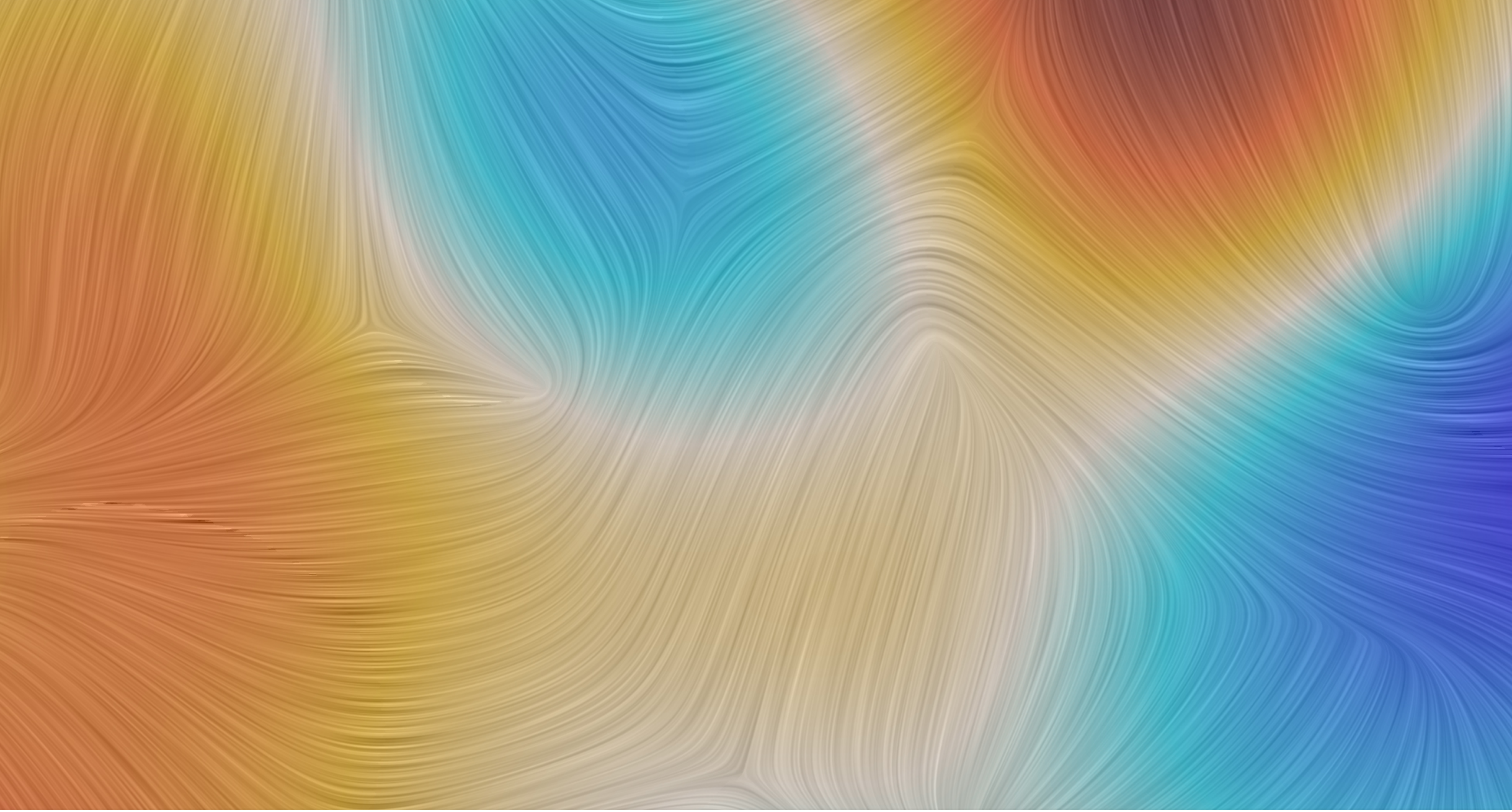

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Inflation beyond GR

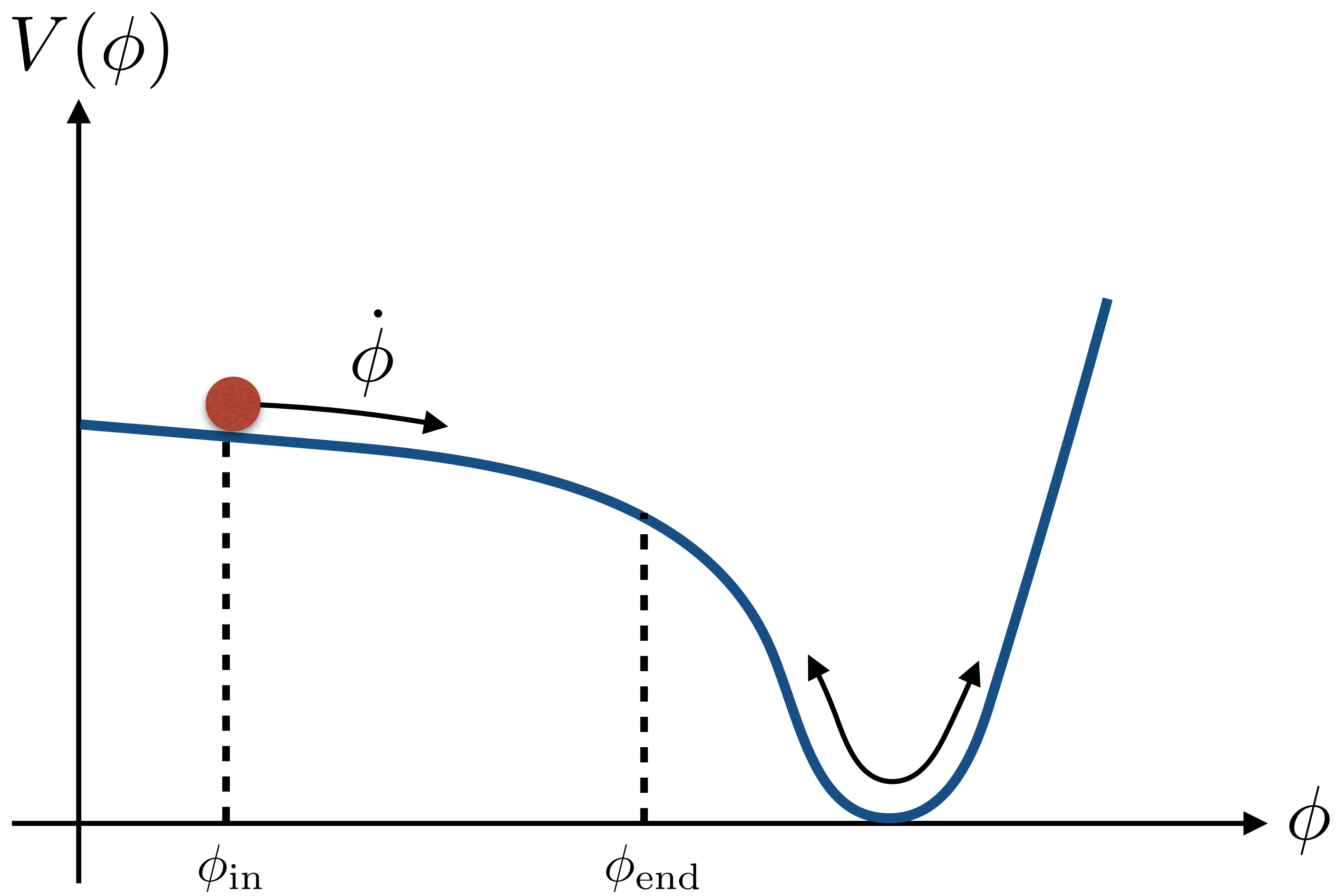
Héctor Ramírez

IFIC - University of Valencia

APC Paris - Oct '18

Single-field slow-roll

— A. Linde; Phys. Lett. **108B** (1982) 389.



Einstein-Hilbert action:

$$\mathcal{S}_{\rm EH} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$

The theory of quantum fluctuations

Power spectra of primordial perturbations:

- $$\Delta^2_\zeta(k) = \frac{1}{8\pi^2 M^2_{\rm pl}} \frac{H^2}{\epsilon_H}\bigg|_{k=aH}$$
- $$\Delta^2_\gamma(k) = \frac{2}{\pi^2} \frac{H^2}{M^2_{\rm pl}}\bigg|_{k=aH}$$

Tensor-to-scalar ratio

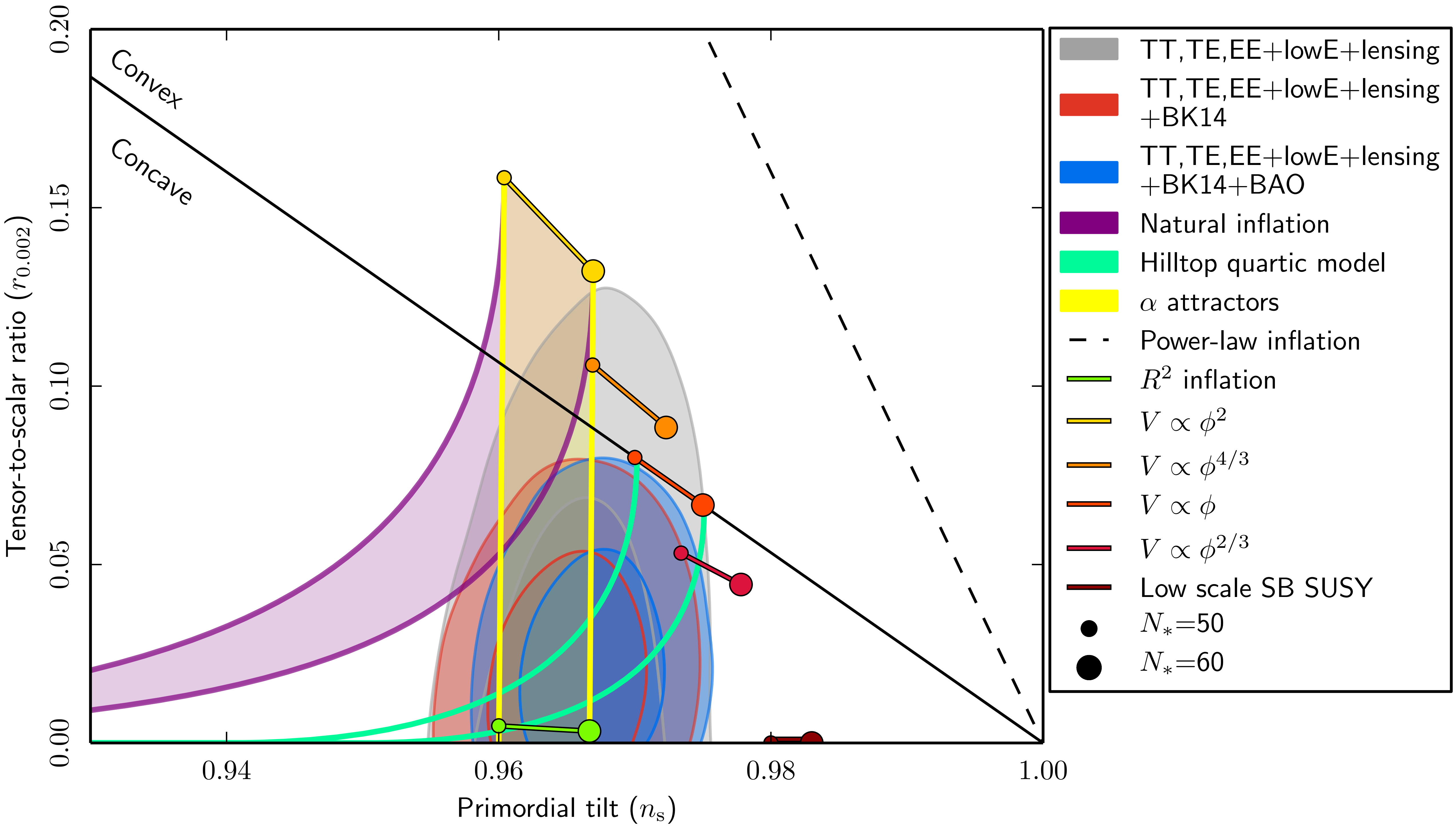
- $$r \equiv \frac{\Delta^2_\gamma}{\Delta^2_\zeta}$$

Spectral index

- $$n_s - 1 \equiv \frac{\mathrm{d}\ \ln \Delta^2_\zeta}{\mathrm{d}\ \ln k}$$

And others: $$n_t \equiv \frac{\mathrm{d}\ \ln\ \Delta^2_h}{\mathrm{d}\ \ln k}\ , \quad \alpha_s \equiv \frac{\mathrm{d}\, n_s}{\mathrm{d}\ln k}\ , \quad \beta_s \equiv \frac{\mathrm{d}\, \alpha_s}{\mathrm{d}\ln k}\ , \ldots$$

The most favored models



— Y. Akrami *et al.* [Planck Collaboration]; arXiv:1807.06211 [astro-ph.CO].

Inflation beyond GR

Outline:

1. Comment on scalar-tensor and vector-tensor theories.

2. Scalar-vector-tensor theories for cosmology and inflation.

3. G-inflation status.

4. Generalized slow-roll (GSR) techniques.

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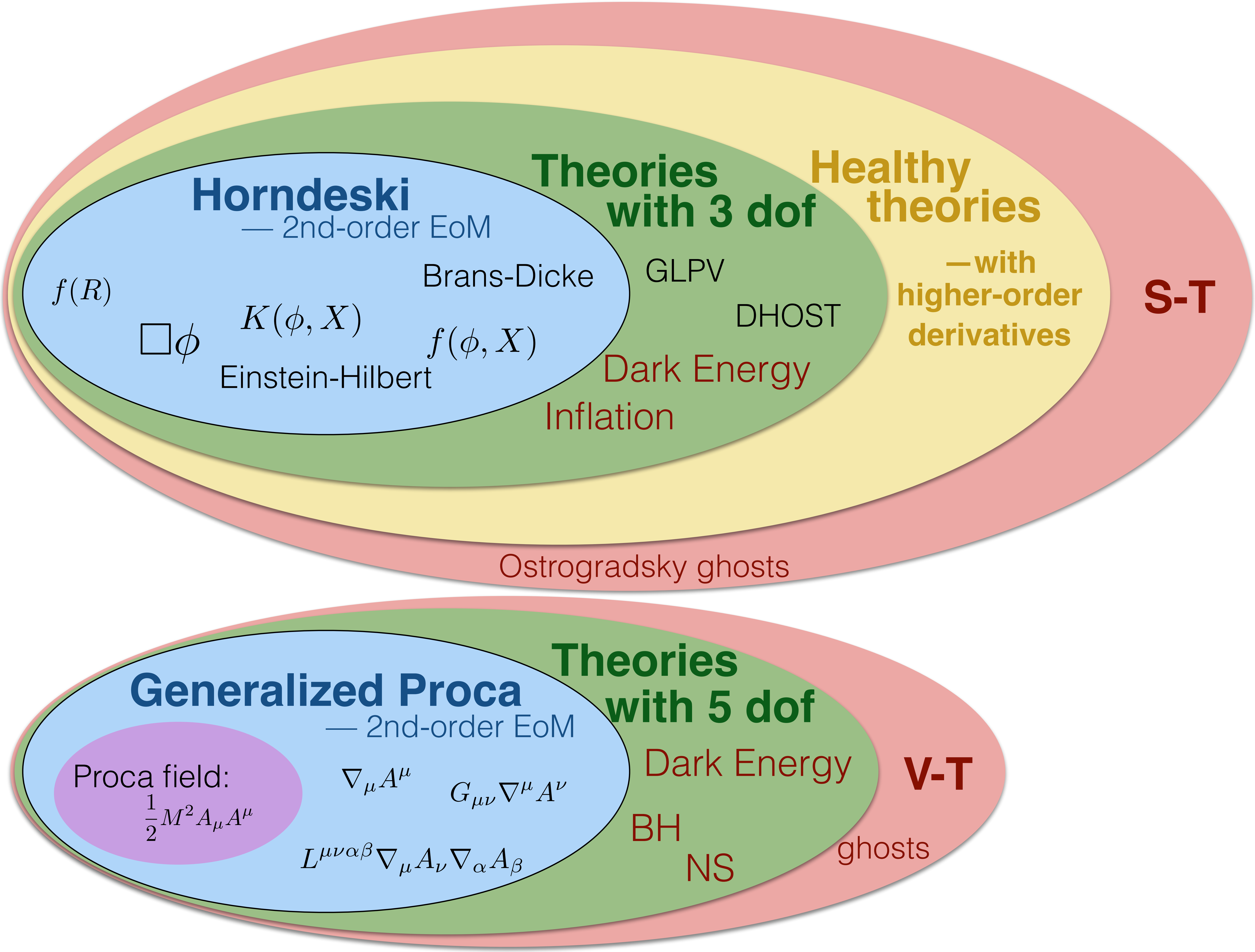
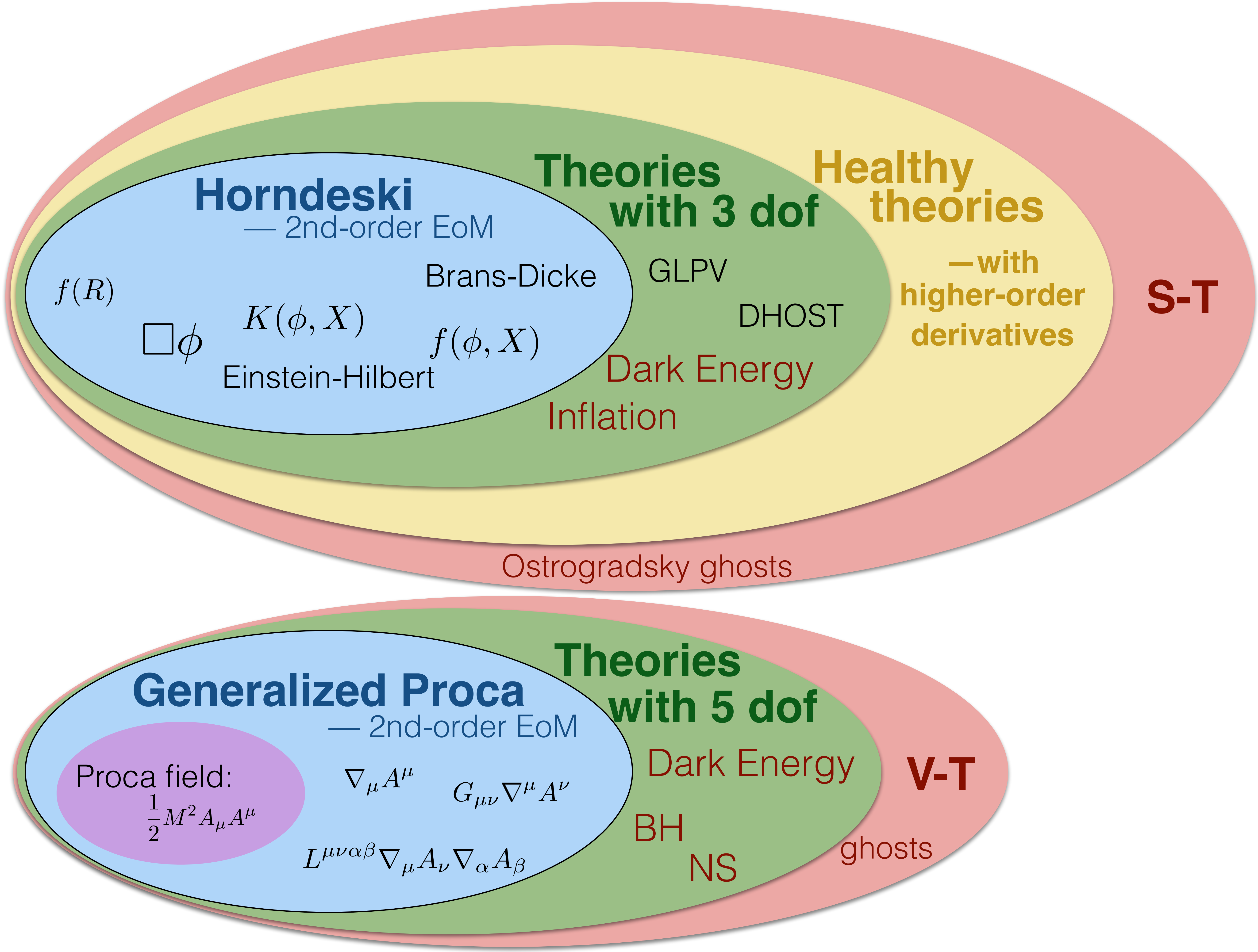

Horndeski
— 2nd-order EoM
$f(R)$
$\Box\phi$
$K(\phi, X)$
Brans-Dicke
$f(\phi, X)$
Einstein-Hilbert
Theories with 3 dof
GLPV
DHOST
Dark Energy
Inflation
Healthy theories
—with higher-order derivatives
S-T
Ostrogradsky ghosts
Generalized Proca
— 2nd-order EoM
Proca field:
$\frac{1}{2}M^2 A_\mu A^\mu$
$\nabla_\mu A^\mu$
$G_{\mu\nu}\nabla^\mu A^\nu$
$L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta$
Theories with 5 dof
Dark Energy
BH
NS
V-T
ghosts



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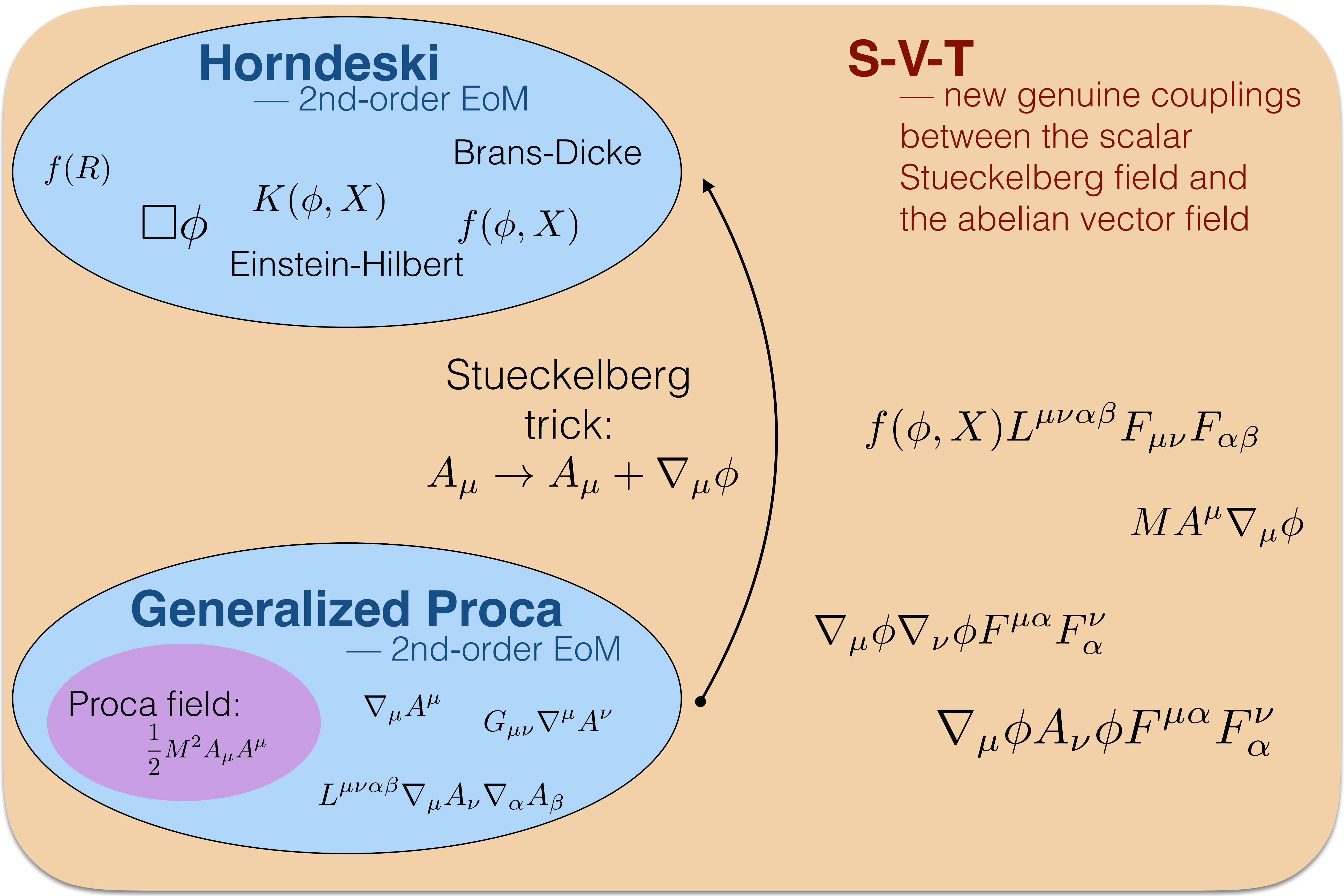
Horndeski
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$L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta$
S-V-T
— new genuine couplings between the scalar Stueckelberg field and the abelian vector field
$f(\phi, X) L^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$
$M A^\mu \nabla_\mu \phi$
$\nabla_\mu \phi \nabla_\nu \phi F^{\mu\alpha} F^\nu_\alpha$
$\nabla_\mu \phi A_\nu \phi F^{\mu\alpha} F^\nu_\alpha$

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Scalar-Vector-Tensor theories

— L. Heisenberg; arXiv:1801.01523 [gr-qc]

Generalized Proca

— Broken gauge invariance

Stueckelberg trick

$$A_\mu \to A_\mu + \nabla_\mu \phi$$

Shift symmetric Horndeski

$$(\phi \to \phi - \varphi)$$

+

SV interactions

— Gauge invariant

[breaking gauge invariance]

$U(1)$-broken SVT theories

+

soft shift-symm. breaking

5 dof:

1 S, **2** transverse **V**, **2 T** polarizations

6 dof:

2 S, **2** transverse **V**, **2 T** polarizations (some part of) The Action:

$$\mathcal{S}_{\rm SVT} = \int d^4x\sqrt{-g}\sum_{i=2}^{6}\mathcal{L}_i$$

- $\mathcal{L}_2 = f_2(\phi, X1, X2, X3, F, Y_1, Y_2, Y_3)$
  - $X_1 = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$
  - $X_2 = -\frac{1}{2}A^\mu\nabla_\mu\phi$
  - $X_3 = -\frac{1}{2}A_\mu A^\mu$
  - $F \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
  - $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$
  - $Y_1 \equiv \nabla_\mu\phi\nabla_\nu\phi F^{\mu\alpha}F^\nu_\alpha$
  - $Y_2 \equiv \nabla_\mu\phi A_\nu F^{\mu\alpha}F^\nu_\alpha$
  - $Y_3 \equiv A_\mu A_\nu F^{\mu\alpha}F^\nu_\alpha$
- $\mathcal{L}_{3,4,5,6} \propto$ (highly-) nontrivial couplings between the above terms and
  - R
  - $G_{\mu\nu}$
  - $L^{\mu\nu\alpha\beta}$

…and other combinations.

- New type of hairy black hole solutions for the gauge-invariant Lagrangian have been found.

L. Heisenberg and S. Tsujikawa;
doi:10.1016/j.physletb.2018.03.059
[arXiv:1802.07035 [gr-qc]].

- For a FLRW space-time, the EoM and the conditions for the absence of ghosts and Laplacian instabilities, for the full gauge-broken, parity-invariant SVT action have been computed.

L. Heisenberg, R. Kase and S. Tsujikawa;
doi:10.1103/PhysRevD.98.024038 [arXiv:1805.01066 [gr-qc]].

Dark Energy implications

- From tensor perturbations:
  - $$c_t^2 = \frac{2f_4 - A_0\dot{\phi}f_{5,\phi} - \dot{A}_0 A_0^2 f_{5,X_3}}{2f_4 - 2A_0^2 f_{4,X_3} + A_0\dot{\phi}f_{5,\phi} - HA_0^3 f_{5,X3}}$$
- Then, given the bound[†] $-3\times 10^{-15} \leq c_t - 1 \leq 7\times 10^{-16}$:
  - $f_4(\phi, X_3) = f_4(\phi)$
  - $f_5(\phi, X_3) = \text{constant}$

R. Kase and S. Tsujikawa; arXiv:1805.11919 [gr-qc].

†
- B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations]; PRL **119** (2017) no.16, 161101.
- A. Goldstein *et al.*; Astrophys. J. **848** (2017) no.2, L14.

A particular model for **inflation**

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Let's consider the simplest SVT Lagrangian!

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + F + X_1 - V(\phi) + \beta_m M X_2 + \beta_A M^2 X_3\right]$$

- Apart from the 3 modified Friedmann equations, there's a fourth EoM:

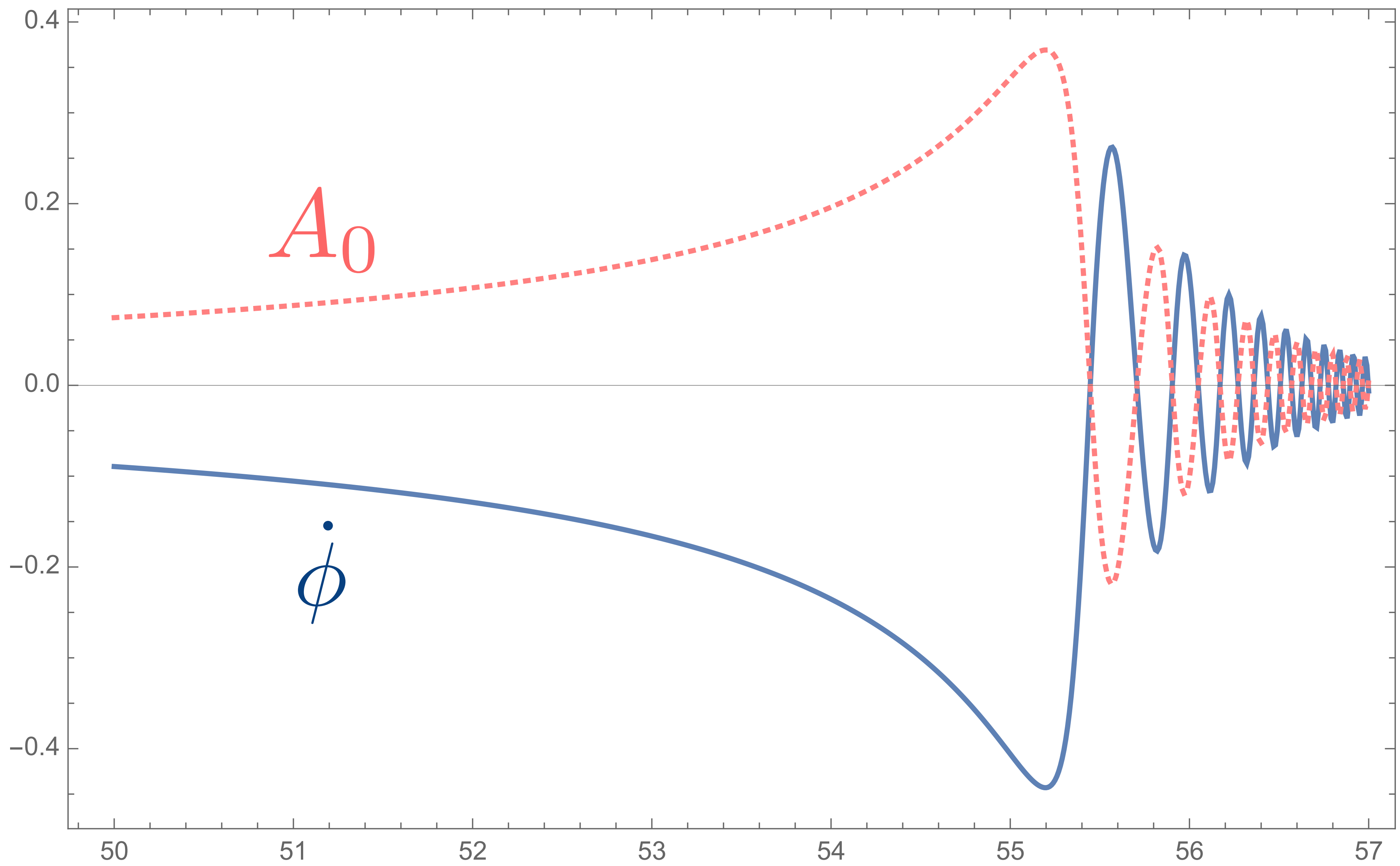
$$A_0 = -\frac{\beta_m}{2\beta_A M}\dot{\phi}$$

Cheat sheet

- $X_1 = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$
- $\boxed{X_2 = -\frac{1}{2}A^\mu\nabla_\mu\phi}$
- $X_3 = -\frac{1}{2}A_\mu A^\mu$
- $F \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
- $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$

A particular model for **inflation**

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*



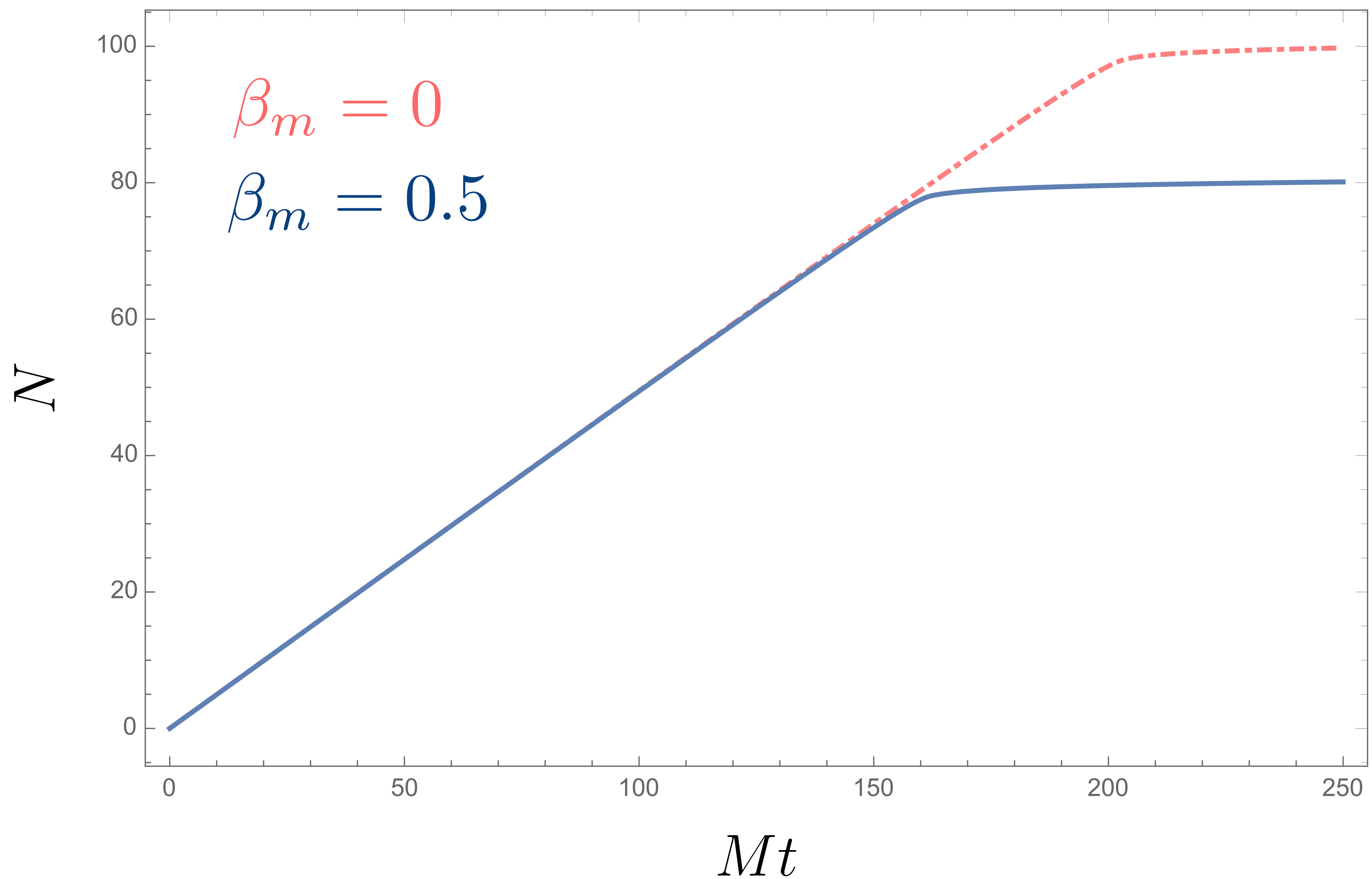
N

*Computed for Starobinsky inflation: $$V(\phi) = \frac{3}{4}\alpha_c M^2 M_{\rm pl}^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3\alpha_c}}\frac{\phi}{M_{\rm pl}}\right)\right]^2 \quad \text{with} \quad \alpha_c = \frac{\sqrt{6}}{3}$$

- A. A. Starobinsky; Phys. Lett. B **91** (1980) 99.
- R. Kallosh, *et al.*; JHEP **1311** (2013) 198.

A particular model for **inflation**

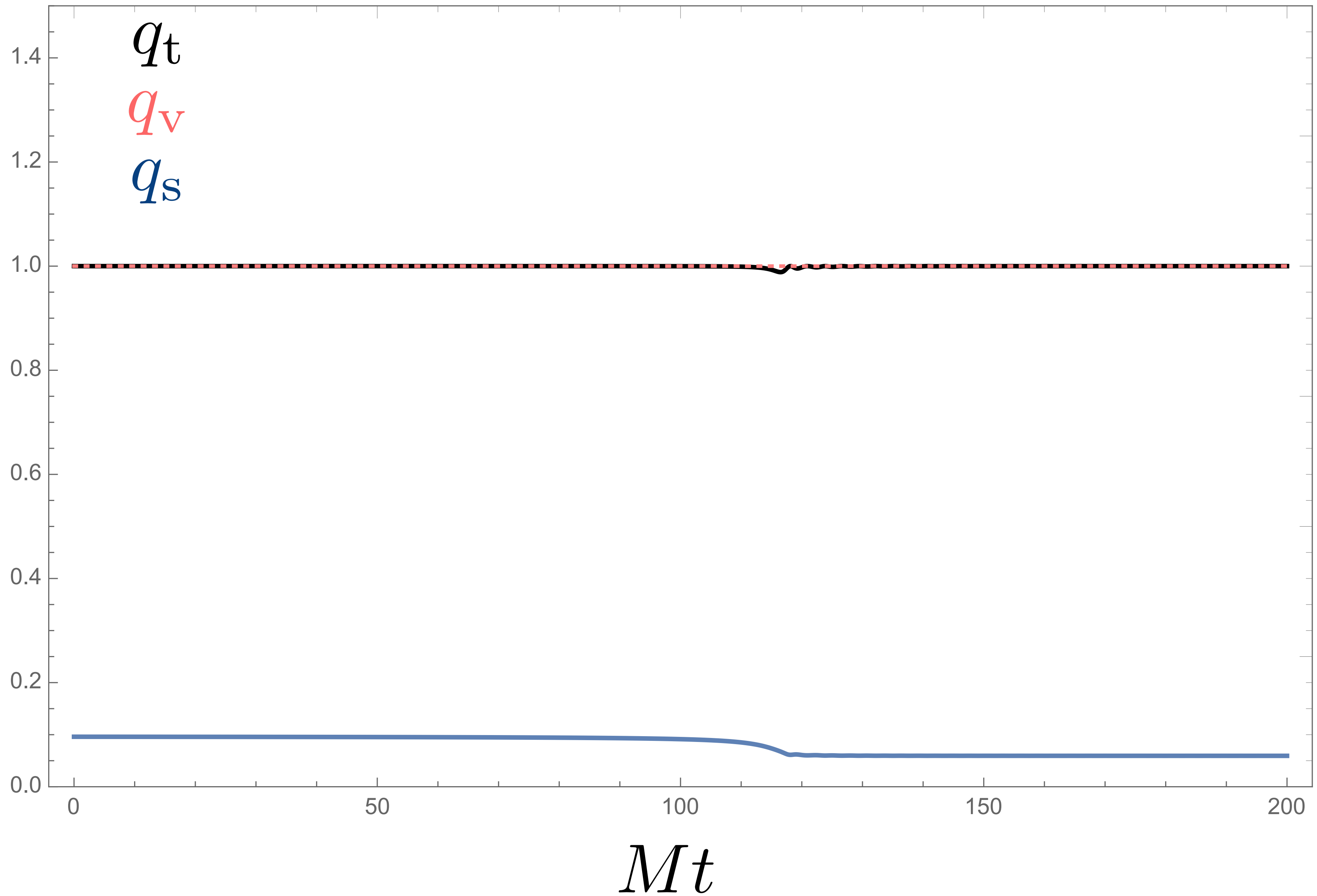
— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*



- The non-vanishing coupling leads to a smaller amount of inflation due to the additional evolution of A_0.

A particular model for **inflation**

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

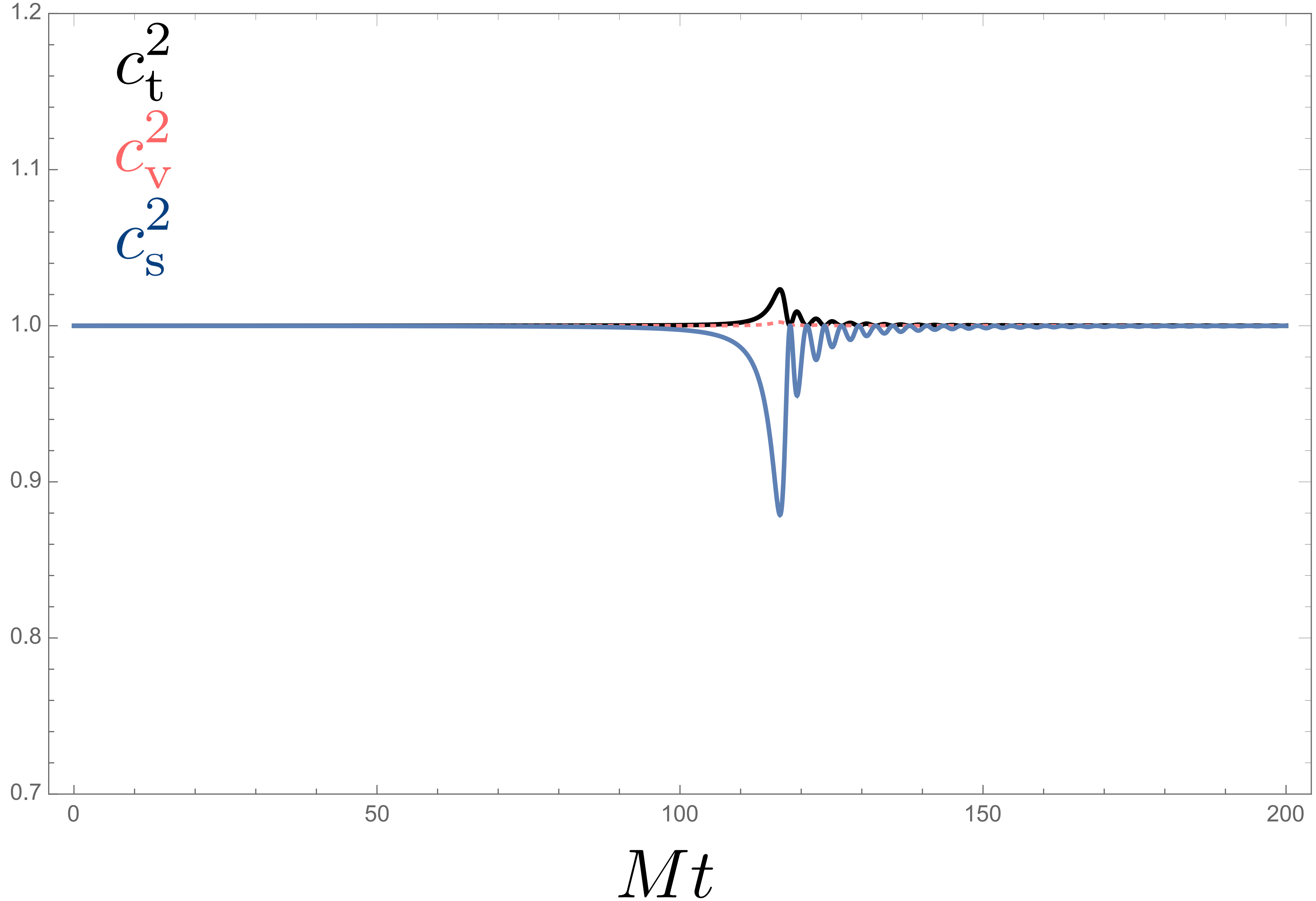


- The condition for the absence of scalar ghosts implies

$$4\beta_A > \beta_m^2 \geq 0$$

A particular model for **inflation**

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*



A particular model for **inflation**

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Given that $A_0 \propto \dot{\phi}$, we can substitute it in the EoM:
  - $$3M_{\rm pl}^2 H^2 = \frac{1}{2}\beta\dot{\phi}^2 + V\,,$$
  - $$-2M_{\rm pl}^2 \dot{H} = \beta\dot{\phi}^2\,,$$
  - $$\ddot{\phi} + 3H\dot{\phi} + \frac{V_{,\phi}}{\beta} = 0$$

where $$\boxed{\beta \equiv 1 - \frac{\beta_m^2}{4\beta_A}}$$

- Then the background dynamics is equivalent to a effective single-field dynamics driven by a rescaled field $d\varphi = \sqrt{\beta}d\phi$.

A particular model for **inflation**

Perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- The fully-general quadratic actions for tensor, vector and scalar perturbations were also computed in:

L. Heisenberg, R. Kase and S. Tsujikawa; doi:10.1103/PhysRevD.98.024038 [arXiv:1805.01066 [gr-qc]].

A particular model for **inflation**

Perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Tensors are not modified (same as in canonical inflation).
- For vectors, $A_i = Z_i + \nabla_i \psi$, the intrinsic vector pert., Z_i , has a solution given by

$$Z_i = A_+ e^{\lambda_+ t} + A_- e^{\lambda_- t}$$

where

$$\lambda_\pm = \frac{H}{2}\left[-1 \pm \sqrt{1 - \frac{4\beta_A M^2}{H^2}}\right]$$

Exponential suppression!

$(\beta_A > 0)$

Then, the amplitud of V perturbations at the end of inflation is negligible.

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- For scalars we have $\delta\phi$ and ψ .
- The 2nd-order action is highly nontrivial even for this simplified case (computed fully generally in 1805.01066).
- We define an isocurvature perturbation as

$$\delta\chi \equiv \psi + \frac{\beta_m}{2\beta_A M}\delta\phi$$

We found that this perturbation experiments the same suppression as vectors, *i.e.*,

$$\psi \simeq -\frac{\beta_m}{2\beta_A M}\delta\phi$$

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- We introduce the gauge-invariant curvature perturbation in flat gauge as

$$\mathcal{R} = \mathcal{R}_{\delta\phi} + \mathcal{R}_{\psi} \xrightarrow{\psi = \delta\chi + 2(\beta-1)\delta\phi} \mathcal{R} = \mathcal{R}_{\delta\phi} + \mathcal{R}_{\delta\chi}$$

Because $\delta\chi$ is suppressed, we only need to compute $\mathcal{R}_{\delta\phi}$.

- In the end, the scalar power spectrum is given by

$$\mathcal{P}_{\mathcal{R}_{\delta\phi}} = \left.\frac{H^4}{4\pi^2\dot{\phi}^2\beta}\right|_{k=aH}$$

As long as $\delta\chi$ is negligibly small compared to $\delta\phi$, the effective single-field description also works for curvature perturbations.

A particular model for **inflation**

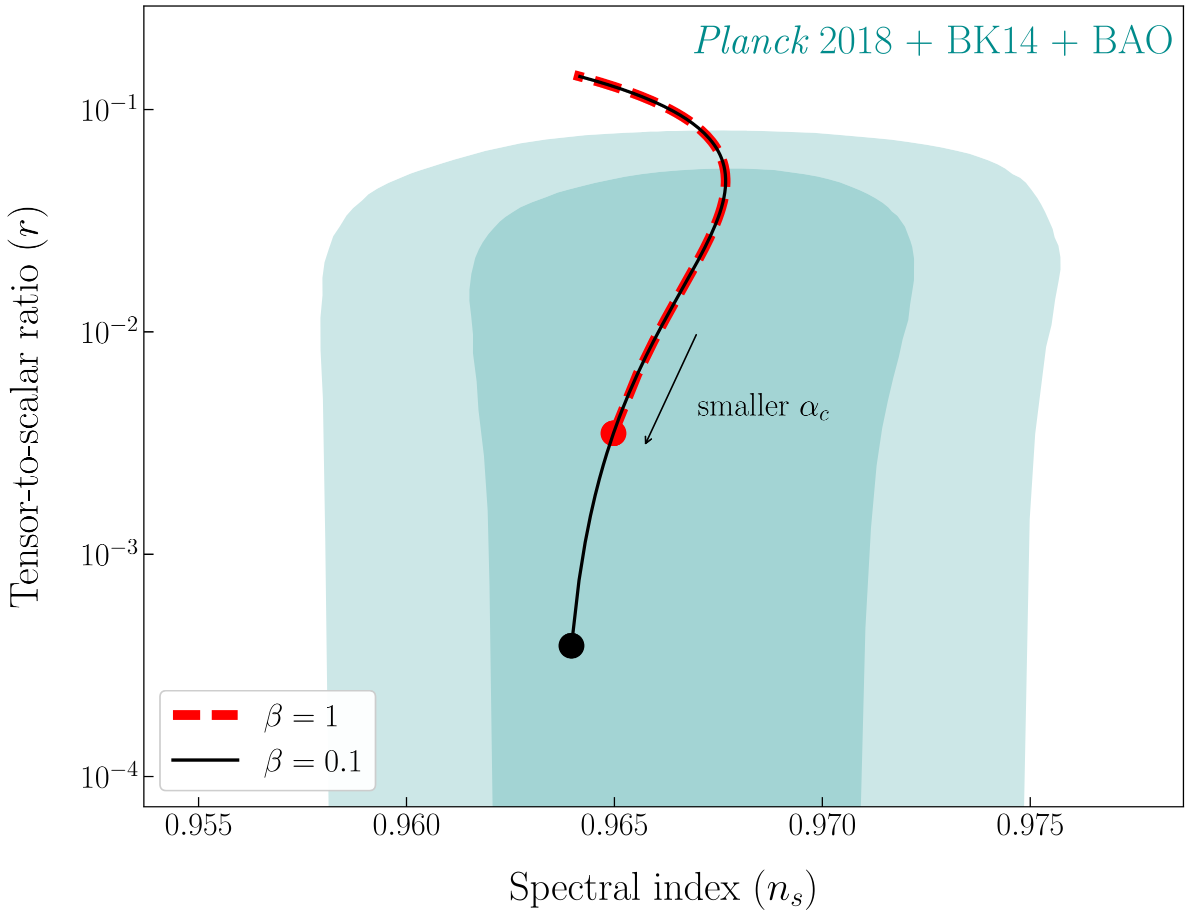
Perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Following the slow-roll approximation,
  - $n_s = 1 - \frac{1}{\beta}(6\epsilon_V - 2\eta_V)$
  - $r = \frac{16\epsilon_V}{\beta}$
- We confronted different inflaton potentials with CMB data:
  - $V(\phi) = \frac{3}{4}\alpha_c M^2 M_{\rm pl}^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3\alpha_c}}\frac{\phi}{M_{\rm pl}}\right)\right]^2$ — α-attractors

    — R. Kallosh, *et al.*; JHEP **1311** (2013) 198.
  - $V(\phi) = M^2 M_{\rm pl}^2 \left[1 - \left(\frac{\mu}{\phi}\right)^p + \cdots\right]$ — Brane inflation

    - J. Garcia-Bellido, *et al.*; JHEP **0201** (2002) 036.
    - G. R. Dvali, *et al.*; hep-th/0105203.
    - S. Kachru, *et al.*; JCAP **0310** (2003) 013.

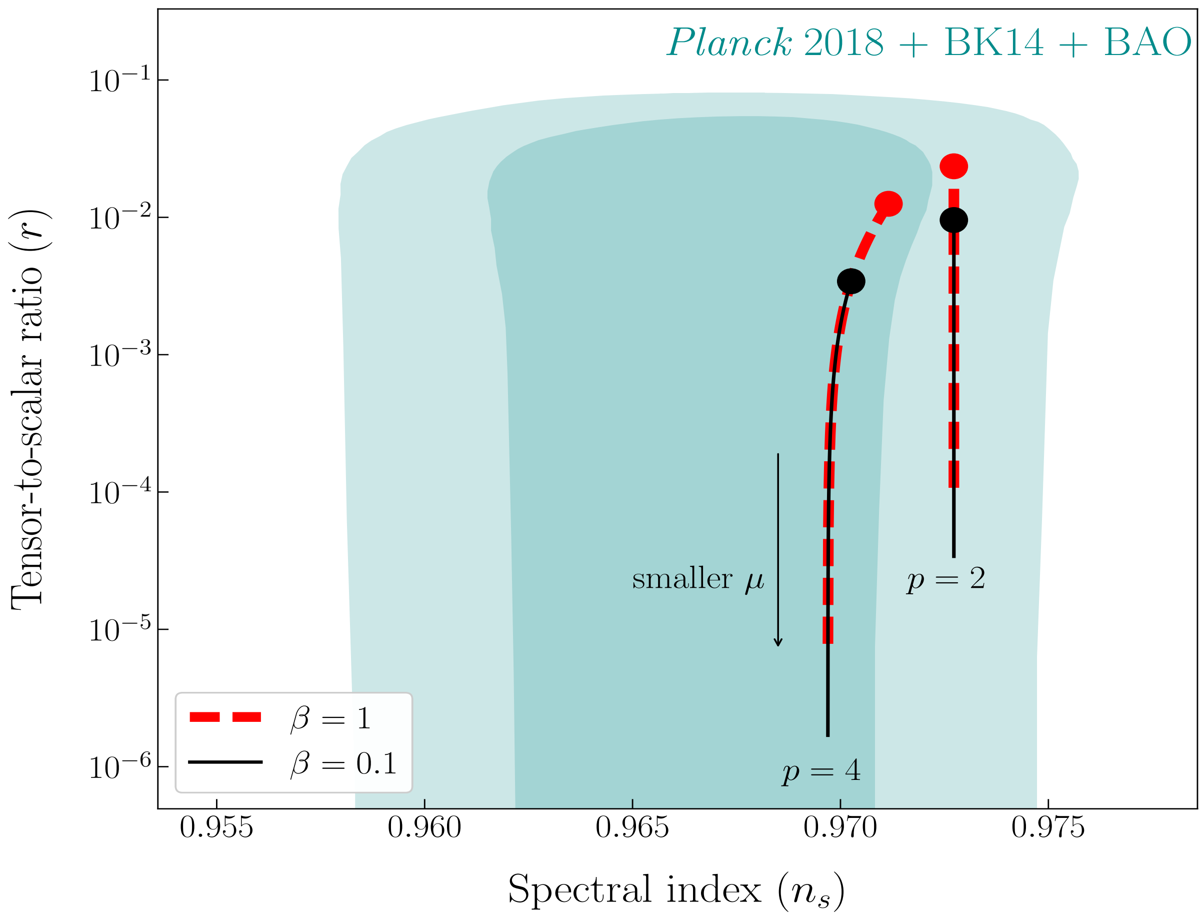


α-attractors

The coupling β can lead to the suppression of

$$r = 16\epsilon$$

compared to the canonical case.



Brane inflation

For smaller β , the total field velocity gets larger and hence the inflaton needs to start from a flatter region to get enough ΔN.

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

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- G. W. Horndeski; Int. J. Theor. Phys. **10** (1974) 363.
- A. Nicolis *et al.*; PRD **79** (2009) 064036.
- C. Deffayet *et al.*; PRD **84** (2011) 064039.

Horndeski theory:

$$S = \sum_{i=2}^{5} \int \mathrm{d}^4x \sqrt{-g}\mathcal{L}_i$$

where

$$\mathcal{L}_2 = G_2 \ ,$$

$$\mathcal{L}_3 = G_3 \Box\phi \ ,$$

$$\mathcal{L}_4 = G_4 R - 2G_{4,X}[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \ ,$$

$$\mathcal{L}_5 = G_5 G^{\mu\nu}\phi_{;\mu\nu} + \frac{G_{5,X}}{3}[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}] \ .$$

Canonical inflation:

$$G_2 = -\frac{X}{2} - V(\phi) \ , \qquad G_4 = \frac{M_{\rm pl}^2}{2} \ ,$$

$$G_3 = 0 \ , \qquad G_5 = 0 \ .$$

$\boxed{G_3 + \text{chaotic inflation} = \text{G-inflation}}$

— J. Ohashi and S. Tsujikawa; JCAP **1210** (2012) 035.

$$\mathcal{L}_2 = X - V(\phi) = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_3 = M^{-3} X \Box \phi$$

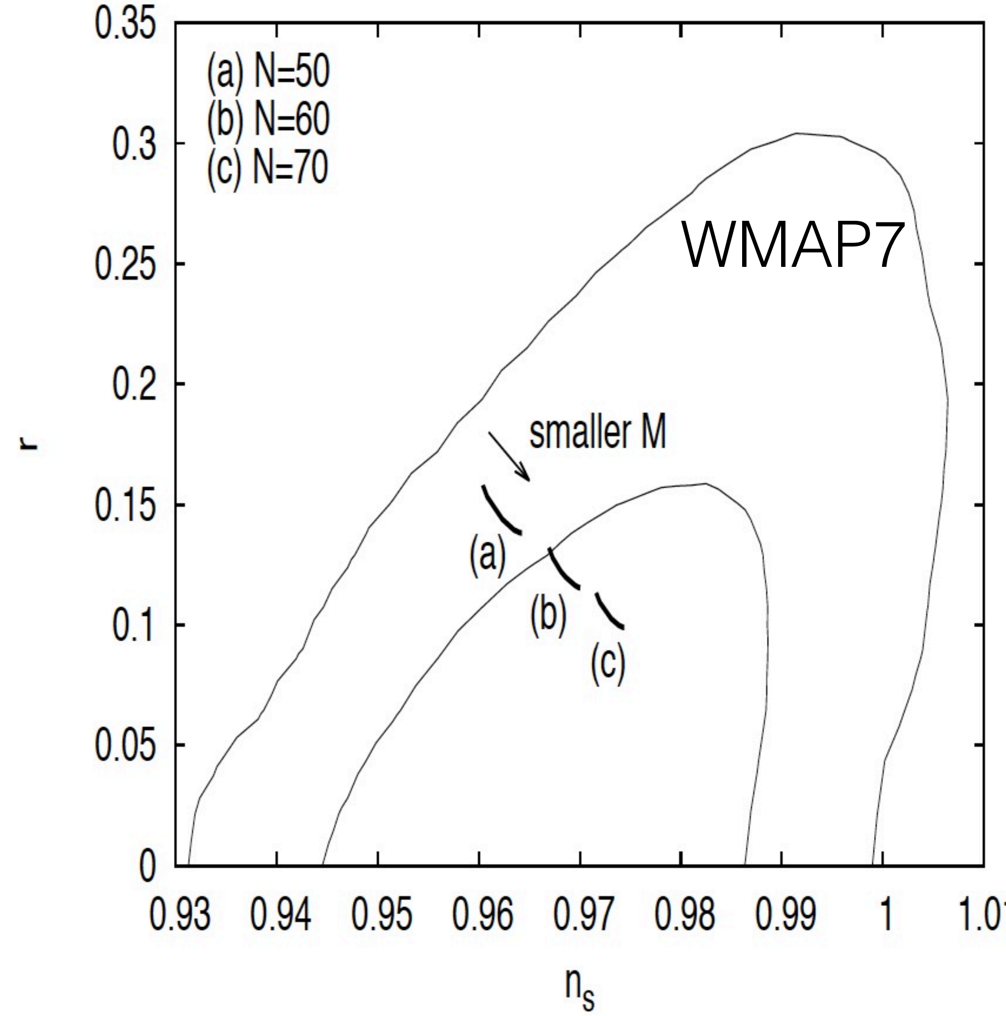
$$\mathcal{L}_4 = \frac{1}{2} M_{\rm pl}^2 \mathcal{R}$$

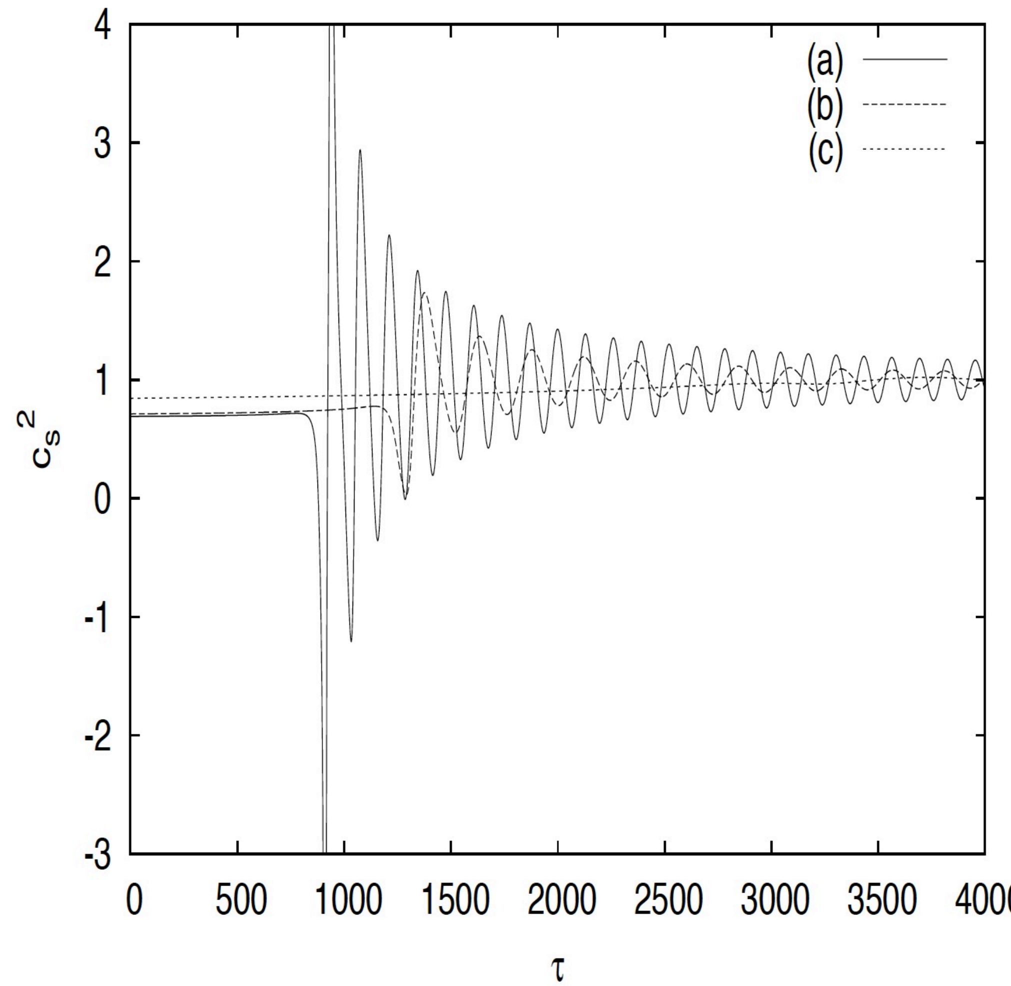
a) $M = 3 \times 10^{-4} M_{\rm pl}$

b) $M = 4.2 \times 10^{-4} M_{\rm pl}$

c) $M = 1 \times 10^{-3} M_{\rm pl}$

For $c_s^2 > 0$, $M > 4.2 \times 10^{-4} M_{\rm pl}$





$\boxed{G_3 + \text{chaotic inflation} = \text{G-inflation}}$

— J. Ohashi and S. Tsujikawa; JCAP **1210** (2012) 035.

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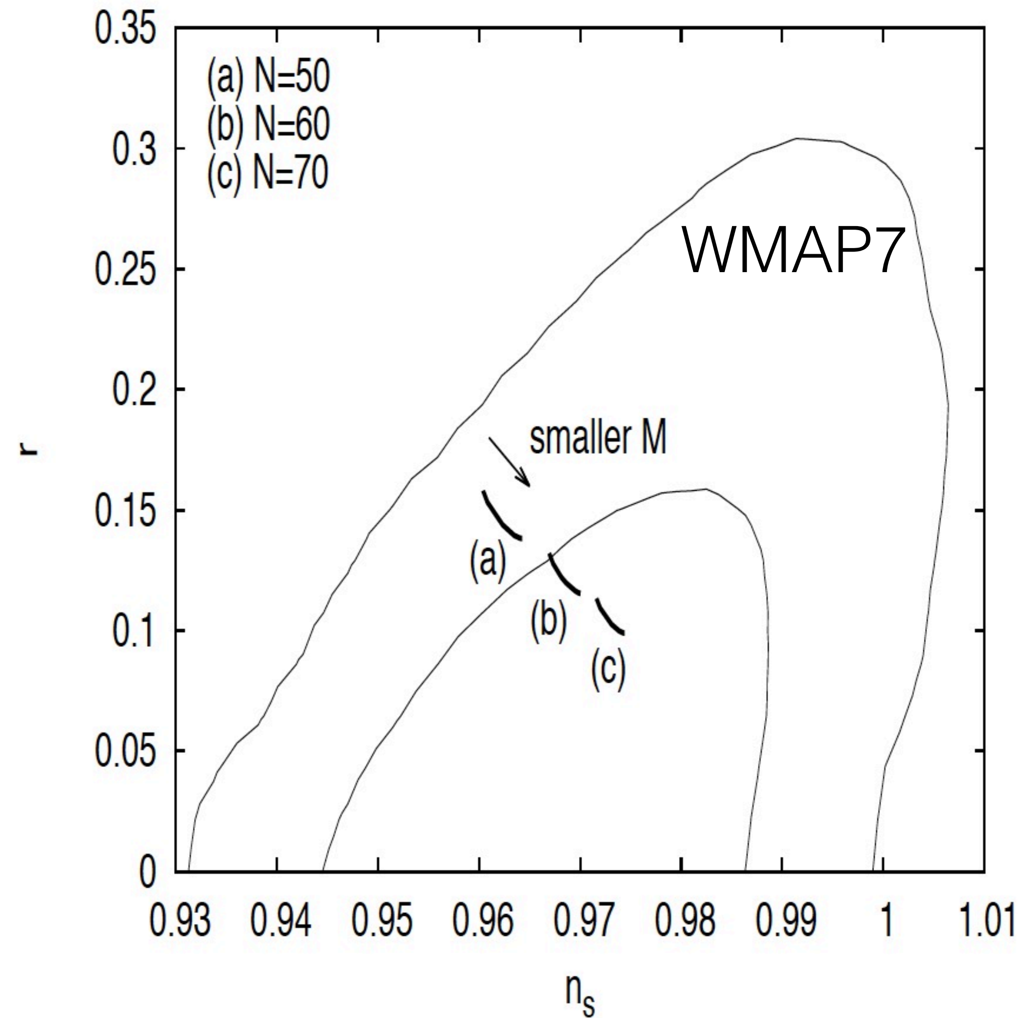
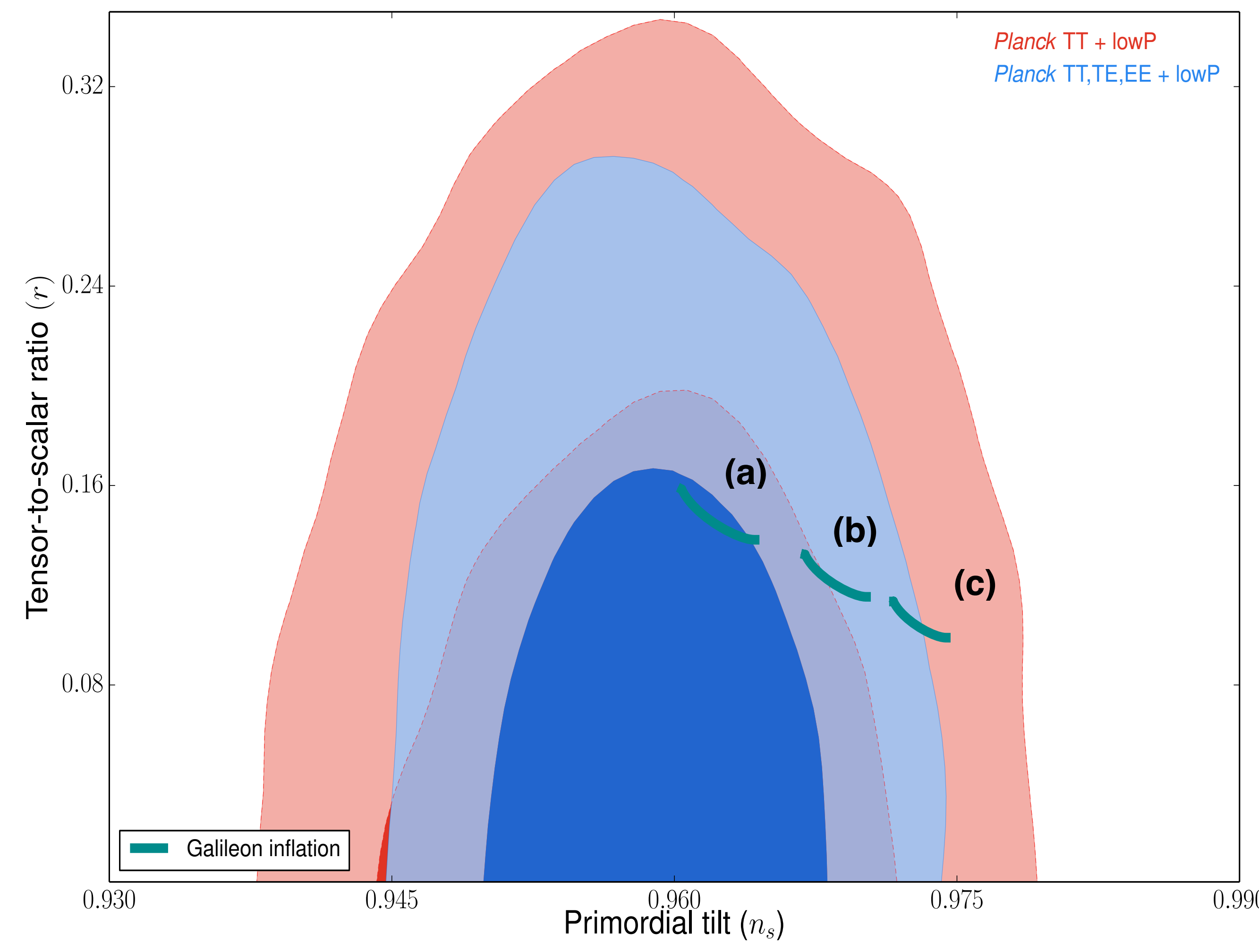
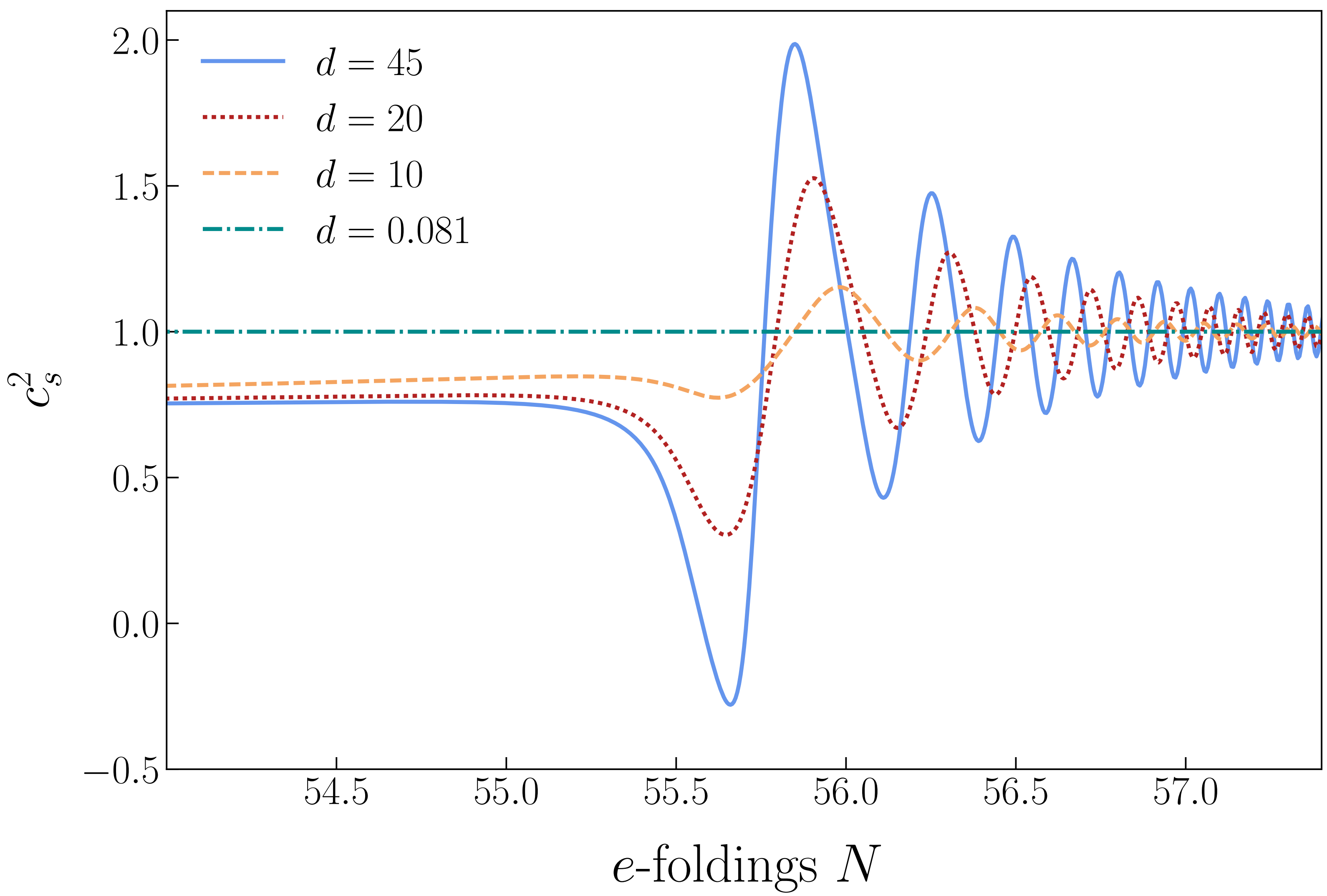




— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena; JCAP **1804** (2018) no.04, 039.

G_3 + *tanh* + chaotic inflation = transient G-inflation

$$\mathcal{L}_3 = M^{-3}\left[1 + \tanh\left(\frac{\phi - \phi_r}{d}\right)\right] X\Box\phi$$



$\phi_r = 13$

$M = 1.3 \times 10^{-4}$

No gradient instabilities for any mass scale M.

— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena;
JCAP **1804** (2018) no.04, 039.

G_3 + *tanh* + chaotic inflation
= transient G-inflation

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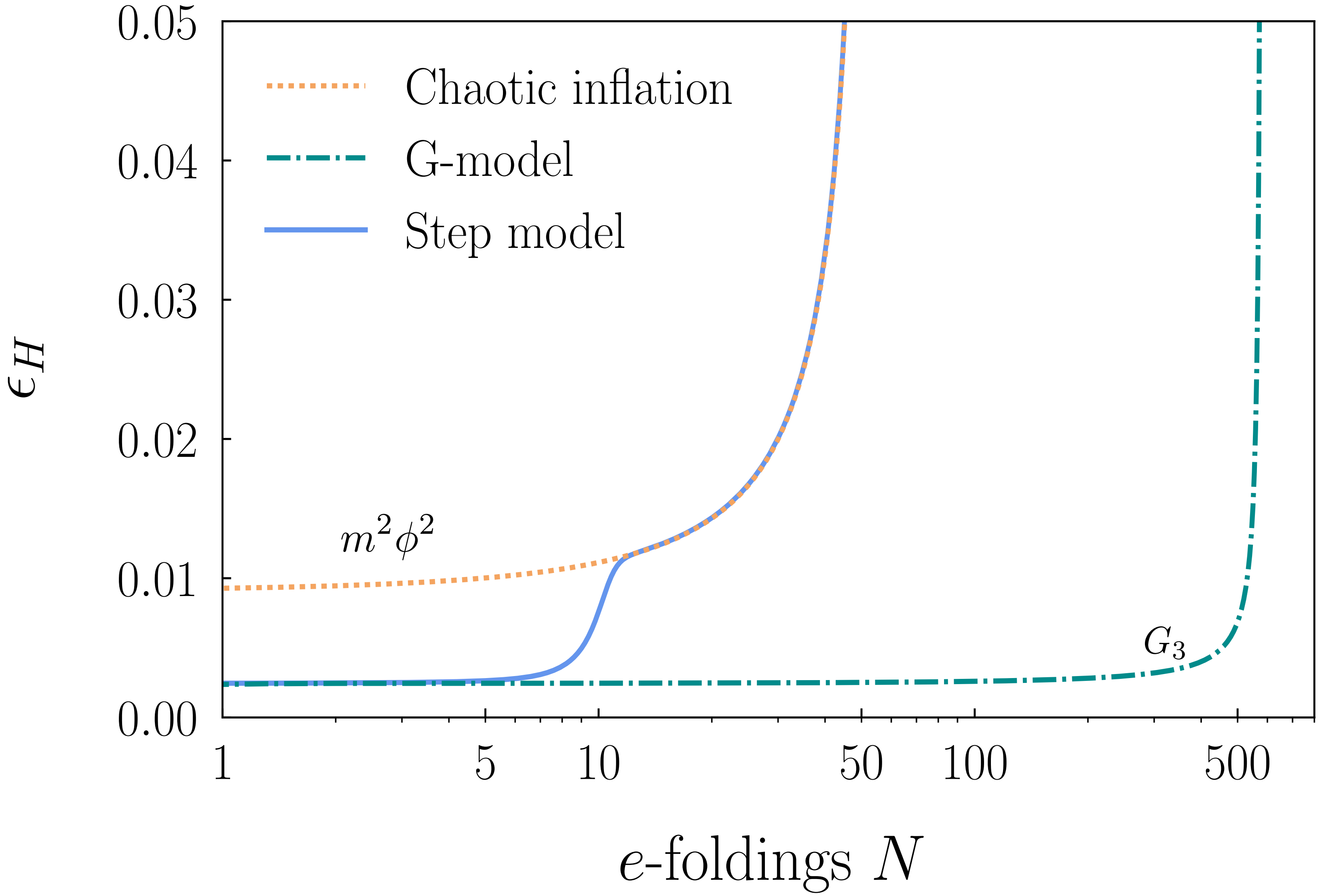
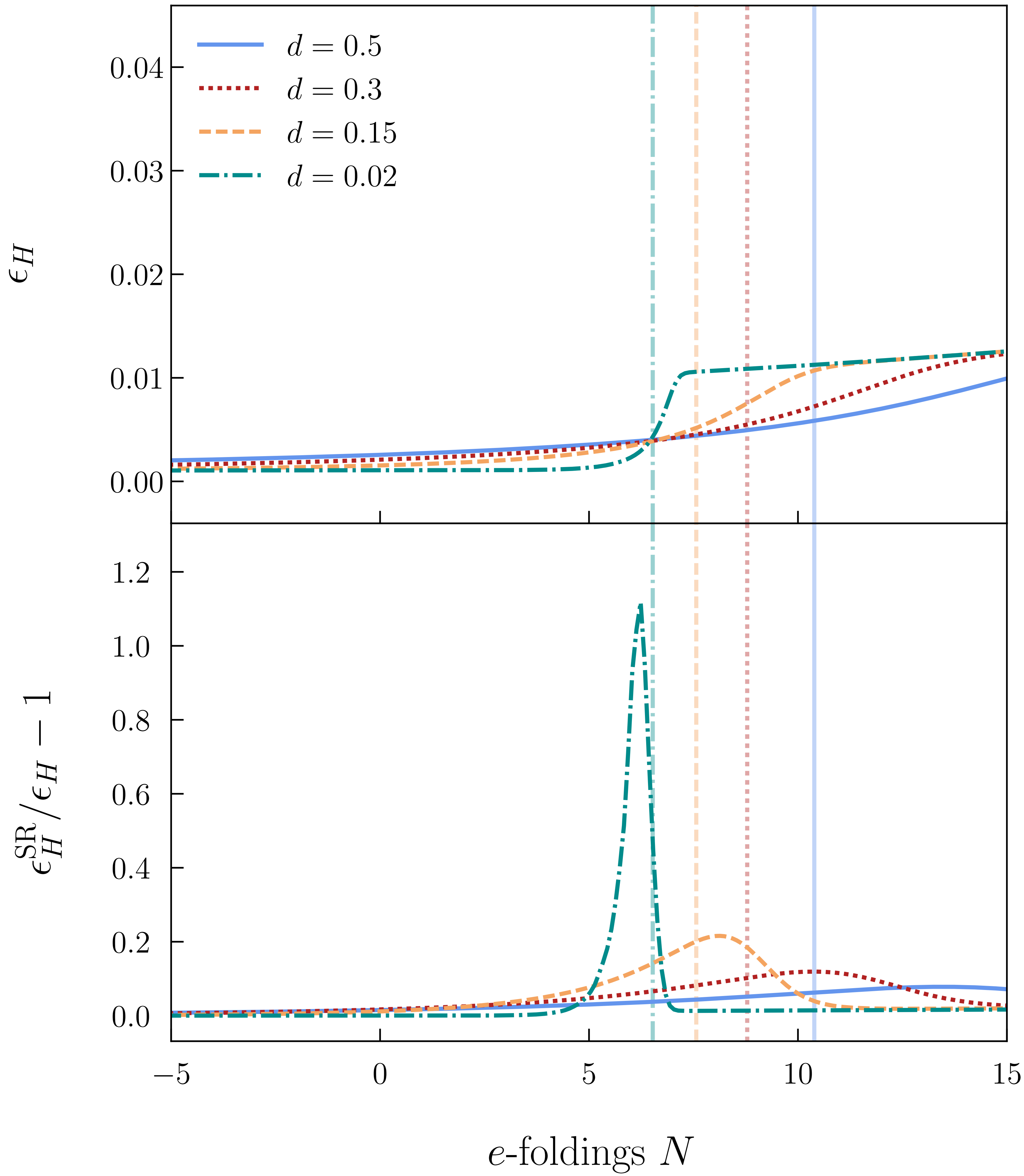


A transition between the two regimes before inflation ends.

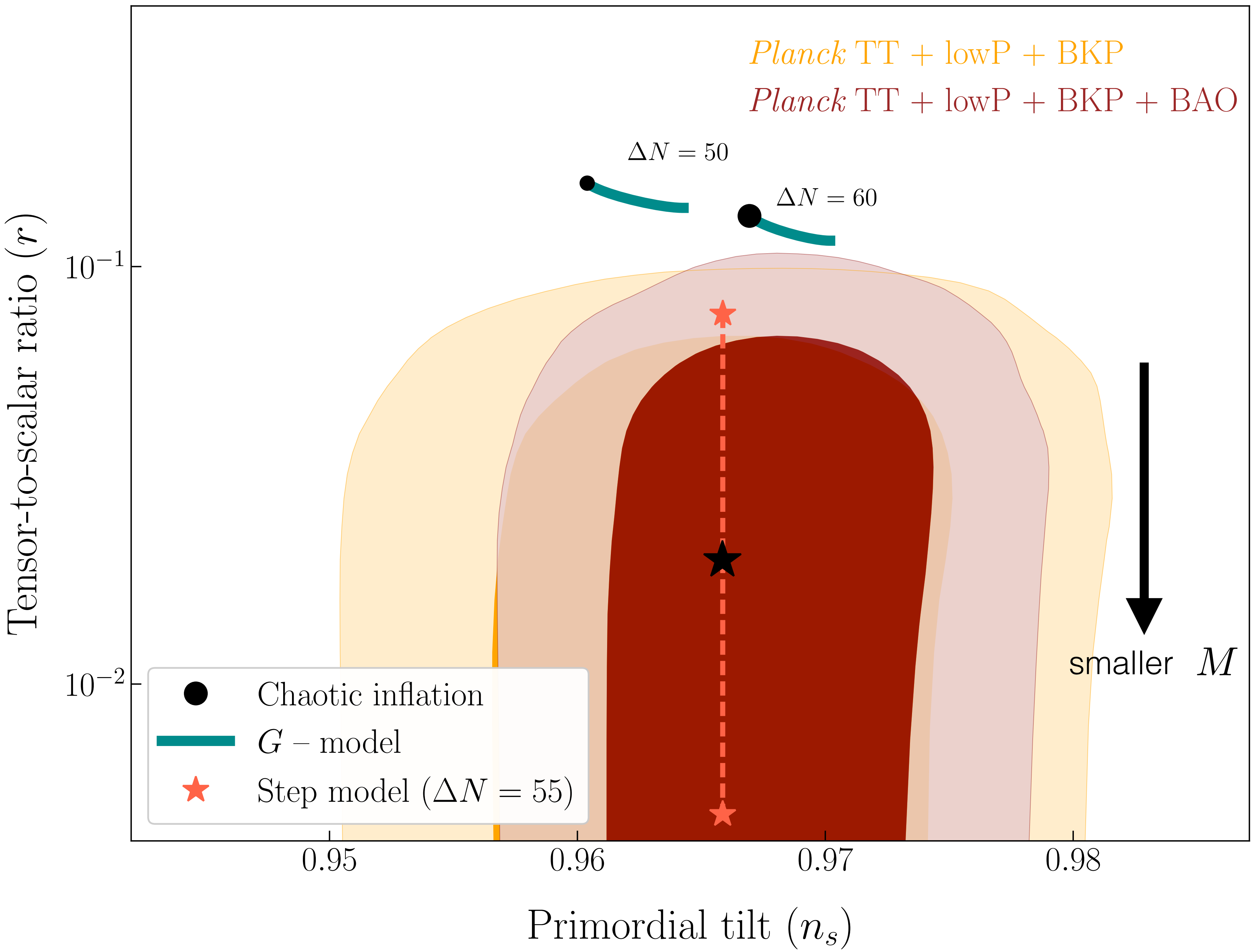


Slow - roll violation:

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

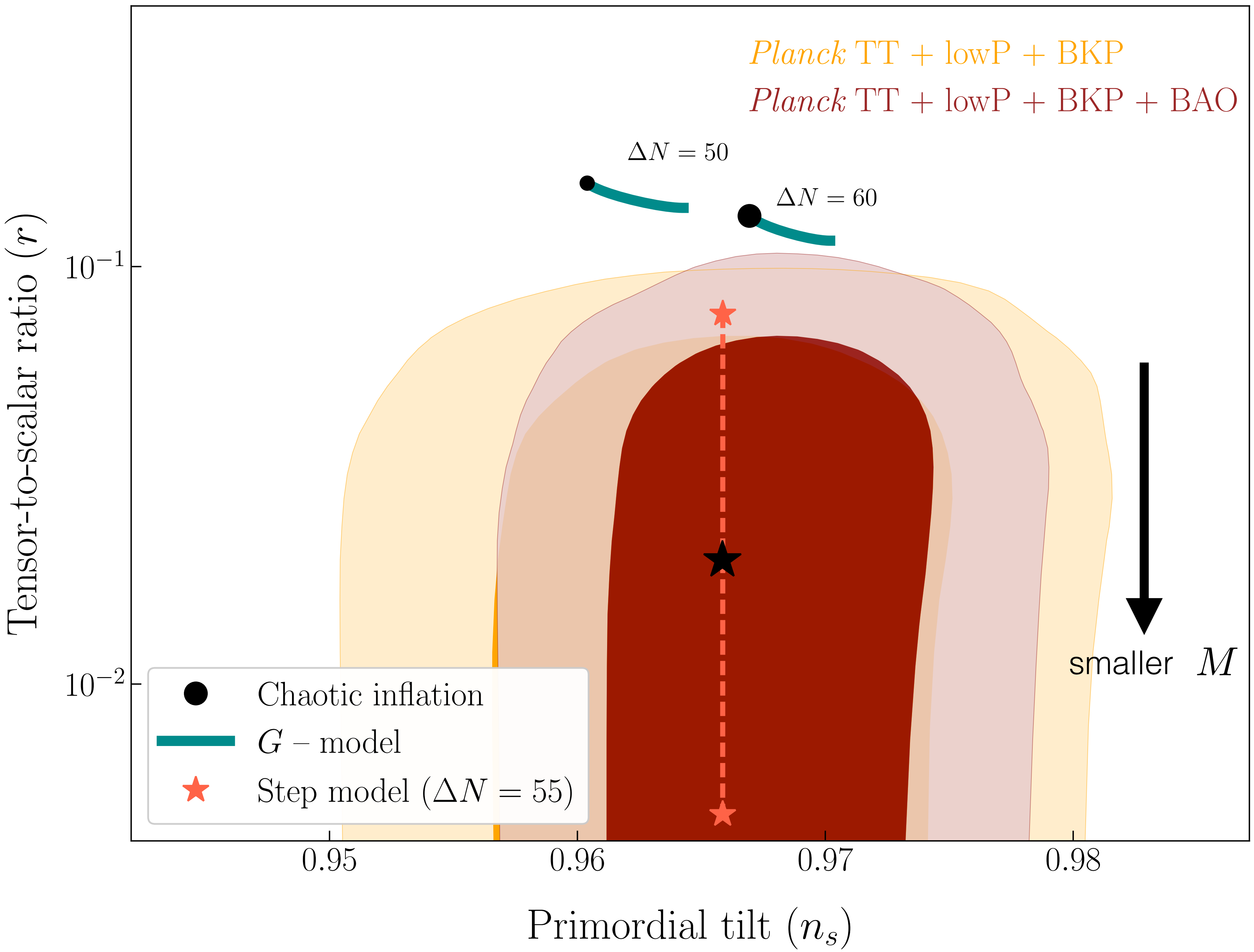
- $N = 0$: CMB scales.
- Vertical lines: where the transition occurs.
- SR violation is maximal around the transition.

Observables



$$n_s = 9.66$$
$$\alpha_s = -0.011$$

- n_s and α_s fixed.
- Find a set of values for d and ϕ_r.
- This places lower and upper bounds on r.



Observables

$$n_s = 9.66$$
$$\alpha_s = -0.011$$

$$\boxed{r \gtrsim 0.005}$$

- A smaller α_s would shift the line upwards because the step gets wider.
- A larger α_s would be in tension with measurements.

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Inflation

- T. Kobayashi, *et al.*; Prog. Theor. Phys. **126** (2011) 511.
- H. Motohashi and W. Hu; PRD **96** (2017) no.2, 023502.

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$S_\gamma^{(2)} = \sum_{\lambda=+,\times} \int \mathrm{d}^4x \frac{a^3 b_t}{4c_t^2} \left(\dot{\gamma}_\lambda^2 - \frac{c_t^2 k^2}{a^2} \gamma_\lambda^2 \right)$$

In canonical inflation:

$$b_s = b_t = c_s = c_t = 1$$

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

Scalar parameters:

- $$b_s = \frac{c_s^2}{\epsilon_H}\left[\frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2}\right]$$

- $$c_s^2 = \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1\dot{w}_1w_2 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)}$$

Tensor parameters:

- $$b_t = w_1c_t^2$$

- $$c_t^2 = \frac{w_4}{w_1}$$

$$w_1 = M_{\rm pl}^2 - 2\left(3G_4 + 2HG_5\dot{\phi}\right) + 2G_{5,\phi}X\,,$$

$$w_2 = 2M_{\rm pl}^2H - 2G_3\dot{\phi} - 2\left(30HG_4 - 5G_{4,\phi}\dot{\phi} + 14H^2G_5\dot{\phi}\right) + 28HG_{5,\phi}X\,,$$

$$w_3 = -9M_{\rm pl}^2H^2 + 3\left(X + 12HG_3\dot{\phi}\right) + 6\left(135H^2G_4 - 2G_{3,\phi}X - 45HG_{4,\phi}\dot{\phi} + 56H^3G_5\dot{\phi}\right) - 504H^2G_{5,\phi}X\,,$$

$$w_4 = M_{\rm pl}^2 + 2\left(G_4 - 2G_5\ddot{\phi}\right) - 2G_{5,\phi}X\,.$$

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \, \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$\frac{\mathrm{d}^2 v}{\mathrm{d}\tau^2} + \left(c_s^2 k^2 - \frac{1}{z} \frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2} \right) v = 0$$

* Mukhanov - Sasaki equation

- $v = z\zeta$
- $z = a\sqrt{\dfrac{2 b_s \epsilon_H}{c_s^2}}$

- Assume slow-roll approximation.
- Solve numerically.
- Use GSR techniques.

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

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Generalized Slow-Roll

- E. D. Stewart; PRD **65** (2002) 103508.
- C. Dvorkin and W. Hu; PRD **81** (2010) 023518.
- W. Hu; PRD **84** (2011) 027303.
- W. Hu; PRD **89** (2014) no.12, 123503.
- H. Motohashi and W. Hu; PRD **92** (2015) no.4, 043501.
- H. Motohashi and W. Hu; PRD **96** (2017) no.2, 023502.

… and others.

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

de Sitter background

deviations from dS

- $y \equiv \sqrt{2 c_s k} v$
- $x \equiv k s_s$
- $s_s \equiv \int c_s \mathrm{d}\tau$

- $f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2} \frac{aHs_s}{c_s}}$
- $\Delta_\zeta^2(k) = \lim_{x\to 0} \left| \frac{xy}{f} \right|^2$

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

3. Apply Green function techniques (GSR).

$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

- $$G = -2\ln f + \frac{2}{3}\left(\ln f\right)'$$ Source function
- $$W(u) = \frac{3\sin(2u)}{2u^3} - \frac{3\cos(2u)}{u^2} - \frac{3\sin(2u)}{2u}$$

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

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$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

4. Taylor expand GSR formula and write down analytic equations (OSR).

Optimized Slow-Roll

If $\Delta N > 1$ we can Taylor expand the GSR formula around some epoch x_f:

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f)\, G^{(p)}(\ln x_f)$$

- $q_1(\ln x_f) = \ln x_1 - \ln x_f$

$$\boxed{\ln x_1 \equiv \frac{7}{3} - \ln 2 - \gamma_{\mathrm{E}}}$$

- $q_p(\ln x_f) = -\dfrac{1}{p!}\displaystyle\int_{x_m}^{\infty} W'(x)\left(\ln \frac{x}{x_f}\right)^p$

Optimized SR for Horndeski (leading order):

— H. Motohashi and W. Hu; PRD **96** (2017) no.2, 023502.

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p (\ln x_f)\, G^{(p)}(\ln x_f)$$

$$G = -2\ln f + \frac{2}{3}(\ln f)'$$

$$f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2}} \frac{a H s_s}{c_s}$$

Scalars

$$\ln \Delta_\zeta^2 \approx \ln\left(\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 - \frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1}\Big|_{x=x_1}$$

$$n_s - 1 \approx -4\epsilon_H - 2\delta_1 - \sigma_{s1} - \xi_{s1} - \frac{2}{3}\delta_2 - \frac{7}{3}\sigma_{s2} - \frac{1}{3}\xi_{s2}\Big|_{x=x_1}$$

$$\alpha_s \approx -2\delta_2 - \sigma_{s2} - \xi_{s2} - \frac{2}{3}\delta_3 - \frac{7}{3}\sigma_{s3} - \frac{1}{3}\xi_{s3} - 8\epsilon_H^2 - 10\epsilon_H\delta_1 + 2\delta_1^2\Big|_{x=x_1}$$

Tensors

$$\ln \Delta_\gamma^2 \approx \ln\left(\frac{H^2}{2\pi^2 b_t c_t}\right) - \frac{8}{3}\epsilon_H - \frac{7}{3}\sigma_{t1} - \frac{1}{3}\xi_{t1}\Big|_{x=x_1}$$

$$n_t \approx -2\epsilon_H - \sigma_{t1} - \xi_{t1} - \frac{7}{3}\sigma_{t2} - \frac{1}{3}\xi_{t2}\Big|_{x=x_1}$$

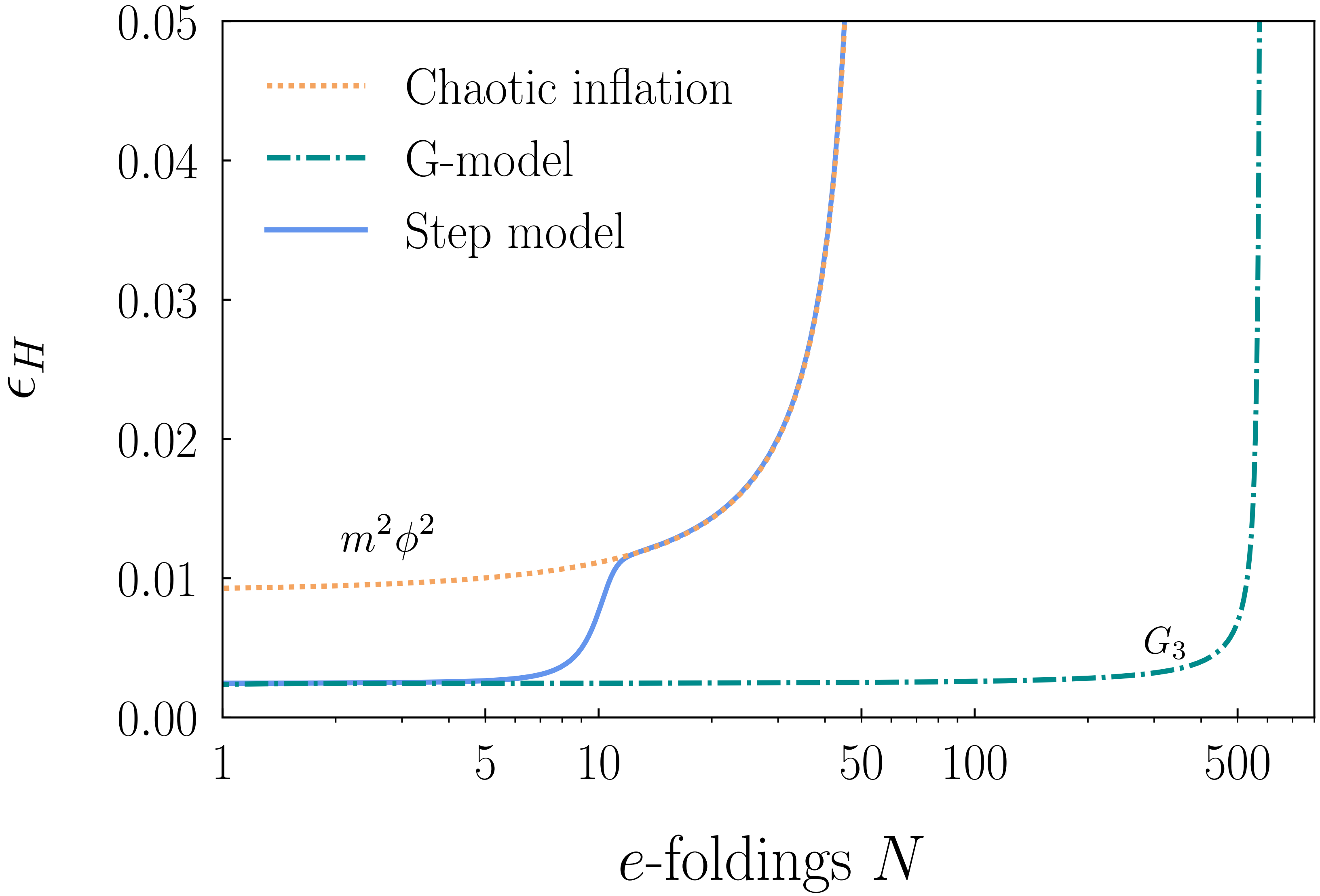
$$\alpha_t \approx -\sigma_{t2} - \xi_{t2} - \frac{7}{3}\sigma_{t3} - \frac{1}{3}\xi_{t3} - 4\epsilon_H^2 - 4\epsilon_H\delta_1\Big|_{x=x_1}$$

$$\ln x_1 \equiv \frac{7}{3} - \ln 2 - \gamma_{\mathrm{E}}$$

— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena; JCAP **1804** (2018) no.04, 039.

G_3 + *tanh* + chaotic inflation = transient G-inflation

$$\mathcal{L}_3 = M^{-3}\left[1 + \tanh\left(\frac{\phi - \phi_r}{d}\right)\right] X\Box\phi$$



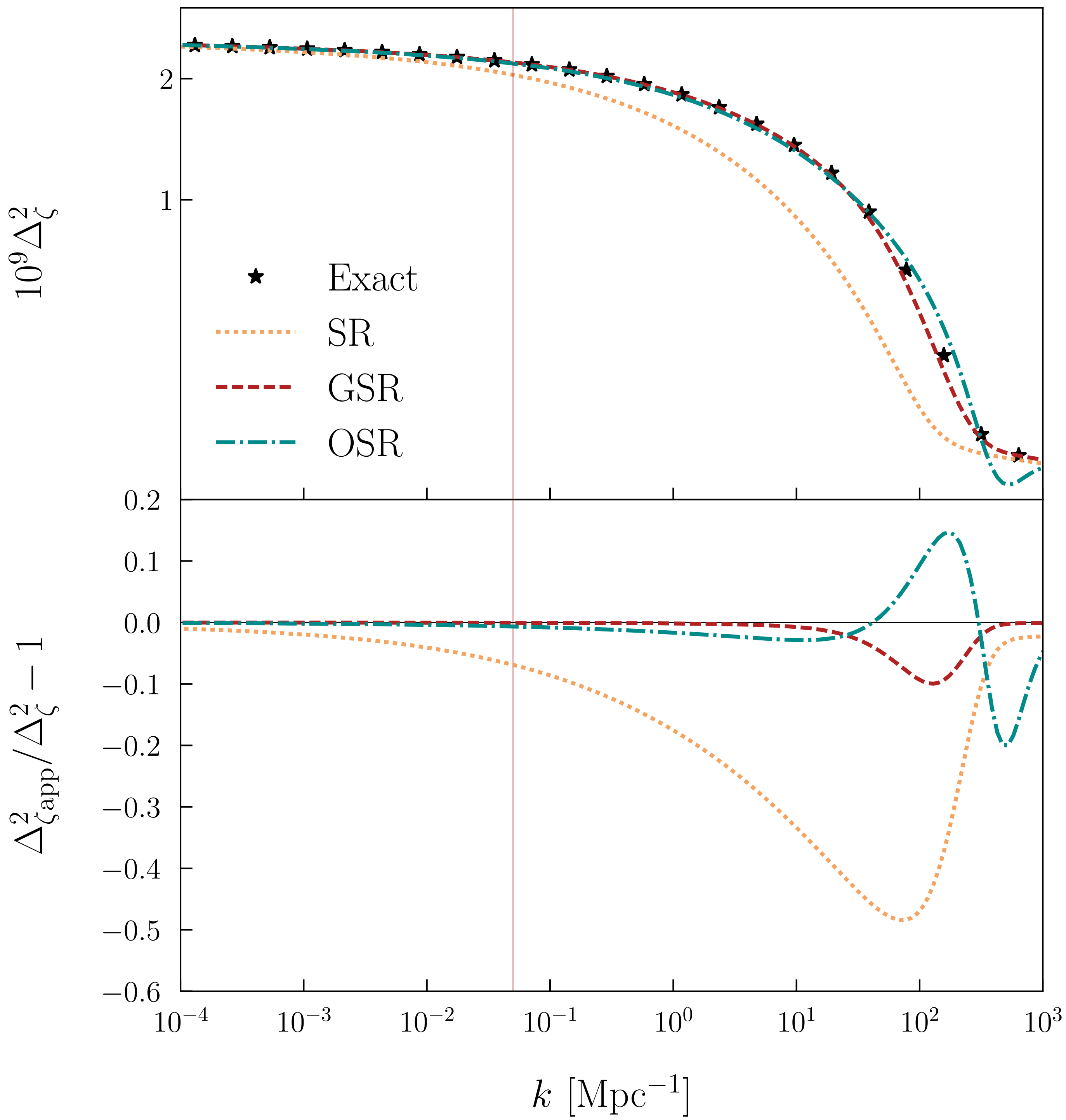
Step size ~ 4 *e*-folds

Ideal for OSR

— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena; JCAP **1804** (2018) no.04, 039.

G_3 + *tanh* + chaotic inflation = transient G-inflation

Different approximations



$$d = 0.086$$

$$M = 1.3 \times 10^{-4}$$

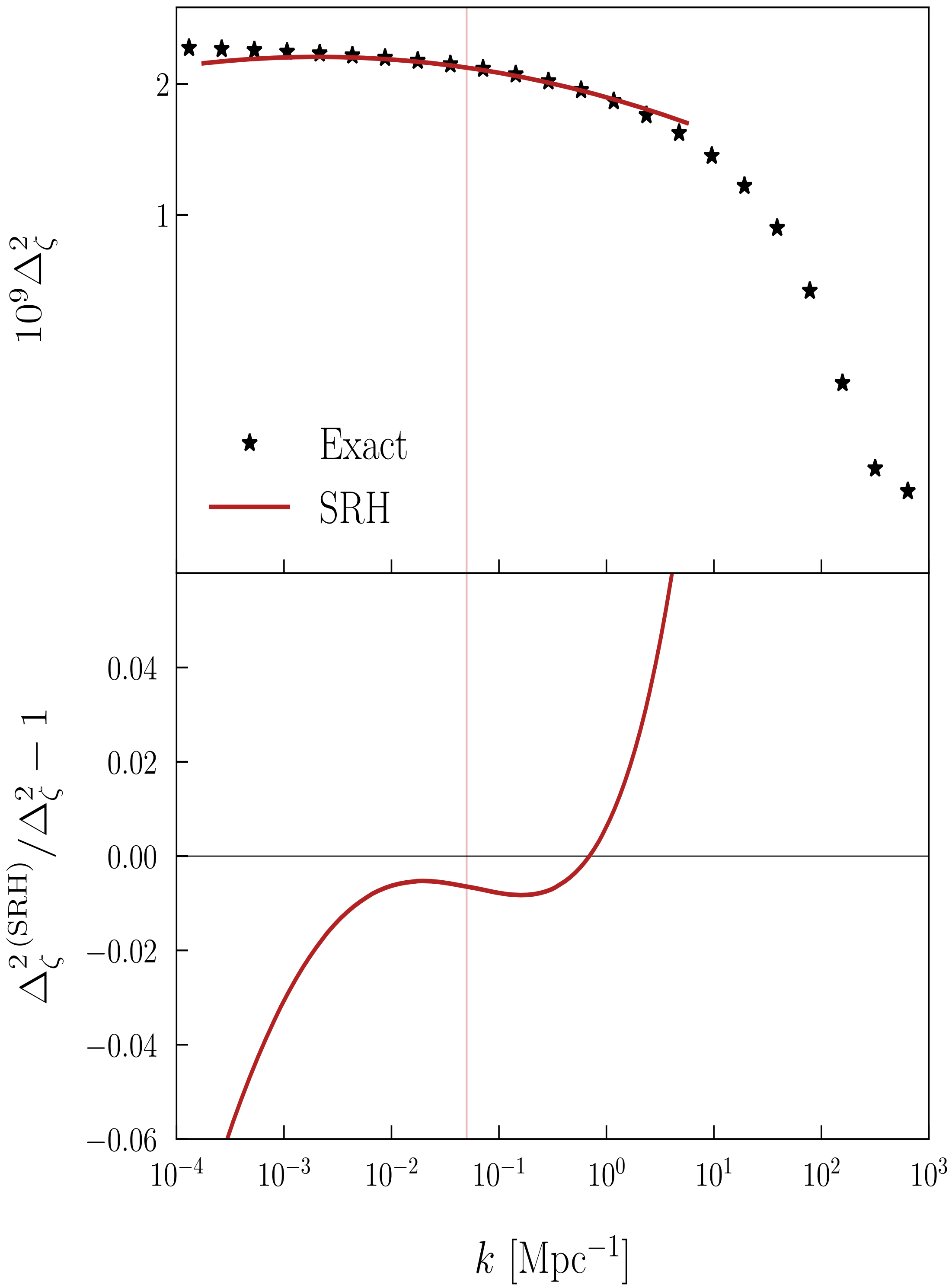
SR ✗

GSR ✓

OSR ✓

— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena; JCAP **1804** (2018) no.04, 039.

G_3 + *tanh* + chaotic inflation = transient G-inflation



Slow - roll hierarchy formula:

$$\Delta_\zeta^{2(\mathrm{SRH})}(k) = A_s\left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k_*)}$$

- Using OSR parameters

Deviations of less than 1%

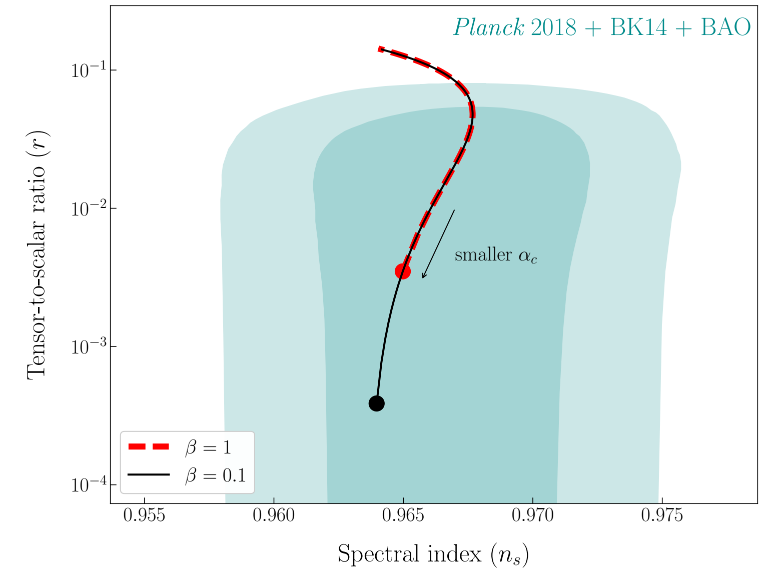
Summary

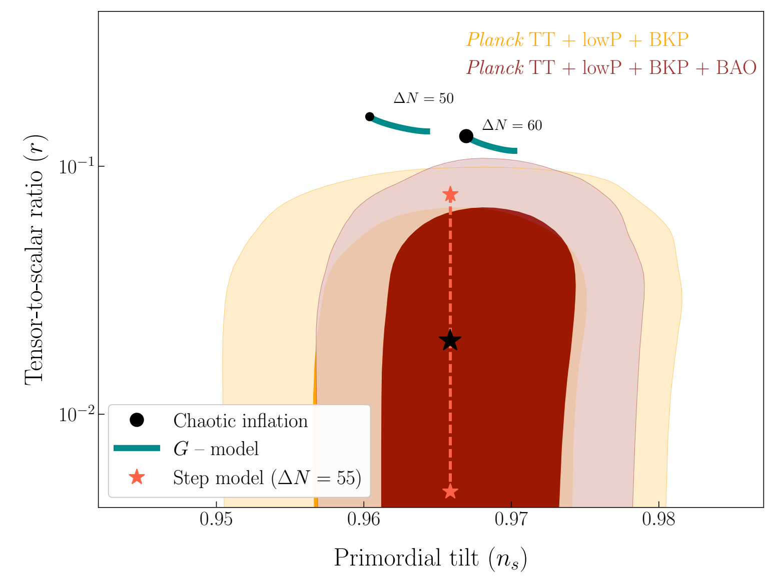
- Scalar-Vector-Tensor theories represent a new and very rich framework to study several cosmological and astrophysical phenomena.

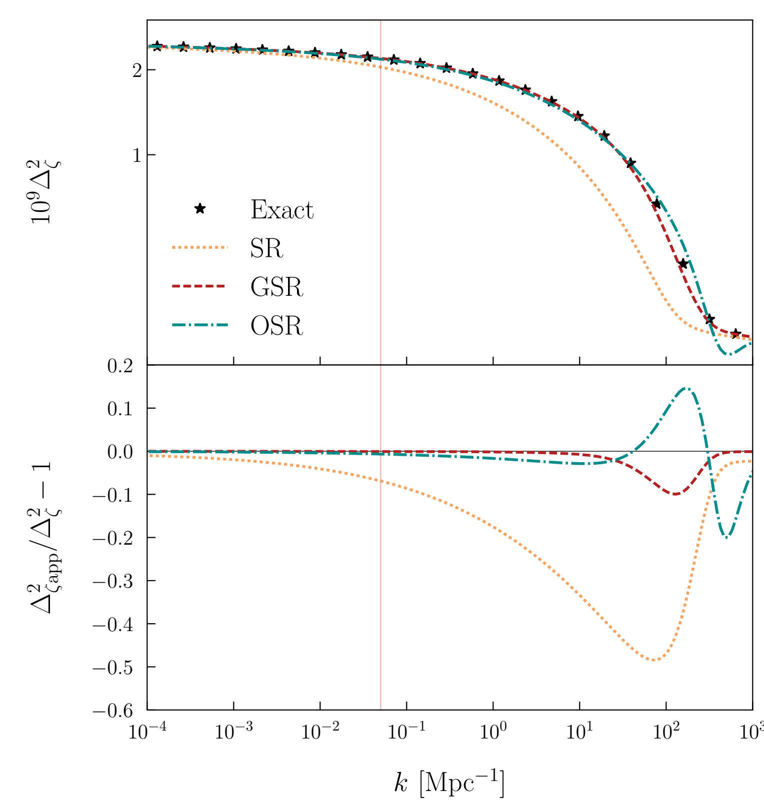
    - The simplest addition of a vector field modifies the amount of inflation and reduces the tensor-to-scalar ratio for small-field models.

- G-inflation, the simplest scalar-tensor model in the framework of Horndeski gravity, is back to life with its full phenomenology.

- Generalized slow-roll and Optimized slow-roll techniques are efficient tools for this type of models to compute the power spectra, and also the bispectrum!







Back-up slides

A particular model for **inflation**

Vector perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Vector quadratic action:

$$\mathcal{S}_{\rm V}^{(2)} = \int d^4x \sum_{i=1}^{2} \frac{a}{2}\left[\dot{Z}_i^2 - \frac{1}{a^2}\left(\partial Z_i\right)^2 - \beta_A M^2 Z_i^2\right]$$

$$\boxed{Z_i'' + \left(k^2 + a^2\beta_A M^2\right) Z_i = 0}$$

- For $k^2 \gg a^2\beta_A M^2$: $\quad Z_i = e^{-ik\tau}/\sqrt{2k}$ *Bunch-Davies vacuum

- For $a^2\beta_A M^2 > k^2$:

$$Z_i = A_+ e^{\lambda_+ t} + A_- e^{\lambda_- t} \qquad \text{with} \qquad \lambda_\pm = \frac{H}{2}\left[-1 \pm \sqrt{1 - \frac{4\beta_A M^2}{H^2}}\right]$$

*Exponential suppression

- For $4\beta_A M^2 > H^2$: $\quad |Z_i| \propto e^{-Ht/2}$

*Decay with damped oscillations

A particular model for **inflation**

Scalar perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

$$\mathcal{S}_{\rm s}^{(2)} = \int d^4x\, a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} - \frac{k^2}{a^2} \vec{\mathcal{X}}^t \boldsymbol{G} \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} \right) \quad , \qquad \mathcal{X}^t = (\psi_k, \delta\phi_k)$$

where

- $K_{11} = \dfrac{k^2 \beta_A M^2}{2(k^2 + a^2 \beta_A M^2)}$
- $K_{12} = K_{21} = \dfrac{\beta_m}{2\beta_A M} K_{11}$
- $K_{22} = \dfrac{1}{2} - \dfrac{a^2 \beta_m^2 M^2}{8(k^2 + a^2 \beta_A M^2)}$
- $G_{11} = \dfrac{\beta_A M^2}{2}$
- $G_{12} = G_{21} = \dfrac{\beta_m M}{4}$
- $G_{22} = \dfrac{1}{2}$
- $M_{22} = \dfrac{V_{,\phi\phi}}{2} - \dfrac{(1-\delta_\phi^2) V_{,\phi}^2}{6 M_{\rm pl}^2 H^2} - \dfrac{(1+\delta_\phi)^4 V_{,\phi}^4}{324 \beta H^6 M_{\rm pl}^4}$

and

- $\delta_\phi \equiv \dfrac{\beta \ddot{\phi}}{V_{,\phi}} = -\dfrac{3\beta H \dot{\phi} + V_{,\phi}}{V_{,\phi}}$

A particular model for **inflation**

Perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Varying the action wrt ψ:

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 K_{11}\delta\dot{\chi}_k\right) + \frac{k^2}{a^2}G_{11}\delta\chi_k = 0$$

- For $k^2 \gg a^2\beta_A M^2$: $\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \frac{k^2}{a^2}\delta\chi_k = 0$

*EoM of a massless field

  - For $k^2/a^2 \gg H^2$: $v_k = \sqrt{2}a\delta\chi_k = e^{-ik\int dt/a}/\sqrt{2k}$ *BD vacuum

- For $k^2/a^2 \ll \beta_A M^2$: $\delta\ddot{\chi}_k + H\delta\dot{\chi}_k + \beta_A M^2\delta\chi_k = 0$

$$\delta\chi_k = A_+ e^{\lambda_+ t} + A_- e^{\lambda_- t}$$

*Exponential suppression

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Varying the action wrt $\delta\phi$:

$$\frac{1}{a^3}\frac{d}{dt}\left[a^3\left(K_{22}\delta\dot{\phi}_k + K_{12}\dot{\psi}_k\right)\right] + \frac{k^2}{a^2}\left(G_{22}\delta\phi_k + G_{12}\psi_k\right) + M_{22}\delta\phi_k = 0$$

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3\tilde{K}_{22}\delta\dot{\phi}_k\right) + \left(\frac{k^2}{a^2}\tilde{G}_{22} + M_{22}\right)\delta\phi_k = 0$$

$$\delta\sigma_k'' + \left(k^2 - \frac{a''}{a} + \frac{2a^2M_{22}}{\beta}\right)\delta\sigma_k = 0$$

with

$$\delta\sigma_k \equiv a\sqrt{\beta}\,\delta\phi_k$$

$$\delta\sigma_k'' + \left[k^2 - 2(aH)^2\left(1 + \frac{5\epsilon_V - 3\eta_V}{2\beta}\right)\right]\delta\sigma_k = 0$$

A particular model for **inflation**

Perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

$$\delta\sigma_k'' + \left[k^2 - 2(aH)^2\left(1 + \frac{5\epsilon_V - 3\eta_V}{2\beta}\right)\right]\delta\sigma_k = 0$$

has as solution:

$$\delta\sigma_k = \frac{\sqrt{\pi|\tau|}}{2} e^{i(1+2\nu)\pi/4} H_\nu^{(1)}(k|\tau|)$$ where $$\nu = \frac{3}{2} + \frac{3\epsilon_V - \eta_V}{\beta}$$

- For $k\tau \to 0$: $$\delta\phi_k = i\frac{H(1-\epsilon)}{k^{3/2}\sqrt{2\beta}}\frac{\Gamma(\nu)}{\Gamma(3/2)}\left(\frac{k|\tau|}{2}\right)^{3/2-\nu}$$
- Since $\mathcal{R}_\phi = -\frac{H\delta\phi_k}{\dot\phi}$, $$\frac{1}{a^3\epsilon}\frac{d}{dt}\left(a^3\epsilon\dot{\mathcal{R}}_\phi\right) + \frac{k^2}{a^2}\mathcal{R}_\phi = 0$$

■ $$\mathcal{R}_\phi = c_1 + c_2\int\frac{dt}{a^3\epsilon} \longrightarrow \mathcal{P}_{\mathcal{R}_\phi} \equiv \frac{k^3}{2\pi^2}\left|\mathcal{R}_\phi\right|^2 = \left.\frac{H^4}{4\pi^2\dot\phi^2\beta}\right|_{k=aH}$$

OSR slow-roll params

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2}\left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2}\zeta^2\right)$$

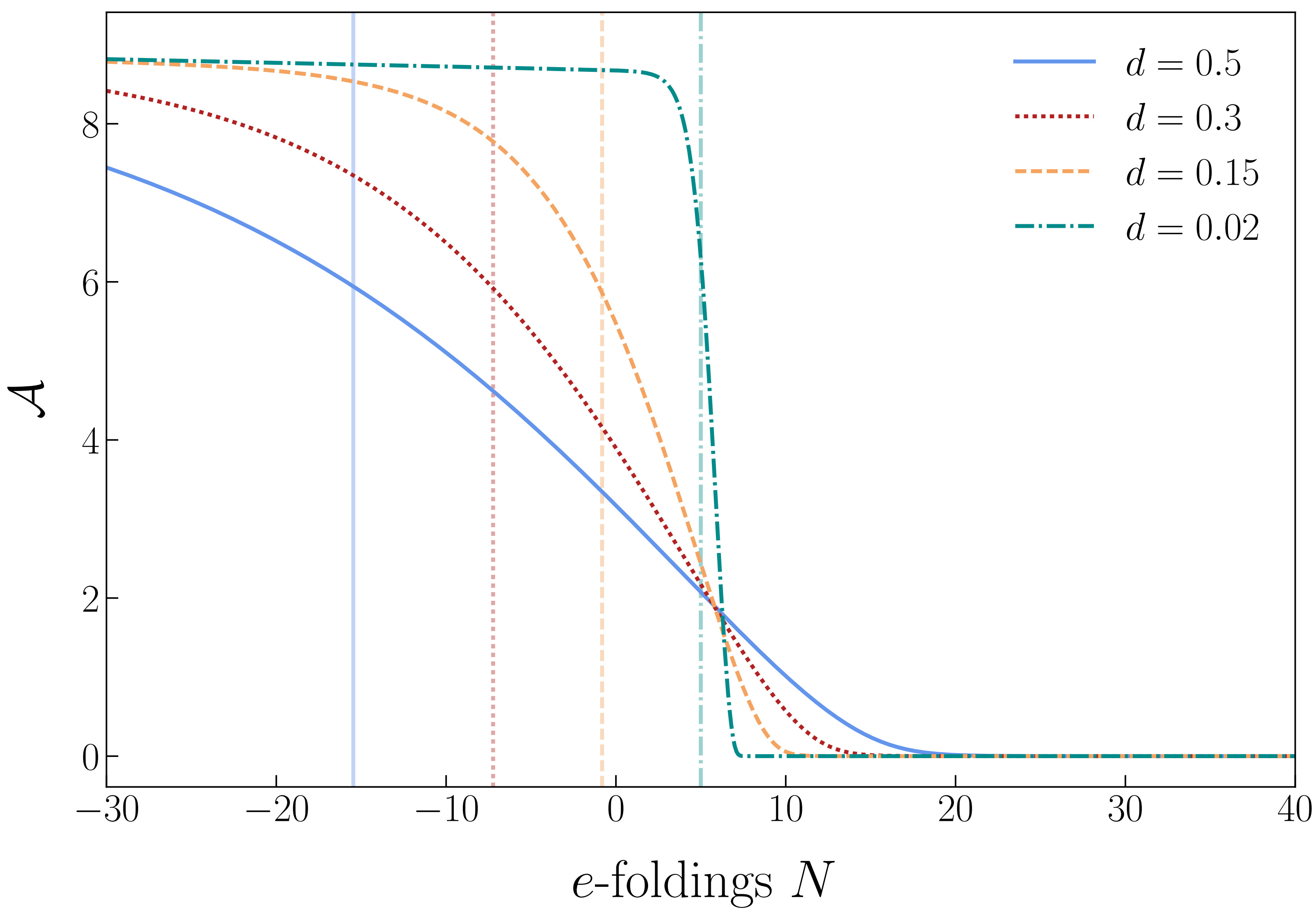
- $\epsilon_H = -\dfrac{\mathrm{d}\ \ln\ H}{\mathrm{d}N}$,
- $\delta_1 \equiv \dfrac{1}{2}\dfrac{\mathrm{d}\ln\epsilon_H}{\mathrm{d}N} - \epsilon_H,$
- $\delta_{p+1} \equiv \dfrac{\mathrm{d}\delta_p}{\mathrm{d}N} + \delta_p(\delta_1 - p\epsilon_H).$

- $\sigma_{i,1} \equiv \dfrac{\mathrm{d}\ln c_i}{\mathrm{d}N},$
- $\sigma_{i,p+1} \equiv \dfrac{\mathrm{d}\sigma_{i,p}}{\mathrm{d}N},$

- $\xi_{i,1} \equiv \dfrac{\mathrm{d}\ln b_i}{\mathrm{d}N},$
- $\xi_{i,p+1} \equiv \dfrac{d\xi_{i,p}}{dN}$

Transition parameter

- $3H^2 \approx V,$
- $\epsilon_H \equiv -\frac{H'}{H} \approx \frac{1}{2(1+\mathcal{A})}\left(\frac{V_{,\phi}}{V}\right)^2 .$
- $3H^2\phi'(1+\mathcal{A}) + V_{,\phi} \approx 0\ ,$



- Vertical lines: the position of the step.