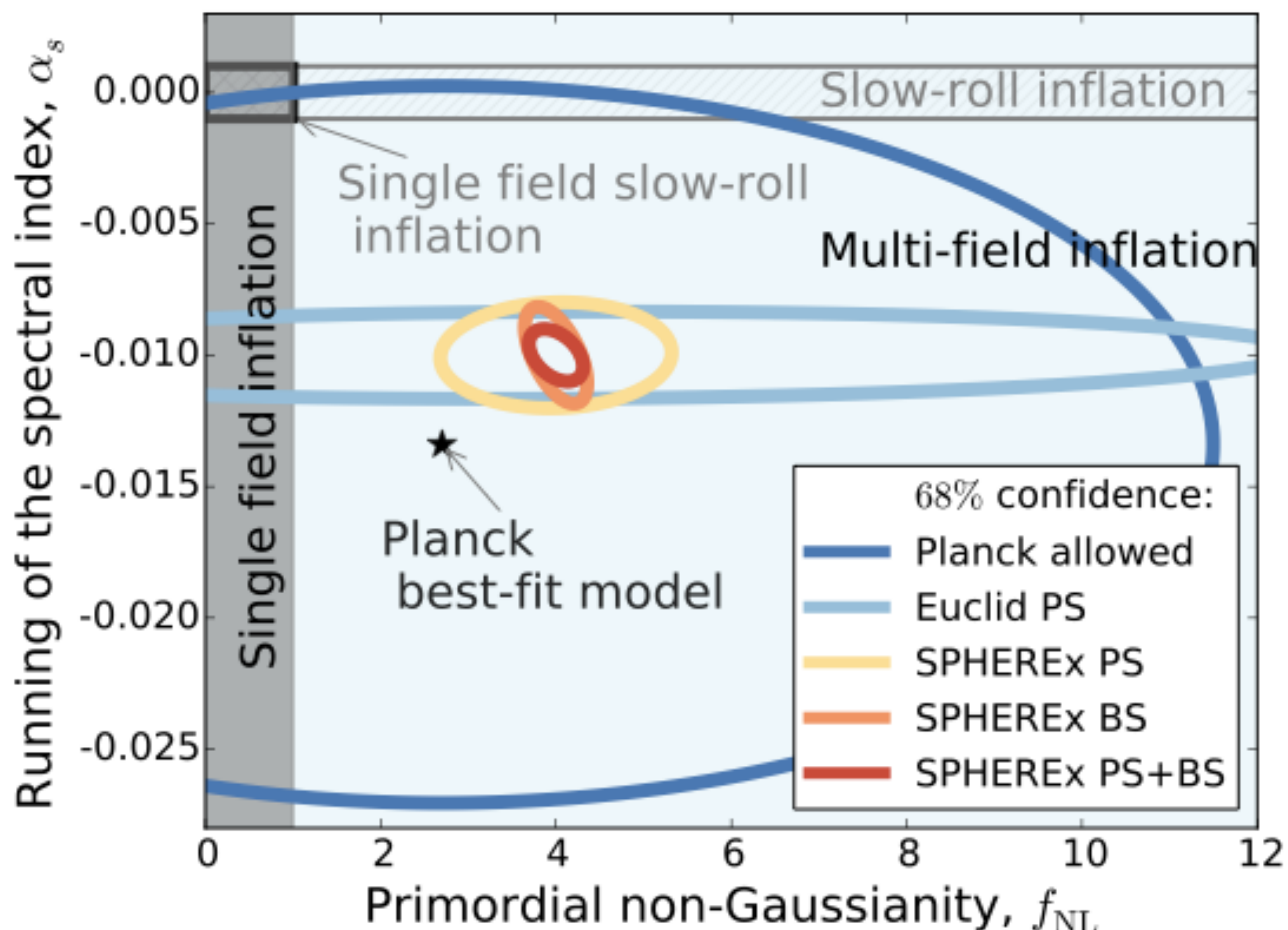
Results

	Planck TT+WP		BK+Planck TT+WP	
N	60	50	60	50
ξ	$0.0028^{+0.0023}_{-0.0025}$	$0.0024^{+0.0023}_{-0.0023}$	$0.0027^{+0.0023}_{-0.0022}$	$0.0027^{+0.0020}_{-0.0019}$
n_s	$0.958^{+0.010}_{-0.011}$	$0.954^{+0.007}_{-0.009}$	$0.958^{+0.009}_{-0.011}$	$0.953^{+0.007}_{-0.009}$
r	$0.038^{+0.051}_{-0.031}$	$0.063^{+0.056}_{-0.048}$	$0.038^{+0.039}_{-0.030}$	$0.053^{+0.038}_{-0.037}$
$\alpha \equiv \mathrm{d}n_s/\mathrm{d}\ln k$	$-0.0005^{+0.0001}_{-0.0001}$	$-0.0007^{+0.0001}_{-0.0001}$	$-0.0005^{+0.0001}_{-0.0001}$	$-0.0007^{+0.0001}_{-0.0001}$

Future constraints

- Negative coupling gives significant running (and higher r).
- The running is a good discriminator.
- Future constraints might falsify this model.





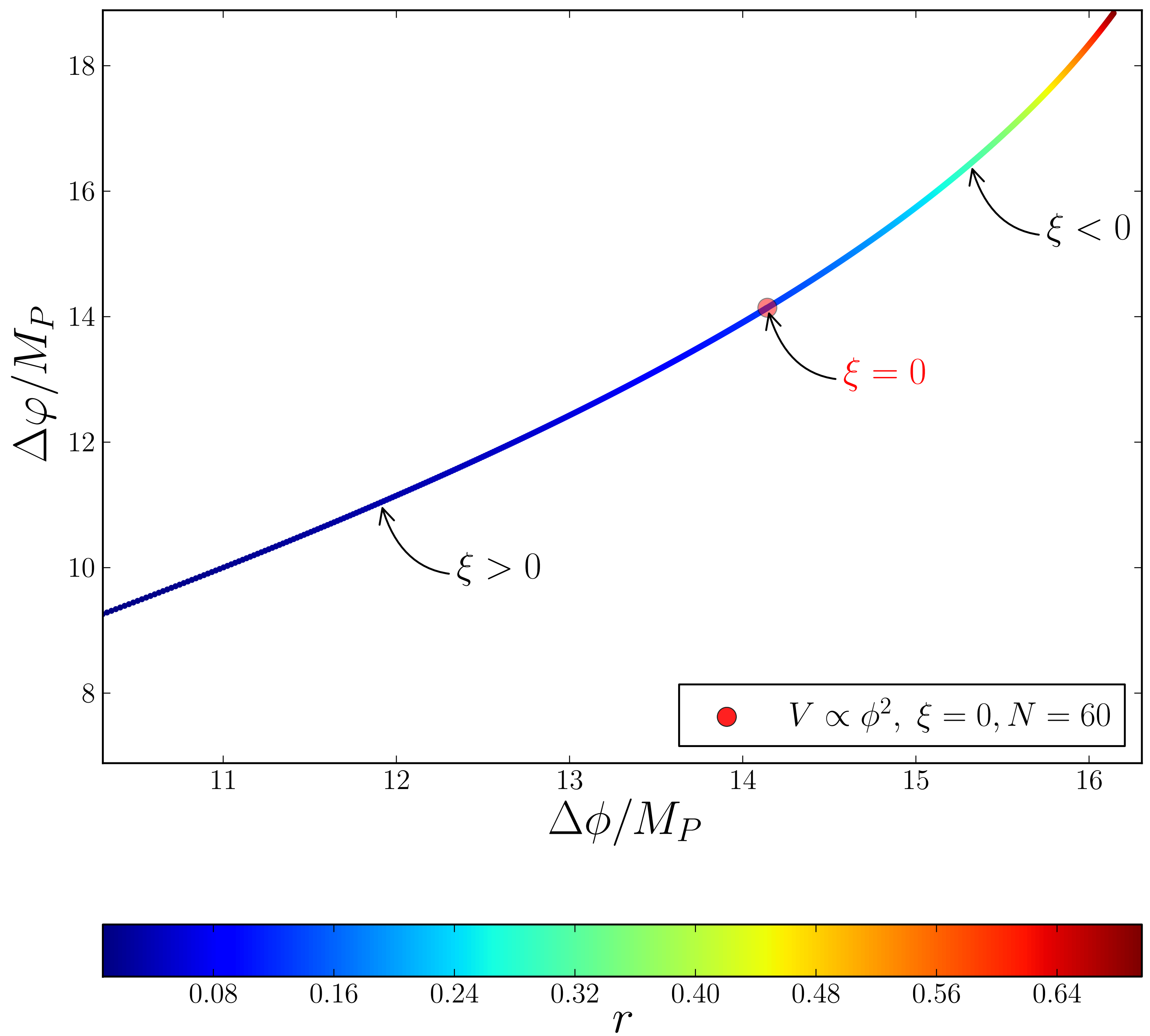
e.g. SPHEREx

(Spectrophotometer for the History of Universe, Epoch of Reionization and ices Explorer)

Results

$$\left(\frac{\mathrm{d}\varphi}{\mathrm{d}\phi}\right)^2 = \frac{1}{\Omega} + \frac{3}{2} M_P^2 \left(\frac{\Omega'}{\Omega}\right)^2$$

The excursion of the non-minimally coupled inflaton is bit smaller but still super-Planckian.



Do current data prefer a non-minimally coupled inflaton?

The answer is YES!

Phenomenological approaches of inflation

L. Boubekeur, E. Giusarma, O. Mena and H.R.,

arXiv:1411.7237 [astro-ph.CO].
Phys. Rev. D 91 (2015) 8, 083006.

- The smocking gun: measuring the gravitational waves (GWs), *i.e.*, r .

- Additional information, the scale dependence of the power spectrum of inflationary GWs (consistency relation):

$$n_t = -\frac{r}{8}$$

✦ Measurement of might turn out to be very challenging: may be it is not the best way to test single-field slow-roll inflation.

✦ Testing the consistency relation in each model (*e.g.* $r = -4(n_s - 1)$ for $V \propto \phi^2$) assumes particular inflationary potentials: this is not model-independent (obviously).

✦ It would be more appealing to try to work out the inflationary predictions in pictures where the inflationary potential does not play any crucial role: the shape of the potential is difficult to understand in a fundamental theory.

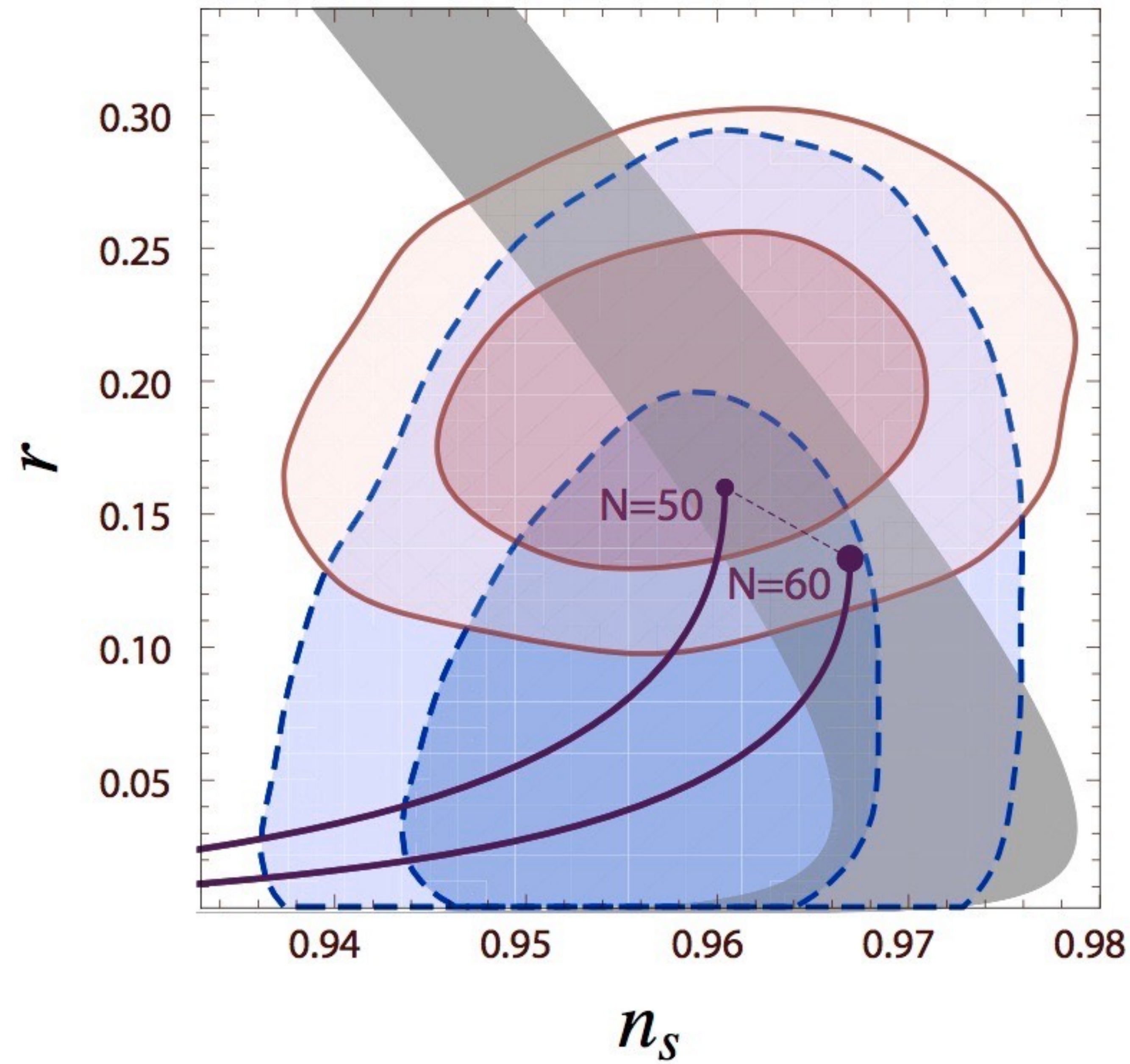
Two approaches

Mukhanov Parametrization:

$$\boxed{\frac{p}{\rho}+1=\frac{\beta}{(1+N_e)^{\alpha}}}$$

$$n_s-1=-\frac{3\beta}{(N_*+1)^{\alpha}}-\frac{\alpha}{N_*+1}$$

$$r=\frac{24\beta}{(N_*+1)^{\alpha}}$$



L. Barranco, L. Boubekeur and O. Mena, PRD 90 (2014) 6, 063007

Two approaches

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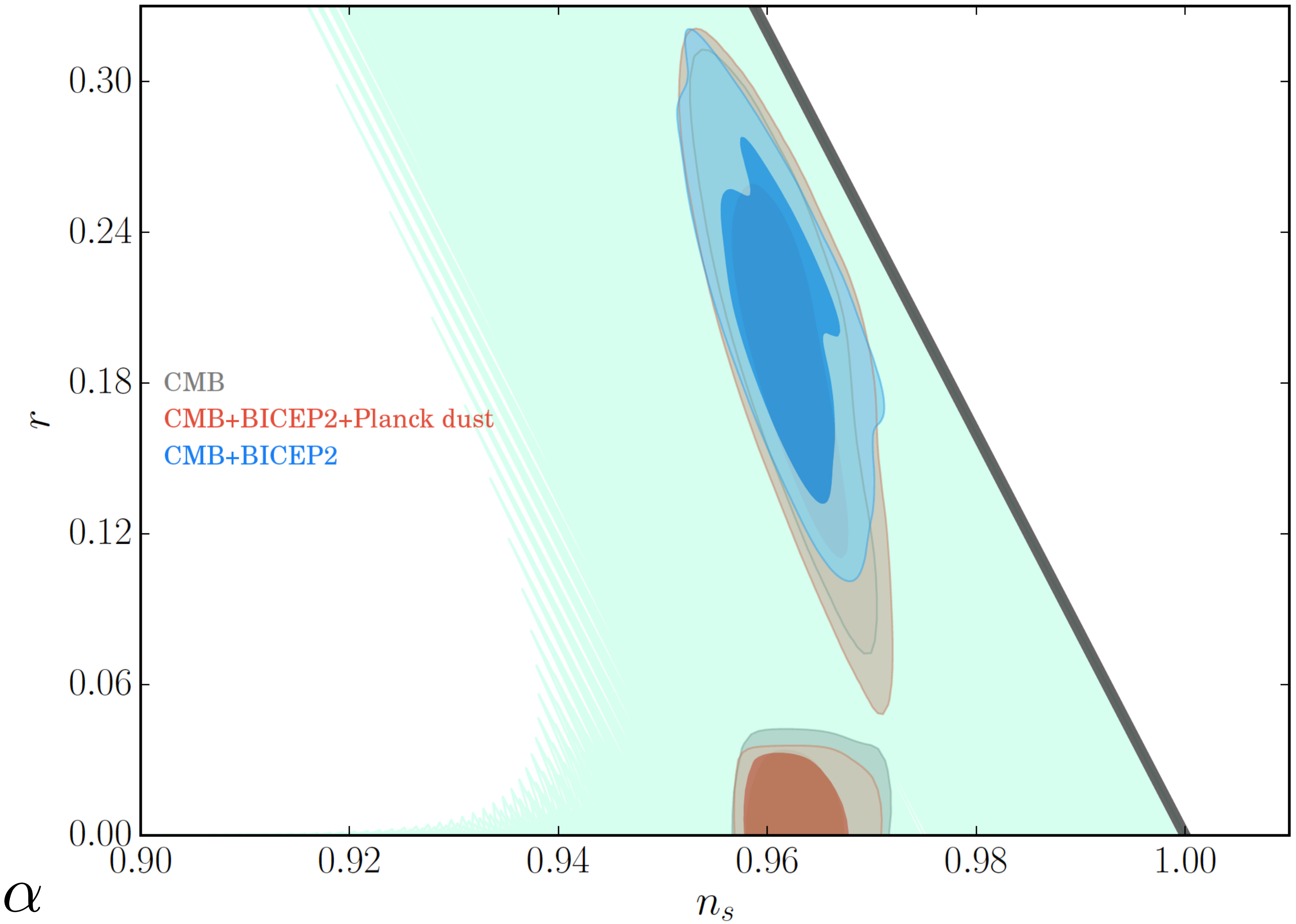


$$\Longrightarrow \quad n_s=1-\frac{r}{8}-\frac{\alpha}{N_*+1}$$

Two approaches

The Hubble Flow Formalism

$$\epsilon_{\rm H} \equiv 2M_{\rm pl}^2 \left(\frac{H'(\phi)}{H(\phi)}\right)^2 \qquad \eta_{\rm H} \equiv 2M_{\rm pl}^2 \left(\frac{H''(\phi)}{H(\phi)}\right)$$

$${}^{\ell}\lambda_{\rm H} \equiv (2M_{\rm pl}^2)^{\ell} \frac{(H')^{\ell-1}}{H^{\ell}} \frac{{\rm d}^{(\ell+1)} H}{{\rm d}\phi^{(\ell+1)}}$$

SR parameters obey the infinite system of first-order differential equations:

$$\frac{{\rm d}\epsilon_{\rm H}}{{\rm d}N} = \epsilon_{\rm H}(\sigma_{\rm H} + 2\epsilon_{\rm H}) \qquad \frac{{\rm d}\sigma_{\rm H}}{{\rm d}N} = -5\epsilon_{\rm H}\sigma_{\rm H} - 12\epsilon_{\rm H}^2 + 2({}^2\lambda_{\rm H})$$

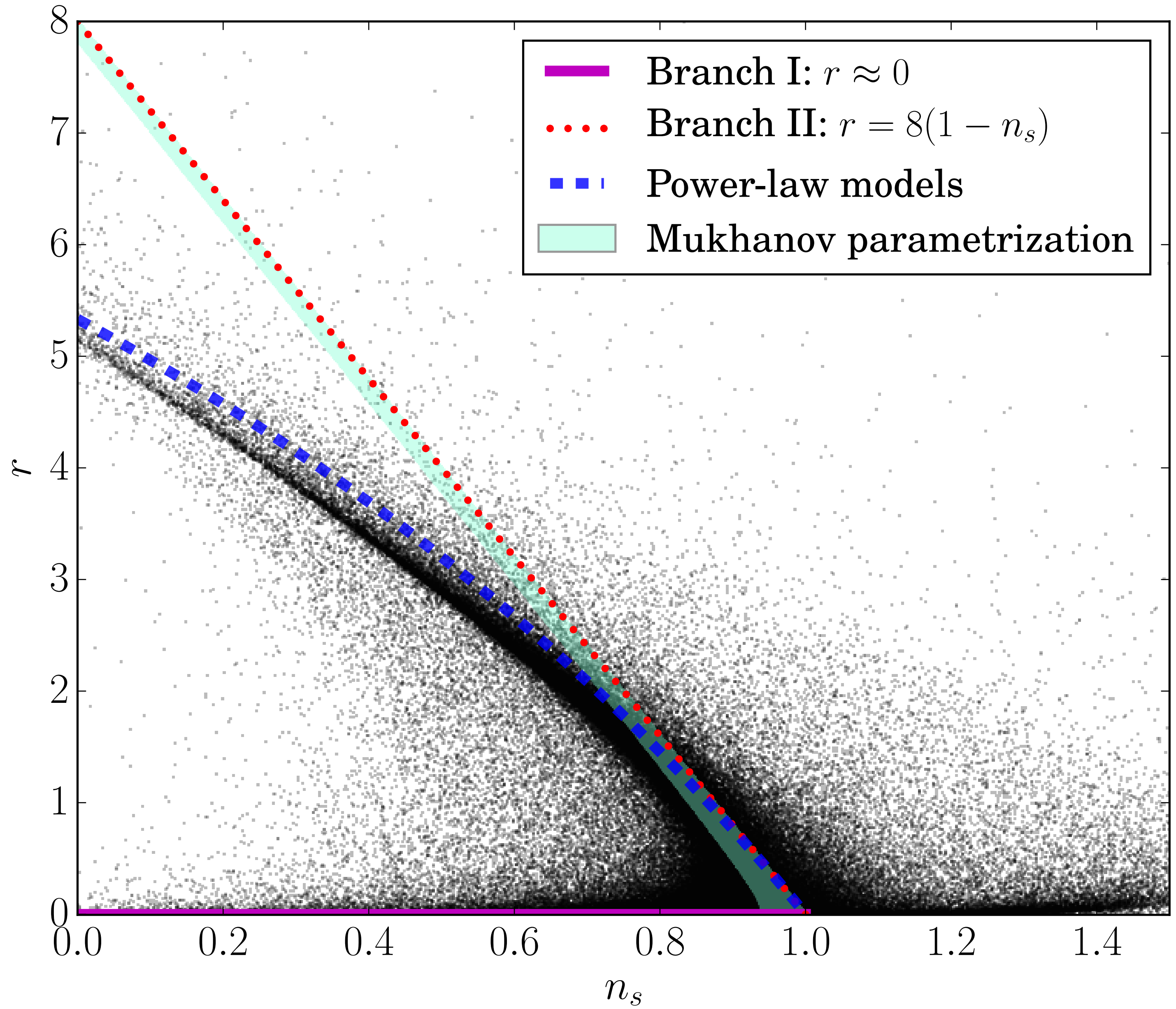
$$\frac{d({}^{\ell}\lambda_{\rm H})}{dN} = \left[\frac{\ell-1}{2}\sigma_{\rm H} + (\ell-2)\epsilon_{\rm H}\right] {}^{\ell}\lambda_{\rm H} + {}^{\ell+1}\lambda_{\rm H}$$

The Hubble Flow Formalism

- To solve the flow equations we used Flowcode1.0 (Monte Carlo approach).
- We generate a total of 6x10^6 inflationary models.
- Slow-roll hierarchy is truncated at order M =8.
- The models cluster around the attractors given by the fixed points:

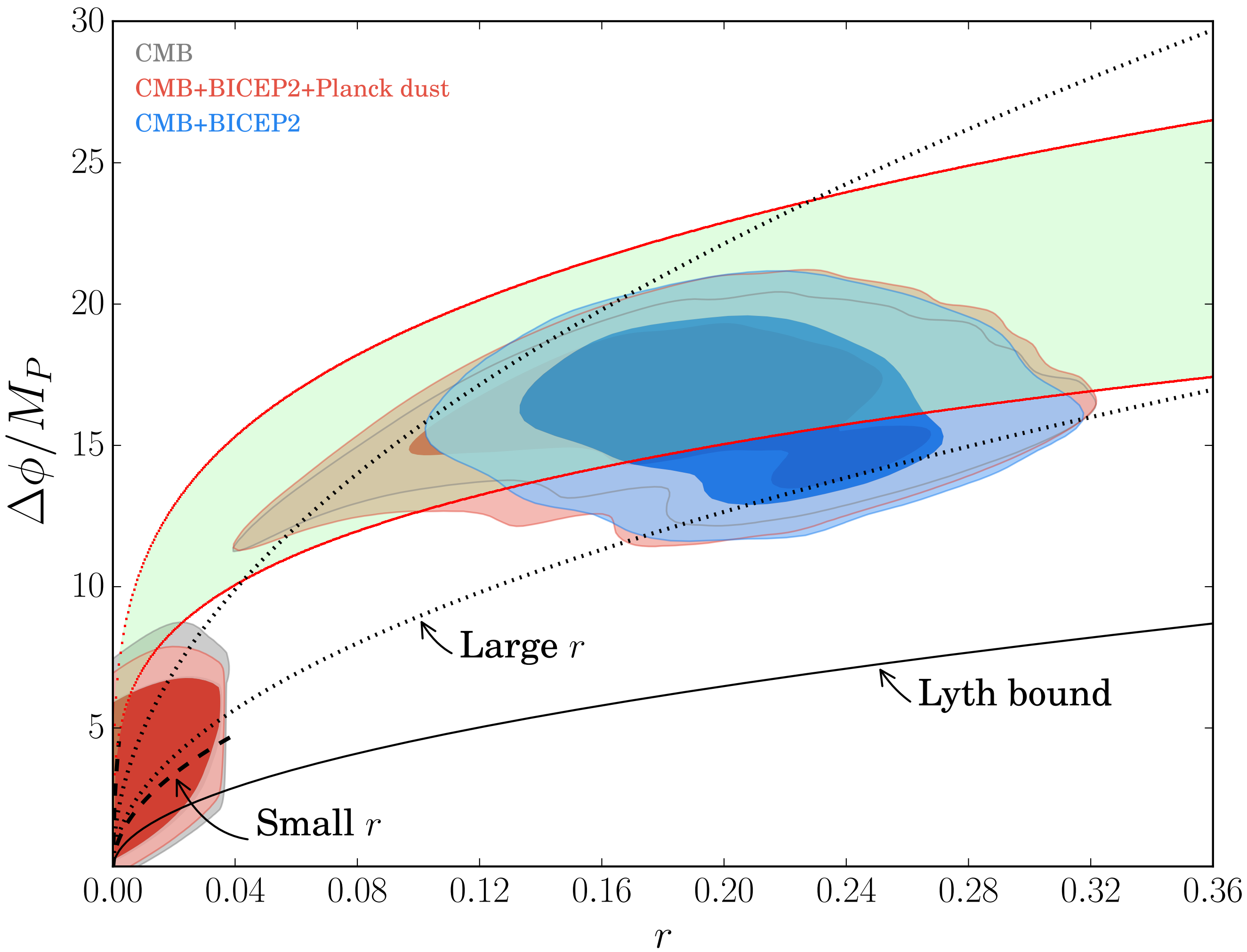
I: $r = 0$ and $n_s = \text{const.}$

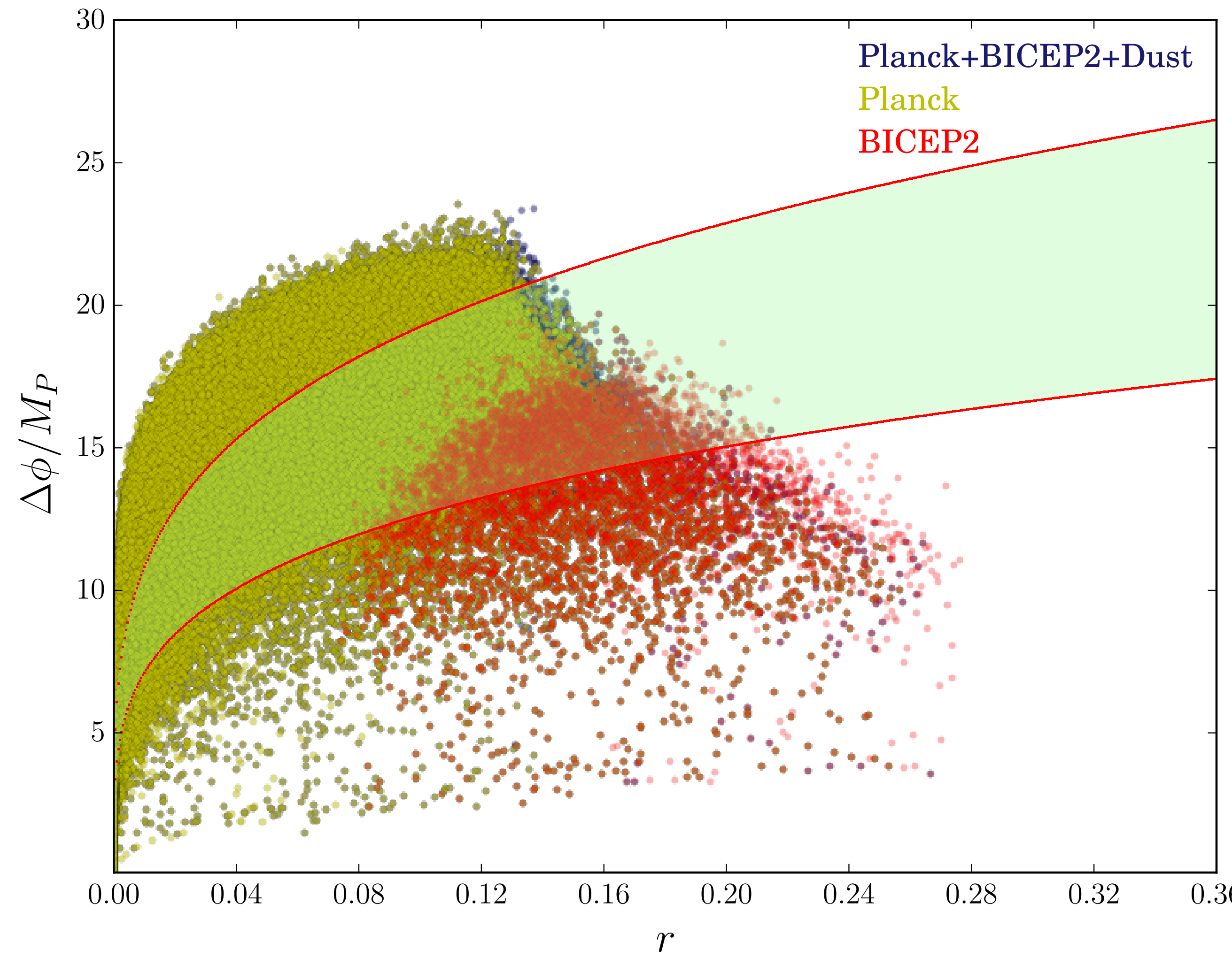
II: $n_s = 1 - \frac{r}{8} \times \left[\frac{1}{1 - r/16}\right]$



The inflaton excursion

$$\frac{\Delta\phi}{M_{\rm pl}} = \int_0^{N_*} dN \sqrt{\frac{3\beta}{(1+N)^\alpha}}$$





A summary

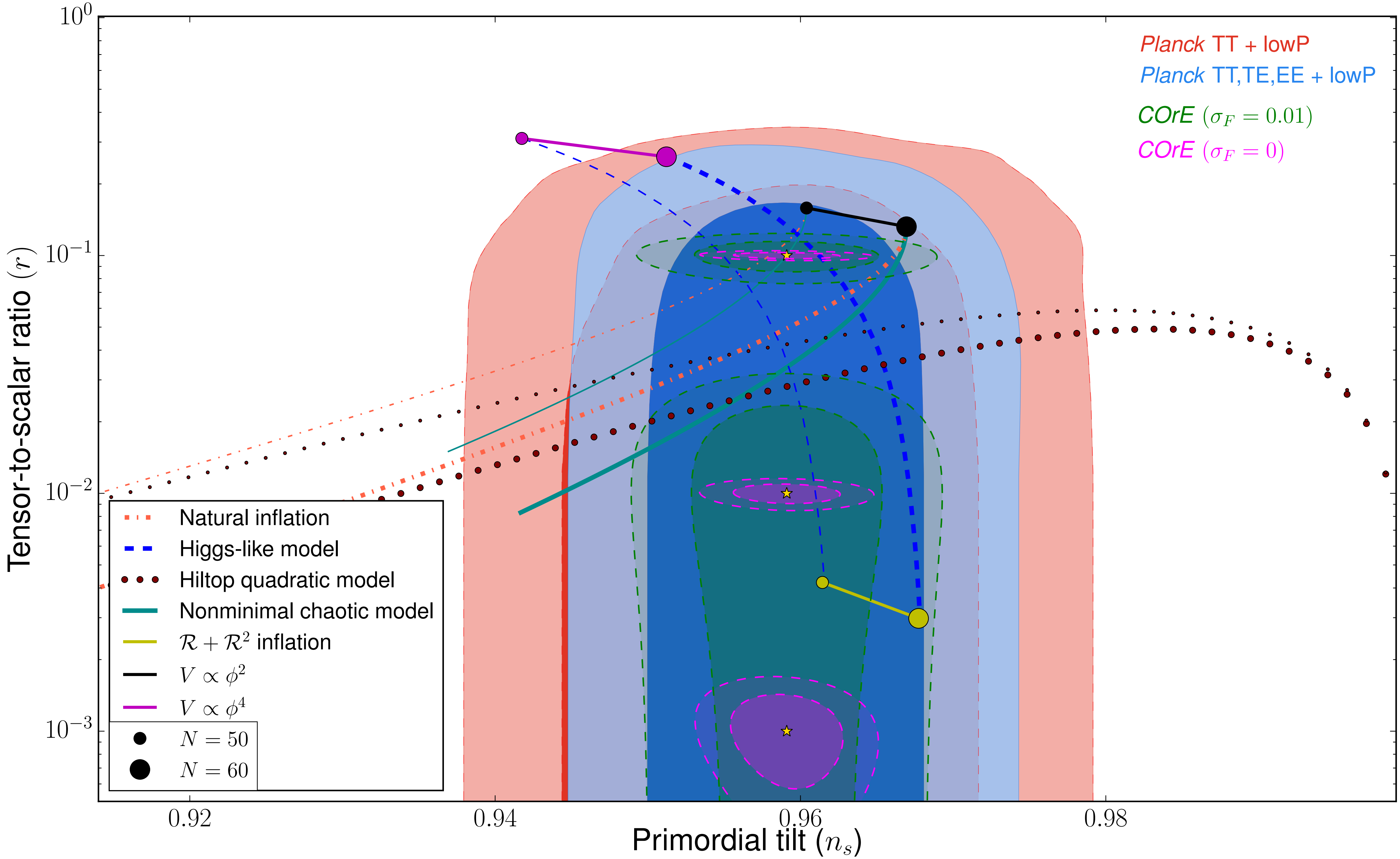
- One approach is a pure theoretical formulation and the other one is pure phenomenological one.
- The parameters regions recovered from both model-independent methods are almost identical.
- Inflaton excursions take sub-Planckian values once PLANCK+BICEP2+Dust is accounted for.

The future of the single-field slow-roll models

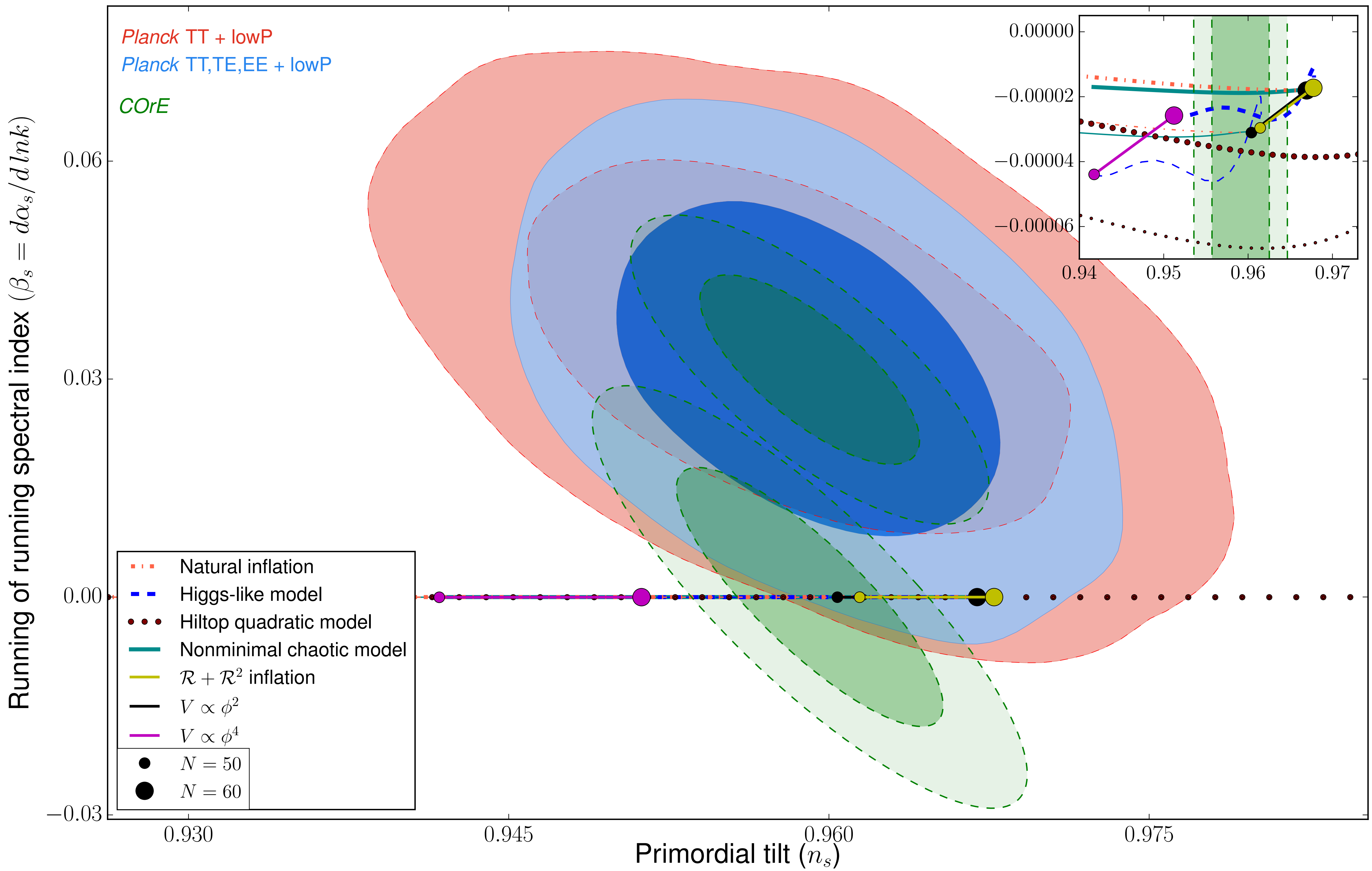
M. Escudero, H.R., L. Boubekeur, E. Giusarma and O. Mena,

arXiv:1509.05419 [astro-ph.CO].

COrE: Cosmic Origins Explorer



$$\beta_s \equiv \frac{\mathrm{d}^2 n_s}{\mathrm{d}\,(\ln k)^2}$$



Thanks