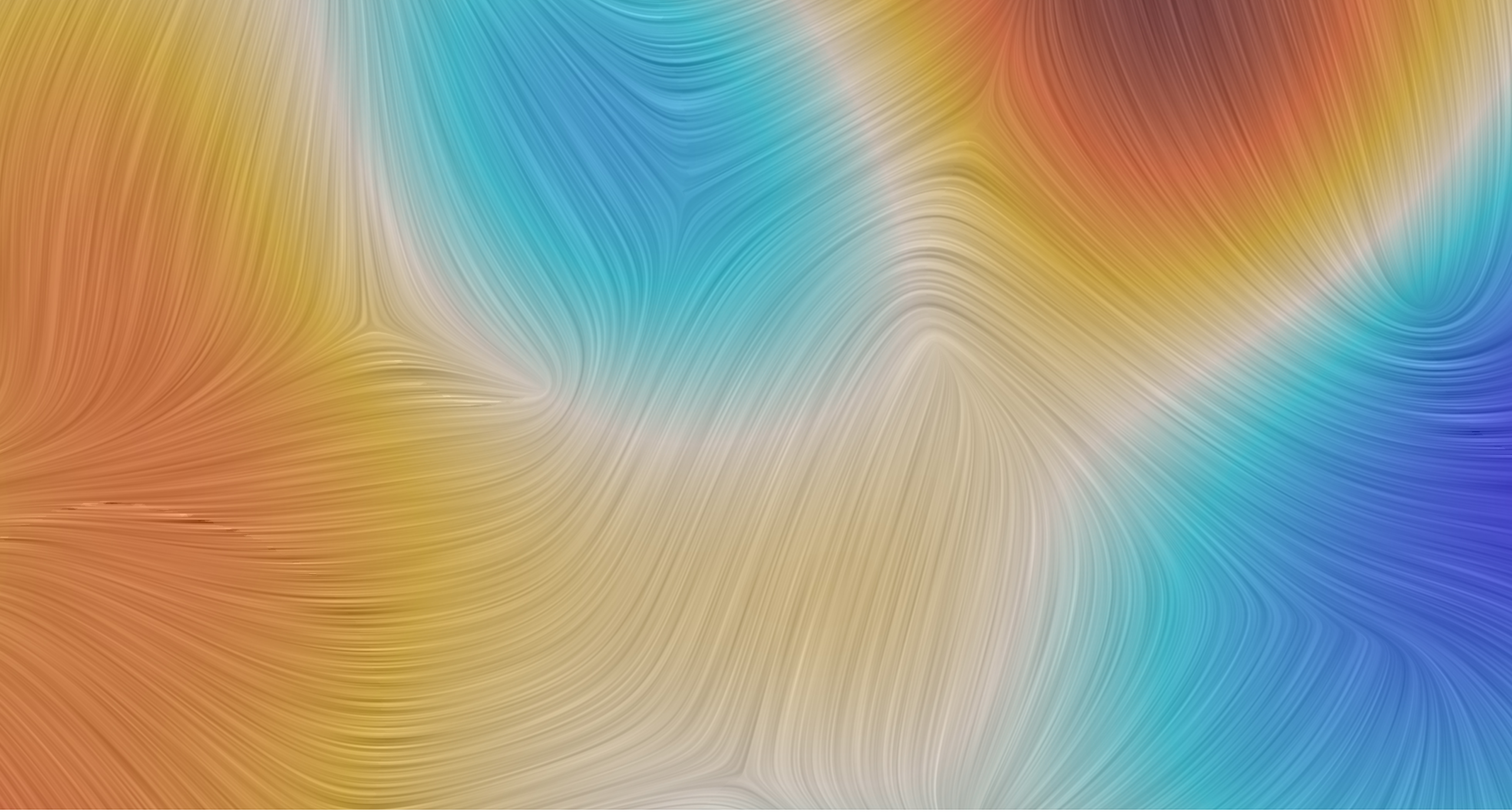


Noncanonical Approaches to Inflation

Thesis defense

Héctor Ramírez

Universitat de València

IFIC Institut de Física Corpuscular

CSIC Consejo Superior de Investigaciones Científicas

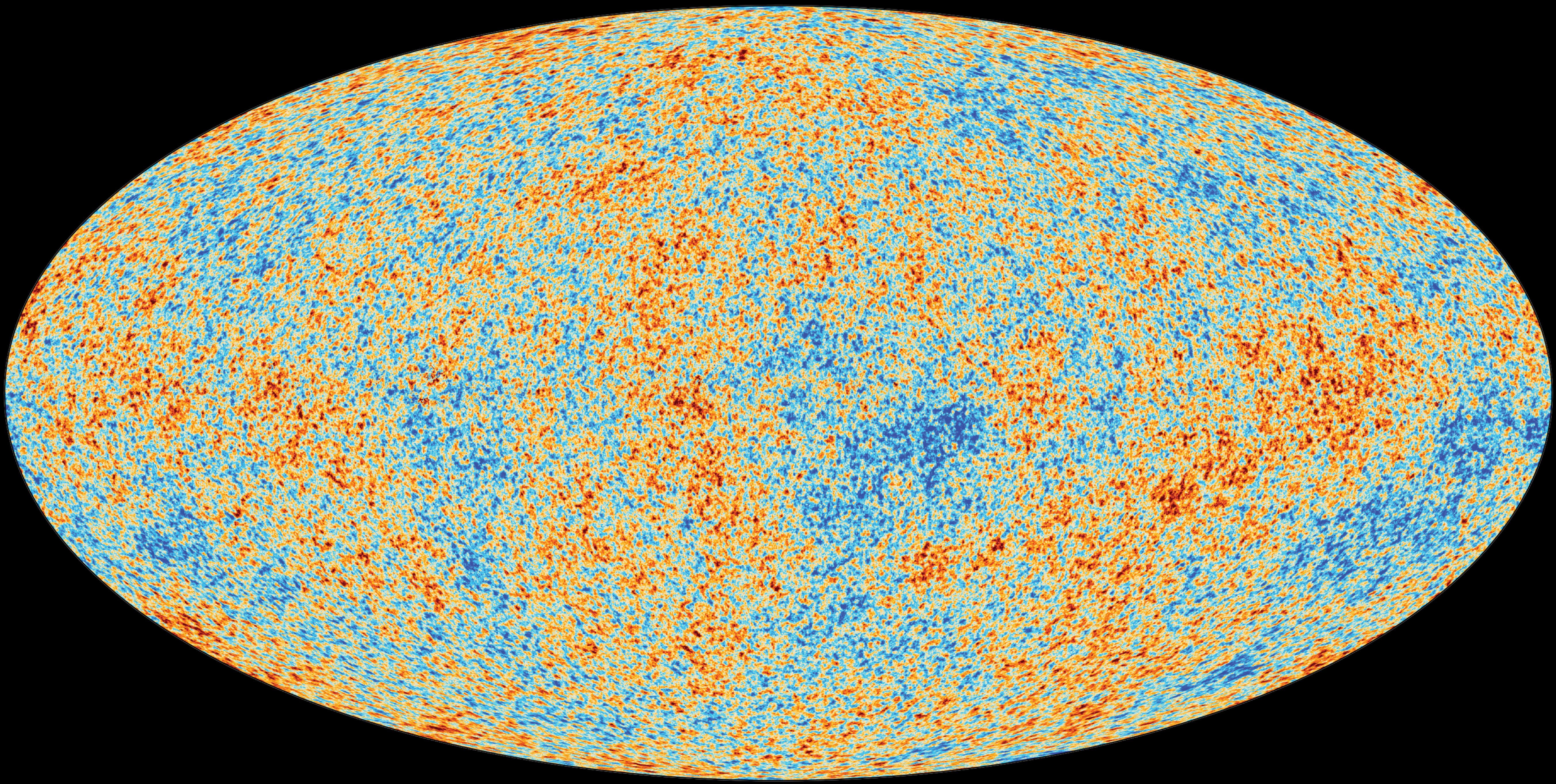
Excelencia Severo Ochoa

invisiblesPlus neutrinos, dark matter & dark energy physics

elusives neutrinos, dark matter & dark energy physics

The **C**osmic **M**icrowave **B**ackground

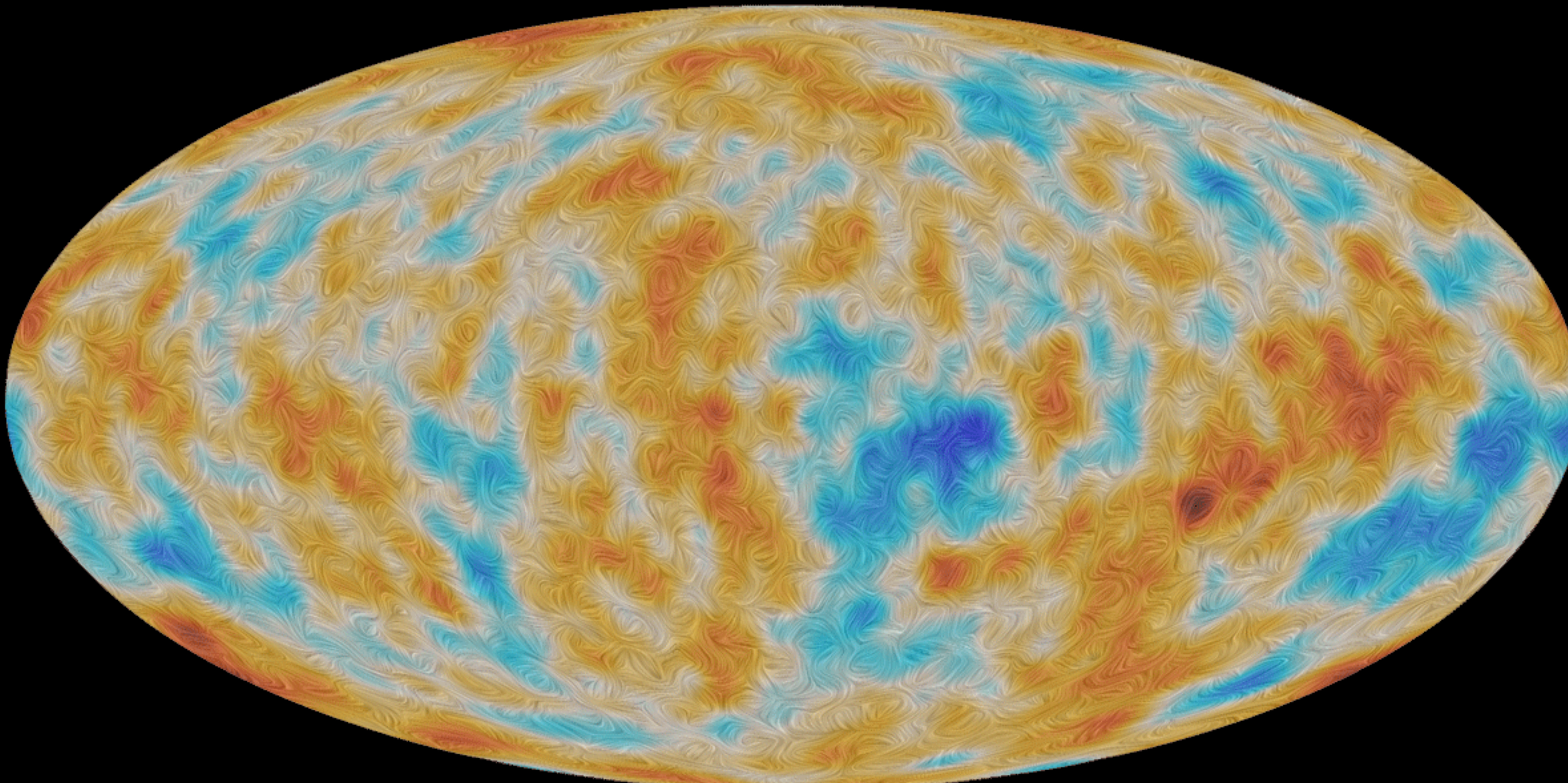
- ESA and the Planck Collaboration



Anisotropies at $\sim 10^{-5}$

The **C**osmic **M**icrowave **B**ackground

- ESA and the Planck Collaboration



E-mode polarization

The **C**osmic **M**icrowave **B**ackground

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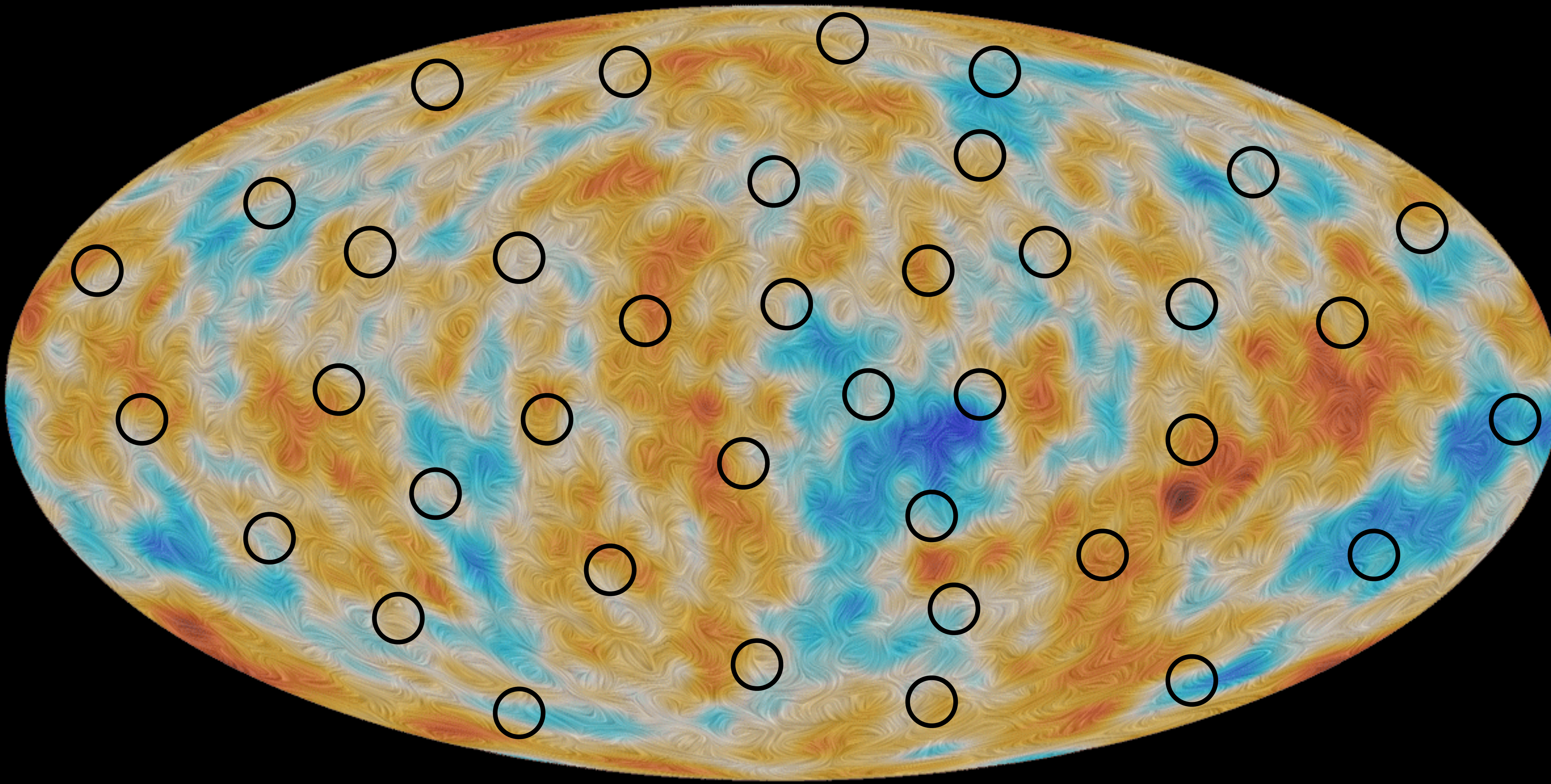
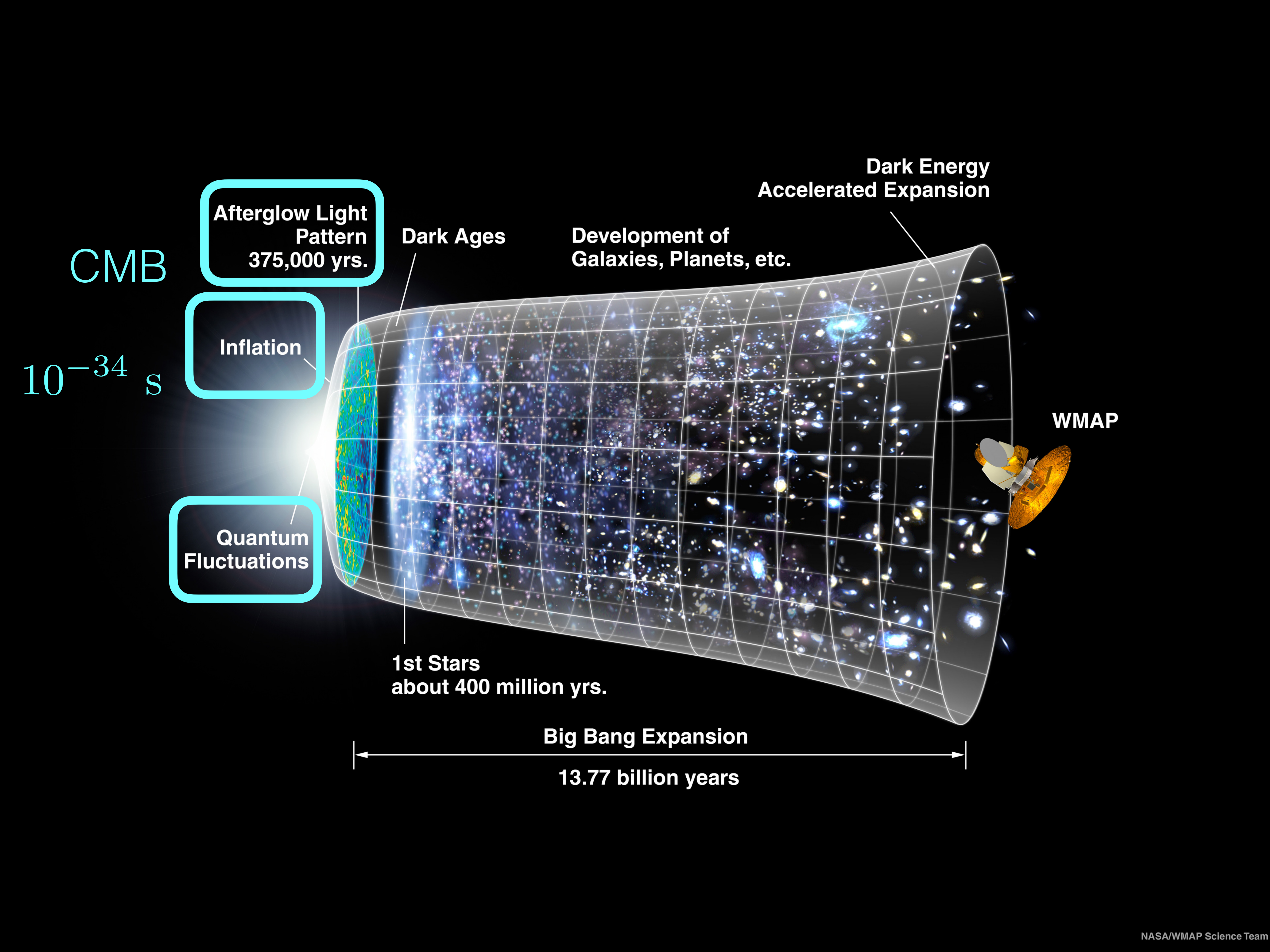


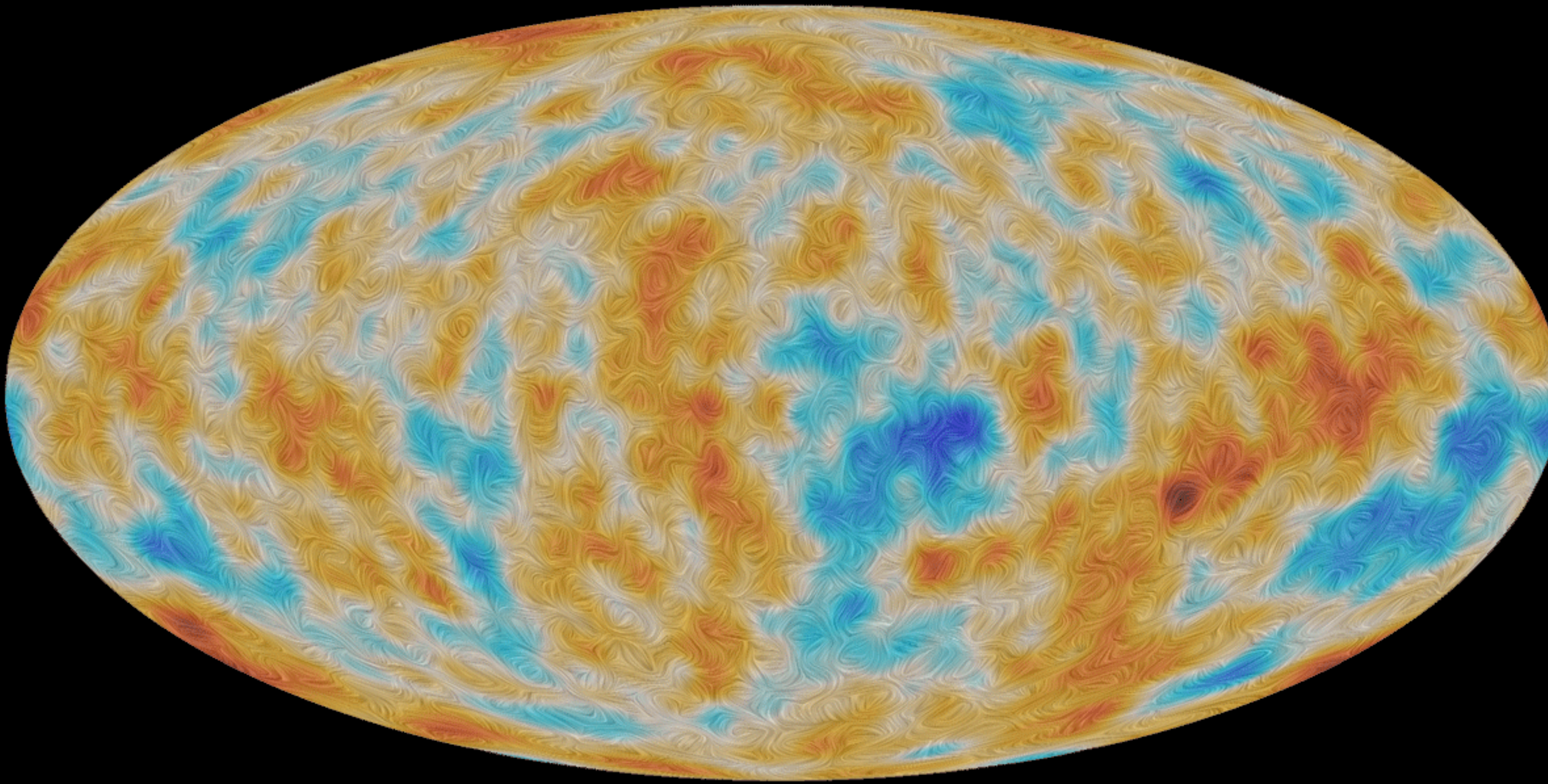
10^4 Causally independent patches



Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
CMB
Inflation
10^{-34} s
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years
NASA/WMAP Science Team

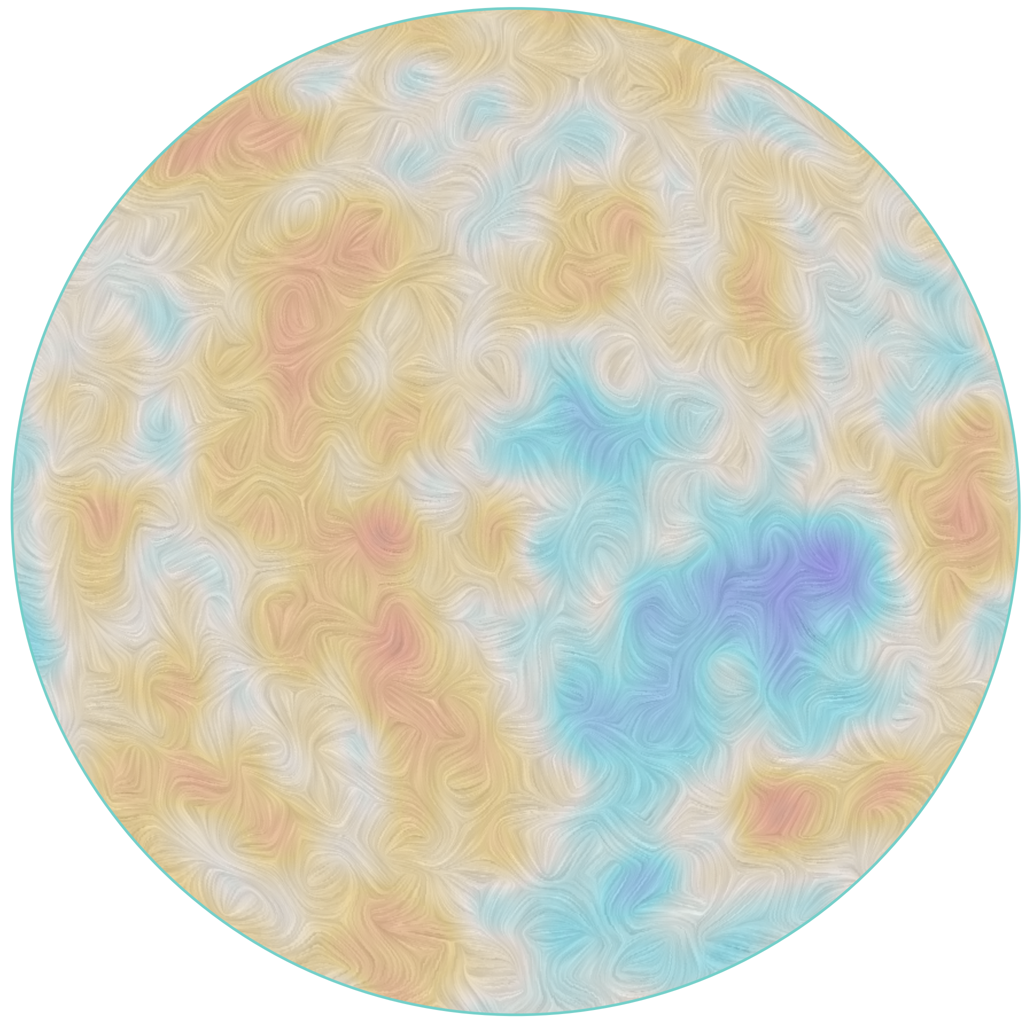
The **C**osmic **M**icrowave **B**ackground

\- ESA and the Planck Collaboration



B-mode polarization?

The comoving Hubble radius



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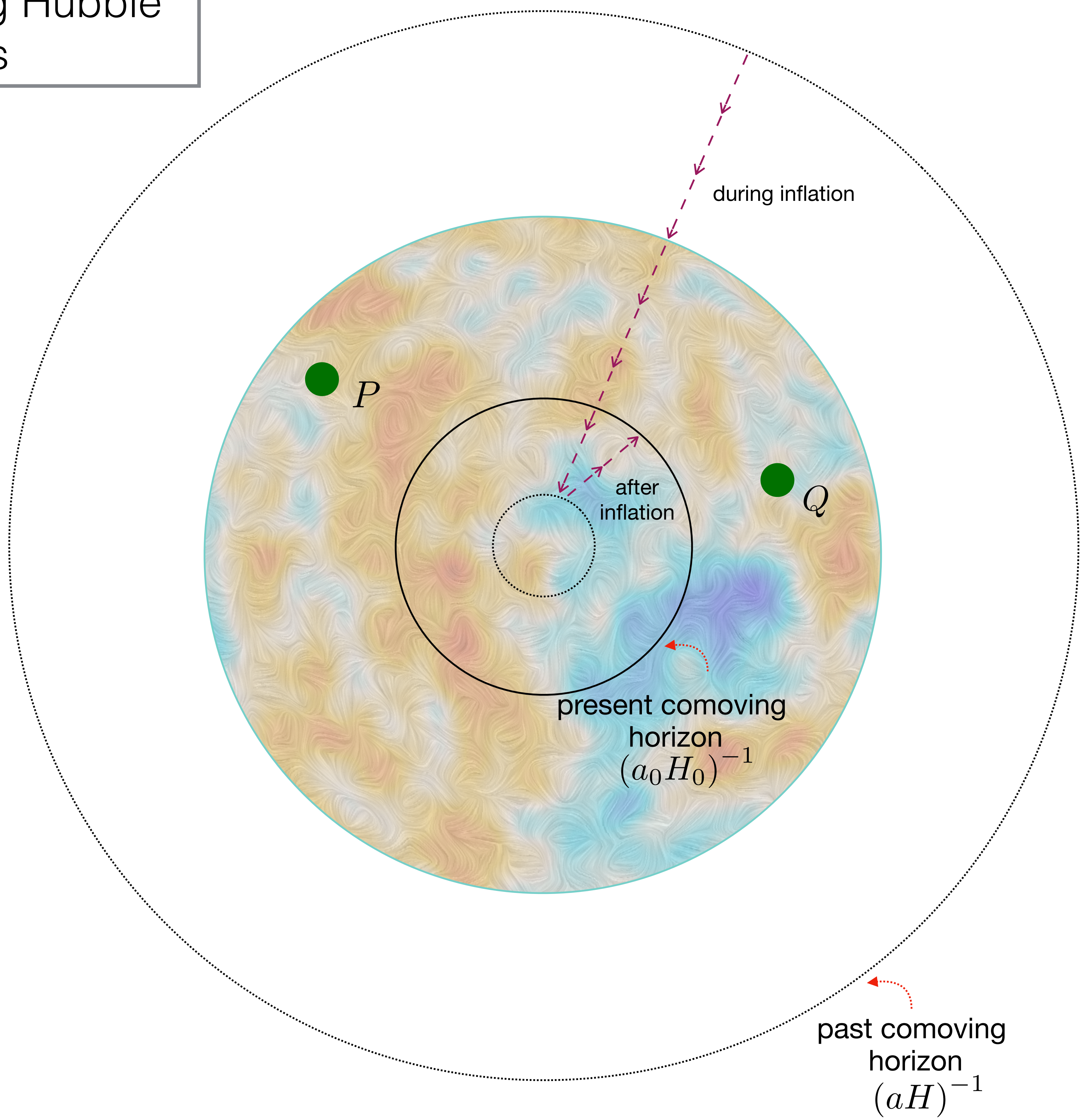

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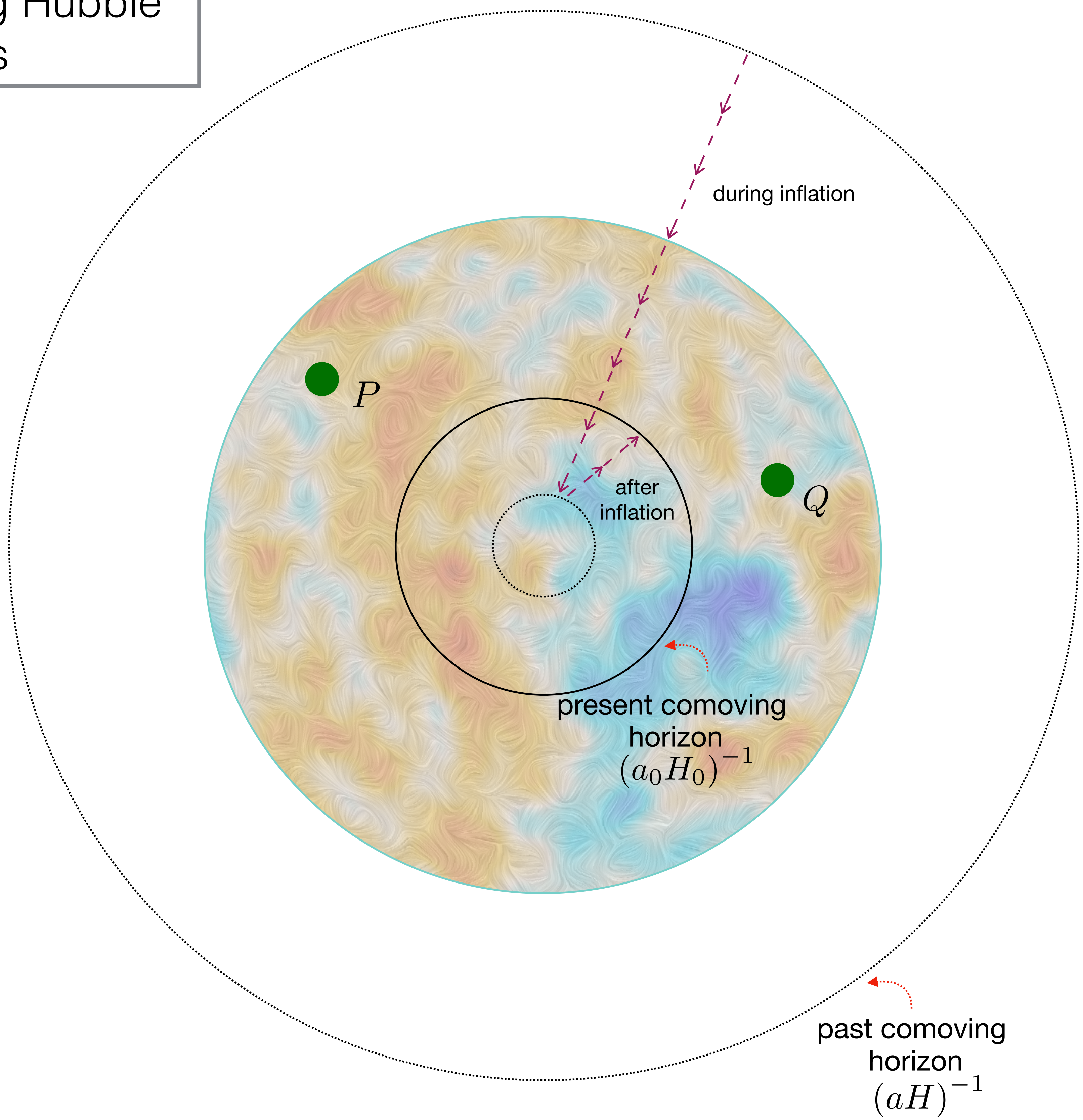
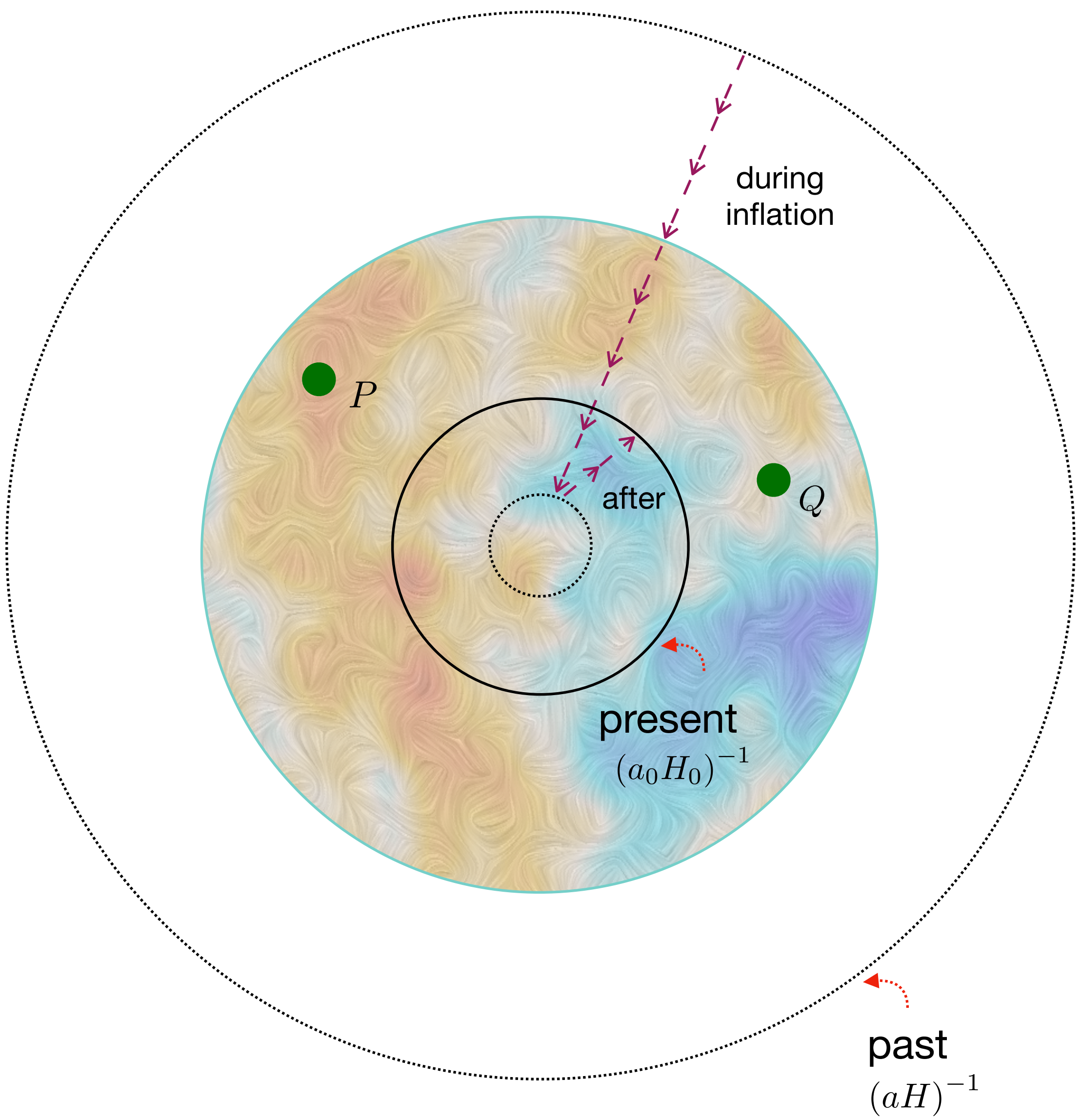


The comoving Hubble radius

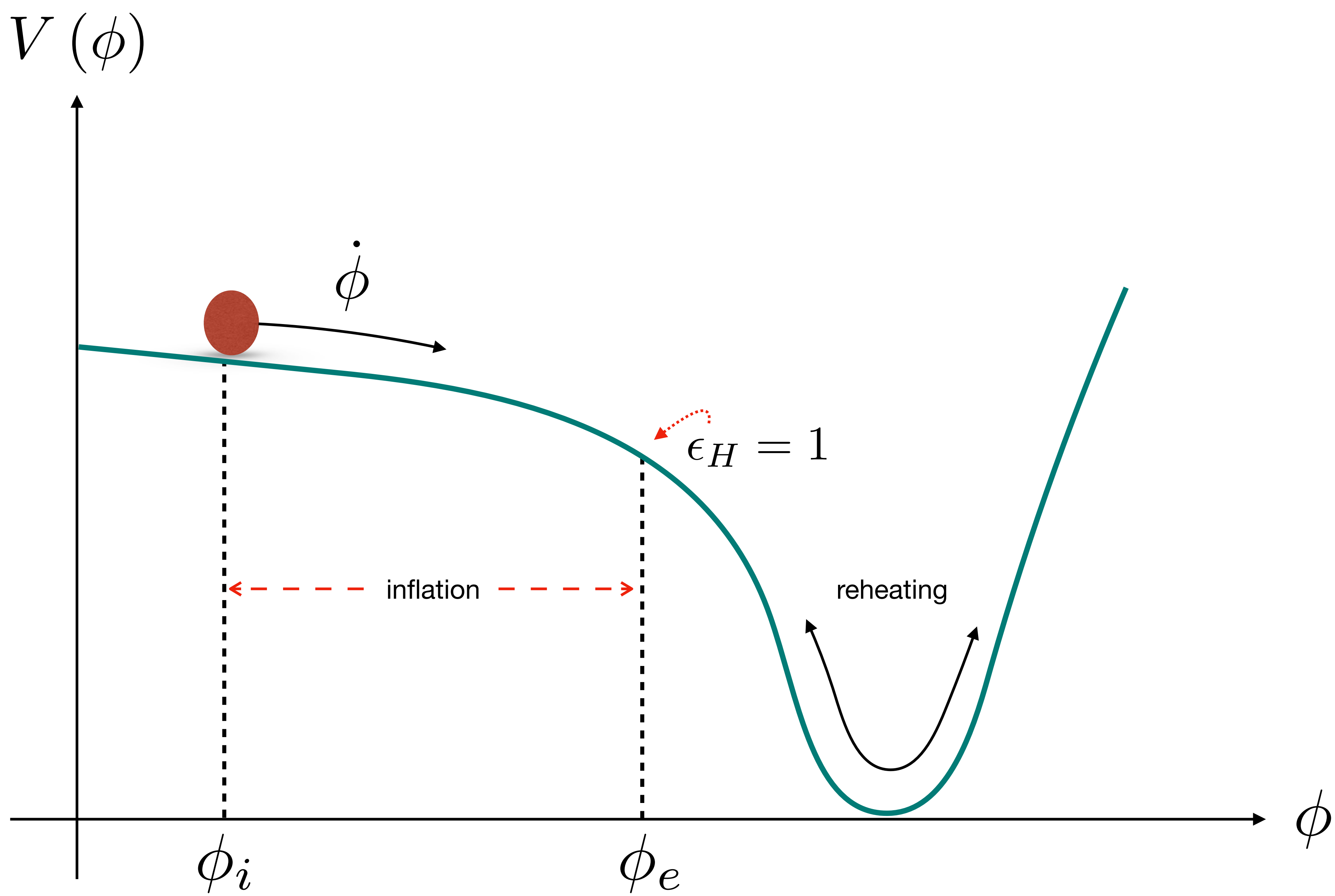


Conditions for inflation:

- Accelerated expansion:
  - $$\ddot{a} > 0$$
- Almost constant H:
  - $$\epsilon_H \equiv -\frac{\dot{H}}{H^2} < 1$$
- Fluid with a negative pressure:
  - $$p < -\frac{1}{3}\rho$$

Single-field slow-roll inflation

— A. Linde; Phys. Lett. **108B** (1982) 389.



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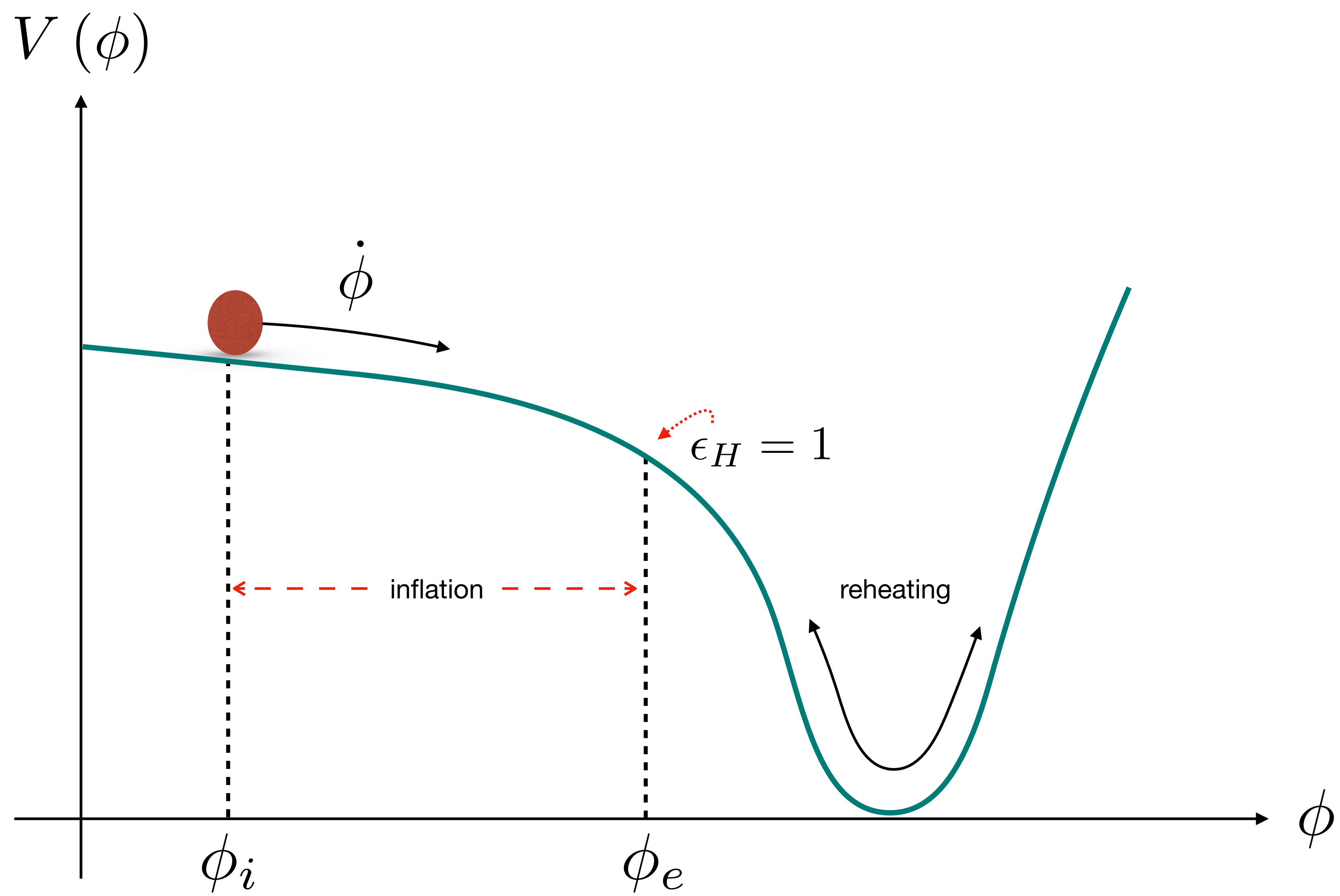

Action:

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\mathrm{pl}}^2}{2}R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - V(\phi)\right]$$

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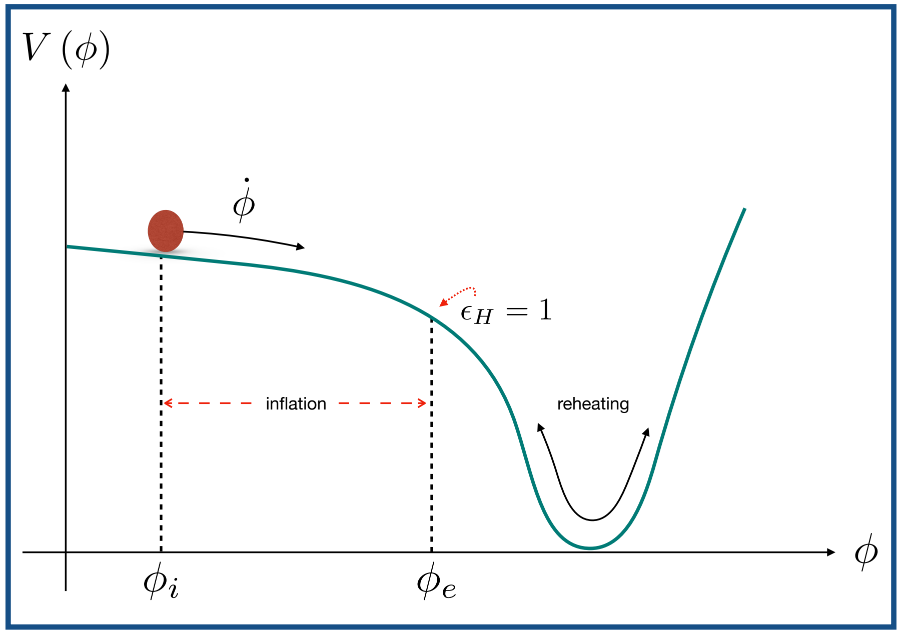
— A. Linde; Phys. Lett. **108B** (1982) 389.



Conditions for inflation:

- $$\epsilon_H = -\frac{\dot{H}}{H^2} \longleftrightarrow \epsilon_H \ll 1$$
- $$\eta_H = \frac{1}{2}\frac{\mathrm{d}\,\ln\epsilon_H}{\mathrm{d}N} - \epsilon_H \longleftrightarrow |\eta_H| \ll 1$$

Number of *e*-folds of inflation:

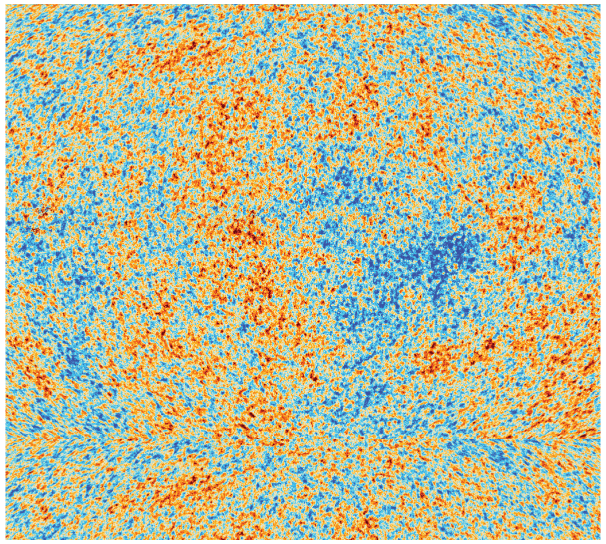
$$N_{\rm CMB} = \int_{t_{\rm CMB}}^{t_{\rm end}} H \mathrm{d}t \approx 40 - 60$$

The theory of quantum fluctuations

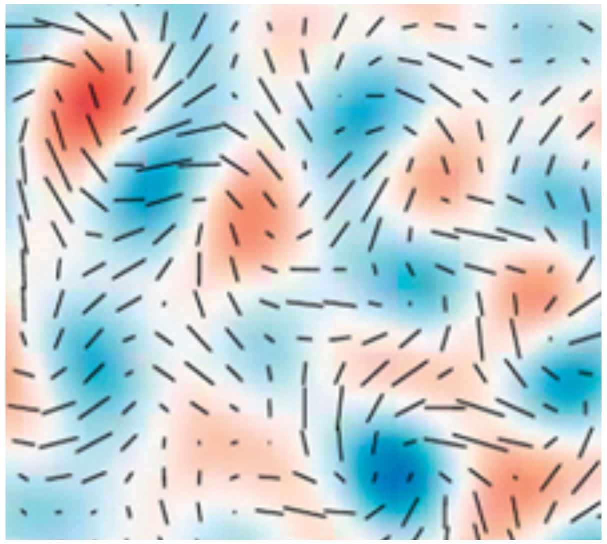
— V. Mukhanov & G. Chibisov; *JETP Lett.* **33** (1981) 532–535.

Scalar and metric perturbations

$$g_{ij} = a^2(t)\left[\left(1 + 2\,\boxed{\zeta(t,\vec{x})}\,\right)\delta_{ij} + 2\,\boxed{h_{ij}(t,\vec{x})}\,\right]$$



Temperature anisotropies



B-mode polarization

The theory of quantum fluctuations

— V. Mukhanov & G. Chibisov; *JETP Lett.* **33** (1981) 532–535.

Scalars

- $$S_\zeta^{(2)} = \int \mathrm{d}^4x a^3 \epsilon_H \left(\dot{\zeta}^2 - \frac{k^2}{a^2}\zeta^2 \right)$$

Tensors

- $$S_\gamma^{(2)} = \sum_{\lambda=+,\times} \int \mathrm{d}^4x \frac{a^3}{4} \left(\dot{\gamma}_\lambda^2 - \frac{k^2}{a^2}\gamma_\lambda^2 \right)$$

Mukhanov - Sasaki equation

$$u_p'' + \left(k^2 - \frac{z_p''}{z_p} \right) u_p = 0$$

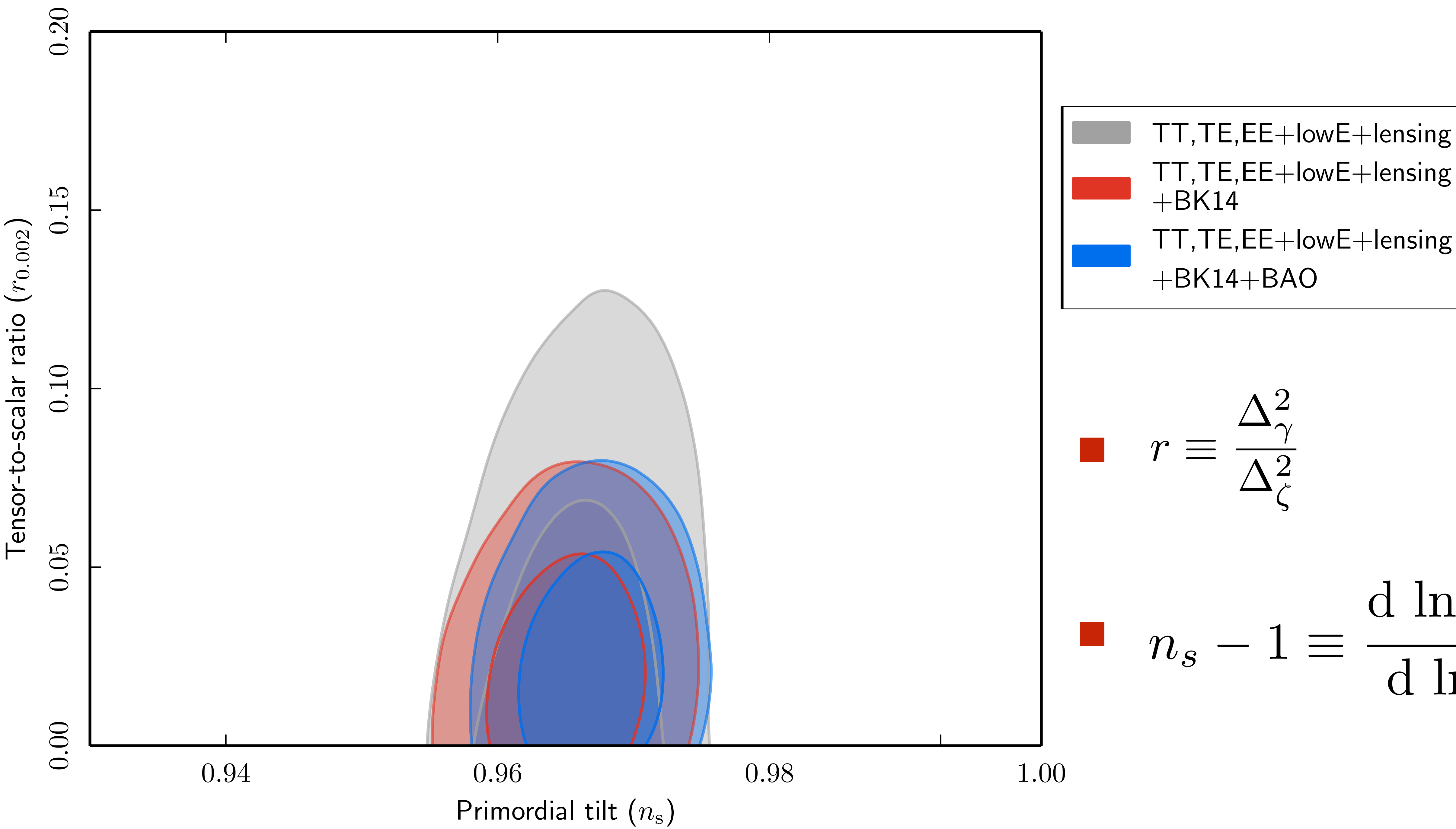
$$u_s = a\sqrt{2\epsilon_H}\zeta$$

$$u_t = a\gamma/\sqrt{2}$$

Dimensionless power spectra

$$\Delta_{u_p}^2(k) \equiv \frac{k^3}{2\pi^2} |u_p(\tau)|^2$$

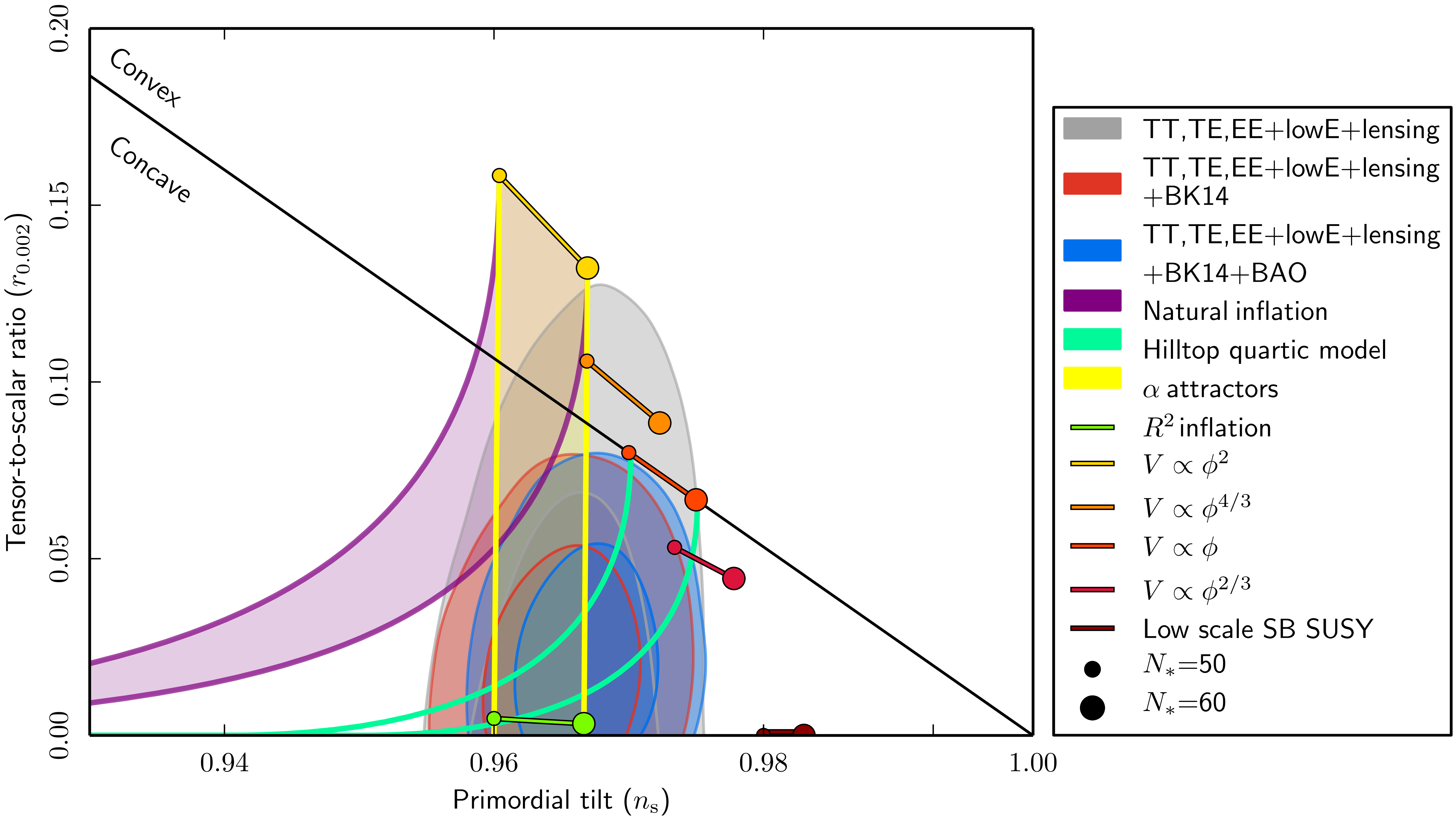
The observables



- $$r \equiv \frac{\Delta_\gamma^2}{\Delta_\zeta^2}$$
- $$n_s - 1 \equiv \frac{\mathrm{d}\ \ln \Delta_\zeta^2}{\mathrm{d}\ \ln k}$$

— Y. Akrami *et al.* [Planck Collaboration]; arXiv:1807.06211 [astro-ph.CO].

The models



— Y. Akrami *et al.* [Planck Collaboration]; arXiv:1807.06211 [astro-ph.CO].

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Further parameters and parametrizations

Noncanonical models beyond General Relativity

- Future constraints

$$\alpha_s \equiv \frac{\mathrm{d}\, n_s}{\mathrm{d}\ln k} \qquad \beta_s \equiv \frac{\mathrm{d}\, \alpha_s}{\mathrm{d}\ln k}$$

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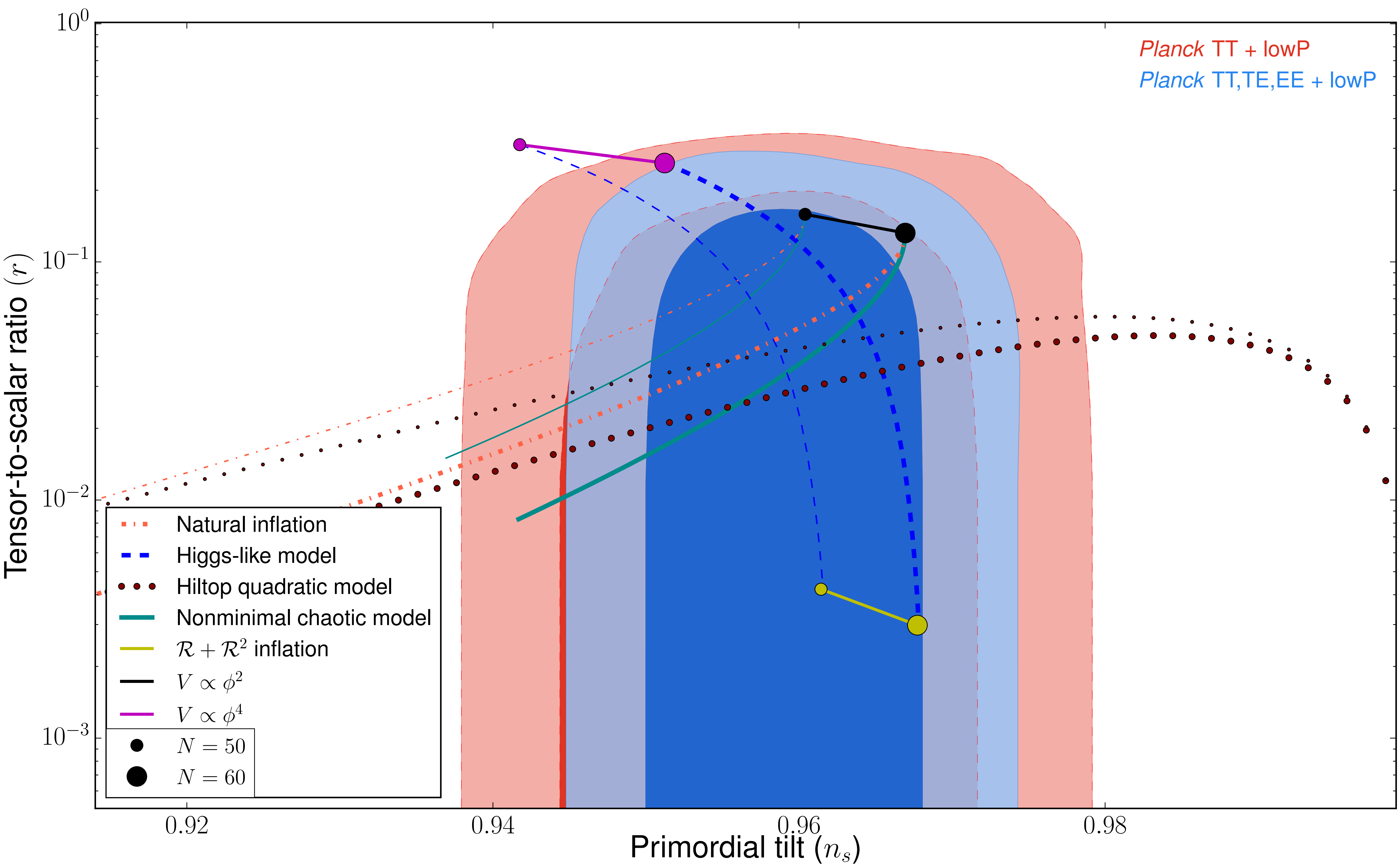
- Future constraints

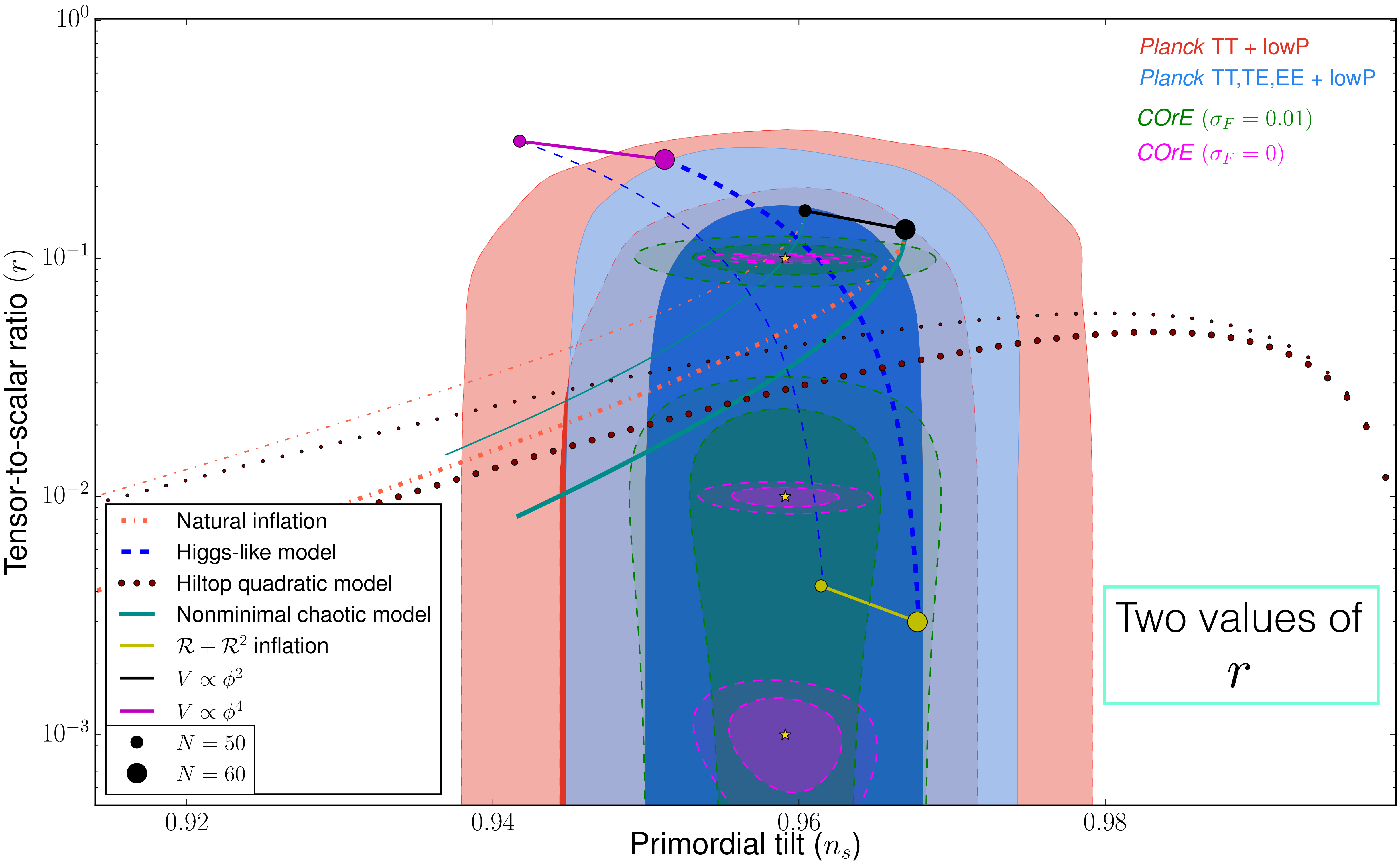
– M. Escudero, **HR**, L. Boubekeur, E. Giusarma and O. Mena; JCAP **1602** (2016) no.02, 020.



COrE: Cosmic Origins Explorer

- Future constraints

– M. Escudero, **HR**, L. Boubekeur, E. Giusarma and O. Mena; JCAP **1602** (2016) no.02, 020.

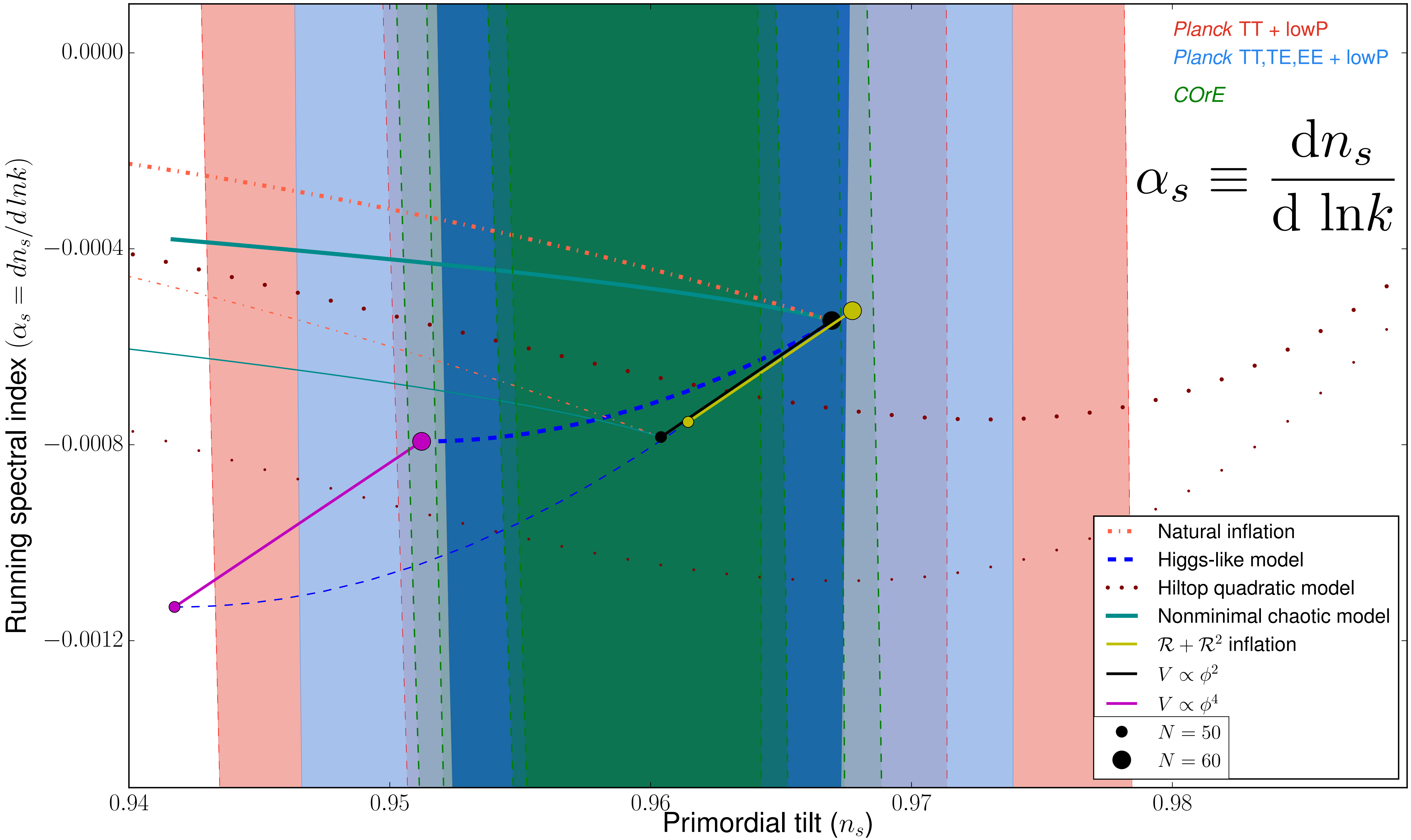


Two values of r

COrE: Cosmic Origins Explorer

- Future constraints

– M. Escudero, **HR**, L. Boubekeur, E. Giusarma and O. Mena; JCAP **1602** (2016) no.02, 020.



COrE: Cosmic Origins Explorer

- Future constraints

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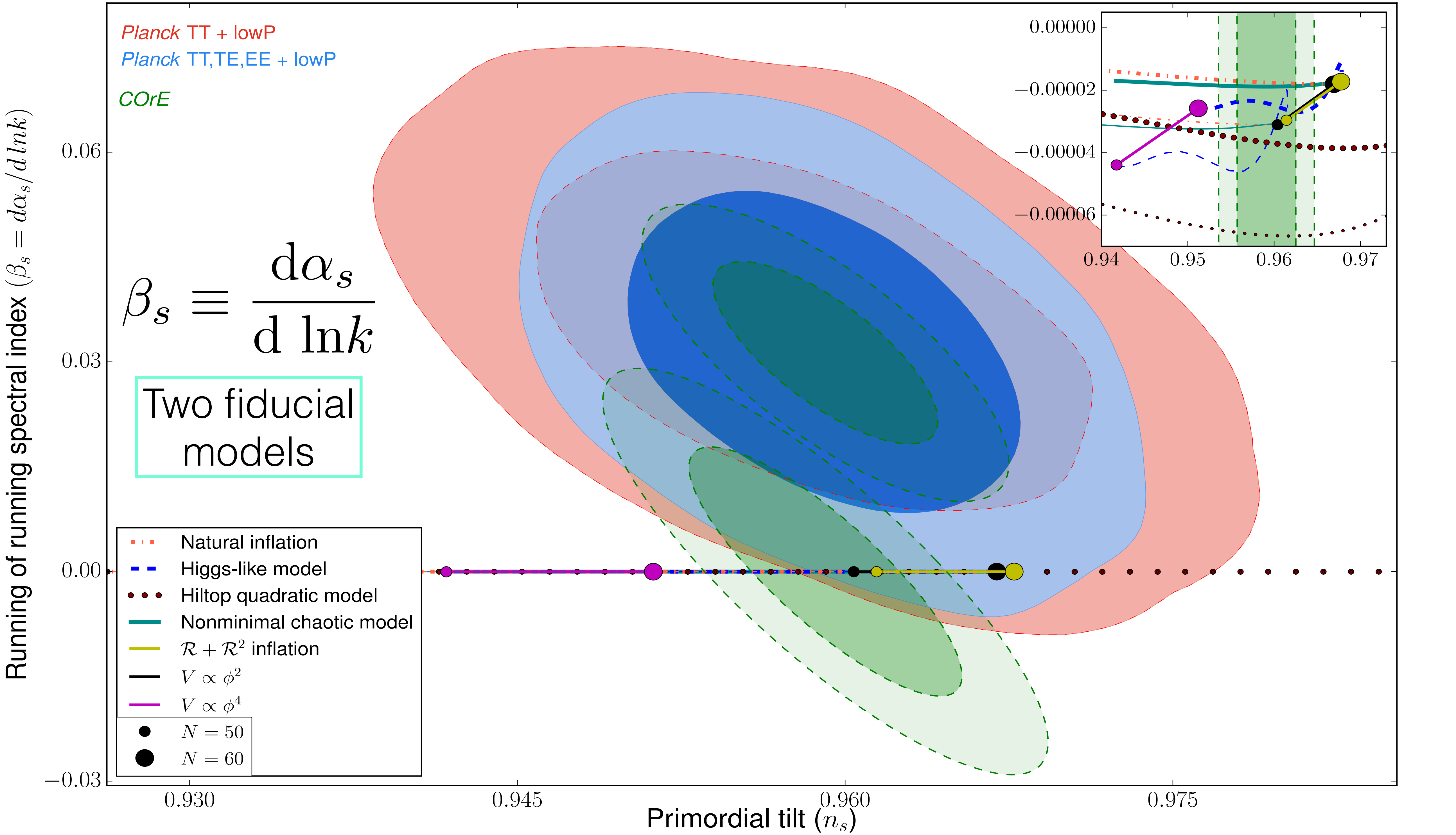

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- Phenomenological approaches

- What is a proposed model/potential telling us about the inflationary epoch?
  - The shape of the potential is difficult to understand in a fundamental theory.
- We can test the slow-roll consistency relation:

$$r = -8n_t$$

  - Doing it for every model doesn't help us with the task.
  - Doing it experimentally entails a huge challenge.

★ How can we test the paradigm then?

  - ➡ How about taking the conditions for inflation and combine them?

Mukhanov parametrization

— V. Mukhanov; Eur. Phys. J. C73 (2013) 2486.

From the slow-roll approximation:

$$\omega \equiv \frac{p}{\rho} = -1 + \frac{2}{3}\epsilon_H$$

$$0 < \epsilon_H \leq 1$$

We can take the following ansatz:

$$\omega + 1 = \frac{\beta}{(\mathcal{N}+1)^{\alpha}}$$

where $\alpha, \beta \sim \mathcal{O}(1)$

This leads to:

- $$n_s - 1 = -\frac{3\beta}{(\mathcal{N}+1)^{\alpha}} - \frac{\alpha}{\mathcal{N}+1}$$
- $$r = \frac{24\beta}{(\mathcal{N}+1)^{\alpha}}$$

This model-independent approach covers most of the favored models previously discussed. For instance:

➡ Chaotic scenarios: $\alpha = 1$ (Branch I)

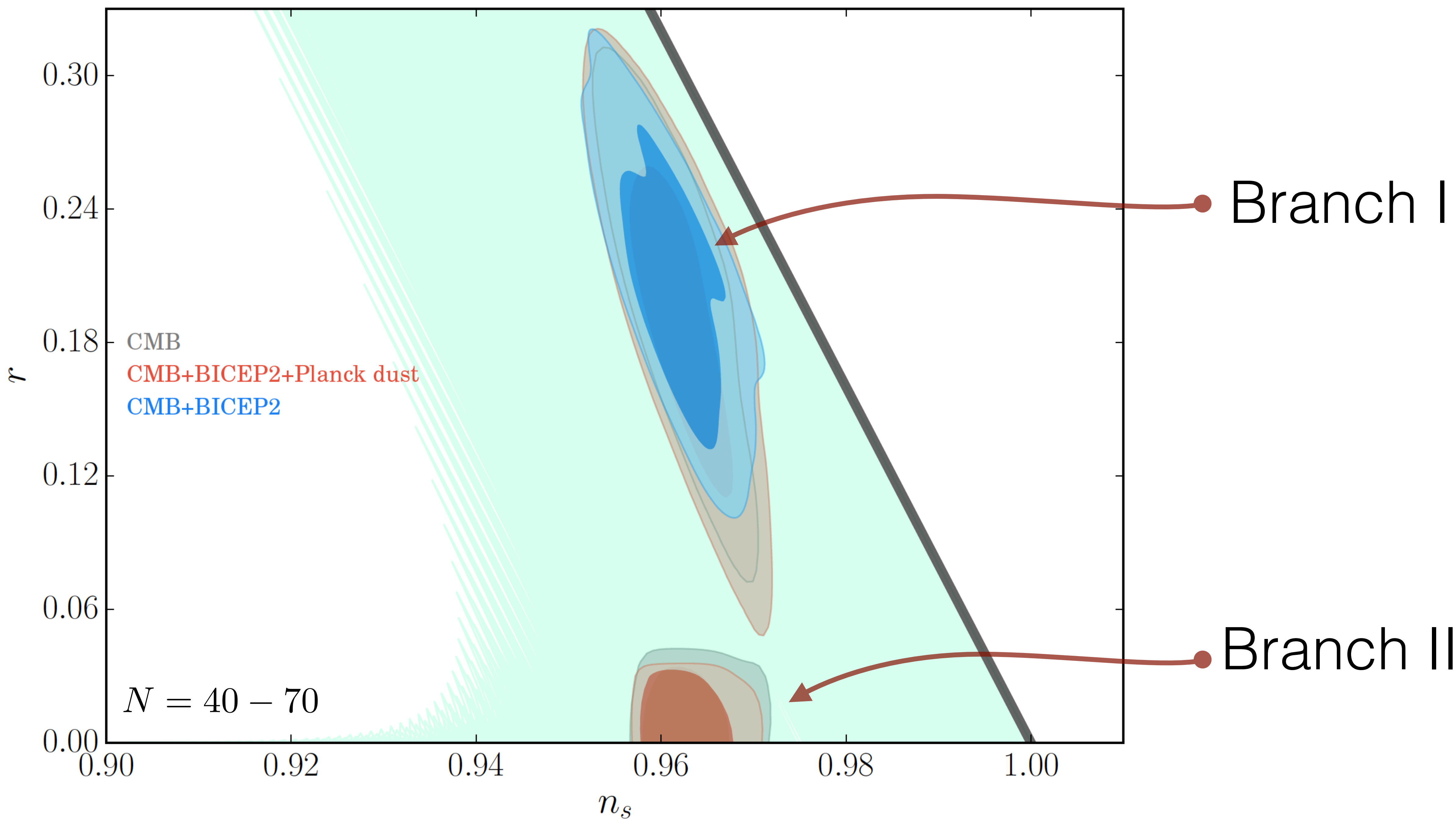
$$n_s - 1 = -\frac{3\beta+1}{\mathcal{N}+1} \qquad r = \frac{24\beta}{\mathcal{N}+1}$$

➡ Starobinsky inflation: $\alpha = 2\ ,\quad \beta = 1/2$ (Branch II)

$$n_s - 1 \simeq -\frac{2}{\mathcal{N}} \qquad r \simeq \frac{12}{\mathcal{N}^2}$$

Mukhanov parametrization

— V. Mukhanov; Eur. Phys. J. C73 (2013) 2486.
— L. Barranco et al.; Phys. Rev. D90 (2014) 063007.
— L. Boubekeur, E. Giusarma, O. Mena, **HR**; Phys. Rev. D91 (2015) 083006.



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Noncanonical models
beyond General Relativity

(Part I)

Noncanonical models beyond General Relativity

- The simplest scenarios are in tension with the CMB data.
- There may also be needs to modify GR at late times.

⋆ What is the most general modification respecting GR's symmetries?

Towards the most general modification of GR

(a recipe)

- Ingredients:
  - Lorentz invariance
  - Locality
  - Unitarity
  - A (pseudo-) Riemaniann spacetime

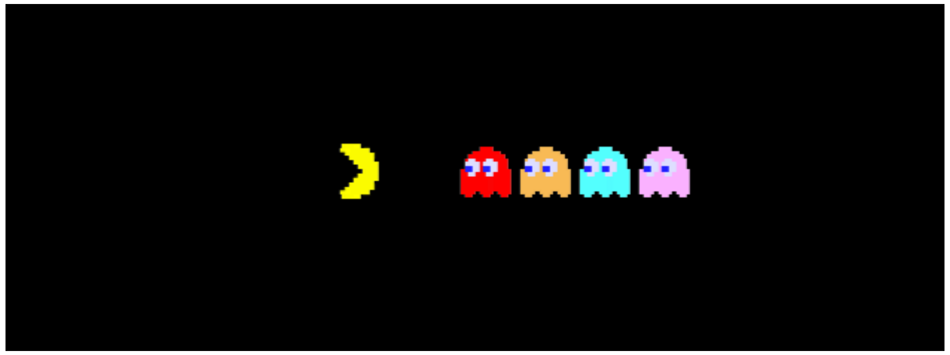
Towards the most general modification of GR

(a recipe)

- Ingredients:
  - Lorentz invariance
  - Unitarity
  - Locality
  - A (pseudo-) Riemaniann spacetime
- ➡ Combinations of arbitrarily higher-order derivatives are allowed.
- ➡ Derivatives higher than order 1 lead to EoM higher than order 2.
  - ➡ The system can excite negative dof's!
- A rule:

Ostrogradsky's theorem

"Higher-derivative theories contain extra degrees of freedom, and are usually plagued by negative energies and related instabilities."



Towards the most general modification of GR

(a recipe)

- Instructions:

1. Construct combinations order by order in derivatives:

$$\partial^\mu\phi\partial_\mu\phi\,, \quad (\partial^\mu\phi\partial_\mu\phi)\partial^\nu\partial_\nu\phi\,, \quad (\partial^\mu\partial_\mu\phi)^2\,, \quad \partial^\mu\partial^\nu\phi\partial_\mu\partial_\nu\phi, \ldots$$

2. Find constraints that remove *ghostly* terms according to our rule.

3. Covariantize the theory:

$$\partial_\mu \to \nabla_\mu$$

4. Restore the correct order with nonminimal couplings to R and $G_{\mu\nu}$.

- G. W. Horndeski; Int. J. Theor. Phys. **10** (1974) 363.
- A. Nicolis *et al.*; PRD **79** (2009) 064036.
- C. Deffayet *et al.*; PRD **84** (2011) 064039.

- Result: Horndeski theory

$$S = \sum_{i=2}^{5} \int \mathrm{d}^4x \sqrt{-g}\mathcal{L}_i$$

where

$$X \equiv g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi$$
$$G_i = G_i(\phi, X)$$

$$\mathcal{L}_2 = G_2 \ ,$$
$$\mathcal{L}_3 = G_3\Box\phi \ ,$$
$$\mathcal{L}_4 = G_4R - 2G_{4,X}[(\Box\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}] \ ,$$
$$\mathcal{L}_5 = G_5G^{\mu\nu}\phi_{;\mu\nu} + \frac{G_{5,X}}{3}[(\Box\phi)^3 - 3(\Box\phi)\phi_{;\mu\nu}\phi^{;\mu\nu} + 2\phi_{;\mu\nu}\phi^{;\mu\sigma}\phi^{;\nu}{}_{;\sigma}] \ .$$

Canonical inflation:

$$G_2 = -\frac{X}{2} - V(\phi) \ , \qquad G_4 = \frac{M_{\rm pl}^2}{2} \ ,$$
$$G_3 = 0 \ , \qquad G_5 = 0 \ .$$

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- Nonminimal couplings

$$\xi\phi^2 R$$

$$\left(\quad G_4(\phi, X) = f(\phi) \quad \right)$$

- Nonminimal couplings

Nonminimal chaotic inflation

– A. Linde, M. Noorbala, A. Westphal; *JCAP* **1103** (2011) 013.
– L. Boubekeur, E. Giusarma, O. Mena, **HR**; Phys. Rev. **D91** (2015) 103004.

$$\mathcal{S}_{\rm NM} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}\left(1+\xi\phi^2\right)R - \frac{1}{2}g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - U(\phi)\right]$$

Performing a conformal transformation:

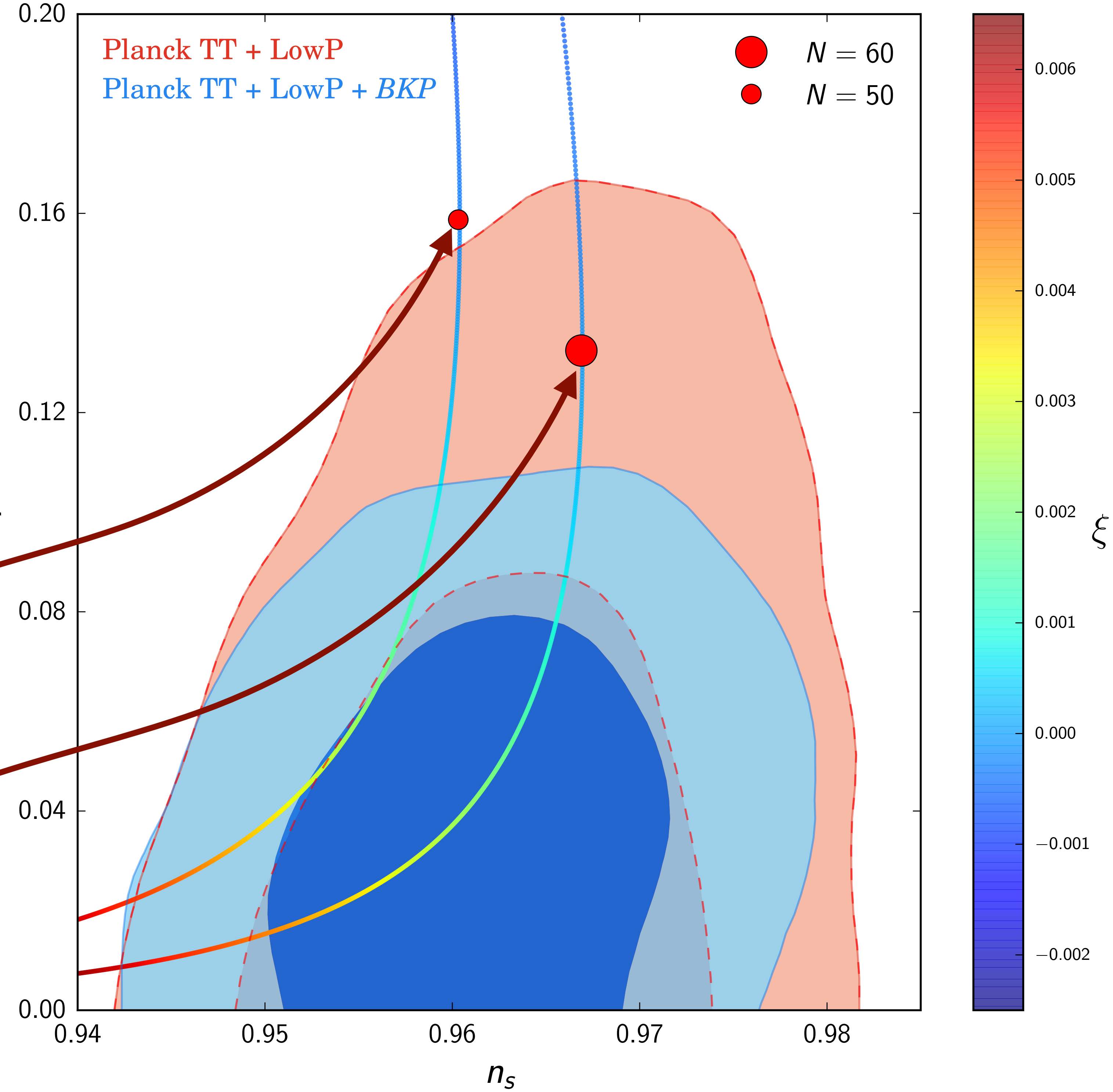
$$g^{\rm E}_{\mu\nu} = \Omega(\phi)g_{\mu\nu} \qquad \text{where} \qquad \Omega(\phi) \equiv 1+\xi\phi^2$$

We recast the action in canonical form with

$$V\left[\varphi(\phi)\right] = \frac{U(\phi)}{\Omega^2(\phi)} \longrightarrow U(\phi) = \frac{1}{2}m^2\phi^2$$

Nonminimal chaotic inflation

— L. Boubekeur, E. Giusarma, O. Mena, **HR**; Phys. Rev. **D91** (2015) 103004.



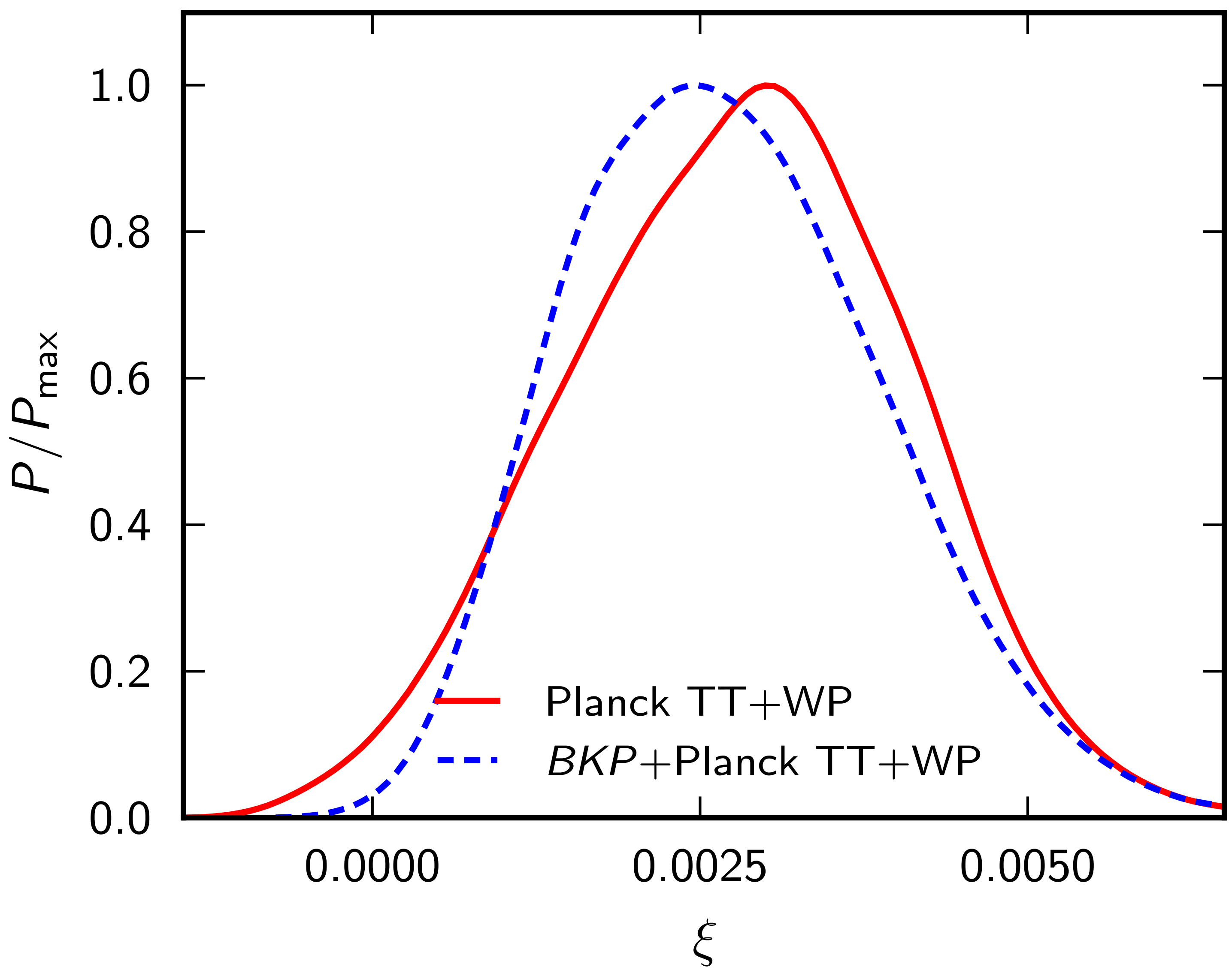
Ruled out at more than 99% C.L.

Disfavoured at 95% C.L.

Nonminimal chaotic inflation

— L. Boubekeur, E. Giusarma, O. Mena, **HR**; Phys. Rev. **D91** (2015) 103004.

One-dimensional posterior probability distributions.



- Slight preference for nonzero ξ .

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$$G_3(\phi, X)\Box\phi$$

(G-inflation)

G-inflation

— J. Ohashi and S. Tsujikawa; JCAP **1210** (2012) 035.

$$\mathcal{L}_2 = X - V(\phi) = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}m^2\phi^2$$

$$\mathcal{L}_3 = M^{-3} X \Box \phi$$

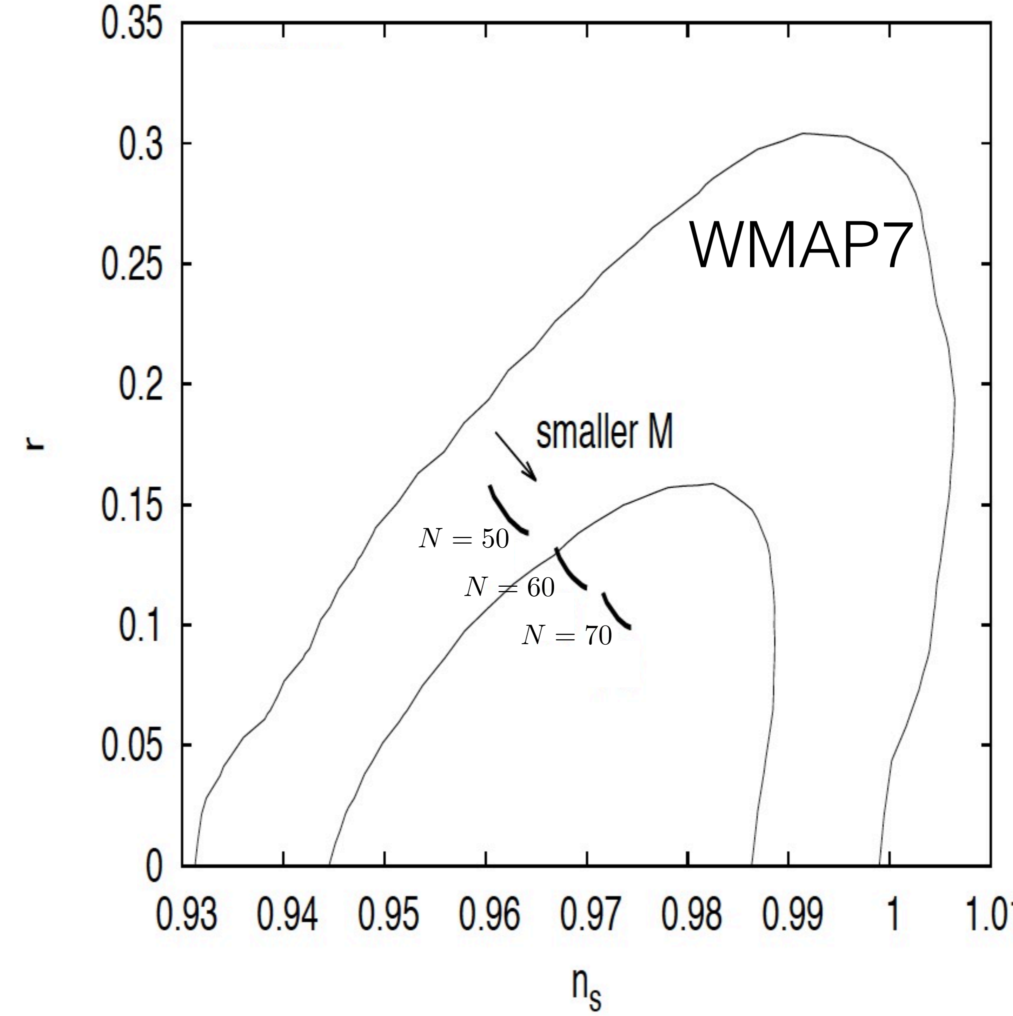
$$\mathcal{L}_4 = \frac{1}{2} M_{\rm pl}^2 \mathcal{R}$$

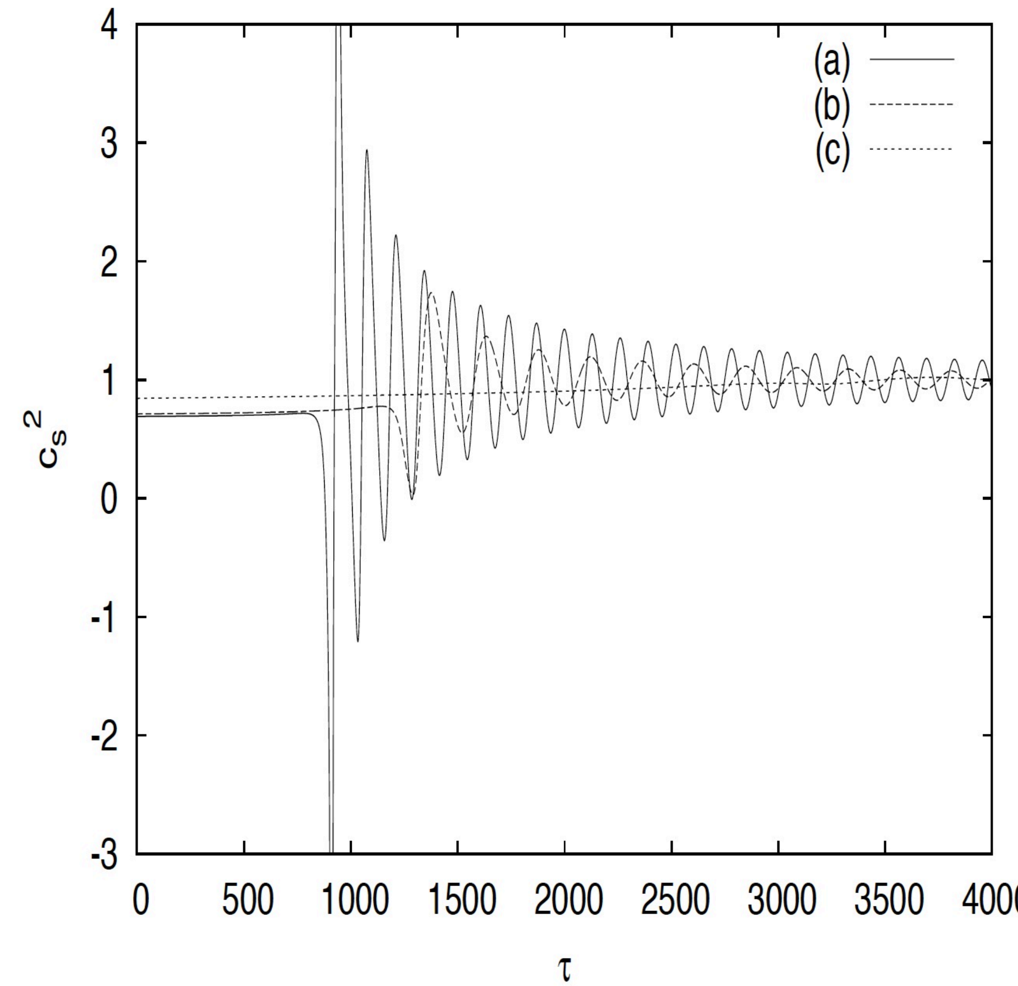
a) $M = 3 \times 10^{-4} M_{\rm pl}$

b) $M = 4.2 \times 10^{-4} M_{\rm pl}$

c) $M = 1 \times 10^{-3} M_{\rm pl}$

For $c_s^2 > 0$, $M > 4.2 \times 10^{-4} M_{\rm pl}$





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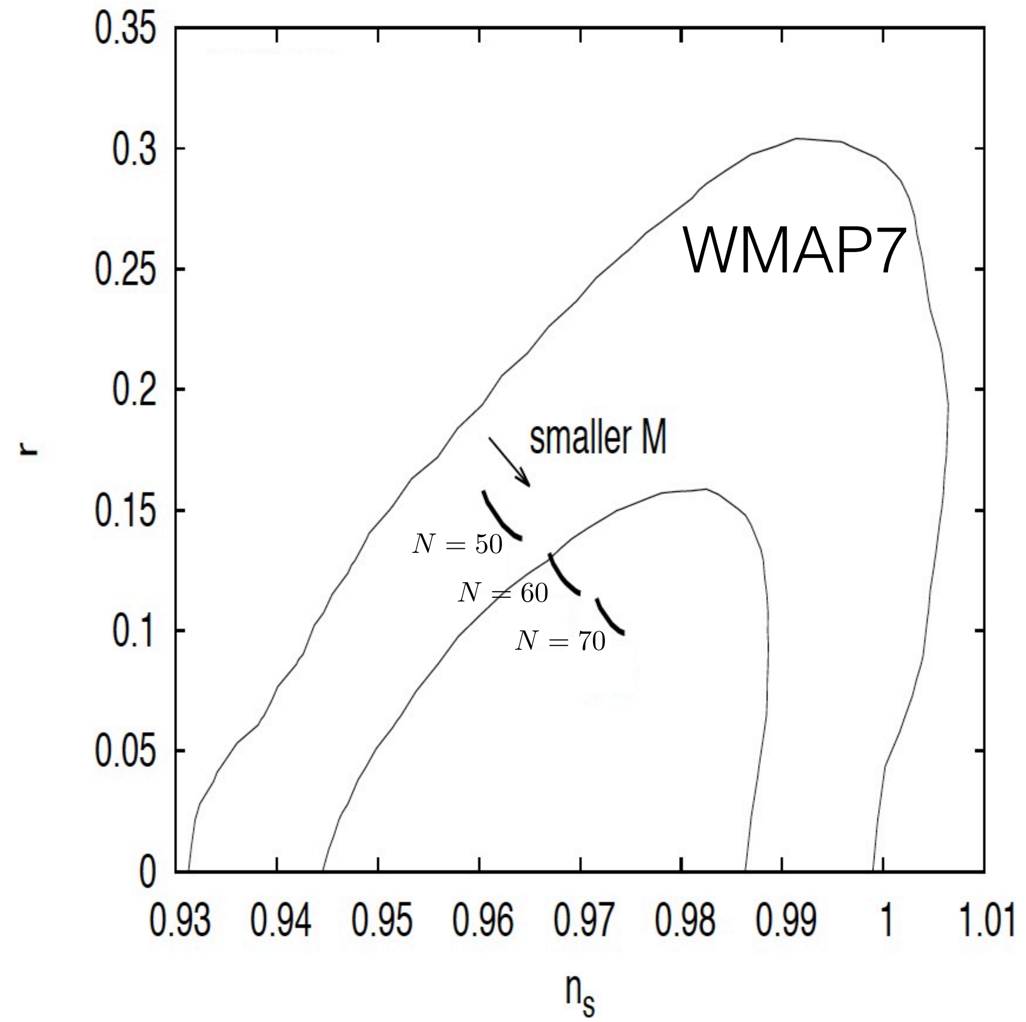
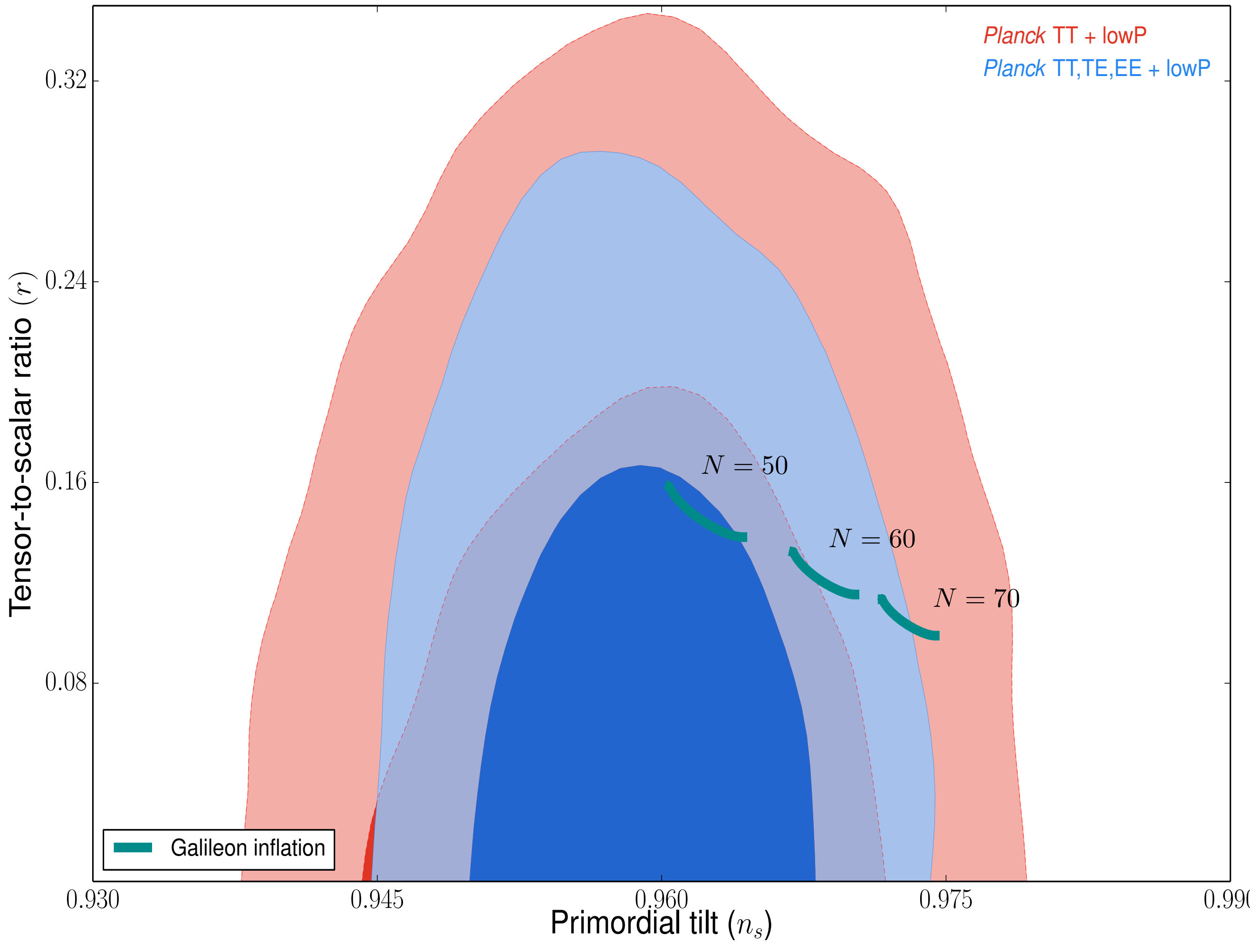


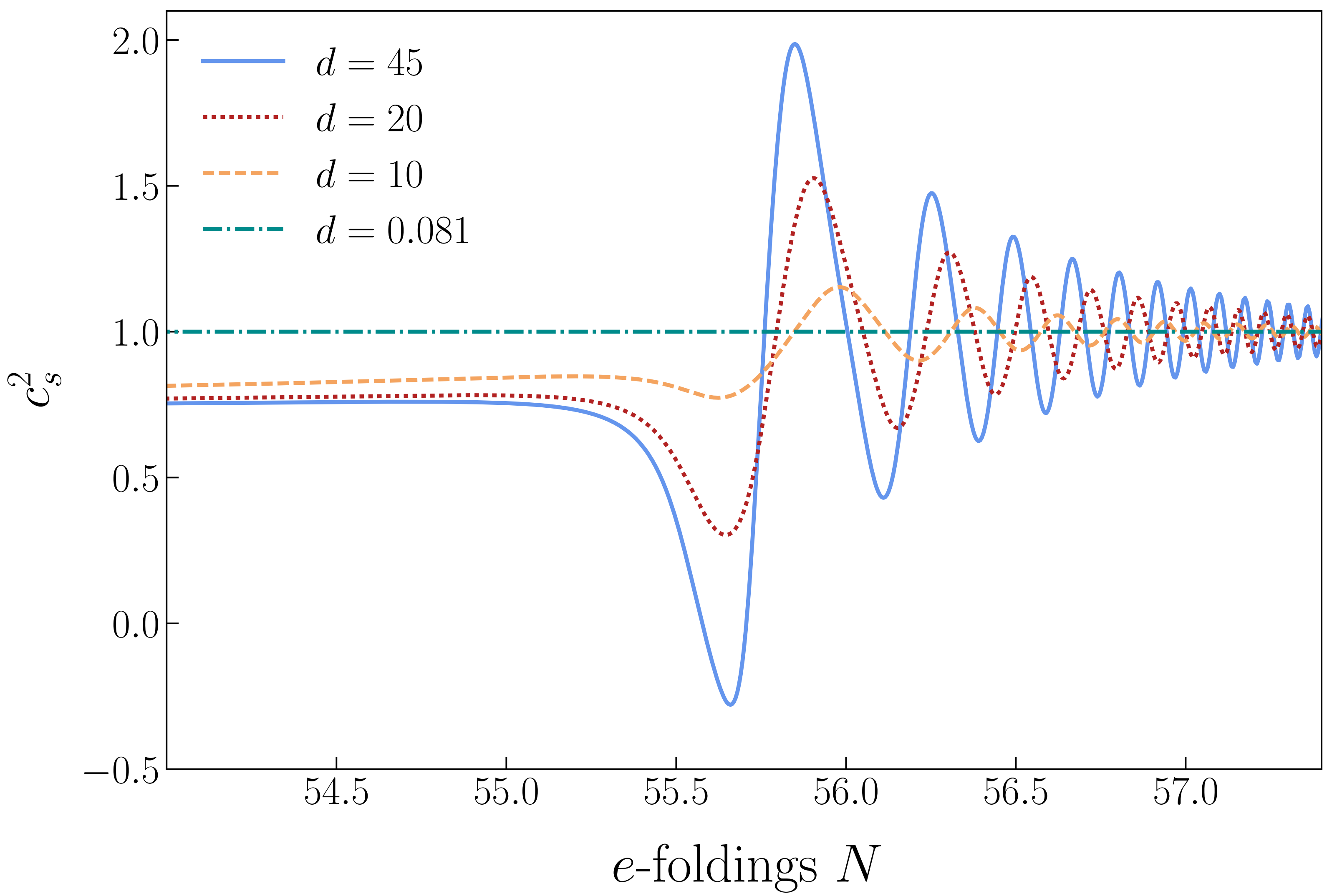


— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena; JCAP **1804** (2018) no.04, 039.

G-inflation

G_3 + *tanh* + chaotic inflation
= transient G-inflation

$$\mathcal{L}_3 = M^{-3}\left[1+\tanh\left(\frac{\phi-\phi_r}{d}\right)\right]X\Box\phi$$



$\phi_r = 13$

$M = 1.3\times10^{-4}$

No gradient instabilities for any mass scale M.

— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena;
JCAP **1804** (2018) no.04, 039.

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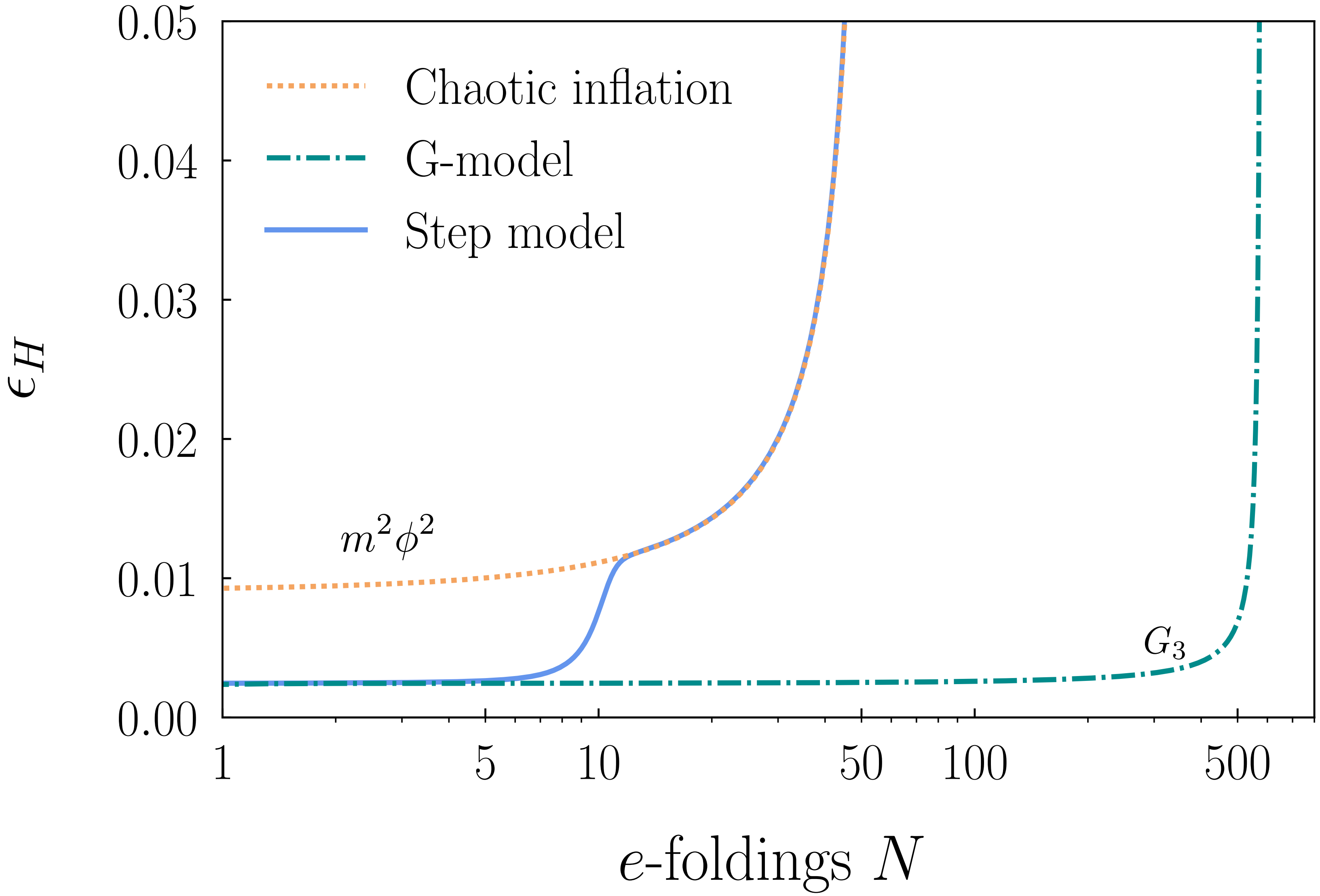
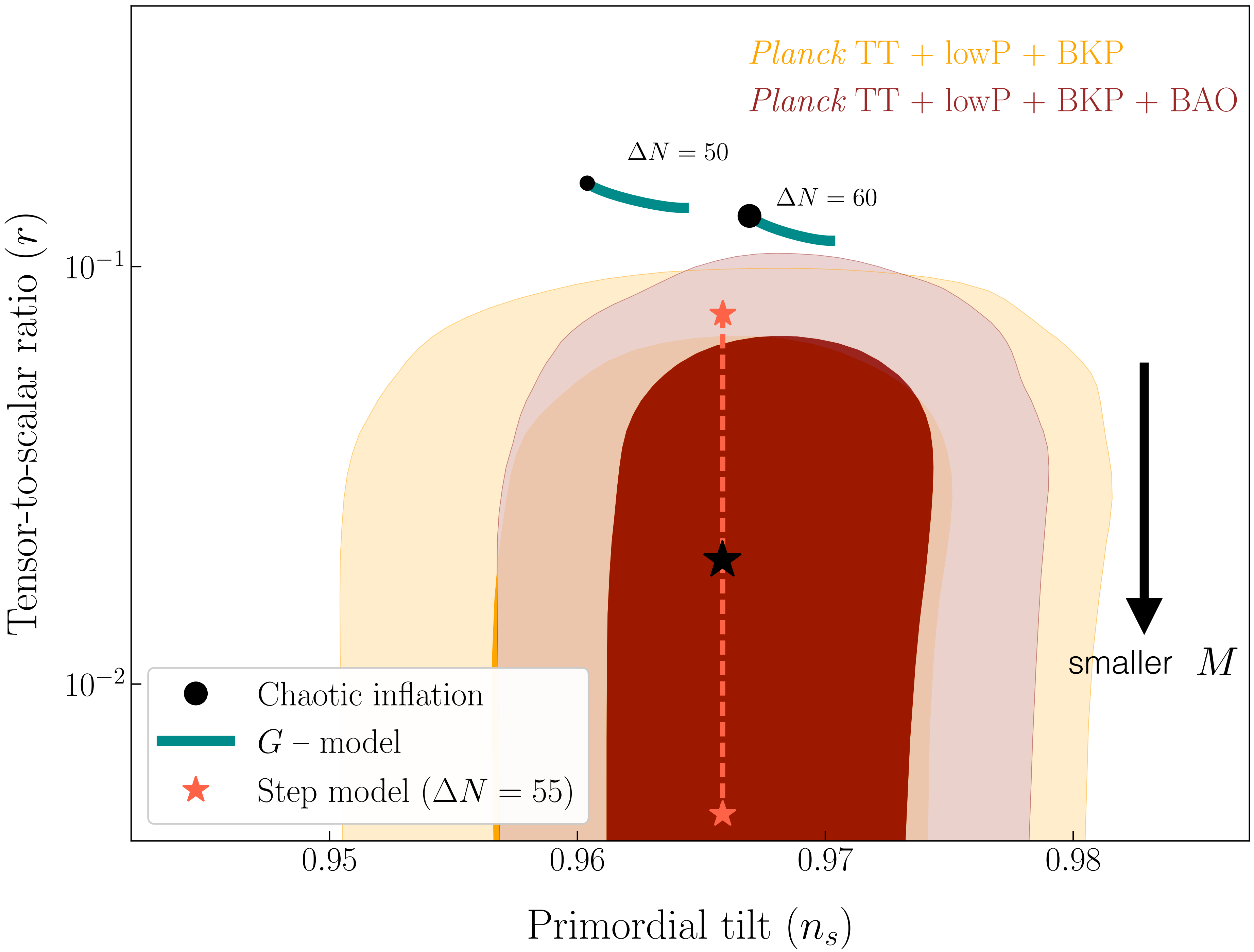
= transient G-inflation

$$\mathcal{L}_3 = M^{-3}\left[1+\tanh\left(\frac{\phi-\phi_r}{d}\right)\right]X\Box\phi$$



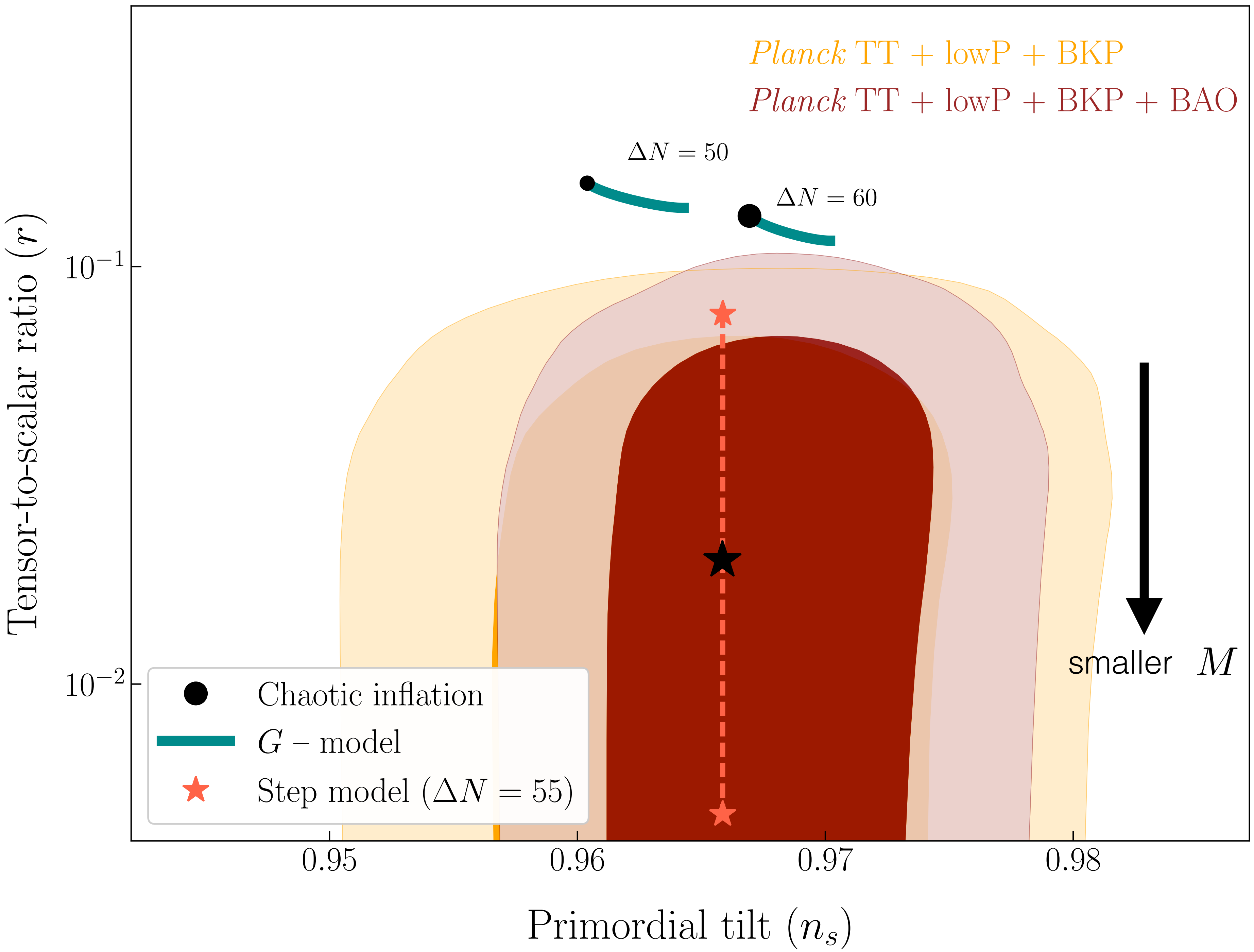
A transition between the two regimes before inflation ends.



Observables

$$n_s = 0.966$$
$$\alpha_s = -0.011$$

- n_s and α_s fixed.
- Find a set of values for d and ϕ_r.
- This places lower and upper bounds on r.



Observables

$$n_s = 0.966$$
$$\alpha_s = -0.011$$

$$\boxed{r \gtrsim 0.005}$$

- A smaller α_s would shift the line upwards because the step gets wider.
- A larger α_s would be in tension with measurements.

In this Thesis

Further parameters and parametrizations

Noncanonical models beyond General Relativity

- Future constraints

$$\alpha_s \equiv \frac{\mathrm{d}\, n_s}{\mathrm{d} \ln k} \qquad \beta_s \equiv \frac{\mathrm{d}\, \alpha_s}{\mathrm{d} \ln k}$$

- Phenomenological approaches

$$p/\rho \propto \mathcal{N}^{\alpha}$$

- Nonminimal couplings

$$\xi \phi^2 R$$

- Higher-order couplings

$$G_3(\phi, X) \Box \phi$$

- Couplings to vector fields

$$A^{\mu} \nabla_{\mu} \phi$$

In this Thesis

Noncanonical models
beyond General Relativity

(Part II)

Noncanonical models beyond General Relativity

Horndeski

— 2nd-order EoM

$f(R)$

Brans-Dicke

$\Box\phi$

$K(\phi, X)$

$f(\phi, X)$

Einstein-Hilbert

Generalized Proca

— 2nd-order EoM

Proca field:

$\frac{1}{2}M^2 A_\mu A^\mu$

$\nabla_\mu A^\mu$

$G_{\mu\nu}\nabla^\mu A^\nu$

$L^{\mu\nu\alpha\beta}\nabla_\mu A_\nu \nabla_\alpha A_\beta$

Noncanonical models beyond General Relativity

Horndeski

— 2nd-order EoM

$f(R)$

Brans-Dicke

$K(\phi, X)$

$\Box\phi$

$f(\phi, X)$

Einstein-Hilbert

Stueckelberg trick:

$A_\mu \rightarrow A_\mu + \nabla_\mu \phi$

Generalized Proca

— 2nd-order EoM

Proca field:

$\frac{1}{2} M^2 A_\mu A^\mu$

$\nabla_\mu A^\mu$

$G_{\mu\nu} \nabla^\mu A^\nu$

$L^{\mu\nu\alpha\beta} \nabla_\mu A_\nu \nabla_\alpha A_\beta$

Scalar-Vector-Tensor

— new genuine couplings between the scalar Stueckelberg field and the abelian vector field.

$f(\phi, X) L^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$

$M A^\mu \nabla_\mu \phi$

$\nabla_\mu \phi \nabla_\nu \phi F^{\mu\alpha} F^\nu_\alpha$

$\nabla_\mu \phi A_\nu \phi F^{\mu\alpha} F^\nu_\alpha$

— L. Heisenberg; JCAP **1810** (2018) no.10, 054.

Scalar-Vector-Tensor theories

(some part of) The Action:

$$\mathcal{S}_{\rm SVT} = \int d^4x\sqrt{-g}\sum_{i=2}^{6}\mathcal{L}_i$$

- $\mathcal{L}_2 = f_2(\phi, X_1, \boxed{X_2,} X_3, F, Y_1, Y_2, Y_3)$
  - $X_1 = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$
  - $\boxed{X_2 = -\frac{1}{2}A^\mu\nabla_\mu\phi}$
  - $X_3 = -\frac{1}{2}A_\mu A^\mu$
  - $F \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
  - $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$
  - $Y_1 \equiv \nabla_\mu\phi\nabla_\nu\phi F^{\mu\alpha}F^\nu_\alpha$
  - $Y_2 \equiv \nabla_\mu\phi A_\nu F^{\mu\alpha}F^\nu_\alpha$
  - $Y_3 \equiv A_\mu A_\nu F^{\mu\alpha}F^\nu_\alpha$
- $\mathcal{L}_{3,4,5,6} \propto$ (highly-) nontrivial couplings between the above terms and
  - R
  - $G_{\mu\nu}$
  - $L^{\mu\nu\alpha\beta}$

…and other combinations.

In this Thesis

Noncanonical models
beyond General Relativity

- Couplings to vector fields

$$A^\mu \nabla_\mu \phi$$

• Couplings to vector fields

— L. Heisenberg, **HR**, S. Tsujikawa; PRD **99** (2019) no.2, 023505.

- Let's consider the simplest SVT Lagrangian!

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + F + X_1 - V(\phi) + \beta_m M X_2 + \beta_A M^2 X_3\right]$$

- Apart from the 3 modified Einstein equations, there's a fourth EoM:

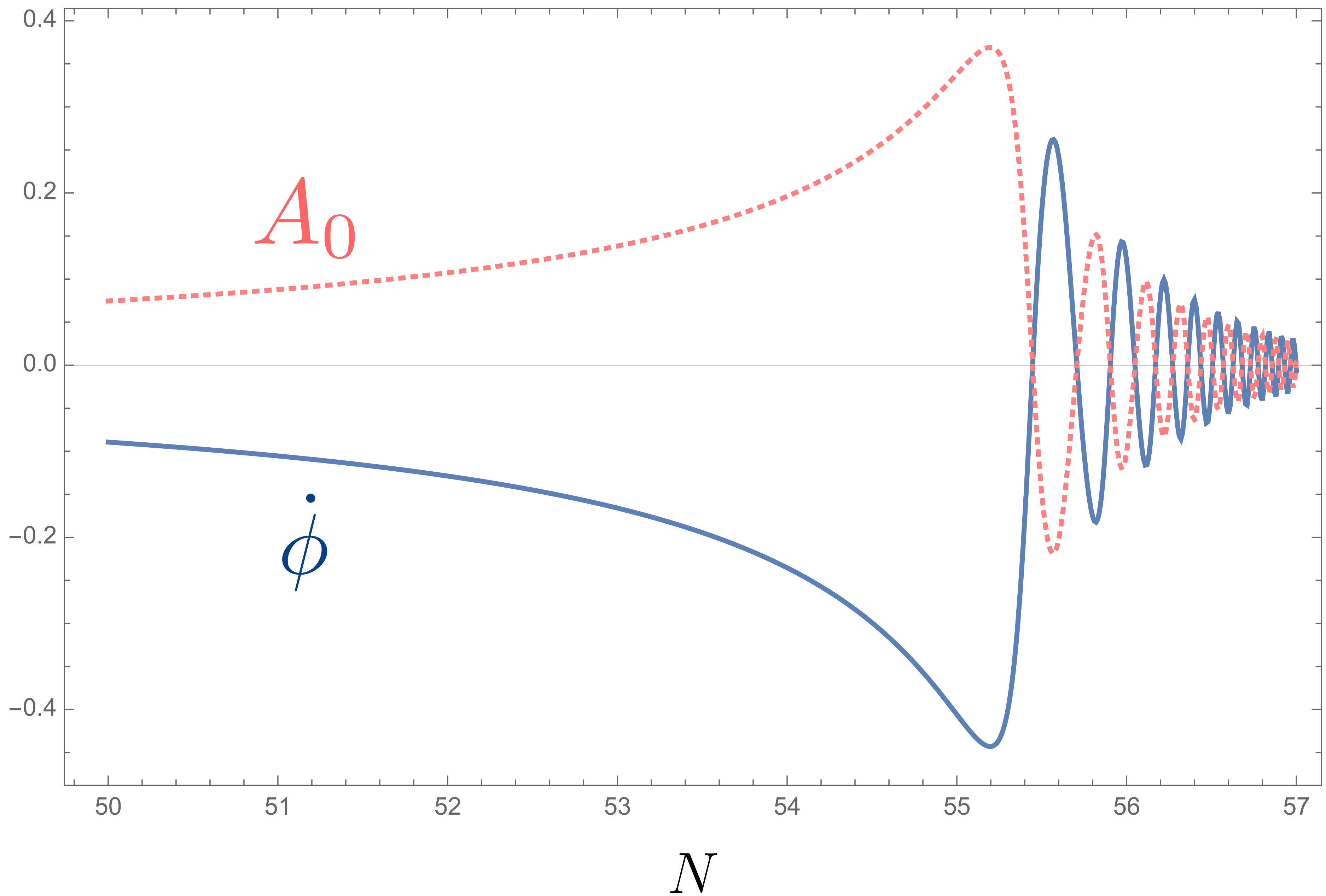
$$A_0 = -\frac{\beta_m}{2\beta_A M}\dot{\phi}$$

Cheat sheet

- $X_1 = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$
- $\boxed{X_2 = -\frac{1}{2}A^\mu\nabla_\mu\phi}$
- $X_3 = -\frac{1}{2}A_\mu A^\mu$
- $F \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
- $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$

A particular model for **inflation**

— L. Heisenberg, **HR**, S. Tsujikawa; PRD **99** (2019) no.2, 023505.

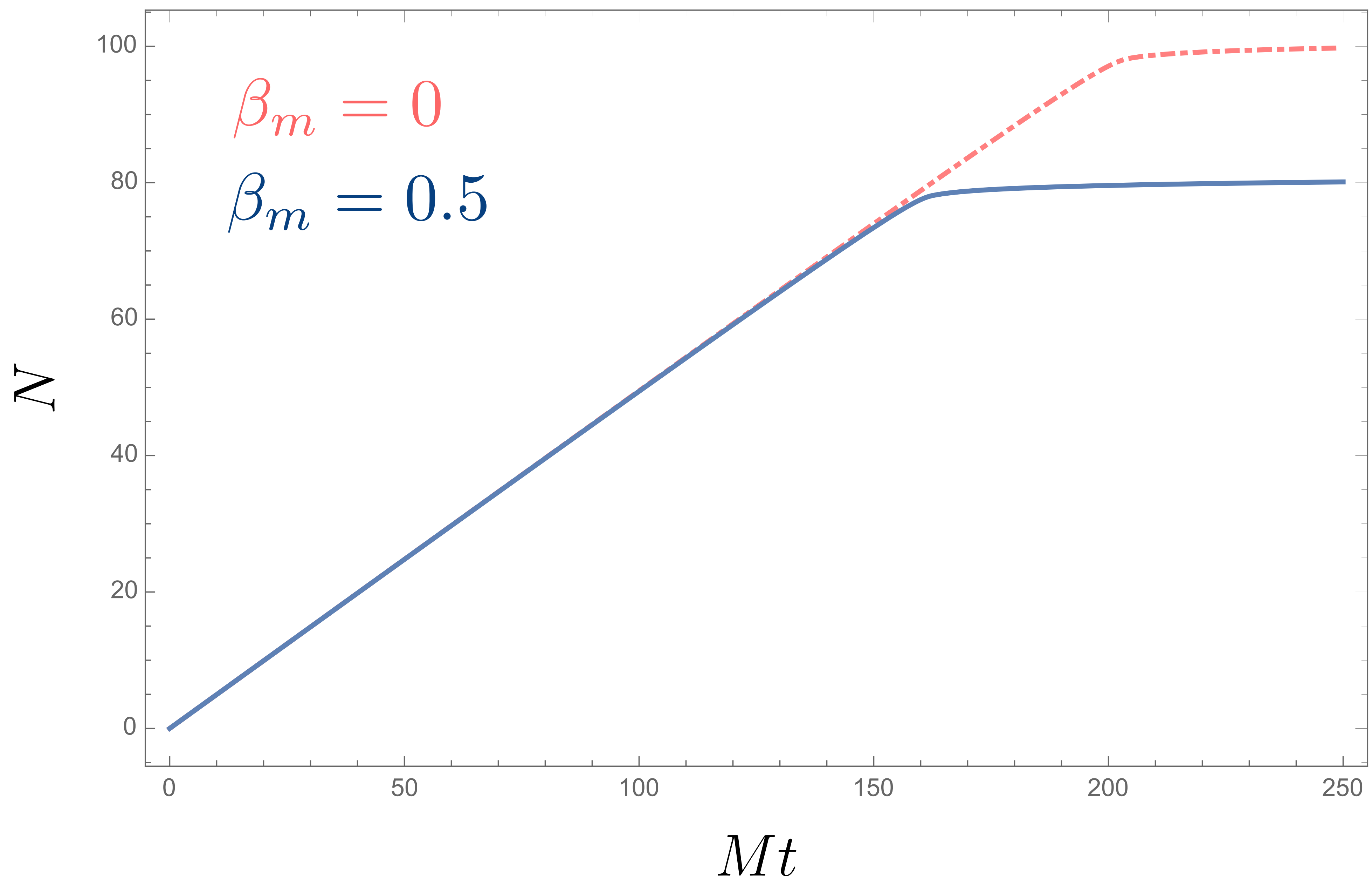


*Computed for Starobinsky inflation: $V(\phi) = \frac{3}{4}\alpha_c M^2 M_{\rm pl}^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3\alpha_c}}\frac{\phi}{M_{\rm pl}}\right)\right]^2$ with $\alpha_c = \frac{\sqrt{6}}{3}$

- A. A. Starobinsky; Phys. Lett. B **91** (1980) 99.
- R. Kallosh, *et al.*; JHEP **1311** (2013) 198.

A particular model for **inflation**

— L. Heisenberg, **HR**, S. Tsujikawa; PRD **99** (2019) no.2, 023505.



- The non-vanishing coupling leads to a smaller amount of inflation due to the additional evolution of A_0.

A particular model for **inflation**

— L. Heisenberg, **HR**, S. Tsujikawa; PRD **99** (2019) no.2, 023505.

- Given that $A_0 \propto \dot{\phi}$, we can substitute it in the EoM:

  - $$3M_{\rm pl}^2 H^2 = \frac{1}{2}\beta\dot{\phi}^2 + V\,,$$
  - $$-2M_{\rm pl}^2 \dot{H} = \beta\dot{\phi}^2\,,$$
  - $$\ddot{\phi} + 3H\dot{\phi} + \frac{V_{,\phi}}{\beta} = 0$$

where

$$\boxed{\beta \equiv 1 - \frac{\beta_m^2}{4\beta_A}}$$

- Then the background dynamics is equivalent to an effective single-field dynamics driven by a rescaled field $d\varphi = \sqrt{\beta}d\phi$.

A particular model for **inflation**

Perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; PRD **99** (2019) no.2, 023505.

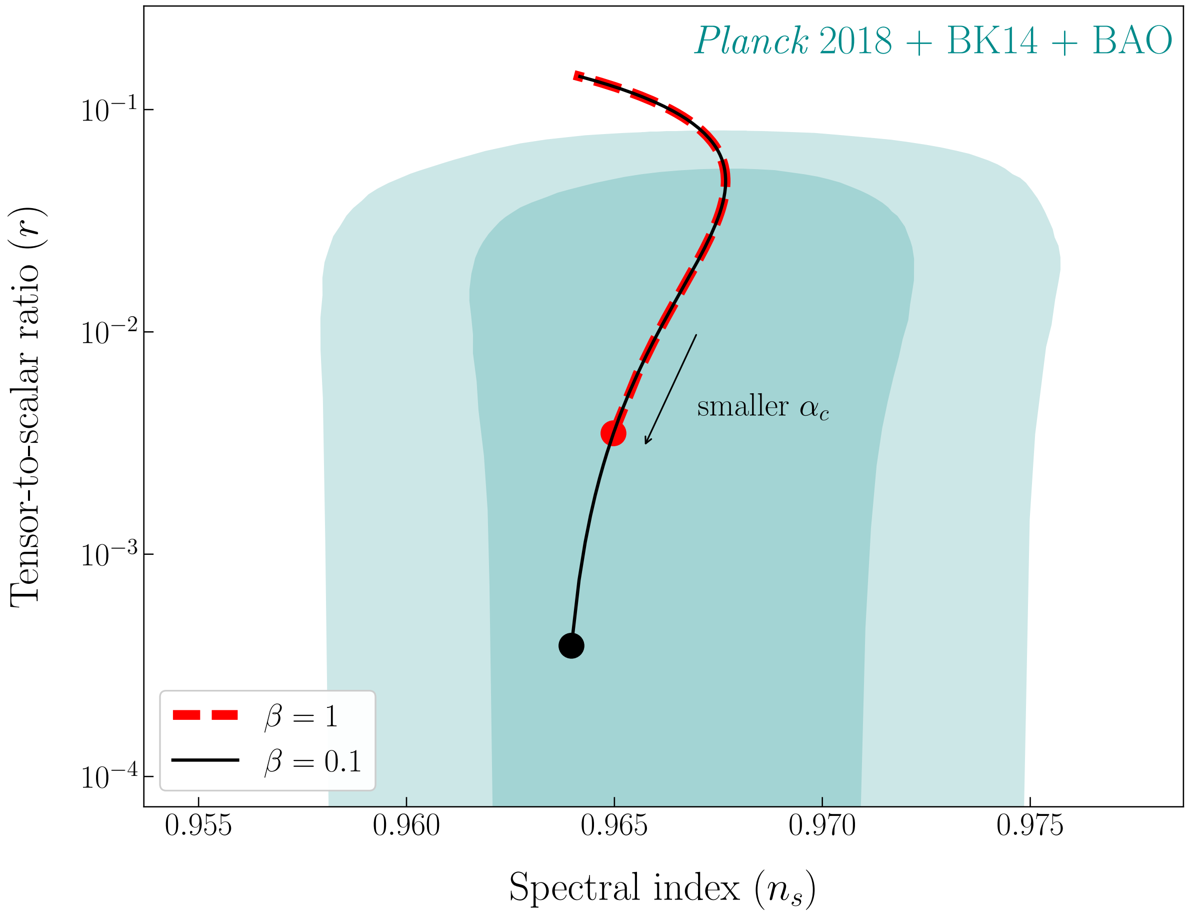
- We confronted different inflaton potentials with CMB data:

■ $$V(\phi) = \frac{3}{4}\alpha_c M^2 M_{\rm pl}^2 \left[1 - \exp\left(-\sqrt{\frac{2}{3\alpha_c}}\frac{\phi}{M_{\rm pl}}\right)\right]^2$$ α-attractors

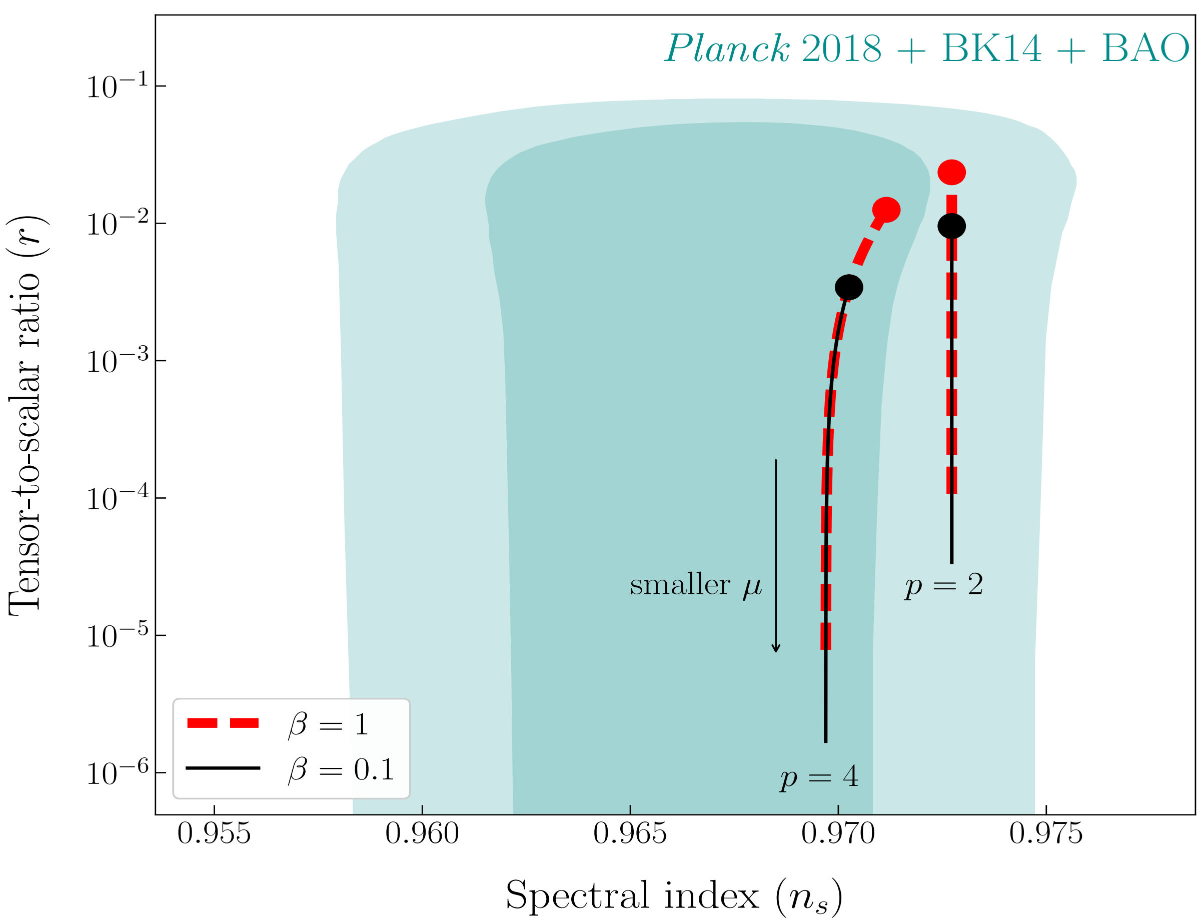
— R. Kallosh, *et al.*; JHEP **1311** (2013) 198.

■ $$V(\phi) = M^2 M_{\rm pl}^2 \left[1 - \left(\frac{\mu}{\phi}\right)^p + \cdots\right]$$ Brane inflation

- J. Garcia-Bellido, *et al.*; JHEP **0201** (2002) 036.
- G. R. Dvali, *et al.*; hep-th/0105203.
- S. Kachru, *et al.*; JCAP **0310** (2003) 013.



α-attractors



Brane inflation

The coupling β can lead to the suppression of

$$r = 16\epsilon$$

compared to the canonical case.

For smaller β , the total field velocity gets larger and hence the inflaton needs to start from a flatter region to get an enough number of *e*-folds.

— L. Heisenberg, **HR**, S. Tsujikawa; PRD **99** (2019) no.2, 023505.

In this Thesis

Further parameters and parametrizations

Noncanonical models beyond General Relativity

- Future constraints

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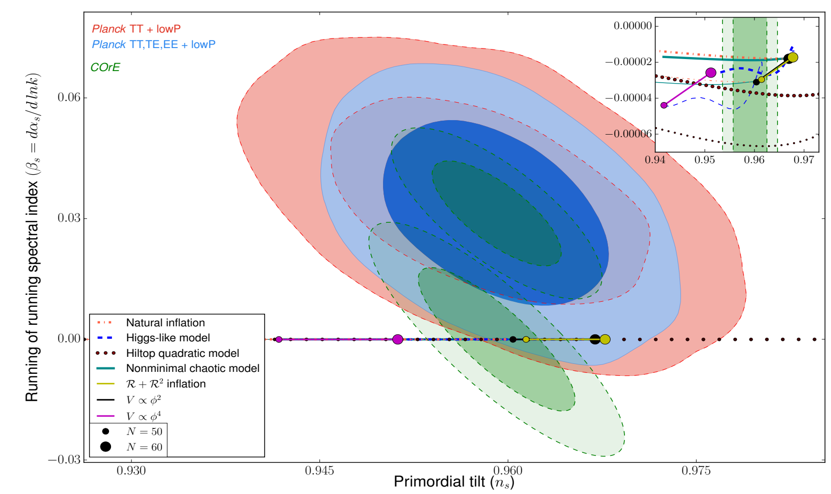
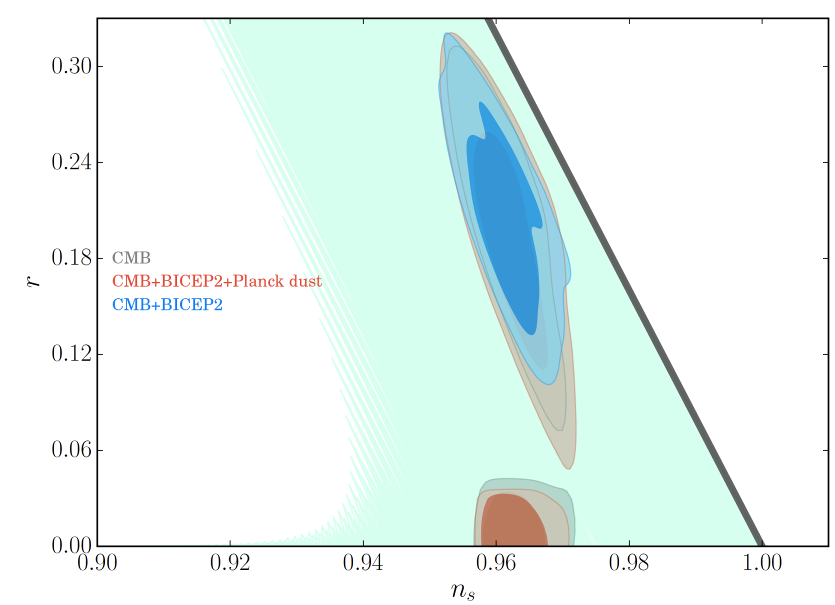
Summary

- ★ It is possible that future observations won't help us in disentangling between models using the parameter space given by n_s and r . Possibly not even with α_s. However, β_s is a potential good discriminator and could rule-out the full class of canonical inflation.



- ★ Predictions on n_s or r can be obtained by constraining the Mukhanov phenomenological parameters without any reference to a specific potential function.



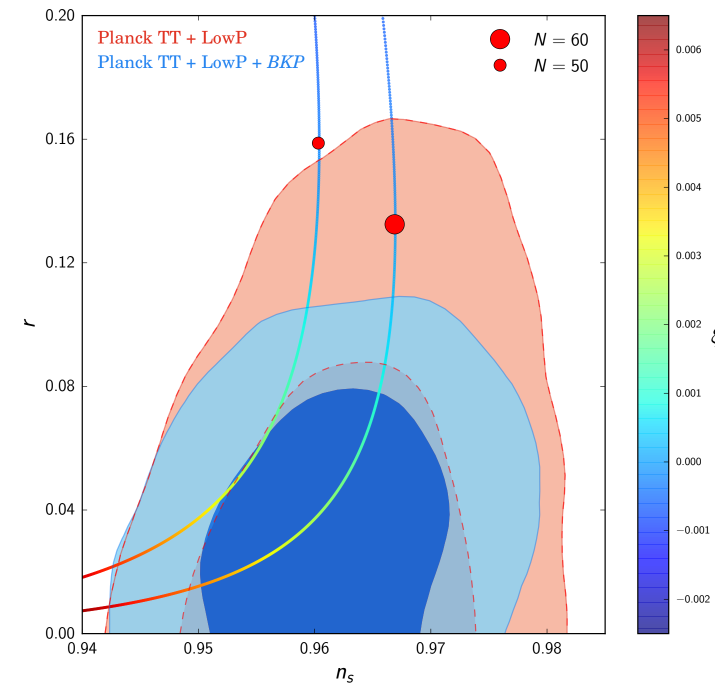
Summary II

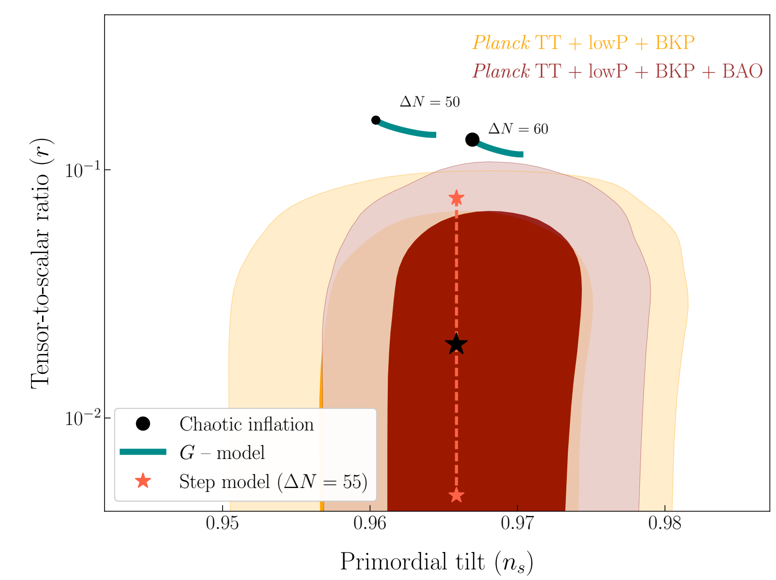
- In the context of chaotic inflation, a nonminimal coupling $\xi\phi^2 R$ is favored at the 2σ level.

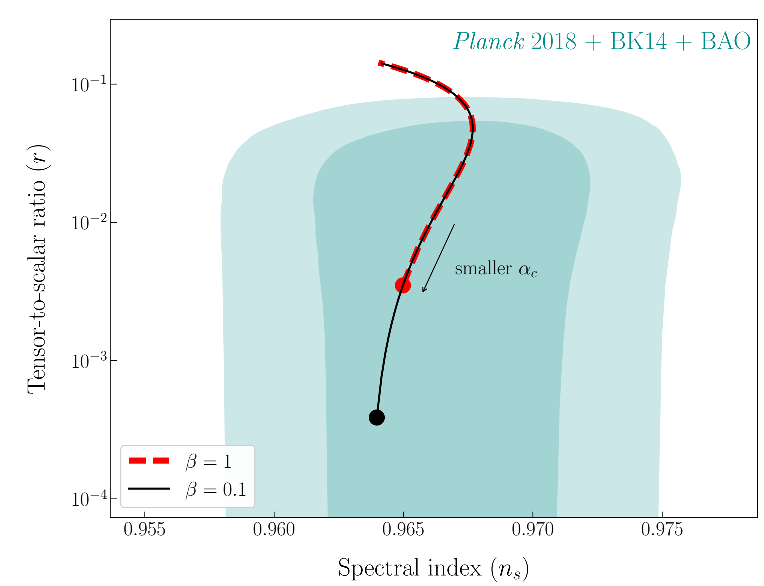
- G-inflation, the simplest scalar-tensor model in the framework of Horndeski gravity, is back to life with its full and rich phenomenology.

- Scalar-Vector-Tensor theories represent a new and very promising framework to study several cosmological and astrophysical phenomena.

  - The simplest addition of a vector field reduces the tensor-to-scalar ratio.







Back-up slides

Generalized Slow-Roll for Horndeski gravity

Quadratic actions in Horndeski gravity

- T. Kobayashi, *et al.*; Prog. Theor. Phys. **126** (2011) 511.
- H. Motohashi and W. Hu; PRD **96** (2017) no.2, 023502.

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \, \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$S_\gamma^{(2)} = \sum_{\lambda=+,\times} \int \mathrm{d}^4x \, \frac{a^3 b_t}{4c_t^2} \left(\dot{\gamma}_\lambda^2 - \frac{c_t^2 k^2}{a^2} \gamma_\lambda^2 \right)$$

In canonical inflation:

$$b_s = b_t = c_s = c_t = 1$$

Scalar parameters:

$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

- $$b_s = \frac{c_s^2}{\epsilon_H}\left[\frac{w_1(4w_1w_3 + 9w_2^2)}{3w_2^2}\right]$$

- $$c_s^2 = \frac{3(2w_1^2w_2H - w_2^2w_4 + 4w_1\dot{w}_1w_2 - 2w_1^2\dot{w}_2)}{w_1(4w_1w_3 + 9w_2^2)}$$

Tensor parameters:

- $$b_t = w_1 c_t^2$$
- $$c_t^2 = \frac{w_4}{w_1}$$

$$w_1 = M_{\rm pl}^2 - 2\left(3G_4 + 2HG_5\dot{\phi}\right) + 2G_{5,\phi}X\,,$$

$$w_2 = 2M_{\rm pl}^2H - 2G_3\dot{\phi} - 2\left(30HG_4 - 5G_{4,\phi}\dot{\phi} + 14H^2G_5\dot{\phi}\right) + 28HG_{5,\phi}X\,,$$

$$w_3 = -9M_{\rm pl}^2H^2 + 3\left(X + 12HG_3\dot{\phi}\right) + 6\left(135H^2G_4 - 2G_{3,\phi}X - 45HG_{4,\phi}\dot{\phi} + 56H^3G_5\dot{\phi}\right) - 504H^2G_{5,\phi}X\,,$$

$$w_4 = M_{\rm pl}^2 + 2\left(G_4 - 2G_5\ddot{\phi}\right) - 2G_{5,\phi}X\,.$$

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

$$\frac{\mathrm{d}^2 v}{\mathrm{d}\tau^2} + \left(c_s^2 k^2 - \frac{1}{z} \frac{\mathrm{d}^2 z}{\mathrm{d}\tau^2} \right) v = 0$$

* Mukhanov - Sasaki equation

- $v = z\zeta$
- $z = a\sqrt{\dfrac{2b_s\epsilon_H}{c_s^2}}$

- Assume slow-roll approximation.
- Solve numerically.
- Use GSR techniques.

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2} \left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2} \zeta^2 \right)$$

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Generalized Slow-Roll

- E. D. Stewart; PRD **65** (2002) 103508.
- C. Dvorkin and W. Hu; PRD **81** (2010) 023518.
- W. Hu; PRD **84** (2011) 027303.
- W. Hu; PRD **89** (2014) no.12, 123503.
- H. Motohashi and W. Hu; PRD **92** (2015) no.4, 043501.
- H. Motohashi and W. Hu; PRD **96** (2017) no.2, 023502.

… and others.

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

de Sitter background

deviations from dS

- $y \equiv \sqrt{2c_s k v}$
- $x \equiv k s_s$
- $s_s \equiv \int c_s \mathrm{d}\tau$

- $f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2} \frac{aH s_s}{c_s}}$
- $\Delta_\zeta^2(k) = \lim_{x\to 0} \left| \frac{xy}{f} \right|^2$

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

3. Apply Green function techniques (GSR).

$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

- $$G = -2\ln f + \frac{2}{3}\left(\ln f\right)'$$ Source function
- $$W(u) = \frac{3\sin(2u)}{2u^3} - \frac{3\cos(2u)}{u^2} - \frac{3\sin(2u)}{2u}$$

Generalized slow-roll recipe:

1. Write down Mukhanov-Sasaki equation.
2. Isolate deviations from de Sitter background.

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(1 - \frac{2}{x^2}\right) y = \frac{f'' - 3f'}{f} \frac{y}{x^2}$$

3. Apply Green function techniques (GSR).

$$\ln \Delta^{2(1)} = G\left(\ln x_m\right) + \int_{x_m}^{\infty} \mathrm{d}(\ln x) W(kx) G'\left(\ln x\right)$$

4. Taylor expand GSR formula and write down analytic equations (OSR).

Optimized Slow-Roll

If $\Delta N > 1$ we can Taylor expand the GSR formula around some epoch x_f:

$$\ln\,\Delta^2 \simeq G\left(\ln\,x_f\right) + \sum_{p=1}^{\infty} q_p\left(\ln\,x_f\right) G^{(p)}\left(\ln\,x_f\right)$$

- $q_1(\ln\,x_f) = \ln\,x_1 - \ln\,x_f$

$$\boxed{\ln\,x_1 \equiv \frac{7}{3} - \ln\,2 - \gamma_{\mathrm{E}}}$$

- $q_p(\ln x_f) = -\dfrac{1}{p!}\displaystyle\int_{x_m}^{\infty} W'(x)\left(\ln\,\frac{x}{x_f}\right)^p$

Optimized SR for Horndeski (leading order):

— H. Motohashi and W. Hu; PRD **96** (2017) no.2, 023502.

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p (\ln x_f) G^{(p)}(\ln x_f)$$

$$G = -2\ln f + \frac{2}{3}(\ln f)'$$

$$f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2}} \frac{aHs_s}{c_s}$$

$$\ln \Delta_\zeta^2 \approx \ln\left(\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 - \frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1}\bigg|_{x=x_1}$$

$$n_s - 1 \approx -4\epsilon_H - 2\delta_1 - \sigma_{s1} - \xi_{s1} - \frac{2}{3}\delta_2 - \frac{7}{3}\sigma_{s2} - \frac{1}{3}\xi_{s2}\bigg|_{x=x_1}$$

$$\alpha_s \approx -2\delta_2 - \sigma_{s2} - \xi_{s2} - \frac{2}{3}\delta_3 - \frac{7}{3}\sigma_{s3} - \frac{1}{3}\xi_{s3} - 8\epsilon_H^2 - 10\epsilon_H\delta_1 + 2\delta_1^2\bigg|_{x=x_1}$$

Scalars

$$\ln \Delta_\gamma^2 \approx \ln\left(\frac{H^2}{2\pi^2 b_t c_t}\right) - \frac{8}{3}\epsilon_H - \frac{7}{3}\sigma_{t1} - \frac{1}{3}\xi_{t1}\bigg|_{x=x_1}$$

$$n_t \approx -2\epsilon_H - \sigma_{t1} - \xi_{t1} - \frac{7}{3}\sigma_{t2} - \frac{1}{3}\xi_{t2}\bigg|_{x=x_1}$$

$$\alpha_t \approx -\sigma_{t2} - \xi_{t2} - \frac{7}{3}\sigma_{t3} - \frac{1}{3}\xi_{t3} - 4\epsilon_H^2 - 4\epsilon_H\delta_1\bigg|_{x=x_1}$$

Tensors

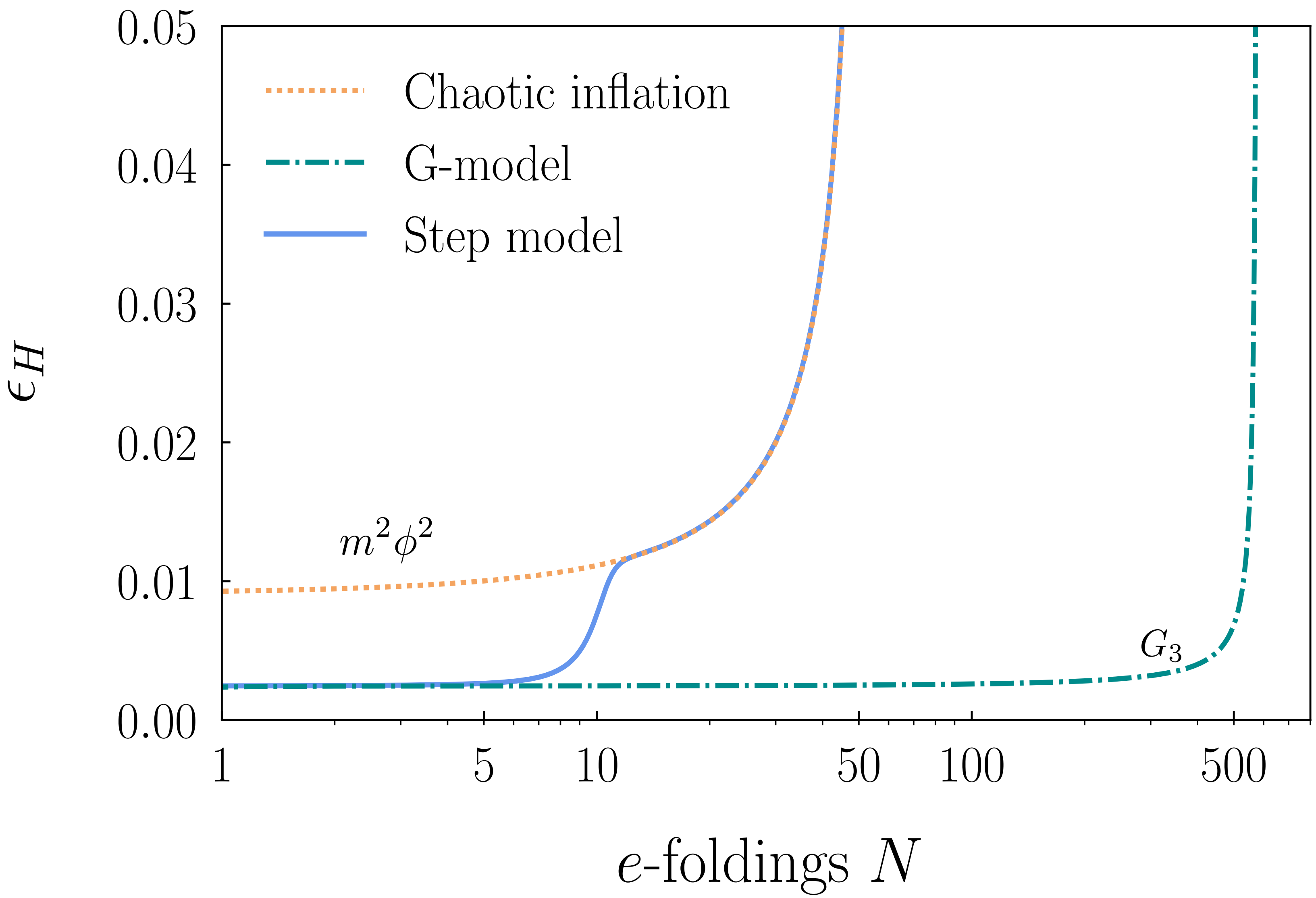
$$\ln x_1 \equiv \frac{7}{3} - \ln 2 - \gamma_{\mathrm{E}}$$

— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena;
JCAP **1804** (2018) no.04, 039.

G-inflation

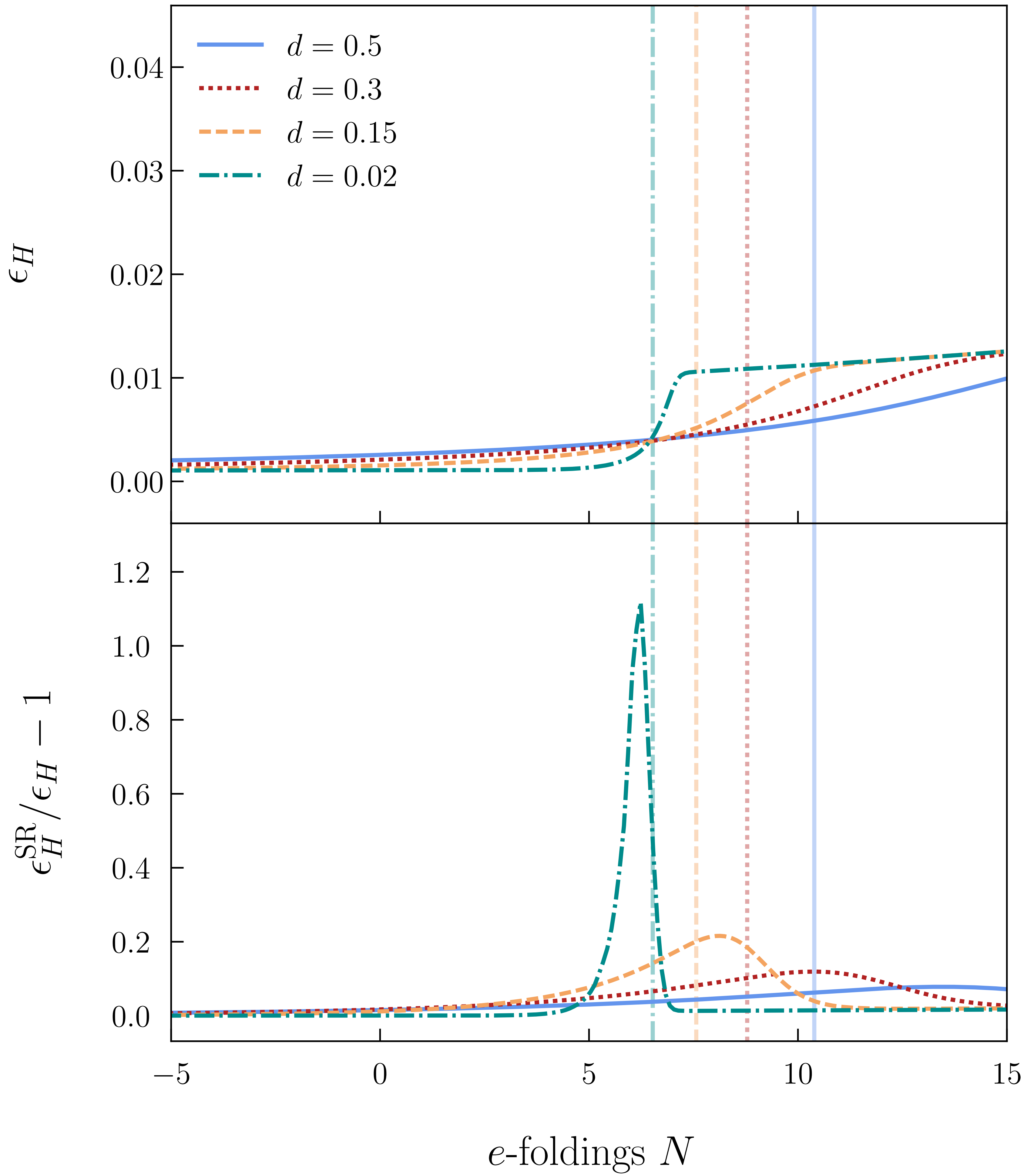
G_3 + *tanh* + chaotic inflation
= transient G-inflation

$$\mathcal{L}_3 = M^{-3}\left[1+\tanh\left(\frac{\phi-\phi_r}{d}\right)\right]X\Box\phi$$



Step size ~ 4 *e*-folds

Ideal for OSR



Slow - roll violation:

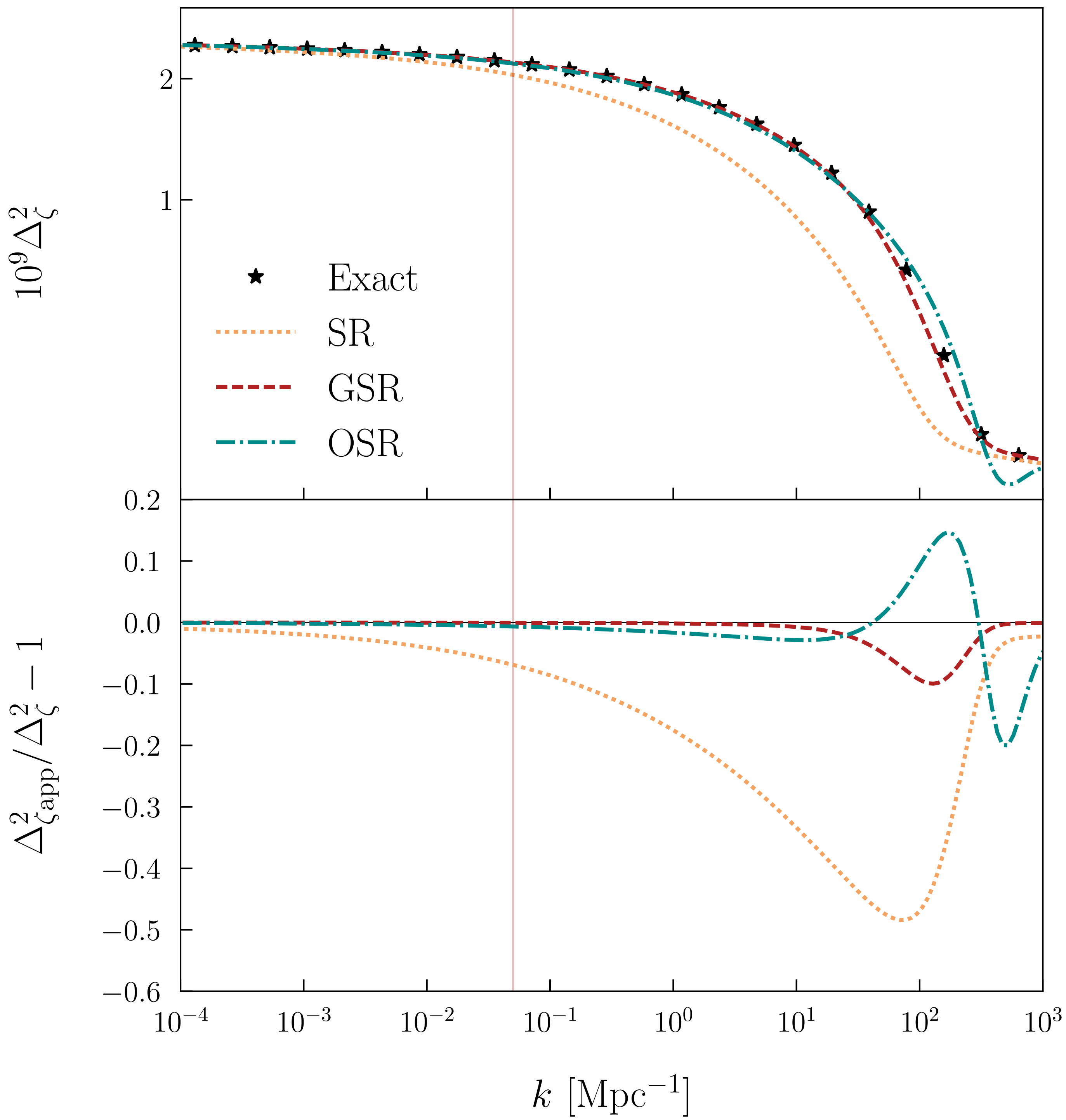
$$\epsilon_H = -\frac{\dot{H}}{H^2}$$

- $N = 0$: CMB scales.
- Vertical lines: where the transition occurs.
- SR violation is maximal around the transition.

— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena;
JCAP **1804** (2018) no.04, 039.

G-inflation

G_3 + *tanh* + chaotic inflation
= transient G-inflation



$$d = 0.086$$

$$M = 1.3 \times 10^{-4}$$

SR ✗

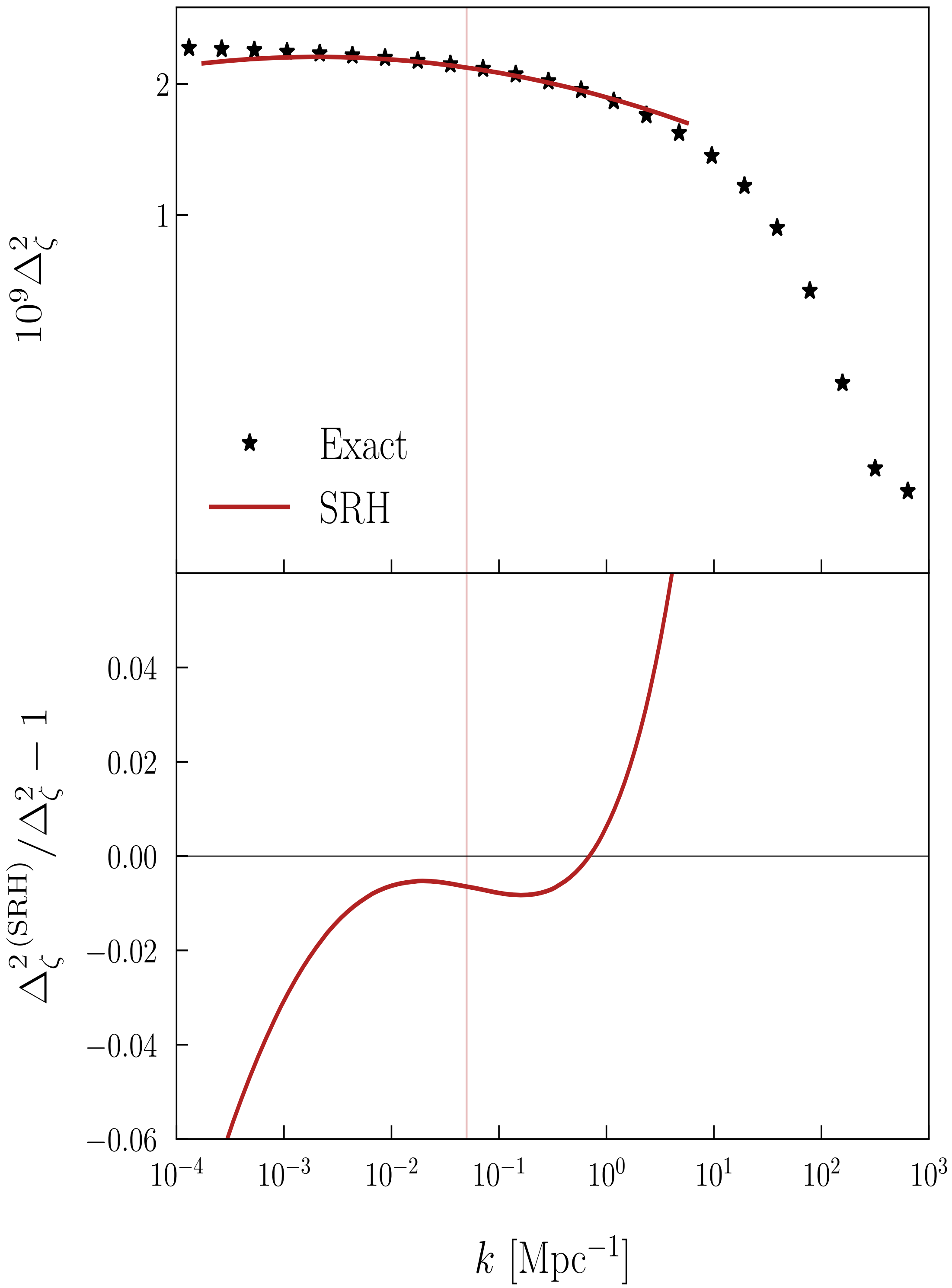
GSR ✓

OSR ✓

— **HR**, S. Passaglia, H. Motohashi, W. Hu and O. Mena;
JCAP **1804** (2018) no.04, 039.

G-inflation

G_3 + *tanh* + chaotic inflation = transient G-inflation



Slow - roll hierarchy formula:

$$\Delta_\zeta^{2(\mathrm{SRH})}(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1 + \frac{1}{2}\alpha_s \ln(k/k_*)}$$

- Using OSR parameters

Deviations of less than 1%

Optimized Slow-Roll

OSR

If $\Delta N > 1$ we can Taylor expand the GSR formula around some epoch x_f:

- $$\ln\,\Delta^2 \simeq G\left(\ln\,x_f\right) + \sum_{p=1}^{\infty} q_p\left(\ln\,x_f\right) G^{(p)}\left(\ln\,x_f\right)$$

- $$\frac{\mathrm{d}\,\ln\,\Delta^2}{\mathrm{d}\,\ln\,k} \simeq -G'\left(\ln\,x_f\right) - \sum_{p=1}^{\infty} q_p\left(\ln\,x_f\right) G^{(p+1)}\left(\ln\,x_f\right)$$

- $$\alpha \simeq G''\left(\ln\,x_f\right) + \sum_{p=1}^{\infty} q_p\left(\ln\,x_f\right) G^{(p+2)}\left(\ln\,x_f\right)$$

- $$\frac{\mathrm{d}G^{(p)}(\ln\,x)}{\mathrm{d}\,\ln\,k} = -G^{(p+1)}(\ln\,x)$$

OSR

If $\Delta N > 1$ we can Taylor expand the GSR formula around some epoch x_f:

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f)\, G^{(p)}(\ln x_f)$$

$$\boxed{\ln x_1 \equiv \frac{7}{3} - \ln 2 - \gamma_{\mathrm{E}}}$$

- $q_1(\ln x_f) = \ln x_1 - \ln x_f$
- $q_p(\ln x_f) = -\frac{1}{p!} \int_{x_m}^{\infty} W'(x) \left(\ln \frac{x}{x_f} \right)^p$

$$\lim_{p\to\infty} \frac{q_p}{q_{p-1}} = -\frac{1}{2} \xrightarrow{\text{convergence criterion}} \lim_{p\to\infty} \left| \frac{G^{(p+1)}}{G^{(p)}} \right| < 2$$

Leading order of standard SR ($\ln x_f = 0$):

- $$\ln \Delta^2 \simeq G(0)$$
- $$\frac{\mathrm{d} \ln \Delta^2}{\mathrm{d} \ln k} \simeq -G'(0)$$
- $$\alpha \simeq G''(0)$$

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p (\ln x_f) G^{(p)} (\ln x_f)$$

$$q_1(\ln x_f) = \ln x_1 - \ln x_f$$

$$G = -2\ln f + \frac{2}{3} (\ln f)'$$

Leading order of standard SR ($\ln\, x_f = 0$):

- $\ln\, \Delta^2 \simeq G(0)$
- $\dfrac{\mathrm{d}\ \ln\ \Delta^2}{\mathrm{d}\ \ln\ k} \simeq -G'(0)$
- $\alpha \simeq G''(0)$

NLO SR correction ($p = 1$):

- $q_1(0) = \ln\, x_1 = 1.06$
- $G'(0) \sim (\Delta N)^{-1}$

If $\Delta N \sim 3$,
the correction is ~ 0.35 .

$$\ln\, \Delta^2 \simeq G\left(\ln\, x_f\right) + \sum_{p=1}^{\infty} q_p\left(\ln\, x_f\right) G^{(p)}\left(\ln\, x_f\right)$$

$$q_1(\ln\, x_f) = \ln\, x_1 - \ln\, x_f$$

$$G = -2\ln\, f + \frac{2}{3}\left(\ln\, f\right)'$$

Leading order of standard SR ($\ln x_f = 0$):

- $\ln \Delta^2 \simeq G(0)$
- $\dfrac{d \ln \Delta^2}{d \ln k} \simeq -G'(0)$
- $\alpha \simeq G''(0)$

Leading order of OSR ($\ln x_f = \ln x_1$):

- $\ln \Delta^2 \simeq G(\ln x_1)$
- $\dfrac{d \ln \Delta^2}{d \ln k} \simeq -G'(\ln x_1)$
- $\alpha \simeq G''(\ln x_1)$

NLO SR correction ($p = 1$):

- $q_1(0) = \ln x_1 = 1.06$
- $G'(0) \sim (\Delta N)^{-1}$

If $\Delta N \sim 3$,
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$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p(\ln x_f)\, G^{(p)}(\ln x_f)$$

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Leading order of standard SR ($\ln x_f = 0$):

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- $\dfrac{d \ln \Delta^2}{d \ln k} \simeq -G'(0)$
- $\alpha \simeq G''(0)$

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- $\dfrac{d \ln \Delta^2}{d \ln k} \simeq -G'(\ln x_1)$
- $\alpha \simeq G''(\ln x_1)$

NLO SR correction ($p=1$):

- $q_1(0) = \ln x_1 = 1.06$
- $G'(0) \sim (\Delta N)^{-1}$

If $\Delta N \sim 3$,
the correction is ~ 0.35 .

NNLO OSR correction ($p=2$):

- $q_2(\ln x_1) \simeq -0.36$
- $G''(\ln x_1) \sim (\Delta N)^{-2}$

If $\Delta N \sim 3$,
the correction is ~ 0.04 .

$$\ln \Delta^2 \simeq G(\ln x_f) + \sum_{p=1}^{\infty} q_p (\ln x_f)\, G^{(p)}(\ln x_f)$$

$$G = -2\ln f + \frac{2}{3}(\ln f)'$$

$$f = \sqrt{8\pi^2 \frac{b_s \epsilon_H c_s}{H^2}} \frac{aHs_s}{c_s}$$

Optimized SR for Horndeski (leading order):

— H. Motohashi and W. Hu; PRD **96** (2017) no.2, 023502.

$$\ln \Delta_\zeta^2 \approx \ln\left(\frac{H^2}{8\pi^2 b_s c_s \epsilon_H}\right) - \frac{10}{3}\epsilon_H - \frac{2}{3}\delta_1 - \frac{7}{3}\sigma_{s1} - \frac{1}{3}\xi_{s1}\Big|_{x=x_1}$$

$$n_s - 1 \approx -4\epsilon_H - 2\delta_1 - \sigma_{s1} - \xi_{s1} - \frac{2}{3}\delta_2 - \frac{7}{3}\sigma_{s2} - \frac{1}{3}\xi_{s2}\Big|_{x=x_1}$$

$$\alpha_s \approx -2\delta_2 - \sigma_{s2} - \xi_{s2} - \frac{2}{3}\delta_3 - \frac{7}{3}\sigma_{s3} - \frac{1}{3}\xi_{s3} - 8\epsilon_H^2 - 10\epsilon_H\delta_1 + 2\delta_1^2\Big|_{x=x_1}$$

Scalars

$$\ln \Delta_\gamma^2 \approx \ln\left(\frac{H^2}{2\pi^2 b_t c_t}\right) - \frac{8}{3}\epsilon_H - \frac{7}{3}\sigma_{t1} - \frac{1}{3}\xi_{t1}\Big|_{x=x_1}$$

$$n_t \approx -2\epsilon_H - \sigma_{t1} - \xi_{t1} - \frac{7}{3}\sigma_{t2} - \frac{1}{3}\xi_{t2}\Big|_{x=x_1}$$

Tensors

$$\alpha_t \approx -\sigma_{t2} - \xi_{t2} - \frac{7}{3}\sigma_{t3} - \frac{1}{3}\xi_{t3} - 4\epsilon_H^2 - 4\epsilon_H\delta_1\Big|_{x=x_1}$$

$$\ln x_1 \equiv \frac{7}{3} - \ln 2 - \gamma_{\mathrm{E}}$$

OSR slow-roll params

$$S_\zeta^{(2)} = \int \mathrm{d}^4x \frac{a^3 b_s \epsilon_H}{c_s^2}\left(\dot{\zeta}^2 - \frac{c_s^2 k^2}{a^2}\zeta^2\right)$$

- $$\epsilon_H = -\frac{\mathrm{d}\ \ln\ H}{\mathrm{d}N} \quad ,$$
- $$\delta_1 \equiv \frac{1}{2}\frac{\mathrm{d}\ln\epsilon_H}{\mathrm{d}N} - \epsilon_H ,$$
- $$\delta_{p+1} \equiv \frac{\mathrm{d}\delta_p}{\mathrm{d}N} + \delta_p(\delta_1 - p\epsilon_H).$$

- $$\sigma_{i,1} \equiv \frac{\mathrm{d}\ln c_i}{\mathrm{d}N},$$
- $$\sigma_{i,p+1} \equiv \frac{\mathrm{d}\sigma_{i,p}}{\mathrm{d}N},$$

- $$\xi_{i,1} \equiv \frac{\mathrm{d}\ln b_i}{\mathrm{d}N},$$
- $$\xi_{i,p+1} \equiv \frac{d\xi_{i,p}}{dN}$$

Summary:

- $\Delta N \sim N \sim 60$ $\longrightarrow$ Standard slow-roll
- $1 < \Delta N < N$ $\longrightarrow$ Optimized slow-roll
- $\Delta N \sim 1$ $\longrightarrow$ Generalized slow-roll
- $\Delta N < 1$ $\longrightarrow$ Stationary phase approximation
  (Miranda et al.; 1510.07580)

Slow-Roll

Slow-roll approximation:

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} = -V'(\phi) \qquad\qquad H^2 = -\frac{1}{3M_{\rm pl}^2}\left[\cancel{\frac{\dot{\phi}^2}{2}} + V(\phi)\right]$$

- $$\epsilon_H = -\frac{\dot{H}}{H^2}$$
- $$\eta_H = \frac{1}{2}\frac{\mathrm{d}\ \ln\epsilon_H}{\mathrm{d}N} - \epsilon_H$$

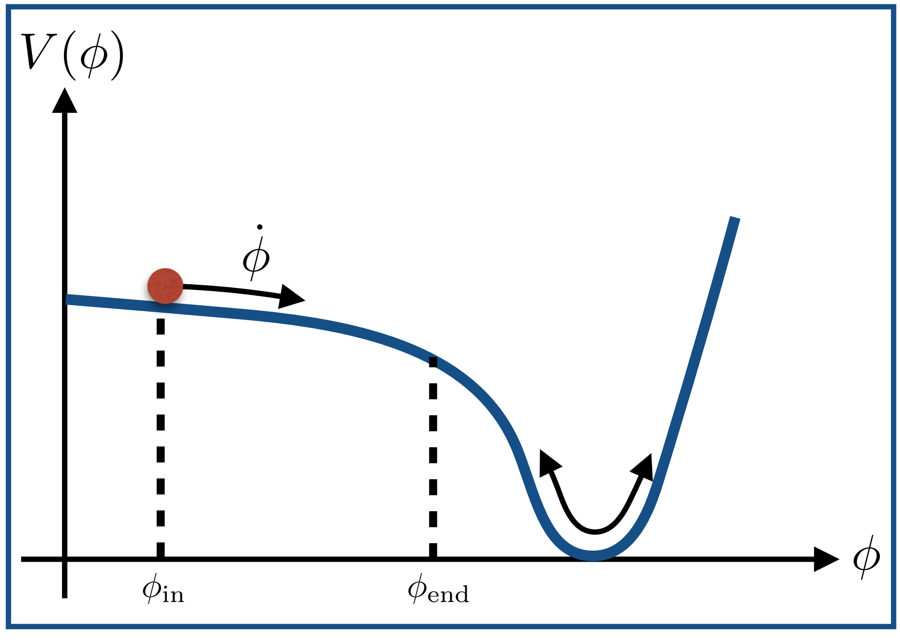
$\longleftrightarrow$ Slow-roll conditions:

$$\epsilon_H \ll 1$$

$$|\eta_H| \ll 1$$

Number of *e*-folds of inflation:

$$N_{\rm CMB} = \int_{\phi_{\rm CMB}}^{\phi_{\rm end}} \frac{\mathrm{d}\phi}{M_{\rm pl}\sqrt{2\epsilon}} \approx 40 - 60$$



The theory of quantum fluctuations

Perturbation theory in the comoving gauge:

- $\delta\phi = 0$
- $\delta g_{ij} = a^2(1-2\zeta)\delta_{ij} + a^2 h_{ij}$

- $$S_\zeta^{(2)} = \int \mathrm{d}^4x a^3 \epsilon_H \left(\dot{\zeta}^2 - \frac{k^2}{a^2}\zeta^2\right)$$ Scalars
- $$S_\gamma^{(2)} = \sum_{\lambda=+,\times} \int \mathrm{d}^4x \frac{a^3}{4} \left(\dot{\gamma}_\lambda^2 - \frac{k^2}{a^2}\gamma_\lambda^2\right)$$ Tensors

The theory of quantum fluctuations

Mukhanov - Sasaki equation:

$$\boxed{v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0}$$

- $v_k = z\zeta$ Scalars
- $v_k = z\gamma_{+,\times}$ Tensors

- $a\mathrm{d}\tau = \mathrm{d}t$
- $z^2 = 2a^2\epsilon_H$

Bunch-Davies vacuum:

$$v_k = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right)$$

Mukhanov - Sasaki equation:

$$v''_{\vec{k}} + \left(k^2 - \frac{z''}{z}\right) v_{\vec{k}} = 0$$

$$v_{\vec{k}}(\tau) \equiv \int \mathrm{d}^3\vec{x}\, e^{-i\vec{k}\cdot\vec{x}} v(\tau, \vec{x})$$

$$v_k(\tau) = \frac{e^{-ik\tau}}{\sqrt{2k}}\left(1 - \frac{i}{k\tau}\right) \overset{\lim_{k\tau\to 0}}{=} \frac{1}{i\sqrt{2}}\left(\frac{1}{k^{\frac{3}{2}}\tau}\right)$$

Gravitational waves:

$$S_{(2)} = \frac{M_{\rm pl}}{8}\int \mathrm{d}^4x\, a^2\left[(h'_{ij})^2 - (\nabla h_{ij})^2\right]$$

$$v_{\mathbf{k},\gamma} \equiv \frac{a}{2} M_{\rm pl} h_{\mathbf{k},\gamma}$$

$$h_{ij}(\tau, \mathbf{x}) = \int \frac{\mathrm{d}^3\mathbf{x}}{(2\pi^{\frac{3}{2}})} \sum_{\gamma=+,\times} \epsilon^{\gamma}_{ij}(k) h_{\mathbf{k},\gamma}(\tau) e^{i\mathbf{k}\cdot\mathbf{x}}$$

The observables:

$$\langle \hat{v}_{\vec{k}} \hat{v}_{\vec{k}'} \rangle \equiv P_v(k)\delta(\vec{k} + \vec{k}')$$

$$P_v(k) \equiv |v_k|^2$$

$$P_\zeta(k) = \frac{1}{z^2} P_v(k) = \frac{1}{z^2}\frac{1}{2k^3}\frac{1}{\tau^2} = \frac{1}{4k^3}\frac{H^2}{\epsilon}$$

Power spectra

$$\Delta_s^2(k) \equiv \frac{k^3}{2\pi^2} P_\zeta(k) = \frac{1}{8\pi^2}\frac{H^2}{\epsilon}\bigg|_{k=aH}$$

$$\Delta_t^2(k) = \frac{2}{\pi^2}\frac{H^2}{M_{\mathrm{pl}}}\bigg|_{k=aH}$$

Tensor-to-scalar ratio

$$\boxed{r \equiv \frac{\Delta_t^2}{\Delta_s^2} = 16\epsilon}$$

Primordial tilt

$$\boxed{n_s - 1 \equiv \frac{\mathrm{d}\ln \Delta_s^2}{\mathrm{d}\ln k} = -2\epsilon - \eta}$$

Slow-Roll hierarchy

Slow-roll hierarchy

By the SR parameters, $\epsilon_H \sim \eta_H = \mathcal{O}(\xi)$ ← Assumption

$$\Delta_\zeta^2(k) = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_H} \left[1 + \mathcal{O}(\xi)\right]\Bigg|_{k=aH}$$

Slow-roll hierarchy

By the SR parameters, $\epsilon_H \sim \eta_H = \mathcal{O}(\xi)$ Assumption

$$\Delta_\zeta^2(k) = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_H}\left[1 + \mathcal{O}(\xi)\right]\Big|_{k=aH}$$

So

$$n_s \equiv 1 + \frac{\mathrm{d}\ \ln\ \Delta_\zeta^2}{\mathrm{d}\ \ln\ k} = 1 + \mathcal{O}(\xi)\Big|_{k=aH}$$

is approximately scale invariant, as required by observations.

Slow-roll hierarchy

By the SR parameters, $\epsilon_H \sim \eta_H = \mathcal{O}(\xi)$ ← Assumption ✓

$$\Delta_\zeta^2(k) = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_H} \left[1 + \mathcal{O}(\xi)\right] \Big|_{k=aH}$$

Extra assumption: $\mathcal{O}(\xi)$ terms are also approximately scale invariant. Then,

$$n_s = 1 - 4\epsilon_H - 2\eta_H + \mathcal{O}\left(\xi^2\right) \Big|_{k=aH}$$

Slow-roll hierarchy

By the SR parameters, $\epsilon_H \sim \eta_H = \mathcal{O}(\xi)$ ← $\boxed{\text{Assumption}}$ ✓

$$\Delta_\zeta^2(k) = \frac{H^2}{8\pi^2 M_{\rm pl}^2 \epsilon_H}\left[1+\mathcal{O}(\xi)\right]\Bigg|_{k=aH}$$

Extra assumption: $\mathcal{O}(\xi)$ terms are also approximately scale invariant. Then,

$$n_s = 1 - 4\epsilon_H - 2\eta_H + \mathcal{O}\left(\xi^2\right)\Bigg|_{k=aH}$$

So, $$\alpha_s \equiv \frac{\mathrm{d}n_s}{\mathrm{d}\ \ln\ k} = \mathcal{O}(\xi^2)$$

which is NOT required by observations, nor is it consequence of SR.

Stewart; 0110322

Hubble-flow formalism

Two approaches

The Hubble Flow Formalism

$$\epsilon_{\rm H} \equiv 2M_{\rm pl}^2 \left(\frac{H'(\phi)}{H(\phi)}\right)^2 \qquad \eta_{\rm H} \equiv 2M_{\rm pl}^2 \left(\frac{H''(\phi)}{H(\phi)}\right)$$

$$^{\ell}\lambda_{\rm H} \equiv (2M_{\rm pl}^2)^{\ell} \frac{(H')^{\ell-1}}{H^{\ell}} \frac{{\rm d}^{(\ell+1)}H}{{\rm d}\phi^{(\ell+1)}}$$

SR parameters obey the infinite system of first-order differential equations:

$$\frac{{\rm d}\epsilon_{\rm H}}{{\rm d}N} = \epsilon_{\rm H}(\sigma_{\rm H} + 2\epsilon_{\rm H}) \qquad \frac{{\rm d}\sigma_{\rm H}}{{\rm d}N} = -5\epsilon_{\rm H}\sigma_{\rm H} - 12\epsilon_{\rm H}^2 + 2(^2\lambda_{\rm H})$$

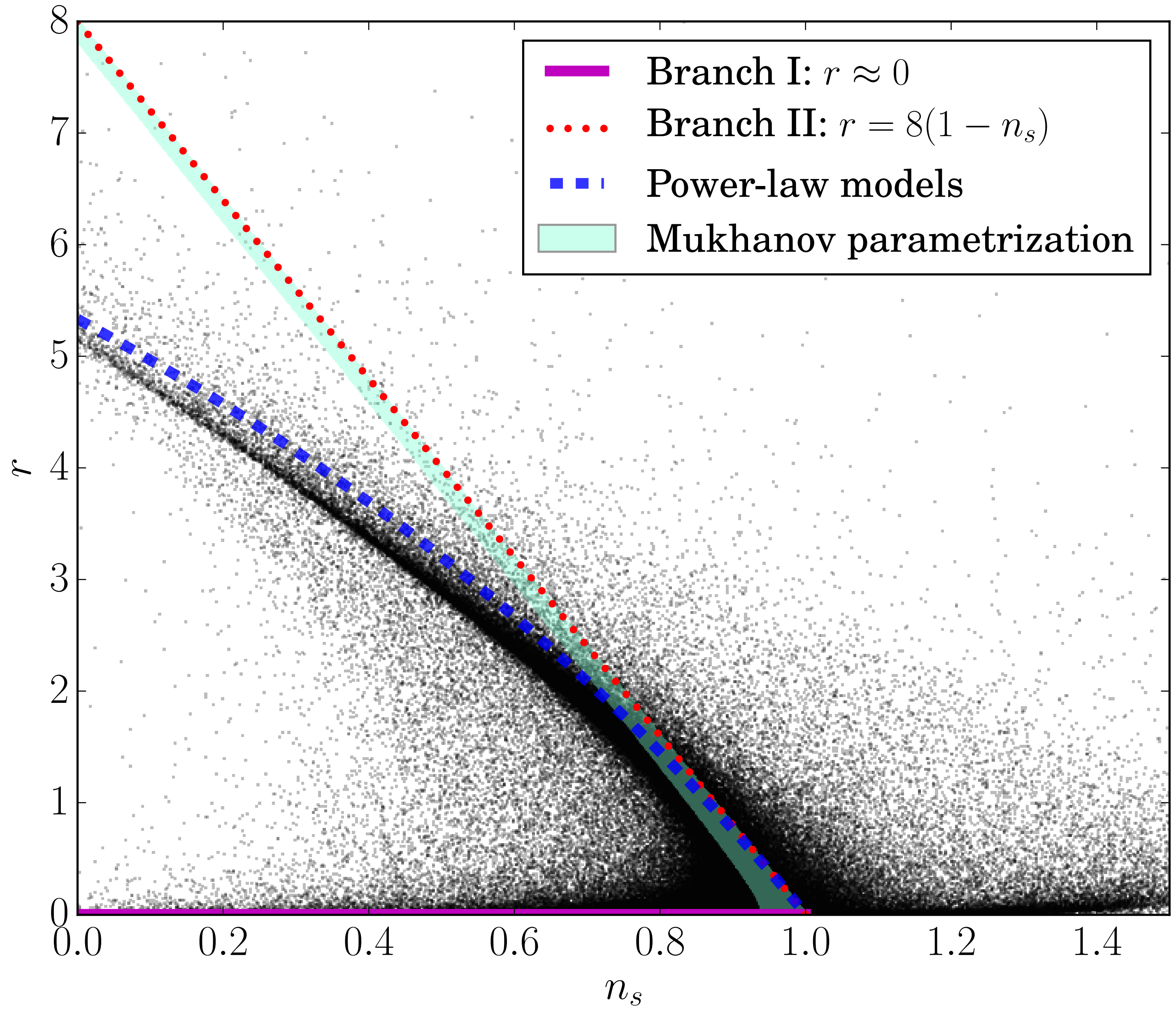
$$\frac{d(^{\ell}\lambda_{\rm H})}{dN} = \left[\frac{\ell-1}{2}\sigma_{\rm H} + (\ell-2)\epsilon_{\rm H}\right]{}^{\ell}\lambda_{\rm H} + {}^{\ell+1}\lambda_{\rm H}$$

The Hubble Flow Formalism

- To solve the flow equations we used Flowcode1.0 (Monte Carlo approach).
- We generate a total of 6×10^6 inflationary models.
- Slow-roll hierarchy is truncated at order $M = 8$.
- The models cluster around the attractors given by the fixed points:

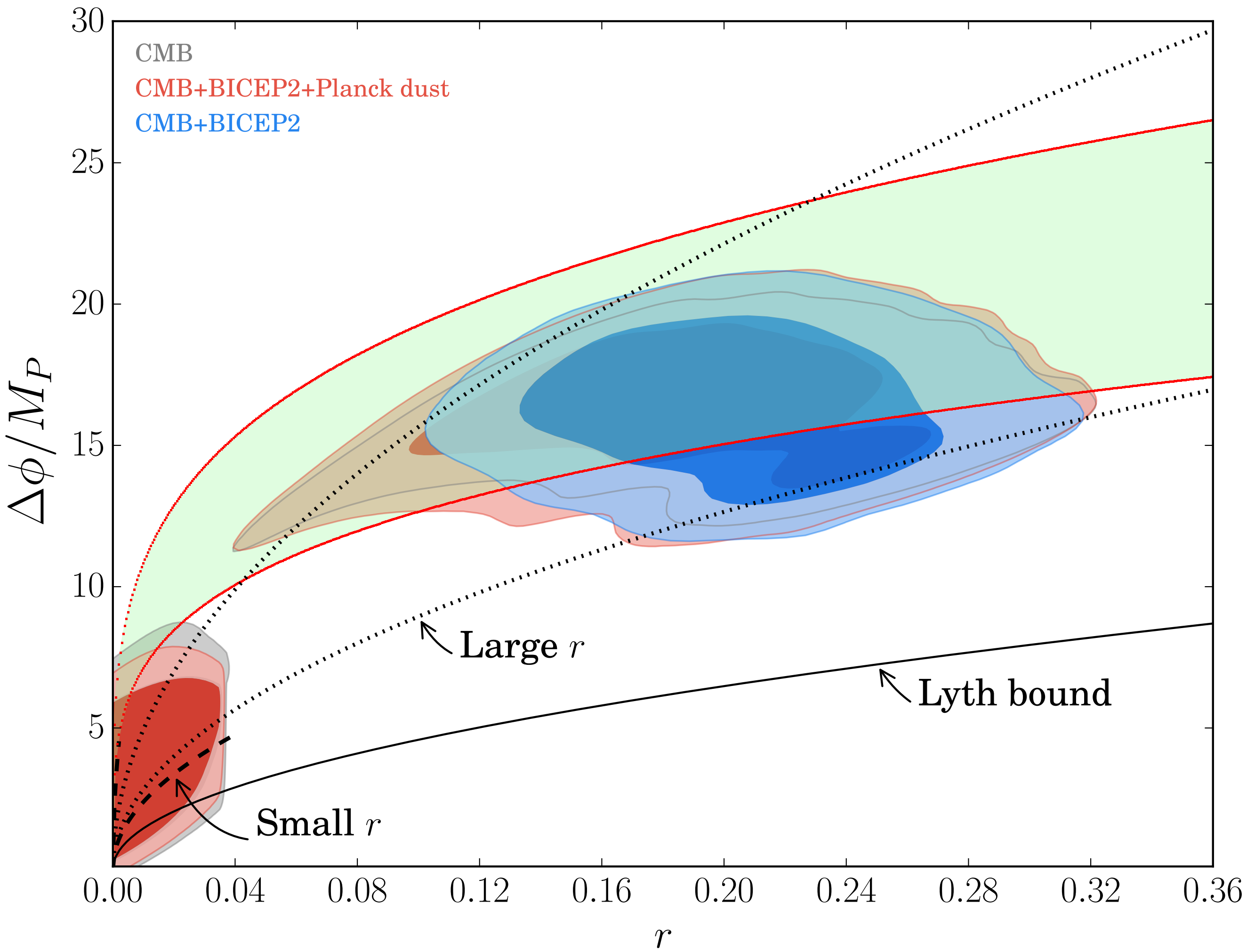
I: $r = 0$ and $n_s = \text{const.}$

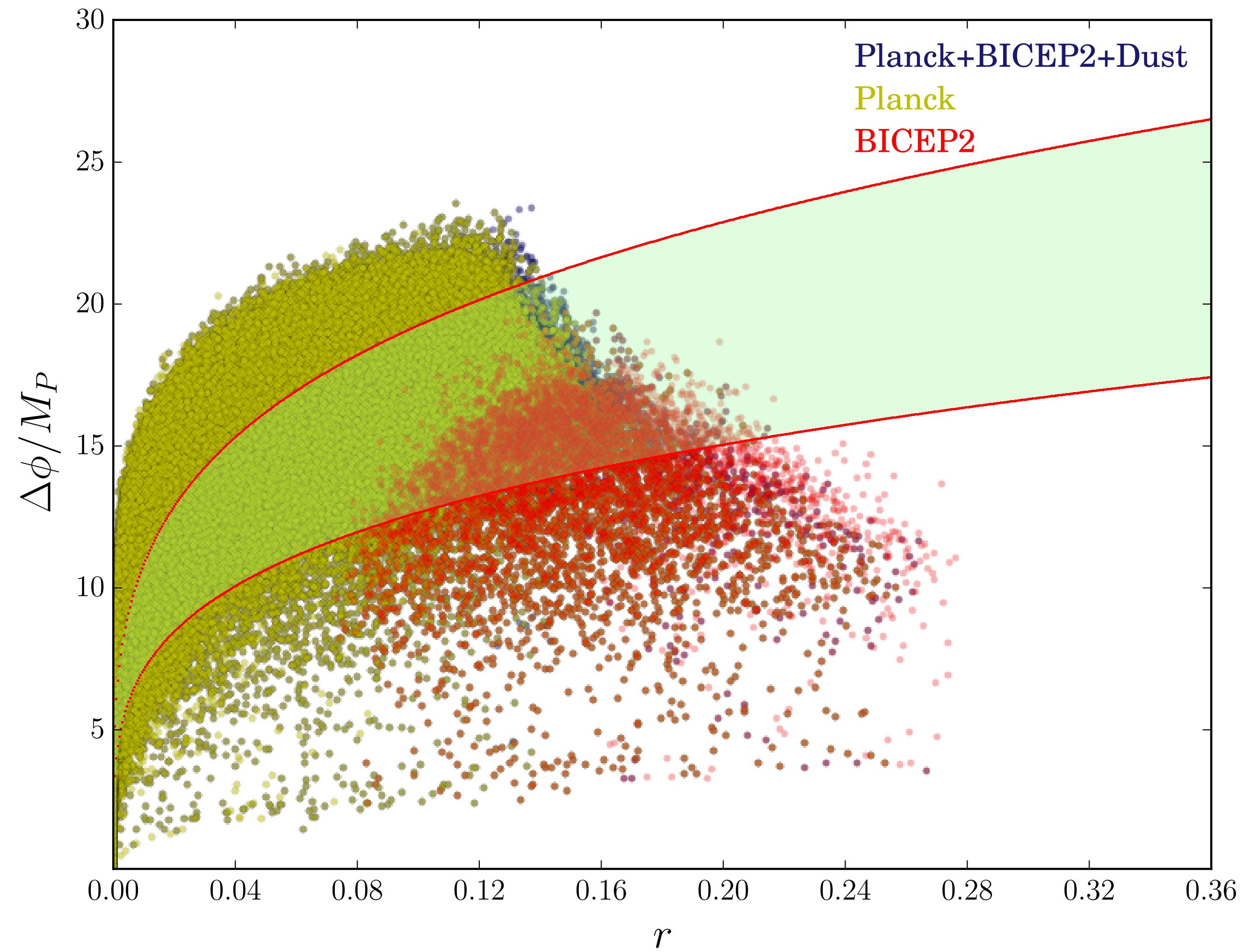
II: $n_s = 1 - \frac{r}{8} \times \left[\frac{1}{1 - r/16}\right]$



The inflaton excursion

$$\frac{\Delta\phi}{M_{\rm pl}} = \int_0^{N_*} dN \sqrt{\frac{3\beta}{(1+N)^\alpha}}$$





Nonminimal coupling

Motivation

- It is usually assumed that a term of the form $\xi \mathcal{R} \phi^2$ vanishes.

- Since the inflaton is coupled to light degrees of freedom (during reheating),

$$\mathcal{L}_{\text{reheating}} \simeq \lambda \phi^2/4! + y_\psi \phi \bar{\psi} \psi + \lambda_\chi \chi^2 \phi^2 + \ldots$$

the RGE of ξ is nontrivial. One can make it vanish at some scale, but it will be nonzero at some point because of its running:

$$\beta_\xi = \frac{\xi - \frac{1}{6}}{(4\pi)^2} \left[\lambda + \lambda_\xi + 4y_\psi^2 + \ldots\right]$$

- What about nonminimally scenarios?:

- D. S. Salopek, J. R. Bond and J. M. Bardeen, PRD 40, 1753 (1989).
- T. Futamase and K. i. Maeda, PRD 39 (1989) 399.
- R. Fakir and W. G. Unruh, PRD 41, 1783 (1990).
- D. I. Kaiser, PRD 52, 4295 (1995), [astro-ph/9408044].
- E. Komatsu and T. Futamase, PRD 59, 064029 (1999).
- M. P. Hertzberg, JHEP 1011, 023 (2010).
- N. Okada, M. U. Rehman and Q. Shafi,. PRD 82 (2010) 043502.
- A. Linde, M. Noorbala and A. Westphal, JCAP 1103, 013 (2011).
- D. I. Kaiser and E. I. Sfakianakis, PRL 112 (2014) 1, 011302.
- T. Chiba and K. Kohri, PTEP 2015, no. 2, 023E01.
- C. Pallis and Q. Shafi, JCAP 1503, no. 03, 023 (2015).
- S. Tsujikawa, J. Ohashi, S. Kuroyanagi and A. De Felice, PRD 88 (2013) 2, 023529.

Future constraints

- We can construct a combination of first order slow-roll observables:

$$n_s - 1 + \frac{r}{4} = -20\,\xi\,,$$

- This combination vanishes for $\xi = 0$, in the context of the chaotic scenario.

- Future observations from PIXIE, Euclid, COrE, and PRISM are targeting

$$\sigma_r = \sigma_{n_s - 1} = 10^{-3}$$

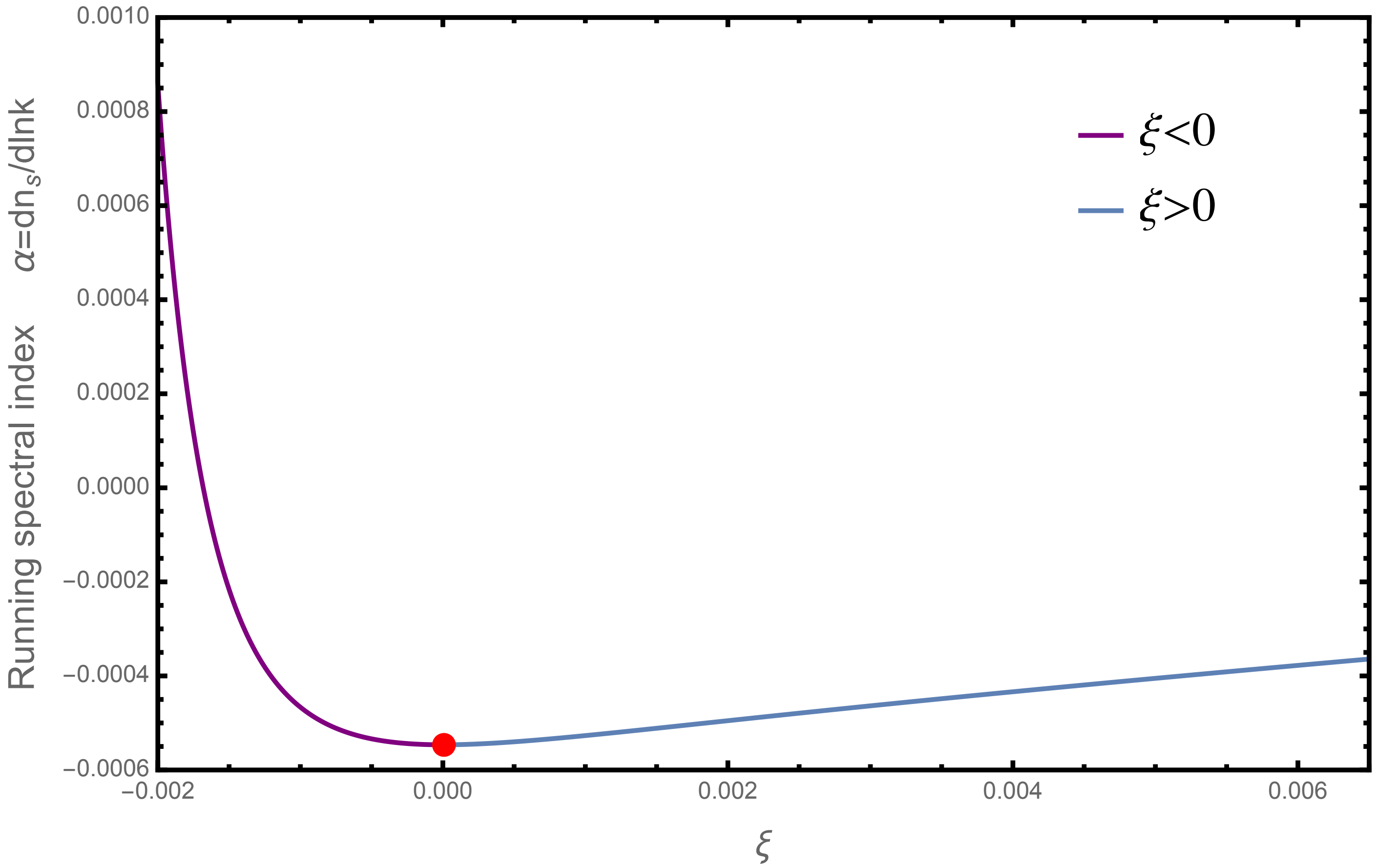
$$\downarrow$$

$$\sigma_\xi \leq 10^{-4}$$

- Combined with future accurate measurements of n_s , this might rule-out this model due to its nontrivial correlation with r.

Future constraints

- Negative coupling gives significant running (and higher r).
- The running is a good discriminator.
- Future constraints might falsify this model.



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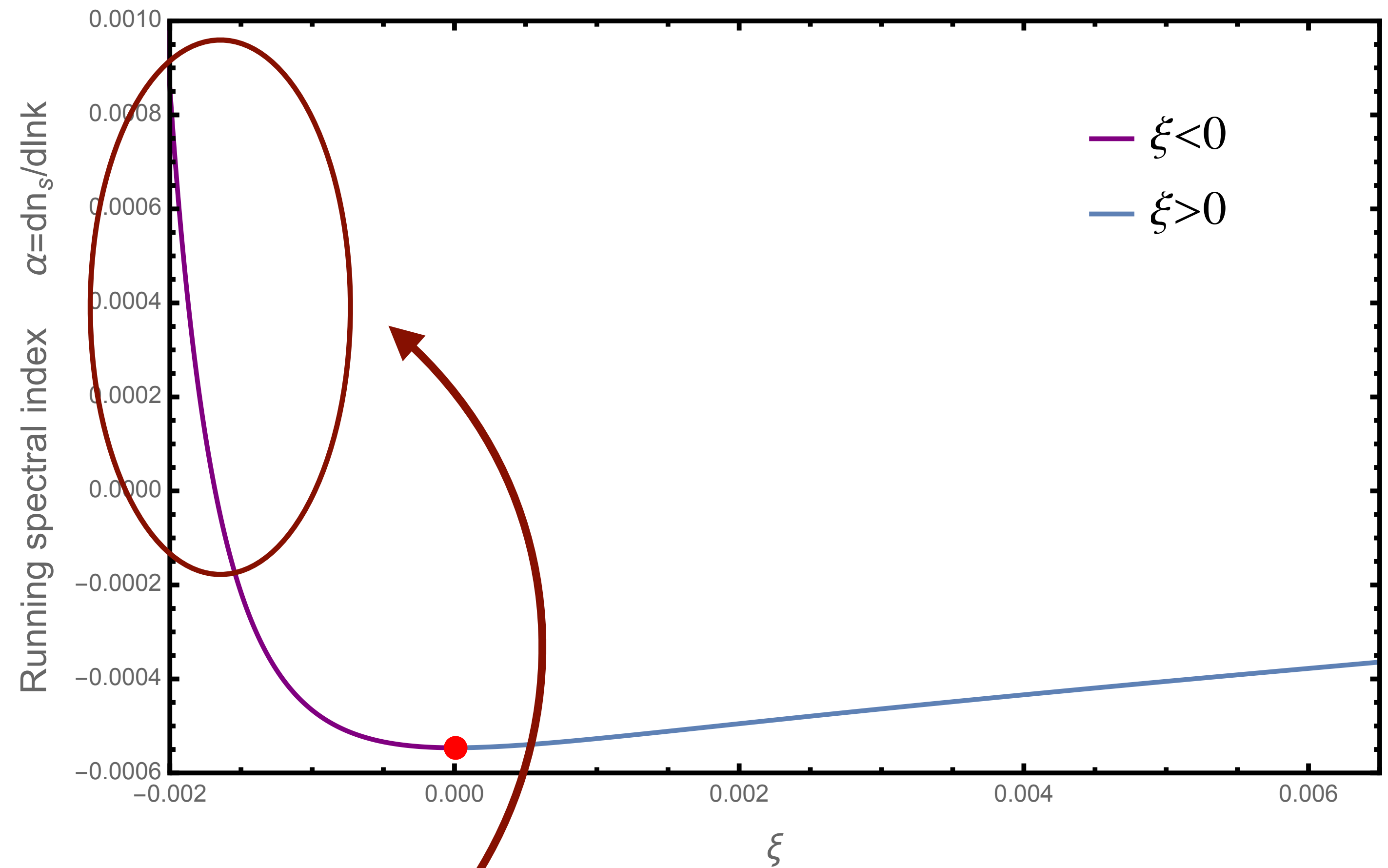

Compatible with Planck '15, but $r > 0.2$

Future constraints

- Negative coupling gives significant running (and higher r).
- The running is a good discriminator.
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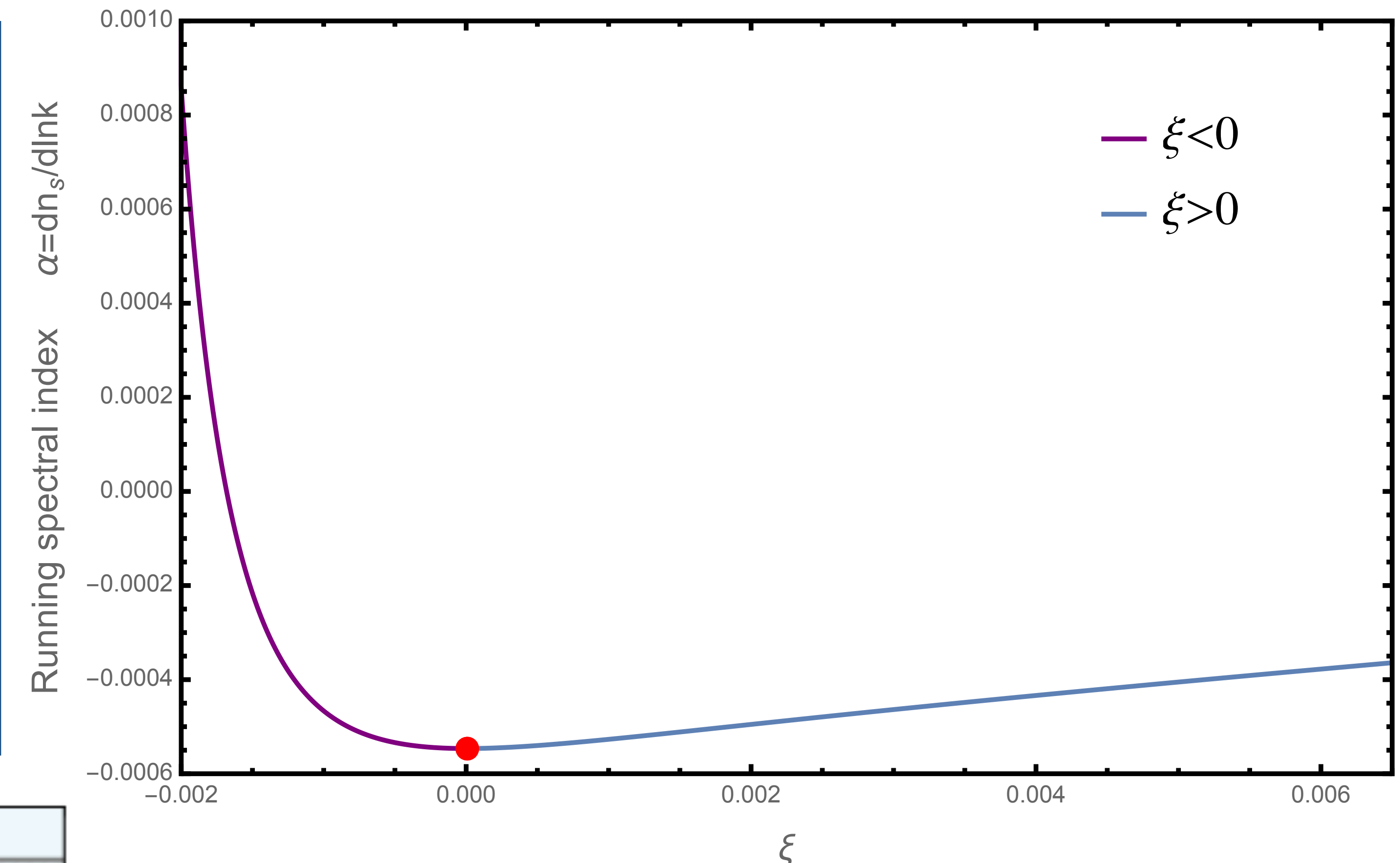
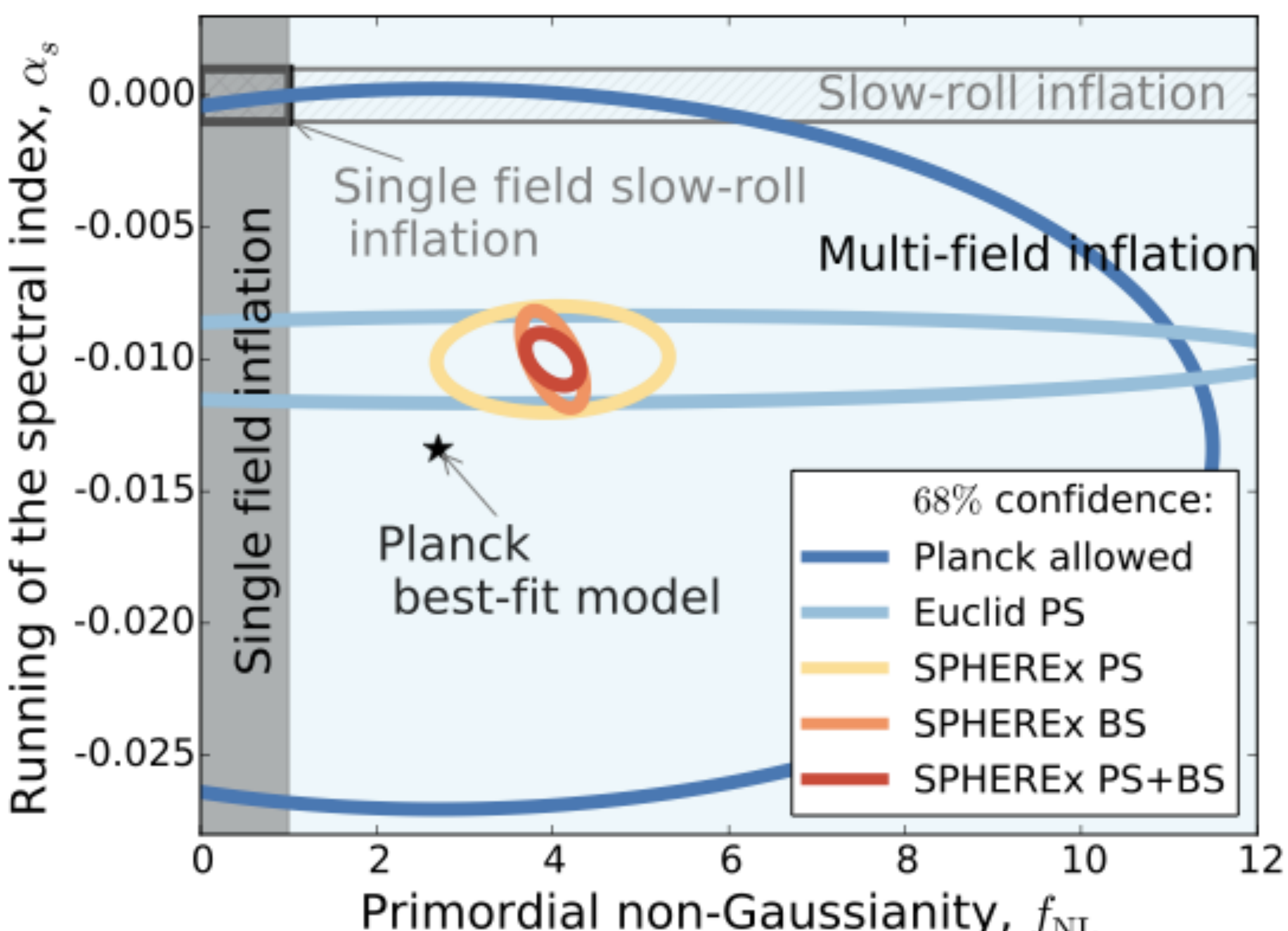




SPHEREx (Caltech)

(Spectrophotometer for the History of Universe, Epoch of Reionization and ices Explorer)

$$\sigma_{\alpha_s} \simeq 0.00065$$

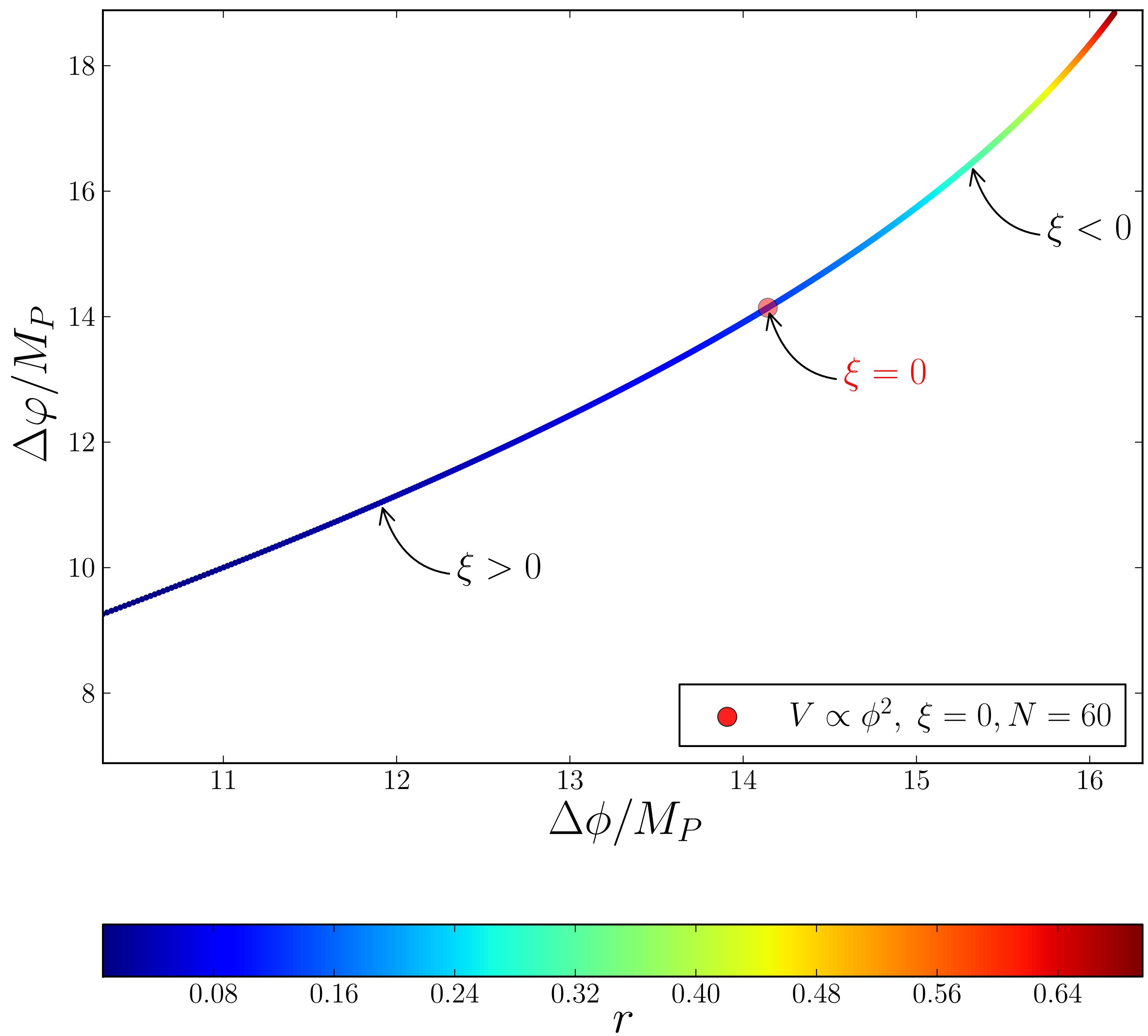
O. Doré *et al.*, arXiv:1412.4872 [astro-ph.CO].

Results

	Planck TT+WP		BK+Planck TT+WP	
N	60	50	60	50
ξ	$0.0028^{+0.0023}_{-0.0025}$	$0.0024^{+0.0023}_{-0.0023}$	$0.0027^{+0.0023}_{-0.0022}$	$0.0027^{+0.0020}_{-0.0019}$
n_s	$0.958^{+0.010}_{-0.011}$	$0.954^{+0.007}_{-0.009}$	$0.958^{+0.009}_{-0.011}$	$0.953^{+0.007}_{-0.009}$
r	$0.038^{+0.051}_{-0.031}$	$0.063^{+0.056}_{-0.048}$	$0.038^{+0.039}_{-0.030}$	$0.053^{+0.038}_{-0.037}$
$\alpha \equiv \mathrm{d}n_s/\mathrm{d}\ln k$	$-0.0005^{+0.0001}_{-0.0001}$	$-0.0007^{+0.0001}_{-0.0001}$	$-0.0005^{+0.0001}_{-0.0001}$	$-0.0007^{+0.0001}_{-0.0001}$

Results

$$\boxed{\left(\frac{\mathrm{d}\varphi}{\mathrm{d}\phi}\right)^2 = \frac{1}{\Omega} + \frac{3}{2} M_P^2 \left(\frac{\Omega'}{\Omega}\right)^2}$$



The excursion of the non-minimally coupled inflaton is bit smaller but still super-Planckian.

Inflation with mixed helicities

Scalar-Vector-Tensor theories

— L. Heisenberg; arXiv:1801.01523 [gr-qc]

Generalized Proca

— Broken gauge invariance

Stueckelberg trick

$$A_\mu \to A_\mu + \nabla_\mu \phi$$

Shift symmetric Horndeski

$$(\phi \to \phi - \varphi)$$

+

SV interactions

— Gauge invariant

[breaking gauge invariance]

$U(1)$-broken SVT theories

+

soft shift-symm. breaking

5 dof:

1 S, **2** transverse **V**, **2 T** polarizations

6 dof:

2 S, **2** transverse **V**, **2 T** polarizations

- New type of hairy black hole solutions for the gauge-invariant Lagrangian have been found.

L. Heisenberg and S. Tsujikawa; doi:10.1016/j.physletb.2018.03.059 [arXiv:1802.07035 [gr-qc]].

- For a FLRW space-time, the EoM and the conditions for the absence of ghosts and Laplacian instabilities, for the full gauge-broken, parity-invariant SVT action have been computed.

L. Heisenberg, R. Kase and S. Tsujikawa; doi:10.1103/PhysRevD.98.024038 [arXiv:1805.01066 [gr-qc]].

Dark Energy implications

- From tensor perturbations:

  - $$c_t^2 = \frac{2f_4 - A_0\dot{\phi}f_{5,\phi} - \dot{A}_0A_0^2 f_{5,X_3}}{2f_4 - 2A_0^2 f_{4,X_3} + A_0\dot{\phi}f_{5,\phi} - HA_0^3 f_{5,X3}}$$

- Then, given the bound[†] $-3\times10^{-15} \le c_t - 1 \le 7\times10^{-16}$:

  - $f_4(\phi, X_3) = f_4(\phi)$
  - $f_5(\phi, X_3) = \text{constant}$

R. Kase and S. Tsujikawa; arXiv:1805.11919 [gr-qc].

†
- B. P. Abbott *et al.* [LIGO Scientific and Virgo Collaborations]; PRL **119** (2017) no.16, 161101.
- A. Goldstein *et al.*; Astrophys. J. **848** (2017) no.2, L14.

A particular model for **inflation**

Vector perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Vector quadratic action:

$$\mathcal{S}_{\rm V}^{(2)} = \int d^4x \sum_{i=1}^{2} \frac{a}{2} \left[\dot{Z}_i^2 - \frac{1}{a^2} (\partial Z_i)^2 - \beta_A M^2 Z_i^2 \right]$$

$$\boxed{Z_i'' + \left(k^2 + a^2 \beta_A M^2\right) Z_i = 0}$$

- For $k^2 \gg a^2 \beta_A M^2$: $Z_i = e^{-ik\tau}/\sqrt{2k}$ *Bunch-Davies vacuum

- For $a^2 \beta_A M^2 > k^2$:

$$Z_i = A_+ e^{\lambda_+ t} + A_- e^{\lambda_- t} \quad \text{with} \quad \lambda_\pm = \frac{H}{2}\left[-1 \pm \sqrt{1 - \frac{4\beta_A M^2}{H^2}}\right]$$

*Exponential suppression

- For $4\beta_A M^2 > H^2$: $|Z_i| \propto e^{-Ht/2}$

*Decay with damped oscillations

A particular model for **inflation**

Scalar perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

$$\mathcal{S}_{\rm s}^{(2)} = \int d^4x\, a^3 \left(\dot{\vec{\mathcal{X}}}^t \boldsymbol{K} \dot{\vec{\mathcal{X}}} - \frac{k^2}{a^2} \vec{\mathcal{X}}^t \boldsymbol{G} \vec{\mathcal{X}} - \vec{\mathcal{X}}^t \boldsymbol{M} \vec{\mathcal{X}} \right) \quad , \qquad \mathcal{X}^t = (\psi_k, \delta\phi_k)$$

where

- $K_{11} = \dfrac{k^2 \beta_A M^2}{2(k^2 + a^2 \beta_A M^2)}$
- $K_{12} = K_{21} = \dfrac{\beta_m}{2\beta_A M} K_{11}$
- $K_{22} = \dfrac{1}{2} - \dfrac{a^2 \beta_m^2 M^2}{8(k^2 + a^2 \beta_A M^2)}$
- $G_{11} = \dfrac{\beta_A M^2}{2}$
- $G_{12} = G_{21} = \dfrac{\beta_m M}{4}$
- $G_{22} = \dfrac{1}{2}$
- $M_{22} = \dfrac{V_{,\phi\phi}}{2} - \dfrac{(1-\delta_\phi^2) V_{,\phi}^2}{6 M_{\rm pl}^2 H^2} - \dfrac{(1+\delta_\phi)^4 V_{,\phi}^4}{324 \beta H^6 M_{\rm pl}^4}$

and

- $\delta_\phi \equiv \dfrac{\beta \ddot{\phi}}{V_{,\phi}} = -\dfrac{3\beta H \dot{\phi} + V_{,\phi}}{V_{,\phi}}$

A particular model for **inflation**

Perturbations

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Varying the action wrt ψ:

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3 K_{11}\delta\dot{\chi}_k\right) + \frac{k^2}{a^2}G_{11}\delta\chi_k = 0$$

- For $k^2 \gg a^2\beta_A M^2$: $\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \frac{k^2}{a^2}\delta\chi_k = 0$

*EoM of a massless field

  - For $k^2/a^2 \gg H^2$: $v_k = \sqrt{2}a\delta\chi_k = e^{-ik\int dt/a}/\sqrt{2k}$ *BD vacuum

- For $k^2/a^2 \ll \beta_A M^2$: $\delta\ddot{\chi}_k + H\delta\dot{\chi}_k + \beta_A M^2\delta\chi_k = 0$

$$\delta\chi_k = A_+ e^{\lambda_+ t} + A_- e^{\lambda_- t}$$

*Exponential suppression

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

- Varying the action wrt $\delta\phi$:

$$\frac{1}{a^3}\frac{d}{dt}\left[a^3\left(K_{22}\delta\dot{\phi}_k + K_{12}\dot{\psi}_k\right)\right] + \frac{k^2}{a^2}\left(G_{22}\delta\phi_k + G_{12}\psi_k\right) + M_{22}\delta\phi_k = 0$$

$$\frac{1}{a^3}\frac{d}{dt}\left(a^3\tilde{K}_{22}\delta\dot{\phi}_k\right) + \left(\frac{k^2}{a^2}\tilde{G}_{22} + M_{22}\right)\delta\phi_k = 0$$

$$\delta\sigma_k'' + \left(k^2 - \frac{a''}{a} + \frac{2a^2 M_{22}}{\beta}\right)\delta\sigma_k = 0 \qquad \text{with} \qquad \delta\sigma_k \equiv a\sqrt{\beta}\,\delta\phi_k$$

$$\delta\sigma_k'' + \left[k^2 - 2(aH)^2\left(1 + \frac{5\epsilon_V - 3\eta_V}{2\beta}\right)\right]\delta\sigma_k = 0$$

— L. Heisenberg, **HR**, S. Tsujikawa; *in prep.*

$$\delta\sigma_k'' + \left[k^2 - 2(aH)^2\left(1 + \frac{5\epsilon_V - 3\eta_V}{2\beta}\right)\right]\delta\sigma_k = 0$$

has as solution:

$$\delta\sigma_k = \frac{\sqrt{\pi|\tau|}}{2} e^{i(1+2\nu)\pi/4} H_\nu^{(1)}(k|\tau|)$$ where $$\nu = \frac{3}{2} + \frac{3\epsilon_V - \eta_V}{\beta}$$

- For $k\tau \to 0$: $$\delta\phi_k = i\frac{H(1-\epsilon)}{k^{3/2}\sqrt{2\beta}}\frac{\Gamma(\nu)}{\Gamma(3/2)}\left(\frac{k|\tau|}{2}\right)^{3/2-\nu}$$
- Since $\mathcal{R}_\phi = -\frac{H\delta\phi_k}{\dot{\phi}}$, $$\frac{1}{a^3\epsilon}\frac{d}{dt}\left(a^3\epsilon\dot{\mathcal{R}}_\phi\right) + \frac{k^2}{a^2}\mathcal{R}_\phi = 0$$

■ $\mathcal{R}_\phi = c_1 + c_2\int\frac{dt}{a^3\epsilon}$ $\longrightarrow$ $$\mathcal{P}_{\mathcal{R}_\phi} \equiv \frac{k^3}{2\pi^2}\left|\mathcal{R}_\phi\right|^2 = \left.\frac{H^4}{4\pi^2\dot{\phi}^2\beta}\right|_{k=aH}$$