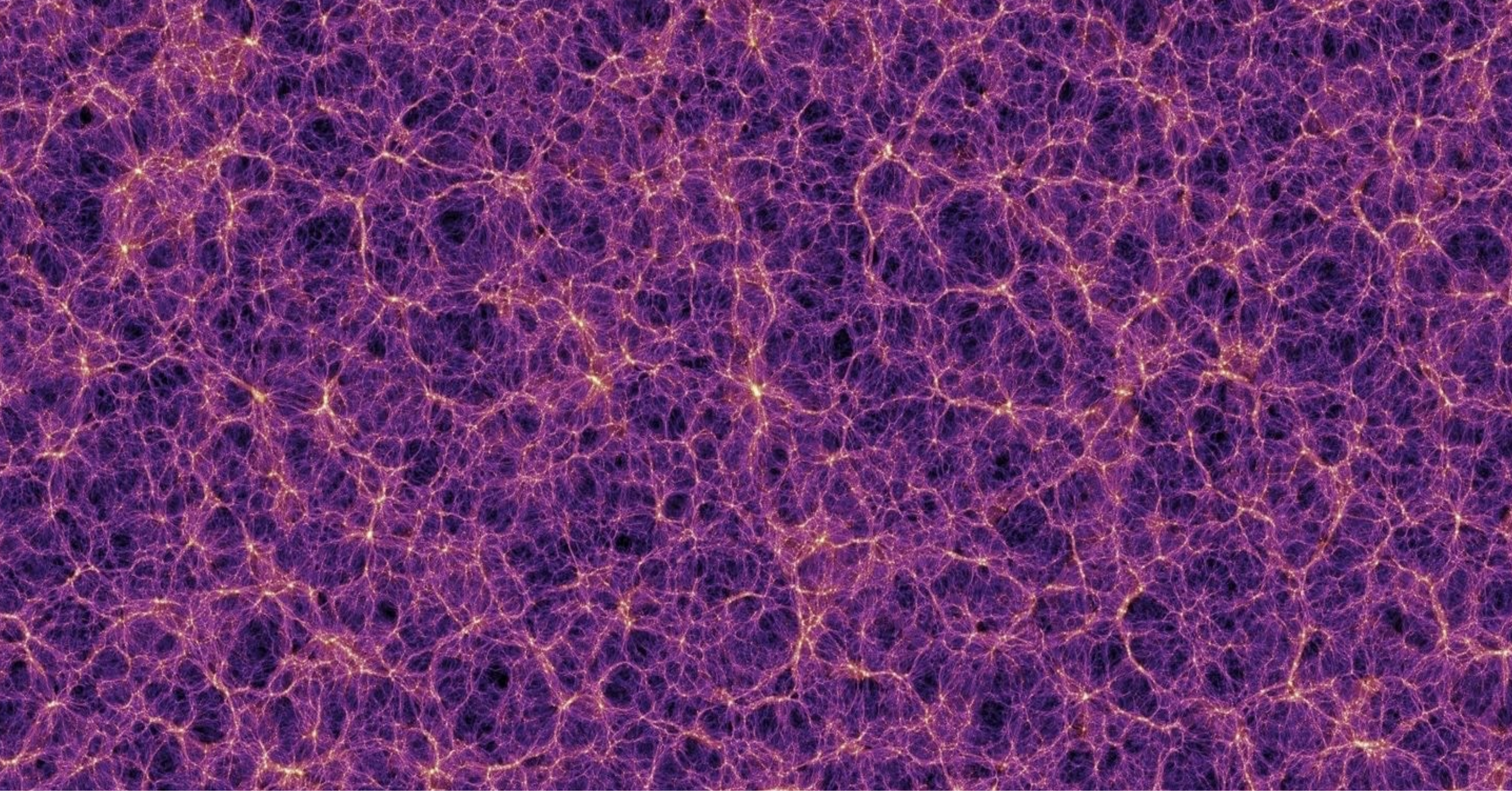

Computing inflationary predictions in general scalar-tensor theories

Héctor Ramírez

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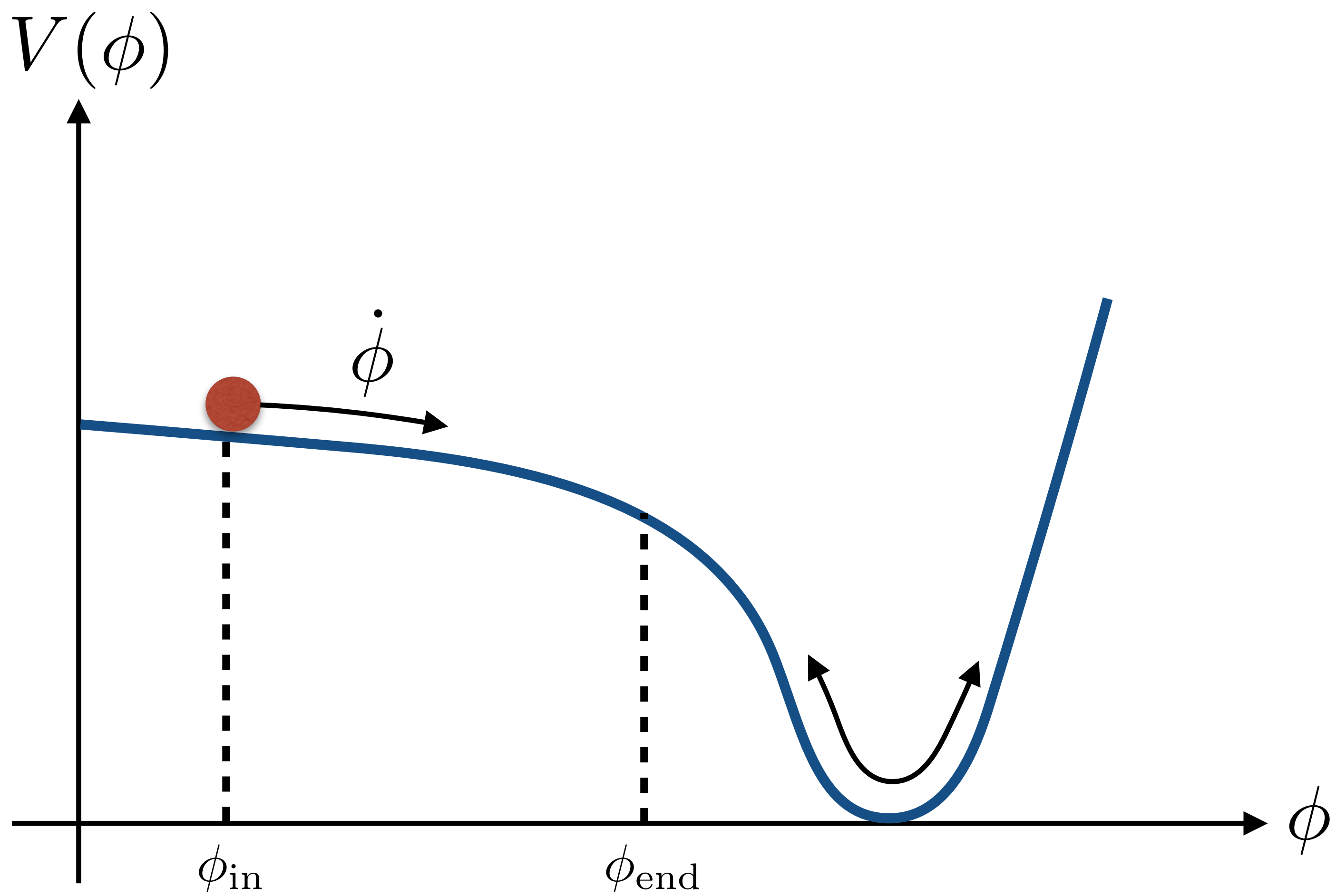
Outline:

1. Slow - roll inflation.
2. Scalar - Tensor theories.
3. Generalized slow-roll (GSR) techniques.
4. G-inflation.

Single-field slow-roll



Einstein-Hilbert action:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Slow-roll approximation:

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} = -V'(\phi)$$

$$H^2 = -\frac{1}{3M_{\rm pl}^2}\left[\cancel{\frac{\dot{\phi}^2}{2}} + V(\phi)\right]$$

- $$\epsilon_H = -\frac{\dot{H}}{H^2}$$
- $$\eta_H = \frac{1}{2}\frac{\mathrm{d}\ \ln\epsilon_H}{\mathrm{d}N} - \epsilon_H$$

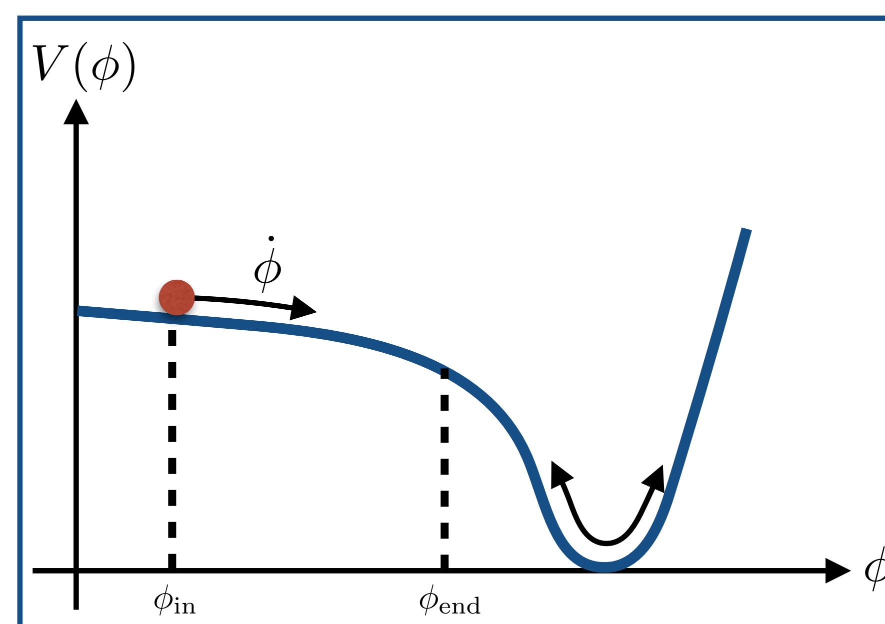
$\longleftrightarrow$ Slow-roll conditions:

$$\epsilon_H \ll 1$$

$$|\eta_H| \ll 1$$

Number of *e*-folds of inflation:

$$N_{\rm CMB} = \int_{\phi_{\rm CMB}}^{\phi_{\rm end}} \frac{\mathrm{d}\phi}{M_{\rm pl}\sqrt{2\epsilon}} \approx 40 - 60$$



The theory of quantum fluctuations

Perturbation theory in the comoving gauge:

- $\delta\phi = 0$
- $\delta g_{ij} = a^2(1-2\zeta)\delta_{ij} + a^2 h_{ij}$

- $$S_\zeta^{(2)} = \int \mathrm{d}^4x a^3 \epsilon_H \left(\dot{\zeta}^2 - \frac{k^2}{a^2}\zeta^2\right)$$ Scalars
- $$S_\gamma^{(2)} = \sum_{\lambda=+,\times} \int \mathrm{d}^4x \frac{a^3}{4} \left(\dot{\gamma}_\lambda^2 - \frac{k^2}{a^2}\gamma_\lambda^2\right)$$ Tensors

The theory of quantum fluctuations

Mukhanov - Sasaki equation:

$$\boxed{v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0}$$

- $v_k = z\zeta$ Scalars
- $v_k = z\gamma_{+,\times}$ Tensors

- $a\mathrm{d}\tau = \mathrm{d}t$
- $z^2 = 2a^2\epsilon_H$

Bunch-Davies vacuum:

$$v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

The theory of quantum fluctuations

Power spectra of primordial perturbations:

- $$\Delta_\zeta^2(k) = \frac{1}{8\pi^2 M_{\rm pl}^2}\frac{H^2}{\epsilon_H}\bigg|_{k=aH}$$
- $$\Delta_\gamma^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\rm pl}^2}\bigg|_{k=aH}$$

Tensor-to-scalar ratio

- $$r \equiv \frac{\Delta_\gamma^2}{\Delta_\zeta^2}$$

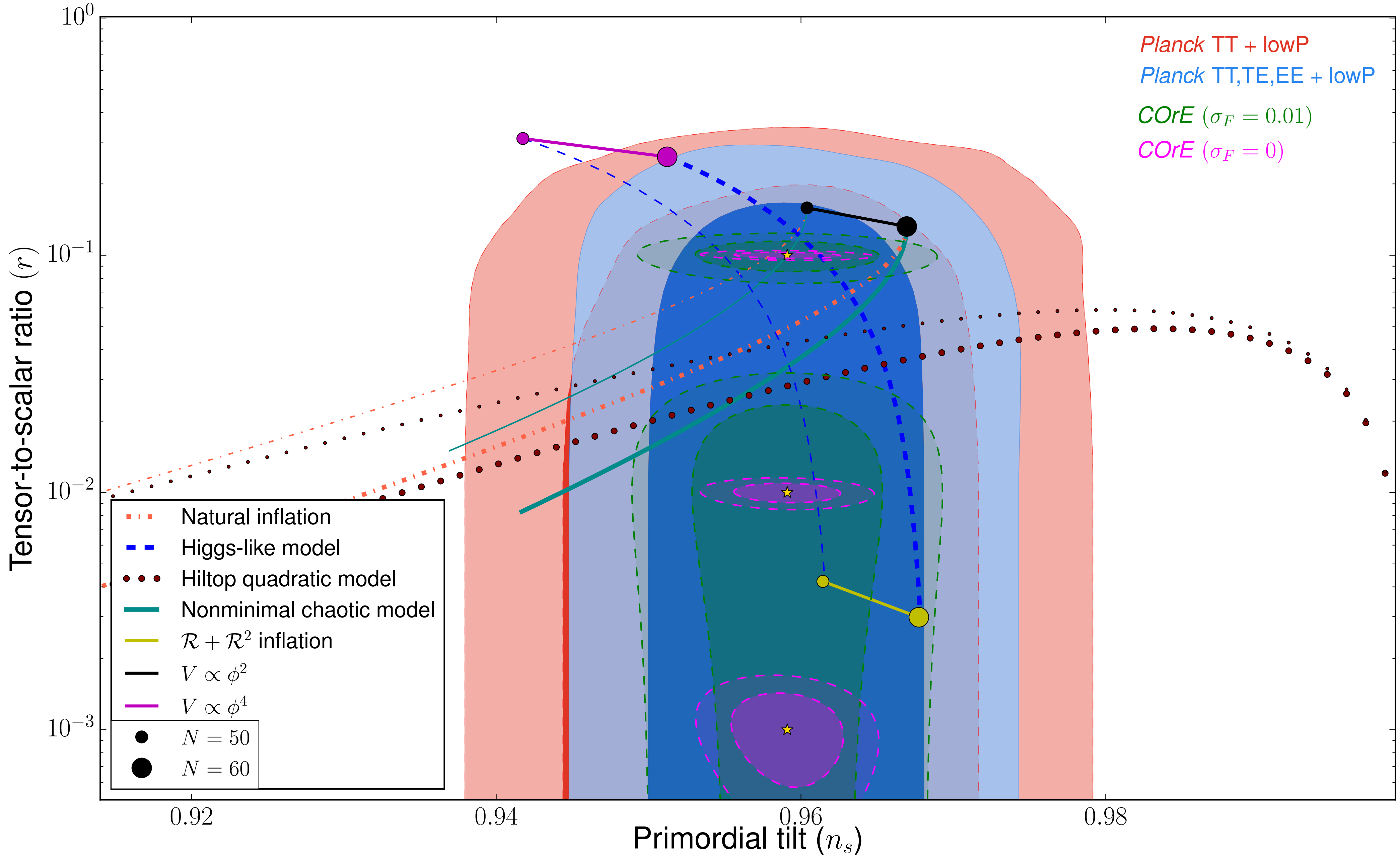
Spectral index

- $$n_s - 1 \equiv \frac{\mathrm{d}\ \ln\Delta_\zeta^2}{\mathrm{d}\ \ln k}$$

And others: $$n_t \equiv \frac{\mathrm{d}\ \ln\ \Delta_h^2}{\mathrm{d}\ \ln k}\ ,\quad \alpha_s \equiv \frac{\mathrm{d}\, n_s}{\mathrm{d}\ln k}\quad ,\quad \beta_s \equiv \frac{\mathrm{d}\,\alpha_s}{\mathrm{d}\ln k}\quad ,\ldots$$

The most favored models

M. Escudero, **HR**, L. Boubekeur, E. Giusarma and O. Mena; 1509.05419



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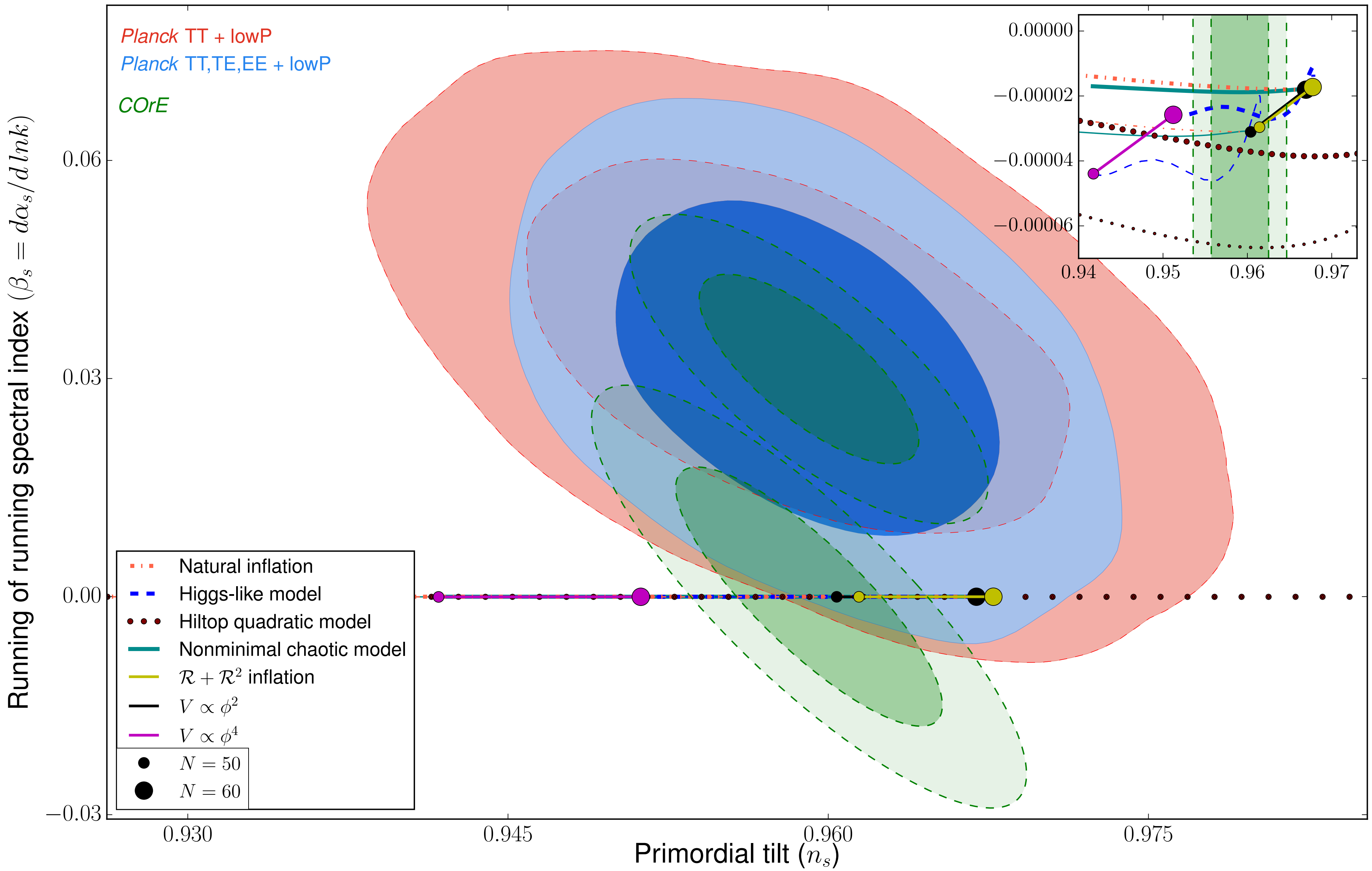

$$\beta_s \equiv \frac{\mathrm{d}^2 n_s}{\mathrm{d}\,(\ln k)^2}$$

Slow-roll hierarchy

By the SR parameters, $\epsilon_H \sim \eta_H = \mathcal{O}(\xi)$ ← Assumption

$$\Delta_\zeta^2(k) = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_H} \left[1 + \mathcal{O}(\xi)\right]\Big|_{k=aH}$$

Slow-roll hierarchy

By the SR parameters, $\epsilon_H \sim \eta_H = \mathcal{O}(\xi)$ ← Assumption ✓

$$\Delta_\zeta^2(k) = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_H} \left[1 + \mathcal{O}(\xi)\right]\Bigg|_{k=aH}$$

So

$$n_s \equiv 1 + \frac{\mathrm{d}\ \ln\ \Delta_\zeta^2}{\mathrm{d}\ \ln\ k} = 1 + \mathcal{O}(\xi)\Bigg|_{k=aH}$$

is approximately scale invariant, as required by observations.

$\boxed{\text{Slow-roll hierarchy}}$

By the SR parameters, $\epsilon_H \sim \eta_H = \mathcal{O}(\xi)$ $\leftarrow$ $\boxed{\text{Assumption}}$

$$\Delta_\zeta^2(k) = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_H} \left[1 + \mathcal{O}(\xi)\right]\Big|_{k=aH}$$

$\boxed{\text{Extra assumption: } \mathcal{O}(\xi) \text{ terms are also approximately scale invariant. Then,}}$

$$n_s = 1 - 4\epsilon_H - 2\eta_H + \mathcal{O}\left(\xi^2\right)\Big|_{k=aH}$$

Slow-roll hierarchy

By the SR parameters, $\epsilon_H \sim \eta_H = \mathcal{O}(\xi)$ ← Assumption

$$\Delta^2_\zeta(k) = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_H}\left[1+\mathcal{O}(\xi)\right]\Big|_{k=aH}$$

Extra assumption: $\mathcal{O}(\xi)$ terms are also approximately scale invariant. Then,

$$n_s = 1 - 4\epsilon_H - 2\eta_H + \mathcal{O}\left(\xi^2\right)\Big|_{k=aH}$$

So, $\alpha_s \equiv \dfrac{\mathrm{d}n_s}{\mathrm{d}\ \ln\ k} = \mathcal{O}(\xi^2)$

which is NOT required by observations, nor is it consequence of SR. Stewart; 0110322

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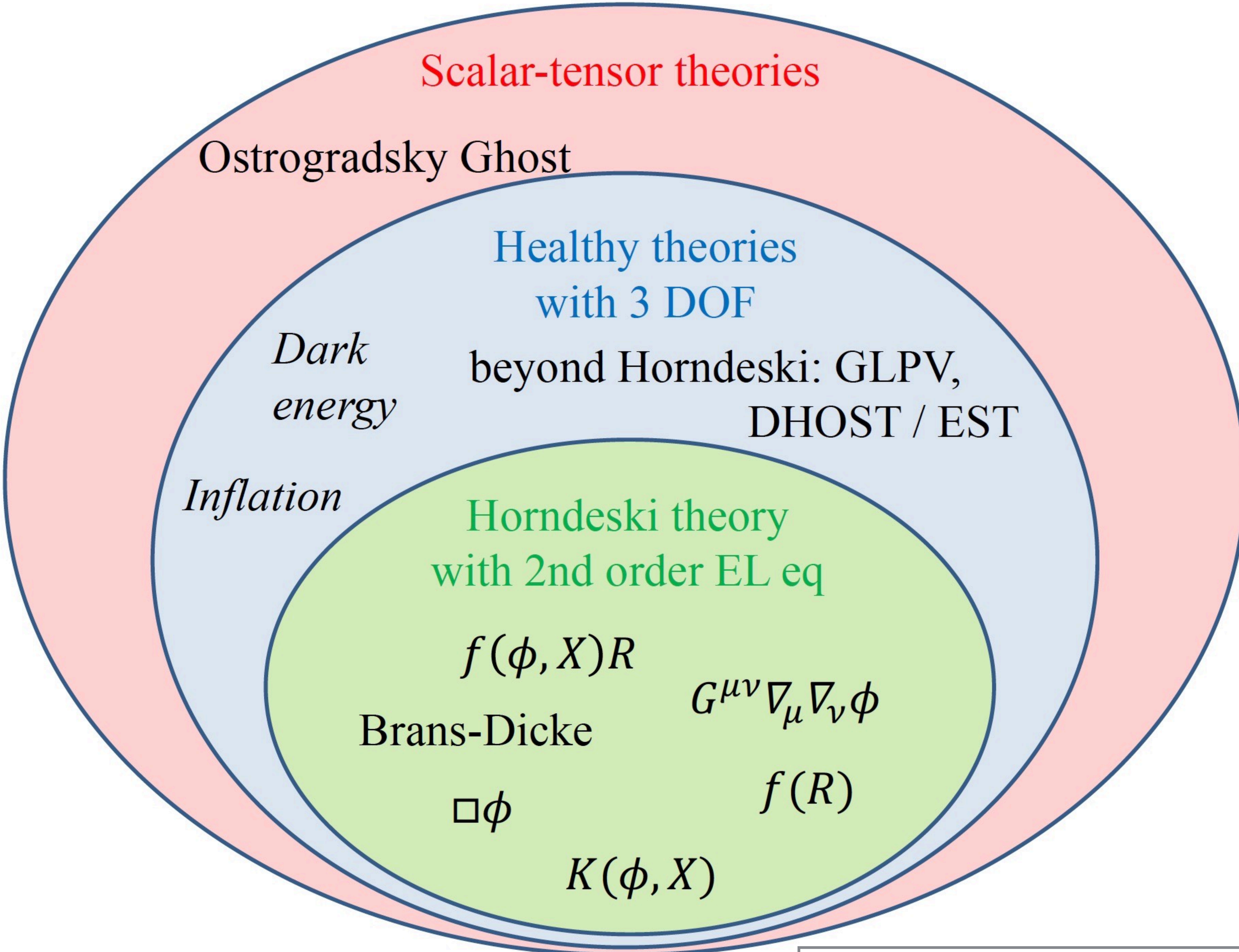
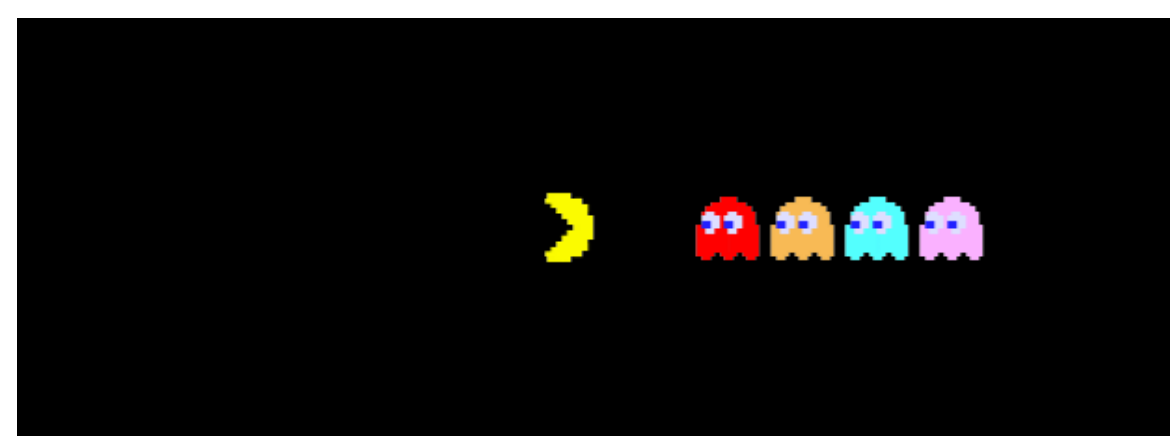


Borrowed from **Hayato Motohashi**'s talks

Ostrogradski's theorem:

"Higher-derivative theories contain extra degrees of freedom, and are usually plagued by negative energies and related instabilities."



Scalar - Tensor theories:

- G. Horndeski (1974)
- A. Nicolis *et al.*; 0811.2197
- C. Deffayet *et al.*; 1103.3260

$$S = \sum_{i=2}^{5} \int \mathrm{d}^4x \sqrt{-g}\mathcal{L}_i$$

$$X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$G_i = G_i(\phi, X)$$

$$\mathcal{L}_2 = G_2 \ ,$$

$$\mathcal{L}_3 = G_3\Box\phi \ ,$$

$$\mathcal{L}_4 = G_4 R - 2G_{4,X}[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \ ,$$

$$\mathcal{L}_5 = G_5 G^{\mu\nu}\phi_{;\mu\nu} + \frac{G_{5,X}}{3}[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}] \ .$$

Applications to:

- Late-time Cosmology (dark energy).
- Early-universe Cosmology (inflation).

Not only scalar-tensor theories, but also...

Vector - Tensor theories:

- G. Horndeski (1976)
- L. Heisenberg; 1402.7026

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[F + \sum_{i=2}^{6}\mathcal{L}_i + \mathcal{L}_m\right]$$

$$\boxed{X \equiv -\frac{1}{2}A_\mu A^\mu}$$

$$\begin{aligned}
\mathcal{L}_2 =& G_2(X, F, Y)\,,\\
\mathcal{L}_3 =& G_3(X)\nabla_\mu A^\mu\,,\\
\mathcal{L}_4 =& G_4(X)R + G_{4,X}(X)\left[(\nabla_\mu A^\mu)^2 - \nabla_\mu A_\nu \nabla^\nu A^\mu\right]\,,\\
\mathcal{L}_5 =& G_5(X)G_{\mu\nu}\nabla^\mu A^\nu - \frac{1}{6}G_{5,X}(X)\left[(\nabla_\mu A^\mu)^3 - 3\nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2\nabla_\rho A_\sigma \nabla^\nu A^\rho \nabla^\sigma A_\nu\right]\\
&- g_5(X)\tilde{F}^{\alpha\mu}\tilde{F}^{\beta}_{\ \mu}\nabla_\alpha A_\beta\,,\\
\mathcal{L}_6 =& G_6(X)L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2}G_{6,X}(X)\tilde{F}^{\alpha\beta}\tilde{F}^{\mu\nu}\nabla_\alpha A_\mu \nabla_\beta A_\nu\,,
\end{aligned}$$

- $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$
- $\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$
- $L \equiv \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\epsilon^{\alpha\beta\gamma\delta}R_{\rho\sigma\gamma\delta}$
- $F \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
- $Y \equiv A^\mu A^\nu F_\mu^{\ \alpha}F_{\nu\alpha}$

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\mathcal{L}_5 =& G_5(X) G_{\mu\nu} \nabla^\mu A^\nu - \frac{1}{6} G_{5,X}(X) \left[(\nabla_\mu A^\mu)^3 - 3 \nabla_\mu A^\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho + 2 \nabla_\rho A_\sigma \nabla^\nu A^\rho \nabla^\sigma A_\nu \right] \\
& - g_5(X) \tilde{F}^{\alpha\mu} \tilde{F}^\beta_{\ \mu} \nabla_\alpha A_\beta\,, \\
\mathcal{L}_6 =& G_6(X) L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta + \frac{1}{2} G_{6,X}(X) \tilde{F}^{\alpha\beta} \tilde{F}^{\mu\nu} \nabla_\alpha A_\mu \nabla_\beta A_\nu\,,
\end{aligned}$$

Applications to:

- Late-time Cosmology.
- Compact objects (hairy black holes, neutron stars,...)

And...

Scalar - Vector - Tensor theories:

- L. Heisenberg; 1801.01523

"The resulting Lagrangians consist of new genuine couplings…"

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\underbrace{\sum_{i=3}^{5}\mathcal{L}^i_{\mathrm{ST}}}_{\text{Horndeski Lagrangian}} + \sum_{i=2}^{4}\mathcal{L}^i_{\mathrm{SVT}}\right)$$

$$\mathcal{L}^2_{\mathrm{SVT}} = f_2(\pi, X, F, \tilde{F}, Y)\ ,$$
$$\mathcal{L}^3_{\mathrm{SVT}} = \mathcal{M}^{\mu\nu}_3 \nabla_\mu \partial_\nu \pi\ ,$$
$$\mathcal{L}^4_{\mathrm{SVT}} = \mathcal{M}^{\mu\nu\alpha\beta}_4 \nabla_\mu \partial_\alpha \pi \nabla_\nu \partial_\beta \pi + f_4(\pi, X) L^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}\ .$$

$$\boxed{X = -\frac{1}{2}(\partial\pi)^2}$$

- $Y \equiv \partial_\mu \pi \partial_\nu \pi F^{\mu\alpha} F^\nu_\alpha$
- $\mathcal{M}^{\mu\nu\alpha\beta}_4 = \left(\frac{1}{2} f_{4,X} + \tilde{f}_4(\pi)\right) \tilde{F}^{\mu\nu} \tilde{F}^{\alpha\beta}$
- $F \equiv F_{\mu\nu} F^{\mu\nu}$
- $\tilde{F} \equiv F_{\mu\nu} \tilde{F}^{\mu\nu}$
- $\mathcal{M}^{\mu\nu}_3 = \left(f_3(\pi, X) g_{\rho\sigma} + \tilde{f}_3(\pi, X) \partial_\rho \pi \partial_\sigma \pi\right) \tilde{F}^{\mu\rho} \tilde{F}^{\nu\sigma}$

And...

Scalar - Vector - Tensor theories:

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$$\mathcal{L}^4_{\mathrm{SVT}} = \mathcal{M}_4^{\mu\nu\alpha\beta}\nabla_\mu\partial_\alpha\pi\nabla_\nu\partial_\beta\pi + f_4(\pi, X)L^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}\ .$$

$$\boxed{X = -\frac{1}{2}(\partial\pi)^2}$$

Applications to:

- New black hole and neutron star solutions.
  - L. Heisenberg & S. Tsujikawa; 1802.07035
- Inflation and magnetogenesis.
  - H. Motohashi, **HR** and S. Tsujikawa; in progress.
- Dark energy and dark matter.

We focus on scalar-tensor theories...

Horndeski theory:

$$S=\sum_{i=2}^{5}\int \mathrm{d}^4x\sqrt{-g}\mathcal{L}_i$$

$$X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$$

$$G_i = G_i(\phi, X)$$

$$\mathcal{L}_2 = G_2 \ ,$$

$$\mathcal{L}_3 = G_3\Box\phi \ ,$$

$$\mathcal{L}_4 = G_4 R - 2G_{4,X}[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \ ,$$

$$\mathcal{L}_5 = G_5 G^{\mu\nu}\phi_{;\mu\nu} + \frac{G_{5,X}}{3}[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}] \ .$$

Canonical inflation:

$$G_2 = -\frac{X}{2} - V(\phi) \ , \qquad G_4 = \frac{M_{\rm pl}^2}{2} \ ,$$

$$G_3 = 0 \ , \qquad G_5 = 0 \ .$$

Inflation

- T. Kobayashi *et al.*; 1105.5723
- H. Motohashi, W. Hu; 1704.01128

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$S_\gamma^{(2)} = \sum_{\lambda=+,\times} \int \mathrm{d}^4x \frac{a^3 b_t}{4c_t^2} \left(\dot{\gamma}_\lambda^2 - \frac{c_t^2 k^2}{a^2} \gamma_\lambda^2 \right)$$

In canonical inflation:

$$b_s = b_t = c_s = c_t = 1$$

Scalar parameters:

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

- $$b_s = \frac{c_s^2}{\epsilon_H}\left[\frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2}\right]$$

- $$c_s^2 = \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1\dot{w}_1w_2 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)}$$

Tensor parameters:

- $$b_t = w_1c_t^2$$

- $$c_t^2 = \frac{w_4}{w_1}$$

$$w_1 = M_{\rm pl}^2 - 2\left(3G_4 + 2HG_5\dot{\phi}\right) + 2G_{5,\phi}X\,,$$

$$w_2 = 2M_{\rm pl}^2H - 2G_3\dot{\phi} - 2\left(30HG_4 - 5G_{4,\phi}\dot{\phi} + 14H^2G_5\dot{\phi}\right) + 28HG_{5,\phi}X\,,$$

$$w_3 = -9M_{\rm pl}^2H^2 + 3\left(X + 12HG_3\dot{\phi}\right) + 6\left(135H^2G_4 - 2G_{3,\phi}X - 45HG_{4,\phi}\dot{\phi} + 56H^3G_5\dot{\phi}\right) - 504H^2G_{5,\phi}X\,,$$

$$w_4 = M_{\rm pl}^2 + 2\left(G_4 - 2G_5\ddot{\phi}\right) - 2G_{5,\phi}X\,.$$

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$\frac{\mathrm{d}^2 v}{\mathrm{d}\tau^2} + \left(c_s^2 k^2 - \frac{1}{z} \frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2} \right) v = 0$$

* Mukhanov - Sasaki equation

- $v = z\zeta$
- $z = a\sqrt{\dfrac{2b_s\epsilon_H}{c_s^2}}$

- Assume slow-roll approximation.
- Solve numerically.
- Use GSR techniques.

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

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Generalized Slow-Roll

- E. Stewart; 0110322
- C. Dvorkin, W. Hu; 0910.2237
- W. Hu; 1104.4500
- W. Hu; 1405.2020
- H. Motohashi, W. Hu; 1503.04810
- H. Motohashi, W. Hu; 1704.01128

… and others.

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

de Sitter background

deviations from dS

- $y \equiv \sqrt{2c_s k} v$
- $x \equiv k s_s$
- $s_s \equiv \int c_s \mathrm{d}\tau$

- $f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2}} \frac{aHs_s}{c_s}$
- $\Delta_\zeta^2(k) = \lim_{x\to 0} \left| \frac{xy}{f} \right|^2$

Generalized slow-roll recipe:

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$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

3. Apply Green function techniques (GSR).

$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

- $$G = -2 \ln f + \frac{2}{3} (\ln f)'$$ Source function
- $$W(u) = \frac{3 \sin(2u)}{2u^3} - \frac{3 \cos(2u)}{u^2} - \frac{3 \sin(2u)}{2u}$$

Generalized slow-roll recipe:

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$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

4. Taylor expand GSR formula and write down analytic equations (OSR).

Optimized Slow-Roll

If $\Delta N > 1$ we can Taylor expand the GSR formula around some epoch x_f:

$$\ln\,\Delta^2 \simeq G\,(\ln\,x_f) + \sum_{p=1}^{\infty} q_p\,(\ln\,x_f)\,G^{(p)}\,(\ln\,x_f)$$

- $q_1(\ln\,x_f) = \ln\,x_1 - \ln\,x_f$

$$\boxed{\ln\,x_1 \equiv \frac{7}{3} - \ln\,2 - \gamma_{\mathrm{E}}}$$

- $q_p(\ln x_f) = -\dfrac{1}{p!}\displaystyle\int_{x_m}^{\infty} W'(x)\left(\ln\,\frac{x}{x_f}\right)^p$

Leading order of standard SR ($\ln x_f = 0$):

- $$\ln \Delta^2 \simeq G(0)$$
- $$\frac{\mathrm{d} \ln \Delta^2}{\mathrm{d} \ln k} \simeq -G'(0)$$
- $$\alpha \simeq G''(0)$$

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f)\, G^{(p)}(\ln x_f)$$

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$$G = -2 \ln f + \frac{2}{3}(\ln f)'$$

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NLO SR correction ($p = 1$):

- $q_1(0) = \ln x_1 = 1.06$
- $G'(0) \sim (\Delta N)^{-1}$

If $\Delta N \sim 3$,
the correction is ~ 0.35 .

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p (\ln x_f)\, G^{(p)}(\ln x_f)$$

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- $\alpha \simeq G''(\ln x_1)$

NLO SR correction ($p = 1$):

- $q_1(0) = \ln x_1 = 1.06$
- $G'(0) \sim (\Delta N)^{-1}$

If $\Delta N \sim 3$,
the correction is ~ 0.35 .

NNLO OSR correction ($p = 2$):

- $q_2(\ln x_1) \simeq -0.36$
- $G''(\ln x_1) \sim (\Delta N)^{-2}$

If $\Delta N \sim 3$,
the correction is ~ 0.04 .

Summary:

- $\Delta N \sim N \sim 60$ $\longrightarrow$ Standard slow-roll
- $1 < \Delta N < N$ $\longrightarrow$ Optimized slow-roll
- $\Delta N \sim 1$ $\longrightarrow$ Generalized slow-roll
- $\Delta N < 1$ $\longrightarrow$ Stationary phase approximation (Miranda et al.; 1510.07580)

Optimized SR for Horndeski (leading order):

- H. Motohashi, W. Hu; 1704.01128

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p (\ln x_f)\, G^{(p)}(\ln x_f)$$

$$G = -2\ln f + \frac{2}{3}(\ln f)'$$

$$f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2}}\, \frac{aHs_s}{c_s}$$

Scalars

$$\ln \Delta_\zeta^2 \approx \ln\left(\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 - \frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1}\Big|_{x=x_1}$$

$$n_s - 1 \approx -4\epsilon_H - 2\delta_1 - \sigma_{s1} - \xi_{s1} - \frac{2}{3}\delta_2 - \frac{7}{3}\sigma_{s2} - \frac{1}{3}\xi_{s2}\Big|_{x=x_1}$$

$$\alpha_s \approx -2\delta_2 - \sigma_{s2} - \xi_{s2} - \frac{2}{3}\delta_3 - \frac{7}{3}\sigma_{s3} - \frac{1}{3}\xi_{s3} - 8\epsilon_H^2 - 10\epsilon_H\delta_1 + 2\delta_1^2\Big|_{x=x_1}$$

Tensors

$$\ln \Delta_\gamma^2 \approx \ln\left(\frac{H^2}{2\pi^2 b_t c_t}\right) - \frac{8}{3}\epsilon_H - \frac{7}{3}\sigma_{t1} - \frac{1}{3}\xi_{t1}\Big|_{x=x_1}$$

$$n_t \approx -2\epsilon_H - \sigma_{t1} - \xi_{t1} - \frac{7}{3}\sigma_{t2} - \frac{1}{3}\xi_{t2}\Big|_{x=x_1}$$

$$\alpha_t \approx -\sigma_{t2} - \xi_{t2} - \frac{7}{3}\sigma_{t3} - \frac{1}{3}\xi_{t3} - 4\epsilon_H^2 - 4\epsilon_H\delta_1\Big|_{x=x_1}$$

$$\ln x_1 \equiv \frac{7}{3} - \ln 2 - \gamma_{\rm E}$$

Optimized SR for Horndeski (leading order):

- H. Motohashi, W. Hu; 1704.01128

$$r \equiv \frac{4\Delta_\gamma^2}{\Delta_\zeta^2} \approx 16\epsilon_H \frac{b_s c_s}{b_t c_t} \approx -\frac{8 b_s c_s}{b_t c_t} n_t,$$

Deviations from the standard consistency relation in the observations could be checked in this context.

Outline:

1. Slow - roll inflation.

2. Scalar - Tensor theories.

3. Generalized slow-roll (GSR) techniques.

4. G-inflation.

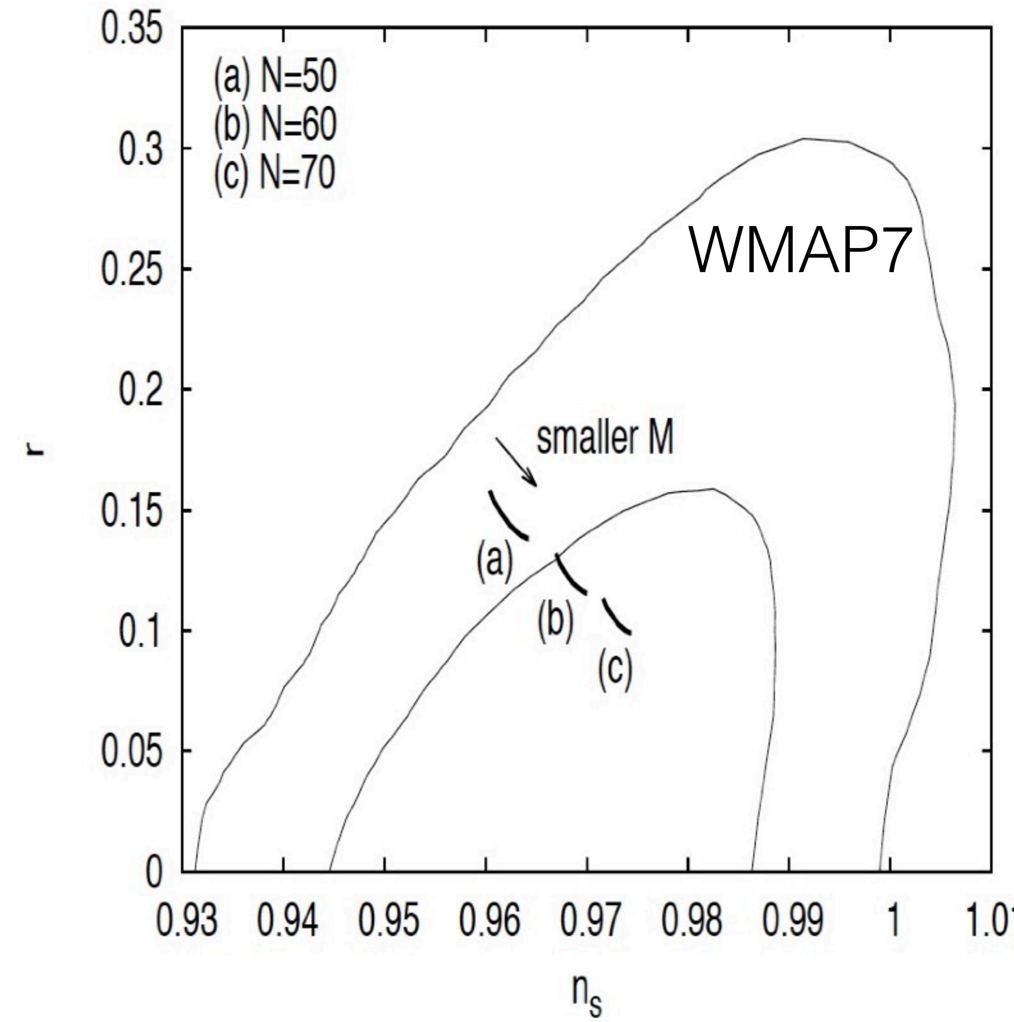
$\boxed{G_3 \text{ + chaotic inflation = G-inflation}}$

J. Ohashi, S. Tsujikawa; 1207.4879

$$\mathcal{L}_2 = X - V(\phi) = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_3 = M^{-3} X \Box \phi$$

$$\mathcal{L}_4 = \frac{1}{2} M_{\rm pl}^2 \mathcal{R}$$



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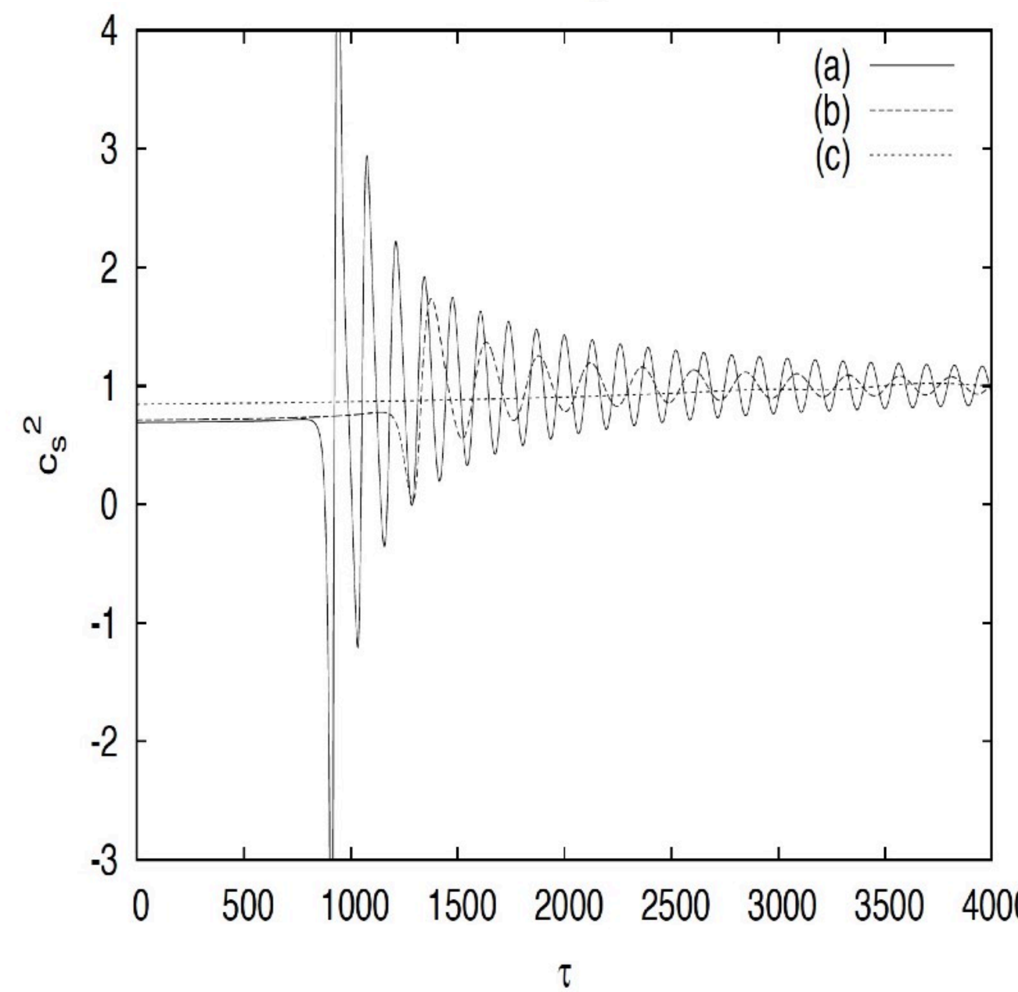
a) $M = 3 \times 10^{-4} M_{\rm pl}$

b) $M = 4.2 \times 10^{-4} M_{\rm pl}$

c) $M = 1 \times 10^{-3} M_{\rm pl}$



For $c_s^2 > 0$, $M > 4.2 \times 10^{-4} M_{\rm pl}$

$$\boxed{G_3 + \text{chaotic inflation} = \text{G-inflation}}$$

J. Ohashi, S. Tsujikawa; 1207.4879

$$\mathcal{L}_2 = X - V(\phi) = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$$

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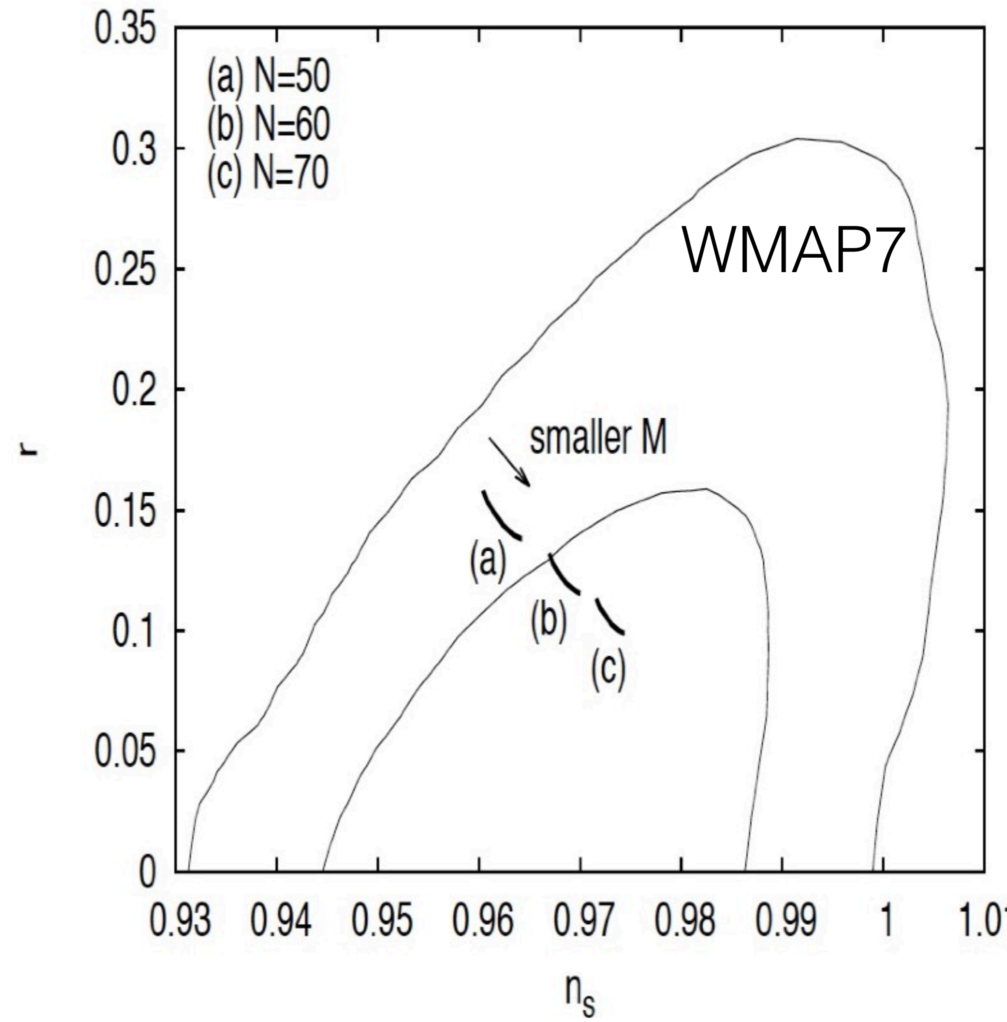
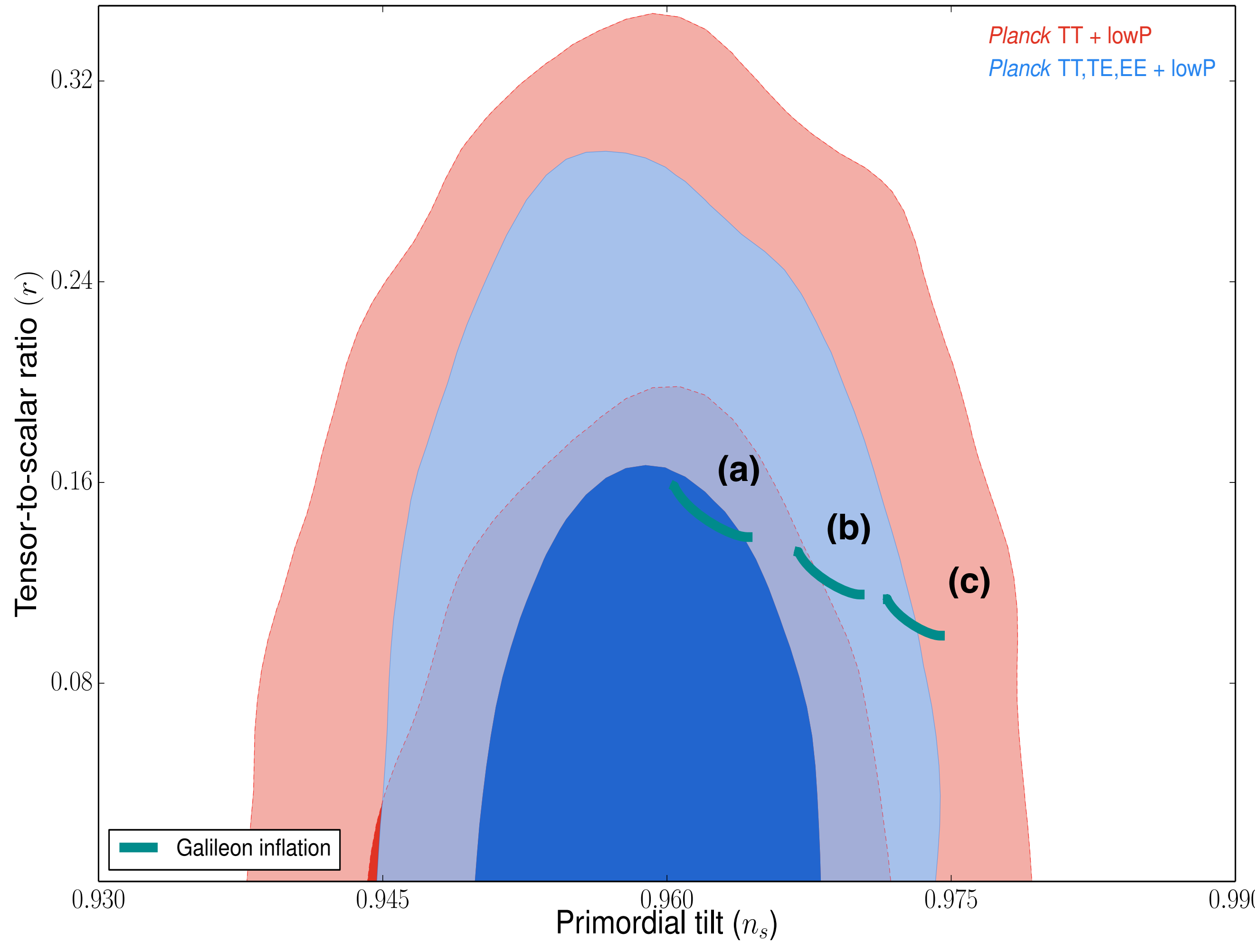
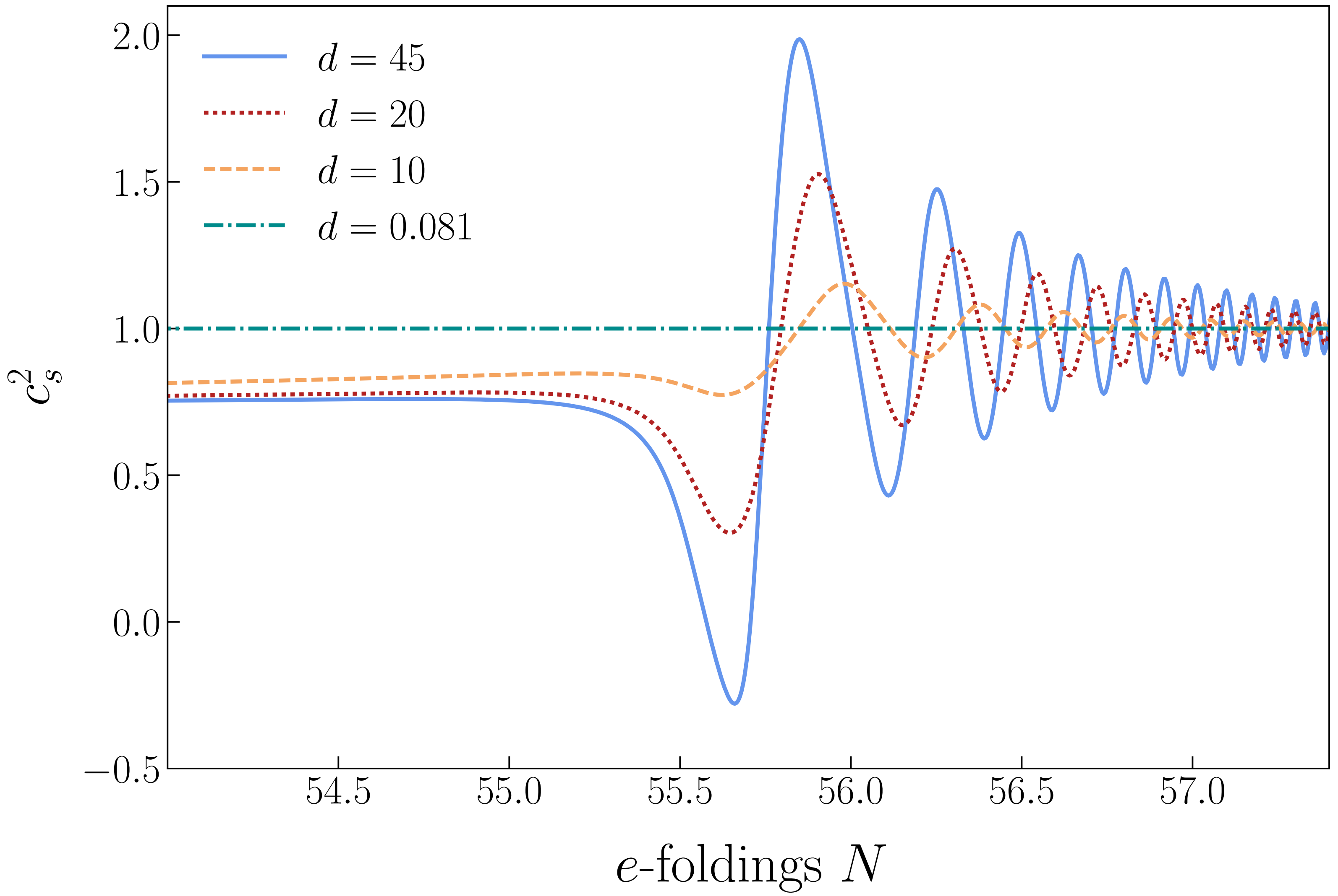




G_3 + *tanh* + chaotic inflation = transient G-inflation

HR, S. Passaglia, H. Motohashi, W. Hu, O. Mena; 1207.4879

$$\mathcal{L}_3 = M^{-3}\left[1+\tanh\left(\frac{\phi-\phi_r}{d}\right)\right]X\Box\phi$$



$\phi_r = 13$

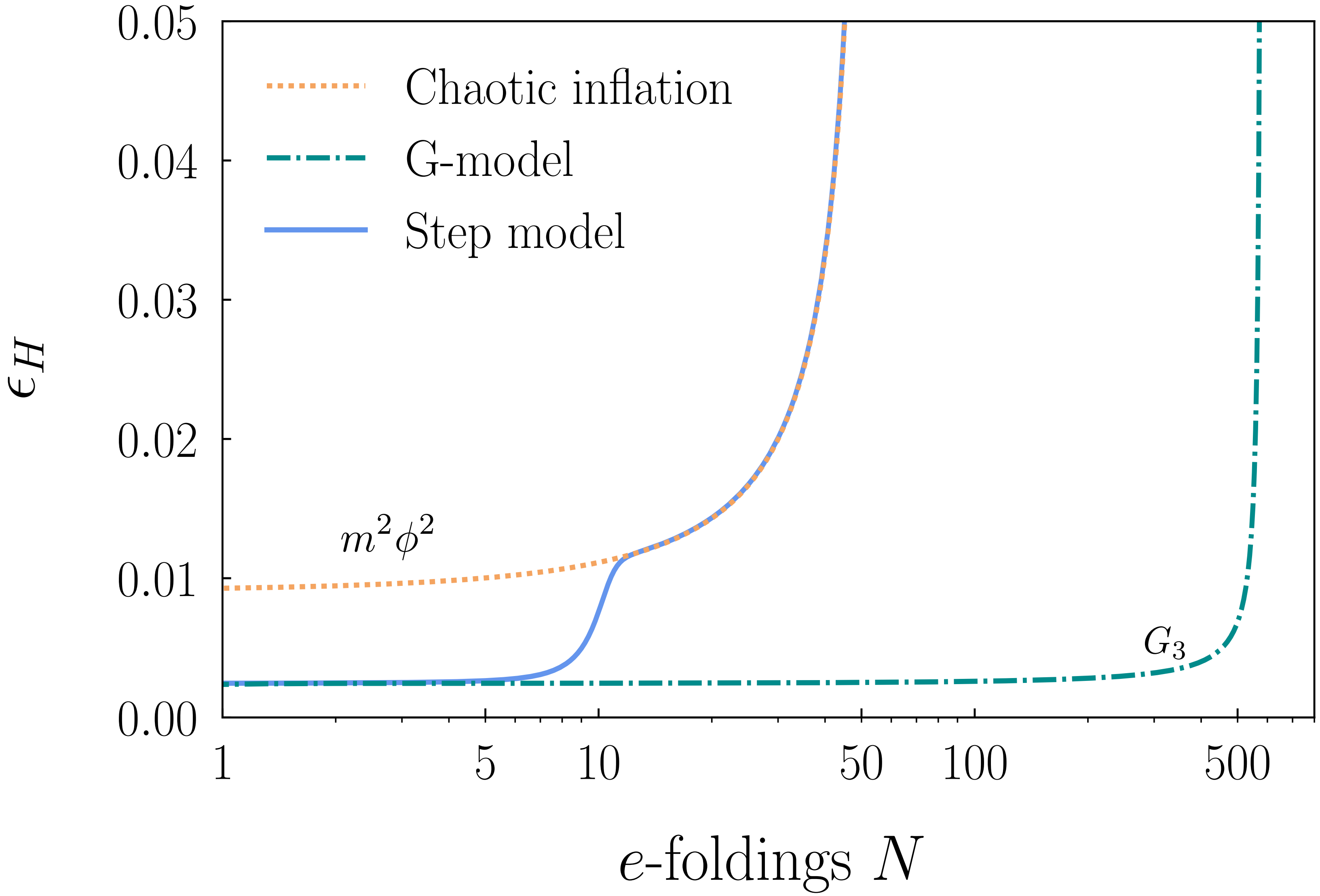
$M = 1.3\times10^{-4}$

No gradient instabilities for any mass scale M.

G_3 + *tanh* + chaotic inflation = transient G-inflation

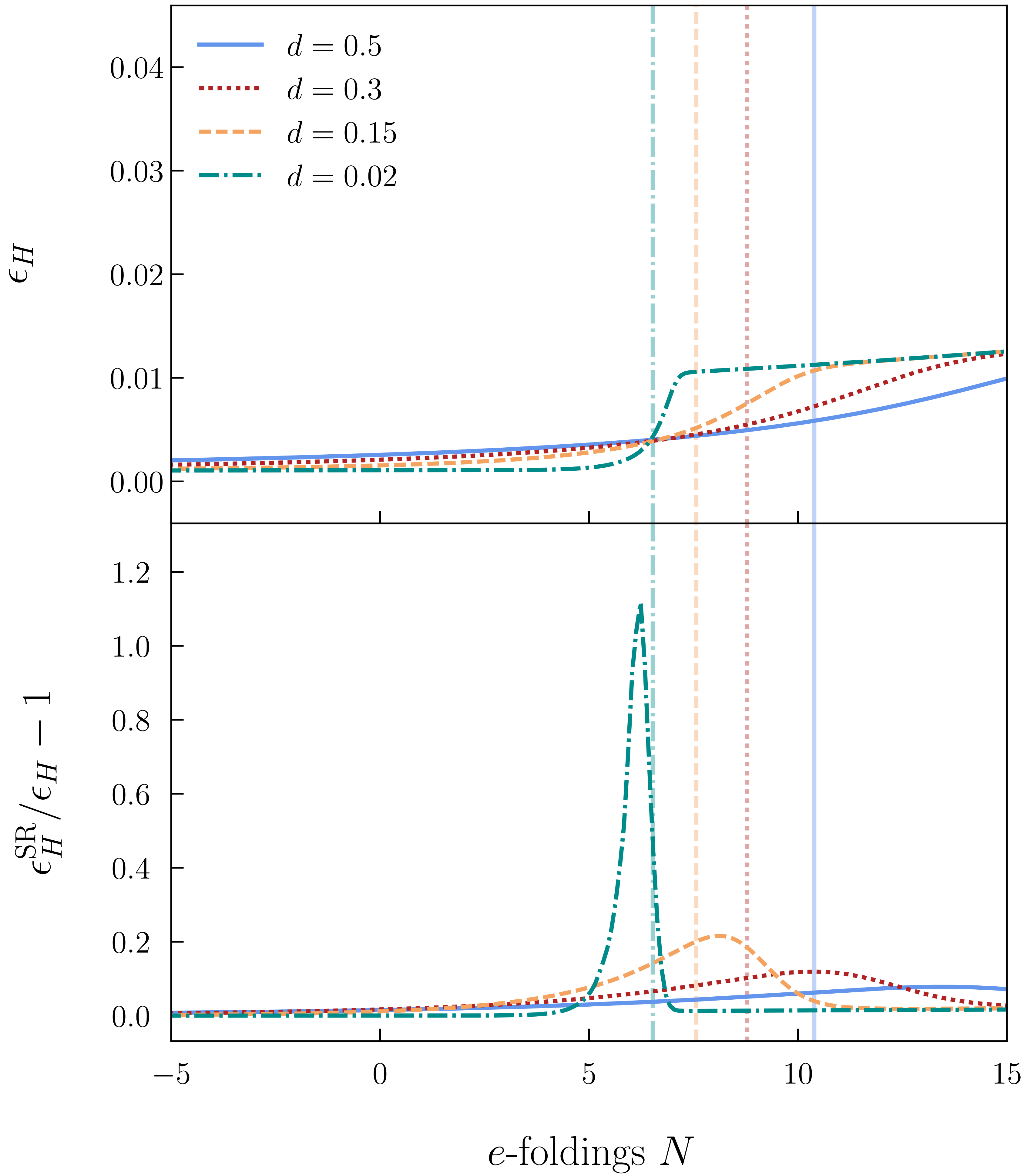
HR, S. Passaglia, H. Motohashi, W. Hu, O. Mena; 1207.4879

$$\mathcal{L}_3 = M^{-3}\left[1 + \tanh\left(\frac{\phi - \phi_r}{d}\right)\right] X\Box\phi$$



Step size ~ 4 *e*-folds

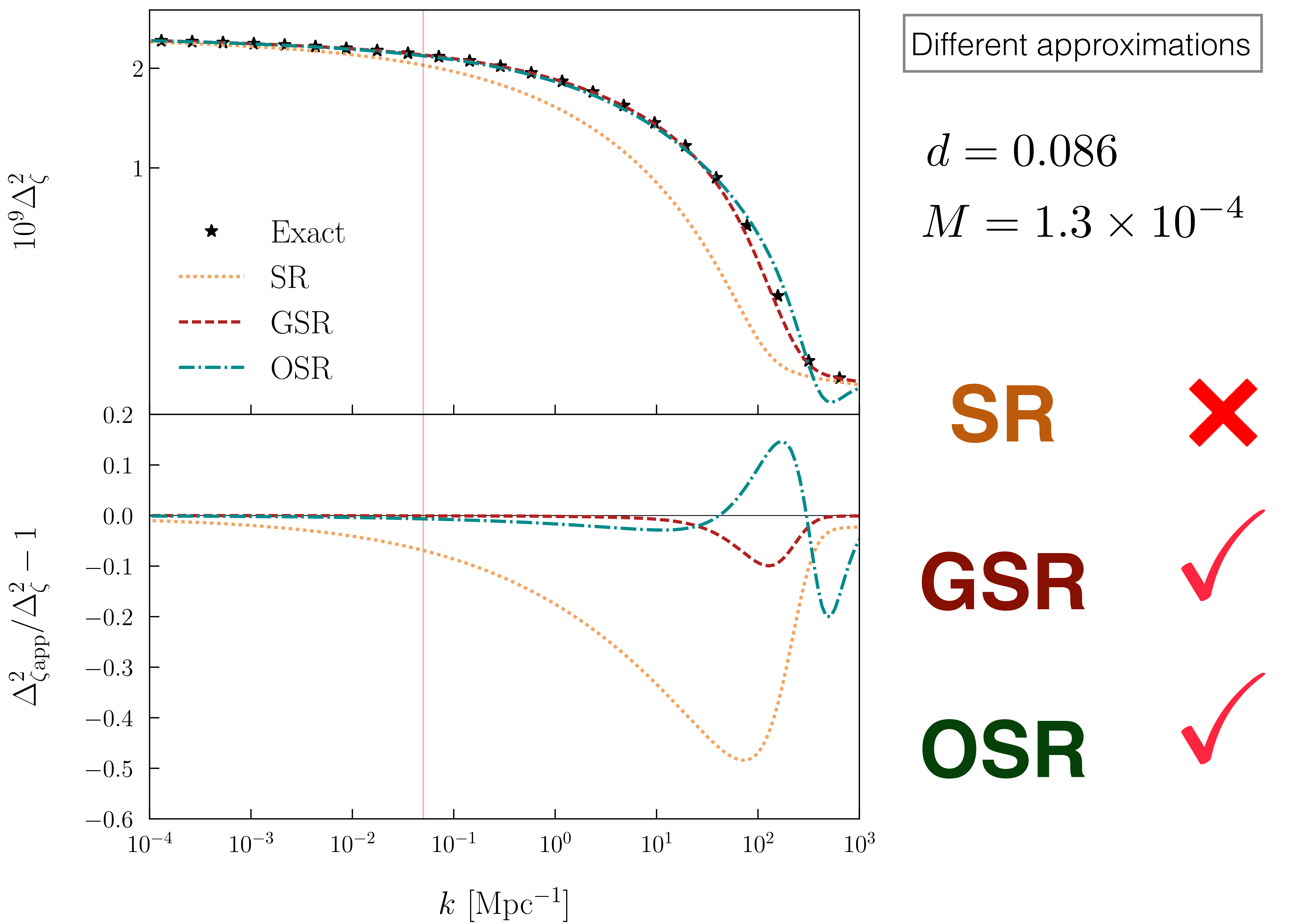
Ideal for OSR



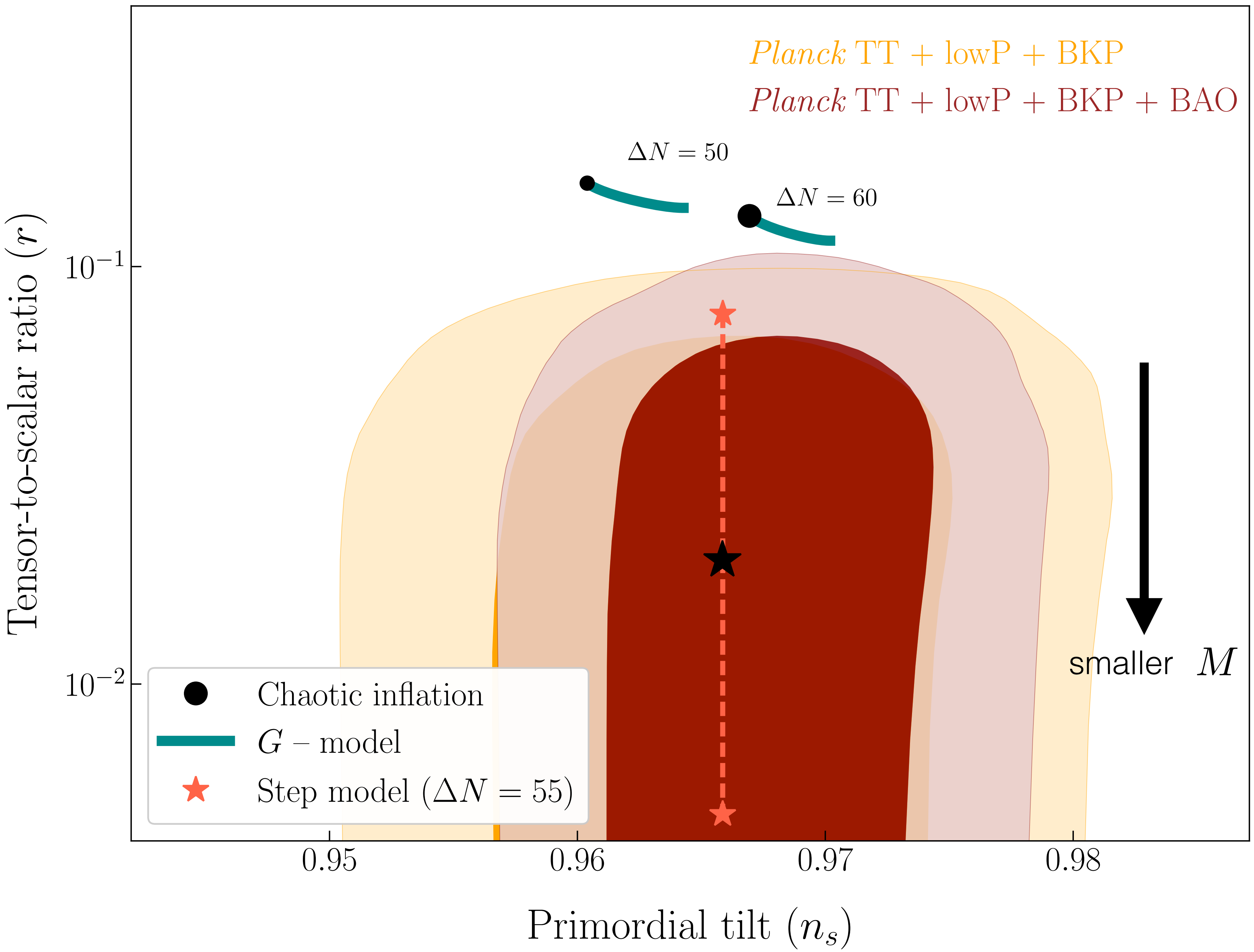
Slow - roll violation:

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

- $N = 0$: CMB scales.
- Vertical lines: where the transition occurs.
- SR violation is maximal around the transition.



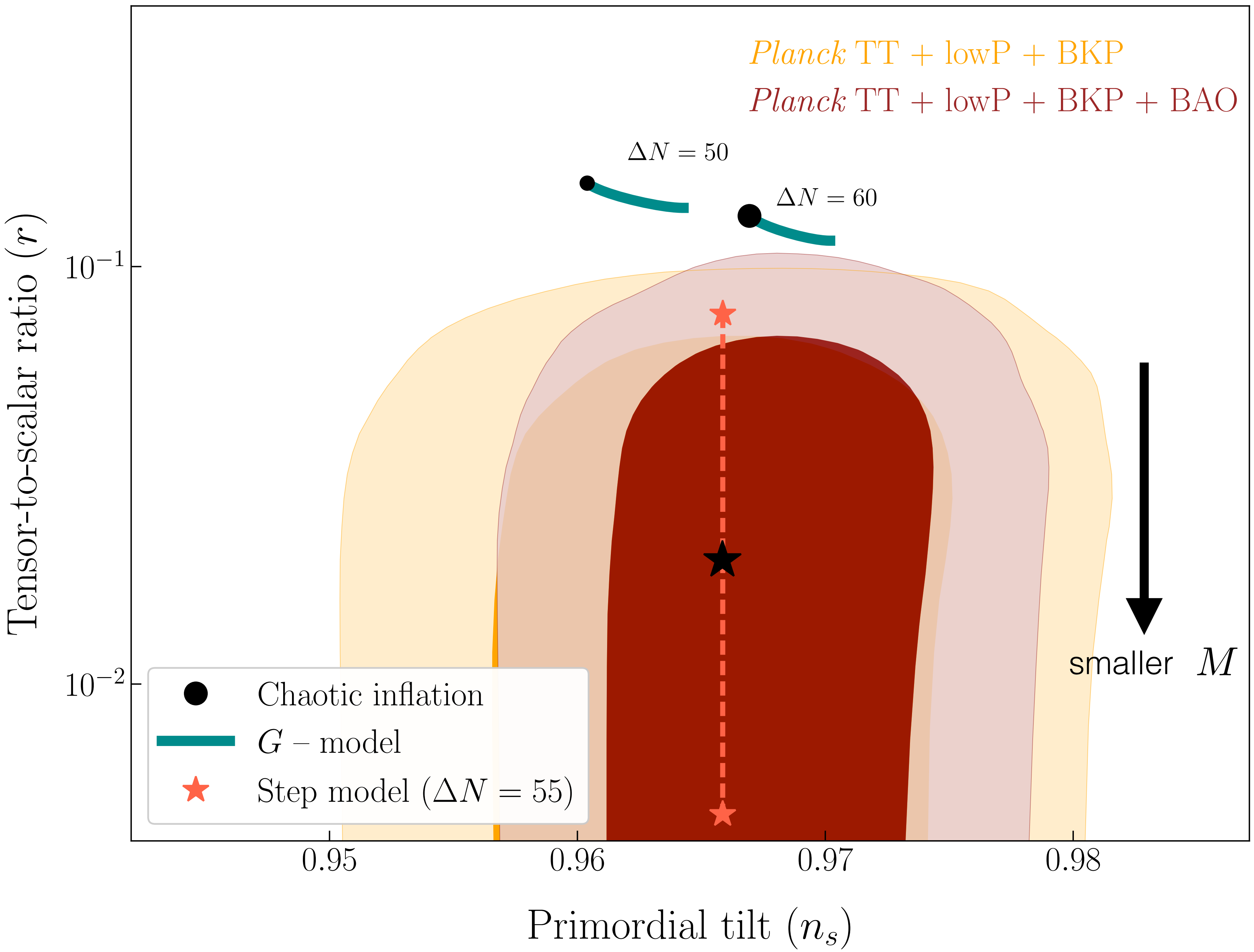
Different approximations
$d = 0.086$
$M = 1.3 \times 10^{-4}$
SR
GSR
OSR
Exact
SR
GSR
OSR
$10^9 \Delta_\zeta^2$
$\Delta^2_{\zeta\mathrm{app}}/\Delta^2_\zeta - 1$
k [Mpc^{-1}]



Observables

$$n_s = 9.66$$
$$\alpha_s = -0.011$$

- n_s and α_s fixed.
- Find a set of values for d and ϕ_r.
- This places lower and upper bounds on r.

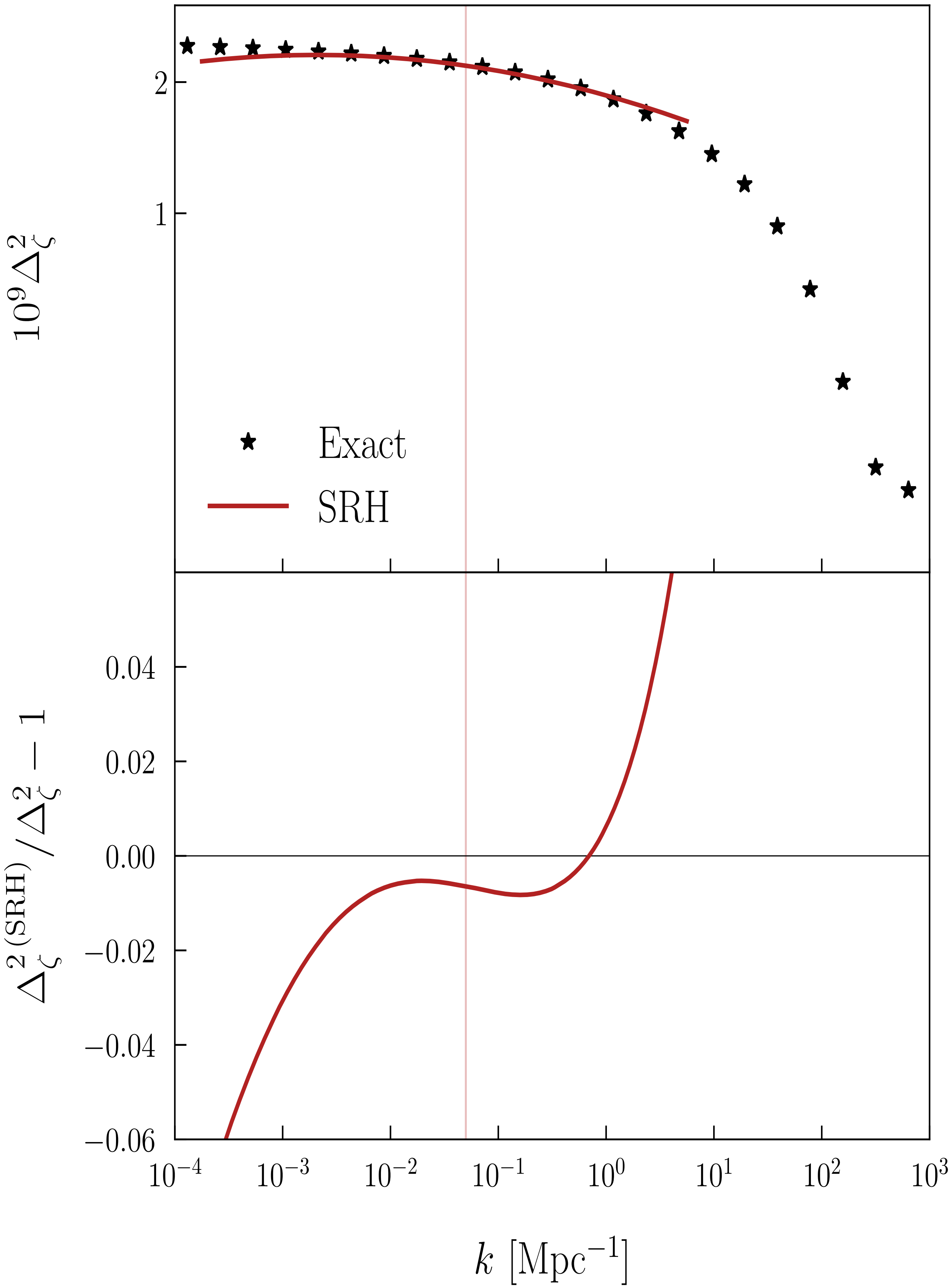


Observables

$$n_s = 9.66$$
$$\alpha_s = -0.011$$

$$\boxed{r \gtrsim 0.005}$$

- A smaller α_s would shift the line upwards because the step gets wider.
- A larger α_s would be in tension with measurements.



Slow - roll hierarchy formula:

$$\Delta_\zeta^{2(\mathrm{SRH})}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k_*)}$$

- Using OSR parameters

Deviations of less than 1%

Summary

- ★ Inflation in the Horndeski framework is viable and can *cure* some popular models.

- ★ The transient G-inflation model allows us to compute observables during a G-inflation period and to end inflation as canonical.

- ★ Generalized slow-roll and Optimized slow-roll techniques are efficient tools for this type of models to compute the power spectra an also the bispectrum.